

# Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "4 Trig functions\4.2 Cosine"

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## Test results for the 294 problems in "4.2.0 (a cos)^m (b trig)^n.m"

- **Problem 1: Result more than twice size of optimal antiderivative.**

$$\int \cos[a + b x] dx$$

Optimal (type 3, 10 leaves, 1 step):

$$\frac{\sin[a + b x]}{b}$$

Result (type 3, 21 leaves):

$$\frac{\cos[b x] \sin[a]}{b} + \frac{\cos[a] \sin[b x]}{b}$$

- **Problem 42: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{a \cos[x]^2}} dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\sin[x]] \cos[x]}{\sqrt{a \cos[x]^2}}$$

Result (type 3, 46 leaves):

$$\frac{\cos[x] \left( -\operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] \right)}{\sqrt{a \cos[x]^2}}$$

■ **Problem 43: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a \cos[x]^2)^{3/2}} dx$$

Optimal (type 3, 42 leaves, 3 steps) :

$$\frac{\text{ArcTanh}[\sin[x]] \cos[x]}{2 a \sqrt{a \cos[x]^2}} + \frac{\tan[x]}{2 a \sqrt{a \cos[x]^2}}$$

Result (type 3, 91 leaves) :

$$\frac{\cos[x] \left( \log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \cos[2x] \left( \log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] \right) - \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] - 2 \sin[x]}{4 (a \cos[x]^2)^{3/2}}$$

■ **Problem 145: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{b \cos[c + dx]}}{\cos[c + dx]^{3/2}} dx$$

Optimal (type 3, 33 leaves, 2 steps) :

$$\frac{\text{ArcTanh}[\sin[c + dx]] \sqrt{b \cos[c + dx]}}{d \sqrt{\cos[c + dx]}}$$

Result (type 3, 75 leaves) :

$$\frac{\sqrt{b \cos[c + dx]} \left( -\log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right)}{d \sqrt{\cos[c + dx]}}$$

■ **Problem 155: Result more than twice size of optimal antiderivative.**

$$\int \frac{(b \cos[c + dx])^{3/2}}{\cos[c + dx]^{5/2}} dx$$

Optimal (type 3, 34 leaves, 2 steps) :

$$\frac{b \text{ArcTanh}[\sin[c + dx]] \sqrt{b \cos[c + dx]}}{d \sqrt{\cos[c + dx]}}$$

Result (type 3, 75 leaves) :

$$\frac{(b \cos[c + dx])^{3/2} \left( -\log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right)}{d \cos[c + dx]^{3/2}}$$

■ **Problem 166: Result more than twice size of optimal antiderivative.**

$$\int \frac{(b \cos [c + d x])^{5/2}}{\cos [c + d x]^{7/2}} dx$$

Optimal (type 3, 36 leaves, 2 steps) :

$$\frac{b^2 \operatorname{ArcTanh}[\sin [c + d x]] \sqrt{b \cos [c + d x]}}{d \sqrt{\cos [c + d x]}}$$

Result (type 3, 75 leaves) :

$$\frac{(b \cos [c + d x])^{5/2} \left( -\operatorname{Log}\left[\cos \left[\frac{1}{2}(c + d x)\right] - \sin \left[\frac{1}{2}(c + d x)\right]\right] + \operatorname{Log}\left[\cos \left[\frac{1}{2}(c + d x)\right] + \sin \left[\frac{1}{2}(c + d x)\right]\right] \right)}{d \cos [c + d x]^{5/2}}$$

■ **Problem 177: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\cos [c + d x]} \sqrt{b \cos [c + d x]}} dx$$

Optimal (type 3, 33 leaves, 2 steps) :

$$\frac{\operatorname{ArcTanh}[\sin [c + d x]] \sqrt{\cos [c + d x]}}{d \sqrt{b \cos [c + d x]}}$$

Result (type 3, 75 leaves) :

$$\frac{\sqrt{\cos [c + d x]} \left( -\operatorname{Log}\left[\cos \left[\frac{1}{2}(c + d x)\right] - \sin \left[\frac{1}{2}(c + d x)\right]\right] + \operatorname{Log}\left[\cos \left[\frac{1}{2}(c + d x)\right] + \sin \left[\frac{1}{2}(c + d x)\right]\right] \right)}{d \sqrt{b \cos [c + d x]}}$$

■ **Problem 187: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos [c + d x]}}{(b \cos [c + d x])^{3/2}} dx$$

Optimal (type 3, 36 leaves, 2 steps) :

$$\frac{\operatorname{ArcTanh}[\sin [c + d x]] \sqrt{\cos [c + d x]}}{b d \sqrt{b \cos [c + d x]}}$$

Result (type 3, 75 leaves) :

$$\frac{\cos [c + d x]^{3/2} \left( -\operatorname{Log}\left[\cos \left[\frac{1}{2}(c + d x)\right] - \sin \left[\frac{1}{2}(c + d x)\right]\right] + \operatorname{Log}\left[\cos \left[\frac{1}{2}(c + d x)\right] + \sin \left[\frac{1}{2}(c + d x)\right]\right] \right)}{d (b \cos [c + d x])^{3/2}}$$

■ **Problem 197: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^{3 / 2}}{(b \cos [c+d x])^{5 / 2}} d x$$

Optimal (type 3, 36 leaves, 2 steps) :

$$\frac{\text{ArcTanh}[\text{Sin}[c+d x]] \sqrt{\cos [c+d x]}}{b^2 d \sqrt{b \cos [c+d x]}}$$

Result (type 3, 78 leaves) :

$$\frac{\sqrt{\cos [c+d x]} \left( -\text{Log} \left[ \cos \left[ \frac{1}{2} (c+d x) \right] - \text{Sin} \left[ \frac{1}{2} (c+d x) \right] \right] + \text{Log} \left[ \cos \left[ \frac{1}{2} (c+d x) \right] + \text{Sin} \left[ \frac{1}{2} (c+d x) \right] \right] \right)}{b^2 d \sqrt{b \cos [c+d x]}}$$

■ **Problem 284: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [e+f x]^m \csc [e+f x]^n d x$$

Optimal (type 5, 85 leaves, 2 steps) :

$$\frac{\cos [e+f x]^{-1+m} \left( \cos [e+f x] \right)^{\frac{1-m}{2}} \csc [e+f x]^{-1+n} \text{Hypergeometric2F1} \left[ \frac{1-m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \text{Sin}[e+f x]^2 \right]}{f (1-n)}$$

Result (type 6, 3229 leaves) :

$$\begin{aligned} & - \left( \left( 2 (-3+n) \text{AppellF1} \left[ \frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right] \right. \right. \\ & \quad \left. \left( \cos \left[ \frac{1}{2} (e+f x) \right]^2 \right)^{1+m} \cos [e+f x]^m \csc [e+f x]^{2n} \left( \cos [e+f x] \text{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^m \text{Tan} \left[ \frac{1}{2} (e+f x) \right] \right) \right) / \\ & \left( f (-1+n) \left( (-3+n) \text{AppellF1} \left[ \frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right] + \right. \right. \\ & \quad \left. \left. 2 \left( m \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, 1-m, 1+m-n, \frac{5}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right] + \right. \right. \right. \\ & \quad \left. \left. (1+m-n) \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, -m, 2+m-n, \frac{5}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right] \right) \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \\ & \left. - \left( \left( (-3+n) \text{AppellF1} \left[ \frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right] \right) \left( \cos \left[ \frac{1}{2} (e+f x) \right]^2 \right)^{1+m} \csc [e+f x]^n \right. \right. \\ & \quad \left. \left. \text{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \left( \cos [e+f x] \text{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^m \right) \right) / \left( (-1+n) \left( (-3+n) \text{AppellF1} \left[ \frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 + 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \Big) \Big) + \\
& \left( 2(1+m)(-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\cos\left[\frac{1}{2}(e+fx)\right]^2\right)^m \right. \\
& \quad \left. \operatorname{Csc}[e+fx]^n \left(\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \sin\left[\frac{1}{2}(e+fx)\right]^2 \right) / \\
& \left( (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\
& \left( 2(-3+n)n \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\cos\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+m} \right. \\
& \quad \left. \cos[e+fx] \operatorname{Csc}[e+fx]^{1+n} \left(\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \tan\left[\frac{1}{2}(e+fx)\right] \right) / \\
& \left( (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
& \left( 2(-3+n) \left(\cos\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+m} \operatorname{Csc}[e+fx]^n \left(\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \tan\left[\frac{1}{2}(e+fx)\right] \right. \\
& \quad \left. \left( -\frac{1}{\frac{3}{2}-\frac{n}{2}} m \left(\frac{1}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \right. \right. \\
& \quad \left. \frac{1}{\frac{3}{2}-\frac{n}{2}} (1+m-n) \left(\frac{1}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) / \left( (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right.
\end{aligned}$$



$$\begin{aligned}
& -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + (1+m-n) \left( -\frac{1}{\frac{5}{2}-\frac{n}{2}} m \left(\frac{3}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1-m, \right. \right. \\
& \left. \left. 2+m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{1}{\frac{5}{2}-\frac{n}{2}} (2+m-n) \left(\frac{3}{2}-\frac{n}{2}\right) \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, -m, 3+m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) + \right. \\
& \left. 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] + \right. \right. \\
& \left. \left. (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 285: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a \operatorname{Cos}[e+fx])^m \operatorname{Csc}[e+fx]^n dx$$

Optimal (type 5, 88 leaves, 2 steps):

$$\frac{1}{f(1-n)} a (a \operatorname{Cos}[e+fx])^{-1+m} (\operatorname{Cos}[e+fx]^2)^{\frac{1-m}{2}} \operatorname{Csc}[e+fx]^{-1+n} \operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Sin}[e+fx]^2\right]$$

Result (type 6, 3231 leaves):

$$\begin{aligned}
& - \left( \left( 2(-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \right. \\
& \quad \left. \left( \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{1+m} (a \operatorname{Cos}[e+fx])^m \operatorname{Csc}[e+fx]^{2n} \left( \operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \Bigg) / \\
& \left( f(-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) + \right. \\
& \quad 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] + \right. \\
& \quad \left. \left. (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \\
& \quad \left. \left( - \left( \left( (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \left( \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{1+m} \operatorname{Csc}[e+fx]^n \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \left(\text{Cos}[e+fx] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \Big/ \left( (-1+n) \left( (-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left( m \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\
& \quad \left. \left. \left. (1+m-n) \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \right) \Big) + \\
& \left( 2(1+m)(-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\text{Cos}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \right. \\
& \quad \left. \text{Csc}[e+fx]^n \left(\text{Cos}[e+fx] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \text{Sin}\left[\frac{1}{2}(e+fx)\right]^2 \right) \Big/ \\
& \left( (-1+n) \left( (-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. 2 \left( m \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \right. \right. \right. \\
& \quad \left. \left. \left. \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \right) \Big) + \\
& \left( 2(-3+n)n \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\text{Cos}\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+m} \right. \\
& \quad \left. \text{Cos}[e+fx] \text{Csc}[e+fx]^{1+n} \left(\text{Cos}[e+fx] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \Big/ \\
& \left( (-1+n) \left( (-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. 2 \left( m \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \right. \right. \right. \\
& \quad \left. \left. \left. \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \right) \Big) - \\
& \left( 2(-3+n) \left(\text{Cos}\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+m} \text{Csc}[e+fx]^n \left(\text{Cos}[e+fx] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right. \\
& \quad \left. \left( -\frac{1}{\frac{3}{2}-\frac{n}{2}} m \left(\frac{1}{2}-\frac{n}{2}\right) \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] - \right. \right. \\
& \quad \left. \left. \frac{1}{\frac{3}{2}-\frac{n}{2}} (1+m-n) \left(\frac{1}{2}-\frac{n}{2}\right) \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right)
\end{aligned}$$





$$\begin{aligned}
& \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{5}{2}-\frac{n}{2}}(1-m)\left(\frac{3}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 2-m, 1+m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right], \\
& -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + (1+m-n)\left(-\frac{1}{\frac{5}{2}-\frac{n}{2}}m\left(\frac{3}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1-m, \right.\right. \\
& \left.\left.2+m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{1}{\frac{5}{2}-\frac{n}{2}}(2+m-n)\left(\frac{3}{2}-\frac{n}{2}\right)\right. \\
& \left.\left.\operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, -m, 3+m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)\right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( (-1+n)\left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right.\right. \\
& \left.2\left(m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right.\right. \\
& \left.\left.(1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)\right)\right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 286: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^m (b \operatorname{Csc}[e+fx])^n dx$$

Optimal (type 5, 88 leaves, 2 steps):

$$\frac{1}{f(1-n)} b \cos[e+fx]^{-1+m} (\cos[e+fx]^2)^{\frac{1-m}{2}} (b \operatorname{Csc}[e+fx])^{-1+n} \operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin[e+fx]^2\right]$$

Result (type 6, 3237 leaves):

$$\begin{aligned}
& -\left(2(-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \\
& \left(\cos\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+m} \cos[e+fx]^m \operatorname{Csc}[e+fx]^n (b \operatorname{Csc}[e+fx])^n \left(\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \Bigg) / \\
& \left(f(-1+n)\left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right.\right. \\
& \left.2\left(m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right.\right. \\
& \left.\left.(1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)\right)\right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{\frac{3}{2} - \frac{n}{2}} (1+m-n) \left( \frac{1}{2} - \frac{n}{2} \right) \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, -m, 2+m-n, \frac{5}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \\
& \text{Tan} \left[ \frac{1}{2} (e+fx) \right] \Bigg) / \left( (-1+n) \left( (-3+n) \text{AppellF1} \left[ \frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& 2 \left( m \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, 1-m, 1+m-n, \frac{5}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right] + (1+m-n) \right. \\
& \left. \left. \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, -m, 2+m-n, \frac{5}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right] \right) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) - \\
& \left( 2m(-3+n) \text{AppellF1} \left[ \frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right] \left( \text{Cos} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^{1+m} \right. \\
& \left. \text{Csc}[e+fx]^n \left( \text{Cos}[e+fx] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^{-1+m} \text{Tan} \left[ \frac{1}{2} (e+fx) \right] \right. \\
& \left. \left( -\text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \text{Sin}[e+fx] + \text{Cos}[e+fx] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \text{Tan} \left[ \frac{1}{2} (e+fx) \right] \right) \right) / \\
& \left( (-1+n) \left( (-3+n) \text{AppellF1} \left[ \frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& 2 \left( m \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, 1-m, 1+m-n, \frac{5}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right] + (1+m-n) \right. \\
& \left. \left. \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, -m, 2+m-n, \frac{5}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right] \right) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) + \\
& \left( 2(-3+n) \text{AppellF1} \left[ \frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right] \left( \text{Cos} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^{1+m} \right. \\
& \left. \text{Csc}[e+fx]^n \left( \text{Cos}[e+fx] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^m \text{Tan} \left[ \frac{1}{2} (e+fx) \right] \right. \\
& \left. \left( 2 \left( m \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, 1-m, 1+m-n, \frac{5}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right] + (1+m-n) \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, -m, 2+m-n, \right. \right. \right. \\
& \left. \left. \frac{5}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right] \right) \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \text{Tan} \left[ \frac{1}{2} (e+fx) \right] + (-3+n) \left( -\frac{1}{\frac{3}{2} - \frac{n}{2}} m \left( \frac{1}{2} - \frac{n}{2} \right) \text{AppellF1} \left[ \right. \right. \\
& \left. \left. \frac{3}{2} - \frac{n}{2}, 1-m, 1+m-n, \frac{5}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \text{Tan} \left[ \frac{1}{2} (e+fx) \right] - \frac{1}{\frac{3}{2} - \frac{n}{2}} (1+m-n) \right. \\
& \left. \left. \left( \frac{1}{2} - \frac{n}{2} \right) \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, -m, 2+m-n, \frac{5}{2} - \frac{n}{2}, \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \text{Tan} \left[ \frac{1}{2} (e+fx) \right] \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left( m \left( -\frac{1}{\frac{5}{2}-\frac{n}{2}}(1+m-n) \left( \frac{3}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1-m, 2+m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{5}{2}-\frac{n}{2}}(1-m) \left( \frac{3}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 2-m, 1+m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) + (1+m-n) \left( -\frac{1}{\frac{5}{2}-\frac{n}{2}} m \left( \frac{3}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1-m, \right. \right. \right. \\
& \quad \left. \left. \left. 2+m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{1}{\frac{5}{2}-\frac{n}{2}}(2+m-n) \left( \frac{3}{2}-\frac{n}{2} \right) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, -m, 3+m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \right) \Bigg/ \\
& \left( (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) + \right. \\
& \quad \left. 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \Bigg)
\end{aligned}$$

■ **Problem 287: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a \operatorname{Cos}[e+fx])^m (b \operatorname{Csc}[e+fx])^n dx$$

Optimal (type 5, 91 leaves, 2 steps):

$$\frac{1}{f(1-n)} a b (a \operatorname{Cos}[e+fx])^{-1+m} (\operatorname{Cos}[e+fx]^2)^{\frac{1-m}{2}} (b \operatorname{Csc}[e+fx])^{-1+n} \operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Sin}[e+fx]^2\right]$$

Result (type 6, 3239 leaves):

$$\begin{aligned}
& - \left( \left( 2 (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \right. \\
& \quad \left. \left( \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{1+m} (a \operatorname{Cos}[e+fx])^m \operatorname{Csc}[e+fx]^n (b \operatorname{Csc}[e+fx])^n \left( \operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \Bigg/ \\
& \left( f (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) + \right. \\
& \quad \left. 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{\frac{3}{2} - \frac{n}{2}} (1+m-n) \left( \frac{1}{2} - \frac{n}{2} \right) \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, -m, 2+m-n, \frac{5}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \\
& \tan \left[ \frac{1}{2} (e+fx) \right] \Bigg) \Bigg/ \left( (-1+n) \left( (-3+n) \text{AppellF1} \left[ \frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& 2 \left( m \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, 1-m, 1+m-n, \frac{5}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + (1+m-n) \right. \\
& \left. \left. \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, -m, 2+m-n, \frac{5}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (e+fx) \right]^2 \right) \Bigg) - \\
& \left( 2m(-3+n) \text{AppellF1} \left[ \frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \left( \cos \left[ \frac{1}{2} (e+fx) \right]^2 \right)^{1+m} \right. \\
& \left. \text{Csc}[e+fx]^n \left( \cos[e+fx] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^{-1+m} \tan \left[ \frac{1}{2} (e+fx) \right] \right. \\
& \left. \left( -\text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \sin[e+fx] + \cos[e+fx] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \tan \left[ \frac{1}{2} (e+fx) \right] \right) \right) \Bigg/ \\
& \left( (-1+n) \left( (-3+n) \text{AppellF1} \left[ \frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& 2 \left( m \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, 1-m, 1+m-n, \frac{5}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + (1+m-n) \right. \\
& \left. \left. \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, -m, 2+m-n, \frac{5}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (e+fx) \right]^2 \right) \Bigg) + \\
& \left( 2(-3+n) \text{AppellF1} \left[ \frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \left( \cos \left[ \frac{1}{2} (e+fx) \right]^2 \right)^{1+m} \right. \\
& \left. \text{Csc}[e+fx]^n \left( \cos[e+fx] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^m \tan \left[ \frac{1}{2} (e+fx) \right] \right. \\
& \left. \left( 2 \left( m \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, 1-m, 1+m-n, \frac{5}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + (1+m-n) \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, -m, 2+m-n, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{5}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \right) \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \tan \left[ \frac{1}{2} (e+fx) \right] + (-3+n) \left( -\frac{1}{\frac{3}{2} - \frac{n}{2}} m \left( \frac{1}{2} - \frac{n}{2} \right) \text{AppellF1} \left[ \right. \right. \right. \\
& \left. \left. \left. \frac{3}{2} - \frac{n}{2}, 1-m, 1+m-n, \frac{5}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \tan \left[ \frac{1}{2} (e+fx) \right] - \frac{1}{\frac{3}{2} - \frac{n}{2}} (1+m-n) \right. \right. \\
& \left. \left. \left. \left( \frac{1}{2} - \frac{n}{2} \right) \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, -m, 2+m-n, \frac{5}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \tan \left[ \frac{1}{2} (e+fx) \right] \right) \right) \right) \Bigg) +
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left( m \left( -\frac{1}{\frac{5}{2}-\frac{n}{2}}(1+m-n) \left( \frac{3}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1-m, 2+m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{5}{2}-\frac{n}{2}}(1-m) \left( \frac{3}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 2-m, 1+m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) + (1+m-n) \left( -\frac{1}{\frac{5}{2}-\frac{n}{2}} m \left( \frac{3}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1-m, \right. \right. \right. \\
& \quad \left. \left. \left. 2+m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{1}{\frac{5}{2}-\frac{n}{2}}(2+m-n) \left( \frac{3}{2}-\frac{n}{2} \right) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, -m, 3+m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \right) / \\
& \left( (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) + \right. \\
& \quad \left. 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \right)
\end{aligned}$$

## Test results for the 189 problems in "4.2.10 (c+d x)^m (a+b cos)^n.m"

- **Problem 31: Result more than twice size of optimal antiderivative.**

$$\int (c+dx) \operatorname{Sec}[a+bx] dx$$

Optimal (type 4, 75 leaves, 5 steps):

$$-\frac{2i(c+dx) \operatorname{ArcTan}\left[e^{i(a+bx)}\right]}{b} + \frac{id \operatorname{PolyLog}\left[2, -ie^{i(a+bx)}\right]}{b^2} - \frac{id \operatorname{PolyLog}\left[2, ie^{i(a+bx)}\right]}{b^2}$$

Result (type 4, 220 leaves):



$$-\frac{c \operatorname{Log}\left[\cos\left[\frac{a}{2} + \frac{bx}{2}\right] - \sin\left[\frac{a}{2} + \frac{bx}{2}\right]\right]}{b} + \frac{c \operatorname{Log}\left[\cos\left[\frac{a}{2} + \frac{bx}{2}\right] + \sin\left[\frac{a}{2} + \frac{bx}{2}\right]\right]}{b} + \frac{1}{b^2}$$

$$d \left( \left( -a + \frac{\pi}{2} - bx \right) \left( \operatorname{Log}\left[1 - e^{i\left(-a + \frac{\pi}{2} - bx\right)}\right] - \operatorname{Log}\left[1 + e^{i\left(-a + \frac{\pi}{2} - bx\right)}\right]\right) - \left( -a + \frac{\pi}{2} \right) \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2} \left( -a + \frac{\pi}{2} - bx \right)\right]\right] \right) +$$

$$i \left( \operatorname{PolyLog}\left[2, -e^{i\left(-a + \frac{\pi}{2} - bx\right)}\right] - \operatorname{PolyLog}\left[2, e^{i\left(-a + \frac{\pi}{2} - bx\right)}\right] \right)$$

■ **Problem 33: Result more than twice size of optimal antiderivative.**

$$\int (c + dx)^3 \operatorname{Sec}[a + bx]^2 dx$$

Optimal (type 4, 114 leaves, 6 steps):

$$-\frac{i(c+dx)^3}{b} + \frac{3d(c+dx)^2 \operatorname{Log}\left[1 + e^{2i(a+bx)}\right]}{b^2} - \frac{3id^2(c+dx) \operatorname{PolyLog}\left[2, -e^{2i(a+bx)}\right]}{b^3} + \frac{3d^3 \operatorname{PolyLog}\left[3, -e^{2i(a+bx)}\right]}{2b^4} + \frac{(c+dx)^3 \operatorname{Tan}[a+bx]}{b}$$

Result (type 4, 397 leaves):

$$-\frac{1}{4b^4}$$

$$d^3 e^{-ia} \left( 2ib^2 x^2 \left( 2b e^{2ia} x + 3i \left( 1 + e^{2ia} \right) \operatorname{Log}\left[1 + e^{2i(a+bx)}\right] \right) + 6ib \left( 1 + e^{2ia} \right) x \operatorname{PolyLog}\left[2, -e^{2i(a+bx)}\right] - 3 \left( 1 + e^{2ia} \right) \operatorname{PolyLog}\left[3, -e^{2i(a+bx)}\right] \right)$$

$$\operatorname{Sec}[a] + \frac{3c^2 d \operatorname{Sec}[a] \left( \cos[a] \operatorname{Log}\left[\cos[a] \cos[bx] - \sin[a] \sin[bx]\right] + bx \sin[a] \right)}{b^2 \left( \cos[a]^2 + \sin[a]^2 \right)} +$$

$$\left( 3cd^2 \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - 1 \right) / \left( \sqrt{1 + \operatorname{Cot}[a]^2} \right) \operatorname{Cot}[a] \right.$$

$$\left. \left( i b x \left( -\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \right) - \pi \operatorname{Log}\left[1 + e^{-2ibx}\right] - 2 \left( bx - \operatorname{ArcTan}[\operatorname{Cot}[a]] \right) \operatorname{Log}\left[1 - e^{2i(bx - \operatorname{ArcTan}[\operatorname{Cot}[a]])}\right] + \pi \operatorname{Log}[\cos[bx]] - \right.$$

$$\left. 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}[\sin[bx - \operatorname{ArcTan}[\operatorname{Cot}[a]]] + i \operatorname{PolyLog}\left[2, e^{2i(bx - \operatorname{ArcTan}[\operatorname{Cot}[a]])}\right] \right) \operatorname{Sec}[a] \right) /$$

$$\left( b^3 \sqrt{\operatorname{Csc}[a]^2 \left( \cos[a]^2 + \sin[a]^2 \right)} \right) + \frac{\operatorname{Sec}[a] \operatorname{Sec}[a+bx] \left( c^3 \sin[bx] + 3c^2 dx \sin[bx] + 3cd^2 x^2 \sin[bx] + d^3 x^3 \sin[bx] \right)}{b}$$

■ **Problem 34: Result more than twice size of optimal antiderivative.**

$$\int (c + dx)^2 \operatorname{Sec}[a + bx]^2 dx$$

Optimal (type 4, 82 leaves, 5 steps):

$$-\frac{i(c+dx)^2}{b} + \frac{2d(c+dx) \operatorname{Log}\left[1 + e^{2i(a+bx)}\right]}{b^2} - \frac{id^2 \operatorname{PolyLog}\left[2, -e^{2i(a+bx)}\right]}{b^3} + \frac{(c+dx)^2 \operatorname{Tan}[a+bx]}{b}$$

Result (type 4, 253 leaves):

$$\frac{2 c d \operatorname{Sec}[a] (\operatorname{Cos}[a] \operatorname{Log}[\operatorname{Cos}[a] \operatorname{Cos}[b x] - \operatorname{Sin}[a] \operatorname{Sin}[b x]] + b x \operatorname{Sin}[a])}{b^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} +$$

$$\left( d^2 \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - 1 \right) / \left( \sqrt{1 + \operatorname{Cot}[a]^2} \right) \operatorname{Cot}[a] (i b x (-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]]) - \right.$$

$$\pi \operatorname{Log}[1 + e^{-2 i b x}] - 2 (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \operatorname{Log}[1 - e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]])}] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] -$$

$$\left. 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}[\operatorname{Sin}[b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]] + i \operatorname{PolyLog}[2, e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}]] \right) \operatorname{Sec}[a] \Big/$$

$$\left( b^3 \sqrt{\operatorname{Csc}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) + \frac{\operatorname{Sec}[a] \operatorname{Sec}[a + b x] (c^2 \operatorname{Sin}[b x] + 2 c d x \operatorname{Sin}[b x] + d^2 x^2 \operatorname{Sin}[b x])}{b}$$

■ **Problem 38: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^2 \operatorname{Sec}[a + b x]^3 dx$$

Optimal (type 4, 193 leaves, 9 steps):

$$-\frac{i (c + d x)^2 \operatorname{ArcTan}[e^{i (a + b x)}]}{b} + \frac{d^2 \operatorname{ArcTanh}[\operatorname{Sin}[a + b x]]}{b^3} + \frac{i d (c + d x) \operatorname{PolyLog}[2, -i e^{i (a + b x)}]}{b^2} - \frac{i d (c + d x) \operatorname{PolyLog}[2, i e^{i (a + b x)}]}{b^2}$$

$$\frac{d^2 \operatorname{PolyLog}[3, -i e^{i (a + b x)}]}{b^3} + \frac{d^2 \operatorname{PolyLog}[3, i e^{i (a + b x)}]}{b^3} - \frac{d (c + d x) \operatorname{Sec}[a + b x]}{b^2} + \frac{(c + d x)^2 \operatorname{Sec}[a + b x] \operatorname{Tan}[a + b x]}{2 b}$$

Result (type 4, 526 leaves):

$$\frac{1}{b^2} \left( -i b c^2 \operatorname{ArcTan}[e^{i (a + b x)}] - \frac{2 i d^2 \operatorname{ArcTan}[e^{i (a + b x)}]}{b} + b c d x \operatorname{Log}[1 - i e^{i (a + b x)}] + \right.$$

$$\frac{1}{2} b d^2 x^2 \operatorname{Log}[1 - i e^{i (a + b x)}] - b c d x \operatorname{Log}[1 + i e^{i (a + b x)}] - \frac{1}{2} b d^2 x^2 \operatorname{Log}[1 + i e^{i (a + b x)}] + i d (c + d x) \operatorname{PolyLog}[2, -i e^{i (a + b x)}] -$$

$$\left. i d (c + d x) \operatorname{PolyLog}[2, i e^{i (a + b x)}] - \frac{d^2 \operatorname{PolyLog}[3, -i e^{i (a + b x)}]}{b} + \frac{d^2 \operatorname{PolyLog}[3, i e^{i (a + b x)}]}{b} \right) -$$

$$\frac{d (c + d x) \operatorname{Sec}[a]}{b^2} + \frac{c^2 + 2 c d x + d^2 x^2}{4 b (\operatorname{Cos}[\frac{a}{2} + \frac{b x}{2}] - \operatorname{Sin}[\frac{a}{2} + \frac{b x}{2}])^2} + \frac{-c d \operatorname{Sin}[\frac{b x}{2}] - d^2 x \operatorname{Sin}[\frac{b x}{2}]}{b^2 (\operatorname{Cos}[\frac{a}{2}] - \operatorname{Sin}[\frac{a}{2}]) (\operatorname{Cos}[\frac{a}{2} + \frac{b x}{2}] - \operatorname{Sin}[\frac{a}{2} + \frac{b x}{2}])}$$

$$\frac{-c^2 - 2 c d x - d^2 x^2}{4 b (\operatorname{Cos}[\frac{a}{2} + \frac{b x}{2}] + \operatorname{Sin}[\frac{a}{2} + \frac{b x}{2}])^2} + \frac{c d \operatorname{Sin}[\frac{b x}{2}] + d^2 x \operatorname{Sin}[\frac{b x}{2}]}{b^2 (\operatorname{Cos}[\frac{a}{2}] + \operatorname{Sin}[\frac{a}{2}]) (\operatorname{Cos}[\frac{a}{2} + \frac{b x}{2}] + \operatorname{Sin}[\frac{a}{2} + \frac{b x}{2}])}$$

■ **Problem 39: Result more than twice size of optimal antiderivative.**

$$\int (c + d x) \operatorname{Sec}[a + b x]^3 dx$$

Optimal (type 4, 117 leaves, 6 steps):

$$-\frac{i(c+dx)\operatorname{ArcTan}\left[e^{i(a+bx)}\right]}{b} + \frac{id\operatorname{PolyLog}\left[2, -ie^{i(a+bx)}\right]}{2b^2} - \frac{id\operatorname{PolyLog}\left[2, ie^{i(a+bx)}\right]}{2b^2} - \frac{d\operatorname{Sec}[a+bx]}{2b^2} + \frac{(c+dx)\operatorname{Sec}[a+bx]\operatorname{Tan}[a+bx]}{2b}$$

Result (type 4, 480 leaves):

$$\begin{aligned} & -\frac{c\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{2b} + \frac{c\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{2b} + \\ & \frac{1}{2b^2}d\left(\left(-a + \frac{\pi}{2} - bx\right)\left(\operatorname{Log}\left[1 - e^{i(-a+\frac{\pi}{2}-bx)}\right] - \operatorname{Log}\left[1 + e^{i(-a+\frac{\pi}{2}-bx)}\right]\right) - \right. \\ & \quad \left. \left(-a + \frac{\pi}{2}\right)\operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}\left(-a + \frac{\pi}{2} - bx\right)\right]\right] + i\left(\operatorname{PolyLog}\left[2, -e^{i(-a+\frac{\pi}{2}-bx)}\right] - \operatorname{PolyLog}\left[2, e^{i(-a+\frac{\pi}{2}-bx)}\right]\right)\right) + \\ & \frac{dx}{4b\left(\operatorname{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] - \operatorname{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right]\right)^2} - \frac{d\operatorname{Sin}\left[\frac{bx}{2}\right]}{2b^2\left(\operatorname{Cos}\left[\frac{a}{2}\right] - \operatorname{Sin}\left[\frac{a}{2}\right]\right)\left(\operatorname{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] - \operatorname{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right]\right)} - \frac{dx}{4b\left(\operatorname{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] + \operatorname{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right]\right)^2} + \\ & \frac{d\operatorname{Sin}\left[\frac{bx}{2}\right]}{2b^2\left(\operatorname{Cos}\left[\frac{a}{2}\right] + \operatorname{Sin}\left[\frac{a}{2}\right]\right)\left(\operatorname{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] + \operatorname{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right]\right)} + \frac{c}{4b\left(\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right)^2} - \frac{c}{4b\left(\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right)^2} \end{aligned}$$

■ **Problem 55: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[a+bx]^2}{(c+dx)^{9/2}} dx$$

Optimal (type 4, 247 leaves, 11 steps):

$$\begin{aligned} & -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\operatorname{Cos}[a+bx]^2}{7d(c+dx)^{7/2}} + \frac{32b^2\operatorname{Cos}[a+bx]^2}{105d^3(c+dx)^{3/2}} + \frac{128b^{7/2}\sqrt{\pi}\operatorname{Cos}\left[2a - \frac{2bc}{d}\right]\operatorname{FresnelC}\left[\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right]}{105d^{9/2}} - \\ & \frac{128b^{7/2}\sqrt{\pi}\operatorname{FresnelS}\left[\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right]\operatorname{Sin}\left[2a - \frac{2bc}{d}\right]}{105d^{9/2}} + \frac{8b\operatorname{Cos}[a+bx]\operatorname{Sin}[a+bx]}{35d^2(c+dx)^{5/2}} - \frac{128b^3\operatorname{Cos}[a+bx]\operatorname{Sin}[a+bx]}{105d^4\sqrt{c+dx}} \end{aligned}$$

Result (type 4, 987 leaves):

$$\begin{aligned}
& - \frac{1}{7d(c+dx)^{7/2}} + \\
& \frac{1}{2} \left( \cos[2a] \left( -\frac{1}{7d} 32\sqrt{2} \left(\frac{b}{d}\right)^{7/2} \cos\left[\frac{bc}{d}\right] \sin\left[\frac{bc}{d}\right] \left( \frac{\sin\left[\frac{2b(c+dx)}{d}\right]}{8\sqrt{2}\left(\frac{b}{d}\right)^{7/2}(c+dx)^{7/2}} + \frac{2}{5} \left( \frac{\cos\left[\frac{2b(c+dx)}{d}\right]}{4\sqrt{2}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2}} - \frac{2}{3} \left( 2 \frac{\cos\left[\frac{2b(c+dx)}{d}\right]}{\sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx}} + \sqrt{2}\pi \right. \right. \right. \right. \\
& \left. \left. \left. \left. \text{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \frac{\sin\left[\frac{2b(c+dx)}{d}\right]}{2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2}} \right) \right) - \frac{1}{7d} 16\sqrt{2} \left(\frac{b}{d}\right)^{7/2} \cos\left[\frac{2bc}{d}\right] \left( \frac{\cos\left[\frac{2b(c+dx)}{d}\right]}{8\sqrt{2}\left(\frac{b}{d}\right)^{7/2}(c+dx)^{7/2}} - \right. \right. \\
& \left. \left. \left. \left. \frac{2}{5} \left( \frac{\sin\left[\frac{2b(c+dx)}{d}\right]}{4\sqrt{2}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2}} + \frac{2}{3} \left( \frac{\cos\left[\frac{2b(c+dx)}{d}\right]}{2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2}} - 2 \left( -\sqrt{2}\pi \text{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \frac{\sin\left[\frac{2b(c+dx)}{d}\right]}{\sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx}} \right) \right) \right) \right) \right) - \\
& 2\cos[a]\sin[a] \left( -\frac{1}{7d} 16\sqrt{2} \left(\frac{b}{d}\right)^{7/2} \left( \cos\left[\frac{bc}{d}\right] - \sin\left[\frac{bc}{d}\right] \right) \left( \cos\left[\frac{bc}{d}\right] + \sin\left[\frac{bc}{d}\right] \right) \left( \frac{\sin\left[\frac{2b(c+dx)}{d}\right]}{8\sqrt{2}\left(\frac{b}{d}\right)^{7/2}(c+dx)^{7/2}} + \right. \right. \\
& \left. \left. \left. \left. \frac{2}{5} \left( \frac{\cos\left[\frac{2b(c+dx)}{d}\right]}{4\sqrt{2}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2}} - \frac{2}{3} \left( \frac{\cos\left[\frac{2b(c+dx)}{d}\right]}{\sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx}} + \sqrt{2}\pi \text{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \frac{\sin\left[\frac{2b(c+dx)}{d}\right]}{2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2}} \right) \right) \right) \right) \right) + \\
& \frac{1}{7d} 16\sqrt{2} \left(\frac{b}{d}\right)^{7/2} \sin\left[\frac{2bc}{d}\right] \left( \frac{\cos\left[\frac{2b(c+dx)}{d}\right]}{8\sqrt{2}\left(\frac{b}{d}\right)^{7/2}(c+dx)^{7/2}} - \frac{2}{5} \left( \frac{\sin\left[\frac{2b(c+dx)}{d}\right]}{4\sqrt{2}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2}} + \right. \right. \\
& \left. \left. \left. \left. \frac{2}{3} \left( \frac{\cos\left[\frac{2b(c+dx)}{d}\right]}{2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2}} - 2 \left( -\sqrt{2}\pi \text{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \frac{\sin\left[\frac{2b(c+dx)}{d}\right]}{\sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx}} \right) \right) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 62: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[a + bx]^3}{(c + dx)^{7/2}} dx$$

Optimal (type 4, 356 leaves, 19 steps):

$$\begin{aligned} & -\frac{16b^2 \cos[a + bx]}{5d^3 \sqrt{c + dx}} - \frac{2 \cos[a + bx]^3}{5d(c + dx)^{5/2}} + \frac{24b^2 \cos[a + bx]^3}{5d^3 \sqrt{c + dx}} + \frac{2b^{5/2} \sqrt{2\pi} \cos\left[a - \frac{bc}{d}\right] \operatorname{FresnelS}\left[\frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c + dx}}{\sqrt{d}}\right]}{5d^{7/2}} + \\ & \frac{6b^{5/2} \sqrt{6\pi} \cos\left[3a - \frac{3bc}{d}\right] \operatorname{FresnelS}\left[\frac{\sqrt{b} \sqrt{\frac{6}{\pi}} \sqrt{c + dx}}{\sqrt{d}}\right]}{5d^{7/2}} + \frac{6b^{5/2} \sqrt{6\pi} \operatorname{FresnelC}\left[\frac{\sqrt{b} \sqrt{\frac{6}{\pi}} \sqrt{c + dx}}{\sqrt{d}}\right] \sin\left[3a - \frac{3bc}{d}\right]}{5d^{7/2}} + \\ & \frac{2b^{5/2} \sqrt{2\pi} \operatorname{FresnelC}\left[\frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c + dx}}{\sqrt{d}}\right] \sin\left[a - \frac{bc}{d}\right]}{5d^{7/2}} + \frac{4b \cos[a + bx]^2 \sin[a + bx]}{5d^2 (c + dx)^{3/2}} \end{aligned}$$

Result (type 4, 1429 leaves):

$$\begin{aligned} & \frac{3}{4} \left( -\sin[a] \left( \frac{1}{5d} 2 \left(\frac{b}{d}\right)^{5/2} \sin\left[\frac{bc}{d}\right] \left( \frac{\cos\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2}} - \frac{2}{3} \left( \frac{\cos\left[\frac{b(c+dx)}{d}\right]}{\sqrt{\frac{b}{d}} \sqrt{c+dx}} + \sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] + \frac{\sin\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2}} \right) \right) - \frac{1}{5d} \right. \\ & \left. 2 \left(\frac{b}{d}\right)^{5/2} \cos\left[\frac{bc}{d}\right] \left( \frac{\sin\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2}} + \frac{2}{3} \left( \frac{\cos\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2}} - 2 \left( -\sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] + \frac{\sin\left[\frac{b(c+dx)}{d}\right]}{\sqrt{\frac{b}{d}} \sqrt{c+dx}} \right) \right) \right) \right) + \\ & \left. \cos[a] \left( -\frac{1}{5d} 2 \left(\frac{b}{d}\right)^{5/2} \cos\left[\frac{bc}{d}\right] \left( \frac{\cos\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2}} - \frac{2}{3} \left( \frac{\cos\left[\frac{b(c+dx)}{d}\right]}{\sqrt{\frac{b}{d}} \sqrt{c+dx}} + \sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] + \frac{\sin\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2}} \right) \right) - \right. \\ & \left. \frac{1}{5d} 2 \left(\frac{b}{d}\right)^{5/2} \sin\left[\frac{bc}{d}\right] \left( \frac{\sin\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2}} + \frac{2}{3} \left( \frac{\cos\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2}} - 2 \left( -\sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] + \frac{\sin\left[\frac{b(c+dx)}{d}\right]}{\sqrt{\frac{b}{d}} \sqrt{c+dx}} \right) \right) \right) \right) \right) + \end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} \left( -\text{Sin}[3 a] \left( \frac{1}{5 d} 18 \sqrt{3} \left( \frac{b}{d} \right)^{5/2} \text{Sin}\left[\frac{3 b c}{d}\right] \left( \frac{\text{Cos}\left[\frac{3 b (c+d x)}{d}\right]}{9 \sqrt{3} \left( \frac{b}{d} \right)^{5/2} (c+d x)^{5/2}} - \right. \right. \right. \\
& \left. \left. \left. \frac{2}{3} \left( 2 \left( \frac{\text{Cos}\left[\frac{3 b (c+d x)}{d}\right]}{\sqrt{3} \sqrt{\frac{b}{d} \sqrt{c+d x}}} + \sqrt{2 \pi} \text{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}\right] \right) + \frac{\text{Sin}\left[\frac{3 b (c+d x)}{d}\right]}{3 \sqrt{3} \left( \frac{b}{d} \right)^{3/2} (c+d x)^{3/2}} \right) - \frac{1}{5 d} 18 \sqrt{3} \left( \frac{b}{d} \right)^{5/2} \text{Cos}\left[\frac{3 b c}{d}\right] \right. \right. \\
& \left. \left. \left( \frac{\text{Sin}\left[\frac{3 b (c+d x)}{d}\right]}{9 \sqrt{3} \left( \frac{b}{d} \right)^{5/2} (c+d x)^{5/2}} + \frac{2}{3} \left( \frac{\text{Cos}\left[\frac{3 b (c+d x)}{d}\right]}{3 \sqrt{3} \left( \frac{b}{d} \right)^{3/2} (c+d x)^{3/2}} - 2 \left( -\sqrt{2 \pi} \text{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}\right] + \frac{\text{Sin}\left[\frac{3 b (c+d x)}{d}\right]}{\sqrt{3} \sqrt{\frac{b}{d} \sqrt{c+d x}}}\right) \right) \right) \right) \right) + \\
& \text{Cos}[3 a] \left( -\frac{1}{5 d} 18 \sqrt{3} \left( \frac{b}{d} \right)^{5/2} \text{Cos}\left[\frac{3 b c}{d}\right] \left( \frac{\text{Cos}\left[\frac{3 b (c+d x)}{d}\right]}{9 \sqrt{3} \left( \frac{b}{d} \right)^{5/2} (c+d x)^{5/2}} - \right. \right. \\
& \left. \left. \left. \frac{2}{3} \left( 2 \left( \frac{\text{Cos}\left[\frac{3 b (c+d x)}{d}\right]}{\sqrt{3} \sqrt{\frac{b}{d} \sqrt{c+d x}}} + \sqrt{2 \pi} \text{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}\right] \right) + \frac{\text{Sin}\left[\frac{3 b (c+d x)}{d}\right]}{3 \sqrt{3} \left( \frac{b}{d} \right)^{3/2} (c+d x)^{3/2}} \right) - \frac{1}{5 d} 18 \sqrt{3} \left( \frac{b}{d} \right)^{5/2} \text{Sin}\left[\frac{3 b c}{d}\right] \right. \right. \\
& \left. \left. \left( \frac{\text{Sin}\left[\frac{3 b (c+d x)}{d}\right]}{9 \sqrt{3} \left( \frac{b}{d} \right)^{5/2} (c+d x)^{5/2}} + \frac{2}{3} \left( \frac{\text{Cos}\left[\frac{3 b (c+d x)}{d}\right]}{3 \sqrt{3} \left( \frac{b}{d} \right)^{3/2} (c+d x)^{3/2}} - 2 \left( -\sqrt{2 \pi} \text{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}\right] + \frac{\text{Sin}\left[\frac{3 b (c+d x)}{d}\right]}{\sqrt{3} \sqrt{\frac{b}{d} \sqrt{c+d x}}}\right) \right) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 75: Attempted integration timed out after 120 seconds.**

$$\int x \sqrt{\text{Cos}[a + b x]} dx$$

Optimal (type 9, 14 leaves, 0 steps):

Unintegrable  $\left[ x \sqrt{\text{Cos}[a + b x]}, x \right]$

Result (type 1, 1 leaves):

???

- **Problem 86: Attempted integration timed out after 120 seconds.**

$$\int \frac{x}{\cos[a + b x]^{3/2}} dx$$

Optimal (type 9, 54 leaves, 1 step):

$$\frac{4 \sqrt{\cos[a + b x]}}{b^2} + \frac{2 x \sin[a + b x]}{b \sqrt{\cos[a + b x]}} - \text{Unintegrable}\left[x \sqrt{\cos[a + b x]}, x\right]$$

Result (type 1, 1 leaves):

???

- **Problem 129: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^2}{a + a \cos[e + f x]} dx$$

Optimal (type 4, 101 leaves, 6 steps):

$$-\frac{i (c + d x)^2}{a f} + \frac{4 d (c + d x) \log\left[1 + e^{i(e + f x)}\right]}{a f^2} - \frac{4 i d^2 \text{PolyLog}\left[2, -e^{i(e + f x)}\right]}{a f^3} + \frac{(c + d x)^2 \tan\left[\frac{e}{2} + \frac{f x}{2}\right]}{a f}$$

Result (type 4, 454 leaves):

$$\frac{8 c d \cos\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \sec\left[\frac{e}{2}\right] \left(\cos\left[\frac{e}{2}\right] \log\left[\cos\left[\frac{e}{2}\right] \cos\left[\frac{f x}{2}\right] - \sin\left[\frac{e}{2}\right] \sin\left[\frac{f x}{2}\right]\right] + \frac{1}{2} f x \sin\left[\frac{e}{2}\right])}{f^2 (a + a \cos[e + f x]) \left(\cos\left[\frac{e}{2}\right]^2 + \sin\left[\frac{e}{2}\right]^2\right)} +$$

$$\left(8 d^2 \cos\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \csc\left[\frac{e}{2}\right] \left(\frac{1}{4} e^{-i \text{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right]} f^2 x^2 - 1 / \left(\sqrt{1 + \cot\left[\frac{e}{2}\right]^2}\right) \cot\left[\frac{e}{2}\right]\right.\right.$$

$$\left.\left(\frac{1}{2} i f x \left(-\pi - 2 \text{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right]\right) - \pi \log\left[1 + e^{-i f x}\right] - 2 \left(\frac{f x}{2} - \text{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right]\right) \log\left[1 - e^{2 i \left(\frac{f x}{2} - \text{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right)}\right] + \right.\right.$$

$$\left.\left.\pi \log\left[\cos\left[\frac{f x}{2}\right]\right] - 2 \text{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right] \log\left[\sin\left[\frac{f x}{2} - \text{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right]\right] + i \text{PolyLog}\left[2, e^{2 i \left(\frac{f x}{2} - \text{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right)}\right]\right]\right) \sec\left[\frac{e}{2}\right]\right) /$$

$$\left(f^3 (a + a \cos[e + f x]) \sqrt{\csc\left[\frac{e}{2}\right]^2 \left(\cos\left[\frac{e}{2}\right]^2 + \sin\left[\frac{e}{2}\right]^2\right)} + \frac{2 \cos\left[\frac{e}{2} + \frac{f x}{2}\right] \sec\left[\frac{e}{2}\right] \left(c^2 \sin\left[\frac{f x}{2}\right] + 2 c d x \sin\left[\frac{f x}{2}\right] + d^2 x^2 \sin\left[\frac{f x}{2}\right]\right)}{f (a + a \cos[e + f x])}\right)$$

- **Problem 133: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^3}{(a + a \cos[e + f x])^2} dx$$

Optimal (type 4, 271 leaves, 10 steps):

$$\begin{aligned}
& - \frac{i(c+dx)^3}{3a^2f} + \frac{2d(c+dx)^2 \operatorname{Log}[1+e^{i(e+fx)}]}{a^2f^2} + \frac{4d^3 \operatorname{Log}\left[\cos\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{a^2f^4} - \frac{4id^2(c+dx) \operatorname{PolyLog}[2, -e^{i(e+fx)}]}{a^2f^3} + \frac{4d^3 \operatorname{PolyLog}[3, -e^{i(e+fx)}]}{a^2f^4} \\
& \frac{d(c+dx)^2 \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]^2}{2a^2f^2} + \frac{2d^2(c+dx) \operatorname{Tan}\left[\frac{e}{2} + \frac{fx}{2}\right]}{a^2f^3} + \frac{(c+dx)^3 \operatorname{Tan}\left[\frac{e}{2} + \frac{fx}{2}\right]}{3a^2f} + \frac{(c+dx)^3 \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \operatorname{Tan}\left[\frac{e}{2} + \frac{fx}{2}\right]}{6a^2f}
\end{aligned}$$

Result (type 4, 1016 leaves):

$$\begin{aligned}
& - \frac{1}{3f^4(a+a\cos[e+fx])^2} 4d^3 e^{-\frac{ie}{2}} \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^4 \\
& \left( if^2x^2(e^{ie}fx + 3i(1+e^{ie})\operatorname{Log}[1+e^{i(e+fx)}]) + 6i(1+e^{ie})fx \operatorname{PolyLog}[2, -e^{i(e+fx)}] - 6(1+e^{ie}) \operatorname{PolyLog}[3, -e^{i(e+fx)}] \right) \operatorname{Sec}\left[\frac{e}{2}\right] + \\
& \frac{16d^3 \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^4 \operatorname{Sec}\left[\frac{e}{2}\right] \left( \cos\left[\frac{e}{2}\right] \operatorname{Log}\left[\cos\left[\frac{e}{2}\right] \cos\left[\frac{fx}{2}\right] - \sin\left[\frac{e}{2}\right] \sin\left[\frac{fx}{2}\right]\right] + \frac{1}{2}fx \sin\left[\frac{e}{2}\right] \right)}{f^4(a+a\cos[e+fx])^2 \left( \cos\left[\frac{e}{2}\right]^2 + \sin\left[\frac{e}{2}\right]^2 \right)} + \\
& \frac{8c^2d \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^4 \operatorname{Sec}\left[\frac{e}{2}\right] \left( \cos\left[\frac{e}{2}\right] \operatorname{Log}\left[\cos\left[\frac{e}{2}\right] \cos\left[\frac{fx}{2}\right] - \sin\left[\frac{e}{2}\right] \sin\left[\frac{fx}{2}\right]\right] + \frac{1}{2}fx \sin\left[\frac{e}{2}\right] \right)}{f^2(a+a\cos[e+fx])^2 \left( \cos\left[\frac{e}{2}\right]^2 + \sin\left[\frac{e}{2}\right]^2 \right)} + \\
& \left( 16cd^2 \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^4 \operatorname{Csc}\left[\frac{e}{2}\right] \left( \frac{1}{4} e^{-i \operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right]} f^2x^2 - 1 / \left( \sqrt{1 + \cot\left[\frac{e}{2}\right]^2} \right) \cot\left[\frac{e}{2}\right] \right. \right. \\
& \left. \left. \left( \frac{1}{2} ifx \left( -\pi - 2 \operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right] \right) - \pi \operatorname{Log}[1+e^{-ifx}] - 2 \left( \frac{fx}{2} - \operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right] \right) \operatorname{Log}\left[1 - e^{2i\left(\frac{fx}{2} - \operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right)}\right]} \right) + \right. \right. \\
& \left. \left. \pi \operatorname{Log}\left[\cos\left[\frac{fx}{2}\right]\right] - 2 \operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right] \operatorname{Log}\left[\sin\left[\frac{fx}{2} - \operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right]\right] \right] + i \operatorname{PolyLog}\left[2, e^{2i\left(\frac{fx}{2} - \operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right)}\right]} \right) \right) \operatorname{Sec}\left[\frac{e}{2}\right] \right) / \\
& \left( f^3(a+a\cos[e+fx])^2 \sqrt{\operatorname{Csc}\left[\frac{e}{2}\right]^2 \left( \cos\left[\frac{e}{2}\right]^2 + \sin\left[\frac{e}{2}\right]^2 \right)} \right) + \frac{1}{3f^3(a+a\cos[e+fx])^2} \cos\left[\frac{e}{2} + \frac{fx}{2}\right] \operatorname{Sec}\left[\frac{e}{2}\right] \\
& \left( -3c^2df \cos\left[\frac{fx}{2}\right] - 6cd^2fx \cos\left[\frac{fx}{2}\right] - 3d^3fx^2 \cos\left[\frac{fx}{2}\right] - 3c^2df \cos\left[e + \frac{fx}{2}\right] - 6cd^2fx \cos\left[e + \frac{fx}{2}\right] - \right. \\
& \left. 3d^3fx^2 \cos\left[e + \frac{fx}{2}\right] + 12cd^2 \sin\left[\frac{fx}{2}\right] + 3c^3f^2 \sin\left[\frac{fx}{2}\right] + 12d^3x \sin\left[\frac{fx}{2}\right] + 9c^2df^2x \sin\left[\frac{fx}{2}\right] + \right. \\
& \left. 9cd^2f^2x^2 \sin\left[\frac{fx}{2}\right] + 3d^3f^2x^3 \sin\left[\frac{fx}{2}\right] - 6cd^2 \sin\left[e + \frac{fx}{2}\right] - 6d^3x \sin\left[e + \frac{fx}{2}\right] + 6cd^2 \sin\left[e + \frac{3fx}{2}\right] + \right. \\
& \left. c^3f^2 \sin\left[e + \frac{3fx}{2}\right] + 6d^3x \sin\left[e + \frac{3fx}{2}\right] + 3c^2df^2x \sin\left[e + \frac{3fx}{2}\right] + 3cd^2f^2x^2 \sin\left[e + \frac{3fx}{2}\right] + d^3f^2x^3 \sin\left[e + \frac{3fx}{2}\right] \right)
\end{aligned}$$

■ **Problem 134: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+dx)^2}{(a+a\cos[e+fx])^2} dx$$



Optimal (type 4, 212 leaves, 9 steps) :

$$-\frac{i(c+dx)^2}{3a^2f} + \frac{4d(c+dx)\operatorname{Log}[1+e^{i(e+fx)}]}{3a^2f^2} - \frac{4id^2\operatorname{PolyLog}[2, -e^{i(e+fx)}]}{3a^2f^3} - \frac{d(c+dx)\operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]^2}{3a^2f^2} + \frac{2d^2\tan\left[\frac{e}{2} + \frac{fx}{2}\right]}{3a^2f^3} + \frac{(c+dx)^2\tan\left[\frac{e}{2} + \frac{fx}{2}\right]}{3a^2f} + \frac{(c+dx)^2\operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]^2\tan\left[\frac{e}{2} + \frac{fx}{2}\right]}{6a^2f}$$

Result (type 4, 619 leaves) :

$$\frac{16cd\cos\left[\frac{e}{2} + \frac{fx}{2}\right]^4\operatorname{Sec}\left[\frac{e}{2}\right]\left(\cos\left[\frac{e}{2}\right]\operatorname{Log}\left[\cos\left[\frac{e}{2}\right]\cos\left[\frac{fx}{2}\right] - \sin\left[\frac{e}{2}\right]\sin\left[\frac{fx}{2}\right]\right) + \frac{1}{2}fx\sin\left[\frac{e}{2}\right]}{3f^2(a+a\cos[e+fx])^2\left(\cos\left[\frac{e}{2}\right]^2 + \sin\left[\frac{e}{2}\right]^2\right)} + \left(16d^2\cos\left[\frac{e}{2} + \frac{fx}{2}\right]^4\operatorname{Csc}\left[\frac{e}{2}\right]\left(\frac{1}{4}e^{-i\operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right]}f^2x^2 - 1\right) / \left(\sqrt{1 + \cot\left[\frac{e}{2}\right]^2}\right)\cot\left[\frac{e}{2}\right] + \left(\frac{1}{2}ifx\left(-\pi - 2\operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right]\right) - \pi\operatorname{Log}\left[1 + e^{-ifx}\right] - 2\left(\frac{fx}{2} - \operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right]\right)\operatorname{Log}\left[1 - e^{2i\left(\frac{fx}{2} - \operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right)}\right]\right) + \pi\operatorname{Log}\left[\cos\left[\frac{fx}{2}\right]\right] - 2\operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right]\operatorname{Log}\left[\sin\left[\frac{fx}{2} - \operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right]\right] + i\operatorname{PolyLog}\left[2, e^{2i\left(\frac{fx}{2} - \operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right)}\right]\right)\right)\operatorname{Sec}\left[\frac{e}{2}\right]\right) / \left(3f^3(a+a\cos[e+fx])^2\sqrt{\operatorname{Csc}\left[\frac{e}{2}\right]^2\left(\cos\left[\frac{e}{2}\right]^2 + \sin\left[\frac{e}{2}\right]^2\right)}\right) + \frac{1}{3f^3(a+a\cos[e+fx])^2}\cos\left[\frac{e}{2} + \frac{fx}{2}\right] \operatorname{Sec}\left[\frac{e}{2}\right] \left(-2cdf\cos\left[\frac{fx}{2}\right] - 2d^2fx\cos\left[\frac{fx}{2}\right] - 2cdf\cos\left[e + \frac{fx}{2}\right] - 2d^2fx\cos\left[e + \frac{fx}{2}\right] + 4d^2\sin\left[\frac{fx}{2}\right] + 3c^2f^2\sin\left[\frac{fx}{2}\right] + 6cdf^2x\sin\left[\frac{fx}{2}\right] + 3d^2f^2x^2\sin\left[\frac{fx}{2}\right] - 2d^2\sin\left[e + \frac{fx}{2}\right] + 2d^2\sin\left[e + \frac{3fx}{2}\right] + c^2f^2\sin\left[e + \frac{3fx}{2}\right] + 2cdf^2x\sin\left[e + \frac{3fx}{2}\right] + d^2f^2x^2\sin\left[e + \frac{3fx}{2}\right])$$

■ **Problem 139: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+dx)^2}{a-a\cos[e+fx]} dx$$

Optimal (type 4, 102 leaves, 6 steps) :

$$-\frac{i(c+dx)^2}{af} - \frac{(c+dx)^2\cot\left[\frac{e}{2} + \frac{fx}{2}\right]}{af} + \frac{4d(c+dx)\operatorname{Log}[1-e^{i(e+fx)}]}{af^2} - \frac{4id^2\operatorname{PolyLog}[2, e^{i(e+fx)}]}{af^3}$$

Result (type 4, 447 leaves) :

$$\frac{2 \operatorname{Csc}\left[\frac{e}{2}\right] \left(c^2 \operatorname{Sin}\left[\frac{f x}{2}\right] + 2 c d x \operatorname{Sin}\left[\frac{f x}{2}\right] + d^2 x^2 \operatorname{Sin}\left[\frac{f x}{2}\right]\right) \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]}{f (a - a \operatorname{Cos}[e + f x])} +$$

$$\frac{8 c d \operatorname{Csc}\left[\frac{e}{2}\right] \left(-\frac{1}{2} f x \operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{f x}{2}\right] \operatorname{Sin}\left[\frac{e}{2}\right] + \operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{f x}{2}\right]\right] \operatorname{Sin}\left[\frac{e}{2}\right]\right) \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^2}{f^2 (a - a \operatorname{Cos}[e + f x]) \left(\operatorname{Cos}\left[\frac{e}{2}\right]^2 + \operatorname{Sin}\left[\frac{e}{2}\right]^2\right)} -$$

$$\left(8 d^2 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \left(\frac{1}{4} e^{i \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{e}{2}\right]\right]} f^2 x^2 + 1 / \left(\sqrt{1 + \operatorname{Tan}\left[\frac{e}{2}\right]^2}\right) \left(\frac{1}{2} i f x \left(-\pi + 2 \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{e}{2}\right]\right]\right) - \pi \operatorname{Log}\left[1 + e^{-i f x}\right] -\right.\right.\right.$$

$$\left.\left.2 \left(\frac{f x}{2} + \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{e}{2}\right]\right]\right) \operatorname{Log}\left[1 - e^{2 i \left(\frac{f x}{2} + \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{e}{2}\right]\right)}\right] + \pi \operatorname{Log}\left[\operatorname{Cos}\left[\frac{f x}{2}\right]\right] + 2 \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{e}{2}\right]\right] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{f x}{2} + \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{e}{2}\right]\right]\right]\right] +\right.\right.$$

$$\left.\left. i \operatorname{PolyLog}\left[2, e^{2 i \left(\frac{f x}{2} + \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{e}{2}\right]\right)}\right]\right] \operatorname{Tan}\left[\frac{e}{2}\right]\right) / \left(f^3 (a - a \operatorname{Cos}[e + f x]) \sqrt{\operatorname{Sec}\left[\frac{e}{2}\right]^2 \left(\operatorname{Cos}\left[\frac{e}{2}\right]^2 + \operatorname{Sin}\left[\frac{e}{2}\right]^2\right)}\right)$$

■ **Problem 172: Result more than twice size of optimal antiderivative.**

$$\int \frac{x}{\sqrt{a + a \operatorname{Cos}[c + d x]}} dx$$

Optimal (type 4, 156 leaves, 6 steps):

$$-\frac{4 i x \operatorname{ArcTan}\left[e^{\frac{1}{2} i (c+d x)}\right] \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]}{d \sqrt{a + a \operatorname{Cos}[c + d x]}} + \frac{4 i \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] \operatorname{PolyLog}\left[2, -i e^{\frac{1}{2} i (c+d x)}\right]}{d^2 \sqrt{a + a \operatorname{Cos}[c + d x]}} - \frac{4 i \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] \operatorname{PolyLog}\left[2, i e^{\frac{1}{2} i (c+d x)}\right]}{d^2 \sqrt{a + a \operatorname{Cos}[c + d x]}}$$

Result (type 4, 333 leaves):

$$-\frac{1}{d^2 \sqrt{a (1 + \operatorname{Cos}[c + d x])}} 2 \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] \left(d x \operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]\right] + 2 i \operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]\right] \operatorname{Log}\left[\left(\frac{1}{2} + \frac{i}{2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]\right)\right] -\right.$$

$$2 i \operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]\right] \operatorname{Log}\left[\left(\frac{1}{2} - \frac{i}{2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]\right)\right] - d x \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]\right] -$$

$$2 i \operatorname{Log}\left[\frac{1}{2} \left((1 + i) - (1 - i) \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]\right)\right] \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]\right] + 2 i \operatorname{Log}\left[\left(-\frac{1}{2} - \frac{i}{2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]\right)\right] \left.\right]$$

$$\operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]\right] + 2 i \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} - \frac{i}{2}\right) \left(-1 + \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]\right)\right] - 2 i \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} + \frac{i}{2}\right) \left(-1 + \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]\right)\right] -$$

$$2 i \operatorname{PolyLog}\left[2, \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]\right)\right] + 2 i \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]\right)\right] \left.\right]$$

■ **Problem 180: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^3}{(a + a \operatorname{Cos}[x])^{3/2}} dx$$

Optimal (type 4, 423 leaves, 16 steps):

$$\begin{aligned}
& - \frac{3 x^2}{a \sqrt{a + a \cos[x]}} - \frac{24 i x \operatorname{ArcTan}\left[e^{\frac{i x}{2}}\right] \cos\left[\frac{x}{2}\right]}{a \sqrt{a + a \cos[x]}} - \frac{i x^3 \operatorname{ArcTan}\left[e^{\frac{i x}{2}}\right] \cos\left[\frac{x}{2}\right]}{a \sqrt{a + a \cos[x]}} + \frac{24 i \cos\left[\frac{x}{2}\right] \operatorname{PolyLog}\left[2, -i e^{\frac{i x}{2}}\right]}{a \sqrt{a + a \cos[x]}} + \\
& \frac{3 i x^2 \cos\left[\frac{x}{2}\right] \operatorname{PolyLog}\left[2, -i e^{\frac{i x}{2}}\right]}{a \sqrt{a + a \cos[x]}} - \frac{24 i \cos\left[\frac{x}{2}\right] \operatorname{PolyLog}\left[2, i e^{\frac{i x}{2}}\right]}{a \sqrt{a + a \cos[x]}} - \frac{3 i x^2 \cos\left[\frac{x}{2}\right] \operatorname{PolyLog}\left[2, i e^{\frac{i x}{2}}\right]}{a \sqrt{a + a \cos[x]}} - \frac{12 x \cos\left[\frac{x}{2}\right] \operatorname{PolyLog}\left[3, -i e^{\frac{i x}{2}}\right]}{a \sqrt{a + a \cos[x]}} + \\
& \frac{12 x \cos\left[\frac{x}{2}\right] \operatorname{PolyLog}\left[3, i e^{\frac{i x}{2}}\right]}{a \sqrt{a + a \cos[x]}} - \frac{24 i \cos\left[\frac{x}{2}\right] \operatorname{PolyLog}\left[4, -i e^{\frac{i x}{2}}\right]}{a \sqrt{a + a \cos[x]}} + \frac{24 i \cos\left[\frac{x}{2}\right] \operatorname{PolyLog}\left[4, i e^{\frac{i x}{2}}\right]}{a \sqrt{a + a \cos[x]}} + \frac{x^3 \tan\left[\frac{x}{2}\right]}{2 a \sqrt{a + a \cos[x]}}
\end{aligned}$$

Result (type 4, 1391 leaves):

$$\begin{aligned}
& - \frac{6 x^2 \operatorname{Cos}\left[\frac{x}{2}\right]^3}{(a(1+\operatorname{Cos}[x]))^{3/2}} + \frac{48 \operatorname{Cos}\left[\frac{x}{2}\right]^3 \left(\frac{1}{2} x \left(\operatorname{Log}\left[1-i e^{\frac{i x}{2}}\right]-\operatorname{Log}\left[1+i e^{\frac{i x}{2}}\right]\right)+i \left(\operatorname{PolyLog}\left[2,-i e^{\frac{i x}{2}}\right]-\operatorname{PolyLog}\left[2,i e^{\frac{i x}{2}}\right]\right)\right)}{(a(1+\operatorname{Cos}[x]))^{3/2}} + \frac{1}{(a(1+\operatorname{Cos}[x]))^{3/2}} \\
& 8 \operatorname{Cos}\left[\frac{x}{2}\right]^3 \left(\frac{1}{8} \pi^3 \operatorname{Log}\left[\operatorname{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2}-\frac{x}{2}\right)\right]\right]+\frac{3}{4} \pi^2 \left(\left(\frac{\pi}{2}-\frac{x}{2}\right)\left(\operatorname{Log}\left[1-e^{i\left(\frac{\pi}{2}-\frac{x}{2}\right)}\right]-\operatorname{Log}\left[1+e^{i\left(\frac{\pi}{2}-\frac{x}{2}\right)}\right]\right)+i \left(\operatorname{PolyLog}\left[2,-e^{i\left(\frac{\pi}{2}-\frac{x}{2}\right)}\right]-\operatorname{PolyLog}\left[2,e^{i\left(\frac{\pi}{2}-\frac{x}{2}\right)}\right]\right)\right) - \\
& \frac{3}{2} \pi \left(\left(\frac{\pi}{2}-\frac{x}{2}\right)^2 \left(\operatorname{Log}\left[1-e^{i\left(\frac{\pi}{2}-\frac{x}{2}\right)}\right]-\operatorname{Log}\left[1+e^{i\left(\frac{\pi}{2}-\frac{x}{2}\right)}\right]\right)+2 i \left(\frac{\pi}{2}-\frac{x}{2}\right) \left(\operatorname{PolyLog}\left[2,-e^{i\left(\frac{\pi}{2}-\frac{x}{2}\right)}\right]-\operatorname{PolyLog}\left[2,e^{i\left(\frac{\pi}{2}-\frac{x}{2}\right)}\right]\right)+\right. \\
& \left.2\left(-\operatorname{PolyLog}\left[3,-e^{i\left(\frac{\pi}{2}-\frac{x}{2}\right)}\right]+\operatorname{PolyLog}\left[3,e^{i\left(\frac{\pi}{2}-\frac{x}{2}\right)}\right]\right)\right)+8\left(\frac{1}{4} i \left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)^4+\frac{1}{64} i \left(\frac{\pi}{2}-\frac{x}{2}\right)^4-\right. \\
& \left.\frac{1}{8} \pi^3 \left(i \left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)-\operatorname{Log}\left[1+e^{2 i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)}\right]\right)-\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)^3 \operatorname{Log}\left[1+e^{2 i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)}\right]-\frac{1}{8}\left(\frac{\pi}{2}-\frac{x}{2}\right)^3 \operatorname{Log}\left[1+e^{i\left(\frac{\pi}{2}-\frac{x}{2}\right)}\right]+\right. \\
& \left.\frac{3}{4} \pi^2 \left(\frac{1}{2} i \left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)^2-\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right) \operatorname{Log}\left[1+e^{2 i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)}\right]+\frac{1}{2} i \operatorname{PolyLog}\left[2,-e^{2 i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)}\right]\right)+ \\
& \frac{3}{2} i \left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)^2 \operatorname{PolyLog}\left[2,-e^{2 i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)}\right]+\frac{3}{8} i \left(\frac{\pi}{2}-\frac{x}{2}\right)^2 \operatorname{PolyLog}\left[2,-e^{i\left(\frac{\pi}{2}-\frac{x}{2}\right)}\right]- \\
& \frac{3}{2} \pi \left(\frac{1}{3} i \left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)^3-\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)^2 \operatorname{Log}\left[1+e^{2 i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)}\right]+i \left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right) \operatorname{PolyLog}\left[2,-e^{2 i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)}\right]-\right. \\
& \left.\frac{1}{2} \operatorname{PolyLog}\left[3,-e^{2 i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)}\right]\right)-\frac{3}{2}\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right) \operatorname{PolyLog}\left[3,-e^{2 i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)}\right]- \\
& \left.\frac{3}{4}\left(\frac{\pi}{2}-\frac{x}{2}\right) \operatorname{PolyLog}\left[3,-e^{i\left(\frac{\pi}{2}-\frac{x}{2}\right)}\right]-\frac{3}{4} i \operatorname{PolyLog}\left[4,-e^{2 i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)}\right]-\frac{3}{4} i \operatorname{PolyLog}\left[4,-e^{i\left(\frac{\pi}{2}-\frac{x}{2}\right)}\right]\right)\right)+ \\
& \frac{x^3 \operatorname{Cos}\left[\frac{x}{2}\right]^3}{2(a(1+\operatorname{Cos}[x]))^{3/2}(\operatorname{Cos}\left[\frac{x}{4}\right]-\operatorname{Sin}\left[\frac{x}{4}\right])^2}-\frac{6 x^2 \operatorname{Cos}\left[\frac{x}{2}\right]^3 \operatorname{Sin}\left[\frac{x}{4}\right]}{(a(1+\operatorname{Cos}[x]))^{3/2}(\operatorname{Cos}\left[\frac{x}{4}\right]-\operatorname{Sin}\left[\frac{x}{4}\right])} - \\
& \frac{x^3 \operatorname{Cos}\left[\frac{x}{2}\right]^3}{2(a(1+\operatorname{Cos}[x]))^{3/2}(\operatorname{Cos}\left[\frac{x}{4}\right]+\operatorname{Sin}\left[\frac{x}{4}\right])^2} + \\
& \frac{6 x^2 \operatorname{Cos}\left[\frac{x}{2}\right]^3 \operatorname{Sin}\left[\frac{x}{4}\right]}{(a(1+\operatorname{Cos}[x]))^{3/2}(\operatorname{Cos}\left[\frac{x}{4}\right]+\operatorname{Sin}\left[\frac{x}{4}\right])}
\end{aligned}$$

■ **Problem 187: Result more than twice size of optimal antiderivative.**

$$\int \frac{x}{a+b \operatorname{Cos}[c+d x]} dx$$

Optimal (type 4, 214 leaves, 8 steps):

$$-\frac{i x \operatorname{Log}\left[1 + \frac{b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2} d} + \frac{i x \operatorname{Log}\left[1 + \frac{b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2} d} - \frac{\operatorname{PolyLog}\left[2, -\frac{b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2} d^2} + \frac{\operatorname{PolyLog}\left[2, -\frac{b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2} d^2}$$

Result (type 4, 756 leaves):

$$\begin{aligned} & \frac{1}{\sqrt{-a^2 + b^2} d^2} \left( 2 (c + dx) \operatorname{ArcTanh}\left[\frac{(a + b) \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{-a^2 + b^2}}\right] - 2 \left(c + \operatorname{ArcCos}\left[-\frac{a}{b}\right]\right) \operatorname{ArcTanh}\left[\frac{(-a + b) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{-a^2 + b^2}}\right] + \right. \\ & \left. \left( \operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(a + b) \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{-a^2 + b^2}}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-a + b) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{-a^2 + b^2}}\right] \right) \operatorname{Log}\left[\frac{\sqrt{-a^2 + b^2} e^{-\frac{1}{2} i (c + dx)}}{\sqrt{2} \sqrt{b} \sqrt{a + b \operatorname{Cos}[c + dx]}}\right] + \right. \\ & \left. \left( \operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2 i \left( \operatorname{ArcTanh}\left[\frac{(a + b) \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{-a^2 + b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-a + b) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{-a^2 + b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2 + b^2} e^{\frac{1}{2} i (c + dx)}}{\sqrt{2} \sqrt{b} \sqrt{a + b \operatorname{Cos}[c + dx]}}\right] - \right. \\ & \left. \left( \operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(-a + b) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{-a^2 + b^2}}\right] \right) \operatorname{Log}\left[\frac{(a + b) \left(-a + b - i \sqrt{-a^2 + b^2}\right) \left(1 + i \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right)}{b \left(a + b + \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right)}\right] - \right. \\ & \left. \left( \operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-a + b) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{-a^2 + b^2}}\right] \right) \operatorname{Log}\left[\frac{(a + b) \left(i a - i b + \sqrt{-a^2 + b^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right)}{b \left(a + b + \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right)}\right] + \right. \\ & \left. i \left( \operatorname{PolyLog}\left[2, \frac{\left(a - i \sqrt{-a^2 + b^2}\right) \left(a + b - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right)}{b \left(a + b + \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right)}\right] - \operatorname{PolyLog}\left[2, \frac{\left(a + i \sqrt{-a^2 + b^2}\right) \left(a + b - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right)}{b \left(a + b + \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right)}\right] \right) \right) \end{aligned}$$

## Test results for the 62 problems in "4.2.1.1 (a+b cos)^n.m"

- Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(3 + 5 \operatorname{Cos}[c + dx])^4} dx$$

Optimal (type 3, 140 leaves, 6 steps):

$$\frac{279 \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 279 \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{32768 d} + \frac{5 \operatorname{Sin}[c+dx]}{48 d (3+5 \operatorname{Cos}[c+dx])^3} - \frac{25 \operatorname{Sin}[c+dx]}{512 d (3+5 \operatorname{Cos}[c+dx])^2} + \frac{995 \operatorname{Sin}[c+dx]}{24576 d (3+5 \operatorname{Cos}[c+dx])}$$

Result (type 3, 296 leaves):

$$\frac{1}{393216 d (3+5 \operatorname{Cos}[c+dx])^3} \left( 467046 \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 104625 \operatorname{Cos}[3(c+dx)] \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 765855 \operatorname{Cos}[c+dx] \left( \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) + 376650 \operatorname{Cos}[2(c+dx)] \left( \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) - 467046 \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 104625 \operatorname{Cos}[3(c+dx)] \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 226140 \operatorname{Sin}[c+dx] + 190800 \operatorname{Sin}[2(c+dx)] + 99500 \operatorname{Sin}[3(c+dx)] \right)$$

■ **Problem 41: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(3-5 \operatorname{Cos}[c+dx])^4} dx$$

Optimal (type 3, 138 leaves, 6 steps):

$$-\frac{279 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{32768 d} + \frac{279 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{32768 d} - \frac{5 \operatorname{Sin}[c+dx]}{48 d (3-5 \operatorname{Cos}[c+dx])^3} + \frac{25 \operatorname{Sin}[c+dx]}{512 d (3-5 \operatorname{Cos}[c+dx])^2} - \frac{995 \operatorname{Sin}[c+dx]}{24576 d (3-5 \operatorname{Cos}[c+dx])}$$

Result (type 3, 288 leaves):

$$\frac{1}{393\,216\,d\,(-3+5\cos[c+dx])^3} \left( 467\,046 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - 2\sin\left[\frac{1}{2}(c+dx)\right]\right] - 104\,625 \cos[3(c+dx)] \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - 2\sin\left[\frac{1}{2}(c+dx)\right]\right] - 765\,855 \cos[c+dx] \left( \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - 2\sin\left[\frac{1}{2}(c+dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + 2\sin\left[\frac{1}{2}(c+dx)\right]\right] \right) + 376\,650 \cos[2(c+dx)] \left( \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - 2\sin\left[\frac{1}{2}(c+dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + 2\sin\left[\frac{1}{2}(c+dx)\right]\right] \right) - 467\,046 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + 2\sin\left[\frac{1}{2}(c+dx)\right]\right] + 104\,625 \cos[3(c+dx)] \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + 2\sin\left[\frac{1}{2}(c+dx)\right]\right] + 226\,140 \sin[c+dx] - 190\,800 \sin[2(c+dx)] + 99\,500 \sin[3(c+dx)] \right)$$

■ **Problem 45: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(-3+5\cos[c+dx])^4} dx$$

Optimal (type 3, 138 leaves, 6 steps):

$$\frac{279 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - 2\sin\left[\frac{1}{2}(c+dx)\right]\right]}{32\,768\,d} + \frac{279 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + 2\sin\left[\frac{1}{2}(c+dx)\right]\right]}{32\,768\,d} - \frac{5 \sin[c+dx]}{48\,d\,(3-5\cos[c+dx])^3} + \frac{25 \sin[c+dx]}{512\,d\,(3-5\cos[c+dx])^2} - \frac{995 \sin[c+dx]}{24\,576\,d\,(3-5\cos[c+dx])}$$

Result (type 3, 288 leaves):

$$\frac{1}{393\,216\,d\,(-3+5\cos[c+dx])^3} \left( 467\,046 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - 2\sin\left[\frac{1}{2}(c+dx)\right]\right] - 104\,625 \cos[3(c+dx)] \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - 2\sin\left[\frac{1}{2}(c+dx)\right]\right] - 765\,855 \cos[c+dx] \left( \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - 2\sin\left[\frac{1}{2}(c+dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + 2\sin\left[\frac{1}{2}(c+dx)\right]\right] \right) + 376\,650 \cos[2(c+dx)] \left( \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - 2\sin\left[\frac{1}{2}(c+dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + 2\sin\left[\frac{1}{2}(c+dx)\right]\right] \right) - 467\,046 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + 2\sin\left[\frac{1}{2}(c+dx)\right]\right] + 104\,625 \cos[3(c+dx)] \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + 2\sin\left[\frac{1}{2}(c+dx)\right]\right] + 226\,140 \sin[c+dx] - 190\,800 \sin[2(c+dx)] + 99\,500 \sin[3(c+dx)] \right)$$

■ **Problem 49: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(-3 - 5 \cos[c + dx])^4} dx$$

Optimal (type 3, 140 leaves, 6 steps):

$$\frac{279 \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - 279 \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{32768 d} - \frac{5 \sin[c+dx]}{48 d (3+5 \cos[c+dx])^3} - \frac{25 \sin[c+dx]}{512 d (3+5 \cos[c+dx])^2} + \frac{995 \sin[c+dx]}{24576 d (3+5 \cos[c+dx])}$$

Result (type 3, 296 leaves):

$$\frac{1}{393216 d (3+5 \cos[c+dx])^3} \left( 467046 \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + 104625 \cos[3(c+dx)] \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + 765855 \cos[c+dx] \left( \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) + 376650 \cos[2(c+dx)] \left( \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) - 467046 \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] - 104625 \cos[3(c+dx)] \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] + 226140 \sin[c+dx] + 190800 \sin[2(c+dx)] + 99500 \sin[3(c+dx)] \right)$$

■ **Problem 56: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + dx])^{4/3} dx$$

Optimal (type 6, 108 leaves, 3 steps):

$$\left( \sqrt{2} (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{4}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos[c+dx]), \frac{b(1 - \cos[c+dx])}{a+b}\right] (a+b \cos[c+dx])^{1/3} \sin[c+dx] \right) / \left( d \sqrt{1 + \cos[c+dx]} \left( \frac{a+b \cos[c+dx]}{a+b} \right)^{1/3} \right)$$

Result (type 6, 246 leaves):



$$-\frac{1}{16bd} 3 (a + b \cos[c + dx])^{1/3} \operatorname{Csc}[c + dx]$$

$$\left( 4 (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, \frac{a + b \cos[c + dx]}{a - b}, \frac{a + b \cos[c + dx]}{a + b}\right] \sqrt{-\frac{b(-1 + \cos[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \cos[c + dx])}{a - b}} + 5a \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, \frac{a + b \cos[c + dx]}{a - b}, \frac{a + b \cos[c + dx]}{a + b}\right] \sqrt{-\frac{b(-1 + \cos[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \cos[c + dx])}{a - b}} (a + b \cos[c + dx]) - 4b^2 \sin^2[c + dx] \right)$$

■ **Problem 61: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \cos[c + dx])^{4/3}} dx$$

Optimal (type 6, 110 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{4}{3}, \frac{3}{2}, \frac{1}{2} (1 - \cos[c + dx]), \frac{b(1 - \cos[c + dx])}{a + b}\right] \left(\frac{a + b \cos[c + dx]}{a + b}\right)^{1/3} \sin[c + dx]}{(a + b) d \sqrt{1 + \cos[c + dx]} (a + b \cos[c + dx])^{1/3}}$$

Result (type 6, 268 leaves):

$$\left( 15a \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, \frac{a + b \cos[c + dx]}{a - b}, \frac{a + b \cos[c + dx]}{a + b}\right] \sqrt{-\frac{b(-1 + \cos[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \cos[c + dx])}{a - b}} (a + b \cos[c + dx]) \operatorname{Csc}[c + dx] - 6 \left( 5b^2 + 2 \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, \frac{1}{2}, \frac{8}{3}, \frac{a + b \cos[c + dx]}{a - b}, \frac{a + b \cos[c + dx]}{a + b}\right] \sqrt{-\frac{b(-1 + \cos[c + dx])}{a + b}} \sqrt{\frac{b(1 + \cos[c + dx])}{-a + b}} (a + b \cos[c + dx])^2 \operatorname{Csc}[c + dx]^2 \right) \sin[c + dx] \right) / (10b(a^2 - b^2) d (a + b \cos[c + dx])^{1/3})$$

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## Test results for the 99 problems in "4.2.12 (e x)^m (a+b cos(c+d x^n))^p.m"

■ **Problem 3: Result more than twice size of optimal antiderivative.**

$$\int x \cos[a + bx^2] dx$$

Optimal (type 3, 15 leaves, 2 steps) :

$$\frac{\sin[a + b x^2]}{2 b}$$

Result (type 3, 31 leaves) :

$$\frac{\cos[b x^2] \sin[a]}{2 b} + \frac{\cos[a] \sin[b x^2]}{2 b}$$

■ **Problem 90: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^2 \cos[a + b \sqrt{c + d x}] dx$$

Optimal (type 3, 346 leaves, 14 steps) :

$$\begin{aligned} & \frac{240 \cos[a + b \sqrt{c + d x}]}{b^6 d^3} + \frac{24 c \cos[a + b \sqrt{c + d x}]}{b^4 d^3} + \frac{2 c^2 \cos[a + b \sqrt{c + d x}]}{b^2 d^3} - \frac{120 (c + d x) \cos[a + b \sqrt{c + d x}]}{b^4 d^3} - \\ & \frac{12 c (c + d x) \cos[a + b \sqrt{c + d x}]}{b^2 d^3} + \frac{10 (c + d x)^2 \cos[a + b \sqrt{c + d x}]}{b^2 d^3} + \frac{240 \sqrt{c + d x} \sin[a + b \sqrt{c + d x}]}{b^5 d^3} + \frac{24 c \sqrt{c + d x} \sin[a + b \sqrt{c + d x}]}{b^3 d^3} + \\ & \frac{2 c^2 \sqrt{c + d x} \sin[a + b \sqrt{c + d x}]}{b d^3} - \frac{40 (c + d x)^{3/2} \sin[a + b \sqrt{c + d x}]}{b^3 d^3} - \frac{4 c (c + d x)^{3/2} \sin[a + b \sqrt{c + d x}]}{b d^3} + \frac{2 (c + d x)^{5/2} \sin[a + b \sqrt{c + d x}]}{b d^3} \end{aligned}$$

Result (type 3, 224 leaves) :

$$\begin{aligned} & \frac{1}{b^6 d^3} e^{-i (a + b \sqrt{c + d x})} \left( 120 + 120 i b \sqrt{c + d x} + i b^5 d^2 x^2 \sqrt{c + d x} - 4 i b^3 \sqrt{c + d x} (2 c + 5 d x) - 12 b^2 (4 c + 5 d x) + b^4 d x (4 c + 5 d x) + \right. \\ & \left. e^{2 i (a + b \sqrt{c + d x})} \left( 120 - 120 i b \sqrt{c + d x} - i b^5 d^2 x^2 \sqrt{c + d x} + 4 i b^3 \sqrt{c + d x} (2 c + 5 d x) - 12 b^2 (4 c + 5 d x) + b^4 d x (4 c + 5 d x) \right) \right) \end{aligned}$$

■ **Problem 93: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[a + b \sqrt{c + d x}]}{x} dx$$

Optimal (type 4, 126 leaves, 8 steps) :

$$\begin{aligned} & \cos[a - b \sqrt{c}] \operatorname{CosIntegral}[b (\sqrt{c} + \sqrt{c + d x})] + \cos[a + b \sqrt{c}] \operatorname{CosIntegral}[b \sqrt{c} - b \sqrt{c + d x}] - \\ & \sin[a - b \sqrt{c}] \operatorname{SinIntegral}[b (\sqrt{c} + \sqrt{c + d x})] + \sin[a + b \sqrt{c}] \operatorname{SinIntegral}[b \sqrt{c} - b \sqrt{c + d x}] \end{aligned}$$

Result (type 4, 145 leaves) :

$$\begin{aligned} & \frac{1}{2} e^{-i (a + b \sqrt{c})} \left( \operatorname{ExpIntegralEi}[-i b (-\sqrt{c} + \sqrt{c + d x})] + e^{2 i (a + b \sqrt{c})} \operatorname{ExpIntegralEi}[i b (-\sqrt{c} + \sqrt{c + d x})] \right) + \\ & e^{2 i b \sqrt{c}} \operatorname{ExpIntegralEi}[-i b (\sqrt{c} + \sqrt{c + d x})] + e^{2 i a} \operatorname{ExpIntegralEi}[i b (\sqrt{c} + \sqrt{c + d x})] \end{aligned}$$

■ **Problem 94: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Cos}\left[a + b\sqrt{c + dx}\right]}{x^2} dx$$

Optimal (type 4, 184 leaves, 10 steps):

$$\begin{aligned} & -\frac{\text{Cos}\left[a + b\sqrt{c + dx}\right]}{x} + \frac{bd \text{CosIntegral}\left[b\left(\sqrt{c} + \sqrt{c + dx}\right)\right] \text{Sin}\left[a - b\sqrt{c}\right]}{2\sqrt{c}} - \frac{bd \text{CosIntegral}\left[b\sqrt{c} - b\sqrt{c + dx}\right] \text{Sin}\left[a + b\sqrt{c}\right]}{2\sqrt{c}} + \\ & \frac{bd \text{Cos}\left[a - b\sqrt{c}\right] \text{SinIntegral}\left[b\left(\sqrt{c} + \sqrt{c + dx}\right)\right]}{2\sqrt{c}} + \frac{bd \text{Cos}\left[a + b\sqrt{c}\right] \text{SinIntegral}\left[b\sqrt{c} - b\sqrt{c + dx}\right]}{2\sqrt{c}} \end{aligned}$$

Result (type 4, 240 leaves):

$$\begin{aligned} & \frac{1}{4\sqrt{c}x} i \left( e^{-ia} \left( 2i\sqrt{c} e^{-ib\sqrt{c+dx}} - bd e^{-ib\sqrt{c}} x \text{ExpIntegralEi}\left[-ib\left(-\sqrt{c} + \sqrt{c+dx}\right)\right] \right) + bde^{ib\sqrt{c}} x \text{ExpIntegralEi}\left[-ib\left(\sqrt{c} + \sqrt{c+dx}\right)\right] \right) + \\ & e^{i(a-b\sqrt{c})} \left( 2i\sqrt{c} e^{ib\left(\sqrt{c} + \sqrt{c+dx}\right)} + bde^{2ib\sqrt{c}} x \text{ExpIntegralEi}\left[ib\left(-\sqrt{c} + \sqrt{c+dx}\right)\right] - bdx \text{ExpIntegralEi}\left[ib\left(\sqrt{c} + \sqrt{c+dx}\right)\right] \right) \end{aligned}$$

■ **Problem 95: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^2 \text{Cos}\left[a + b(c + dx)^{1/3}\right] dx$$

Optimal (type 3, 537 leaves, 20 steps):

$$\begin{aligned} & -\frac{720c \text{Cos}\left[a + b(c + dx)^{1/3}\right]}{b^6 d^3} - \frac{120960(c + dx)^{1/3} \text{Cos}\left[a + b(c + dx)^{1/3}\right]}{b^8 d^3} + \frac{6c^2(c + dx)^{1/3} \text{Cos}\left[a + b(c + dx)^{1/3}\right]}{b^2 d^3} + \\ & \frac{360c(c + dx)^{2/3} \text{Cos}\left[a + b(c + dx)^{1/3}\right]}{b^4 d^3} + \frac{20160(c + dx) \text{Cos}\left[a + b(c + dx)^{1/3}\right]}{b^6 d^3} - \frac{30c(c + dx)^{4/3} \text{Cos}\left[a + b(c + dx)^{1/3}\right]}{b^2 d^3} - \\ & \frac{1008(c + dx)^{5/3} \text{Cos}\left[a + b(c + dx)^{1/3}\right]}{b^4 d^3} + \frac{24(c + dx)^{7/3} \text{Cos}\left[a + b(c + dx)^{1/3}\right]}{b^2 d^3} + \frac{120960 \text{Sin}\left[a + b(c + dx)^{1/3}\right]}{b^9 d^3} - \\ & \frac{6c^2 \text{Sin}\left[a + b(c + dx)^{1/3}\right]}{b^3 d^3} - \frac{720c(c + dx)^{1/3} \text{Sin}\left[a + b(c + dx)^{1/3}\right]}{b^5 d^3} - \frac{60480(c + dx)^{2/3} \text{Sin}\left[a + b(c + dx)^{1/3}\right]}{b^7 d^3} + \\ & \frac{3c^2(c + dx)^{2/3} \text{Sin}\left[a + b(c + dx)^{1/3}\right]}{bd^3} + \frac{120c(c + dx) \text{Sin}\left[a + b(c + dx)^{1/3}\right]}{b^3 d^3} + \frac{5040(c + dx)^{4/3} \text{Sin}\left[a + b(c + dx)^{1/3}\right]}{b^5 d^3} - \\ & \frac{6c(c + dx)^{5/3} \text{Sin}\left[a + b(c + dx)^{1/3}\right]}{bd^3} - \frac{168(c + dx)^2 \text{Sin}\left[a + b(c + dx)^{1/3}\right]}{b^3 d^3} + \frac{3(c + dx)^{8/3} \text{Sin}\left[a + b(c + dx)^{1/3}\right]}{bd^3} \end{aligned}$$

Result (type 3, 382 leaves):

$$\frac{1}{2 b^9 d^3} \left( 3 e^{-i (a+b (c+dx)^{1/3})} \left( -40 320 i \left( -1 + e^{2 i (a+b (c+dx)^{1/3})} \right) - 40 320 b \left( 1 + e^{2 i (a+b (c+dx)^{1/3})} \right) (c+dx)^{1/3} + 20 160 i b^2 \left( -1 + e^{2 i (a+b (c+dx)^{1/3})} \right) (c+dx)^{2/3} - \right. \right. \\ \left. i b^8 d^2 \left( -1 + e^{2 i (a+b (c+dx)^{1/3})} \right) x^2 (c+dx)^{2/3} + 2 b^7 d \left( 1 + e^{2 i (a+b (c+dx)^{1/3})} \right) x (c+dx)^{1/3} (3 c + 4 d x) - \right. \\ \left. 240 i b^4 \left( -1 + e^{2 i (a+b (c+dx)^{1/3})} \right) (c+dx)^{1/3} (6 c + 7 d x) - 24 b^5 \left( 1 + e^{2 i (a+b (c+dx)^{1/3})} \right) (c+dx)^{2/3} (9 c + 14 d x) + \right. \\ \left. 240 b^3 \left( 1 + e^{2 i (a+b (c+dx)^{1/3})} \right) (27 c + 28 d x) + 2 i b^6 \left( -1 + e^{2 i (a+b (c+dx)^{1/3})} \right) (9 c^2 + 36 c d x + 28 d^2 x^2) \right)$$

■ **Problem 98: Result is not expressed in closed-form.**

$$\int \frac{\cos[a + b (c + d x)^{1/3}]}{x} dx$$

Optimal (type 4, 234 leaves, 11 steps):

$$\cos[a + b c^{1/3}] \operatorname{CosIntegral}[b c^{1/3} - b (c + d x)^{1/3}] + \cos[a + (-1)^{2/3} b c^{1/3}] \operatorname{CosIntegral}[(-1)^{2/3} b c^{1/3} - b (c + d x)^{1/3}] + \\ \cos[a - (-1)^{1/3} b c^{1/3}] \operatorname{CosIntegral}[(-1)^{1/3} b c^{1/3} + b (c + d x)^{1/3}] + \sin[a + b c^{1/3}] \operatorname{SinIntegral}[b c^{1/3} - b (c + d x)^{1/3}] + \\ \sin[a + (-1)^{2/3} b c^{1/3}] \operatorname{SinIntegral}[(-1)^{2/3} b c^{1/3} - b (c + d x)^{1/3}] - \sin[a - (-1)^{1/3} b c^{1/3}] \operatorname{SinIntegral}[(-1)^{1/3} b c^{1/3} + b (c + d x)^{1/3}]$$

Result (type 7, 243 leaves):

$$\frac{1}{2} \left( \operatorname{RootSum}[c - \#1^3 \&, \operatorname{Cos}[a + b \#1] \operatorname{CosIntegral}[b ((c + d x)^{1/3} - \#1)] - i \operatorname{CosIntegral}[b ((c + d x)^{1/3} - \#1)] \operatorname{Sin}[a + b \#1] - \right. \\ \left. i \operatorname{Cos}[a + b \#1] \operatorname{SinIntegral}[b ((c + d x)^{1/3} - \#1)] - \operatorname{Sin}[a + b \#1] \operatorname{SinIntegral}[b ((c + d x)^{1/3} - \#1)] \& \right] + \\ \operatorname{RootSum}[c - \#1^3 \&, \operatorname{Cos}[a + b \#1] \operatorname{CosIntegral}[b ((c + d x)^{1/3} - \#1)] + i \operatorname{CosIntegral}[b ((c + d x)^{1/3} - \#1)] \operatorname{Sin}[a + b \#1] + \\ \left. i \operatorname{Cos}[a + b \#1] \operatorname{SinIntegral}[b ((c + d x)^{1/3} - \#1)] - \operatorname{Sin}[a + b \#1] \operatorname{SinIntegral}[b ((c + d x)^{1/3} - \#1)] \& \right])$$

■ **Problem 99: Result is not expressed in closed-form.**

$$\int \frac{\cos[a + b (c + d x)^{1/3}]}{x^2} dx$$

Optimal (type 4, 332 leaves, 13 steps):

$$-\frac{\cos[a + b (c + d x)^{1/3}]}{x} - \frac{b d \operatorname{CosIntegral}[b c^{1/3} - b (c + d x)^{1/3}] \operatorname{Sin}[a + b c^{1/3}]}{3 c^{2/3}} + \\ \frac{(-1)^{1/3} b d \operatorname{CosIntegral}[(-1)^{1/3} b c^{1/3} + b (c + d x)^{1/3}] \operatorname{Sin}[a - (-1)^{1/3} b c^{1/3}]}{3 c^{2/3}} - \\ \frac{(-1)^{2/3} b d \operatorname{CosIntegral}[(-1)^{2/3} b c^{1/3} - b (c + d x)^{1/3}] \operatorname{Sin}[a + (-1)^{2/3} b c^{1/3}]}{3 c^{2/3}} + \\ \frac{b d \operatorname{Cos}[a + b c^{1/3}] \operatorname{SinIntegral}[b c^{1/3} - b (c + d x)^{1/3}]}{3 c^{2/3}} + \frac{(-1)^{2/3} b d \operatorname{Cos}[a + (-1)^{2/3} b c^{1/3}] \operatorname{SinIntegral}[(-1)^{2/3} b c^{1/3} - b (c + d x)^{1/3}]}{3 c^{2/3}} + \\ \frac{(-1)^{1/3} b d \operatorname{Cos}[a - (-1)^{1/3} b c^{1/3}] \operatorname{SinIntegral}[(-1)^{1/3} b c^{1/3} + b (c + d x)^{1/3}]}{3 c^{2/3}}$$

Result (type 7, 138 leaves) :

$$-\frac{\cos[a + b(c + dx)^{1/3}]}{x} - \frac{1}{6} i b d \operatorname{RootSum}\left[c - \#1^3 \&, \frac{e^{-i a - i b \#1} \operatorname{ExpIntegralEi}\left[-i b \left((c + dx)^{1/3} - \#1\right)\right]}{\#1^2}\right] +$$

$$\frac{1}{6} i b d \operatorname{RootSum}\left[c - \#1^3 \&, \frac{e^{i a + i b \#1} \operatorname{ExpIntegralEi}\left[i b \left((c + dx)^{1/3} - \#1\right)\right]}{\#1^2}\right]$$

## Test results for the 88 problems in "4.2.1.2 (g sin)^p (a+b cos)^m.m"

- Problem 58: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(e \sin[c + dx])^{11/2}}{a + b \cos[c + dx]} dx$$

Optimal (type 4, 544 leaves, 15 steps) :

$$\frac{(-a^2 + b^2)^{9/4} e^{11/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right]}{b^{11/2} d} + \frac{(-a^2 + b^2)^{9/4} e^{11/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right]}{b^{11/2} d} +$$

$$\frac{2 a (21 a^4 - 49 a^2 b^2 + 33 b^4) e^6 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c + dx]}}{21 b^6 d \sqrt{e \sin[c + dx]}} -$$

$$\frac{a (a^2 - b^2)^3 e^6 \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c + dx]}}{b^6 \left(a^2 - b \left(b - \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \sin[c + dx]}} -$$

$$\frac{a (a^2 - b^2)^3 e^6 \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c + dx]}}{b^6 \left(a^2 - b \left(b + \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \sin[c + dx]}} - \frac{2 e^5 (21 (a^2 - b^2)^2 - a b (7 a^2 - 12 b^2) \cos[c + dx]) \sqrt{e \sin[c + dx]}}{21 b^5 d} +$$

$$\frac{2 e^3 (7 (a^2 - b^2) - 5 a b \cos[c + dx]) (e \sin[c + dx])^{5/2}}{35 b^3 d} - \frac{2 e (e \sin[c + dx])^{9/2}}{9 b d}$$

Result (type 6, 2235 leaves) :

$$\frac{1}{d} \left( \frac{a (28 a^2 - 51 b^2) \cos[c + dx]}{42 b^4} + \frac{(-9 a^2 + 14 b^2) \cos[2(c + dx)]}{45 b^3} + \frac{a \cos[3(c + dx)]}{14 b^2} - \frac{\cos[4(c + dx)]}{36 b} \right) \operatorname{Csc}[c + dx]^5 (e \sin[c + dx])^{11/2} -$$

$$\frac{1}{1680 b^4 d \sin[c + dx]^{11/2}}$$

$$\begin{aligned}
& (e \sin[c + dx])^{11/2} \left( \frac{1}{(a + b \cos[c + dx]) (1 - \sin[c + dx]^2)} 2 (392 a^3 b - 722 a b^3) \cos[c + dx]^2 (a + b \sqrt{1 - \sin[c + dx]^2}) \right. \\
& \left( \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left( -2 \operatorname{ArcTan} \left[ 1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + 2 \operatorname{ArcTan} \left[ 1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[ \sqrt{a^2 - b^2} - \right. \right. \\
& \left. \left. \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx] \right] + \operatorname{Log} \left[ \sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx] \right] \right) + \\
& \left( 5 b (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\sin[c + dx]} \sqrt{1 - \sin[c + dx]^2} \right) / \left( \left( -5 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \Bigg) + \\
& \frac{1}{(a + b \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} 2 (-280 a^4 + 636 a^2 b^2 - 721 b^4) \cos[c + dx] (a + b \sqrt{1 - \sin[c + dx]^2}) \left( -\frac{1}{(-a^2 + b^2)^{3/4}} \right. \\
& \left( \frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left( 2 \operatorname{ArcTan} \left[ 1 - \frac{(1 + i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[ 1 + \frac{(1 + i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] + \operatorname{Log} \left[ \sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} \right. \right. \\
& \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] - \operatorname{Log} \left[ \sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] \right) \Bigg) + \\
& \left( 5 a (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\sin[c + dx]} \right) / \left( \sqrt{1 - \sin[c + dx]^2} \left( 5 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \Bigg) + \\
& \frac{1}{(a + b \cos[c + dx]) (1 - 2 \sin[c + dx]^2) \sqrt{1 - \sin[c + dx]^2}} (840 a^4 - 1764 a^2 b^2 + 959 b^4) \cos[c + dx] \cos[2(c + dx)]
\end{aligned}$$

$$\begin{aligned}
& \left( a + b \sqrt{1 - \sin[c + dx]^2} \right) \left( \frac{\left( \frac{1}{2} - \frac{i}{2} \right) (-2a^2 + b^2) \operatorname{ArcTan} \left[ 1 - \frac{(1+i) \sqrt{b} \sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}} \right]}{b^{3/2} (-a^2+b^2)^{3/4}} - \frac{\left( \frac{1}{2} - \frac{i}{2} \right) (-2a^2 + b^2) \operatorname{ArcTan} \left[ 1 + \frac{(1+i) \sqrt{b} \sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}} \right]}{b^{3/2} (-a^2+b^2)^{3/4}} \right) + \\
& \frac{1}{b^{3/2} (-a^2+b^2)^{3/4}} \left( \frac{1}{4} - \frac{i}{4} \right) (-2a^2 + b^2) \operatorname{Log} \left[ \sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin[c+dx]} + i b \sin[c+dx] \right] - \frac{1}{b^{3/2} (-a^2+b^2)^{3/4}} \\
& \left( \frac{1}{4} - \frac{i}{4} \right) (-2a^2 + b^2) \operatorname{Log} \left[ \sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin[c+dx]} + i b \sin[c+dx] \right] + \frac{4 \sqrt{\sin[c+dx]}}{b} + \\
& \left( 10 a (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \sqrt{\sin[c+dx]} \right) / \left( \sqrt{1 - \sin[c+dx]^2} \left( 5 (a^2 - b^2) \operatorname{AppellF1} \left[ \right. \right. \right. \\
& \left. \left. \left. \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] - 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \right) \sin[c+dx]^2 \right) (a^2 + b^2 (-1 + \sin[c+dx]^2)) \right) - \\
& \left( 36 a (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \sin[c+dx]^{5/2} \right) / \left( 5 \sqrt{1 - \sin[c+dx]^2} \left( 9 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] - 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \right) \sin[c+dx]^2 \right) (a^2 + b^2 (-1 + \sin[c+dx]^2)) \right) \right) \right)
\end{aligned}$$

■ **Problem 59: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin[c + dx])^{9/2}}{a + b \cos[c + dx]} dx$$

Optimal (type 4, 461 leaves, 14 steps):

$$\begin{aligned}
& - \frac{(-a^2 + b^2)^{7/4} e^{9/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{b^{9/2} d} + \frac{(-a^2 + b^2)^{7/4} e^{9/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{b^{9/2} d} + \\
& \frac{a (a^2 - b^2)^2 e^5 \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{b^5 \left(b - \sqrt{-a^2 + b^2}\right) d \sqrt{e \sin[c+dx]}} + \frac{a (a^2 - b^2)^2 e^5 \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{b^5 \left(b + \sqrt{-a^2 + b^2}\right) d \sqrt{e \sin[c+dx]}} - \\
& \frac{2 a (5 a^2 - 8 b^2) e^4 \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{e \sin[c+dx]}}{5 b^4 d \sqrt{\sin[c+dx]}} + \frac{2 e^3 (5 (a^2 - b^2) - 3 a b \cos[c+dx]) (e \sin[c+dx])^{3/2}}{15 b^3 d} - \frac{2 e (e \sin[c+dx])^{7/2}}{7 b d}
\end{aligned}$$

Result (type 6, 1228 leaves):



$$\begin{aligned}
& \frac{1}{5 b^3 d \operatorname{Sin}[c+d x]^{9/2}} \\
& (e \operatorname{Sin}[c+d x])^{9/2} \left( \frac{1}{(a+b \operatorname{Cos}[c+d x]) (1-\operatorname{Sin}[c+d x]^2)} 2 (5 a^3 - 8 a b^2) \operatorname{Cos}[c+d x]^2 (a+b \sqrt{1-\operatorname{Sin}[c+d x]^2}) \left( \frac{1}{4 \sqrt{2} b^{3/2} (a^2-b^2)^{1/4}} \right. \right. \\
& a \left( -2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c+d x]}}{(a^2-b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c+d x]}}{(a^2-b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{a^2-b^2} - \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \right. \right. \\
& \left. \left. \sqrt{\operatorname{Sin}[c+d x]} + b \operatorname{Sin}[c+d x]\right] - \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\operatorname{Sin}[c+d x]} + b \operatorname{Sin}[c+d x]\right] \right) + \\
& \left( 7 b (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2+b^2}\right] \operatorname{Sin}[c+d x]^{3/2} \sqrt{1-\operatorname{Sin}[c+d x]^2} \right) / \left( 3 \left( -7 (a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2+b^2}\right] + 2 \left( 2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2+b^2}\right] \right. \right. \right. \\
& \left. \left. \left. (a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2+b^2}\right] \right) \operatorname{Sin}[c+d x]^2 (a^2+b^2 (-1+\operatorname{Sin}[c+d x]^2)) \right) \right) \right) + \\
& \frac{1}{12 (a+b \operatorname{Cos}[c+d x]) \sqrt{1-\operatorname{Sin}[c+d x]^2}} (2 a^2 b - 5 b^3) \operatorname{Cos}[c+d x] (a+b \sqrt{1-\operatorname{Sin}[c+d x]^2}) \left( \frac{1}{\sqrt{b} (-a^2+b^2)^{1/4}} \right. \\
& (3+3 i) \left( 2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Sin}[c+d x]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Sin}[c+d x]}}{(-a^2+b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \right. \right. \\
& \left. \left. (-a^2+b^2)^{1/4} \sqrt{\operatorname{Sin}[c+d x]} + i b \operatorname{Sin}[c+d x]\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\operatorname{Sin}[c+d x]} + i b \operatorname{Sin}[c+d x]\right] \right) \right) + \\
& \left( 56 a (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2+b^2}\right] \operatorname{Sin}[c+d x]^{3/2} \right) / \left( \sqrt{1-\operatorname{Sin}[c+d x]^2} \left( 7 (a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2+b^2}\right] - 2 \left( 2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2+b^2}\right] \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2+b^2}\right] \right) \operatorname{Sin}[c+d x]^2 (a^2+b^2 (-1+\operatorname{Sin}[c+d x]^2)) \right) \right) \right) \right) + \\
& \frac{\operatorname{Csc}[c+d x]^4 (e \operatorname{Sin}[c+d x])^{9/2} \left( -\frac{(-28 a^2+37 b^2) \operatorname{Sin}[c+d x]}{42 b^3} - \frac{a \operatorname{Sin}[2(c+d x)]}{5 b^2} + \frac{\operatorname{Sin}[3(c+d x)]}{14 b} \right)}{d}
\end{aligned}$$

- **Problem 60: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin[c + dx])^{7/2}}{a + b \cos[c + dx]} dx$$

Optimal (type 4, 474 leaves, 14 steps):

$$\frac{(-a^2 + b^2)^{5/4} e^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{b^{7/2} d} + \frac{(-a^2 + b^2)^{5/4} e^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{b^{7/2} d} -$$

$$\frac{2 a (3 a^2 - 4 b^2) e^4 \operatorname{EllipticF}\left[\frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\sin[c + dx]}}{3 b^4 d \sqrt{e \sin[c + dx]}} + \frac{a (a^2 - b^2)^2 e^4 \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\sin[c + dx]}}{b^4 (a^2 - b (b - \sqrt{-a^2 + b^2})) d \sqrt{e \sin[c + dx]}} +$$

$$\frac{a (a^2 - b^2)^2 e^4 \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\sin[c + dx]}}{b^4 (a^2 - b (b + \sqrt{-a^2 + b^2})) d \sqrt{e \sin[c + dx]}} +$$

$$\frac{2 e^3 (3 (a^2 - b^2) - a b \cos[c + dx]) \sqrt{e \sin[c + dx]}}{3 b^3 d} - \frac{2 e (e \sin[c + dx])^{5/2}}{5 b d}$$

Result (type 6, 2155 leaves):

$$\frac{\left(-\frac{2 a \cos[c+dx]}{3 b^2} + \frac{\cos[2(c+dx)]}{5 b}\right) \operatorname{Csc}[c + dx]^3 (e \sin[c + dx])^{7/2}}{d} +$$

$$\frac{1}{60 b^2 d \sin[c + dx]^{7/2}} (e \sin[c + dx])^{7/2} \left( \frac{1}{(a + b \cos[c + dx]) (1 - \sin[c + dx])^2} 28 a b \cos[c + dx]^2 (a + b \sqrt{1 - \sin[c + dx]^2}) \right.$$

$$\left. \left( \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left( -2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2 - b^2} - \right. \right. \right.$$

$$\left. \left. \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx]\right] + \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx]\right] \right) +$$

$$\left( 5 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] \sqrt{\sin[c + dx]} \sqrt{1 - \sin[c + dx]^2} \right) / \left( \left( -5 (a^2 - b^2) \right. \right.$$

$$\left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] + 2 \left( 2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] \right) \right)$$

$$\begin{aligned}
& \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^2 \left( a^2 + b^2 (-1 + \sin[c + dx]^2) \right) \right) \right) \right) + \\
& \frac{1}{(a + b \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} 2 (-10 a^2 + 27 b^2) \cos[c + dx] \left( a + b \sqrt{1 - \sin[c + dx]^2} \right) \left( -\frac{1}{(-a^2 + b^2)^{3/4}} \right. \\
& \left. \left( \frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left( 2 \operatorname{ArcTan} \left[ 1 - \frac{(1+i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[ 1 + \frac{(1+i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] + \operatorname{Log} \left[ \sqrt{-a^2 + b^2} - (1+i) \sqrt{b} \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] - \operatorname{Log} \left[ \sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] \right) \right) \right) + \\
& \left( 5 a (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\sin[c + dx]} \right) / \left( \sqrt{1 - \sin[c + dx]^2} \left( 5 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \left( a^2 + b^2 (-1 + \sin[c + dx]^2) \right) \right) \right) \right) + \\
& \frac{1}{(a + b \cos[c + dx]) (1 - 2 \sin[c + dx]^2) \sqrt{1 - \sin[c + dx]^2}} (30 a^2 - 33 b^2) \cos[c + dx] \cos[2 (c + dx)] \left( a + b \sqrt{1 - \sin[c + dx]^2} \right) \\
& \left( \frac{\left( \frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[ 1 - \frac{(1+i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} - \frac{\left( \frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[ 1 + \frac{(1+i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} \right) + \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left( \frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[ \sqrt{-a^2 + b^2} - (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] - \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left( \frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[ \sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] + \\
& \frac{4 \sqrt{\sin[c + dx]}}{b} + \left( 10 a (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\sin[c + dx]} \right) / \\
& \left( \sqrt{1 - \sin[c + dx]^2} \left( 5 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \right) \right)
\end{aligned}$$



$$\begin{aligned}
& - \frac{2 \operatorname{Csc}[c + d x] (e \operatorname{Sin}[c + d x])^{5/2}}{3 b d} + \\
& \frac{1}{b d \operatorname{Sin}[c + d x]^{5/2}} (e \operatorname{Sin}[c + d x])^{5/2} \left( \frac{1}{(a + b \operatorname{Cos}[c + d x]) (1 - \operatorname{Sin}[c + d x])^2} 2 a \operatorname{Cos}[c + d x]^2 \left( a + b \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) \right. \\
& \left. \left( \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left( -2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{a^2 - b^2} - \right. \right. \right. \\
& \left. \left. \left. \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + b \operatorname{Sin}[c + d x]\right] - \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + b \operatorname{Sin}[c + d x]\right] \right) \right) + \\
& \left( 7 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] \operatorname{Sin}[c + d x]^{3/2} \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) / \left( 3 \left( -7 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] + 2 \left( 2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] \right) \operatorname{Sin}[c + d x]^2 \right) (a^2 + b^2 (-1 + \operatorname{Sin}[c + d x]^2)) \right) \right) + \\
& \frac{1}{12 (a + b \operatorname{Cos}[c + d x]) \sqrt{1 - \operatorname{Sin}[c + d x]^2}} b \operatorname{Cos}[c + d x] \left( a + b \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) \left( \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} \right. \\
& \left. (3 + 3 i) \left( 2 \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + i b \operatorname{Sin}[c + d x]\right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + i b \operatorname{Sin}[c + d x]\right] \right) \right) + \\
& \left( 56 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] \operatorname{Sin}[c + d x]^{3/2} \right) / \left( \sqrt{1 - \operatorname{Sin}[c + d x]^2} \left( 7 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] - 2 \left( 2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] \right) \operatorname{Sin}[c + d x]^2 \right) (a^2 + b^2 (-1 + \operatorname{Sin}[c + d x]^2)) \right) \right) \right)
\end{aligned}$$

■ **Problem 62: Result unnecessarily involves higher level functions.**

$$\int \frac{(e \operatorname{Sin}[c + d x])^{3/2}}{a + b \operatorname{Cos}[c + d x]} dx$$

Optimal (type 4, 410 leaves, 13 steps):

$$\frac{(-a^2 + b^2)^{1/4} e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{b^{3/2} d} + \frac{(-a^2 + b^2)^{1/4} e^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{b^{3/2} d} +$$

$$\frac{2 a e^2 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{b^2 d \sqrt{e \operatorname{Sin}[c+dx]}} - \frac{a (a^2 - b^2) e^2 \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{b^2 \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \operatorname{Sin}[c+dx]}} -$$

$$\frac{a (a^2 - b^2) e^2 \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{b^2 \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \operatorname{Sin}[c+dx]}} - \frac{2 e \sqrt{e \operatorname{Sin}[c+dx]}}{b d}$$

Result (type 6, 624 leaves):

$$\frac{1}{20 d (a + b \operatorname{Cos}[c + dx]) \operatorname{Sin}[c + dx]^{3/2} \sqrt{1 - \operatorname{Sin}[c + dx]^2}} \operatorname{Cos}[c + dx] (e \operatorname{Sin}[c + dx])^{3/2} \left(a + b \sqrt{1 - \operatorname{Sin}[c + dx]^2}\right)$$

$$\left(-\frac{1}{b^{3/2}} (5 - 5i) \left(2 (-a^2 + b^2)^{1/4} \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - 2 (-a^2 + b^2)^{1/4} \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right]\right) +$$

$$\left(-a^2 + b^2\right)^{1/4} \operatorname{Log}\left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c+dx]} + i b \operatorname{Sin}[c+dx]\right] -$$

$$\left(-a^2 + b^2\right)^{1/4} \operatorname{Log}\left[\sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c+dx]} + i b \operatorname{Sin}[c+dx]\right] + (4+4i) \sqrt{b} \sqrt{\operatorname{Sin}[c+dx]}\right)$$

$$\left(72 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sin}[c+dx]^{5/2}\right) / \left(\sqrt{1 - \operatorname{Sin}[c+dx]^2}\right)$$

$$\left(9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] +$$

$$\left(-a^2 + b^2\right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sin}[c+dx]^2 (a^2 + b^2 (-1 + \operatorname{Sin}[c+dx]^2))\right)\right)$$

■ **Problem 63: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{e \operatorname{Sin}[c+dx]}}{a + b \operatorname{Cos}[c+dx]} dx$$

Optimal (type 4, 302 leaves, 9 steps):

$$\begin{aligned}
& - \frac{\sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{\sqrt{b} (-a^2+b^2)^{1/4} d} + \frac{\sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{\sqrt{b} (-a^2+b^2)^{1/4} d} + \\
& \frac{a e \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{b \left(b - \sqrt{-a^2+b^2}\right) d \sqrt{e \sin[c+dx]}} + \frac{a e \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{b \left(b + \sqrt{-a^2+b^2}\right) d \sqrt{e \sin[c+dx]}}
\end{aligned}$$

Result (type 6, 556 leaves):

$$\begin{aligned}
& \frac{1}{12 d \sqrt{\cos[c+dx]^2} (a+b \cos[c+dx]) \sqrt{\sin[c+dx]}} \cos[c+dx] \left(a+b \sqrt{\cos[c+dx]^2}\right) \sqrt{e \sin[c+dx]} \\
& \left( \frac{1}{\sqrt{b} (-a^2+b^2)^{1/4}} (3+3i) \left( 2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - \right. \right. \right. \\
& \quad \left. \left. (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin[c+dx]} + i b \sin[c+dx]\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin[c+dx]} + i b \sin[c+dx]\right] \right) + \\
& \left( 56 a (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] \sin[c+dx]^{3/2} \right) / \left( \sqrt{\cos[c+dx]^2} (a^2-b^2+b^2 \sin[c+dx]^2) \right) \\
& \left( 7 (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] - 2 \left( 2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \\
& \quad \left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] \right) \sin[c+dx]^2 \right) \Big) \Big)
\end{aligned}$$

■ **Problem 64: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+b \cos[c+dx]) \sqrt{e \sin[c+dx]}} dx$$

Optimal (type 4, 307 leaves, 9 steps):

$$\begin{aligned}
& \frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{3/4} d \sqrt{e}} + \frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{3/4} d \sqrt{e}} + \\
& \frac{a \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{\left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \sin[c+dx]}} + \frac{a \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{\left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \sin[c+dx]}}
\end{aligned}$$

Result (type 6, 567 leaves):

$$\frac{1}{d (a + b \cos[c + dx]) \sqrt{e \sin[c + dx]} \sqrt{1 - \sin[c + dx]^2}} (a + b \sqrt{1 - \sin[c + dx]^2})$$

$$\left( -\frac{1}{(-a^2 + b^2)^{3/4}} \left( \frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left( 2 \operatorname{ArcTan} \left[ 1 - \frac{(1+i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[ 1 + \frac{(1+i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] + \operatorname{Log} \left[ \sqrt{-a^2 + b^2} - (1+i) \sqrt{b} \right. \right. \right.$$

$$\left. \left. (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] - \operatorname{Log} \left[ \sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] \right) +$$

$$\left( 5 a (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\sin[c + dx]} \right) / \left( \sqrt{1 - \sin[c + dx]^2} \right.$$

$$\left. \left( 5 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right.$$

$$\left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right)$$

■ **Problem 65: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \cos[c + dx]) (e \sin[c + dx])^{3/2}} dx$$

Optimal (type 4, 426 leaves, 13 steps):

$$-\frac{b^{3/2} \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{(-a^2 + b^2)^{5/4} d e^{3/2}} + \frac{b^{3/2} \operatorname{ArcTanh} \left[ \frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{(-a^2 + b^2)^{5/4} d e^{3/2}} +$$

$$\frac{2 (b - a \cos[c + dx])}{(a^2 - b^2) d e \sqrt{e \sin[c + dx]}} - \frac{a b \operatorname{EllipticPi} \left[ \frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2 \right] \sqrt{\sin[c + dx]}}{(a^2 - b^2) (b - \sqrt{-a^2 + b^2}) d e \sqrt{e \sin[c + dx]}} -$$

$$\frac{a b \operatorname{EllipticPi} \left[ \frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2 \right] \sqrt{\sin[c + dx]}}{(a^2 - b^2) (b + \sqrt{-a^2 + b^2}) d e \sqrt{e \sin[c + dx]}} - \frac{2 a \operatorname{EllipticE} \left[ \frac{1}{2} (c - \frac{\pi}{2} + dx), 2 \right] \sqrt{e \sin[c + dx]}}{(a^2 - b^2) d e^2 \sqrt{\sin[c + dx]}}$$

Result (type 6, 1186 leaves):



$$\begin{aligned}
& - \frac{2(-b + a \cos[c + dx]) \sin[c + dx]}{(a^2 - b^2) d (e \sin[c + dx])^{3/2}} - \\
& \frac{1}{(a-b)(a+b)d(e \sin[c + dx])^{3/2}} \sin[c + dx]^{3/2} \left( \frac{1}{(a+b \cos[c + dx])(1 - \sin[c + dx]^2)} 2ab \cos[c + dx]^2 (a + b \sqrt{1 - \sin[c + dx]^2}) \right. \\
& \left. \left( \frac{1}{4\sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left( -2 \operatorname{ArcTan} \left[ 1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + 2 \operatorname{ArcTan} \left[ 1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[ \sqrt{a^2 - b^2} - \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx] \right] - \operatorname{Log} \left[ \sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx] \right] \right) \right) + \\
& \left( 7b (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^{3/2} \sqrt{1 - \sin[c + dx]^2} \right) / \left( 3 \left( -7 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + 2 \left( 2b^2 \operatorname{AppellF1} \left[ \frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right) \right) + \\
& \frac{1}{12(a+b \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} (a^2 + b^2) \cos[c + dx] (a + b \sqrt{1 - \sin[c + dx]^2}) \left( \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} \right. \\
& \left. (3 + 3i) \left( 2 \operatorname{ArcTan} \left[ 1 - \frac{(1+i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[ 1 + \frac{(1+i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - \operatorname{Log} \left[ \sqrt{-a^2 + b^2} - (1+i) \sqrt{b} \right. \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] + \operatorname{Log} \left[ \sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] \right) \right) + \\
& \left( 56a (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^{3/2} \right) / \left( \sqrt{1 - \sin[c + dx]^2} \left( 7 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left( 2b^2 \operatorname{AppellF1} \left[ \frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 66: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \cos[c + dx]) (e \sin[c + dx])^{5/2}} dx$$

Optimal (type 4, 447 leaves, 13 steps):

$$\begin{aligned}
 & \frac{b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{7/4} d e^{5/2}} + \frac{b^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{7/4} d e^{5/2}} + \\
 & \frac{2(b-a \cos[c+dx])}{3(a^2-b^2) d e (e \sin[c+dx])^{3/2}} + \frac{2 a \operatorname{EllipticF}\left[\frac{1}{2}(c-\frac{\pi}{2}+dx), 2\right] \sqrt{\sin[c+dx]}}{3(a^2-b^2) d e^2 \sqrt{e \sin[c+dx]}} - \\
 & \frac{a b^2 \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(c-\frac{\pi}{2}+dx), 2\right] \sqrt{\sin[c+dx]}}{(a^2-b^2) \left(a^2-b \left(b-\sqrt{-a^2+b^2}\right)\right) d e^2 \sqrt{e \sin[c+dx]}} - \frac{a b^2 \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(c-\frac{\pi}{2}+dx), 2\right] \sqrt{\sin[c+dx]}}{(a^2-b^2) \left(a^2-b \left(b+\sqrt{-a^2+b^2}\right)\right) d e^2 \sqrt{e \sin[c+dx]}}
 \end{aligned}$$

Result (type 6, 1192 leaves):

$$\begin{aligned}
& - \frac{2(-b + a \cos[c + dx]) \sin[c + dx]}{3(a^2 - b^2)d(e \sin[c + dx])^{5/2}} + \\
& \frac{1}{3(a-b)(a+b)d(e \sin[c + dx])^{5/2}} \sin[c + dx]^{5/2} \left( \frac{1}{(a+b \cos[c + dx])(1 - \sin[c + dx]^2)} 2ab \cos[c + dx]^2 (a+b \sqrt{1 - \sin[c + dx]^2}) \right. \\
& \left. \left( \frac{1}{4\sqrt{2}\sqrt{b}(a^2 - b^2)^{3/4}} a \left( -2 \operatorname{ArcTan} \left[ 1 - \frac{\sqrt{2}\sqrt{b}\sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + 2 \operatorname{ArcTan} \left[ 1 + \frac{\sqrt{2}\sqrt{b}\sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[ \sqrt{a^2 - b^2} - \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{2}\sqrt{b}(a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx] \right] + \operatorname{Log} \left[ \sqrt{a^2 - b^2} + \sqrt{2}\sqrt{b}(a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx] \right] \right) \right) + \\
& \left( 5b(a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\sin[c + dx]} \sqrt{1 - \sin[c + dx]^2} \right) / \left( \left( -5(a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + 2 \left( 2b^2 \operatorname{AppellF1} \left[ \frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \right) (a^2 + b^2(-1 + \sin[c + dx]^2)) \right) \right) + \\
& \frac{1}{(a+b \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} 2(a^2 - 3b^2) \cos[c + dx] (a+b \sqrt{1 - \sin[c + dx]^2}) \left( -\frac{1}{(-a^2 + b^2)^{3/4}} \right. \\
& \left. \left( \frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left( 2 \operatorname{ArcTan} \left[ 1 - \frac{(1+i)\sqrt{b}\sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[ 1 + \frac{(1+i)\sqrt{b}\sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] + \operatorname{Log} \left[ \sqrt{-a^2 + b^2} - (1+i)\sqrt{b} \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + ib \sin[c + dx] \right] - \operatorname{Log} \left[ \sqrt{-a^2 + b^2} + (1+i)\sqrt{b}(-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + ib \sin[c + dx] \right] \right) \right) + \\
& \left( 5a(a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\sin[c + dx]} \right) / \left( \sqrt{1 - \sin[c + dx]^2} \left( 5(a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left( 2b^2 \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \right) (a^2 + b^2(-1 + \sin[c + dx]^2)) \right) \right) \right)
\end{aligned}$$

■ **Problem 67: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \cos[c + dx]) (e \sin[c + dx])^{7/2}} dx$$

Optimal (type 4, 501 leaves, 14 steps):

$$\begin{aligned}
 & - \frac{b^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{9/4} d e^{7/2}} + \frac{b^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{9/4} d e^{7/2}} + \frac{2(b-a \cos[c+dx])}{5(a^2-b^2) d e (e \sin[c+dx])^{5/2}} - \\
 & \frac{2(5b^3+a(3a^2-8b^2)\cos[c+dx])}{5(a^2-b^2)^2 d e^3 \sqrt{e \sin[c+dx]}} + \frac{a b^3 \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right] \sqrt{\sin[c+dx]}}{(a^2-b^2)^2 \left(b-\sqrt{-a^2+b^2}\right) d e^3 \sqrt{e \sin[c+dx]}} + \\
 & \frac{a b^3 \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right] \sqrt{\sin[c+dx]}}{(a^2-b^2)^2 \left(b+\sqrt{-a^2+b^2}\right) d e^3 \sqrt{e \sin[c+dx]}} - \frac{2a(3a^2-8b^2) \operatorname{EllipticE}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right] \sqrt{e \sin[c+dx]}}{5(a^2-b^2)^2 d e^4 \sqrt{\sin[c+dx]}}
 \end{aligned}$$

Result (type 6, 1275 leaves):

$$\begin{aligned}
& \left( -\frac{2(5b^3+3a^3\cos[c+dx]-8ab^2\cos[c+dx])\operatorname{Csc}[c+dx]}{5(a^2-b^2)^2} - \frac{2(-b+a\cos[c+dx])\operatorname{Csc}[c+dx]^3}{5(a^2-b^2)} \right) \operatorname{Sin}[c+dx]^4 \\
& \frac{1}{d(e\operatorname{Sin}[c+dx])^{7/2}} - \frac{1}{5(a-b)^2(a+b)^2d(e\operatorname{Sin}[c+dx])^{7/2}} \\
& \operatorname{Sin}[c+dx]^{7/2} \left( \frac{1}{(a+b\cos[c+dx])(1-\operatorname{Sin}[c+dx]^2)} 2(3a^3b-8ab^3)\cos[c+dx]^2(a+b\sqrt{1-\operatorname{Sin}[c+dx]^2}) \right. \\
& \left( \frac{1}{4\sqrt{2}b^{3/2}(a^2-b^2)^{1/4}} a \left( -2\operatorname{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{b}\sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4}}\right] + 2\operatorname{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{b}\sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{a^2-b^2} - \right. \right. \\
& \left. \left. \sqrt{2}\sqrt{b}(a^2-b^2)^{1/4}\sqrt{\operatorname{Sin}[c+dx]} + b\operatorname{Sin}[c+dx]\right] - \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2}\sqrt{b}(a^2-b^2)^{1/4}\sqrt{\operatorname{Sin}[c+dx]} + b\operatorname{Sin}[c+dx]\right] \right) + \\
& \left( 7b(a^2-b^2)\operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2\operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sin}[c+dx]^{3/2}\sqrt{1-\operatorname{Sin}[c+dx]^2} \right) / \left( 3\left(-7(a^2-b^2)\right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2\operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] + 2\left(2b^2\operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2\operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \\
& \left. \left. (a^2-b^2)\operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2\operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sin}[c+dx]^2\right)(a^2+b^2(-1+\operatorname{Sin}[c+dx]^2)) \right) \right) + \\
& \frac{1}{12(a+b\cos[c+dx])\sqrt{1-\operatorname{Sin}[c+dx]^2}} (3a^4-8a^2b^2-5b^4)\cos[c+dx](a+b\sqrt{1-\operatorname{Sin}[c+dx]^2}) \\
& \left( \frac{1}{\sqrt{b}(-a^2+b^2)^{1/4}} (3+3i) \left( 2\operatorname{ArcTan}\left[1 - \frac{(1+i)\sqrt{b}\sqrt{\operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - 2\operatorname{ArcTan}\left[1 + \frac{(1+i)\sqrt{b}\sqrt{\operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \right. \right. \\
& \left. \left. \sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\operatorname{Sin}[c+dx]} + ib\operatorname{Sin}[c+dx]\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\operatorname{Sin}[c+dx]} + ib\operatorname{Sin}[c+dx]\right] \right) + \\
& \left( 56a(a^2-b^2)\operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2\operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sin}[c+dx]^{3/2} \right) / \left( \sqrt{1-\operatorname{Sin}[c+dx]^2} \left( 7(a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2\operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] - 2\left(2b^2\operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2\operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \\
& \left. \left. (-a^2+b^2)\operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2\operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sin}[c+dx]^2\right)(a^2+b^2(-1+\operatorname{Sin}[c+dx]^2)) \right) \right) \right)
\end{aligned}$$

- **Problem 68: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin[c + dx])^{11/2}}{(a + b \cos[c + dx])^2} dx$$

Optimal (type 4, 557 leaves, 15 steps):

$$\frac{9 a (-a^2 + b^2)^{5/4} e^{11/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 b^{11/2} d} + \frac{9 a (-a^2 + b^2)^{5/4} e^{11/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 b^{11/2} d} -$$

$$\frac{3 (21 a^4 - 28 a^2 b^2 + 5 b^4) e^6 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{7 b^6 d \sqrt{e \sin[c+dx]}} +$$

$$\frac{9 a^2 (a^2 - b^2)^2 e^6 \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{2 b^6 \left(a^2 - b \left(b - \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \sin[c+dx]}} +$$

$$\frac{9 a^2 (a^2 - b^2)^2 e^6 \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{2 b^6 \left(a^2 - b \left(b + \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \sin[c+dx]}} +$$

$$\frac{3 e^5 (21 a (a^2 - b^2) - b (7 a^2 - 5 b^2) \cos[c+dx]) \sqrt{e \sin[c+dx]}}{7 b^5 d} - \frac{9 e^3 (7 a - 5 b \cos[c+dx]) (e \sin[c+dx])^{5/2}}{35 b^3 d} + \frac{e (e \sin[c+dx])^{9/2}}{b d (a + b \cos[c+dx])}$$

Result (type 6, 2229 leaves):

$$\frac{\left(\frac{(-28 a^2 + 17 b^2) \cos[c+dx]}{14 b^4} + \frac{(-a^2 + b^2)^2}{b^5 (a + b \cos[c+dx])} + \frac{2 a \cos[2(c+dx)]}{5 b^3} - \frac{\cos[3(c+dx)]}{14 b^2}\right) \operatorname{Csc}[c+dx]^5 (e \sin[c+dx])^{11/2}}{d} - \frac{1}{70 b^5 d \sin[c+dx]^{11/2}}$$

$$(e \sin[c+dx])^{11/2} \left( \frac{1}{(a + b \cos[c+dx]) (1 - \sin[c+dx])^2} 2 (35 a^4 - 126 a^2 b^2 + 75 b^4) \cos[c+dx]^2 \left(a + b \sqrt{1 - \sin[c+dx]}\right)^2 \right.$$

$$\left. \left( \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left( -2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2 - b^2} - \right. \right. \right.$$

$$\left. \left. \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right] + \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right] \right) +$$

$$\left( 5 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2}\right] \sqrt{\sin[c+dx]} \sqrt{1 - \sin[c+dx]^2} \right) / \left( \left( -5 (a^2 - b^2) \right. \right.$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2+b^2}\right] + 2 \left( 2 b^2 \text{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2+b^2}\right] + \right. \\
& \left. (a^2 - b^2) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2+b^2}\right] \text{Sin}[c+dx]^2 \right) (a^2 + b^2 (-1 + \text{Sin}[c+dx]^2)) \Bigg) \Bigg) + \\
& \frac{1}{(a + b \text{Cos}[c + dx]) \sqrt{1 - \text{Sin}[c + dx]^2}} 2 (70 a^3 b - 93 a b^3) \text{Cos}[c + dx] \left( a + b \sqrt{1 - \text{Sin}[c + dx]^2} \right) \left( -\frac{1}{(-a^2 + b^2)^{3/4}} \right. \\
& \left. \left( \frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left( 2 \text{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\text{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \text{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\text{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + \text{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \right. \right. \right. \\
& \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\text{Sin}[c+dx]} + i b \text{Sin}[c+dx] \right] - \text{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\text{Sin}[c+dx]} + i b \text{Sin}[c+dx] \right] \right) \Bigg) + \\
& \left( 5 a (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2+b^2}\right] \sqrt{\text{Sin}[c+dx]} \right) / \left( \sqrt{1 - \text{Sin}[c+dx]^2} \left( 5 (a^2 - b^2) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2+b^2}\right] - 2 \left( 2 b^2 \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
& \left. \left. (-a^2 + b^2) \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2+b^2}\right] \text{Sin}[c+dx]^2 \right) (a^2 + b^2 (-1 + \text{Sin}[c+dx]^2)) \right) \Bigg) \Bigg) + \\
& \frac{1}{(a + b \text{Cos}[c + dx]) (1 - 2 \text{Sin}[c + dx]^2) \sqrt{1 - \text{Sin}[c + dx]^2}} (-140 a^3 b + 147 a b^3) \text{Cos}[c + dx] \text{Cos}[2 (c + dx)] \\
& \left( a + b \sqrt{1 - \text{Sin}[c + dx]^2} \right) \\
& \left( \frac{\left( \frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \text{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\text{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right]}{b^{3/2} (-a^2 + b^2)^{3/4}} - \frac{\left( \frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \text{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\text{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right]}{b^{3/2} (-a^2 + b^2)^{3/4}} \right) + \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left( \frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \text{Log}\left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\text{Sin}[c+dx]} + i b \text{Sin}[c+dx] \right] - \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left( \frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \text{Log}\left[\sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\text{Sin}[c+dx]} + i b \text{Sin}[c+dx] \right] + \\
& \frac{4 \sqrt{\text{Sin}[c+dx]}}{b} + \left( 10 a (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2+b^2}\right] \sqrt{\text{Sin}[c+dx]} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{1 - \sin[c + dx]^2} \left( 5 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \\
& \quad \left. \left. \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \right) \\
& \quad \left. (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) - \left( 36 a (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^{5/2} \right) / \\
& \left( 5 \sqrt{1 - \sin[c + dx]^2} \left( 9 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - \right. \right. \\
& \quad \left. \left. 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \left. \right) \left. \right) \left. \right)
\end{aligned}$$

■ **Problem 69: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin[c + dx])^{9/2}}{(a + b \cos[c + dx])^2} dx$$

Optimal (type 4, 473 leaves, 14 steps):

$$\begin{aligned}
& \frac{7 a (-a^2 + b^2)^{3/4} e^{9/2} \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{2 b^{9/2} d} + \frac{7 a (-a^2 + b^2)^{3/4} e^{9/2} \operatorname{ArcTanh} \left[ \frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{2 b^{9/2} d} - \\
& \frac{7 a^2 (a^2 - b^2) e^5 \operatorname{EllipticPi} \left[ \frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2 \right] \sqrt{\sin[c + dx]}}{2 b^5 (b - \sqrt{-a^2 + b^2}) d \sqrt{e \sin[c + dx]}} - \\
& \frac{7 a^2 (a^2 - b^2) e^5 \operatorname{EllipticPi} \left[ \frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2 \right] \sqrt{\sin[c + dx]}}{2 b^5 (b + \sqrt{-a^2 + b^2}) d \sqrt{e \sin[c + dx]}} + \\
& \frac{7 (5 a^2 - 3 b^2) e^4 \operatorname{EllipticE} \left[ \frac{1}{2} (c - \frac{\pi}{2} + dx), 2 \right] \sqrt{e \sin[c + dx]}}{5 b^4 d \sqrt{\sin[c + dx]}} - \frac{7 e^3 (5 a - 3 b \cos[c + dx]) (e \sin[c + dx])^{3/2}}{15 b^3 d} + \frac{e (e \sin[c + dx])^{7/2}}{b d (a + b \cos[c + dx])}
\end{aligned}$$

Result (type 6, 1229 leaves):



1

 $10 b^3 d \operatorname{Sin}[c + d x]^{9/2}$ 

$$\begin{aligned}
& 7 (e \operatorname{Sin}[c + d x])^{9/2} \left( \frac{1}{(a + b \operatorname{Cos}[c + d x]) (1 - \operatorname{Sin}[c + d x]^2)} 2 (5 a^2 - 3 b^2) \operatorname{Cos}[c + d x]^2 (a + b \sqrt{1 - \operatorname{Sin}[c + d x]^2}) \left( \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} \right. \right. \\
& a \left[ -2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + \right. \\
& \left. \left. b \operatorname{Sin}[c + d x]\right] - \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + b \operatorname{Sin}[c + d x]\right] \right) + \\
& \left( 7 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] \operatorname{Sin}[c + d x]^{3/2} \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) / \left( 3 \left( -7 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] + 2 \left( 2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] \right) \operatorname{Sin}[c + d x]^2 (a^2 + b^2 (-1 + \operatorname{Sin}[c + d x]^2)) \right) \right) \right) + \\
& \frac{1}{6 (a + b \operatorname{Cos}[c + d x]) \sqrt{1 - \operatorname{Sin}[c + d x]^2}} a b \operatorname{Cos}[c + d x] (a + b \sqrt{1 - \operatorname{Sin}[c + d x]^2}) \left( \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} \right. \\
& (3 + 3 i) \left( 2 \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} \right. \right. \\
& \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + i b \operatorname{Sin}[c + d x]\right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + i b \operatorname{Sin}[c + d x]\right] \right) \right) + \\
& \left( 56 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] \operatorname{Sin}[c + d x]^{3/2} \right) / \left( \sqrt{1 - \operatorname{Sin}[c + d x]^2} \left( 7 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] - 2 \left( 2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] \right) \operatorname{Sin}[c + d x]^2 (a^2 + b^2 (-1 + \operatorname{Sin}[c + d x]^2)) \right) \right) \right) \right) + \\
& \frac{\operatorname{Csc}[c + d x]^4 (e \operatorname{Sin}[c + d x])^{9/2} \left( -\frac{4 a \operatorname{Sin}[c + d x]}{3 b^3} + \frac{-a^2 \operatorname{Sin}[c + d x] + b^2 \operatorname{Sin}[c + d x]}{b^3 (a + b \operatorname{Cos}[c + d x])} + \frac{\operatorname{Sin}[2 (c + d x)]}{5 b^2} \right)}{d}
\end{aligned}$$

d

■ **Problem 70: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin[c + dx])^{7/2}}{(a + b \cos[c + dx])^2} dx$$

Optimal (type 4, 487 leaves, 14 steps):

$$\frac{5 a (-a^2 + b^2)^{1/4} e^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 b^{7/2} d} + \frac{5 a (-a^2 + b^2)^{1/4} e^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 b^{7/2} d} +$$

$$\frac{5 (3 a^2 - b^2) e^4 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c + dx]}}{3 b^4 d \sqrt{e \sin[c + dx]}} - \frac{5 a^2 (a^2 - b^2) e^4 \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c + dx]}}{2 b^4 \left(a^2 - b \left(b - \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \sin[c + dx]}} -$$

$$\frac{5 a^2 (a^2 - b^2) e^4 \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c + dx]}}{2 b^4 \left(a^2 - b \left(b + \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \sin[c + dx]}} - \frac{5 e^3 (3 a - b \cos[c + dx]) \sqrt{e \sin[c + dx]}}{3 b^3 d} + \frac{e (e \sin[c + dx])^{5/2}}{b d (a + b \cos[c + dx])}$$

Result (type 6, 2156 leaves):

$$\frac{\left(\frac{2 \cos[c+dx]}{3 b^2} + \frac{-a^2+b^2}{b^3 (a+b \cos[c+dx])}\right) \operatorname{Csc}[c + dx]^3 (e \sin[c + dx])^{7/2}}{d} +$$

$$\frac{1}{6 b^3 d \sin[c + dx]^{7/2}} (e \sin[c + dx])^{7/2} \left( \frac{1}{(a + b \cos[c + dx]) (1 - \sin[c + dx]^2)} 2 (3 a^2 - 5 b^2) \cos[c + dx]^2 \left(a + b \sqrt{1 - \sin[c + dx]^2}\right) \right.$$

$$\left. \left( \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left( -2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2 - b^2} - \right. \right. \right.$$

$$\left. \left. \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx]\right] + \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx]\right] \right) +$$

$$\left( 5 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] \sqrt{\sin[c + dx]} \sqrt{1 - \sin[c + dx]^2} \right) / \left( \left( -5 (a^2 - b^2) \right. \right.$$

$$\left. \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] + 2 \left( 2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] + \right. \right.$$

$$\left. \left. (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right) +$$

$$\begin{aligned}
& \frac{1}{(a+b \cos [c+d x]) \sqrt{1-\sin [c+d x]^2}} 8 a b \cos [c+d x] \left(a+b \sqrt{1-\sin [c+d x]^2}\right) \left(-\frac{1}{\left(-a^2+b^2\right)^{3 / 4}}\left(\frac{1}{8}-\frac{i}{8}\right) \sqrt{b}\right. \\
& \left.\left(2 \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]-2 \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]+\operatorname{Log}\left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4}\right.\right.\right. \\
& \left.\left.\left.\sqrt{\sin [c+d x]}+i b \sin [c+d x]\right]-\operatorname{Log}\left[\sqrt{-a^2+b^2}+(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4}\sqrt{\sin [c+d x]}+i b \sin [c+d x]\right]\right)\right)+ \\
& \left(5 a\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \sqrt{\sin [c+d x]}\right) / \left(\sqrt{1-\sin [c+d x]^2}\left(5\left(a^2-b^2\right)\right.\right. \\
& \left.\left.\operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]-2\left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]+\right.\right.\right. \\
& \left.\left.\left(-a^2+b^2\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]\right) \sin [c+d x]^2\left(a^2+b^2\left(-1+\sin [c+d x]^2\right)\right)\right)\right) \\
& \frac{1}{(a+b \cos [c+d x])\left(1-2 \sin [c+d x]^2\right) \sqrt{1-\sin [c+d x]^2}} 6 a b \cos [c+d x] \cos [2(c+d x)]\left(a+b \sqrt{1-\sin [c+d x]^2}\right) \\
& \left(\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(-2 a^2+b^2\right) \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]}{b^{3 / 2}\left(-a^2+b^2\right)^{3 / 4}}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(-2 a^2+b^2\right) \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]}{b^{3 / 2}\left(-a^2+b^2\right)^{3 / 4}}+\right. \\
& \frac{1}{b^{3 / 2}\left(-a^2+b^2\right)^{3 / 4}}\left(\frac{1}{4}-\frac{i}{4}\right)\left(-2 a^2+b^2\right) \operatorname{Log}\left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4}\sqrt{\sin [c+d x]}+i b \sin [c+d x]\right]- \\
& \frac{1}{b^{3 / 2}\left(-a^2+b^2\right)^{3 / 4}}\left(\frac{1}{4}-\frac{i}{4}\right)\left(-2 a^2+b^2\right) \operatorname{Log}\left[\sqrt{-a^2+b^2}+(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4}\sqrt{\sin [c+d x]}+i b \sin [c+d x]\right]+ \\
& \frac{4 \sqrt{\sin [c+d x]}}{b}+\left(10 a\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \sqrt{\sin [c+d x]}\right) / \\
& \left(\sqrt{1-\sin [c+d x]^2}\left(5\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]-2\left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4},\right.\right.\right. \right. \\
& \left.\left.\left.\sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]+(-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]\right) \sin [c+d x]^2\right) \\
& \left.\left(a^2+b^2\left(-1+\sin [c+d x]^2\right)\right)\right)-\left(36 a\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \sin [c+d x]^{5 / 2}\right) /
\end{aligned}$$

$$\left( 5 \sqrt{1 - \sin[c + dx]^2} \left( 9 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - \right. \right. \\ \left. \left. 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\ \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right)$$

■ **Problem 71: Result unnecessarily involves higher level functions.**

$$\int \frac{(e \sin[c + dx])^{5/2}}{(a + b \cos[c + dx])^2} dx$$

Optimal (type 4, 404 leaves, 13 steps):

$$-\frac{3 a e^{5/2} \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{2 b^{5/2} (-a^2 + b^2)^{1/4} d} + \frac{3 a e^{5/2} \operatorname{ArcTanh} \left[ \frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{2 b^{5/2} (-a^2 + b^2)^{1/4} d} + \frac{3 a^2 e^3 \operatorname{EllipticPi} \left[ \frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left( c - \frac{\pi}{2} + dx \right), 2 \right] \sqrt{\sin[c + dx]}}{2 b^3 \left( b - \sqrt{-a^2 + b^2} \right) d \sqrt{e \sin[c + dx]}} + \\ \frac{3 a^2 e^3 \operatorname{EllipticPi} \left[ \frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left( c - \frac{\pi}{2} + dx \right), 2 \right] \sqrt{\sin[c + dx]}}{2 b^3 \left( b + \sqrt{-a^2 + b^2} \right) d \sqrt{e \sin[c + dx]}} - \frac{3 e^2 \operatorname{EllipticE} \left[ \frac{1}{2} \left( c - \frac{\pi}{2} + dx \right), 2 \right] \sqrt{e \sin[c + dx]}}{b^2 d \sqrt{\sin[c + dx]}} + \frac{e (e \sin[c + dx])^{3/2}}{b d (a + b \cos[c + dx])}$$

Result (type 6, 616 leaves):

$$\frac{\text{Csc}[c + dx] (e \sin[c + dx])^{5/2}}{bd (a + b \cos[c + dx])} -$$

$$\frac{1}{bd (a + b \cos[c + dx]) \sin[c + dx]^{5/2} (1 - \sin[c + dx]^2)} 3 \cos[c + dx]^2 (e \sin[c + dx])^{5/2} \left( a + b \sqrt{1 - \sin[c + dx]^2} \right)$$

$$\left( \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left( -2 \operatorname{ArcTan} \left[ 1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + 2 \operatorname{ArcTan} \left[ 1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[ \right. \right. \right.$$

$$\left. \left. \sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx] \right] - \operatorname{Log} \left[ \sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx] \right] \right) +$$

$$\left( 7 b (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^{3/2} \sqrt{1 - \sin[c + dx]^2} \right) /$$

$$\left( 3 \left( -7 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + 2 \right. \right.$$

$$\left. \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \right)$$

$$\left. \left. \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right)$$

■ **Problem 72: Result unnecessarily involves higher level functions.**

$$\int \frac{(e \sin[c + dx])^{3/2}}{(a + b \cos[c + dx])^2} dx$$

Optimal (type 4, 418 leaves, 13 steps):

$$\frac{a e^{3/2} \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{2 b^{3/2} (-a^2 + b^2)^{3/4} d} + \frac{a e^{3/2} \operatorname{ArcTanh} \left[ \frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{2 b^{3/2} (-a^2 + b^2)^{3/4} d} -$$

$$\frac{e^2 \operatorname{EllipticF} \left[ \frac{1}{2} \left( c - \frac{\pi}{2} + dx \right), 2 \right] \sqrt{\sin[c + dx]}}{b^2 d \sqrt{e \sin[c + dx]}} + \frac{a^2 e^2 \operatorname{EllipticPi} \left[ \frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left( c - \frac{\pi}{2} + dx \right), 2 \right] \sqrt{\sin[c + dx]}}{2 b^2 \left( a^2 - b \left( b - \sqrt{-a^2 + b^2} \right) \right) d \sqrt{e \sin[c + dx]}} +$$

$$\frac{a^2 e^2 \operatorname{EllipticPi} \left[ \frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left( c - \frac{\pi}{2} + dx \right), 2 \right] \sqrt{\sin[c + dx]}}{2 b^2 \left( a^2 - b \left( b + \sqrt{-a^2 + b^2} \right) \right) d \sqrt{e \sin[c + dx]}} + \frac{e \sqrt{e \sin[c + dx]}}{bd (a + b \cos[c + dx])}$$

Result (type 6, 614 leaves):

$$\frac{\text{Csc}[c + dx] (e \text{Sin}[c + dx])^{3/2}}{bd (a + b \text{Cos}[c + dx])} - \frac{1}{bd (a + b \text{Cos}[c + dx]) \text{Sin}[c + dx]^{3/2} (1 - \text{Sin}[c + dx]^2)} \text{Cos}[c + dx]^2 (e \text{Sin}[c + dx])^{3/2} \left( a + b \sqrt{1 - \text{Sin}[c + dx]^2} \right)$$

$$\left( \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left( -2 \text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\text{Sin}[c + dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\text{Sin}[c + dx]}}{(a^2 - b^2)^{1/4}}\right] - \text{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Sin}[c + dx]} + b \text{Sin}[c + dx]\right] + \text{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Sin}[c + dx]} + b \text{Sin}[c + dx]\right] \right) + \left( 5 b (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \text{Sin}[c + dx]^2, \frac{b^2 \text{Sin}[c + dx]^2}{-a^2 + b^2}\right] \sqrt{\text{Sin}[c + dx]} \sqrt{1 - \text{Sin}[c + dx]^2} \right) / \left( \left( -5 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \text{Sin}[c + dx]^2, \frac{b^2 \text{Sin}[c + dx]^2}{-a^2 + b^2}\right] + 2 \left( 2 b^2 \text{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \text{Sin}[c + dx]^2, \frac{b^2 \text{Sin}[c + dx]^2}{-a^2 + b^2}\right] + (a^2 - b^2) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \text{Sin}[c + dx]^2, \frac{b^2 \text{Sin}[c + dx]^2}{-a^2 + b^2}\right] \right) \text{Sin}[c + dx]^2 \right) (a^2 + b^2 (-1 + \text{Sin}[c + dx]^2)) \right) \right)$$

■ **Problem 73: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{e \text{Sin}[c + dx]}}{(a + b \text{Cos}[c + dx])^2} dx$$

Optimal (type 4, 438 leaves, 13 steps):

$$\frac{a \sqrt{e} \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \text{Sin}[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right]}{2 \sqrt{b} (-a^2 + b^2)^{5/4} d} - \frac{a \sqrt{e} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \text{Sin}[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right]}{2 \sqrt{b} (-a^2 + b^2)^{5/4} d} + \frac{a^2 e \text{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\text{Sin}[c + dx]}}{2 b (a^2 - b^2) (b - \sqrt{-a^2 + b^2}) d \sqrt{e \text{Sin}[c + dx]}} + \frac{a^2 e \text{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\text{Sin}[c + dx]}}{2 b (a^2 - b^2) (b + \sqrt{-a^2 + b^2}) d \sqrt{e \text{Sin}[c + dx]}} + \frac{\text{EllipticE}\left[\frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{e \text{Sin}[c + dx]}}{(a^2 - b^2) d \sqrt{\text{Sin}[c + dx]}} - \frac{b (e \text{Sin}[c + dx])^{3/2}}{(a^2 - b^2) d e (a + b \text{Cos}[c + dx])}$$

Result (type 6, 1181 leaves):

$$\begin{aligned}
& \frac{b \sin[c+dx] \sqrt{e \sin[c+dx]}}{(-a^2+b^2) d (a+b \cos[c+dx])} + \\
& \frac{1}{2(a-b)(a+b)d\sqrt{\sin[c+dx]}} \sqrt{e \sin[c+dx]} \left( \frac{1}{(a+b \cos[c+dx])(1-\sin[c+dx]^2)} 2b \cos[c+dx]^2 (a+b\sqrt{1-\sin[c+dx]^2}) \right. \\
& \left. \left( \frac{1}{4\sqrt{2} b^{3/2} (a^2-b^2)^{1/4}} a \left( -2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{a^2-b^2} - \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right] - \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right] \right) \right) + \\
& \left( 7b(a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] \sin[c+dx]^{3/2} \sqrt{1-\sin[c+dx]^2} \right) / \left( 3 \left( -7(a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] + 2 \left( 2b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] \right) \sin[c+dx]^2 (a^2+b^2(-1+\sin[c+dx]^2)) \right) \right) \right) + \\
& \frac{1}{6(a+b \cos[c+dx])\sqrt{1-\sin[c+dx]^2}} a \cos[c+dx] (a+b\sqrt{1-\sin[c+dx]^2}) \left( \frac{1}{\sqrt{b}(-a^2+b^2)^{1/4}} \right. \\
& \left. (3+3i) \left( 2 \operatorname{ArcTan}\left[1 - \frac{(1+i)\sqrt{b}\sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i)\sqrt{b}\sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i)\sqrt{b} \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2)^{1/4} \sqrt{\sin[c+dx]} + ib \sin[c+dx]\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i)\sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin[c+dx]} + ib \sin[c+dx]\right] \right) \right) + \\
& \left( 56a(a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] \sin[c+dx]^{3/2} \right) / \left( \sqrt{1-\sin[c+dx]^2} \left( 7(a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] - 2 \left( 2b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] \right) \sin[c+dx]^2 (a^2+b^2(-1+\sin[c+dx]^2)) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 74: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \cos[c+dx])^2 \sqrt{e \sin[c+dx]}} dx$$

Optimal (type 4, 445 leaves, 13 steps):

$$\begin{aligned}
 & - \frac{3 a \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4} \sqrt{e}}\right]}{2\left(-a^2+b^2\right)^{7 / 4} d \sqrt{e}} - \frac{3 a \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4} \sqrt{e}}\right]}{2\left(-a^2+b^2\right)^{7 / 4} d \sqrt{e}} - \\
 & \frac{\operatorname{EllipticF}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{\left(a^2-b^2\right) d \sqrt{e \sin [c+d x]}} + \frac{3 a^2 \operatorname{EllipticPi}\left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{2\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right) d \sqrt{e \sin [c+d x]}} + \\
 & \frac{3 a^2 \operatorname{EllipticPi}\left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{2\left(a^2-b^2\right)\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right) d \sqrt{e \sin [c+d x]}} - \frac{b \sqrt{e \sin [c+d x]}}{\left(a^2-b^2\right) d e\left(a+b \cos [c+d x]\right)}
 \end{aligned}$$

Result (type 6, 1182 leaves):



$$\begin{aligned}
& - \frac{b \sin[c + dx]}{(a^2 - b^2) d (a + b \cos[c + dx]) \sqrt{e \sin[c + dx]}} + \\
& \frac{1}{2(a-b)(a+b)d\sqrt{e \sin[c + dx]}} \sqrt{\sin[c + dx]} \left( - \frac{1}{(a + b \cos[c + dx]) (1 - \sin[c + dx])^2} 2b \cos[c + dx]^2 (a + b \sqrt{1 - \sin[c + dx]^2}) \right. \\
& \left. \left( \frac{1}{4\sqrt{2}\sqrt{b}(a^2 - b^2)^{3/4}} a \left( -2 \operatorname{ArcTan} \left[ 1 - \frac{\sqrt{2}\sqrt{b}\sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + 2 \operatorname{ArcTan} \left[ 1 + \frac{\sqrt{2}\sqrt{b}\sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[ \sqrt{a^2 - b^2} - \sqrt{2}\sqrt{b} \right. \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx] \right] + \operatorname{Log} \left[ \sqrt{a^2 - b^2} + \sqrt{2}\sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx] \right] \right) + \right. \\
& \left. \left( 5b(a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\sin[c + dx]} \sqrt{1 - \sin[c + dx]^2} \right) / \left( \left( -5(a^2 - b^2) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1} \left[ \frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + 2 \left( 2b^2 \operatorname{AppellF1} \left[ \frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right) \right) + \\
& \frac{1}{(a + b \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} 4a \cos[c + dx] (a + b \sqrt{1 - \sin[c + dx]^2}) \left( - \frac{1}{(-a^2 + b^2)^{3/4}} \left( \frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \right. \\
& \left. \left( 2 \operatorname{ArcTan} \left[ 1 - \frac{(1+i)\sqrt{b}\sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[ 1 + \frac{(1+i)\sqrt{b}\sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] + \operatorname{Log} \left[ \sqrt{-a^2 + b^2} - (1+i)\sqrt{b} (-a^2 + b^2)^{1/4} \right. \right. \right. \\
& \left. \left. \left. \sqrt{\sin[c + dx]} + ib \sin[c + dx] \right] - \operatorname{Log} \left[ \sqrt{-a^2 + b^2} + (1+i)\sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + ib \sin[c + dx] \right] \right) + \right. \\
& \left. \left( 5a(a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\sin[c + dx]} \right) / \left( \sqrt{1 - \sin[c + dx]^2} \left( 5(a^2 - b^2) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left( 2b^2 \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 75: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \cos[c + dx])^2 (e \sin[c + dx])^{3/2}} dx$$

Optimal (type 4, 507 leaves, 14 steps):

$$\begin{aligned}
 & \frac{5 a b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin [c+d x]}}{\left(-a^2+b^2\right)^{1/4} \sqrt{e}}\right]}{2\left(-a^2+b^2\right)^{9/4} d e^{3/2}} - \frac{5 a b^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin [c+d x]}}{\left(-a^2+b^2\right)^{1/4} \sqrt{e}}\right]}{2\left(-a^2+b^2\right)^{9/4} d e^{3/2}} - \frac{b}{\left(a^2-b^2\right) d e\left(a+b \cos [c+d x]\right) \sqrt{e \sin [c+d x]}} + \\
 & \frac{5 a b - \left(2 a^2+3 b^2\right) \cos [c+d x]}{\left(a^2-b^2\right)^2 d e \sqrt{e \sin [c+d x]}} - \frac{5 a^2 b \operatorname{EllipticPi}\left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{2\left(a^2-b^2\right)^2\left(b-\sqrt{-a^2+b^2}\right) d e \sqrt{e \sin [c+d x]}} - \\
 & \frac{5 a^2 b \operatorname{EllipticPi}\left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{2\left(a^2-b^2\right)^2\left(b+\sqrt{-a^2+b^2}\right) d e \sqrt{e \sin [c+d x]}} - \frac{\left(2 a^2+3 b^2\right) \operatorname{EllipticE}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{e \sin [c+d x]}}{\left(a^2-b^2\right)^2 d e^2 \sqrt{\sin [c+d x]}}
 \end{aligned}$$

Result (type 6, 1259 leaves):

$$\frac{\sin[c+dx]^2 \left( -\frac{2(-2ab+a^2\cos[c+dx]+b^2\cos[c+dx])\csc[c+dx]}{(a^2-b^2)^2} + \frac{b^3\sin[c+dx]}{(a^2-b^2)^2(a+b\cos[c+dx])} \right)}{d(e\sin[c+dx])^{3/2}} - \frac{1}{2(a-b)^2(a+b)^2d(e\sin[c+dx])^{3/2}}$$

$$\sin[c+dx]^{3/2} \left( \frac{1}{(a+b\cos[c+dx])(1-\sin[c+dx]^2)} 2(2a^2b+3b^3)\cos[c+dx]^2(a+b\sqrt{1-\sin[c+dx]^2}) \right.$$

$$\left. \left( \frac{1}{4\sqrt{2}b^{3/2}(a^2-b^2)^{1/4}} a \left( -2\operatorname{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{b}\sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}}\right] + 2\operatorname{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{b}\sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{a^2-b^2} - \right. \right. \right.$$

$$\left. \left. \sqrt{2}\sqrt{b}(a^2-b^2)^{1/4}\sqrt{\sin[c+dx]} + b\sin[c+dx]\right] - \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2}\sqrt{b}(a^2-b^2)^{1/4}\sqrt{\sin[c+dx]} + b\sin[c+dx]\right] \right) + \right.$$

$$\left. \left( 7b(a^2-b^2)\operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2\sin[c+dx]^2}{-a^2+b^2}\right] \sin[c+dx]^{3/2}\sqrt{1-\sin[c+dx]^2} \right) / \left( 3\left(-7(a^2-b^2)\right. \right.$$

$$\left. \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2\sin[c+dx]^2}{-a^2+b^2}\right] + 2\left(2b^2\operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2\sin[c+dx]^2}{-a^2+b^2}\right] + \right. \right.$$

$$\left. \left. (a^2-b^2)\operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2\sin[c+dx]^2}{-a^2+b^2}\right] \sin[c+dx]^2 \right) (a^2+b^2(-1+\sin[c+dx]^2)) \right) \right) +$$

$$\frac{1}{12(a+b\cos[c+dx])\sqrt{1-\sin[c+dx]^2}} (2a^3+8ab^2)\cos[c+dx](a+b\sqrt{1-\sin[c+dx]^2})$$

$$\left( \frac{1}{\sqrt{b}(-a^2+b^2)^{1/4}} (3+3i) \left( 2\operatorname{ArcTan}\left[1 - \frac{(1+i)\sqrt{b}\sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - 2\operatorname{ArcTan}\left[1 + \frac{(1+i)\sqrt{b}\sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \right. \right. \right.$$

$$\left. \left. \sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\sin[c+dx]} + ib\sin[c+dx]\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\sin[c+dx]} + ib\sin[c+dx]\right] \right) +$$

$$\left( 56a(a^2-b^2)\operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2\sin[c+dx]^2}{-a^2+b^2}\right] \sin[c+dx]^{3/2} \right) / \left( \sqrt{1-\sin[c+dx]^2} \left( 7(a^2-b^2) \right. \right.$$

$$\left. \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2\sin[c+dx]^2}{-a^2+b^2}\right] - 2\left(2b^2\operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2\sin[c+dx]^2}{-a^2+b^2}\right] + \right. \right.$$

$$\left. \left. (-a^2+b^2)\operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2\sin[c+dx]^2}{-a^2+b^2}\right] \sin[c+dx]^2 \right) (a^2+b^2(-1+\sin[c+dx]^2)) \right) \right) \right)$$

■ **Problem 76: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b\cos[c+dx])^2(e\sin[c+dx])^{5/2}} dx$$

Optimal (type 4, 530 leaves, 14 steps):

$$\begin{aligned}
 & - \frac{7 a b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 (-a^2+b^2)^{11/4} d e^{5/2}} - \frac{7 a b^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 (-a^2+b^2)^{11/4} d e^{5/2}} - \frac{b}{(a^2-b^2) d e (a+b \cos[c+dx]) (e \sin[c+dx])^{3/2}} + \\
 & \frac{7 a b - (2 a^2 + 5 b^2) \cos[c+dx]}{3 (a^2-b^2)^2 d e (e \sin[c+dx])^{3/2}} + \frac{(2 a^2 + 5 b^2) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{3 (a^2-b^2)^2 d e^2 \sqrt{e \sin[c+dx]}} - \\
 & \frac{7 a^2 b^2 \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{2 (a^2-b^2)^2 \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) d e^2 \sqrt{e \sin[c+dx]}} - \frac{7 a^2 b^2 \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{2 (a^2-b^2)^2 \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) d e^2 \sqrt{e \sin[c+dx]}}
 \end{aligned}$$

Result (type 6, 1257 leaves):

$$\begin{aligned}
& \frac{\left( \frac{b^3}{(a^2-b^2)^2 (a+b \cos[c+dx])} - \frac{2(-2ab+a^2 \cos[c+dx]+b^2 \cos[c+dx]) \operatorname{Csc}[c+dx]^2}{3(a^2-b^2)^2} \right) \operatorname{Sin}[c+dx]^3}{d (e \operatorname{Sin}[c+dx])^{5/2}} + \frac{1}{6(a-b)^2 (a+b)^2 d (e \operatorname{Sin}[c+dx])^{5/2}} \\
& \operatorname{Sin}[c+dx]^{5/2} \left( \frac{1}{(a+b \cos[c+dx]) (1-\operatorname{Sin}[c+dx]^2)} 2(2a^2b+5b^3) \cos[c+dx]^2 (a+b\sqrt{1-\operatorname{Sin}[c+dx]^2}) \right. \\
& \left. \left( \frac{1}{4\sqrt{2}\sqrt{b}(a^2-b^2)^{3/4}} a \left( -2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{b}\sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{b}\sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2-b^2} - \right. \right. \right. \\
& \left. \left. \left. \sqrt{2}\sqrt{b}(a^2-b^2)^{1/4}\sqrt{\operatorname{Sin}[c+dx]} + b \operatorname{Sin}[c+dx]\right] + \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2}\sqrt{b}(a^2-b^2)^{1/4}\sqrt{\operatorname{Sin}[c+dx]} + b \operatorname{Sin}[c+dx]\right] \right) \right) + \\
& \left( 5b(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \sqrt{\operatorname{Sin}[c+dx]} \sqrt{1-\operatorname{Sin}[c+dx]^2} \right) / \left( \left( -5(a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] + 2 \left( 2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (a^2-b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sin}[c+dx]^2 \right) (a^2+b^2(-1+\operatorname{Sin}[c+dx]^2)) \right) \right) \right) + \\
& \frac{1}{(a+b \cos[c+dx]) \sqrt{1-\operatorname{Sin}[c+dx]^2}} 2(2a^3-16ab^2) \cos[c+dx] (a+b\sqrt{1-\operatorname{Sin}[c+dx]^2}) \left( -\frac{1}{(-a^2+b^2)^{3/4}} \right. \\
& \left. \left( \frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left( 2 \operatorname{ArcTan}\left[1 - \frac{(1+i)\sqrt{b}\sqrt{\operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i)\sqrt{b}\sqrt{\operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i)\sqrt{b} \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2)^{1/4}\sqrt{\operatorname{Sin}[c+dx]} + ib \operatorname{Sin}[c+dx]\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\operatorname{Sin}[c+dx]} + ib \operatorname{Sin}[c+dx]\right] \right) \right) + \\
& \left( 5a(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \sqrt{\operatorname{Sin}[c+dx]} \right) / \left( \sqrt{1-\operatorname{Sin}[c+dx]^2} \left( 5(a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] - 2 \left( 2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sin}[c+dx]^2 \right) (a^2+b^2(-1+\operatorname{Sin}[c+dx]^2)) \right) \right) \right) \right)
\end{aligned}$$

- **Problem 77: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \cos[c + dx])^2 (e \sin[c + dx])^{7/2}} dx$$

Optimal (type 4, 590 leaves, 15 steps):

$$\frac{9 a b^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right] - 9 a b^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 (-a^2 + b^2)^{13/4} d e^{7/2}} - \frac{b}{(a^2 - b^2) d e (a + b \cos[c + dx]) (e \sin[c + dx])^{5/2}} + \frac{9 a b - (2 a^2 + 7 b^2) \cos[c + dx]}{5 (a^2 - b^2)^2 d e (e \sin[c + dx])^{5/2}} - \frac{3 (15 a b^3 + (2 a^4 - 10 a^2 b^2 - 7 b^4) \cos[c + dx])}{5 (a^2 - b^2)^3 d e^3 \sqrt{e \sin[c + dx]}} + \frac{9 a^2 b^3 \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c + dx]}}{2 (a^2 - b^2)^3 \left(b - \sqrt{-a^2 + b^2}\right) d e^3 \sqrt{e \sin[c + dx]}} + \frac{9 a^2 b^3 \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c + dx]}}{2 (a^2 - b^2)^3 \left(b + \sqrt{-a^2 + b^2}\right) d e^3 \sqrt{e \sin[c + dx]}} - \frac{3 (2 a^4 - 10 a^2 b^2 - 7 b^4) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{e \sin[c + dx]}}{5 (a^2 - b^2)^3 d e^4 \sqrt{\sin[c + dx]}}$$

Result (type 6, 1344 leaves):

$$\begin{aligned}
& \frac{1}{d (e \operatorname{Sin}[c+d x])^{7/2}} \operatorname{Sin}[c+d x]^4 \left( -\frac{2 (20 a b^3 + 3 a^4 \operatorname{Cos}[c+d x] - 15 a^2 b^2 \operatorname{Cos}[c+d x] - 8 b^4 \operatorname{Cos}[c+d x]) \operatorname{Csc}[c+d x]}{5 (a^2 - b^2)^3} - \right. \\
& \left. \frac{2 (-2 a b + a^2 \operatorname{Cos}[c+d x] + b^2 \operatorname{Cos}[c+d x]) \operatorname{Csc}[c+d x]^3}{5 (a^2 - b^2)^2} - \frac{b^5 \operatorname{Sin}[c+d x]}{(a^2 - b^2)^3 (a + b \operatorname{Cos}[c+d x])} \right) - \frac{1}{10 (a-b)^3 (a+b)^3 d (e \operatorname{Sin}[c+d x])^{7/2}} \\
& 3 \operatorname{Sin}[c+d x]^{7/2} \left( \frac{1}{(a + b \operatorname{Cos}[c+d x]) (1 - \operatorname{Sin}[c+d x]^2)} {}_2 F_1 \left( 2 a^4 b - 10 a^2 b^3 - 7 b^5, \operatorname{Cos}[c+d x]^2, a + b \sqrt{1 - \operatorname{Sin}[c+d x]^2} \right) \right. \\
& \left( \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left( -2 \operatorname{ArcTan} \left[ 1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c+d x]}}{(a^2 - b^2)^{1/4}} \right] + 2 \operatorname{ArcTan} \left[ 1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c+d x]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[ \sqrt{a^2 - b^2} - \right. \right. \\
& \left. \left. \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c+d x]} + b \operatorname{Sin}[c+d x] \right] - \operatorname{Log} \left[ \sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c+d x]} + b \operatorname{Sin}[c+d x] \right] \right) + \\
& \left( 7 b (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2 + b^2} \right] \operatorname{Sin}[c+d x]^{3/2} \sqrt{1 - \operatorname{Sin}[c+d x]^2} \right) / \left( 3 \left( -7 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2 + b^2} \right] + 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2 + b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2 + b^2} \right] \right) \operatorname{Sin}[c+d x]^2 \right) (a^2 + b^2 (-1 + \operatorname{Sin}[c+d x]^2)) \left. \right) \left. \right) + \\
& \frac{1}{12 (a + b \operatorname{Cos}[c+d x]) \sqrt{1 - \operatorname{Sin}[c+d x]^2}} (2 a^5 - 10 a^3 b^2 - 22 a b^4) \operatorname{Cos}[c+d x] (a + b \sqrt{1 - \operatorname{Sin}[c+d x]^2}) \\
& \left( \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} (3 + 3 i) \left( 2 \operatorname{ArcTan} \left[ 1 - \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Sin}[c+d x]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[ 1 + \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Sin}[c+d x]}}{(-a^2 + b^2)^{1/4}} \right] - \operatorname{Log} \left[ \sqrt{-a^2 + b^2} - (1 + i) \right. \right. \right. \\
& \left. \left. \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c+d x]} + i b \operatorname{Sin}[c+d x] \right] + \operatorname{Log} \left[ \sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c+d x]} + i b \operatorname{Sin}[c+d x] \right] \right) \left. \right) + \\
& \left( 56 a (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2 + b^2} \right] \operatorname{Sin}[c+d x]^{3/2} \right) / \left( \sqrt{1 - \operatorname{Sin}[c+d x]^2} (7 (a^2 - b^2) \right. \\
& \left. \operatorname{AppellF1} \left[ \frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2 + b^2} \right] - 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2 + b^2} \right] + \right. \right. \\
& \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2 + b^2} \right] \right) \operatorname{Sin}[c+d x]^2 \right) (a^2 + b^2 (-1 + \operatorname{Sin}[c+d x]^2)) \left. \right) \left. \right) \left. \right)
\end{aligned}$$

■ **Problem 78: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin[c + dx])^{13/2}}{(a + b \cos[c + dx])^3} dx$$

Optimal (type 4, 590 leaves, 15 steps):

$$\frac{11 (9 a^4 - 11 a^2 b^2 + 2 b^4) e^{13/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right] - 11 (9 a^4 - 11 a^2 b^2 + 2 b^4) e^{13/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{13/2} (-a^2 + b^2)^{1/4} d} - \frac{11 a (9 a^4 - 11 a^2 b^2 + 2 b^4) e^7 \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\sin[c+dx]}}{8 b^7 (b - \sqrt{-a^2 + b^2}) d \sqrt{e \sin[c+dx]}} - \frac{11 a (9 a^4 - 11 a^2 b^2 + 2 b^4) e^7 \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\sin[c+dx]}}{8 b^7 (b + \sqrt{-a^2 + b^2}) d \sqrt{e \sin[c+dx]}} + \frac{11 a (45 a^2 - 37 b^2) e^6 \operatorname{EllipticE}\left[\frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{e \sin[c+dx]}}{20 b^6 d \sqrt{\sin[c+dx]}} - \frac{11 e^5 (5 (9 a^2 - 2 b^2) - 27 a b \cos[c+dx]) (e \sin[c+dx])^{3/2}}{60 b^5 d} + \frac{11 e^3 (9 a + 2 b \cos[c+dx]) (e \sin[c+dx])^{7/2}}{28 b^3 d (a + b \cos[c+dx])} + \frac{e (e \sin[c+dx])^{11/2}}{2 b d (a + b \cos[c+dx])^2}$$

Result (type 6, 1324 leaves):

$$\frac{1}{40 b^5 d \sin[c+dx]^{13/2}} - 11 (e \sin[c+dx])^{13/2} \left( \frac{1}{(a + b \cos[c+dx]) (1 - \sin[c+dx])^2} 2 (45 a^3 - 37 a b^2) \cos[c+dx]^2 (a + b \sqrt{1 - \sin[c+dx]^2}) \left( \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} \right. \right. \right. \\ \left. \left. \left. a \left( -2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + \right. \right. \right. \right. \\ \left. \left. \left. b \sin[c+dx]\right] - \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right] \right) \right) + \\ \left( 7 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2}\right] \sin[c+dx]^{3/2} \sqrt{1 - \sin[c+dx]^2} \right) / \left( 3 \left( -7 (a^2 - b^2) \right. \right. \\ \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2}\right] + 2 \left( 2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2}\right] + \right. \right. \right.$$



$$\begin{aligned}
& \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \sin[c+dx]^2 (a^2 + b^2 (-1 + \sin[c+dx]^2)) \right) \right) \right) + \\
& \frac{1}{12 (a + b \cos[c+dx]) \sqrt{1 - \sin[c+dx]^2}} (18 a^2 b - 10 b^3) \cos[c+dx] \left( a + b \sqrt{1 - \sin[c+dx]^2} \right) \\
& \left( \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} (3 + 3i) \left( 2 \operatorname{ArcTan} \left[ 1 - \frac{(1+i) \sqrt{b} \sqrt{\sin[c+dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[ 1 + \frac{(1+i) \sqrt{b} \sqrt{\sin[c+dx]}}{(-a^2 + b^2)^{1/4}} \right] - \operatorname{Log} \left[ \sqrt{-a^2 + b^2} - (1+i) \right. \right. \right. \\
& \left. \left. \left. \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c+dx]} + i b \sin[c+dx] \right] + \operatorname{Log} \left[ \sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c+dx]} + i b \sin[c+dx] \right] \right) \right) + \\
& \left( 56 a (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \sin[c+dx]^{3/2} \right) / \left( \sqrt{1 - \sin[c+dx]^2} (7 (a^2 - b^2) \right. \\
& \left. \operatorname{AppellF1} \left[ \frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] - 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] + \right. \right. \\
& \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \right) \sin[c+dx]^2 (a^2 + b^2 (-1 + \sin[c+dx]^2)) \right) \right) \right) + \\
& \frac{1}{d} \operatorname{Csc}[c+dx]^6 (e \sin[c+dx])^{13/2} \left( \frac{(-168 a^2 + 65 b^2) \sin[c+dx]}{42 b^5} - \frac{19 (a^3 \sin[c+dx] - a b^2 \sin[c+dx])}{4 b^5 (a + b \cos[c+dx])} + \right. \\
& \frac{a^4 \sin[c+dx] - 2 a^2 b^2 \sin[c+dx] + b^4 \sin[c+dx]}{2 b^5 (a + b \cos[c+dx])^2} + \\
& \frac{3 a \sin[2 (c + d x)]}{5 b^4} - \\
& \left. \frac{\sin[3 (c + d x)]}{14 b^3} \right)
\end{aligned}$$

- **Problem 79: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin[c+dx])^{11/2}}{(a + b \cos[c+dx])^3} dx$$

Optimal (type 4, 604 leaves, 15 steps):

$$\begin{aligned}
& \frac{9 (7 a^4 - 9 a^2 b^2 + 2 b^4) e^{11/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{11/2} (-a^2+b^2)^{3/4} d} - \\
& \frac{9 (7 a^4 - 9 a^2 b^2 + 2 b^4) e^{11/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{11/2} (-a^2+b^2)^{3/4} d} + \frac{3 a (21 a^2 - 13 b^2) e^6 \operatorname{EllipticF}\left[\frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{4 b^6 d \sqrt{e \operatorname{Sin}[c+dx]}} - \\
& \frac{9 a (7 a^4 - 9 a^2 b^2 + 2 b^4) e^6 \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2+b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{8 b^6 (a^2 - b (b - \sqrt{-a^2+b^2})) d \sqrt{e \operatorname{Sin}[c+dx]}} - \\
& \frac{9 a (7 a^4 - 9 a^2 b^2 + 2 b^4) e^6 \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2+b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{8 b^6 (a^2 - b (b + \sqrt{-a^2+b^2})) d \sqrt{e \operatorname{Sin}[c+dx]}} - \\
& \frac{3 e^5 (3 (7 a^2 - 2 b^2) - 7 a b \operatorname{Cos}[c+dx]) \sqrt{e \operatorname{Sin}[c+dx]}}{4 b^5 d} + \frac{9 e^3 (7 a + 2 b \operatorname{Cos}[c+dx]) (e \operatorname{Sin}[c+dx])^{5/2}}{20 b^3 d (a + b \operatorname{Cos}[c+dx])} + \frac{e (e \operatorname{Sin}[c+dx])^{9/2}}{2 b d (a + b \operatorname{Cos}[c+dx])^2}
\end{aligned}$$

Result (type 6, 2224 leaves):

$$\begin{aligned}
& \frac{\left(\frac{2 a \operatorname{Cos}[c+dx]}{b^4} + \frac{(-a^2+b^2)^2}{2 b^5 (a+b \operatorname{Cos}[c+dx])^2} - \frac{17 a (a^2-b^2)}{4 b^5 (a+b \operatorname{Cos}[c+dx])} - \frac{\operatorname{Cos}[2 (c+dx)]}{5 b^3}\right) \operatorname{Csc}[c+dx]^5 (e \operatorname{Sin}[c+dx])^{11/2}}{d} + \\
& \frac{1}{40 b^5 d \operatorname{Sin}[c+dx]^{11/2}} 3 (e \operatorname{Sin}[c+dx])^{11/2} \left(\frac{1}{(a+b \operatorname{Cos}[c+dx]) (1-\operatorname{Sin}[c+dx]^2)} 2 (25 a^3 - 37 a b^2) \operatorname{Cos}[c+dx]^2 (a+b \sqrt{1-\operatorname{Sin}[c+dx]^2})\right. \\
& \left(\frac{1}{4 \sqrt{2} \sqrt{b} (a^2-b^2)^{3/4}} a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2-b^2} - \right.\right. \\
& \left.\left.\sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\operatorname{Sin}[c+dx]} + b \operatorname{Sin}[c+dx]\right] + \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\operatorname{Sin}[c+dx]} + b \operatorname{Sin}[c+dx]\right]\right) + \\
& \left(5 b (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \sqrt{\operatorname{Sin}[c+dx]} \sqrt{1-\operatorname{Sin}[c+dx]^2}\right) / \left(\left(-5 (a^2-b^2)\right.\right. \\
& \left.\left.\operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] + 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] +\right.\right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left( (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c + dx]^2, \frac{b^2 \operatorname{Sin}[c + dx]^2}{-a^2 + b^2} \right] \operatorname{Sin}[c + dx]^2 \left( a^2 + b^2 (-1 + \operatorname{Sin}[c + dx]^2) \right) \right) \right) + \\
& \frac{1}{(a + b \operatorname{Cos}[c + dx]) \sqrt{1 - \operatorname{Sin}[c + dx]^2}} 2 (30 a^2 b - 16 b^3) \operatorname{Cos}[c + dx] \left( a + b \sqrt{1 - \operatorname{Sin}[c + dx]^2} \right) \left( -\frac{1}{(-a^2 + b^2)^{3/4}} \right. \\
& \left. \left( \frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left( 2 \operatorname{ArcTan} \left[ 1 - \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Sin}[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[ 1 + \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Sin}[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] + \operatorname{Log} \left[ \sqrt{-a^2 + b^2} - (1+i) \sqrt{b} \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + dx]} + i b \operatorname{Sin}[c + dx] \right] - \operatorname{Log} \left[ \sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + dx]} + i b \operatorname{Sin}[c + dx] \right] \right) \right) + \\
& \left( 5 a (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + dx]^2, \frac{b^2 \operatorname{Sin}[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\operatorname{Sin}[c + dx]} \right) / \left( \sqrt{1 - \operatorname{Sin}[c + dx]^2} \left( 5 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + dx]^2, \frac{b^2 \operatorname{Sin}[c + dx]^2}{-a^2 + b^2} \right] - 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c + dx]^2, \frac{b^2 \operatorname{Sin}[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c + dx]^2, \frac{b^2 \operatorname{Sin}[c + dx]^2}{-a^2 + b^2} \right] \right) \operatorname{Sin}[c + dx]^2 \left( a^2 + b^2 (-1 + \operatorname{Sin}[c + dx]^2) \right) \right) \right) + \\
& \frac{1}{(a + b \operatorname{Cos}[c + dx]) (1 - 2 \operatorname{Sin}[c + dx]^2) \sqrt{1 - \operatorname{Sin}[c + dx]^2}} (-40 a^2 b + 14 b^3) \operatorname{Cos}[c + dx] \operatorname{Cos}[2 (c + dx)] \\
& \left( a + b \sqrt{1 - \operatorname{Sin}[c + dx]^2} \right) \\
& \left( \frac{\left( \frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[ 1 - \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Sin}[c + dx]}}{(-a^2 + b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} - \frac{\left( \frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[ 1 + \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Sin}[c + dx]}}{(-a^2 + b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} + \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \right. \\
& \left. \left( \frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[ \sqrt{-a^2 + b^2} - (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + dx]} + i b \operatorname{Sin}[c + dx] \right] - \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \right. \\
& \left. \left( \frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[ \sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + dx]} + i b \operatorname{Sin}[c + dx] \right] + \frac{4 \sqrt{\operatorname{Sin}[c + dx]}}{b} + \right. \\
& \left. \left( 10 a (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + dx]^2, \frac{b^2 \operatorname{Sin}[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\operatorname{Sin}[c + dx]} \right) / \left( \sqrt{1 - \operatorname{Sin}[c + dx]^2} \left( 5 (a^2 - b^2) \operatorname{AppellF1} \left[ \right. \right. \right. \\
& \left. \left. \left. \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + dx]^2, \frac{b^2 \operatorname{Sin}[c + dx]^2}{-a^2 + b^2} \right] - 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c + dx]^2, \frac{b^2 \operatorname{Sin}[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned} & \left. \left( (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) - \\ & \left( 36 a (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^{5/2} \right) / \left( 5 \sqrt{1 - \sin[c + dx]^2} \left( 9 (a^2 - b^2) \right. \right. \\ & \left. \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \\ & \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \left. \right) \end{aligned}$$

■ **Problem 80: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin[c + dx])^{9/2}}{(a + b \cos[c + dx])^3} dx$$

Optimal (type 4, 498 leaves, 14 steps):

$$\begin{aligned} & - \frac{7 (5 a^2 - 2 b^2) e^{9/2} \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{8 b^{9/2} (-a^2 + b^2)^{1/4} d} + \frac{7 (5 a^2 - 2 b^2) e^{9/2} \operatorname{ArcTanh} \left[ \frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{8 b^{9/2} (-a^2 + b^2)^{1/4} d} + \\ & \frac{7 a (5 a^2 - 2 b^2) e^5 \operatorname{EllipticPi} \left[ \frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left( c - \frac{\pi}{2} + dx \right), 2 \right] \sqrt{\sin[c + dx]}}{8 b^5 \left( b - \sqrt{-a^2 + b^2} \right) d \sqrt{e \sin[c + dx]}} + \\ & - \frac{7 a (5 a^2 - 2 b^2) e^5 \operatorname{EllipticPi} \left[ \frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left( c - \frac{\pi}{2} + dx \right), 2 \right] \sqrt{\sin[c + dx]}}{8 b^5 \left( b + \sqrt{-a^2 + b^2} \right) d \sqrt{e \sin[c + dx]}} - \\ & \frac{35 a e^4 \operatorname{EllipticE} \left[ \frac{1}{2} \left( c - \frac{\pi}{2} + dx \right), 2 \right] \sqrt{e \sin[c + dx]}}{4 b^4 d \sqrt{\sin[c + dx]}} + \frac{7 e^3 (5 a + 2 b \cos[c + dx]) (e \sin[c + dx])^{3/2}}{12 b^3 d (a + b \cos[c + dx])} + \frac{e (e \sin[c + dx])^{7/2}}{2 b d (a + b \cos[c + dx])^2} \end{aligned}$$

Result (type 6, 1231 leaves):

$$\begin{aligned}
& \frac{\text{Csc}[c + dx]^4 (e \sin[c + dx])^{9/2} \left( \frac{2 \sin[c + dx]}{3b^3} + \frac{11a \sin[c + dx]}{4b^3 (a + b \cos[c + dx])} + \frac{-a^2 \sin[c + dx] + b^2 \sin[c + dx]}{2b^3 (a + b \cos[c + dx])^2} \right)}{d} \\
& \frac{1}{8b^3 d \sin[c + dx]^{9/2}} \left( \frac{1}{(a + b \cos[c + dx]) (1 - \sin[c + dx])^2} 10a \cos[c + dx]^2 (a + b \sqrt{1 - \sin[c + dx]^2}) \right. \\
& \left. \left( \frac{1}{4\sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left( -2 \text{ArcTan} \left[ 1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + 2 \text{ArcTan} \left[ 1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \text{Log} \left[ \sqrt{a^2 - b^2} - \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx] \right] - \text{Log} \left[ \sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx] \right] \right) \right) + \\
& \left( 7b (a^2 - b^2) \text{AppellF1} \left[ \frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^{3/2} \sqrt{1 - \sin[c + dx]^2} \right) / \left( 3 \left( -7 (a^2 - b^2) \right. \right. \\
& \left. \left. \text{AppellF1} \left[ \frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + 2 \left( 2b^2 \text{AppellF1} \left[ \frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) \text{AppellF1} \left[ \frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right) \right) + \\
& \frac{1}{6 (a + b \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} b \cos[c + dx] (a + b \sqrt{1 - \sin[c + dx]^2}) \left( \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} \right. \\
& \left. (3 + 3i) \left( 2 \text{ArcTan} \left[ 1 - \frac{(1 + i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \text{ArcTan} \left[ 1 + \frac{(1 + i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - \text{Log} \left[ \sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] + \text{Log} \left[ \sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] \right) \right) + \\
& \left( 56a (a^2 - b^2) \text{AppellF1} \left[ \frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^{3/2} \right) / \left( \sqrt{1 - \sin[c + dx]^2} \left( 7 (a^2 - b^2) \right. \right. \\
& \left. \left. \text{AppellF1} \left[ \frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left( 2b^2 \text{AppellF1} \left[ \frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \text{AppellF1} \left[ \frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 81: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin[c + dx])^{7/2}}{(a + b \cos[c + dx])^3} dx$$

Optimal (type 4, 512 leaves, 14 steps):

$$\frac{5 (3 a^2 - 2 b^2) e^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{7/2} (-a^2+b^2)^{3/4} d} + \frac{5 (3 a^2 - 2 b^2) e^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{7/2} (-a^2+b^2)^{3/4} d} -$$

$$\frac{15 a e^4 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{4 b^4 d \sqrt{e \operatorname{Sin}[c+dx]}} + \frac{5 a (3 a^2 - 2 b^2) e^4 \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{8 b^4 \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \operatorname{Sin}[c+dx]}} +$$

$$\frac{5 a (3 a^2 - 2 b^2) e^4 \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{8 b^4 \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \operatorname{Sin}[c+dx]}} +$$

$$\frac{5 e^3 (3 a + 2 b \operatorname{Cos}[c+dx]) \sqrt{e \operatorname{Sin}[c+dx]}}{4 b^3 d (a + b \operatorname{Cos}[c+dx])} + \frac{e (e \operatorname{Sin}[c+dx])^{5/2}}{2 b d (a + b \operatorname{Cos}[c+dx])^2}$$

Result (type 6, 2154 leaves):

$$\frac{\left(\frac{-a^2+b^2}{2 b^3 (a+b \operatorname{Cos}[c+dx])^2} + \frac{9 a}{4 b^3 (a+b \operatorname{Cos}[c+dx])}\right) \operatorname{Csc}[c+dx]^3 (e \operatorname{Sin}[c+dx])^{7/2}}{d} -$$

$$\frac{1}{8 b^3 d \operatorname{Sin}[c+dx]^{7/2}} (e \operatorname{Sin}[c+dx])^{7/2} \left( \frac{1}{(a+b \operatorname{Cos}[c+dx]) (1-\operatorname{Sin}[c+dx])^2} 14 a \operatorname{Cos}[c+dx]^2 \left(a+b \sqrt{1-\operatorname{Sin}[c+dx]^2}\right) \right.$$

$$\left. \left( \frac{1}{4 \sqrt{2} \sqrt{b} (a^2-b^2)^{3/4}} a \left( -2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2-b^2} - \right. \right. \right.$$

$$\left. \left. \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\operatorname{Sin}[c+dx]} + b \operatorname{Sin}[c+dx]\right] + \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\operatorname{Sin}[c+dx]} + b \operatorname{Sin}[c+dx]\right] \right) \right) +$$

$$\left( 5 b (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \sqrt{\operatorname{Sin}[c+dx]} \sqrt{1-\operatorname{Sin}[c+dx]^2} \right) / \left( \left( -5 (a^2-b^2) \right. \right.$$

$$\left. \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] + 2 \left( 2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] + \right. \right.$$

$$\left. \left. (a^2-b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sin}[c+dx]^2 \right) (a^2+b^2 (-1+\operatorname{Sin}[c+dx]^2)) \right) \right) \right) +$$

$$\begin{aligned}
& \frac{1}{(a+b \cos [c+d x]) \sqrt{1-\sin [c+d x]^2}} 12 b \cos [c+d x] \left(a+b \sqrt{1-\sin [c+d x]^2}\right) \left(-\frac{1}{\left(-a^2+b^2\right)^{3 / 4}}\left(\frac{1}{8}-\frac{i}{8}\right) \sqrt{b}\right. \\
& \left.\left(2 \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]-2 \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]+\operatorname{Log}\left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4}\right.\right.\right. \\
& \left.\left.\left.\sqrt{\sin [c+d x]}+i b \sin [c+d x]\right]-\operatorname{Log}\left[\sqrt{-a^2+b^2}+(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4}\sqrt{\sin [c+d x]}+i b \sin [c+d x]\right]\right)\right)+ \\
& \left(5 a\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \sqrt{\sin [c+d x]}\right) / \left(\sqrt{1-\sin [c+d x]^2}\left(5\left(a^2-b^2\right)\right.\right. \\
& \left.\left.\operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]-2\left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]+\right.\right.\right. \\
& \left.\left.\left(-a^2+b^2\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]\right) \sin [c+d x]^2\left(a^2+b^2\left(-1+\sin [c+d x]^2\right)\right)\right)\right) \\
& \frac{1}{(a+b \cos [c+d x])\left(1-2 \sin [c+d x]^2\right) \sqrt{1-\sin [c+d x]^2}} 4 b \cos [c+d x] \cos [2(c+d x)]\left(a+b \sqrt{1-\sin [c+d x]^2}\right) \\
& \left(\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(-2 a^2+b^2\right) \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]}{b^{3 / 2}\left(-a^2+b^2\right)^{3 / 4}}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(-2 a^2+b^2\right) \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]}{b^{3 / 2}\left(-a^2+b^2\right)^{3 / 4}}+\right. \\
& \frac{1}{b^{3 / 2}\left(-a^2+b^2\right)^{3 / 4}}\left(\frac{1}{4}-\frac{i}{4}\right)\left(-2 a^2+b^2\right) \operatorname{Log}\left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4}\sqrt{\sin [c+d x]}+i b \sin [c+d x]\right]- \\
& \frac{1}{b^{3 / 2}\left(-a^2+b^2\right)^{3 / 4}}\left(\frac{1}{4}-\frac{i}{4}\right)\left(-2 a^2+b^2\right) \operatorname{Log}\left[\sqrt{-a^2+b^2}+(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4}\sqrt{\sin [c+d x]}+i b \sin [c+d x]\right]+ \\
& \frac{4 \sqrt{\sin [c+d x]}}{b}+\left(10 a\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \sqrt{\sin [c+d x]}\right) / \\
& \left(\sqrt{1-\sin [c+d x]^2}\left(5\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]-2\left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4},\right.\right.\right. \right. \\
& \left.\left.\left.\sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]+\left(-a^2+b^2\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]\right) \sin [c+d x]^2\right) \\
& \left.\left(a^2+b^2\left(-1+\sin [c+d x]^2\right)\right)\right)-\left(36 a\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \sin [c+d x]^{5 / 2}\right) /
\end{aligned}$$

$$\left( 5 \sqrt{1 - \sin[c + dx]^2} \left( 9 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - \right. \right. \\ \left. \left. 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\ \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right)$$

■ **Problem 82: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin[c + dx])^{5/2}}{(a + b \cos[c + dx])^3} dx$$

Optimal (type 4, 520 leaves, 14 steps):

$$\begin{aligned} & - \frac{3 (a^2 - 2 b^2) e^{5/2} \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{8 b^{5/2} (-a^2 + b^2)^{5/4} d} + \frac{3 (a^2 - 2 b^2) e^{5/2} \operatorname{ArcTanh} \left[ \frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{8 b^{5/2} (-a^2 + b^2)^{5/4} d} - \\ & \frac{3 a (a^2 - 2 b^2) e^3 \operatorname{EllipticPi} \left[ \frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2 \right] \sqrt{\sin[c + dx]}}{8 b^3 (a^2 - b^2) (b - \sqrt{-a^2 + b^2}) d \sqrt{e \sin[c + dx]}} - \\ & \frac{3 a (a^2 - 2 b^2) e^3 \operatorname{EllipticPi} \left[ \frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2 \right] \sqrt{\sin[c + dx]}}{8 b^3 (a^2 - b^2) (b + \sqrt{-a^2 + b^2}) d \sqrt{e \sin[c + dx]}} + \\ & \frac{3 a e^2 \operatorname{EllipticE} \left[ \frac{1}{2} (c - \frac{\pi}{2} + dx), 2 \right] \sqrt{e \sin[c + dx]}}{4 b^2 (a^2 - b^2) d \sqrt{\sin[c + dx]}} + \frac{e (e \sin[c + dx])^{3/2}}{2 b d (a + b \cos[c + dx])^2} - \frac{3 a e (e \sin[c + dx])^{3/2}}{4 b (a^2 - b^2) d (a + b \cos[c + dx])} \end{aligned}$$

Result (type 6, 1225 leaves):



$$\begin{aligned}
& \frac{\text{Csc}[c + d x]^2 (e \text{Sin}[c + d x])^{5/2} \left( \frac{\text{Sin}[c + d x]}{2 b (a + b \text{Cos}[c + d x])^2} + \frac{3 a \text{Sin}[c + d x]}{4 b (-a^2 + b^2) (a + b \text{Cos}[c + d x])} \right)}{d} + \\
& \frac{1}{8 (a - b) b (a + b) d \text{Sin}[c + d x]^{5/2}} 3 (e \text{Sin}[c + d x])^{5/2} \left( \frac{1}{(a + b \text{Cos}[c + d x]) (1 - \text{Sin}[c + d x]^2)} 2 a \text{Cos}[c + d x]^2 (a + b \sqrt{1 - \text{Sin}[c + d x]^2}) \right. \\
& \left. \left( \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left( -2 \text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\text{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] + 2 \text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\text{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] + \text{Log}\left[\sqrt{a^2 - b^2} - \right. \right. \right. \\
& \left. \left. \left. \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Sin}[c + d x]} + b \text{Sin}[c + d x]\right] - \text{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Sin}[c + d x]} + b \text{Sin}[c + d x]\right] \right) \right) + \\
& \left( 7 b (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \text{Sin}[c + d x]^2, \frac{b^2 \text{Sin}[c + d x]^2}{-a^2 + b^2}\right] \text{Sin}[c + d x]^{3/2} \sqrt{1 - \text{Sin}[c + d x]^2} \right) / \left( 3 \left( -7 (a^2 - b^2) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \text{Sin}[c + d x]^2, \frac{b^2 \text{Sin}[c + d x]^2}{-a^2 + b^2}\right] + 2 \left( 2 b^2 \text{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \text{Sin}[c + d x]^2, \frac{b^2 \text{Sin}[c + d x]^2}{-a^2 + b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \text{Sin}[c + d x]^2, \frac{b^2 \text{Sin}[c + d x]^2}{-a^2 + b^2}\right] \right) \text{Sin}[c + d x]^2 (a^2 + b^2 (-1 + \text{Sin}[c + d x]^2)) \right) \right) \right) + \\
& \frac{1}{6 (a + b \text{Cos}[c + d x]) \sqrt{1 - \text{Sin}[c + d x]^2}} b \text{Cos}[c + d x] (a + b \sqrt{1 - \text{Sin}[c + d x]^2}) \left( \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} \right. \\
& \left. (3 + 3 i) \left( 2 \text{ArcTan}\left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\text{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \text{ArcTan}\left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\text{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] - \text{Log}\left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\text{Sin}[c + d x]} + i b \text{Sin}[c + d x]\right] + \text{Log}\left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\text{Sin}[c + d x]} + i b \text{Sin}[c + d x]\right] \right) \right) + \\
& \left( 56 a (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \text{Sin}[c + d x]^2, \frac{b^2 \text{Sin}[c + d x]^2}{-a^2 + b^2}\right] \text{Sin}[c + d x]^{3/2} \right) / \left( \sqrt{1 - \text{Sin}[c + d x]^2} \left( 7 (a^2 - b^2) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \text{Sin}[c + d x]^2, \frac{b^2 \text{Sin}[c + d x]^2}{-a^2 + b^2}\right] - 2 \left( 2 b^2 \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \text{Sin}[c + d x]^2, \frac{b^2 \text{Sin}[c + d x]^2}{-a^2 + b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \text{Sin}[c + d x]^2, \frac{b^2 \text{Sin}[c + d x]^2}{-a^2 + b^2}\right] \right) \text{Sin}[c + d x]^2 (a^2 + b^2 (-1 + \text{Sin}[c + d x]^2)) \right) \right) \right) \right)
\end{aligned}$$

- **Problem 83: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin[c + dx])^{3/2}}{(a + b \cos[c + dx])^3} dx$$

Optimal (type 4, 534 leaves, 14 steps):

$$\begin{aligned} & - \frac{(a^2 + 2b^2) e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{3/2} (-a^2+b^2)^{7/4} d} - \frac{(a^2 + 2b^2) e^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{3/2} (-a^2+b^2)^{7/4} d} - \\ & \frac{a e^2 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{4 b^2 (a^2 - b^2) d \sqrt{e \sin[c+dx]}} + \frac{a (a^2 + 2b^2) e^2 \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{8 b^2 (a^2 - b^2) \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \sin[c+dx]}} + \\ & \frac{a (a^2 + 2b^2) e^2 \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{8 b^2 (a^2 - b^2) \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \sin[c+dx]}} + \frac{e \sqrt{e \sin[c+dx]}}{2 b d (a + b \cos[c+dx])^2} - \frac{a e \sqrt{e \sin[c+dx]}}{4 b (a^2 - b^2) d (a + b \cos[c+dx])} \end{aligned}$$

Result (type 6, 1211 leaves):

$$\begin{aligned}
& \frac{\left( \frac{1}{2b(a+b\cos[c+dx])^2} + \frac{a}{4b(-a^2+b^2)(a+b\cos[c+dx])} \right) \operatorname{Csc}[c+dx] (e \operatorname{Sin}[c+dx])^{3/2}}{d} - \\
& \frac{1}{8(a-b)b(a+b)d \operatorname{Sin}[c+dx]^{3/2}} (e \operatorname{Sin}[c+dx])^{3/2} \left( \frac{1}{(a+b\cos[c+dx])(1-\operatorname{Sin}[c+dx]^2)} 2a \cos[c+dx]^2 (a+b\sqrt{1-\operatorname{Sin}[c+dx]^2}) \right. \\
& \left. \left( \frac{1}{4\sqrt{2}\sqrt{b}(a^2-b^2)^{3/4}} a \left( -2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{b}\sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{b}\sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2-b^2} - \right. \right. \right. \\
& \left. \left. \left. \sqrt{2}\sqrt{b}(a^2-b^2)^{1/4}\sqrt{\operatorname{Sin}[c+dx]} + b \operatorname{Sin}[c+dx]\right] + \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2}\sqrt{b}(a^2-b^2)^{1/4}\sqrt{\operatorname{Sin}[c+dx]} + b \operatorname{Sin}[c+dx]\right] \right) \right) + \\
& \left( 5b(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \sqrt{\operatorname{Sin}[c+dx]} \sqrt{1-\operatorname{Sin}[c+dx]^2} \right) / \left( \left( -5(a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] + 2 \left( 2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \right) \right. \right. \\
& \left. \left. (a^2-b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sin}[c+dx]^2 (a^2+b^2(-1+\operatorname{Sin}[c+dx]^2)) \right) \right) \right) - \\
& \frac{1}{(a+b\cos[c+dx])\sqrt{1-\operatorname{Sin}[c+dx]^2}} 4b \cos[c+dx] (a+b\sqrt{1-\operatorname{Sin}[c+dx]^2}) \left( -\frac{1}{(-a^2+b^2)^{3/4}} \left( \frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \right. \\
& \left. \left( 2 \operatorname{ArcTan}\left[1 - \frac{(1+i)\sqrt{b}\sqrt{\operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i)\sqrt{b}\sqrt{\operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i)\sqrt{b}(-a^2+b^2)^{1/4} \right. \right. \right. \\
& \left. \left. \left. \sqrt{\operatorname{Sin}[c+dx]} + ib \operatorname{Sin}[c+dx]\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\operatorname{Sin}[c+dx]} + ib \operatorname{Sin}[c+dx]\right] \right) \right) + \\
& \left( 5a(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \sqrt{\operatorname{Sin}[c+dx]} \right) / \left( \sqrt{1-\operatorname{Sin}[c+dx]^2} \left( 5(a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] - 2 \left( 2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \right) \right. \right. \\
& \left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sin}[c+dx]^2 (a^2+b^2(-1+\operatorname{Sin}[c+dx]^2)) \right) \right) \right) \right)
\end{aligned}$$

- **Problem 84: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{e \sin[c + d x]}}{(a + b \cos[c + d x])^3} dx$$

Optimal (type 4, 529 leaves, 14 steps):

$$\begin{aligned} & - \frac{(3 a^2 + 2 b^2) \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c + d x]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right]}{8 \sqrt{b} (-a^2 + b^2)^{9/4} d} + \frac{(3 a^2 + 2 b^2) \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c + d x]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right]}{8 \sqrt{b} (-a^2 + b^2)^{9/4} d} + \\ & \frac{a (3 a^2 + 2 b^2) e \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + d x), 2\right] \sqrt{\sin[c + d x]}}{8 b (a^2 - b^2)^2 (b - \sqrt{-a^2 + b^2}) d \sqrt{e \sin[c + d x]}} + \\ & \frac{a (3 a^2 + 2 b^2) e \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + d x), 2\right] \sqrt{\sin[c + d x]}}{8 b (a^2 - b^2)^2 (b + \sqrt{-a^2 + b^2}) d \sqrt{e \sin[c + d x]}} + \\ & \frac{5 a \operatorname{EllipticE}\left[\frac{1}{2} (c - \frac{\pi}{2} + d x), 2\right] \sqrt{e \sin[c + d x]}}{4 (a^2 - b^2)^2 d \sqrt{\sin[c + d x]}} - \frac{b (e \sin[c + d x])^{3/2}}{2 (a^2 - b^2) d e (a + b \cos[c + d x])^2} - \frac{5 a b (e \sin[c + d x])^{3/2}}{4 (a^2 - b^2)^2 d e (a + b \cos[c + d x])} \end{aligned}$$

Result (type 6, 1232 leaves):

$$\begin{aligned}
& \frac{\sqrt{e \sin[c+dx]} \left( -\frac{b \sin[c+dx]}{2(a^2-b^2)(a+b \cos[c+dx])^2} - \frac{5ab \sin[c+dx]}{4(a^2-b^2)^2(a+b \cos[c+dx])} \right)}{d} + \\
& \frac{1}{8(a-b)^2(a+b)^2 d \sqrt{\sin[c+dx]}} \sqrt{e \sin[c+dx]} \left( \frac{1}{(a+b \cos[c+dx])(1-\sin[c+dx])^2} 10ab \cos[c+dx]^2 (a+b \sqrt{1-\sin[c+dx]^2}) \right. \\
& \left. \left( \frac{1}{4\sqrt{2} b^{3/2} (a^2-b^2)^{1/4}} a \left( -2 \operatorname{ArcTan} \left[ 1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}} \right] + 2 \operatorname{ArcTan} \left[ 1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}} \right] + \operatorname{Log} \left[ \sqrt{a^2-b^2} - \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx] \right] - \operatorname{Log} \left[ \sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx] \right] \right) \right) + \\
& \left( 7b(a^2-b^2) \operatorname{AppellF1} \left[ \frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \sin[c+dx]^{3/2} \sqrt{1-\sin[c+dx]^2} \right) / \left( 3 \left( -7(a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] + 2 \left( 2b^2 \operatorname{AppellF1} \left[ \frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2-b^2) \operatorname{AppellF1} \left[ \frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \right) \sin[c+dx]^2 \right) (a^2+b^2(-1+\sin[c+dx]^2)) \right) \left. \right) + \\
& \frac{1}{12(a+b \cos[c+dx]) \sqrt{1-\sin[c+dx]^2}} (8a^2+2b^2) \cos[c+dx] (a+b \sqrt{1-\sin[c+dx]^2}) \\
& \left( \frac{1}{\sqrt{b} (-a^2+b^2)^{1/4}} (3+3i) \left( 2 \operatorname{ArcTan} \left[ 1 - \frac{(1+i) \sqrt{b} \sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[ 1 + \frac{(1+i) \sqrt{b} \sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}} \right] - \operatorname{Log} \left[ \sqrt{-a^2+b^2} - (1+i) \right. \right. \right. \\
& \left. \left. \left. \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin[c+dx]} + i b \sin[c+dx] \right] + \operatorname{Log} \left[ \sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin[c+dx]} + i b \sin[c+dx] \right] \right) \right) + \\
& \left( 56a(a^2-b^2) \operatorname{AppellF1} \left[ \frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \sin[c+dx]^{3/2} \right) / \left( \sqrt{1-\sin[c+dx]^2} \left( 7(a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] - 2 \left( 2b^2 \operatorname{AppellF1} \left[ \frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2) \operatorname{AppellF1} \left[ \frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \right) \sin[c+dx]^2 \right) (a^2+b^2(-1+\sin[c+dx]^2)) \right) \left. \right) \left. \right)
\end{aligned}$$

■ **Problem 85: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \cos[c+dx])^3 \sqrt{e \sin[c+dx]}} dx$$

Optimal (type 4, 535 leaves, 14 steps):

$$\begin{aligned}
 & \frac{3 \sqrt{b} (5 a^2 + 2 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2+b^2)^{11/4} d \sqrt{e}} + \frac{3 \sqrt{b} (5 a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2+b^2)^{11/4} d \sqrt{e}} - \\
 & \frac{7 a \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{4 (a^2 - b^2)^2 d \sqrt{e \operatorname{Sin}[c+dx]}} + \frac{3 a (5 a^2 + 2 b^2) \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{8 (a^2 - b^2)^2 \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \operatorname{Sin}[c+dx]}} + \\
 & \frac{3 a (5 a^2 + 2 b^2) \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{8 (a^2 - b^2)^2 \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \operatorname{Sin}[c+dx]}} - \frac{b \sqrt{e \operatorname{Sin}[c+dx]}}{2 (a^2 - b^2) d e (a + b \operatorname{Cos}[c+dx])^2} - \frac{7 a b \sqrt{e \operatorname{Sin}[c+dx]}}{4 (a^2 - b^2)^2 d e (a + b \operatorname{Cos}[c+dx])}
 \end{aligned}$$

Result (type 6, 1226 leaves):

$$\begin{aligned}
& \frac{\left( -\frac{b}{2(a^2-b^2)(a+b\cos[c+dx])^2} - \frac{7ab}{4(a^2-b^2)^2(a+b\cos[c+dx])} \right) \sin[c+dx]}{d\sqrt{e\sin[c+dx]}} + \\
& \frac{1}{8(a-b)^2(a+b)^2d\sqrt{e\sin[c+dx]}} \sqrt{\sin[c+dx]} \left( -\frac{1}{(a+b\cos[c+dx])(1-\sin[c+dx]^2)} 14ab\cos[c+dx]^2 \left( a+b\sqrt{1-\sin[c+dx]^2} \right) \right. \\
& \left. \left( \frac{1}{4\sqrt{2}\sqrt{b}(a^2-b^2)^{3/4}} a \left( -2\operatorname{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{b}\sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}}\right] + 2\operatorname{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{b}\sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2-b^2} - \sqrt{2}\sqrt{b}\right. \right. \right. \right. \\
& \left. \left. \left. (a^2-b^2)^{1/4}\sqrt{\sin[c+dx]} + b\sin[c+dx]\right] + \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2}\sqrt{b}(a^2-b^2)^{1/4}\sqrt{\sin[c+dx]} + b\sin[c+dx]\right] \right) \right) + \\
& \left( 5b(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2\sin[c+dx]^2}{-a^2+b^2}\right] \sqrt{\sin[c+dx]} \sqrt{1-\sin[c+dx]^2} \right) / \left( \left( -5(a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2\sin[c+dx]^2}{-a^2+b^2}\right] + 2 \left( 2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin[c+dx]^2, \frac{b^2\sin[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (a^2-b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \frac{b^2\sin[c+dx]^2}{-a^2+b^2}\right] \right) \sin[c+dx]^2 \right) (a^2+b^2(-1+\sin[c+dx]^2)) \right) \Bigg) + \\
& \frac{1}{(a+b\cos[c+dx])\sqrt{1-\sin[c+dx]^2}} 2(8a^2+6b^2)\cos[c+dx] \left( a+b\sqrt{1-\sin[c+dx]^2} \right) \left( -\frac{1}{(-a^2+b^2)^{3/4}} \right. \\
& \left. \left( \frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left( 2\operatorname{ArcTan}\left[1 - \frac{(1+i)\sqrt{b}\sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - 2\operatorname{ArcTan}\left[1 + \frac{(1+i)\sqrt{b}\sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i)\sqrt{b}\right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2)^{1/4}\sqrt{\sin[c+dx]} + ib\sin[c+dx]\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\sin[c+dx]} + ib\sin[c+dx]\right] \right) \right) + \\
& \left( 5a(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2\sin[c+dx]^2}{-a^2+b^2}\right] \sqrt{\sin[c+dx]} \right) / \left( \sqrt{1-\sin[c+dx]^2} \left( 5(a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2\sin[c+dx]^2}{-a^2+b^2}\right] - 2 \left( 2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c+dx]^2, \frac{b^2\sin[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \frac{b^2\sin[c+dx]^2}{-a^2+b^2}\right] \right) \sin[c+dx]^2 \right) (a^2+b^2(-1+\sin[c+dx]^2)) \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 86: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \cos[c + dx])^3 (e \sin[c + dx])^{3/2}} dx$$

Optimal (type 4, 611 leaves, 15 steps):

$$\begin{aligned} & -\frac{5 b^{3/2} (7 a^2 + 2 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2+b^2)^{13/4} d e^{3/2}} + \frac{5 b^{3/2} (7 a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2+b^2)^{13/4} d e^{3/2}} - \\ & \frac{b}{2 (a^2 - b^2) d e (a + b \cos[c + dx])^2 \sqrt{e \sin[c + dx]}} - \frac{9 a b}{4 (a^2 - b^2)^2 d e (a + b \cos[c + dx]) \sqrt{e \sin[c + dx]}} + \\ & \frac{5 b (7 a^2 + 2 b^2) - a (8 a^2 + 37 b^2) \cos[c + dx]}{4 (a^2 - b^2)^3 d e \sqrt{e \sin[c + dx]}} - \frac{5 a b (7 a^2 + 2 b^2) \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\sin[c + dx]}}{8 (a^2 - b^2)^3 (b - \sqrt{-a^2 + b^2}) d e \sqrt{e \sin[c + dx]}} - \\ & \frac{5 a b (7 a^2 + 2 b^2) \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\sin[c + dx]}}{8 (a^2 - b^2)^3 (b + \sqrt{-a^2 + b^2}) d e \sqrt{e \sin[c + dx]}} - \frac{a (8 a^2 + 37 b^2) \operatorname{EllipticE}\left[\frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{e \sin[c + dx]}}{4 (a^2 - b^2)^3 d e^2 \sqrt{\sin[c + dx]}} \end{aligned}$$

Result (type 6, 1316 leaves):

$$\begin{aligned} & \frac{1}{d (e \sin[c + dx])^{3/2}} \sin[c + dx]^2 \\ & \left( -\frac{2 (-3 a^2 b - b^3 + a^3 \cos[c + dx] + 3 a b^2 \cos[c + dx]) \operatorname{Csc}[c + dx]}{(a^2 - b^2)^3} + \frac{b^3 \sin[c + dx]}{2 (a^2 - b^2)^2 (a + b \cos[c + dx])^2} + \frac{13 a b^3 \sin[c + dx]}{4 (a^2 - b^2)^3 (a + b \cos[c + dx])} \right) - \\ & \frac{1}{8 (a - b)^3 (a + b)^3 d (e \sin[c + dx])^{3/2}} \\ & \sin[c + dx]^{3/2} \left( \frac{1}{(a + b \cos[c + dx]) (1 - \sin[c + dx])^2} - 2 (8 a^3 b + 37 a b^3) \cos[c + dx]^2 (a + b \sqrt{1 - \sin[c + dx]})^2 \right) \\ & \left( \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left( -2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{a^2 - b^2} - \right. \right. \right. \\ & \left. \left. \left. \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx]\right] - \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx]\right] \right) \right) + \\ & \left( 7 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] \sin[c + dx]^{3/2} \sqrt{1 - \sin[c + dx]^2} \right) / \left( 3 \left( -7 (a^2 - b^2) \right) \right) \end{aligned}$$



$$\begin{aligned}
& \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] + 2 \left( 2b^2 \text{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] + \right. \\
& \left. (a^2 - b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] \right) \sin[c+dx]^2 \left( a^2 + b^2 (-1 + \sin[c+dx]^2) \right) \Bigg) + \\
& \frac{1}{12 (a + b \cos[c+dx]) \sqrt{1 - \sin[c+dx]^2}} (8a^4 + 72a^2b^2 + 10b^4) \cos[c+dx] \left( a + b \sqrt{1 - \sin[c+dx]^2} \right) \\
& \left( \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} (3 + 3i) \left( 2 \text{ArcTan}\left[1 - \frac{(1+i)\sqrt{b}\sqrt{\sin[c+dx]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \text{ArcTan}\left[1 + \frac{(1+i)\sqrt{b}\sqrt{\sin[c+dx]}}{(-a^2 + b^2)^{1/4}}\right] - \text{Log}\left[\sqrt{-a^2 + b^2} - (1+i)\right. \right. \right. \\
& \left. \left. \left. \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c+dx]} + ib \sin[c+dx]\right] + \text{Log}\left[\sqrt{-a^2 + b^2} + (1+i)\sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c+dx]} + ib \sin[c+dx]\right] \right) \right) + \\
& \left( 56a (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] \sin[c+dx]^{3/2} \right) / \left( \sqrt{1 - \sin[c+dx]^2} \left( 7 (a^2 - b^2) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] - 2 \left( 2b^2 \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] \right) \sin[c+dx]^2 \left( a^2 + b^2 (-1 + \sin[c+dx]^2) \right) \right) \right) \Bigg) \Bigg)
\end{aligned}$$

- **Problem 87: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \cos[c + dx])^3 (e \sin[c + dx])^{5/2}} dx$$

Optimal (type 4, 629 leaves, 15 steps):

$$\begin{aligned}
& \frac{7 b^{5/2} (9 a^2 + 2 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2+b^2)^{15/4} d e^{5/2}} + \frac{7 b^{5/2} (9 a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2+b^2)^{15/4} d e^{5/2}} - \frac{b}{2 (a^2-b^2) d e (a+b \cos[c+dx])^2 (e \sin[c+dx])^{3/2}} \\
& \frac{11 a b}{4 (a^2-b^2)^2 d e (a+b \cos[c+dx]) (e \sin[c+dx])^{3/2}} + \frac{7 b (9 a^2 + 2 b^2) - a (8 a^2 + 69 b^2) \cos[c+dx]}{12 (a^2-b^2)^3 d e (e \sin[c+dx])^{3/2}} + \\
& \frac{a (8 a^2 + 69 b^2) \operatorname{EllipticF}\left[\frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\sin[c+dx]}}{12 (a^2-b^2)^3 d e^2 \sqrt{e \sin[c+dx]}} - \frac{7 a b^2 (9 a^2 + 2 b^2) \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2+b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\sin[c+dx]}}{8 (a^2-b^2)^3 \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) d e^2 \sqrt{e \sin[c+dx]}} \\
& \frac{7 a b^2 (9 a^2 + 2 b^2) \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2+b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\sin[c+dx]}}{8 (a^2-b^2)^3 \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) d e^2 \sqrt{e \sin[c+dx]}}
\end{aligned}$$

Result (type 6, 1308 leaves):

$$\begin{aligned}
& \frac{1}{d (e \sin[c+dx])^{5/2}} \\
& \left( \frac{b^3}{2 (a^2-b^2)^2 (a+b \cos[c+dx])^2} + \frac{15 a b^3}{4 (a^2-b^2)^3 (a+b \cos[c+dx])} - \frac{2 (-3 a^2 b - b^3 + a^3 \cos[c+dx] + 3 a b^2 \cos[c+dx]) \operatorname{Csc}[c+dx]^2}{3 (a^2-b^2)^3} \right) \\
& \sin[c+dx]^3 + \frac{1}{24 (a-b)^3 (a+b)^3 d (e \sin[c+dx])^{5/2}} \\
& \sin[c+dx]^{5/2} \left( \frac{1}{(a+b \cos[c+dx]) (1-\sin[c+dx])^2} - 2 (8 a^3 b + 69 a b^3) \cos[c+dx]^2 \left(a + b \sqrt{1-\sin[c+dx]^2}\right) \right) \\
& \left( \frac{1}{4 \sqrt{2} \sqrt{b} (a^2-b^2)^{3/4}} a \left( -2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2-b^2} - \right. \right. \right. \\
& \left. \left. \left. \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right] + \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right] \right) \right) + \\
& \left( 5 b (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] \sqrt{\sin[c+dx]} \sqrt{1-\sin[c+dx]^2} \right) / \left( \left( -5 (a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] + 2 \left( 2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned} & \left. \left( (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^2 (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right) + \\ & \frac{1}{(a + b \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} 2 (8 a^4 - 120 a^2 b^2 - 42 b^4) \cos[c + dx] \left( a + b \sqrt{1 - \sin[c + dx]^2} \right) \left( -\frac{1}{(-a^2 + b^2)^{3/4}} \right. \\ & \left. \left( \frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left( 2 \operatorname{ArcTan} \left[ 1 - \frac{(1+i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[ 1 + \frac{(1+i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] + \operatorname{Log} \left[ \sqrt{-a^2 + b^2} - (1+i) \sqrt{b} \right. \right. \right. \\ & \left. \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] - \operatorname{Log} \left[ \sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] \right) \right) + \\ & \left( 5 a (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\sin[c + dx]} \right) / \left( \sqrt{1 - \sin[c + dx]^2} \left( 5 (a^2 - b^2) \right. \right. \\ & \left. \left. \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\ & \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right) \right) \right) \end{aligned}$$

■ **Problem 88: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \cos[c + dx])^3 (e \sin[c + dx])^{7/2}} dx$$

Optimal (type 4, 700 leaves, 16 steps):

$$\begin{aligned}
& - \frac{9 b^{7/2} (11 a^2 + 2 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2 + b^2)^{17/4} d e^{7/2}} + \frac{9 b^{7/2} (11 a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2 + b^2)^{17/4} d e^{7/2}} - \\
& \frac{b}{2 (a^2 - b^2) d e (a + b \cos[c + dx])^2 (e \sin[c + dx])^{5/2}} - \frac{13 a b}{4 (a^2 - b^2)^2 d e (a + b \cos[c + dx]) (e \sin[c + dx])^{5/2}} + \\
& \frac{9 b (11 a^2 + 2 b^2) - a (8 a^2 + 109 b^2) \cos[c + dx]}{20 (a^2 - b^2)^3 d e (e \sin[c + dx])^{5/2}} - \frac{3 (15 b^3 (11 a^2 + 2 b^2) + a (8 a^4 - 64 a^2 b^2 - 139 b^4) \cos[c + dx])}{20 (a^2 - b^2)^4 d e^3 \sqrt{e \sin[c + dx]}} + \\
& \frac{9 a b^3 (11 a^2 + 2 b^2) \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\sin[c + dx]}}{8 (a^2 - b^2)^4 (b - \sqrt{-a^2 + b^2}) d e^3 \sqrt{e \sin[c + dx]}} + \\
& \frac{9 a b^3 (11 a^2 + 2 b^2) \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\sin[c + dx]}}{8 (a^2 - b^2)^4 (b + \sqrt{-a^2 + b^2}) d e^3 \sqrt{e \sin[c + dx]}} - \\
& \frac{3 a (8 a^4 - 64 a^2 b^2 - 139 b^4) \operatorname{EllipticE}\left[\frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{e \sin[c + dx]}}{20 (a^2 - b^2)^4 d e^4 \sqrt{\sin[c + dx]}}
\end{aligned}$$

Result (type 6, 1408 leaves):

$$\begin{aligned}
& \frac{1}{d (e \sin[c + dx])^{7/2}} \sin[c + dx]^4 \left( - \frac{2 (50 a^2 b^3 + 10 b^5 + 3 a^5 \cos[c + dx] - 24 a^3 b^2 \cos[c + dx] - 39 a b^4 \cos[c + dx]) \operatorname{Csc}[c + dx]}{5 (a^2 - b^2)^4} - \right. \\
& \frac{2 (-3 a^2 b - b^3 + a^3 \cos[c + dx] + 3 a b^2 \cos[c + dx]) \operatorname{Csc}[c + dx]^3}{5 (a^2 - b^2)^3} - \frac{b^5 \sin[c + dx]}{2 (a^2 - b^2)^3 (a + b \cos[c + dx])^2} - \\
& \left. \frac{21 a b^5 \sin[c + dx]}{4 (a^2 - b^2)^4 (a + b \cos[c + dx])} \right) - \frac{1}{40 (a - b)^4 (a + b)^4 d (e \sin[c + dx])^{7/2}} \\
& 3 \sin[c + dx]^{7/2} \left( \frac{1}{(a + b \cos[c + dx]) (1 - \sin[c + dx])^2} 2 (8 a^5 b - 64 a^3 b^3 - 139 a b^5) \cos[c + dx]^2 (a + b \sqrt{1 - \sin[c + dx]^2}) \right. \\
& \left( \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left( -2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{a^2 - b^2} - \right. \right. \\
& \left. \left. \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx]\right] - \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + b \sin[c + dx]\right] \right) \left. \right) +
\end{aligned}$$

$$\begin{aligned}
& \left( 7 b (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^{3/2} \sqrt{1 - \sin[c + dx]^2} \right) / \left( 3 \left( -7 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \Bigg) + \\
& \frac{1}{12 (a + b \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} (8 a^6 - 64 a^4 b^2 - 304 a^2 b^4 - 30 b^6) \cos[c + dx] (a + b \sqrt{1 - \sin[c + dx]^2}) \\
& \left( \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} (3 + 3 i) \left( 2 \operatorname{ArcTan} \left[ 1 - \frac{(1 + i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[ 1 + \frac{(1 + i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - \operatorname{Log} \left[ \sqrt{-a^2 + b^2} - (1 + i) \right. \right. \right. \\
& \left. \left. \left. \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] + \operatorname{Log} \left[ \sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] \right) \right) + \\
& \left( 56 a (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^{3/2} \right) / \left( \sqrt{1 - \sin[c + dx]^2} \left( 7 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \Bigg) \Bigg)
\end{aligned}$$

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## Test results for the 34 problems in "4.2.13 (d+e x)^m cos(a+b x+c x^2)^n.m"

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## Test results for the 22 problems in "4.2.1.3 (g tan)^p (a+b cos)^m.m"

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- Problem 1: Result more than twice size of optimal antiderivative.

$$\int \frac{\tan[x]^4}{a + a \cos[x]} dx$$

Optimal (type 3, 33 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}[\sin[x]]}{2 a} - \frac{\sec[x] \tan[x]}{2 a} + \frac{\tan[x]^3}{3 a}$$

Result (type 3, 105 leaves):

$$-\frac{1}{24a} \operatorname{Sec}[x]^3 \left( 9 \operatorname{Cos}[x] \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right] \right) + \right. \\ \left. 3 \operatorname{Cos}[3x] \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right] \right) + 2(-3 \operatorname{Sin}[x] + 3 \operatorname{Sin}[2x] + \operatorname{Sin}[3x]) \right)$$

- **Problem 3: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[x]^2}{a + a \operatorname{Cos}[x]} dx$$

Optimal (type 3, 15 leaves, 4 steps):

$$-\frac{\operatorname{ArcTanh}[\operatorname{Sin}[x]]}{a} + \frac{\operatorname{Tan}[x]}{a}$$

Result (type 3, 39 leaves):

$$\frac{\operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right] + \operatorname{Tan}[x]}{a}$$

- **Problem 19: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \operatorname{Cos}[x]} \operatorname{Tan}[x] dx$$

Optimal (type 3, 37 leaves, 4 steps):

$$2\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Cos}[x]}}{\sqrt{a}}\right] - 2\sqrt{a + b \operatorname{Cos}[x]}$$

Result (type 3, 75 leaves):

$$-\frac{2\sqrt{a + b \operatorname{Cos}[x]} \left( b + a \operatorname{Sec}[x] - \sqrt{a} \sqrt{b} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[x]}}{\sqrt{b}}\right] \sqrt{\operatorname{Sec}[x]} \sqrt{1 + \frac{a \operatorname{Sec}[x]}{b}} \right)}{b + a \operatorname{Sec}[x]}$$

- **Problem 20: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[x]}{\sqrt{a + b \operatorname{Cos}[x]}} dx$$

Optimal (type 3, 24 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Cos}[x]}}{\sqrt{a}}\right]}{\sqrt{a}}$$

Result (type 3, 60 leaves):

$$\frac{2\sqrt{b} \operatorname{ArcSinh}\left[\frac{\sqrt{a}\sqrt{\sec[x]}}{\sqrt{b}}\right] \sqrt{\frac{b+a\sec[x]}{b}}}{\sqrt{a}\sqrt{a+b\cos[x]}\sqrt{\sec[x]}}$$

- **Problem 21: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{e \tan[c+dx]}}{a+b\cos[c+dx]} dx$$

Optimal (type 4, 204 leaves, 9 steps):

$$\frac{2\sqrt{2}\sqrt{\cos[c+dx]}\operatorname{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{\sin[c+dx]}}{\sqrt{1+\cos[c+dx]}}\right], -1\right]\sqrt{e \tan[c+dx]}}{\sqrt{-a+b}\sqrt{a+b}d\sqrt{\sin[c+dx]}} +$$

$$\frac{2\sqrt{2}\sqrt{\cos[c+dx]}\operatorname{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{\sin[c+dx]}}{\sqrt{1+\cos[c+dx]}}\right], -1\right]\sqrt{e \tan[c+dx]}}{\sqrt{-a+b}\sqrt{a+b}d\sqrt{\sin[c+dx]}}$$

Result (type 6, 584 leaves):

$$\frac{1}{d(a+b\cos[c+dx])\sqrt{\tan[c+dx]}(1+\tan[c+dx])^{3/2}} 2\sec[c+dx]^2\sqrt{e \tan[c+dx]}(b+a\sqrt{1+\tan[c+dx]^2})$$

$$\left(\frac{1}{4\sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}}\left(-2\operatorname{ArcTan}\left[1-\frac{\sqrt{2}\sqrt{a}\sqrt{\tan[c+dx]}}{(a^2-b^2)^{1/4}}\right]+2\operatorname{ArcTan}\left[1+\frac{\sqrt{2}\sqrt{a}\sqrt{\tan[c+dx]}}{(a^2-b^2)^{1/4}}\right]\right)+\right.$$

$$\left.\operatorname{Log}\left[\sqrt{a^2-b^2}-\sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\tan[c+dx]}+a\tan[c+dx]\right]-\operatorname{Log}\left[\sqrt{a^2-b^2}+\sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\tan[c+dx]}+a\tan[c+dx]\right]\right)+$$

$$\left(7b(a^2-b^2)\operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[c+dx]^2, -\frac{a^2\tan[c+dx]^2}{a^2-b^2}\right]\tan[c+dx]^{3/2}\right)/$$

$$\left(3\sqrt{1+\tan[c+dx]^2}\left(-7(a^2-b^2)\operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[c+dx]^2, -\frac{a^2\tan[c+dx]^2}{a^2-b^2}\right]+2\left(2a^2\operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[c+dx]^2, -\frac{a^2\tan[c+dx]^2}{a^2-b^2}\right]+(a^2-b^2)\right.\right.\right.$$

$$\left.\left.\operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\tan[c+dx]^2, -\frac{a^2\tan[c+dx]^2}{a^2-b^2}\right]\tan[c+dx]^2\right)(-b^2+a^2(1+\tan[c+dx]^2))\right)$$

## Test results for the 932 problems in "4.2.2.1 (a+b cos)^m (c+d cos)^n.m"

- **Problem 7: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x]) \sec [c + d x] dx$$

Optimal (type 3, 16 leaves, 2 steps) :

$$a x + \frac{a \operatorname{ArcTanh}[\sin [c + d x]]}{d}$$

Result (type 3, 73 leaves) :

$$a x - \frac{a \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d}$$

- **Problem 8: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x]) \sec [c + d x]^2 dx$$

Optimal (type 3, 24 leaves, 4 steps) :

$$\frac{a \operatorname{ArcTanh}[\sin [c + d x]]}{d} + \frac{a \tan [c + d x]}{d}$$

Result (type 3, 81 leaves) :

$$- \frac{a \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a \tan [c + d x]}{d}$$

- **Problem 9: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x]) \sec [c + d x]^3 dx$$

Optimal (type 3, 47 leaves, 5 steps) :

$$\frac{a \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{a \tan [c + d x]}{d} + \frac{a \sec [c + d x] \tan [c + d x]}{2 d}$$

Result (type 3, 138 leaves) :

$$- \frac{a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} + \frac{a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} + \frac{a \tan [c + d x]}{d} - \frac{a}{4 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{a}{4 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{a \tan [c + d x]}{d}$$



■ **Problem 10: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x]) \sec [c + d x]^4 dx$$

Optimal (type 3, 63 leaves, 5 steps):

$$\frac{a \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{a \tan [c + d x]}{d} + \frac{a \sec [c + d x] \tan [c + d x]}{2 d} + \frac{a \tan [c + d x]^3}{3 d}$$

Result (type 3, 163 leaves):

$$\begin{aligned} & - \frac{a \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right] - \sin \left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \frac{a \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \\ & \frac{a}{4 d \left(\cos \left[\frac{1}{2}(c+d x)\right] - \sin \left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{a}{4 d \left(\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{2 a \tan [c + d x]}{3 d} + \frac{a \sec [c + d x]^2 \tan [c + d x]}{3 d} \end{aligned}$$

■ **Problem 11: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x]) \sec [c + d x]^5 dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$\frac{3 a \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} + \frac{a \tan [c + d x]}{d} + \frac{3 a \sec [c + d x] \tan [c + d x]}{8 d} + \frac{a \sec [c + d x]^3 \tan [c + d x]}{4 d} + \frac{a \tan [c + d x]^3}{3 d}$$

Result (type 3, 227 leaves):

$$\begin{aligned} & - \frac{3 a \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right] - \sin \left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{3 a \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \\ & \frac{a}{16 d \left(\cos \left[\frac{1}{2}(c+d x)\right] - \sin \left[\frac{1}{2}(c+d x)\right]\right)^4} + \frac{3 a}{16 d \left(\cos \left[\frac{1}{2}(c+d x)\right] - \sin \left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{a}{16 d \left(\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right)^4} - \\ & \frac{3 a}{16 d \left(\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{2 a \tan [c + d x]}{3 d} + \frac{a \sec [c + d x]^2 \tan [c + d x]}{3 d} \end{aligned}$$

■ **Problem 12: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x]) \sec [c + d x]^6 dx$$

Optimal (type 3, 101 leaves, 6 steps):

$$\frac{3 a \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} + \frac{a \tan [c + d x]}{d} + \frac{3 a \sec [c + d x] \tan [c + d x]}{8 d} + \frac{a \sec [c + d x]^3 \tan [c + d x]}{4 d} + \frac{2 a \tan [c + d x]^3}{3 d} + \frac{a \tan [c + d x]^5}{5 d}$$

Result (type 3, 249 leaves):

$$\begin{aligned}
& - \frac{3 a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{3 a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \\
& \frac{a}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^4} + \frac{3 a}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{a}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^4} - \\
& \frac{3 a}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{8 a \tan [c+d x]}{15 d} + \frac{4 a \sec [c+d x]^2 \tan [c+d x]}{15 d} + \frac{a \sec [c+d x]^4 \tan [c+d x]}{5 d}
\end{aligned}$$

■ **Problem 18: Result more than twice size of optimal antiderivative.**

$$\int (a+a \cos [c+d x])^2 \sec [c+d x] d x$$

Optimal (type 3, 34 leaves, 3 steps):

$$2 a^2 x + \frac{a^2 \operatorname{ArcTanh}[\sin [c+d x]]}{d} + \frac{a^2 \sin [c+d x]}{d}$$

Result (type 3, 106 leaves):

$$2 a^2 x - \frac{a^2 \operatorname{Log}\left[\cos\left[\frac{c}{2}+\frac{d x}{2}\right]-\sin\left[\frac{c}{2}+\frac{d x}{2}\right]\right]}{d} + \frac{a^2 \operatorname{Log}\left[\cos\left[\frac{c}{2}+\frac{d x}{2}\right]+\sin\left[\frac{c}{2}+\frac{d x}{2}\right]\right]}{d} + \frac{a^2 \cos [d x] \sin [c]}{d} + \frac{a^2 \cos [c] \sin [d x]}{d}$$

■ **Problem 20: Result more than twice size of optimal antiderivative.**

$$\int (a+a \cos [c+d x])^2 \sec [c+d x]^3 d x$$

Optimal (type 3, 54 leaves, 7 steps):

$$\frac{3 a^2 \operatorname{ArcTanh}[\sin [c+d x]]}{2 d} + \frac{2 a^2 \tan [c+d x]}{d} + \frac{a^2 \sec [c+d x] \tan [c+d x]}{2 d}$$

Result (type 3, 119 leaves):

$$\begin{aligned}
& \frac{1}{4 d} a^2 \left( -6 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right] + 6 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right] \right) + \\
& \left( \frac{1}{\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{1}{\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^2} + 8 \tan [c+d x] \right)
\end{aligned}$$

■ **Problem 21: Result more than twice size of optimal antiderivative.**

$$\int (a+a \cos [c+d x])^2 \sec [c+d x]^4 d x$$

Optimal (type 3, 66 leaves, 8 steps):



■ **Problem 28: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^3 \sec [c + d x]^2 dx$$

Optimal (type 3, 48 leaves, 6 steps):

$$3 a^3 x + \frac{3 a^3 \operatorname{ArcTanh}[\sin [c + d x]]}{d} + \frac{a^3 \sin [c + d x]}{d} + \frac{a^3 \tan [c + d x]}{d}$$

Result (type 3, 211 leaves):

$$\frac{1}{8} a^3 (1 + \cos [c + d x])^3 \sec \left[ \frac{1}{2} (c + d x) \right]^6$$

$$\left( 3 x - \frac{3 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{d} + \frac{3 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{d} + \frac{\cos [d x] \sin [c]}{d} + \frac{\cos [c] \sin [d x]}{d} + \frac{\sin \left[ \frac{d x}{2} \right]}{d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)} + \frac{\sin \left[ \frac{d x}{2} \right]}{d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)} \right)$$

■ **Problem 29: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^3 \sec [c + d x]^3 dx$$

Optimal (type 3, 59 leaves, 7 steps):

$$a^3 x + \frac{7 a^3 \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{3 a^3 \tan [c + d x]}{d} + \frac{a^3 \sec [c + d x] \tan [c + d x]}{2 d}$$

Result (type 3, 144 leaves):

$$a^3 \left( \frac{c}{d} + x - \frac{7 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{2 d} + \frac{7 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{2 d} + \frac{1}{4 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \frac{1}{4 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{3 \tan [c + d x]}{d} \right)$$

■ **Problem 30: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^3 \sec [c + d x]^4 dx$$

Optimal (type 3, 72 leaves, 9 steps):

$$\frac{5 a^3 \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{4 a^3 \tan [c + d x]}{d} + \frac{3 a^3 \sec [c + d x] \tan [c + d x]}{2 d} + \frac{a^3 \tan [c + d x]^3}{3 d}$$

Result (type 3, 669 leaves):



Optimal (type 3, 114 leaves, 11 steps):

$$\frac{13 a^3 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{4 a^3 \operatorname{Tan}[c + d x]}{d} + \frac{13 a^3 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{3 a^3 \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d} + \frac{5 a^3 \operatorname{Tan}[c + d x]^3}{3 d} + \frac{a^3 \operatorname{Tan}[c + d x]^5}{5 d}$$

Result (type 3, 487 leaves):

$$-\frac{1}{3840 d} a^3 \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^5 \left( \begin{aligned} & \left( 975 \operatorname{Cos}[2 c + 3 d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 975 \operatorname{Cos}[4 c + 3 d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + \\ & 195 \operatorname{Cos}[4 c + 5 d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 195 \operatorname{Cos}[6 c + 5 d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \\ & 1950 \operatorname{Cos}[d x] \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + \\ & 1950 \operatorname{Cos}[2 c + d x] \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) - \\ & 975 \operatorname{Cos}[2 c + 3 d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 975 \operatorname{Cos}[4 c + 3 d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \\ & 195 \operatorname{Cos}[4 c + 5 d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 195 \operatorname{Cos}[6 c + 5 d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \\ & 4640 \operatorname{Sin}[d x] + 1440 \operatorname{Sin}[2 c + d x] - 1500 \operatorname{Sin}[c + 2 d x] - 1500 \operatorname{Sin}[3 c + 2 d x] - \\ & 3040 \operatorname{Sin}[2 c + 3 d x] - 390 \operatorname{Sin}[3 c + 4 d x] - 390 \operatorname{Sin}[5 c + 4 d x] - 608 \operatorname{Sin}[4 c + 5 d x] \end{aligned} \right)$$

■ **Problem 37: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Cos}[c + d x])^4 \operatorname{Sec}[c + d x]^2 dx$$

Optimal (type 3, 73 leaves, 8 steps):

$$\frac{13 a^4 x}{2} + \frac{4 a^4 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} + \frac{4 a^4 \operatorname{Sin}[c + d x]}{d} + \frac{a^4 \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{2 d} + \frac{a^4 \operatorname{Tan}[c + d x]}{d}$$

Result (type 3, 241 leaves):

$$\frac{1}{64} a^4 (1 + \cos [c + d x])^4 \sec \left[ \frac{1}{2} (c + d x) \right]^8$$

$$\left( 26 x - \frac{16 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{d} + \frac{16 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{d} + \frac{16 \cos [d x] \sin [c]}{d} + \frac{\cos [2 d x] \sin [2 c]}{d} + \frac{16 \cos [c] \sin [d x]}{d} + \frac{\cos [2 c] \sin [2 d x]}{d} + \frac{4 \sin \left[ \frac{d x}{2} \right]}{d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)} + \frac{4 \sin \left[ \frac{d x}{2} \right]}{d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)} \right)$$

■ **Problem 38: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^4 \sec [c + d x]^3 dx$$

Optimal (type 3, 73 leaves, 8 steps):

$$4 a^4 x + \frac{13 a^4 \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{a^4 \sin [c + d x]}{d} + \frac{4 a^4 \tan [c + d x]}{d} + \frac{a^4 \sec [c + d x] \tan [c + d x]}{2 d}$$

Result (type 3, 272 leaves):

$$\frac{1}{64} a^4 (1 + \cos [c + d x])^4 \sec \left[ \frac{1}{2} (c + d x) \right]^8$$

$$\left( 16 x - \frac{26 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{d} + \frac{26 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{d} + \frac{4 \cos [d x] \sin [c]}{d} + \frac{4 \cos [c] \sin [d x]}{d} + \frac{1}{d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{16 \sin \left[ \frac{d x}{2} \right]}{d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)} - \frac{1}{d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{16 \sin \left[ \frac{d x}{2} \right]}{d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)} \right)$$

■ **Problem 39: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^4 \sec [c + d x]^4 dx$$

Optimal (type 3, 73 leaves, 9 steps):

$$a^4 x + \frac{6 a^4 \operatorname{ArcTanh}[\sin [c + d x]]}{d} + \frac{7 a^4 \tan [c + d x]}{d} + \frac{2 a^4 \sec [c + d x] \tan [c + d x]}{d} + \frac{a^4 \tan [c + d x]^3}{3 d}$$

Result (type 3, 178 leaves):

$$\frac{1}{12d} a^4 \operatorname{Sec}[c+dx]^3 \left( 9 \operatorname{Cos}[c+dx] \left( c+dx - 6 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right] \right] + 6 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right] \right] \right) + \right. \\ \left. 3 \operatorname{Cos}[3(c+dx)] \left( c+dx - 6 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right] \right] + 6 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right] \right] \right) + \right. \\ \left. 4 (6 \operatorname{Sin}[c+dx] + 3 \operatorname{Sin}[2(c+dx)] + 5 \operatorname{Sin}[3(c+dx)]) \right)$$

■ **Problem 40: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Cos}[c+dx])^4 \operatorname{Sec}[c+dx]^5 dx$$

Optimal (type 3, 96 leaves, 12 steps):

$$\frac{35 a^4 \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{8d} + \frac{8 a^4 \operatorname{Tan}[c+dx]}{d} + \frac{27 a^4 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{8d} + \frac{a^4 \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx]}{4d} + \frac{4 a^4 \operatorname{Tan}[c+dx]^3}{3d}$$

Result (type 3, 797 leaves):

$$-\frac{35 (a + a \operatorname{Cos}[c+dx])^4 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] - \operatorname{Sin} \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8}{128d} + \frac{35 (a + a \operatorname{Cos}[c+dx])^4 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] + \operatorname{Sin} \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8}{128d} + \\ \frac{(a + a \operatorname{Cos}[c+dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8}{256d \left( \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] - \operatorname{Sin} \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)^4} + \frac{(a + a \operatorname{Cos}[c+dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{24d \left( \operatorname{Cos} \left[ \frac{c}{2} \right] - \operatorname{Sin} \left[ \frac{c}{2} \right] \right) \left( \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] - \operatorname{Sin} \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)^3} + \\ \frac{(a + a \operatorname{Cos}[c+dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8 (97 \operatorname{Cos} \left[ \frac{c}{2} \right] - 65 \operatorname{Sin} \left[ \frac{c}{2} \right])}{768d \left( \operatorname{Cos} \left[ \frac{c}{2} \right] - \operatorname{Sin} \left[ \frac{c}{2} \right] \right) \left( \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] - \operatorname{Sin} \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)^2} + \frac{5 (a + a \operatorname{Cos}[c+dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{12d \left( \operatorname{Cos} \left[ \frac{c}{2} \right] - \operatorname{Sin} \left[ \frac{c}{2} \right] \right) \left( \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] - \operatorname{Sin} \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)} - \\ \frac{(a + a \operatorname{Cos}[c+dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8}{256d \left( \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] + \operatorname{Sin} \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)^4} + \frac{(a + a \operatorname{Cos}[c+dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{24d \left( \operatorname{Cos} \left[ \frac{c}{2} \right] + \operatorname{Sin} \left[ \frac{c}{2} \right] \right) \left( \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] + \operatorname{Sin} \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)^3} + \\ \frac{(a + a \operatorname{Cos}[c+dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8 (-97 \operatorname{Cos} \left[ \frac{c}{2} \right] - 65 \operatorname{Sin} \left[ \frac{c}{2} \right])}{768d \left( \operatorname{Cos} \left[ \frac{c}{2} \right] + \operatorname{Sin} \left[ \frac{c}{2} \right] \right) \left( \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] + \operatorname{Sin} \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)^2} + \frac{5 (a + a \operatorname{Cos}[c+dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{12d \left( \operatorname{Cos} \left[ \frac{c}{2} \right] + \operatorname{Sin} \left[ \frac{c}{2} \right] \right) \left( \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] + \operatorname{Sin} \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)}$$

■ **Problem 41: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Cos}[c+dx])^4 \operatorname{Sec}[c+dx]^6 dx$$

Optimal (type 3, 111 leaves, 13 steps):

$$\frac{7 a^4 \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{2d} + \frac{8 a^4 \operatorname{Tan}[c+dx]}{d} + \frac{7 a^4 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{2d} + \frac{a^4 \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx]}{d} + \frac{8 a^4 \operatorname{Tan}[c+dx]^3}{3d} + \frac{a^4 \operatorname{Tan}[c+dx]^5}{5d}$$

Result (type 3, 498 leaves):



$$\begin{aligned}
& -\frac{1}{960d} a^4 \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^5 \\
& \left( 525 \operatorname{Cos}[2c+3dx] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 525 \operatorname{Cos}[4c+3dx] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \right. \\
& 105 \operatorname{Cos}[4c+5dx] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 105 \operatorname{Cos}[6c+5dx] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \\
& 1050 \operatorname{Cos}[dx] \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) + \\
& 1050 \operatorname{Cos}[2c+dx] \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) - \\
& 525 \operatorname{Cos}[2c+3dx] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 525 \operatorname{Cos}[4c+3dx] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \\
& 105 \operatorname{Cos}[4c+5dx] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 105 \operatorname{Cos}[6c+5dx] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \\
& 2360 \operatorname{Sin}[dx] + 960 \operatorname{Sin}[2c+dx] - 660 \operatorname{Sin}[c+2dx] - 660 \operatorname{Sin}[3c+2dx] - 1600 \operatorname{Sin}[2c+3dx] + \\
& \left. 60 \operatorname{Sin}[4c+3dx] - 210 \operatorname{Sin}[3c+4dx] - 210 \operatorname{Sin}[5c+4dx] - 332 \operatorname{Sin}[4c+5dx] \right)
\end{aligned}$$

■ **Problem 46: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^2}{a+a\operatorname{Cos}[c+dx]} dx$$

Optimal (type 3, 43 leaves, 4 steps):

$$-\frac{x}{a} + \frac{\operatorname{Sin}[c+dx]}{ad} + \frac{\operatorname{Sin}[c+dx]}{ad(1+\operatorname{Cos}[c+dx])}$$

Result (type 3, 89 leaves):

$$\frac{1}{4ad} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left( -2dx \operatorname{Cos}\left[\frac{dx}{2}\right] - 2dx \operatorname{Cos}\left[c + \frac{dx}{2}\right] + 5 \operatorname{Sin}\left[\frac{dx}{2}\right] + \operatorname{Sin}\left[c + \frac{dx}{2}\right] + \operatorname{Sin}\left[c + \frac{3dx}{2}\right] + \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] \right)$$

■ **Problem 49: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]}{a+a\operatorname{Cos}[c+dx]} dx$$

Optimal (type 3, 38 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{ad} - \frac{\operatorname{Sin}[c+dx]}{d(a+a\operatorname{Cos}[c+dx])}$$

Result (type 3, 103 leaves):

$$-\frac{1}{a d (1 + \cos [c + d x])} \\ 2 \cos \left[ \frac{1}{2} (c + d x) \right] \left( \cos \left[ \frac{1}{2} (c + d x) \right] \left( \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] - \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] \right) + \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] \right)$$

■ **Problem 50: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec [c + d x]^2}{a + a \cos [c + d x]} dx$$

Optimal (type 3, 53 leaves, 5 steps) :

$$-\frac{\text{ArcTanh}[\sin [c + d x]]}{a d} + \frac{2 \tan [c + d x]}{a d} - \frac{\tan [c + d x]}{d (a + a \cos [c + d x])}$$

Result (type 3, 188 leaves) :

$$\frac{1}{a d (1 + \cos [c + d x])} \\ 2 \cos \left[ \frac{1}{2} (c + d x) \right] \left( \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + \cos \left[ \frac{1}{2} (c + d x) \right] \left( \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] - \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] \right) + \right. \\ \left. \sin [d x] / \left( \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) \right)$$

■ **Problem 51: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec [c + d x]^3}{a + a \cos [c + d x]} dx$$

Optimal (type 3, 83 leaves, 6 steps) :

$$\frac{3 \text{ArcTanh}[\sin [c + d x]]}{2 a d} - \frac{2 \tan [c + d x]}{a d} + \frac{3 \sec [c + d x] \tan [c + d x]}{2 a d} - \frac{\sec [c + d x] \tan [c + d x]}{d (a + a \cos [c + d x])}$$

Result (type 3, 244 leaves) :

$$\frac{1}{2 a d (1 + \cos [c + d x])} \cos \left[ \frac{1}{2} (c + d x) \right] \\ \left( -4 \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + \cos \left[ \frac{1}{2} (c + d x) \right] \left( -6 \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] + 6 \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) + \right. \\ \left. \frac{1}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \frac{1}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \right. \\ \left. (4 \sin [d x]) / \left( \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) \right)$$

■ **Problem 52: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^4}{a + a \text{Cos}[c + d x]} dx$$

Optimal (type 3, 103 leaves, 6 steps):

$$-\frac{3 \text{ArcTanh}[\text{Sin}[c + d x]]}{2 a d} + \frac{4 \text{Tan}[c + d x]}{a d} - \frac{3 \text{Sec}[c + d x] \text{Tan}[c + d x]}{2 a d} - \frac{\text{Sec}[c + d x]^2 \text{Tan}[c + d x]}{d (a + a \text{Cos}[c + d x])} + \frac{4 \text{Tan}[c + d x]^3}{3 a d}$$

Result (type 3, 706 leaves):

$$\begin{aligned} & \frac{3 \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \text{Cos}[c + d x])} - \frac{3 \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \text{Cos}[c + d x])} + \\ & \frac{2 \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{dx}{2}\right]}{d (a + a \text{Cos}[c + d x])} + \frac{\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Sin}\left[\frac{dx}{2}\right]}{3 d (a + a \text{Cos}[c + d x]) (\text{Cos}\left[\frac{c}{2}\right] - \text{Sin}\left[\frac{c}{2}\right]) (\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right])^3} + \\ & \frac{\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (-\text{Cos}\left[\frac{c}{2}\right] + 2 \text{Sin}\left[\frac{c}{2}\right])}{3 d (a + a \text{Cos}[c + d x]) (\text{Cos}\left[\frac{c}{2}\right] - \text{Sin}\left[\frac{c}{2}\right]) (\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \\ & \frac{10 \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Sin}\left[\frac{dx}{2}\right]}{3 d (a + a \text{Cos}[c + d x]) (\text{Cos}\left[\frac{c}{2}\right] - \text{Sin}\left[\frac{c}{2}\right]) (\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right])} + \\ & \frac{\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Sin}\left[\frac{dx}{2}\right]}{3 d (a + a \text{Cos}[c + d x]) (\text{Cos}\left[\frac{c}{2}\right] + \text{Sin}\left[\frac{c}{2}\right]) (\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right])^3} + \\ & \frac{\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (\text{Cos}\left[\frac{c}{2}\right] + 2 \text{Sin}\left[\frac{c}{2}\right])}{3 d (a + a \text{Cos}[c + d x]) (\text{Cos}\left[\frac{c}{2}\right] + \text{Sin}\left[\frac{c}{2}\right]) (\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \frac{10 \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Sin}\left[\frac{dx}{2}\right]}{3 d (a + a \text{Cos}[c + d x]) (\text{Cos}\left[\frac{c}{2}\right] + \text{Sin}\left[\frac{c}{2}\right]) (\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right])} \end{aligned}$$

■ **Problem 59: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]}{(a + a \text{Cos}[c + d x])^2} dx$$

Optimal (type 3, 66 leaves, 4 steps):

$$\frac{\text{ArcTanh}[\text{Sin}[c + d x]]}{a^2 d} - \frac{4 \text{Sin}[c + d x]}{3 a^2 d (1 + \text{Cos}[c + d x])} - \frac{\text{Sin}[c + d x]}{3 d (a + a \text{Cos}[c + d x])^2}$$

Result (type 3, 152 leaves):

$$-\frac{1}{3 a^2 d (1 + \cos [c + d x])^2} \\ 2 \cos \left[ \frac{1}{2} (c + d x) \right] \left( 6 \cos \left[ \frac{1}{2} (c + d x) \right]^3 \left( \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] - \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] \right) + \right. \\ \left. \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + 8 \cos \left[ \frac{1}{2} (c + d x) \right]^2 \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + \cos \left[ \frac{1}{2} (c + d x) \right] \tan \left[ \frac{c}{2} \right] \right)$$

■ **Problem 60: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec [c + d x]^2}{(a + a \cos [c + d x])^2} dx$$

Optimal (type 3, 81 leaves, 6 steps):

$$-\frac{2 \operatorname{ArcTanh}[\sin [c + d x]]}{a^2 d} + \frac{10 \tan [c + d x]}{3 a^2 d} - \frac{2 \tan [c + d x]}{a^2 d (1 + \cos [c + d x])} - \frac{\tan [c + d x]}{3 d (a + a \cos [c + d x])^2}$$

Result (type 3, 239 leaves):

$$\frac{1}{3 a^2 d (1 + \cos [c + d x])^2} 2 \cos \left[ \frac{1}{2} (c + d x) \right] \left( \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + 14 \cos \left[ \frac{1}{2} (c + d x) \right]^2 \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + \right. \\ \left. 6 \cos \left[ \frac{1}{2} (c + d x) \right]^3 \left( 2 \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] - 2 \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \sin [d x] / \left( \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \right. \right. \right. \\ \left. \left. \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) \right) + \cos \left[ \frac{1}{2} (c + d x) \right] \tan \left[ \frac{c}{2} \right] \right)$$

■ **Problem 61: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec [c + d x]^3}{(a + a \cos [c + d x])^2} dx$$

Optimal (type 3, 119 leaves, 7 steps):

$$\frac{7 \operatorname{ArcTanh}[\sin [c + d x]]}{2 a^2 d} - \frac{16 \tan [c + d x]}{3 a^2 d} + \frac{7 \sec [c + d x] \tan [c + d x]}{2 a^2 d} - \frac{8 \sec [c + d x] \tan [c + d x]}{3 a^2 d (1 + \cos [c + d x])} - \frac{\sec [c + d x] \tan [c + d x]}{3 d (a + a \cos [c + d x])^2}$$

Result (type 3, 292 leaves):

$$\frac{1}{3 a^2 d (1 + \operatorname{Cos}[c + d x])^2} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \left( -2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{d x}{2}\right] - 40 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{d x}{2}\right] + \right. \\ \left. 3 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^3 \left( -14 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 14 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + \right. \\ \left. \frac{1}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{1}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - (8 \operatorname{Sin}[d x]) \right) / \left( \left( \operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\ \left. \left( \operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left( \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \left( \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \right) - 2 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Tan}\left[\frac{c}{2}\right] \right)$$

■ **Problem 62: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^4}{(a + a \operatorname{Cos}[c + d x])^2} dx$$

Optimal (type 3, 133 leaves, 7 steps):

$$-\frac{5 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{a^2 d} + \frac{12 \operatorname{Tan}[c + d x]}{a^2 d} - \frac{5 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{a^2 d} - \frac{10 \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 a^2 d (1 + \operatorname{Cos}[c + d x])} - \frac{\operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d (a + a \operatorname{Cos}[c + d x])^2} + \frac{4 \operatorname{Tan}[c + d x]^3}{a^2 d}$$

Result (type 3, 403 leaves):

$$\frac{20 \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + d x])^2} - \\ \frac{20 \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + d x])^2} + \frac{1}{48 d (a + a \operatorname{Cos}[c + d x])^2} \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 \\ \left( -3 \operatorname{Sin}\left[\frac{d x}{2}\right] + 155 \operatorname{Sin}\left[\frac{3 d x}{2}\right] - 153 \operatorname{Sin}\left[c - \frac{d x}{2}\right] + 21 \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 135 \operatorname{Sin}\left[2 c + \frac{d x}{2}\right] + 25 \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 45 \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] - \right. \\ \left. 85 \operatorname{Sin}\left[3 c + \frac{3 d x}{2}\right] + 99 \operatorname{Sin}\left[c + \frac{5 d x}{2}\right] + 21 \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] + 33 \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] - 45 \operatorname{Sin}\left[4 c + \frac{5 d x}{2}\right] + 57 \operatorname{Sin}\left[2 c + \frac{7 d x}{2}\right] + \right. \\ \left. 18 \operatorname{Sin}\left[3 c + \frac{7 d x}{2}\right] + 24 \operatorname{Sin}\left[4 c + \frac{7 d x}{2}\right] - 15 \operatorname{Sin}\left[5 c + \frac{7 d x}{2}\right] + 24 \operatorname{Sin}\left[3 c + \frac{9 d x}{2}\right] + 11 \operatorname{Sin}\left[4 c + \frac{9 d x}{2}\right] + 13 \operatorname{Sin}\left[5 c + \frac{9 d x}{2}\right] \right)$$

■ **Problem 69: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]}{(a + a \operatorname{Cos}[c + d x])^3} dx$$

Optimal (type 3, 97 leaves, 5 steps):

$$\frac{\text{ArcTanh}[\text{Sin}[c + d x]]}{a^3 d} - \frac{\text{Sin}[c + d x]}{5 d (a + a \text{Cos}[c + d x])^3} - \frac{7 \text{Sin}[c + d x]}{15 a d (a + a \text{Cos}[c + d x])^2} - \frac{22 \text{Sin}[c + d x]}{15 d (a^3 + a^3 \text{Cos}[c + d x])}$$

Result (type 3, 201 leaves):

$$-\frac{1}{15 a^3 d (1 + \text{Cos}[c + d x])^3} 2 \text{Cos}\left[\frac{1}{2} (c + d x)\right] \left(60 \text{Cos}\left[\frac{1}{2} (c + d x)\right]^5 \left(\text{Log}\left[\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right)\right) + 3 \text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{d x}{2}\right] + 14 \text{Cos}\left[\frac{1}{2} (c + d x)\right]^2 \text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{d x}{2}\right] + 88 \text{Cos}\left[\frac{1}{2} (c + d x)\right]^4 \text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{d x}{2}\right] + 3 \text{Cos}\left[\frac{1}{2} (c + d x)\right] \text{Tan}\left[\frac{c}{2}\right] + 14 \text{Cos}\left[\frac{1}{2} (c + d x)\right]^3 \text{Tan}\left[\frac{c}{2}\right]$$

■ **Problem 70: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^2}{(a + a \text{Cos}[c + d x])^3} dx$$

Optimal (type 3, 112 leaves, 7 steps):

$$-\frac{3 \text{ArcTanh}[\text{Sin}[c + d x]]}{a^3 d} + \frac{24 \text{Tan}[c + d x]}{5 a^3 d} - \frac{\text{Tan}[c + d x]}{5 d (a + a \text{Cos}[c + d x])^3} - \frac{3 \text{Tan}[c + d x]}{5 a d (a + a \text{Cos}[c + d x])^2} - \frac{3 \text{Tan}[c + d x]}{d (a^3 + a^3 \text{Cos}[c + d x])}$$

Result (type 3, 286 leaves):

$$\frac{1}{5 a^3 d (1 + \text{Cos}[c + d x])^3} 2 \text{Cos}\left[\frac{1}{2} (c + d x)\right] \left(\text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{d x}{2}\right] + 8 \text{Cos}\left[\frac{1}{2} (c + d x)\right]^2 \text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{d x}{2}\right] + 76 \text{Cos}\left[\frac{1}{2} (c + d x)\right]^4 \text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{d x}{2}\right] + 20 \text{Cos}\left[\frac{1}{2} (c + d x)\right]^5 \left(3 \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right] - 3 \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right) + \text{Sin}[d x] / \left(\left(\text{Cos}\left[\frac{c}{2}\right] - \text{Sin}\left[\frac{c}{2}\right]\right) \left(\text{Cos}\left[\frac{c}{2}\right] + \text{Sin}\left[\frac{c}{2}\right]\right) \left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right) \left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right)\right) + \text{Cos}\left[\frac{1}{2} (c + d x)\right] \text{Tan}\left[\frac{c}{2}\right] + 8 \text{Cos}\left[\frac{1}{2} (c + d x)\right]^3 \text{Tan}\left[\frac{c}{2}\right]$$

■ **Problem 71: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^3}{(a + a \text{Cos}[c + d x])^3} dx$$

Optimal (type 3, 156 leaves, 8 steps):

$$\frac{13 \text{ArcTanh}[\text{Sin}[c + d x]]}{2 a^3 d} - \frac{152 \text{Tan}[c + d x]}{15 a^3 d} + \frac{13 \text{Sec}[c + d x] \text{Tan}[c + d x]}{2 a^3 d} - \frac{\text{Sec}[c + d x] \text{Tan}[c + d x]}{5 d (a + a \text{Cos}[c + d x])^3} - \frac{11 \text{Sec}[c + d x] \text{Tan}[c + d x]}{15 a d (a + a \text{Cos}[c + d x])^2} - \frac{76 \text{Sec}[c + d x] \text{Tan}[c + d x]}{15 d (a^3 + a^3 \text{Cos}[c + d x])}$$

Result (type 3, 403 leaves):

$$\begin{aligned}
& - \frac{52 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + dx])^3} + \\
& \frac{52 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + dx])^3} + \frac{1}{480 d (a + a \operatorname{Cos}[c + dx])^3} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \\
& \left( 1235 \operatorname{Sin}\left[\frac{dx}{2}\right] - 3805 \operatorname{Sin}\left[\frac{3 dx}{2}\right] + 4329 \operatorname{Sin}\left[c - \frac{dx}{2}\right] - 1989 \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 3575 \operatorname{Sin}\left[2c + \frac{dx}{2}\right] + 475 \operatorname{Sin}\left[c + \frac{3 dx}{2}\right] - 2005 \operatorname{Sin}\left[2c + \frac{3 dx}{2}\right] + \right. \\
& 2275 \operatorname{Sin}\left[3c + \frac{3 dx}{2}\right] - 2673 \operatorname{Sin}\left[c + \frac{5 dx}{2}\right] - 105 \operatorname{Sin}\left[2c + \frac{5 dx}{2}\right] - 1593 \operatorname{Sin}\left[3c + \frac{5 dx}{2}\right] + 975 \operatorname{Sin}\left[4c + \frac{5 dx}{2}\right] - 1325 \operatorname{Sin}\left[2c + \frac{7 dx}{2}\right] - \\
& \left. 255 \operatorname{Sin}\left[3c + \frac{7 dx}{2}\right] - 875 \operatorname{Sin}\left[4c + \frac{7 dx}{2}\right] + 195 \operatorname{Sin}\left[5c + \frac{7 dx}{2}\right] - 304 \operatorname{Sin}\left[3c + \frac{9 dx}{2}\right] - 90 \operatorname{Sin}\left[4c + \frac{9 dx}{2}\right] - 214 \operatorname{Sin}\left[5c + \frac{9 dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 80: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^2}{(a + a \operatorname{Cos}[c + dx])^4} dx$$

Optimal (type 3, 135 leaves, 8 steps):

$$\begin{aligned}
& - \frac{4 \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{a^4 d} + \frac{664 \operatorname{Tan}[c + dx]}{105 a^4 d} - \frac{88 \operatorname{Tan}[c + dx]}{105 a^4 d (1 + \operatorname{Cos}[c + dx])^2} - \\
& \frac{4 \operatorname{Tan}[c + dx]}{a^4 d (1 + \operatorname{Cos}[c + dx])} - \frac{\operatorname{Tan}[c + dx]}{7 d (a + a \operatorname{Cos}[c + dx])^4} - \frac{12 \operatorname{Tan}[c + dx]}{35 a d (a + a \operatorname{Cos}[c + dx])^3}
\end{aligned}$$

Result (type 3, 401 leaves):

$$\begin{aligned}
& \frac{64 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + dx])^4} - \frac{64 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + dx])^4} + \\
& \frac{1}{1680 d (a + a \operatorname{Cos}[c + dx])^4} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \left( -10780 \operatorname{Sin}\left[\frac{dx}{2}\right] + 18788 \operatorname{Sin}\left[\frac{3 dx}{2}\right] - 20524 \operatorname{Sin}\left[c - \frac{dx}{2}\right] + \right. \\
& 14644 \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 16660 \operatorname{Sin}\left[2c + \frac{dx}{2}\right] - 4690 \operatorname{Sin}\left[c + \frac{3 dx}{2}\right] + 14378 \operatorname{Sin}\left[2c + \frac{3 dx}{2}\right] - 9100 \operatorname{Sin}\left[3c + \frac{3 dx}{2}\right] + \\
& 11668 \operatorname{Sin}\left[c + \frac{5 dx}{2}\right] - 630 \operatorname{Sin}\left[2c + \frac{5 dx}{2}\right] + 9358 \operatorname{Sin}\left[3c + \frac{5 dx}{2}\right] - 2940 \operatorname{Sin}\left[4c + \frac{5 dx}{2}\right] + 4228 \operatorname{Sin}\left[2c + \frac{7 dx}{2}\right] + \\
& \left. 315 \operatorname{Sin}\left[3c + \frac{7 dx}{2}\right] + 3493 \operatorname{Sin}\left[4c + \frac{7 dx}{2}\right] - 420 \operatorname{Sin}\left[5c + \frac{7 dx}{2}\right] + 664 \operatorname{Sin}\left[3c + \frac{9 dx}{2}\right] + 105 \operatorname{Sin}\left[4c + \frac{9 dx}{2}\right] + 559 \operatorname{Sin}\left[5c + \frac{9 dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 81: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^3}{(a + a \operatorname{Cos}[c + dx])^4} dx$$

Optimal (type 3, 185 leaves, 9 steps):

$$\frac{21 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 a^4 d} - \frac{576 \operatorname{Tan}[c + d x]}{35 a^4 d} + \frac{21 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 a^4 d} -$$

$$\frac{43 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{35 a^4 d (1 + \operatorname{Cos}[c + d x])^2} - \frac{288 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{35 a^4 d (1 + \operatorname{Cos}[c + d x])} - \frac{\operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{7 d (a + a \operatorname{Cos}[c + d x])^4} - \frac{2 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{5 a d (a + a \operatorname{Cos}[c + d x])^3}$$

Result (type 3, 455 leaves):

$$- \frac{168 \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + d x])^4} +$$

$$\frac{168 \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + d x])^4} + \frac{1}{2240 d (a + a \operatorname{Cos}[c + d x])^4} \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2$$

$$\left( 24402 \operatorname{Sin}\left[\frac{d x}{2}\right] - 55556 \operatorname{Sin}\left[\frac{3 d x}{2}\right] + 61054 \operatorname{Sin}\left[c - \frac{d x}{2}\right] - 33614 \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 51842 \operatorname{Sin}\left[2 c + \frac{d x}{2}\right] + 12460 \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] -$$

$$33716 \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 34300 \operatorname{Sin}\left[3 c + \frac{3 d x}{2}\right] - 39788 \operatorname{Sin}\left[c + \frac{5 d x}{2}\right] + 2940 \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] - 26068 \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] + 16660$$

$$\operatorname{Sin}\left[4 c + \frac{5 d x}{2}\right] - 21351 \operatorname{Sin}\left[2 c + \frac{7 d x}{2}\right] - 1295 \operatorname{Sin}\left[3 c + \frac{7 d x}{2}\right] - 14911 \operatorname{Sin}\left[4 c + \frac{7 d x}{2}\right] + 5145 \operatorname{Sin}\left[5 c + \frac{7 d x}{2}\right] - 7329 \operatorname{Sin}\left[3 c + \frac{9 d x}{2}\right] -$$

$$1225 \operatorname{Sin}\left[4 c + \frac{9 d x}{2}\right] - 5369 \operatorname{Sin}\left[5 c + \frac{9 d x}{2}\right] + 735 \operatorname{Sin}\left[6 c + \frac{9 d x}{2}\right] - 1152 \operatorname{Sin}\left[4 c + \frac{11 d x}{2}\right] - 280 \operatorname{Sin}\left[5 c + \frac{11 d x}{2}\right] - 872 \operatorname{Sin}\left[6 c + \frac{11 d x}{2}\right] \right)$$

■ **Problem 91: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Cos}[c + d x])^5} dx$$

Optimal (type 3, 168 leaves, 9 steps):

$$- \frac{5 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{a^5 d} + \frac{496 \operatorname{Tan}[c + d x]}{63 a^5 d} - \frac{\operatorname{Tan}[c + d x]}{9 d (a + a \operatorname{Cos}[c + d x])^5} -$$

$$\frac{5 \operatorname{Tan}[c + d x]}{21 a d (a + a \operatorname{Cos}[c + d x])^4} - \frac{29 \operatorname{Tan}[c + d x]}{63 a^2 d (a + a \operatorname{Cos}[c + d x])^3} - \frac{67 \operatorname{Tan}[c + d x]}{63 a^3 d (a + a \operatorname{Cos}[c + d x])^2} - \frac{5 \operatorname{Tan}[c + d x]}{d (a^5 + a^5 \operatorname{Cos}[c + d x])}$$

Result (type 3, 453 leaves):



$$\frac{160 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^{10} \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \cos[c + dx])^5} -$$

$$\frac{160 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^{10} \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \cos[c + dx])^5} + \frac{1}{2016 d (a + a \cos[c + dx])^5} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + dx]$$

$$\left( -33\,978 \sin\left[\frac{dx}{2}\right] + 52\,002 \sin\left[\frac{3dx}{2}\right] - 56\,952 \sin\left[c - \frac{dx}{2}\right] + 43\,722 \sin\left[c + \frac{dx}{2}\right] - 47\,208 \sin\left[2c + \frac{dx}{2}\right] - 18\,144 \sin\left[c + \frac{3dx}{2}\right] + \right.$$

$$41\,796 \sin\left[2c + \frac{3dx}{2}\right] - 28\,350 \sin\left[3c + \frac{3dx}{2}\right] + 34\,578 \sin\left[c + \frac{5dx}{2}\right] - 5691 \sin\left[2c + \frac{5dx}{2}\right] + 28\,719 \sin\left[3c + \frac{5dx}{2}\right] -$$

$$11\,550 \sin\left[4c + \frac{5dx}{2}\right] + 15\,517 \sin\left[2c + \frac{7dx}{2}\right] - 504 \sin\left[3c + \frac{7dx}{2}\right] + 13\,186 \sin\left[4c + \frac{7dx}{2}\right] - 2835 \sin\left[5c + \frac{7dx}{2}\right] + 4149 \sin\left[3c + \frac{9dx}{2}\right] +$$

$$\left. 252 \sin\left[4c + \frac{9dx}{2}\right] + 3582 \sin\left[5c + \frac{9dx}{2}\right] - 315 \sin\left[6c + \frac{9dx}{2}\right] + 496 \sin\left[4c + \frac{11dx}{2}\right] + 63 \sin\left[5c + \frac{11dx}{2}\right] + 433 \sin\left[6c + \frac{11dx}{2}\right] \right)$$

■ **Problem 92: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^3}{(a + a \cos[c + dx])^5} dx$$

Optimal (type 3, 224 leaves, 10 steps):

$$\frac{31 \operatorname{ArcTanh}[\sin[c + dx]]}{2 a^5 d} - \frac{7664 \tan[c + dx]}{315 a^5 d} + \frac{31 \operatorname{Sec}[c + dx] \tan[c + dx]}{2 a^5 d} - \frac{\operatorname{Sec}[c + dx] \tan[c + dx]}{9 d (a + a \cos[c + dx])^5} -$$

$$\frac{17 \operatorname{Sec}[c + dx] \tan[c + dx]}{63 a d (a + a \cos[c + dx])^4} - \frac{28 \operatorname{Sec}[c + dx] \tan[c + dx]}{45 a^2 d (a + a \cos[c + dx])^3} - \frac{577 \operatorname{Sec}[c + dx] \tan[c + dx]}{315 a^3 d (a + a \cos[c + dx])^2} - \frac{3832 \operatorname{Sec}[c + dx] \tan[c + dx]}{315 d (a^5 + a^5 \cos[c + dx])}$$

Result (type 3, 507 leaves):

$$\begin{aligned}
& - \frac{496 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^{10} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + dx])^5} + \frac{496 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^{10} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + dx])^5} + \\
& \frac{1}{40\,320 d (a + a \operatorname{Cos}[c + dx])^5} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \\
& \left( 1\,472\,562 \operatorname{Sin}\left[\frac{dx}{2}\right] - 2\,822\,886 \operatorname{Sin}\left[\frac{3 dx}{2}\right] + 3\,057\,654 \operatorname{Sin}\left[c - \frac{dx}{2}\right] - 1\,885\,854 \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 2\,644\,362 \operatorname{Sin}\left[2c + \frac{dx}{2}\right] + \right. \\
& 867\,048 \operatorname{Sin}\left[c + \frac{3 dx}{2}\right] - 1\,868\,436 \operatorname{Sin}\left[2c + \frac{3 dx}{2}\right] + 1\,821\,498 \operatorname{Sin}\left[3c + \frac{3 dx}{2}\right] - 2\,083\,537 \operatorname{Sin}\left[c + \frac{5 dx}{2}\right] + 339\,885 \operatorname{Sin}\left[2c + \frac{5 dx}{2}\right] - \\
& 1\,456\,687 \operatorname{Sin}\left[3c + \frac{5 dx}{2}\right] + 966\,735 \operatorname{Sin}\left[4c + \frac{5 dx}{2}\right] - 1\,195\,641 \operatorname{Sin}\left[2c + \frac{7 dx}{2}\right] + 46\,515 \operatorname{Sin}\left[3c + \frac{7 dx}{2}\right] - \\
& 874\,341 \operatorname{Sin}\left[4c + \frac{7 dx}{2}\right] + 367\,815 \operatorname{Sin}\left[5c + \frac{7 dx}{2}\right] - 494\,579 \operatorname{Sin}\left[3c + \frac{9 dx}{2}\right] - 31\,815 \operatorname{Sin}\left[4c + \frac{9 dx}{2}\right] - \\
& 374\,879 \operatorname{Sin}\left[5c + \frac{9 dx}{2}\right] + 87\,885 \operatorname{Sin}\left[6c + \frac{9 dx}{2}\right] - 128\,187 \operatorname{Sin}\left[4c + \frac{11 dx}{2}\right] - 18\,585 \operatorname{Sin}\left[5c + \frac{11 dx}{2}\right] - \\
& \left. 99\,837 \operatorname{Sin}\left[6c + \frac{11 dx}{2}\right] + 9765 \operatorname{Sin}\left[7c + \frac{11 dx}{2}\right] - 15\,328 \operatorname{Sin}\left[5c + \frac{13 dx}{2}\right] - 3150 \operatorname{Sin}\left[6c + \frac{13 dx}{2}\right] - 12\,178 \operatorname{Sin}\left[7c + \frac{13 dx}{2}\right] \right)
\end{aligned}$$

- **Problem 100: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \operatorname{Cos}[c + dx]} \operatorname{Sec}[c + dx] dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$\frac{2 \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{d}$$

Result (type 3, 1294 leaves):

$$\begin{aligned}
& - \left( \left( \left( \frac{1}{4} - \frac{i}{4} \right) (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right. \right. \\
& \quad (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + \\
& \quad \left. \left. \left. 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \right) \right) / \\
& \quad \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right) - \\
& \quad i \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]} \right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \\
& \quad \frac{\sqrt{2} d}{\sqrt{2} d} - \\
& \quad i \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]} \right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \\
& \quad \frac{\sqrt{2} d}{\sqrt{2} d} - \\
& \quad \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{2\sqrt{2} d} - \\
& \quad \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{2\sqrt{2} d} - \\
& \quad \frac{2i \operatorname{ArcTan} \left[ \frac{2i \cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Cot}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right]}{d \sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} + \\
& \quad \frac{1}{d \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right)} \\
& \quad \sqrt{2} \sqrt{a(1 + \cos[c + dx])} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \\
& \quad \left( -dx \cos\left[\frac{c}{2}\right] + 2 \operatorname{Log}\left[\sqrt{2} + 2 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + \frac{4i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2i \cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right)
\end{aligned}$$

- **Problem 101: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos[c + dx]} \operatorname{Sec}[c + dx]^2 dx$$

Optimal (type 3, 62 leaves, 3 steps):

$$\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} + \frac{a \tan[c+dx]}{d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 1426 leaves):

$$\begin{aligned} & - \left( \left( \left( \frac{1}{8} - \frac{i}{8} \right) (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right. \right. \\ & \quad (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + \\ & \quad \left. \left. \left. 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \right) / \right. \\ & \quad \left. \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right) \right) - \\ & \frac{i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{2\sqrt{2}d} - \\ & \frac{i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{2\sqrt{2}d} - \\ & \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{4\sqrt{2}d} - \\ & \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{4\sqrt{2}d} - \\ & \frac{i \operatorname{ArcTan}\left[\frac{2i \cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}\right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Cot}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} + \\ & d \sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2} \end{aligned}$$

$$\left( \sqrt{a(1 + \cos[c + dx])} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \right.$$

$$\left. - dx \cos\left[\frac{c}{2}\right] + 2 \operatorname{Log}\left[\sqrt{2} + 2 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + \frac{4i\sqrt{2} \operatorname{ArcTan}\left[\frac{2i\cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2\sin\left[\frac{c}{2}\right]) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}} \right) /$$

$$\left( \sqrt{2} d \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) \right) + \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{2d \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)} - \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{2d \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)}$$

■ **Problem 102: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos[c + dx]} \operatorname{Sec}[c + dx]^3 dx$$

Optimal (type 3, 102 leaves, 4 steps):

$$\frac{3\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{4d} + \frac{3a \tan[c+dx]}{4d\sqrt{a+a\cos[c+dx]}} + \frac{a \operatorname{Sec}[c+dx] \tan[c+dx]}{2d\sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 1671 leaves):

$$- \left( \left( \left( \frac{3}{32} - \frac{3i}{32} \right) (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right.$$

$$\left. (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + \right.$$

$$\left. 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] /$$

$$\left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right) -$$

$$\frac{3i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{8\sqrt{2}d}$$

$$\begin{aligned}
& \frac{3 \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] \sqrt{a (1 + \cos [c + dx])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{8 \sqrt{2} d} \\
& \frac{3 \sqrt{a (1 + \cos [c + dx])} \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{16 \sqrt{2} d} \\
& \frac{3 \sqrt{a (1 + \cos [c + dx])} \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{16 \sqrt{2} d} \\
& \frac{3 \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \sqrt{a (1 + \cos [c + dx])} \operatorname{Cot} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{4 d \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} +
\end{aligned}$$

$$\left( \begin{aligned}
& 3 \sqrt{a (1 + \cos [c + dx])} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] \\
& \left( \begin{aligned}
& -dx \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right) \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) /
\end{aligned} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( 4 \sqrt{2} d \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right) \right) + \frac{\sqrt{a (1 + \cos [c + dx])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] \sin \left[ \frac{dx}{2} \right]}{4 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)^2} + \\
& \frac{\sqrt{a (1 + \cos [c + dx])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] \left( 3 \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right)}{8 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)} + \\
& \frac{\sqrt{a (1 + \cos [c + dx])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] \sin \left[ \frac{dx}{2} \right]}{4 d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)^2} +
\end{aligned}$$

$$\frac{\sqrt{a(1+\cos[c+dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(-3\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right)}{8d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}$$

■ **Problem 103: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+a\cos[c+dx]} \operatorname{Sec}[c+dx]^4 dx$$

Optimal (type 3, 138 leaves, 5 steps):

$$\frac{5\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{8d} + \frac{5a \operatorname{Tan}[c+dx]}{8d\sqrt{a+a\cos[c+dx]}} + \frac{5a \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{12d\sqrt{a+a\cos[c+dx]}} + \frac{a \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3d\sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 1799 leaves):

$$\begin{aligned} & - \left( \left( \left( \frac{5}{64} - \frac{5i}{64} \right) (1 + e^{ic}) \left( \sqrt{2} - (1-i)e^{\frac{ic}{2}} + (16-16i)e^{\frac{3ic}{2}+idx} + (20+20i)\sqrt{2}e^{2ic+\frac{3idx}{2}} - (34-34i)e^{\frac{5ic}{2}+2idx} - (20+20i)\sqrt{2}e^{3ic+\frac{5idx}{2}} + \right. \right. \\ & \quad (16-16i)e^{\frac{7ic}{2}+3idx} + (4+4i)\sqrt{2}e^{4ic+\frac{7idx}{2}} - (1-i)e^{\frac{9ic}{2}+4idx} + 8ie^{\frac{1}{2}i(c+dx)} - 16\sqrt{2}e^{i(c+dx)} - 40ie^{\frac{3}{2}i(c+dx)} + 34\sqrt{2}e^{2i(c+dx)} + \\ & \quad \left. \left. 40ie^{\frac{5}{2}i(c+dx)} - 16\sqrt{2}e^{3i(c+dx)} - 8ie^{\frac{7}{2}i(c+dx)} + \sqrt{2}e^{4i(c+dx)} - (4+4i)\sqrt{2}e^{\frac{1}{2}i(2c+dx)} \right) x \sqrt{a(1+\cos[c+dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \right) / \\ & \quad \left( \left( (-1-i) + \sqrt{2}e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2}e^{\frac{1}{2}i(c+dx)} - 4ie^{i(c+dx)} + 2\sqrt{2}e^{\frac{3}{2}i(c+dx)} + ie^{2i(c+dx)} \right)^2 \right) \right) - \\ & \frac{5i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2}\sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2}\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1+\cos[c+dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{16\sqrt{2}d} - \\ & \frac{5i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2}\sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2}\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1+\cos[c+dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{16\sqrt{2}d} - \\ & \frac{5\sqrt{a(1+\cos[c+dx])} \operatorname{Log}\left[2 - \sqrt{2}\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2}\sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{32\sqrt{2}d} - \\ & \frac{5\sqrt{a(1+\cos[c+dx])} \operatorname{Log}\left[2 + \sqrt{2}\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2}\sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{32\sqrt{2}d} - \\ & \frac{5i \operatorname{ArcTan}\left[\frac{2i\cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2\sin\left[\frac{c}{2}\right]) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2+4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}}\right] \sqrt{a(1+\cos[c+dx])} \operatorname{Cot}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{\sqrt{-2+4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}} + \end{aligned}$$

$$\left( 5 \sqrt{a (1 + \cos [c + d x])} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)$$

$$\left( -d x \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{d x}{2} \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \operatorname{Tan} \left[ \frac{d x}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) /$$

$$\left( 8 \sqrt{2} d \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right) \right) + \frac{\sqrt{a (1 + \cos [c + d x])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]}{12 d \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^3} + \frac{\sqrt{a (1 + \cos [c + d x])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] \sin \left[ \frac{d x}{2} \right]}{8 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^2} +$$

$$\frac{\sqrt{a (1 + \cos [c + d x])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] \left( 5 \cos \left[ \frac{c}{2} \right] - 3 \sin \left[ \frac{c}{2} \right] \right)}{16 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)}$$

$$\frac{\sqrt{a (1 + \cos [c + d x])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]}{12 d \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^3} +$$

$$\frac{\sqrt{a (1 + \cos [c + d x])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] \sin \left[ \frac{d x}{2} \right]}{8 d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^2} +$$

$$\frac{\sqrt{a (1 + \cos [c + d x])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] \left( -5 \cos \left[ \frac{c}{2} \right] - 3 \sin \left[ \frac{c}{2} \right] \right)}{16 d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)}$$

- **Problem 108: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{3/2} \operatorname{Sec} [c + d x] dx$$

Optimal (type 3, 66 leaves, 4 steps):

$$\frac{2 a^{3/2} \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right]}{d} + \frac{2 a^2 \sin [c + d x]}{d \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 1404 leaves):



$$\begin{aligned}
& - \left( \left( \left( \frac{1}{8} - \frac{i}{8} \right) (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right. \right. \\
& \quad (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + \\
& \quad \left. \left. 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x (a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \right) / \\
& \quad \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right) - \\
& \quad \frac{i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] (a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2\sqrt{2}d} - \\
& \quad \frac{i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] (a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2\sqrt{2}d} - \\
& \quad \frac{(a(1 + \cos[c + dx]))^{3/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{4\sqrt{2}d} - \\
& \quad \frac{(a(1 + \cos[c + dx]))^{3/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{4\sqrt{2}d} + \\
& \quad \frac{\cos\left[\frac{dx}{2}\right] (a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{c}{2}\right]}{d} - \\
& \quad \frac{i \operatorname{ArcTan}\left[\frac{2i \cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}\right] (a(1 + \cos[c + dx]))^{3/2} \operatorname{Cot}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{d \sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} + \\
& \quad \left( (a(1 + \cos[c + dx]))^{3/2} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \right)
\end{aligned}$$

$$\left( -d x \cos\left[\frac{c}{2}\right] + 2 \log\left[\sqrt{2} + 2 \cos\left[\frac{d x}{2}\right]\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{d x}{2}\right] \right) \sin\left[\frac{c}{2}\right] + \frac{4 i \sqrt{2} \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right] - i \left(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]\right) \tan\left[\frac{d x}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right) /$$

$$\left( \sqrt{2} d \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) \right) + \frac{\cos\left[\frac{c}{2}\right] \left( a \left( 1 + \cos[c + d x] \right) \right)^{3/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^3 \sin\left[\frac{d x}{2}\right]}{d}$$

- **Problem 109: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + d x])^{3/2} \sec[c + d x]^2 dx$$

Optimal (type 3, 65 leaves, 4 steps):

$$\frac{3 a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + d x]}{\sqrt{a + a \cos[c + d x]}}\right]}{d} + \frac{a^2 \tan[c + d x]}{d \sqrt{a + a \cos[c + d x]}}$$

Result (type 3, 1449 leaves):

$$\begin{aligned} & - \left( \left( \left( \frac{3}{16} - \frac{3 i}{16} \right) \left( 1 + e^{i c} \right) \left( \sqrt{2} - (1 - i) e^{\frac{i c}{2}} + (16 - 16 i) e^{\frac{3 i c}{2} + i d x} + (20 + 20 i) \sqrt{2} e^{2 i c + \frac{3 i d x}{2}} - (34 - 34 i) e^{\frac{5 i c}{2} + 2 i d x} - (20 + 20 i) \sqrt{2} e^{3 i c + \frac{5 i d x}{2}} + \right. \right. \\ & \quad \left. \left. (16 - 16 i) e^{\frac{7 i c}{2} + 3 i d x} + (4 + 4 i) \sqrt{2} e^{4 i c + \frac{7 i d x}{2}} - (1 - i) e^{\frac{9 i c}{2} + 4 i d x} + 8 i e^{\frac{1}{2} i (c + d x)} - 16 \sqrt{2} e^{i (c + d x)} - 40 i e^{\frac{3}{2} i (c + d x)} + 34 \sqrt{2} e^{2 i (c + d x)} + \right. \right. \\ & \quad \left. \left. 40 i e^{\frac{5}{2} i (c + d x)} - 16 \sqrt{2} e^{3 i (c + d x)} - 8 i e^{\frac{7}{2} i (c + d x)} + \sqrt{2} e^{4 i (c + d x)} - (4 + 4 i) \sqrt{2} e^{\frac{1}{2} i (2 c + d x)} \right) x \left( a \left( 1 + \cos[c + d x] \right) \right)^{3/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^3 \right) / \\ & \left( \left( (-1 - i) + \sqrt{2} e^{\frac{i c}{2}} \right) \left( -1 + e^{i c} \right) \left( i - 2 \sqrt{2} e^{\frac{1}{2} i (c + d x)} - 4 i e^{i (c + d x)} + 2 \sqrt{2} e^{\frac{3}{2} i (c + d x)} + i e^{2 i (c + d x)} \right)^2 \right) - \\ & \frac{3 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{d x}{4}\right] - \sin\left[\frac{c}{4} + \frac{d x}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{d x}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{d x}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{d x}{4}\right] - \sin\left[\frac{c}{4} + \frac{d x}{4}\right]}\right] \left( a \left( 1 + \cos[c + d x] \right) \right)^{3/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^3}{4 \sqrt{2} d} - \\ & \frac{3 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{d x}{4}\right] + \sin\left[\frac{c}{4} + \frac{d x}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{d x}{4}\right]}{\cos\left[\frac{c}{4} + \frac{d x}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{d x}{4}\right] - \sin\left[\frac{c}{4} + \frac{d x}{4}\right]}\right] \left( a \left( 1 + \cos[c + d x] \right) \right)^{3/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^3}{4 \sqrt{2} d} - \\ & \frac{3 \left( a \left( 1 + \cos[c + d x] \right) \right)^{3/2} \log\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{d x}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^3}{8 \sqrt{2} d} - \end{aligned}$$

$$\frac{3 (a (1 + \cos [c + d x]))^{3/2} \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3}{8 \sqrt{2} d}$$

$$\frac{3 i \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \operatorname{Tan} \left[ \frac{d x}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] (a (1 + \cos [c + d x]))^{3/2} \operatorname{Cot} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3}{2 d \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} +$$

$$\left( 3 (a (1 + \cos [c + d x]))^{3/2} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \right)$$

$$\left( -d x \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{d x}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] \right) \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \operatorname{Tan} \left[ \frac{d x}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) /$$

$$\left( 2 \sqrt{2} d \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right) \right) + \frac{(a (1 + \cos [c + d x]))^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3}{4 d (\cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right])} - \frac{(a (1 + \cos [c + d x]))^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3}{4 d (\cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right])}$$

■ **Problem 110: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{3/2} \operatorname{Sec} [c + d x]^3 dx$$

Optimal (type 3, 106 leaves, 5 steps):

$$\frac{7 a^{3/2} \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right]}{4 d} + \frac{7 a^2 \operatorname{Tan} [c + d x]}{4 d \sqrt{a + a \cos [c + d x]}} + \frac{a^2 \operatorname{Sec} [c + d x] \operatorname{Tan} [c + d x]}{2 d \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 1693 leaves):

$$-\left( \left( \frac{7}{64} - \frac{7 i}{64} \right) (1 + e^{i c}) \left( \sqrt{2} - (1 - i) e^{\frac{i c}{2}} + (16 - 16 i) e^{\frac{3 i c}{2} + i d x} + (20 + 20 i) \sqrt{2} e^{2 i c + \frac{3 i d x}{2}} - (34 - 34 i) e^{\frac{5 i c}{2} + 2 i d x} - (20 + 20 i) \sqrt{2} e^{3 i c + \frac{5 i d x}{2}} + (16 - 16 i) e^{\frac{7 i c}{2} + 3 i d x} + (4 + 4 i) \sqrt{2} e^{4 i c + \frac{7 i d x}{2}} - (1 - i) e^{\frac{9 i c}{2} + 4 i d x} + 8 i e^{\frac{1}{2} i (c + d x)} - 16 \sqrt{2} e^{i (c + d x)} - 40 i e^{\frac{3}{2} i (c + d x)} + 34 \sqrt{2} e^{2 i (c + d x)} + \right.$$

$$\begin{aligned}
& \left( 40 i e^{\frac{5}{2} i (c+dx)} - 16 \sqrt{2} e^{3 i (c+dx)} - 8 i e^{\frac{7}{2} i (c+dx)} + \sqrt{2} e^{4 i (c+dx)} - (4 + 4 i) \sqrt{2} e^{\frac{1}{2} i (2c+dx)} \right) x (a (1 + \cos [c + dx]))^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \Big/ \\
& \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2 \sqrt{2} e^{\frac{1}{2} i (c+dx)} - 4 i e^{i (c+dx)} + 2 \sqrt{2} e^{\frac{3}{2} i (c+dx)} + i e^{2 i (c+dx)} \right)^2 \right) \Big) - \\
& \frac{7 i \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{-\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] (a (1 + \cos [c + dx]))^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{16 \sqrt{2} d} - \\
& \frac{7 i \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] (a (1 + \cos [c + dx]))^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{16 \sqrt{2} d} - \\
& \frac{7 (a (1 + \cos [c + dx]))^{3/2} \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{32 \sqrt{2} d} - \\
& \frac{7 (a (1 + \cos [c + dx]))^{3/2} \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{32 \sqrt{2} d} - \\
& \frac{7 i \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i (-\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right]) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] (a (1 + \cos [c + dx]))^{3/2} \operatorname{Cot} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{8 d \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} + \\
& \left( 7 (a (1 + \cos [c + dx]))^{3/2} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \right. \\
& \left. \left( -dx \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right) \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i (-\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right]) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) \Big/
\end{aligned}$$

$$\begin{aligned} & \left( 8\sqrt{2} d \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) \right) + \frac{(a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{8 d \left( \cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^2} + \\ & \frac{(a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left( 7 \cos\left[\frac{c}{2}\right] - 5 \sin\left[\frac{c}{2}\right] \right)}{16 d \left( \cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)} + \\ & \frac{(a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{8 d \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^2} + \\ & \frac{(a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left( -7 \cos\left[\frac{c}{2}\right] - 5 \sin\left[\frac{c}{2}\right] \right)}{16 d \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)} \end{aligned}$$

■ **Problem 111: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{3/2} \sec[c + dx]^4 dx$$

Optimal (type 3, 144 leaves, 6 steps):

$$\frac{11 a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{8 d} + \frac{11 a^2 \tan[c+dx]}{8 d \sqrt{a+a \cos[c+dx]}} + \frac{11 a^2 \sec[c+dx] \tan[c+dx]}{12 d \sqrt{a+a \cos[c+dx]}} + \frac{a^2 \sec[c+dx]^2 \tan[c+dx]}{3 d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 1825 leaves):

$$\begin{aligned} & - \left( \left( \left( \frac{11}{128} - \frac{11 i}{128} \right) (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16 i) e^{\frac{3ic}{2} + idx} + (20 + 20 i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34 i) e^{\frac{5ic}{2} + 2idx} - (20 + 20 i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right. \\ & (16 - 16 i) e^{\frac{7ic}{2} + 3idx} + (4 + 4 i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8 i e^{\frac{1}{2}i(c+dx)} - 16 \sqrt{2} e^{i(c+dx)} - 40 i e^{\frac{3}{2}i(c+dx)} + 34 \sqrt{2} e^{2i(c+dx)} + \\ & \left. \left. 40 i e^{\frac{5}{2}i(c+dx)} - 16 \sqrt{2} e^{3i(c+dx)} - 8 i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4 i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \right) / \\ & \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2 \sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4 i e^{i(c+dx)} + 2 \sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right) - \\ & \frac{11 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c+dx}{4}\right] - \sin\left[\frac{c+dx}{4}\right] - \sqrt{2} \sin\left[\frac{c+dx}{4}\right]}{-\cos\left[\frac{c+dx}{4}\right] + \sqrt{2} \cos\left[\frac{c+dx}{4}\right] - \sin\left[\frac{c+dx}{4}\right]} \right]}{32 \sqrt{2} d} (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \\ & \frac{11 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c+dx}{4}\right] + \sin\left[\frac{c+dx}{4}\right] - \sqrt{2} \sin\left[\frac{c+dx}{4}\right]}{\cos\left[\frac{c+dx}{4}\right] + \sqrt{2} \cos\left[\frac{c+dx}{4}\right] - \sin\left[\frac{c+dx}{4}\right]} \right]}{32 \sqrt{2} d} (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \end{aligned}$$

$$\frac{11 (a (1 + \cos [c + dx]))^{3/2} \log \left[ 2 - \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{64 \sqrt{2} d}$$

$$\frac{11 (a (1 + \cos [c + dx]))^{3/2} \log \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{64 \sqrt{2} d}$$

$$\frac{11 i \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i (-\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right]) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] (a (1 + \cos [c + dx]))^{3/2} \cot \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{16 d \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} +$$

$$\left( 11 (a (1 + \cos [c + dx]))^{3/2} \operatorname{Csc} \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \right)$$

$$\left( -dx \cos \left[ \frac{c}{2} \right] + 2 \log \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right) \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i (-\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right]) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) /$$

$$\left( 16 \sqrt{2} d \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right) \right) + \frac{(a (1 + \cos [c + dx]))^{3/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{24 d (\cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right])^3} + \frac{3 (a (1 + \cos [c + dx]))^{3/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sin \left[ \frac{dx}{2} \right]}{16 d (\cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right])^2} +$$

$$\frac{(a (1 + \cos [c + dx]))^{3/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 (11 \cos \left[ \frac{c}{2} \right] - 5 \sin \left[ \frac{c}{2} \right])}{32 d (\cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right])}$$

$$\frac{(a (1 + \cos [c + dx]))^{3/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{24 d (\cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right])^3} +$$

$$\frac{3 (a (1 + \cos [c + dx]))^{3/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sin \left[ \frac{dx}{2} \right]}{16 d (\cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right])^2} +$$

$$\frac{(a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-11 \cos\left[\frac{c}{2}\right] - 5 \sin\left[\frac{c}{2}\right])}{32 d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])}$$

■ **Problem 116: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{5/2} \operatorname{Sec}[c + dx] dx$$

Optimal (type 3, 98 leaves, 4 steps):

$$\frac{2 a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} + \frac{14 a^3 \sin[c+dx]}{3 d \sqrt{a+a \cos[c+dx]}} + \frac{2 a^2 \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{3 d}$$

Result (type 3, 1513 leaves):

$$\begin{aligned} & - \left( \left( \left( \frac{1}{16} - \frac{i}{16} \right) (1 + e^{ic}) \left( \sqrt{2} - (1-i) e^{\frac{ic}{2}} + (16-16i) e^{\frac{3ic}{2}+idx} + (20+20i) \sqrt{2} e^{2ic+\frac{3idx}{2}} - (34-34i) e^{\frac{5ic}{2}+2idx} - (20+20i) \sqrt{2} e^{3ic+\frac{5idx}{2}} + \right. \right. \right. \\ & \quad (16-16i) e^{\frac{7ic}{2}+3idx} + (4+4i) \sqrt{2} e^{4ic+\frac{7idx}{2}} - (1-i) e^{\frac{9ic}{2}+4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + \\ & \quad \left. \left. 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4+4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \right) / \\ & \quad \left( \left( (-1-i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right) - \\ & \frac{i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{4\sqrt{2}d} (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \\ & \frac{i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{4\sqrt{2}d} (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \\ & \frac{(a(1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8\sqrt{2}d} - \\ & \frac{(a(1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8\sqrt{2}d} + \\ & \frac{5 \cos\left[\frac{dx}{2}\right] (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{c}{2}\right]}{4d} - \end{aligned}$$

$$\begin{aligned}
& \frac{i \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right]-i\left(-\sqrt{2}+2 \sin\left[\frac{c}{2}\right]\right) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}}\right] \left(a\left(1+\cos\left[c+dx\right]\right)\right)^{5/2} \cot\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^5}{2 d \sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}} + \\
& \left( \left(a\left(1+\cos\left[c+dx\right]\right)\right)^{5/2} \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \right. \\
& \left. \left( -dx \cos\left[\frac{c}{2}\right] + 2 \operatorname{Log}\left[\sqrt{2}+2 \cos\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] \right) \sin\left[\frac{c}{2}\right] + \frac{4 i \sqrt{2} \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right]-i\left(-\sqrt{2}+2 \sin\left[\frac{c}{2}\right]\right) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}} \right) \right) / \\
& \left( 2 \sqrt{2} d \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) + \frac{\cos\left[\frac{3dx}{2}\right] \left(a\left(1+\cos\left[c+dx\right]\right)\right)^{5/2} \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \sin\left[\frac{3c}{2}\right]}{12 d} + \right. \\
& \left. \frac{5 \cos\left[\frac{c}{2}\right] \left(a\left(1+\cos\left[c+dx\right]\right)\right)^{5/2} \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{4 d} + \right. \\
& \left. \frac{\cos\left[\frac{3c}{2}\right] \left(a\left(1+\cos\left[c+dx\right]\right)\right)^{5/2} \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \sin\left[\frac{3dx}{2}\right]}{12 d} \right)
\end{aligned}$$

■ **Problem 117: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{5/2} \sec[c + dx]^2 dx$$

Optimal (type 3, 92 leaves, 4 steps):

$$\frac{5 a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} + \frac{a^3 \sin[c+dx]}{d \sqrt{a+a \cos[c+dx]}} + \frac{a^2 \sqrt{a+a \cos[c+dx]} \tan[c+dx]}{d}$$

Result (type 3, 1547 leaves):

$$\begin{aligned}
& - \left( \left( \frac{5}{32} - \frac{5i}{32} \right) (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right. \\
& \left. \left. (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16 \sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34 \sqrt{2} e^{2i(c+dx)} + \right. \right.
\end{aligned}$$



$$\begin{aligned}
& 40 i e^{\frac{5}{2} i (c+dx)} - 16 \sqrt{2} e^{3 i (c+dx)} - 8 i e^{\frac{7}{2} i (c+dx)} + \sqrt{2} e^{4 i (c+dx)} - (4 + 4 i) \sqrt{2} e^{\frac{1}{2} i (2c+dx)} \Big) x (a (1 + \cos [c + dx]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \Big) / \\
& \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2 \sqrt{2} e^{\frac{1}{2} i (c+dx)} - 4 i e^{i (c+dx)} + 2 \sqrt{2} e^{\frac{3}{2} i (c+dx)} + i e^{2 i (c+dx)} \right)^2 \right) \Big) - \\
& \frac{5 i \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{-\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] (a (1 + \cos [c + dx]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{8 \sqrt{2} d} - \\
& \frac{5 i \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] (a (1 + \cos [c + dx]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{8 \sqrt{2} d} - \\
& \frac{5 (a (1 + \cos [c + dx]))^{5/2} \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{16 \sqrt{2} d} - \\
& \frac{5 (a (1 + \cos [c + dx]))^{5/2} \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{16 \sqrt{2} d} + \\
& \frac{\cos \left[ \frac{dx}{2} \right] (a (1 + \cos [c + dx]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sin \left[ \frac{c}{2} \right]}{2 d} - \\
& \frac{5 i \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] (a (1 + \cos [c + dx]))^{5/2} \cot \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{4 d \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} + \\
& \left( \frac{5 (a (1 + \cos [c + dx]))^{5/2} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{\left( \begin{aligned} & -dx \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) \right) /
\end{aligned}$$

$$\left( 4 \sqrt{2} d \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) \right) + \frac{\cos\left[\frac{c}{2}\right] (a (1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{2 d} +$$

$$\frac{(a (1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8 d (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])} - \frac{(a (1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8 d (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])}$$

■ **Problem 118: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{5/2} \sec[c + dx]^3 dx$$

Optimal (type 3, 106 leaves, 4 steps):

$$\frac{19 a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{4 d} + \frac{9 a^3 \tan[c+dx]}{4 d \sqrt{a+a \cos[c+dx]}} + \frac{a^2 \sqrt{a+a \cos[c+dx]} \sec[c+dx] \tan[c+dx]}{2 d}$$

Result (type 3, 1693 leaves):

$$- \left( \left( \left( \frac{19}{128} - \frac{19 i}{128} \right) (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16 i) e^{\frac{3ic}{2} + idx} + (20 + 20 i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34 i) e^{\frac{5ic}{2} + 2idx} - (20 + 20 i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right.$$

$$(16 - 16 i) e^{\frac{7ic}{2} + 3idx} + (4 + 4 i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8 i e^{\frac{1}{2}i(c+dx)} - 16 \sqrt{2} e^{i(c+dx)} - 40 i e^{\frac{3}{2}i(c+dx)} + 34 \sqrt{2} e^{2i(c+dx)} +$$

$$\left. \left. 40 i e^{\frac{5}{2}i(c+dx)} - 16 \sqrt{2} e^{3i(c+dx)} - 8 i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4 i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x (a (1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \right) /$$

$$\left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2 \sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4 i e^{i(c+dx)} + 2 \sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right) -$$

$$\frac{19 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c+dx}{4}\right] - \sin\left[\frac{c+dx}{4}\right] - \sqrt{2} \sin\left[\frac{c+dx}{4}\right]}{-\cos\left[\frac{c+dx}{4}\right] + \sqrt{2} \cos\left[\frac{c+dx}{4}\right] - \sin\left[\frac{c+dx}{4}\right]}\right]}{32 \sqrt{2} d} (a (1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 -$$

$$\frac{19 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c+dx}{4}\right] + \sin\left[\frac{c+dx}{4}\right] - \sqrt{2} \sin\left[\frac{c+dx}{4}\right]}{\cos\left[\frac{c+dx}{4}\right] + \sqrt{2} \cos\left[\frac{c+dx}{4}\right] - \sin\left[\frac{c+dx}{4}\right]}\right]}{32 \sqrt{2} d} (a (1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 -$$

$$\frac{19 (a (1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{64 \sqrt{2} d} -$$

$$\frac{19 (a (1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{64 \sqrt{2} d}$$

$$19 i \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right]-i\left(-\sqrt{2}+2 \sin\left[\frac{c}{2}\right]\right) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}}\right] \left(a\left(1+\cos [c+d x]\right)\right)^{5 / 2} \cot \left[\frac{c}{2}\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^5$$


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$$16 d \sqrt{-2+4 \cos \left[\frac{c}{2}\right]^2+4 \sin \left[\frac{c}{2}\right]^2}$$

$$\left(19\left(a\left(1+\cos [c+d x]\right)\right)^{5 / 2} \operatorname{Csc}\left[\frac{c}{2}\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^5\right)$$

$$\left(-d x \cos \left[\frac{c}{2}\right]+2 \operatorname{Log}\left[\sqrt{2}+2 \cos \left[\frac{d x}{2}\right]\right] \sin \left[\frac{c}{2}\right]+2 \cos \left[\frac{c}{2}\right] \sin \left[\frac{d x}{2}\right]\right) \sin \left[\frac{c}{2}\right]+ \frac{4 i \sqrt{2} \operatorname{ArcTan}\left[\frac{2 i \cos \left[\frac{c}{2}\right]-i\left(-\sqrt{2}+2 \sin \left[\frac{c}{2}\right]\right) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2+4 \cos \left[\frac{c}{2}\right]^2+4 \sin \left[\frac{c}{2}\right]^2}}\right] \cos \left[\frac{c}{2}\right]}{\sqrt{-2+4 \cos \left[\frac{c}{2}\right]^2+4 \sin \left[\frac{c}{2}\right]^2}}\right) /$$

$$\left(16 \sqrt{2} d\left(4 \cos \left[\frac{c}{2}\right]^2+4 \sin \left[\frac{c}{2}\right]^2\right)\right) + \frac{\left(a\left(1+\cos [c+d x]\right)\right)^{5 / 2} \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^5 \sin \left[\frac{d x}{2}\right]}{16 d\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2} +$$

$$\frac{\left(a\left(1+\cos [c+d x]\right)\right)^{5 / 2} \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^5\left(11 \cos \left[\frac{c}{2}\right]-9 \sin \left[\frac{c}{2}\right]\right)}{32 d\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)} +$$

$$\frac{\left(a\left(1+\cos [c+d x]\right)\right)^{5 / 2} \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^5 \sin \left[\frac{d x}{2}\right]}{16 d\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2} +$$

$$\frac{\left(a\left(1+\cos [c+d x]\right)\right)^{5 / 2} \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^5\left(-11 \cos \left[\frac{c}{2}\right]-9 \sin \left[\frac{c}{2}\right]\right)}{32 d\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)}$$

■ **Problem 119: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\left(a+a \cos [c+d x]\right)^{5 / 2} \sec [c+d x]^4 d x$$

Optimal (type 3, 144 leaves, 5 steps):

$$\frac{25 a^{5 / 2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{8 d} + \frac{25 a^3 \operatorname{Tan}[c+d x]}{8 d \sqrt{a+a \cos [c+d x]}} + \frac{13 a^3 \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{12 d \sqrt{a+a \cos [c+d x]}} + \frac{a^2 \sqrt{a+a \cos [c+d x]} \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{3 d}$$

Result (type 3, 1825 leaves) :

$$- \left( \left( \left( \frac{25}{256} - \frac{25 i}{256} \right) (1 + e^{i c}) \left( \sqrt{2} - (1 - i) e^{\frac{i c}{2}} + (16 - 16 i) e^{\frac{3 i c}{2} + i d x} + (20 + 20 i) \sqrt{2} e^{2 i c + \frac{3 i d x}{2}} - (34 - 34 i) e^{\frac{5 i c}{2} + 2 i d x} - (20 + 20 i) \sqrt{2} e^{3 i c + \frac{5 i d x}{2}} + \right. \right. \right. \\ \left. \left. (16 - 16 i) e^{\frac{7 i c}{2} + 3 i d x} + (4 + 4 i) \sqrt{2} e^{4 i c + \frac{7 i d x}{2}} - (1 - i) e^{\frac{9 i c}{2} + 4 i d x} + 8 i e^{\frac{1}{2} i (c + d x)} - 16 \sqrt{2} e^{i (c + d x)} - 40 i e^{\frac{3}{2} i (c + d x)} + 34 \sqrt{2} e^{2 i (c + d x)} + \right. \right. \\ \left. \left. 40 i e^{\frac{5}{2} i (c + d x)} - 16 \sqrt{2} e^{3 i (c + d x)} - 8 i e^{\frac{7}{2} i (c + d x)} + \sqrt{2} e^{4 i (c + d x)} - (4 + 4 i) \sqrt{2} e^{\frac{1}{2} i (2 c + d x)} \right) x (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \right) / \\ \left( \left( (-1 - i) + \sqrt{2} e^{\frac{i c}{2}} \right) (-1 + e^{i c}) \left( i - 2 \sqrt{2} e^{\frac{1}{2} i (c + d x)} - 4 i e^{i (c + d x)} + 2 \sqrt{2} e^{\frac{3}{2} i (c + d x)} + i e^{2 i (c + d x)} \right)^2 \right) -$$

$$\frac{25 i \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{d x}{4} \right] - \sin \left[ \frac{c}{4} + \frac{d x}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{d x}{4} \right]}{-\cos \left[ \frac{c}{4} + \frac{d x}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{d x}{4} \right] - \sin \left[ \frac{c}{4} + \frac{d x}{4} \right]} \right] (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5}{64 \sqrt{2} d} -$$

$$\frac{25 i \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{d x}{4} \right] + \sin \left[ \frac{c}{4} + \frac{d x}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{d x}{4} \right]}{\cos \left[ \frac{c}{4} + \frac{d x}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{d x}{4} \right] - \sin \left[ \frac{c}{4} + \frac{d x}{4} \right]} \right] (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5}{64 \sqrt{2} d} -$$

$$\frac{25 (a (1 + \cos [c + d x]))^{5/2} \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5}{128 \sqrt{2} d} -$$

$$\frac{25 (a (1 + \cos [c + d x]))^{5/2} \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5}{128 \sqrt{2} d} -$$

$$\frac{25 i \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \tan \left[ \frac{d x}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] (a (1 + \cos [c + d x]))^{5/2} \operatorname{Cot} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5}{32 d \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} +$$

$$\left( 25 (a (1 + \cos [c + d x]))^{5/2} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \right)$$

$$\left( -dx \cos\left[\frac{c}{2}\right] + 2 \log\left[\sqrt{2} + 2 \cos\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] \right) \sin\left[\frac{c}{2}\right] + \frac{4i\sqrt{2} \operatorname{ArcTan}\left[\frac{2i\cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2\sin\left[\frac{c}{2}\right])\tan\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}} \right) /$$

$$\begin{aligned} & \left( 32\sqrt{2} d \left( 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2 \right) \right) + \frac{(a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{48d \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^3} + \frac{5(a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{32d \left( \cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^2} + \\ & \frac{5(a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( 5\cos\left[\frac{c}{2}\right] - 3\sin\left[\frac{c}{2}\right] \right)}{64d \left( \cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)} - \\ & \frac{(a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{48d \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^3} + \\ & \frac{5(a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{32d \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^2} - \\ & \frac{5(a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( 5\cos\left[\frac{c}{2}\right] + 3\sin\left[\frac{c}{2}\right] \right)}{64d \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)} \end{aligned}$$

■ **Problem 120: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{5/2} \operatorname{Sec}[c + dx]^5 dx$$

Optimal (type 3, 182 leaves, 6 steps):

$$\begin{aligned} & \frac{163 a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{64 d} + \frac{163 a^3 \tan[c+dx]}{64 d \sqrt{a+a \cos[c+dx]}} + \frac{163 a^3 \operatorname{Sec}[c+dx] \tan[c+dx]}{96 d \sqrt{a+a \cos[c+dx]}} + \\ & \frac{17 a^3 \operatorname{Sec}[c+dx]^2 \tan[c+dx]}{24 d \sqrt{a+a \cos[c+dx]}} + \frac{a^2 \sqrt{a+a \cos[c+dx]} \operatorname{Sec}[c+dx]^3 \tan[c+dx]}{4 d} \end{aligned}$$

Result (type 3, 2069 leaves):

$$\begin{aligned} & - \left( \left( \frac{163}{2048} - \frac{163i}{2048} \right) (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right. \\ & \left. \left. (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + \right. \right. \end{aligned}$$

$$\begin{aligned}
& 40 i e^{\frac{5}{2} i (c+dx)} - 16 \sqrt{2} e^{3 i (c+dx)} - 8 i e^{\frac{7}{2} i (c+dx)} + \sqrt{2} e^{4 i (c+dx)} - (4 + 4 i) \sqrt{2} e^{\frac{1}{2} i (2c+dx)} \Big) x (a (1 + \cos [c + dx]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \Big) / \\
& \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2 \sqrt{2} e^{\frac{1}{2} i (c+dx)} - 4 i e^{i (c+dx)} + 2 \sqrt{2} e^{\frac{3}{2} i (c+dx)} + i e^{2 i (c+dx)} \right)^2 \right) \Big) - \\
& \frac{163 i \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{-\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] (a (1 + \cos [c + dx]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{512 \sqrt{2} d} \\
& \frac{163 i \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] (a (1 + \cos [c + dx]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{512 \sqrt{2} d} \\
& \frac{163 (a (1 + \cos [c + dx]))^{5/2} \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{1024 \sqrt{2} d} \\
& \frac{163 (a (1 + \cos [c + dx]))^{5/2} \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{1024 \sqrt{2} d} \\
& \frac{163 i \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] (a (1 + \cos [c + dx]))^{5/2} \operatorname{Cot} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{256 d \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} + \\
& \left( 163 (a (1 + \cos [c + dx]))^{5/2} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \right. \\
& \left. \left( -dx \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right) \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( 256 \sqrt{2} d \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) \right) + \frac{(a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{64 d \left( \cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^4} + \\
& \frac{(a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( 23 \cos\left[\frac{c}{2}\right] - 17 \sin\left[\frac{c}{2}\right] \right)}{384 d \left( \cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^3} + \\
& \frac{43 (a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{256 d \left( \cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^2} + \\
& \frac{(a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( 163 \cos\left[\frac{c}{2}\right] - 77 \sin\left[\frac{c}{2}\right] \right)}{512 d \left( \cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)} + \\
& \frac{(a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{64 d \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^4} + \\
& \frac{(a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( -23 \cos\left[\frac{c}{2}\right] - 17 \sin\left[\frac{c}{2}\right] \right)}{384 d \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^3} + \\
& \frac{43 (a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{256 d \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^2} + \\
& \frac{(a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( -163 \cos\left[\frac{c}{2}\right] - 77 \sin\left[\frac{c}{2}\right] \right)}{512 d \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)}
\end{aligned}$$

■ **Problem 127: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]}{\sqrt{a + a \cos[c + dx]}} dx$$

Optimal (type 3, 85 leaves, 5 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} - \frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d}$$

Result (type 3, 1413 leaves):

$$\begin{aligned}
& - \left( \left( \left( \frac{1}{2} - \frac{i}{2} \right) (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right. \right. \\
& \left. \left. (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16 \sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 34 \sqrt{2} e^{2i(c+dx)} + 40 i e^{\frac{5}{2}i(c+dx)} - 16 \sqrt{2} e^{3i(c+dx)} - 8 i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \Big) x \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \Big) / \\
& \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \sqrt{a(1 + \operatorname{Cos}[c + dx])} \right) \Big) - \\
& \frac{i \sqrt{2} \operatorname{ArcTan} \left[ \frac{\operatorname{Cos} \left[ \frac{c}{4} + \frac{dx}{4} \right] - \operatorname{Sin} \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \operatorname{Sin} \left[ \frac{c}{4} + \frac{dx}{4} \right]}{-\operatorname{Cos} \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \operatorname{Cos} \left[ \frac{c}{4} + \frac{dx}{4} \right] - \operatorname{Sin} \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{d \sqrt{a(1 + \operatorname{Cos}[c + dx])}} - \\
& \frac{i \sqrt{2} \operatorname{ArcTan} \left[ \frac{\operatorname{Cos} \left[ \frac{c}{4} + \frac{dx}{4} \right] + \operatorname{Sin} \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \operatorname{Sin} \left[ \frac{c}{4} + \frac{dx}{4} \right]}{\operatorname{Cos} \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \operatorname{Cos} \left[ \frac{c}{4} + \frac{dx}{4} \right] - \operatorname{Sin} \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{d \sqrt{a(1 + \operatorname{Cos}[c + dx])}} + \\
& \frac{2 \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{c}{4} + \frac{dx}{4} \right] - \operatorname{Sin} \left[ \frac{c}{4} + \frac{dx}{4} \right] \right]}{d \sqrt{a(1 + \operatorname{Cos}[c + dx])}} - \\
& \frac{2 \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{c}{4} + \frac{dx}{4} \right] + \operatorname{Sin} \left[ \frac{c}{4} + \frac{dx}{4} \right] \right]}{d \sqrt{a(1 + \operatorname{Cos}[c + dx])}} - \\
& \frac{\operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Log} \left[ 2 - \sqrt{2} \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \operatorname{Sin} \left[ \frac{c}{2} + \frac{dx}{2} \right] \right]}{\sqrt{2} d \sqrt{a(1 + \operatorname{Cos}[c + dx])}} - \\
& \frac{\operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Log} \left[ 2 + \sqrt{2} \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \operatorname{Sin} \left[ \frac{c}{2} + \frac{dx}{2} \right] \right]}{\sqrt{2} d \sqrt{a(1 + \operatorname{Cos}[c + dx])}} - \\
& \frac{4 i \operatorname{ArcTan} \left[ \frac{2 i \operatorname{Cos} \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \operatorname{Sin} \left[ \frac{c}{2} \right] \right) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \operatorname{Cos} \left[ \frac{c}{2} \right]^2 + 4 \operatorname{Sin} \left[ \frac{c}{2} \right]^2}} \right] \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Cot} \left[ \frac{c}{2} \right]}{d \sqrt{a(1 + \operatorname{Cos}[c + dx])} \sqrt{-2 + 4 \operatorname{Cos} \left[ \frac{c}{2} \right]^2 + 4 \operatorname{Sin} \left[ \frac{c}{2} \right]^2}} + \\
& \left( 2 \sqrt{2} \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Csc} \left[ \frac{c}{2} \right] \right)
\end{aligned}$$



$$\left( -dx \cos\left[\frac{c}{2}\right] + 2 \log\left[\sqrt{2} + 2 \cos\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] \right) \sin\left[\frac{c}{2}\right] + \frac{4i\sqrt{2} \operatorname{ArcTan}\left[\frac{2i\cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2\sin\left[\frac{c}{2}\right])\tan\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}} \right) /$$

$$\left( d \sqrt{a(1 + \cos[c + dx])} \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) \right)$$

■ **Problem 128: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^2}{\sqrt{a + a \cos[c + dx]}} dx$$

Optimal (type 3, 108 leaves, 6 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} + \frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} + \frac{\tan[c + dx]}{d \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 1540 leaves):

$$\left( \left( \frac{1}{4} - \frac{i}{4} \right) (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right.$$

$$\left. (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + \right.$$

$$\left. 34\sqrt{2} e^{2i(c+dx)} + 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \Big/$$

$$\left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \sqrt{a(1 + \cos[c + dx])} \right) +$$

$$\frac{i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c+dx}{4}\right] - \sin\left[\frac{c+dx}{4}\right] - \sqrt{2} \sin\left[\frac{c+dx}{4}\right]}{-\cos\left[\frac{c+dx}{4}\right] + \sqrt{2} \cos\left[\frac{c+dx}{4}\right] - \sin\left[\frac{c+dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]}{\sqrt{2} d \sqrt{a(1 + \cos[c + dx])}} +$$

$$i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c+dx}{4}\right] + \sin\left[\frac{c+dx}{4}\right] - \sqrt{2} \sin\left[\frac{c+dx}{4}\right]}{\cos\left[\frac{c+dx}{4}\right] + \sqrt{2} \cos\left[\frac{c+dx}{4}\right] - \sin\left[\frac{c+dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right] -$$

$$\frac{\sqrt{2} d \sqrt{a(1 + \cos[c + dx])}}{2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \log\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]} +$$

$$\frac{2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \log\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{d \sqrt{a(1 + \cos[c + dx])}} +$$

$$\begin{aligned}
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{2\sqrt{2} d \sqrt{a} (1 + \cos[c + dx])} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{2\sqrt{2} d \sqrt{a} (1 + \cos[c + dx])} + \\
& \frac{2i \operatorname{ArcTan}\left[\frac{2i \cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Cot}\left[\frac{c}{2}\right]}{d \sqrt{a} (1 + \cos[c + dx]) \sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} - \left( \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \left. \left( -dx \cos\left[\frac{c}{2}\right] + 2 \operatorname{Log}\left[\sqrt{2} + 2 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + \frac{4i\sqrt{2} \operatorname{ArcTan}\left[\frac{2i \cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right) \right) / \\
& \frac{\left( d \sqrt{a} (1 + \cos[c + dx]) \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) \right) + \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]}{d \sqrt{a} (1 + \cos[c + dx]) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)} - \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]}{d \sqrt{a} (1 + \cos[c + dx]) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)}
\end{aligned}$$

■ **Problem 129: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^3}{\sqrt{a + a \cos[c + dx]}} dx$$

Optimal (type 3, 147 leaves, 7 steps):

$$\frac{7 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{4 \sqrt{a} d} - \frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} - \frac{\operatorname{Tan}[c+dx]}{4 d \sqrt{a+a \cos[c+dx]}} + \frac{\operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{2 d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 1791 leaves):

$$\begin{aligned}
& - \left( \left( \frac{7}{16} - \frac{7i}{16} \right) (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right. \\
& \left. \left. (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16 \sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 34 \sqrt{2} e^{2i(c+dx)} + 40 i e^{\frac{5}{2}i(c+dx)} - 16 \sqrt{2} e^{3i(c+dx)} - 8 i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \Big) x \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \Big) / \\
& \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2 \sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4 i e^{i(c+dx)} + 2 \sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \sqrt{a(1 + \operatorname{Cos}[c + dx])} \right) \Big) - \\
& \frac{7 i \operatorname{ArcTan} \left[ \frac{\operatorname{Cos} \left[ \frac{c}{4} + \frac{dx}{4} \right] - \operatorname{Sin} \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \operatorname{Sin} \left[ \frac{c}{4} + \frac{dx}{4} \right]}{-\operatorname{Cos} \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \operatorname{Cos} \left[ \frac{c}{4} + \frac{dx}{4} \right] - \operatorname{Sin} \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{4 \sqrt{2} d \sqrt{a(1 + \operatorname{Cos}[c + dx])}} - \\
& \frac{7 i \operatorname{ArcTan} \left[ \frac{\operatorname{Cos} \left[ \frac{c}{4} + \frac{dx}{4} \right] + \operatorname{Sin} \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \operatorname{Sin} \left[ \frac{c}{4} + \frac{dx}{4} \right]}{\operatorname{Cos} \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \operatorname{Cos} \left[ \frac{c}{4} + \frac{dx}{4} \right] - \operatorname{Sin} \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{4 \sqrt{2} d \sqrt{a(1 + \operatorname{Cos}[c + dx])}} + \\
& \frac{2 \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{c}{4} + \frac{dx}{4} \right] - \operatorname{Sin} \left[ \frac{c}{4} + \frac{dx}{4} \right] \right]}{d \sqrt{a(1 + \operatorname{Cos}[c + dx])}} - \\
& \frac{2 \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{c}{4} + \frac{dx}{4} \right] + \operatorname{Sin} \left[ \frac{c}{4} + \frac{dx}{4} \right] \right]}{d \sqrt{a(1 + \operatorname{Cos}[c + dx])}} - \\
& \frac{7 \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Log} \left[ 2 - \sqrt{2} \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \operatorname{Sin} \left[ \frac{c}{2} + \frac{dx}{2} \right] \right]}{8 \sqrt{2} d \sqrt{a(1 + \operatorname{Cos}[c + dx])}} - \\
& \frac{7 \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Log} \left[ 2 + \sqrt{2} \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \operatorname{Sin} \left[ \frac{c}{2} + \frac{dx}{2} \right] \right]}{8 \sqrt{2} d \sqrt{a(1 + \operatorname{Cos}[c + dx])}} - \\
& \frac{7 i \operatorname{ArcTan} \left[ \frac{2 i \operatorname{Cos} \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \operatorname{Sin} \left[ \frac{c}{2} \right] \right) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \operatorname{Cos} \left[ \frac{c}{2} \right]^2 + 4 \operatorname{Sin} \left[ \frac{c}{2} \right]^2}} \right] \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Cot} \left[ \frac{c}{2} \right]}{2 d \sqrt{a(1 + \operatorname{Cos}[c + dx])} \sqrt{-2 + 4 \operatorname{Cos} \left[ \frac{c}{2} \right]^2 + 4 \operatorname{Sin} \left[ \frac{c}{2} \right]^2}} + \\
& \left( 7 \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Csc} \left[ \frac{c}{2} \right] \right)
\end{aligned}$$

$$\left( -dx \cos\left[\frac{c}{2}\right] + 2 \operatorname{Log}\left[\sqrt{2} + 2 \cos\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] \right) \sin\left[\frac{c}{2}\right] + \frac{4i\sqrt{2} \operatorname{ArcTan}\left[\frac{2i\cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2\sin\left[\frac{c}{2}\right])\tan\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}} \right) /$$

$$\begin{aligned} & \left( 2\sqrt{2} d \sqrt{a(1 + \cos[c + dx])} \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) \right) + \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{2 d \sqrt{a(1 + \cos[c + dx])} \left( \cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^2} + \\ & \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \left( -\cos\left[\frac{c}{2}\right] + 3 \sin\left[\frac{c}{2}\right] \right)}{4 d \sqrt{a(1 + \cos[c + dx])} \left( \cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)} + \\ & \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{2 d \sqrt{a(1 + \cos[c + dx])} \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^2} + \\ & \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \left( \cos\left[\frac{c}{2}\right] + 3 \sin\left[\frac{c}{2}\right] \right)}{4 d \sqrt{a(1 + \cos[c + dx])} \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)} \end{aligned}$$

■ **Problem 130: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^4}{\sqrt{a + a \cos[c + dx]}} dx$$

Optimal (type 3, 181 leaves, 8 steps):

$$-\frac{9 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{8 \sqrt{a} d} + \frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} + \frac{7 \tan[c + dx]}{8 d \sqrt{a + a \cos[c + dx]}} - \frac{\operatorname{Sec}[c + dx] \tan[c + dx]}{12 d \sqrt{a + a \cos[c + dx]}} + \frac{\operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 d \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 1921 leaves):

$$\begin{aligned} & \left( \left( \frac{9}{32} - \frac{9i}{32} \right) (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right. \\ & \quad (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + \\ & \quad \left. 34\sqrt{2} e^{2i(c+dx)} + 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \Big) / \\ & \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \sqrt{a(1 + \cos[c + dx])} \right) + \end{aligned}$$

$$\begin{aligned}
& \frac{9 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]-\sqrt{2} \sin\left[\frac{c}{4}+\frac{dx}{4}\right]}{-\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sqrt{2} \cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2}+\frac{dx}{2}\right]}{8 \sqrt{2} d \sqrt{a} (1+\cos [c+d x])} + \\
& \frac{9 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sin\left[\frac{c}{4}+\frac{dx}{4}\right]-\sqrt{2} \sin\left[\frac{c}{4}+\frac{dx}{4}\right]}{\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sqrt{2} \cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2}+\frac{dx}{2}\right]}{8 \sqrt{2} d \sqrt{a} (1+\cos [c+d x])} - \\
& \frac{2 \cos\left[\frac{c}{2}+\frac{dx}{2}\right] \operatorname{Log}\left[\cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]\right]}{d \sqrt{a} (1+\cos [c+d x])} + \\
& \frac{2 \cos\left[\frac{c}{2}+\frac{dx}{2}\right] \operatorname{Log}\left[\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sin\left[\frac{c}{4}+\frac{dx}{4}\right]\right]}{d \sqrt{a} (1+\cos [c+d x])} + \\
& \frac{9 \cos\left[\frac{c}{2}+\frac{dx}{2}\right] \operatorname{Log}\left[2-\sqrt{2} \cos\left[\frac{c}{2}+\frac{dx}{2}\right]-\sqrt{2} \sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right]}{16 \sqrt{2} d \sqrt{a} (1+\cos [c+d x])} + \\
& \frac{9 \cos\left[\frac{c}{2}+\frac{dx}{2}\right] \operatorname{Log}\left[2+\sqrt{2} \cos\left[\frac{c}{2}+\frac{dx}{2}\right]-\sqrt{2} \sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right]}{16 \sqrt{2} d \sqrt{a} (1+\cos [c+d x])} + \\
& \frac{9 i \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right]-i\left(-\sqrt{2}+2 \sin\left[\frac{c}{2}\right]\right) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}+\frac{dx}{2}\right] \cot\left[\frac{c}{2}\right]}{4 d \sqrt{a} (1+\cos [c+d x]) \sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}} - \left( 9 \cos\left[\frac{c}{2}+\frac{dx}{2}\right] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \left. \left( -d x \cos\left[\frac{c}{2}\right]+2 \operatorname{Log}\left[\sqrt{2}+2 \cos\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right]+2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]\right) \sin\left[\frac{c}{2}\right]+ \frac{4 i \sqrt{2} \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right]-i\left(-\sqrt{2}+2 \sin\left[\frac{c}{2}\right]\right) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}} \right) \right) / \\
& \left( 4 \sqrt{2} d \sqrt{a} (1+\cos [c+d x]) \left( 4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2 \right) \right) + \frac{\cos\left[\frac{c}{2}+\frac{dx}{2}\right]}{6 d \sqrt{a} (1+\cos [c+d x]) \left(\cos\left[\frac{c}{2}+\frac{dx}{2}\right]-\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right)^3} - \\
& \frac{\cos\left[\frac{c}{2}+\frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{4 d \sqrt{a} (1+\cos [c+d x]) \left(\cos\left[\frac{c}{2}\right]-\sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2}+\frac{dx}{2}\right]-\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right)^2} +
\end{aligned}$$

$$\frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \left(7 \cos\left[\frac{c}{2}\right] - 9 \sin\left[\frac{c}{2}\right]\right)}{8 d \sqrt{a} (1 + \cos[c + dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} -$$

$$\frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]}{6 d \sqrt{a} (1 + \cos[c + dx]) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} -$$

$$\frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{4 d \sqrt{a} (1 + \cos[c + dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} +$$

$$\frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \left(-7 \cos\left[\frac{c}{2}\right] - 9 \sin\left[\frac{c}{2}\right]\right)}{8 d \sqrt{a} (1 + \cos[c + dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}$$

■ **Problem 136: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]}{(a + a \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 114 leaves, 6 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{3/2} d} - \frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{\sin[c + dx]}{2 d (a + a \cos[c + dx])^{3/2}}$$

Result (type 3, 1787 leaves):

$$- \left( \left( (1 - i) (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right. \right.$$

$$\left. \left. (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + \right. \right.$$

$$\left. \left. 34\sqrt{2} e^{2i(c+dx)} + 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \Bigg) /$$

$$\left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 (a (1 + \cos[c + dx]))^{3/2} \right) -$$

$$\frac{2i \sqrt{2} \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{d (a (1 + \cos[c + dx]))^{3/2}} +$$

$$\frac{5 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{d (a (1 + \cos[c + dx]))^{3/2}} -$$

$$\frac{5 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{d (a (1 + \cos[c + dx]))^{3/2}} -$$

$$\begin{aligned}
& \frac{\sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a (1 + \cos[c + dx]))^{3/2}} + \\
& \left( (1 - i) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left( (1 + i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1 - i) \sin\left[\frac{c}{4}\right] - i \sqrt{2} \sin\left[\frac{c}{4}\right] \right) \right. \\
& \quad \left. \left( (-1 - i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] + (1 - i) \sin\left[\frac{c}{4}\right] - i \sqrt{2} \sin\left[\frac{c}{4}\right] \right) \right) / \left( \sqrt{2} d (a (1 + \cos[c + dx]))^{3/2} \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right) - \\
& \left( \left( \frac{1}{2} + \frac{i}{2} \right) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \left( (1 + i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1 - i) \sin\left[\frac{c}{4}\right] - i \sqrt{2} \sin\left[\frac{c}{4}\right] \right) \right. \\
& \quad \left. \left( (-1 - i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] + (1 - i) \sin\left[\frac{c}{4}\right] - i \sqrt{2} \sin\left[\frac{c}{4}\right] \right) \right) / \left( \sqrt{2} d (a (1 + \cos[c + dx]))^{3/2} \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right) - \\
& \frac{8 i \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right] - i (-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Cot}\left[\frac{c}{2}\right]}{d (a (1 + \cos[c + dx]))^{3/2} \sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right) + \left( 4 \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left( -dx \cos\left[\frac{c}{2}\right] + 2 \operatorname{Log}\left[\sqrt{2} + 2 \cos\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] \right) \sin\left[\frac{c}{2}\right] + \frac{4 i \sqrt{2} \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right] - i (-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right) \right) / \\
& \left( d (a (1 + \cos[c + dx]))^{3/2} \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) \right) - \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2 d (a (1 + \cos[c + dx]))^{3/2} \left( \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] \right)^2} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2 d (a (1 + \cos[c + dx]))^{3/2} \left( \cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] \right)^2}
\end{aligned}$$

■ **Problem 137: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^2}{(a + a \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 144 leaves, 7 steps):

$$-\frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{3/2} d} + \frac{9 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{\operatorname{Tan}[c+dx]}{2 d (a+a \cos[c+dx])^{3/2}} + \frac{3 \operatorname{Tan}[c+dx]}{2 a d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 1691 leaves):

$$\begin{aligned} & \left( \left( \frac{3}{2} - \frac{3i}{2} \right) (1 + e^{ic}) \left( \sqrt{2} - (1-i) e^{\frac{ic}{2}} + (16-16i) e^{\frac{3ic}{2}+idx} + (20+20i) \sqrt{2} e^{2ic+\frac{3idx}{2}} - (34-34i) e^{\frac{5ic}{2}+2idx} - (20+20i) \sqrt{2} e^{3ic+\frac{5idx}{2}} + \right. \right. \\ & \quad (16-16i) e^{\frac{7ic}{2}+3idx} + (4+4i) \sqrt{2} e^{4ic+\frac{7idx}{2}} - (1-i) e^{\frac{9ic}{2}+4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + \\ & \quad \left. \left. 34\sqrt{2} e^{2i(c+dx)} + 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4+4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \right) / \\ & \left( \left( (-1-i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 (a(1+\cos[c+dx]))^{3/2} \right) + \\ & \frac{3i\sqrt{2} \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{d (a(1+\cos[c+dx]))^{3/2}} + \\ & \frac{3i\sqrt{2} \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{d (a(1+\cos[c+dx]))^{3/2}} - \\ & \frac{9 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{d (a(1+\cos[c+dx]))^{3/2}} + \\ & \frac{9 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{d (a(1+\cos[c+dx]))^{3/2}} + \\ & \frac{3 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{\sqrt{2} d (a(1+\cos[c+dx]))^{3/2}} + \\ & \frac{3 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{\sqrt{2} d (a(1+\cos[c+dx]))^{3/2}} + \\ & \frac{12i \operatorname{ArcTan}\left[\frac{2i \cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Cot}\left[\frac{c}{2}\right]}{d (a(1+\cos[c+dx]))^{3/2} \sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} - \left( 6\sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Csc}\left[\frac{c}{2}\right] \right) \end{aligned}$$



$$\left( -dx \cos\left[\frac{c}{2}\right] + 2 \log\left[\sqrt{2} + 2 \cos\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] \right) \sin\left[\frac{c}{2}\right] + \frac{4i\sqrt{2} \operatorname{ArcTan}\left[\frac{2i\cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2\sin\left[\frac{c}{2}\right])\tan\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}} \right) /$$

$$\left( d(a(1 + \cos[c + dx]))^{3/2} \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) \right) + \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2d(a(1 + \cos[c + dx]))^{3/2} \left( \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] \right)^2} -$$

$$\frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2d(a(1 + \cos[c + dx]))^{3/2} \left( \cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] \right)^2} + \frac{2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{d(a(1 + \cos[c + dx]))^{3/2} \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)} -$$

$$\frac{2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{d(a(1 + \cos[c + dx]))^{3/2} \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)}$$

■ **Problem 138: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^3}{(a + a \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 185 leaves, 8 steps):

$$\frac{19 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{4 a^{3/2} d} - \frac{13 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{7 \tan[c + dx]}{4 a d \sqrt{a + a \cos[c + dx]}} - \frac{\sec[c + dx] \tan[c + dx]}{2 d (a + a \cos[c + dx])^{3/2}} + \frac{\sec[c + dx] \tan[c + dx]}{a d \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 1941 leaves):

$$- \left( \left( \left( \frac{19}{8} - \frac{19i}{8} \right) (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right. \right.$$

$$\left. \left. (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + \right. \right.$$

$$\left. \left. 34\sqrt{2} e^{2i(c+dx)} + 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \right) /$$

$$\left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 (a(1 + \cos[c + dx]))^{3/2} \right) -$$

$$\frac{19i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2 \sqrt{2} d (a(1 + \cos[c + dx]))^{3/2}}$$

$$\begin{aligned}
& \frac{19 i \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{2 \sqrt{2} d (a (1 + \cos [c + dx]))^{3/2}} + \\
& \frac{13 \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \operatorname{Log} \left[ \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] \right]}{d (a (1 + \cos [c + dx]))^{3/2}} - \\
& \frac{13 \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \operatorname{Log} \left[ \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] \right]}{d (a (1 + \cos [c + dx]))^{3/2}} - \\
& \frac{19 \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right]}{4 \sqrt{2} d (a (1 + \cos [c + dx]))^{3/2}} - \\
& \frac{19 \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right]}{4 \sqrt{2} d (a (1 + \cos [c + dx]))^{3/2}} - \\
& \frac{19 i \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i (-\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right]) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \operatorname{Cot} \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} + \\
& d (a (1 + \cos [c + dx]))^{3/2} \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}
\end{aligned}$$

$$\left( 19 \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \operatorname{Csc} \left[ \frac{c}{2} \right] \right)$$

$$\left( -dx \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right) \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i (-\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right]) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) /$$

$$\left( \sqrt{2} d (a (1 + \cos [c + dx]))^{3/2} \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right) \right) - \frac{\cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{2 d (a (1 + \cos [c + dx]))^{3/2} \left( \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] \right)^2} +$$

$$\begin{aligned}
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2d (a (1 + \cos[c + dx]))^{3/2} \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{d (a (1 + \cos[c + dx]))^{3/2} \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(-5 \cos\left[\frac{c}{2}\right] + 7 \sin\left[\frac{c}{2}\right]\right)}{2d (a (1 + \cos[c + dx]))^{3/2} \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{d (a (1 + \cos[c + dx]))^{3/2} \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(5 \cos\left[\frac{c}{2}\right] + 7 \sin\left[\frac{c}{2}\right]\right)}{2d (a (1 + \cos[c + dx]))^{3/2} \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 139: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^4}{(a + a \cos[c + dx])^{5/2}} dx$$

Optimal (type 3, 183 leaves, 7 steps):

$$\begin{aligned}
& \frac{163 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{\cos[c + dx]^3 \sin[c + dx]}{4 d (a + a \cos[c + dx])^{5/2}} - \\
& \frac{17 \cos[c + dx]^2 \sin[c + dx]}{16 a d (a + a \cos[c + dx])^{3/2}} - \frac{197 \sin[c + dx]}{24 a^2 d \sqrt{a + a \cos[c + dx]}} + \frac{95 \sqrt{a + a \cos[c + dx]} \sin[c + dx]}{48 a^3 d}
\end{aligned}$$

Result (type 3, 587 leaves):

$$\begin{aligned}
& - \frac{163 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{4d(a(1 + \cos[c + dx]))^{5/2}} + \frac{163 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{4d(a(1 + \cos[c + dx]))^{5/2}} - \frac{40 \cos\left[\frac{dx}{2}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{c}{2}\right]}{d(a(1 + \cos[c + dx]))^{5/2}} + \\
& \frac{8 \cos\left[\frac{3dx}{2}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{3c}{2}\right]}{3d(a(1 + \cos[c + dx]))^{5/2}} - \frac{40 \cos\left[\frac{c}{2}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{d(a(1 + \cos[c + dx]))^{5/2}} + \frac{8 \cos\left[\frac{3c}{2}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{3dx}{2}\right]}{3d(a(1 + \cos[c + dx]))^{5/2}} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8d(a(1 + \cos[c + dx]))^{5/2} \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^4} - \frac{29 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8d(a(1 + \cos[c + dx]))^{5/2} \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2} \\
& + \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8d(a(1 + \cos[c + dx]))^{5/2} \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^4} + \frac{29 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8d(a(1 + \cos[c + dx]))^{5/2} \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2}
\end{aligned}$$

■ **Problem 140: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^3}{(a + a \cos[c + dx])^{5/2}} dx$$

Optimal (type 3, 145 leaves, 6 steps):

$$\begin{aligned}
& - \frac{75 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{\cos[c + dx]^2 \sin[c + dx]}{4d(a + a \cos[c + dx])^{5/2}} + \frac{13 \sin[c + dx]}{16ad(a + a \cos[c + dx])^{3/2}} + \frac{9 \sin[c + dx]}{4a^2 d \sqrt{a + a \cos[c + dx]}}
\end{aligned}$$

Result (type 3, 489 leaves):

$$\begin{aligned}
& \frac{75 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{4d(a(1 + \cos[c + dx]))^{5/2}} - \frac{75 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{4d(a(1 + \cos[c + dx]))^{5/2}} + \\
& \frac{16 \cos\left[\frac{dx}{2}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{c}{2}\right]}{d(a(1 + \cos[c + dx]))^{5/2}} + \frac{16 \cos\left[\frac{c}{2}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{d(a(1 + \cos[c + dx]))^{5/2}} - \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8d(a(1 + \cos[c + dx]))^{5/2} \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^4} + \frac{21 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8d(a(1 + \cos[c + dx]))^{5/2} \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2} \\
& - \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8d(a(1 + \cos[c + dx]))^{5/2} \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^4} - \frac{21 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8d(a(1 + \cos[c + dx]))^{5/2} \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2}
\end{aligned}$$

■ **Problem 144: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]}{(a + a \cos[c + dx])^{5/2}} dx$$

Optimal (type 3, 144 leaves, 7 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{5/2} d} - \frac{43 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{\sin[c+dx]}{4 d (a+a \cos[c+dx])^{5/2}} - \frac{11 \sin[c+dx]}{16 a d (a+a \cos[c+dx])^{3/2}}$$

Result (type 3, 1919 leaves):

$$\begin{aligned} & - \left( \left( (2-2i) (1+e^{ic}) \left( \sqrt{2} - (1-i) e^{\frac{ic}{2}} + (16-16i) e^{\frac{3ic}{2}+idx} + (20+20i) \sqrt{2} e^{2ic+\frac{3idx}{2}} - (34-34i) e^{\frac{5ic}{2}+2idx} - (20+20i) \sqrt{2} e^{3ic+\frac{5idx}{2}} + \right. \right. \right. \\ & \quad (16-16i) e^{\frac{7ic}{2}+3idx} + (4+4i) \sqrt{2} e^{4ic+\frac{7idx}{2}} - (1-i) e^{\frac{9ic}{2}+4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + \\ & \quad \left. \left. 34\sqrt{2} e^{2i(c+dx)} + 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4+4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \right) / \\ & \quad \left( \left( (-1-i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1+e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 (a(1+\cos[c+dx]))^{5/2} \right) \Big) - \\ & \quad \frac{4i\sqrt{2} \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{d (a(1+\cos[c+dx]))^{5/2}} + \\ & \quad \frac{43 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{4d (a(1+\cos[c+dx]))^{5/2}} - \\ & \quad \frac{43 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{4d (a(1+\cos[c+dx]))^{5/2}} - \\ & \quad \frac{2\sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a(1+\cos[c+dx]))^{5/2}} + \\ & \quad \left( (1-i) \sqrt{2} \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( (1+i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1-i) \sin\left[\frac{c}{4}\right] - i \sqrt{2} \sin\left[\frac{c}{4}\right] \right) \right) \\ & \quad \left( (-1-i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] + (1-i) \sin\left[\frac{c}{4}\right] - i \sqrt{2} \sin\left[\frac{c}{4}\right] \right) \Big) / \left( d (a(1+\cos[c+dx]))^{5/2} \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right) - \\ & \quad \left( (1+i) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \left( (1+i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1-i) \sin\left[\frac{c}{4}\right] - i \sqrt{2} \sin\left[\frac{c}{4}\right] \right) \right) \\ & \quad \left( (-1-i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] + (1-i) \sin\left[\frac{c}{4}\right] - i \sqrt{2} \sin\left[\frac{c}{4}\right] \right) \Big) / \left( \sqrt{2} d (a(1+\cos[c+dx]))^{5/2} \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right) - \end{aligned}$$

$$\begin{aligned}
& \frac{16 i \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right]-i\left(-\sqrt{2}+2 \sin\left[\frac{c}{2}\right]\right) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \operatorname{Cot}\left[\frac{c}{2}\right]}{d\left(a\left(1+\cos [c+dx]\right)\right)^{5 / 2} \sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}}+8 \sqrt{2} \cos\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \operatorname{Csc}\left[\frac{c}{2}\right] \\
& \left(\frac{-dx \cos\left[\frac{c}{2}\right]+2 \operatorname{Log}\left[\sqrt{2}+2 \cos\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right]+2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{\sin\left[\frac{c}{2}\right]}+\frac{4 i \sqrt{2} \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right]-i\left(-\sqrt{2}+2 \sin\left[\frac{c}{2}\right]\right) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}}\right) / \\
& \left(d\left(a\left(1+\cos [c+dx]\right)\right)^{5 / 2}\left(4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2\right)\right)-\frac{\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^5}{8 d\left(a\left(1+\cos [c+dx]\right)\right)^{5 / 2}\left(\cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]\right)^4}- \\
& \frac{11 \cos\left[\frac{c}{2}+\frac{dx}{2}\right]^5}{8 d\left(a\left(1+\cos [c+dx]\right)\right)^{5 / 2}\left(\cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]\right)^2}+ \\
& \frac{\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^5}{8 d\left(a\left(1+\cos [c+dx]\right)\right)^{5 / 2}\left(\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sin\left[\frac{c}{4}+\frac{dx}{4}\right]\right)^4}+ \\
& \frac{11 \cos\left[\frac{c}{2}+\frac{dx}{2}\right]^5}{8 d\left(a\left(1+\cos [c+dx]\right)\right)^{5 / 2}\left(\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sin\left[\frac{c}{4}+\frac{dx}{4}\right]\right)^2}
\end{aligned}$$

■ **Problem 145: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^2}{(a+a \cos [c+dx])^{5 / 2}} dx$$

Optimal (type 3, 174 leaves, 8 steps):

$$\begin{aligned}
& -\frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+dx]}{\sqrt{a+a \cos [c+dx]}}\right]}{a^{5 / 2} d}+\frac{115 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+dx]}{\sqrt{2} \sqrt{a+a \cos [c+dx]}}\right]}{16 \sqrt{2} a^{5 / 2} d}- \\
& \frac{\operatorname{Tan}[c+dx]}{4 d\left(a+a \cos [c+dx]\right)^{5 / 2}}-\frac{15 \operatorname{Tan}[c+dx]}{16 a d\left(a+a \cos [c+dx]\right)^{3 / 2}}+\frac{35 \operatorname{Tan}[c+dx]}{16 a^2 d \sqrt{a+a \cos [c+dx]}}
\end{aligned}$$

Result (type 3, 2051 leaves):

$$\begin{aligned}
& \left( (5 - 5i) (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right. \\
& \quad (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + \\
& \quad \left. \left. 34\sqrt{2} e^{2i(c+dx)} + 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \right) / \\
& \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 (a(1 + \cos[c + dx]))^{5/2} \right) + \\
& \frac{10i\sqrt{2} \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{d(a(1 + \cos[c + dx]))^{5/2}} - \\
& \frac{115 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{4d(a(1 + \cos[c + dx]))^{5/2}} + \\
& \frac{115 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{4d(a(1 + \cos[c + dx]))^{5/2}} + \\
& \frac{5\sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d(a(1 + \cos[c + dx]))^{5/2}} - \\
& \left( (5 - 5i) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( (1 + i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1 - i) \sin\left[\frac{c}{4}\right] - i\sqrt{2} \sin\left[\frac{c}{4}\right] \right) \right. \\
& \quad \left. \left( (-1 - i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] + (1 - i) \sin\left[\frac{c}{4}\right] - i\sqrt{2} \sin\left[\frac{c}{4}\right] \right) \right) / \left( \sqrt{2} d(a(1 + \cos[c + dx]))^{5/2} \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right) + \\
& \left( \left( \frac{5}{2} + \frac{5i}{2} \right) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \left( (1 + i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1 - i) \sin\left[\frac{c}{4}\right] - i\sqrt{2} \sin\left[\frac{c}{4}\right] \right) \right. \\
& \quad \left. \left( (-1 - i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] + (1 - i) \sin\left[\frac{c}{4}\right] - i\sqrt{2} \sin\left[\frac{c}{4}\right] \right) \right) / \left( \sqrt{2} d(a(1 + \cos[c + dx]))^{5/2} \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right) + \\
& \frac{40i \operatorname{ArcTan}\left[\frac{2i \cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Cot}\left[\frac{c}{2}\right]}{d(a(1 + \cos[c + dx]))^{5/2} \sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} - \left( 20\sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Csc}\left[\frac{c}{2}\right] \right)
\end{aligned}$$

$$\left( -dx \cos\left[\frac{c}{2}\right] + 2 \log\left[\sqrt{2} + 2 \cos\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] \right) \sin\left[\frac{c}{2}\right] + \frac{4i\sqrt{2} \operatorname{ArcTan}\left[\frac{2i\cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2\sin\left[\frac{c}{2}\right]) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}} \right) /$$

$$\left( d (a (1 + \cos[c + dx]))^{5/2} \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) \right) + \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8 d (a (1 + \cos[c + dx]))^{5/2} \left( \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] \right)^4} +$$

$$\frac{19 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8 d (a (1 + \cos[c + dx]))^{5/2} \left( \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] \right)^2} - \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8 d (a (1 + \cos[c + dx]))^{5/2} \left( \cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] \right)^4} -$$

$$\frac{19 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8 d (a (1 + \cos[c + dx]))^{5/2} \left( \cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] \right)^2} +$$

$$\frac{4 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{d (a (1 + \cos[c + dx]))^{5/2} \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)} -$$

$$\frac{4 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{d (a (1 + \cos[c + dx]))^{5/2} \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)}$$

- **Problem 146: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^{5/2} (a + a \cos[c + dx]) dx$$

Optimal (type 4, 111 leaves, 6 steps):

$$\frac{6 a \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5 d} + \frac{10 a \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{21 d} +$$

$$\frac{10 a \sqrt{\cos[c + dx]} \sin[c + dx]}{21 d} + \frac{2 a \cos[c + dx]^{3/2} \sin[c + dx]}{5 d} + \frac{2 a \cos[c + dx]^{5/2} \sin[c + dx]}{7 d}$$

Result (type 5, 490 leaves):



$$\begin{aligned}
& a \left( \sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left( -\frac{3 \cot[c]}{5d} + \frac{23 \cos[dx] \sin[c]}{84d} + \right. \right. \\
& \quad \left. \left. \frac{\cos[2dx] \sin[2c]}{10d} + \frac{\cos[3dx] \sin[3c]}{28d} + \frac{23 \cos[c] \sin[dx]}{84d} + \frac{\cos[2c] \sin[2dx]}{10d} + \frac{\cos[3c] \sin[3dx]}{28d} \right) - \right. \\
& \quad \left. \frac{1}{21d \sqrt{1 + \cot[c]^2}} 5 (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right. \\
& \quad \left. \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{10d} 3 (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right. \\
& \quad \left. \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \\
& \quad \left. \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right. \\
& \quad \left. \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) \right)
\end{aligned}$$

- **Problem 147: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^{3/2} (a + a \cos[c+dx]) dx$$

Optimal (type 4, 87 leaves, 5 steps):

$$\frac{6 a \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5 d} + \frac{2 a \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 d} + \frac{2 a \sqrt{\cos[c+dx]} \sin[c+dx]}{3 d} + \frac{2 a \cos[c+dx]^{3/2} \sin[c+dx]}{5 d}$$

Result (type 5, 458 leaves):

$$\begin{aligned}
& a \left( \sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left( -\frac{3 \cot[c]}{5d} + \frac{\cos[dx] \sin[c]}{3d} + \frac{\cos[2dx] \sin[2c]}{10d} + \frac{\cos[c] \sin[dx]}{3d} + \frac{\cos[2c] \sin[2dx]}{10d} \right) \right. \\
& \frac{1}{3d \sqrt{1 + \cot[c]^2}} (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{10d} 3 (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left( \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \\
& \left. \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) \right. \\
& \left. \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) \right)
\end{aligned}$$

■ **Problem 148: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]} (a + a \cos[c+dx]) dx$$

Optimal (type 4, 61 leaves, 4 steps):

$$\frac{2 a \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{2 a \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 d} + \frac{2 a \sqrt{\cos[c+dx]} \sin[c+dx]}{3 d}$$

Result (type 5, 424 leaves):

$$\begin{aligned}
& a \left( \sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left( -\frac{\cot[c]}{d} + \frac{\cos[dx] \sin[c]}{3d} + \frac{\cos[c] \sin[dx]}{3d} \right) - \frac{1}{3d \sqrt{1 + \cot[c]^2}} \right. \\
& (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \\
& \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{2d} (1 + \cos[c+dx]) \\
& \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]\right]^2 \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right)
\end{aligned}$$

■ **Problem 149: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \cos[c+dx]}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 35 leaves, 3 steps):

$$\frac{2 a \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{2 a \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{d}$$

Result (type 5, 155 leaves):

$$\frac{1}{2d} a \sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2$$

$$\left( -2 \sqrt{\cos[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2} \sqrt{\operatorname{Csc}[c]^2} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right.$$

$$\left. \operatorname{Sin}[c] + \operatorname{Tan}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] - \frac{\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \operatorname{Tan}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]}{\sqrt{\operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2}} \right)$$

- **Problem 150: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \cos[c+dx]}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 57 leaves, 4 steps):

$$-\frac{2a \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{2a \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{2a \operatorname{Sin}[c+dx]}{d \sqrt{\cos[c+dx]}}$$

Result (type 5, 413 leaves):

$$\begin{aligned}
& a \left( \sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left( \frac{\operatorname{Csc}[c] \operatorname{Sec}[c]}{d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] \operatorname{Sin}[dx]}{d} \right) - \frac{1}{d \sqrt{1 + \operatorname{Cot}[c]^2}} \right. \\
& (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \frac{1}{2d} (1 + \cos[c+dx]) \\
& \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]\right]^2 \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\
& \left( \sqrt{1 - \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \\
& \left. \frac{\frac{\operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \operatorname{Cos}[c]^2 \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2}}{\sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}} \right)
\end{aligned}$$

■ **Problem 151: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \operatorname{Cos}[c + dx]}{\operatorname{Cos}[c + dx]^{5/2}} dx$$

Optimal (type 4, 83 leaves, 5 steps):

$$-\frac{2 a \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{2 a \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 d} + \frac{2 a \operatorname{Sin}[c+dx]}{3 d \operatorname{Cos}[c+dx]^{3/2}} + \frac{2 a \operatorname{Sin}[c+dx]}{d \sqrt{\operatorname{Cos}[c+dx]}}$$

Result (type 5, 444 leaves):

$$\begin{aligned}
& a \left( \sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left( \frac{\csc[c] \sec[c]}{d} + \frac{\sec[c] \sec[c+dx]^2 \sin[dx]}{3d} + \frac{\sec[c] \sec[c+dx] (\sin[c] + 3 \sin[dx])}{3d} \right) \right) - \\
& \frac{1}{3d \sqrt{1 + \cot[c]^2}} (1 + \cos[c+dx]) \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} + \frac{1}{2d} (1 + \cos[c+dx]) \csc[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left( \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right)
\end{aligned}$$

■ **Problem 152: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \cos[c+dx]}{\cos[c+dx]^{7/2}} dx$$

Optimal (type 4, 111 leaves, 6 steps):

$$-\frac{6a \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5d} + \frac{2a \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} + \frac{2a \sin[c+dx]}{5d \cos[c+dx]^{5/2}} + \frac{2a \sin[c+dx]}{3d \cos[c+dx]^{3/2}} + \frac{6a \sin[c+dx]}{5d \sqrt{\cos[c+dx]}}$$

Result (type 5, 477 leaves):

a

$$\left( \sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left( \frac{3 \operatorname{Csc}[c] \operatorname{Sec}[c]}{5d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^3 \sin[dx]}{5d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (3 \sin[c] + 5 \sin[dx])}{15d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (5 \sin[c] + 9 \sin[dx])}{15d} \right) - \frac{1}{3d \sqrt{1 + \cot[c]^2}} (1 + \cos[c+dx]) \operatorname{Csc}[c] \right.$$

$$\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]}$$

$$\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} + \frac{1}{10d} 3 (1 + \cos[c+dx]) \operatorname{Csc}[c]$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \sqrt{1 + \tan[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right)$$

- **Problem 153: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^{5/2} (a + a \cos[c+dx])^2 dx$$

Optimal (type 4, 147 leaves, 10 steps):

$$\frac{32 a^2 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{15d} + \frac{20 a^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{21d} + \frac{20 a^2 \sqrt{\cos[c+dx]} \sin[c+dx]}{21d} +$$

$$\frac{32 a^2 \cos[c+dx]^{3/2} \sin[c+dx]}{45d} + \frac{4 a^2 \cos[c+dx]^{5/2} \sin[c+dx]}{7d} + \frac{2 a^2 \cos[c+dx]^{7/2} \sin[c+dx]}{9d}$$

Result (type 5, 532 leaves):

$$\begin{aligned}
& \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \left( -\frac{8\cot[c]}{15d} + \frac{23\cos[dx]\sin[c]}{84d} + \frac{37\cos[2dx]\sin[2c]}{360d} + \frac{\cos[3dx]\sin[3c]}{28d} + \right. \\
& \quad \left. \frac{\cos[4dx]\sin[4c]}{144d} + \frac{23\cos[c]\sin[dx]}{84d} + \frac{37\cos[2c]\sin[2dx]}{360d} + \frac{\cos[3c]\sin[3dx]}{28d} + \frac{\cos[4c]\sin[4dx]}{144d} \right) - \\
& \frac{1}{21d\sqrt{1+\cot[c]^2}} 5(a+a\cos[c+dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \\
& \operatorname{Sec}[dx-\operatorname{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]} - \frac{1}{15d} 4(a+a\cos[c+dx])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx+\operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1-\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2} \sqrt{1+\tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1+\tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx+\operatorname{ArcTan}[\tan[c]]] \sqrt{1+\tan[c]^2}}{\cos[c]^2+\sin[c]^2}}{\sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2}}
\end{aligned}$$

- **Problem 154: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^{3/2} (a+a\cos[c+dx])^2 dx$$

Optimal (type 4, 121 leaves, 9 steps):

$$\begin{aligned}
& \frac{12a^2 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5d} + \frac{8a^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{7d} + \\
& \frac{8a^2 \sqrt{\cos[c+dx]} \sin[c+dx]}{7d} + \frac{4a^2 \cos[c+dx]^{3/2} \sin[c+dx]}{5d} + \frac{2a^2 \cos[c+dx]^{5/2} \sin[c+dx]}{7d}
\end{aligned}$$

Result (type 5, 500 leaves):



$$\begin{aligned}
& \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \left( -\frac{3\cot[c]}{5d} + \frac{17\cos[dx]\sin[c]}{56d} + \right. \\
& \quad \left. \frac{\cos[2dx]\sin[2c]}{10d} + \frac{\cos[3dx]\sin[3c]}{56d} + \frac{17\cos[c]\sin[dx]}{56d} + \frac{\cos[2c]\sin[2dx]}{10d} + \frac{\cos[3c]\sin[3dx]}{56d} \right) - \\
& \frac{1}{7d\sqrt{1+\cot[c]^2}} 2(a+a\cos[c+dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \\
& \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{10d} 3(a+a\cos[c+dx])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}}
\end{aligned}$$

- **Problem 155: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^2 dx$$

Optimal (type 4, 95 leaves, 7 steps):

$$\frac{16a^2 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5d} + \frac{4a^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} + \frac{4a^2 \sqrt{\cos[c+dx]} \sin[c+dx]}{3d} + \frac{2a^2 \cos[c+dx]^{3/2} \sin[c+dx]}{5d}$$

Result (type 5, 468 leaves):

$$\begin{aligned}
& \sqrt{\cos [c+d x]} (a+a \cos [c+d x])^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4\left(-\frac{4 \cot [c]}{5 d}+\frac{\cos [d x] \sin [c]}{3 d}+\frac{\cos [2 d x] \sin [2 c]}{20 d}+\frac{\cos [c] \sin [d x]}{3 d}+\frac{\cos [2 c] \sin [2 d x]}{20 d}\right)- \\
& \frac{1}{3 d \sqrt{1+\cot [c]^2}}(a+a \cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \\
& \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}-\frac{1}{5} 2(a+a \cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \\
& \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right)- \\
& \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}}
\end{aligned}$$

■ **Problem 156: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \cos [c+d x])^2}{\sqrt{\cos [c+d x]}} d x$$

Optimal (type 4, 67 leaves, 6 steps):

$$\frac{4 a^2 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{d}+\frac{8 a^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 d}+\frac{2 a^2 \sqrt{\cos [c+d x]} \sin [c+d x]}{3 d}$$

Result (type 5, 434 leaves):

$$\sqrt{\cos [c+d x]} (a+a \cos [c+d x])^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4\left(-\frac{\cot [c]}{d}+\frac{\cos [d x] \sin [c]}{6 d}+\frac{\cos [c] \sin [d x]}{6 d}\right)-\frac{1}{3 d \sqrt{1+\cot [c]^2}}$$

$$2(a+a \cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}-\frac{1}{2 d}(a+a \cos [c+d x])^2$$

$$\operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) /$$

$$\left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right)-$$

$$\frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}}$$

- **Problem 158: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \cos [c+d x])^2}{\cos [c+d x]^{5/2}} d x$$

Optimal (type 4, 91 leaves, 7 steps):

$$-\frac{4 a^2 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{d}+\frac{8 a^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 d}+\frac{2 a^2 \sin [c+d x]}{3 d \cos [c+d x]^{3/2}}+\frac{4 a^2 \sin [c+d x]}{d \sqrt{\cos [c+d x]}}$$

Result (type 5, 454 leaves):

$$\begin{aligned}
& \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \left( \frac{\operatorname{Csc}[c] \operatorname{Sec}[c]}{d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 \sin[dx]}{6d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (\sin[c]+6\sin[dx])}{6d} \right) - \\
& \frac{1}{3d\sqrt{1+\cot[c]^2}} 2(a+a\cos[c+dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \\
& \operatorname{Sec}[dx-\operatorname{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx-\operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]} + \frac{1}{2d} (a+a\cos[c+dx])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx+\operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1-\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2} \sqrt{1+\tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1+\tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx+\operatorname{ArcTan}[\tan[c]]] \sqrt{1+\tan[c]^2}}{\cos[c]^2+\sin[c]^2}}{\sqrt{\cos[c] \cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2}}
\end{aligned}$$

■ **Problem 159: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a\cos[c+dx])^2}{\cos[c+dx]^{7/2}} dx$$

Optimal (type 4, 121 leaves, 9 steps):

$$-\frac{16a^2 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5d} + \frac{4a^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} + \frac{2a^2 \sin[c+dx]}{5d \cos[c+dx]^{5/2}} + \frac{4a^2 \sin[c+dx]}{3d \cos[c+dx]^{3/2}} + \frac{16a^2 \sin[c+dx]}{5d \sqrt{\cos[c+dx]}}$$

Result (type 5, 487 leaves):

$$\begin{aligned}
& \sqrt{\cos [c+d x]} (a+a \cos [c+d x])^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4\left(\frac{4 \operatorname{Csc}[c] \operatorname{Sec}[c]}{5 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^3 \sin [d x]}{10 d}\right. \\
& \quad \left.+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2(3 \sin [c]+10 \sin [d x])}{30 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x](5 \sin [c]+12 \sin [d x])}{15 d}\right)-\frac{1}{3 d \sqrt{1+\cot [c]^2}} \\
& (a+a \cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \\
& \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}+\frac{1}{5 d} 2(a+a \cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \\
& \left(\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right\} \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \right. \\
& \quad \left.\left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right)-\right. \\
& \quad \left.\frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}}\right)
\end{aligned}$$

- **Problem 160: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^{3 / 2}(a+a \cos [c+d x])^3 d x$$

Optimal (type 4, 147 leaves, 12 steps):

$$\begin{aligned}
& \frac{68 a^3 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{15 d}+\frac{44 a^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{21 d}+\frac{44 a^3 \sqrt{\cos [c+d x]} \sin [c+d x]}{21 d}+ \\
& \frac{68 a^3 \cos [c+d x]^{3 / 2} \sin [c+d x]}{45 d}+\frac{6 a^3 \cos [c+d x]^{5 / 2} \sin [c+d x]}{7 d}+\frac{2 a^3 \cos [c+d x]^{7 / 2} \sin [c+d x]}{9 d}
\end{aligned}$$

Result (type 5, 532 leaves):

$$\begin{aligned}
& \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^3 \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \left( -\frac{17\cot[c]}{30d} + \frac{97\cos[dx]\sin[c]}{336d} + \frac{73\cos[2dx]\sin[2c]}{720d} + \frac{3\cos[3dx]\sin[3c]}{112d} + \right. \\
& \left. \frac{\cos[4dx]\sin[4c]}{288d} + \frac{97\cos[c]\sin[dx]}{336d} + \frac{73\cos[2c]\sin[2dx]}{720d} + \frac{3\cos[3c]\sin[3dx]}{112d} + \frac{\cos[4c]\sin[4dx]}{288d} \right) - \\
& \frac{1}{42d\sqrt{1+\cot[c]^2}} 11(a+a\cos[c+dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \\
& \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{60d} 17(a+a\cos[c+dx])^3 \operatorname{Csc}[c] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}}
\end{aligned}$$

- **Problem 161: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^3 dx$$

Optimal (type 4, 121 leaves, 10 steps):

$$\begin{aligned}
& \frac{28a^3 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5d} + \frac{52a^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{21d} + \\
& \frac{52a^3 \sqrt{\cos[c+dx]} \sin[c+dx]}{21d} + \frac{6a^3 \cos[c+dx]^{3/2} \sin[c+dx]}{5d} + \frac{2a^3 \cos[c+dx]^{5/2} \sin[c+dx]}{7d}
\end{aligned}$$

Result (type 5, 500 leaves):

$$\begin{aligned}
& \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^3 \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \left( -\frac{7\cot[c]}{10d} + \frac{107\cos[dx]\sin[c]}{336d} + \right. \\
& \quad \left. \frac{3\cos[2dx]\sin[2c]}{40d} + \frac{\cos[3dx]\sin[3c]}{112d} + \frac{107\cos[c]\sin[dx]}{336d} + \frac{3\cos[2c]\sin[2dx]}{40d} + \frac{\cos[3c]\sin[3dx]}{112d} \right) - \\
& \frac{1}{42d\sqrt{1+\cot[c]^2}} 13 (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \\
& \operatorname{Sec}[dx-\operatorname{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]} - \frac{1}{20d} 7 (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx+\operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1-\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2} \sqrt{1+\tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1+\tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx+\operatorname{ArcTan}[\tan[c]]] \sqrt{1+\tan[c]^2}}{\cos[c]^2+\sin[c]^2}}{\sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2}}
\end{aligned}$$

- **Problem 162: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a\cos[c+dx])^3}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 91 leaves, 8 steps):

$$\frac{36a^3 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5d} + \frac{4a^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{2a^3 \sqrt{\cos[c+dx]} \sin[c+dx]}{d} + \frac{2a^3 \cos[c+dx]^{3/2} \sin[c+dx]}{5d}$$

Result (type 5, 468 leaves):

$$\sqrt{\cos[c+dx]} (a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \left(-\frac{9\cot[c]}{10d}+\frac{\cos[dx]\sin[c]}{4d}+\frac{\cos[2dx]\sin[2c]}{40d}+\frac{\cos[c]\sin[dx]}{4d}+\frac{\cos[2c]\sin[2dx]}{40d}\right) -$$

$$\frac{1}{2d\sqrt{1+\cot[c]^2}}(a+a\cos[c+dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6$$

$$\operatorname{Sec}[dx-\operatorname{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx-\operatorname{ArcTan}[\cot[c]]]}$$

$$\sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]} - \frac{1}{20d} 9 (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx+\operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c]\right) /$$

$$\left(\sqrt{1-\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2} \sqrt{1+\tan[c]^2}\right) -$$

$$\frac{\frac{\sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1+\tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx+\operatorname{ArcTan}[\tan[c]]] \sqrt{1+\tan[c]^2}}{\cos[c]^2+\sin[c]^2}}{\sqrt{\cos[c] \cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2}}$$

■ **Problem 163: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a\cos[c+dx])^3}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 91 leaves, 8 steps):

$$\frac{4a^3 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{20a^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} + \frac{2a^3 \sin[c+dx]}{d\sqrt{\cos[c+dx]}} + \frac{2a^3 \sqrt{\cos[c+dx]} \sin[c+dx]}{3d}$$

Result (type 5, 465 leaves):



$$\sqrt{\cos [c+d x]} (a+a \cos [c+d x])^3 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6$$

$$\left(-\frac{(1+3 \cos [2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{8 d}+\frac{\cos [d x] \sin [c]}{12 d}+\frac{\cos [c] \sin [d x]}{12 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x] \sin [d x]}{4 d}\right)-\frac{1}{6 d \sqrt{1+\cot [c]^2}}$$

$$5(a+a \cos [c+d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}-\frac{1}{4 d}(a+a \cos [c+d x])^3$$

$$\operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) /$$

$$\left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right)-$$

$$\frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}}$$

■ **Problem 164: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \cos [c+d x])^3}{\cos [c+d x]^{5/2}} d x$$

Optimal (type 4, 91 leaves, 8 steps):

$$-\frac{4 a^3 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{d}+\frac{20 a^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 d}+\frac{2 a^3 \sin [c+d x]}{3 d \cos [c+d x]^{3/2}}+\frac{6 a^3 \sin [c+d x]}{d \sqrt{\cos [c+d x]}}$$

Result (type 5, 463 leaves):

$$\sqrt{\cos[c+dx]} (a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6$$

$$\left(-\frac{(-5+\cos[2c])\operatorname{Csc}[c]\operatorname{Sec}[c]}{8d}+\frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^2\sin[dx]}{12d}+\frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx](\sin[c]+9\sin[dx])}{12d}\right)-\frac{1}{6d\sqrt{1+\cot[c]^2}}$$

$$5(a+a\cos[c+dx])^3\operatorname{Csc}[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4},\frac{1}{2}\right\},\left\{\frac{5}{4}\right\},\sin[dx-\operatorname{ArcTan}[\cot[c]]]^2\right]\operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6\operatorname{Sec}[dx-\operatorname{ArcTan}[\cot[c]]]$$

$$\sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]}\sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]}\sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]}+\frac{1}{4d}(a+a\cos[c+dx])^3$$

$$\operatorname{Csc}[c]\operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos[dx+\operatorname{ArcTan}[\tan[c]]]^2\right]\sin[dx+\operatorname{ArcTan}[\tan[c]]]\tan[c]\right)/$$

$$\left(\sqrt{1-\cos[dx+\operatorname{ArcTan}[\tan[c]]]}\sqrt{1+\cos[dx+\operatorname{ArcTan}[\tan[c]]]}\sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]}\sqrt{1+\tan[c]^2}\sqrt{1+\tan[c]^2}\right)-$$

$$\frac{\frac{\sin[dx+\operatorname{ArcTan}[\tan[c]]]\tan[c]}{\sqrt{1+\tan[c]^2}}+\frac{2\cos[c]^2\cos[dx+\operatorname{ArcTan}[\tan[c]]]\sqrt{1+\tan[c]^2}}{\cos[c]^2+\sin[c]^2}}{\sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]}\sqrt{1+\tan[c]^2}}$$

■ **Problem 165: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a\cos[c+dx])^3}{\cos[c+dx]^{7/2}} dx$$

Optimal (type 4, 117 leaves, 10 steps):

$$-\frac{36a^3\operatorname{EllipticE}\left[\frac{1}{2}(c+dx),2\right]}{5d}+\frac{4a^3\operatorname{EllipticF}\left[\frac{1}{2}(c+dx),2\right]}{d}+\frac{2a^3\sin[c+dx]}{5d\cos[c+dx]^{5/2}}+\frac{2a^3\sin[c+dx]}{d\cos[c+dx]^{3/2}}+\frac{36a^3\sin[c+dx]}{5d\sqrt{\cos[c+dx]}}$$

Result (type 5, 485 leaves):

$$\begin{aligned}
& \sqrt{\cos [c+d x]} (a+a \cos [c+d x])^3 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 \\
& \left(\frac{9 \operatorname{Csc}[c] \operatorname{Sec}[c]}{10 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^3 \sin [d x]}{20 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2(\sin [c]+5 \sin [d x])}{20 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x](5 \sin [c]+18 \sin [d x])}{20 d}\right) - \\
& \frac{1}{2 d \sqrt{1+\cot [c]^2}}(a+a \cos [c+d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 \\
& \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}+\frac{1}{20 d} 9(a+a \cos [c+d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]\right]^2 \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \\
& \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right) - \\
& \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}}
\end{aligned}$$

- **Problem 166: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \cos [c+d x])^3}{\cos [c+d x]^{9/2}} d x$$

Optimal (type 4, 147 leaves, 12 steps):

$$\begin{aligned}
& -\frac{28 a^3 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 d}+\frac{52 a^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{21 d}+ \\
& \frac{2 a^3 \sin [c+d x]}{7 d \cos [c+d x]^{7/2}}+\frac{6 a^3 \sin [c+d x]}{5 d \cos [c+d x]^{5/2}}+\frac{52 a^3 \sin [c+d x]}{21 d \cos [c+d x]^{3/2}}+\frac{28 a^3 \sin [c+d x]}{5 d \sqrt{\cos [c+d x]}}
\end{aligned}$$

Result (type 5, 515 leaves):

$$\sqrt{\cos[c+dx]} (a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6$$

$$\left(\frac{7\operatorname{Csc}[c]\operatorname{Sec}[c]}{10d}+\frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^4\operatorname{Sin}[dx]}{28d}+\frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^3(5\operatorname{Sin}[c]+21\operatorname{Sin}[dx])}{140d}+\frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^2(63\operatorname{Sin}[c]+130\operatorname{Sin}[dx])}{420d}+\frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx](65\operatorname{Sin}[c]+147\operatorname{Sin}[dx])}{210d}\right)-\frac{1}{42d\sqrt{1+\operatorname{Cot}[c]^2}}$$

$$13(a+a\cos[c+dx])^3\operatorname{Csc}[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4},\frac{1}{2}\right\},\left\{\frac{5}{4}\right\},\operatorname{Sin}[dx-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right]\operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6$$

$$\operatorname{Sec}[dx-\operatorname{ArcTan}[\operatorname{Cot}[c]]]\sqrt{1-\operatorname{Sin}[dx-\operatorname{ArcTan}[\operatorname{Cot}[c]]]}\sqrt{-\sqrt{1+\operatorname{Cot}[c]^2}\operatorname{Sin}[c]\operatorname{Sin}[dx-\operatorname{ArcTan}[\operatorname{Cot}[c]]]}}$$

$$\sqrt{1+\operatorname{Sin}[dx-\operatorname{ArcTan}[\operatorname{Cot}[c]]]}+\frac{1}{20d}7(a+a\cos[c+dx])^3\operatorname{Csc}[c]\operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\operatorname{Cos}[dx+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right]\operatorname{Sin}[dx+\operatorname{ArcTan}[\operatorname{Tan}[c]]]\operatorname{Tan}[c]\right)/$$

$$\left(\sqrt{1-\operatorname{Cos}[dx+\operatorname{ArcTan}[\operatorname{Tan}[c]]]}\sqrt{1+\operatorname{Cos}[dx+\operatorname{ArcTan}[\operatorname{Tan}[c]]]}\sqrt{\operatorname{Cos}[c]\operatorname{Cos}[dx+\operatorname{ArcTan}[\operatorname{Tan}[c]]]}\sqrt{1+\operatorname{Tan}[c]^2}\sqrt{1+\operatorname{Tan}[c]^2}\right)-$$

$$\frac{\frac{\operatorname{Sin}[dx+\operatorname{ArcTan}[\operatorname{Tan}[c]]]\operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2\operatorname{Cos}[c]^2\operatorname{Cos}[dx+\operatorname{ArcTan}[\operatorname{Tan}[c]]]\sqrt{1+\operatorname{Tan}[c]^2}}{\operatorname{Cos}[c]^2+\operatorname{Sin}[c]^2}}{\sqrt{\operatorname{Cos}[c]\operatorname{Cos}[dx+\operatorname{ArcTan}[\operatorname{Tan}[c]]]}\sqrt{1+\operatorname{Tan}[c]^2}}$$

■ **Problem 167: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^{3/2} (a+a\cos[c+dx])^4 dx$$

Optimal (type 4, 173 leaves, 16 steps):

$$\frac{128a^4\operatorname{EllipticE}\left[\frac{1}{2}(c+dx),2\right]}{15d}+\frac{904a^4\operatorname{EllipticF}\left[\frac{1}{2}(c+dx),2\right]}{231d}+\frac{904a^4\sqrt{\operatorname{Cos}[c+dx]}\operatorname{Sin}[c+dx]}{231d}+$$

$$\frac{128a^4\operatorname{Cos}[c+dx]^{3/2}\operatorname{Sin}[c+dx]}{45d}+\frac{150a^4\operatorname{Cos}[c+dx]^{5/2}\operatorname{Sin}[c+dx]}{77d}+\frac{8a^4\operatorname{Cos}[c+dx]^{7/2}\operatorname{Sin}[c+dx]}{9d}+\frac{2a^4\operatorname{Cos}[c+dx]^{9/2}\operatorname{Sin}[c+dx]}{11d}$$

Result (type 5, 564 leaves):

$$\begin{aligned}
& \sqrt{\cos [c+d x]} (a+a \cos [c+d x])^4 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^8 \\
& \left(-\frac{8 \cot [c]}{15 d}+\frac{4087 \cos [d x] \sin [c]}{14784 d}+\frac{37 \cos [2 d x] \sin [2 c]}{360 d}+\frac{321 \cos [3 d x] \sin [3 c]}{9856 d}+\frac{\cos [4 d x] \sin [4 c]}{144 d}+\frac{\cos [5 d x] \sin [5 c]}{1408 d}+\right. \\
& \left.\frac{4087 \cos [c] \sin [d x]}{14784 d}+\frac{37 \cos [2 c] \sin [2 d x]}{360 d}+\frac{321 \cos [3 c] \sin [3 d x]}{9856 d}+\frac{\cos [4 c] \sin [4 d x]}{144 d}+\frac{\cos [5 c] \sin [5 d x]}{1408 d}\right)- \\
& \frac{1}{462 d \sqrt{1+\cot [c]^2}} 113 (a+a \cos [c+d x])^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^8 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}-\frac{1}{15 d} 4 (a+a \cos [c+d x])^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^8 \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \\
& \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right)- \\
& \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}}
\end{aligned}$$

■ **Problem 168: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos [c+d x]} (a+a \cos [c+d x])^4 dx$$

Optimal (type 4, 147 leaves, 13 steps):

$$\begin{aligned}
& \frac{152 a^4 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{15 d}+\frac{32 a^4 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{7 d}+\frac{32 a^4 \sqrt{\cos [c+d x]} \sin [c+d x]}{7 d}+ \\
& \frac{122 a^4 \cos [c+d x]^{3 / 2} \sin [c+d x]}{45 d}+\frac{8 a^4 \cos [c+d x]^{5 / 2} \sin [c+d x]}{7 d}+\frac{2 a^4 \cos [c+d x]^{7 / 2} \sin [c+d x]}{9 d}
\end{aligned}$$

Result (type 5, 532 leaves):

$$\begin{aligned}
& \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^4 \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^8 \left( -\frac{19\cot[c]}{30d} + \frac{17\cos[dx]\sin[c]}{56d} + \frac{127\cos[2dx]\sin[2c]}{1440d} + \frac{\cos[3dx]\sin[3c]}{56d} + \right. \\
& \left. \frac{\cos[4dx]\sin[4c]}{576d} + \frac{17\cos[c]\sin[dx]}{56d} + \frac{127\cos[2c]\sin[2dx]}{1440d} + \frac{\cos[3c]\sin[3dx]}{56d} + \frac{\cos[4c]\sin[4dx]}{576d} \right) - \\
& \frac{1}{7d\sqrt{1+\cot[c]^2}} 2(a+a\cos[c+dx])^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^8 \\
& \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{60d} 19(a+a\cos[c+dx])^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^8 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right)
\end{aligned}$$

■ **Problem 169: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a\cos[c+dx])^4}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 121 leaves, 11 steps):

$$\begin{aligned}
& \frac{64a^4 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5d} + \frac{136a^4 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{21d} + \\
& \frac{94a^4 \sqrt{\cos[c+dx]} \sin[c+dx]}{21d} + \frac{8a^4 \cos[c+dx]^{3/2} \sin[c+dx]}{5d} + \frac{2a^4 \cos[c+dx]^{5/2} \sin[c+dx]}{7d}
\end{aligned}$$

Result (type 5, 500 leaves):

$$\begin{aligned}
& \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^4 \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^8 \left( -\frac{4\cot[c]}{5d} + \frac{191\cos[dx]\sin[c]}{672d} + \right. \\
& \quad \left. \frac{\cos[2dx]\sin[2c]}{20d} + \frac{\cos[3dx]\sin[3c]}{224d} + \frac{191\cos[c]\sin[dx]}{672d} + \frac{\cos[2c]\sin[2dx]}{20d} + \frac{\cos[3c]\sin[3dx]}{224d} \right) - \\
& \frac{1}{42d\sqrt{1+\cot[c]^2}} 17 (a+a\cos[c+dx])^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^8 \\
& \operatorname{Sec}[dx-\operatorname{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]} - \frac{1}{5d} 2 (a+a\cos[c+dx])^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^8 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx+\operatorname{ArcTan}[\tan[c]]]\right]^2 \sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1-\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2} \sqrt{1+\tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1+\tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx+\operatorname{ArcTan}[\tan[c]]] \sqrt{1+\tan[c]^2}}{\cos[c]^2+\sin[c]^2}}{\sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2}}
\end{aligned}$$

- **Problem 170: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a\cos[c+dx])^4}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 119 leaves, 10 steps):

$$\begin{aligned}
& \frac{56a^4 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5d} + \frac{32a^4 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} + \\
& \frac{2a^4 \sin[c+dx]}{d\sqrt{\cos[c+dx]}} + \frac{8a^4 \sqrt{\cos[c+dx]} \sin[c+dx]}{3d} + \frac{2a^4 \cos[c+dx]^{3/2} \sin[c+dx]}{5d}
\end{aligned}$$

Result (type 5, 497 leaves):

$$\begin{aligned}
& \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^4 \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^8 \left( -\frac{(23+33\cos[2c])\operatorname{Csc}[c]\operatorname{Sec}[c]}{80d} + \right. \\
& \left. \frac{\cos[dx]\sin[c]}{6d} + \frac{\cos[2dx]\sin[2c]}{80d} + \frac{\cos[c]\sin[dx]}{6d} + \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]\sin[dx]}{8d} + \frac{\cos[2c]\sin[2dx]}{80d} \right) - \\
& \frac{1}{3d\sqrt{1+\cot[c]^2}} 2(a+a\cos[c+dx])^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^8 \\
& \operatorname{Sec}[dx-\operatorname{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]} - \frac{1}{20d} 7(a+a\cos[c+dx])^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^8 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx+\operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1-\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2} \sqrt{1+\tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1+\tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx+\operatorname{ArcTan}[\tan[c]]] \sqrt{1+\tan[c]^2}}{\cos[c]^2+\sin[c]^2}}{\sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2}}
\end{aligned}$$

■ **Problem 172: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a\cos[c+dx])^4}{\cos[c+dx]^{7/2}} dx$$

Optimal (type 4, 121 leaves, 11 steps):

$$-\frac{56a^4 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5d} + \frac{32a^4 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} + \frac{2a^4 \sin[c+dx]}{5d \cos[c+dx]^{5/2}} + \frac{8a^4 \sin[c+dx]}{3d \cos[c+dx]^{3/2}} + \frac{66a^4 \sin[c+dx]}{5d \sqrt{\cos[c+dx]}}$$

Result (type 5, 495 leaves):



$$\begin{aligned}
& \sqrt{\cos [c+d x]} (a+a \cos [c+d x])^4 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^8 \left(-\frac{(-61+5 \cos [2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{80 d}+\right. \\
& \quad \left.\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^3 \sin [d x]}{40 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2(3 \sin [c]+20 \sin [d x])}{120 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x](20 \sin [c]+99 \sin [d x])}{120 d}\right) \\
& \frac{1}{3 d \sqrt{1+\cot [c]^2}} 2(a+a \cos [c+d x])^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^8 \\
& \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}+\frac{1}{20 d} 7(a+a \cos [c+d x])^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^8 \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \\
& \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right) \\
& \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}}
\end{aligned}$$

- **Problem 173: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \cos [c+d x])^4}{\cos [c+d x]^{9/2}} d x$$

Optimal (type 4, 147 leaves, 13 steps):

$$\begin{aligned}
& -\frac{64 a^4 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 d}+\frac{136 a^4 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{21 d}+ \\
& \frac{2 a^4 \sin [c+d x]}{7 d \cos [c+d x]^{7/2}}+\frac{8 a^4 \sin [c+d x]}{5 d \cos [c+d x]^{5/2}}+\frac{94 a^4 \sin [c+d x]}{21 d \cos [c+d x]^{3/2}}+\frac{64 a^4 \sin [c+d x]}{5 d \sqrt{\cos [c+d x]}}
\end{aligned}$$

Result (type 5, 515 leaves):

$$\sqrt{\cos [c+d x]} (a+a \cos [c+d x])^4 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^8$$

$$\left(\frac{4 \operatorname{Csc}[c] \operatorname{Sec}[c]}{5 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^4 \sin [d x]}{56 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^3(5 \sin [c]+28 \sin [d x])}{280 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2(84 \sin [c]+235 \sin [d x])}{840 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x](235 \sin [c]+672 \sin [d x])}{840 d}\right)-\frac{1}{42 d \sqrt{1+\cot [c]^2}}$$

$$17(a+a \cos [c+d x])^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^8$$

$$\operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}+\frac{1}{5 d} 2(a+a \cos [c+d x])^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^8$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) /$$

$$\left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right)-$$

$$\frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}}$$

■ **Problem 174: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^{7 / 2}}{a+a \cos [c+d x]} d x$$

Optimal (type 4, 128 leaves, 6 steps):

$$\frac{21 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 a d}-\frac{5 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 a d}-$$

$$\frac{5 \sqrt{\cos [c+d x]} \sin [c+d x]}{3 a d}+\frac{7 \cos [c+d x]^{3 / 2} \sin [c+d x]}{5 a d}-\frac{\cos [c+d x]^{5 / 2} \sin [c+d x]}{d(a+a \cos [c+d x])}$$

Result (type 5, 315 leaves):

$$\frac{1}{15 a (1 + \cos [c + d x])} \cos \left[ \frac{1}{2} (c + d x) \right]^2 \left( \left( 2 i \sqrt{2} e^{-i (c + d x)} \left( 63 (1 + e^{2 i (c + d x)}) + 63 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] + 25 e^{i (c + d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c + d x)} \right] \right) \right) / \left( d (-1 + e^{2 i c}) \sqrt{e^{-i (c + d x)} (1 + e^{2 i (c + d x)})} \right) - \frac{1}{d} 2 \sqrt{\cos [c + d x]} \operatorname{Csc}[c] \left( 15 + 10 \cos [d x] \sin [c]^2 - 6 \cos [c] (-8 + \cos [2 d x] \sin [c]^2) + 30 \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right] \sin \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + 5 \sin [2 c] \sin [d x] - 3 \cos [2 c] \sin [c] \sin [2 d x] \right) \right)$$

- **Problem 175: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^{5/2}}{a + a \cos [c + d x]} dx$$

Optimal (type 4, 100 leaves, 5 steps):

$$-\frac{3 \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right]}{a d} + \frac{5 \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right]}{3 a d} + \frac{5 \sqrt{\cos [c + d x]} \sin [c + d x]}{3 a d} - \frac{\cos [c + d x]^{3/2} \sin [c + d x]}{d (a + a \cos [c + d x])}$$

Result (type 5, 289 leaves):

$$\frac{1}{3 a (1 + \cos [c + d x])} \cos \left[ \frac{1}{2} (c + d x) \right]^2 \left( - \left( 2 i \sqrt{2} e^{-i (c + d x)} \left( 9 (1 + e^{2 i (c + d x)}) + 9 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] + 5 e^{i (c + d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c + d x)} \right] \right) \right) / \left( d (-1 + e^{2 i c}) \sqrt{e^{-i (c + d x)} (1 + e^{2 i (c + d x)})} \right) + 1 / d 2 \sqrt{\cos [c + d x]} \operatorname{Csc}[c] \left( 3 + 6 \cos [c] + 2 \cos [d x] \sin [c]^2 + 6 \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right] \sin \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + \sin [2 c] \sin [d x] \right) \right)$$

- **Problem 176: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^{3/2}}{a + a \cos [c + d x]} dx$$

Optimal (type 4, 72 leaves, 4 steps):

$$\frac{3 \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right]}{a d} - \frac{\operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right]}{a d} - \frac{\sqrt{\cos [c + d x]} \sin [c + d x]}{d (a + a \cos [c + d x])}$$

Result (type 5, 264 leaves):

$$\frac{1}{a(1 + \cos[c + dx])} \cos\left[\frac{1}{2}(c + dx)\right]^2 \left( \left( 2i\sqrt{2} e^{-i(c+dx)} \left( 3(1 + e^{2i(c+dx)}) + 3(-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) + e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \right) / \left( d(-1 + e^{2ic}) \sqrt{e^{-i(c+dx)}(1 + e^{2i(c+dx)})} - \frac{2\sqrt{\cos[c+dx]}(2\cot[c] + \csc[c] + \sec\left[\frac{c}{2}\right]) \sec\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{dx}{2}\right]}{d} \right)$$

- **Problem 177: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]}}{a + a\cos[c+dx]} dx$$

Optimal (type 4, 70 leaves, 4 steps):

$$-\frac{\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{ad} + \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{ad} + \frac{\sqrt{\cos[c+dx]} \sin[c+dx]}{d(a + a\cos[c+dx])}$$

Result (type 5, 256 leaves):

$$\frac{1}{a(1 + \cos[c + dx])} \cos\left[\frac{1}{2}(c + dx)\right]^2 \left( - \left( 2i\sqrt{2} e^{-i(c+dx)} \left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) + e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \right) / \left( d(-1 + e^{2ic}) \sqrt{e^{-i(c+dx)}(1 + e^{2i(c+dx)})} + \frac{2\sqrt{\cos[c+dx]}(\csc[c] + \sec\left[\frac{c}{2}\right]) \sec\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{dx}{2}\right]}{d} \right)$$

- **Problem 178: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\cos[c+dx]}(a + a\cos[c+dx])} dx$$

Optimal (type 4, 70 leaves, 4 steps):

$$\frac{\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{ad} + \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{ad} - \frac{\sqrt{\cos[c+dx]} \sin[c+dx]}{d(a + a\cos[c+dx])}$$

Result (type 5, 257 leaves):

$$\frac{1}{a(1 + \cos[c + dx])} \cos\left[\frac{1}{2}(c + dx)\right]^2 \left( \left( 2i\sqrt{2} e^{-i(c+dx)} \left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - e^{i(c+dx)} \right. \right. \right. \\ \left. \left. \left. (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right]\right) \right) \right) / \\ \left( d(-1 + e^{2ic}) \sqrt{e^{-i(c+dx)}(1 + e^{2i(c+dx)})} - \frac{2\sqrt{\cos[c+dx]} \left( \operatorname{Csc}[c] + \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sin}\left[\frac{dx}{2}\right] \right)}{d} \right)$$

- **Problem 179: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\cos[c + dx]^{3/2} (a + a \cos[c + dx])} dx$$

Optimal (type 4, 96 leaves, 5 steps):

$$-\frac{3 \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} - \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{3 \operatorname{Sin}[c + dx]}{ad \sqrt{\cos[c + dx]}} - \frac{\operatorname{Sin}[c + dx]}{d \sqrt{\cos[c + dx]} (a + a \cos[c + dx])}$$

Result (type 5, 297 leaves):

$$\frac{1}{a(1 + \cos[c + dx])} \cos\left[\frac{1}{2}(c + dx)\right]^2 \left( - \left( 2i\sqrt{2} e^{-i(c+dx)} \left( 3(1 + e^{2i(c+dx)}) + 3(-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - \right. \right. \right. \\ \left. \left. \left. e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right]\right) \right) \right) / \left( d(-1 + e^{2ic}) \sqrt{e^{-i(c+dx)}(1 + e^{2i(c+dx)})} \right) + \\ \frac{\left( 2 \cos\left[\frac{1}{2}(c - dx)\right] + \cos\left[\frac{1}{2}(3c + dx)\right] + 3 \cos\left[\frac{1}{2}(c + 3dx)\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]}{2d \sqrt{\cos[c + dx]}}$$

- **Problem 180: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\cos[c + dx]^{5/2} (a + a \cos[c + dx])} dx$$

Optimal (type 4, 124 leaves, 6 steps):

$$\frac{3 \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{5 \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3ad} + \frac{5 \operatorname{Sin}[c + dx]}{3ad \cos[c + dx]^{3/2}} - \frac{3 \operatorname{Sin}[c + dx]}{ad \sqrt{\cos[c + dx]}} - \frac{\operatorname{Sin}[c + dx]}{d \cos[c + dx]^{3/2} (a + a \cos[c + dx])}$$

Result (type 5, 332 leaves):

$$\frac{1}{3 a (1 + \operatorname{Cos}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^2 \left( \left( 2 i \sqrt{2} e^{-i(c+d x)} \left( 9 (1 + e^{2 i(c+d x)}) + 9 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] - 5 e^{i(c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \right) \right) / \left( d (-1 + e^{2 i c}) \sqrt{e^{-i(c+d x)} (1 + e^{2 i(c+d x)})} \right) - 1 / (4 d \operatorname{Cos}[c + d x]^{3/2}) \left( 10 \operatorname{Cos}\left[\frac{1}{2}(c - d x)\right] + 8 \operatorname{Cos}\left[\frac{1}{2}(3 c + d x)\right] + 4 \operatorname{Cos}\left[\frac{1}{2}(c + 3 d x)\right] + 5 \operatorname{Cos}\left[\frac{1}{2}(5 c + 3 d x)\right] + 9 \operatorname{Cos}\left[\frac{1}{2}(3 c + 5 d x)\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] \right)$$

- **Problem 181: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^{9/2}}{(a + a \operatorname{Cos}[c + d x])^2} dx$$

Optimal (type 4, 160 leaves, 7 steps):

$$\frac{56 \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 a^2 d} - \frac{5 \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{a^2 d} - \frac{5 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{a^2 d} + \frac{56 \operatorname{Cos}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{15 a^2 d} - \frac{3 \operatorname{Cos}[c + d x]^{5/2} \operatorname{Sin}[c + d x]}{a^2 d (1 + \operatorname{Cos}[c + d x])} - \frac{\operatorname{Cos}[c + d x]^{7/2} \operatorname{Sin}[c + d x]}{3 d (a + a \operatorname{Cos}[c + d x])^2}$$

Result (type 5, 367 leaves):

$$\frac{1}{5 a^2 (1 + \operatorname{Cos}[c + d x])^2} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^4 \left( \left( 4 i \sqrt{2} e^{-i(c+d x)} \left( 56 (1 + e^{2 i(c+d x)}) + 56 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + 25 e^{i(c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \right) \right) / \left( d (-1 + e^{2 i c}) \sqrt{e^{-i(c+d x)} (1 + e^{2 i(c+d x)})} \right) - \frac{1}{3 d} 2 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Csc}[c] \left( 120 + 40 \operatorname{Cos}[d x] \operatorname{Sin}[c]^2 - 6 \operatorname{Cos}[2 d x] \operatorname{Sin}[c] \operatorname{Sin}[2 c] + 240 \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sin}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{d x}{2}\right] - 10 \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^3 \operatorname{Sin}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{d x}{2}\right] + 8 \operatorname{Cos}[c] (27 + 5 \operatorname{Sin}[c] \operatorname{Sin}[d x]) - 6 \operatorname{Cos}[2 c] \operatorname{Sin}[c] \operatorname{Sin}[2 d x] - 5 \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Sin}[c] \operatorname{Tan}\left[\frac{c}{2}\right] \right) \right)$$

- **Problem 182: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^{7/2}}{(a + a \operatorname{Cos}[c + d x])^2} dx$$

Optimal (type 4, 138 leaves, 6 steps) :

$$-\frac{7 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} + \frac{10 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 a^2 d} +$$

$$\frac{10 \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{3 a^2 d} - \frac{7 \operatorname{Cos}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{3 a^2 d (1 + \operatorname{Cos}[c+dx])} - \frac{\operatorname{Cos}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{3 d (a + a \operatorname{Cos}[c+dx])^2}$$

Result (type 5, 337 leaves) :

$$\frac{1}{3 a^2 (1 + \operatorname{Cos}[c+dx])^2}$$

$$\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^4 \left( - \left( 4 i \sqrt{2} e^{-i(c+dx)} \left( 21 (1 + e^{2i(c+dx)}) + 21 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + \right. \right.$$

$$\left. \left. 10 e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \right) / \left( d (-1 + e^{2ic}) \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \right) +$$

$$1 / (2d) \sqrt{\operatorname{Cos}[c+dx]} \left( 72 \operatorname{Cos}\left[\frac{1}{2}(c-dx)\right] + 54 \operatorname{Cos}\left[\frac{1}{2}(3c+dx)\right] + 33 \operatorname{Cos}\left[\frac{1}{2}(c+3dx)\right] + 9 \operatorname{Cos}\left[\frac{1}{2}(5c+3dx)\right] + \right.$$

$$\left. \operatorname{Cos}\left[\frac{1}{2}(3c+5dx)\right] - \operatorname{Cos}\left[\frac{1}{2}(7c+5dx)\right] \right) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3$$

■ **Problem 183: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^{5/2}}{(a + a \operatorname{Cos}[c+dx])^2} dx$$

Optimal (type 4, 112 leaves, 5 steps) :

$$\frac{4 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} - \frac{5 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 a^2 d} - \frac{5 \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{3 a^2 d (1 + \operatorname{Cos}[c+dx])} - \frac{\operatorname{Cos}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{3 d (a + a \operatorname{Cos}[c+dx])^2}$$

Result (type 5, 319 leaves) :

$$\frac{1}{3 a^2 (1 + \operatorname{Cos}[c+dx])^2}$$

$$\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^4 \left( \left( 4 i \sqrt{2} e^{-i(c+dx)} \left( 12 (1 + e^{2i(c+dx)}) + 12 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + \right. \right. \right.$$

$$\left. \left. \left. 5 e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \right) / \left( d (-1 + e^{2ic}) \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \right) -$$

$$1 / (2d) \sqrt{\operatorname{Cos}[c+dx]} \left( 20 \operatorname{Cos}\left[\frac{1}{2}(c-dx)\right] + 16 \operatorname{Cos}\left[\frac{1}{2}(3c+dx)\right] + 9 \operatorname{Cos}\left[\frac{1}{2}(c+3dx)\right] + 3 \operatorname{Cos}\left[\frac{1}{2}(5c+3dx)\right] \right)$$

$$\operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3$$

- **Problem 184: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^{3 / 2}}{(a+a \cos [c+d x])^2} d x$$

Optimal (type 4, 109 leaves, 5 steps):

$$-\frac{\text{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{a^2 d} + \frac{2 \text{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 a^2 d} + \frac{\sqrt{\cos [c+d x]} \sin [c+d x]}{a^2 d(1+\cos [c+d x])} - \frac{\sqrt{\cos [c+d x]} \sin [c+d x]}{3 d(a+a \cos [c+d x])^2}$$

Result (type 5, 640 leaves):

$$\begin{aligned} & -\frac{1}{2(a+a \cos [c+d x])^2} i \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ & \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x}(\cos [c]+i \sin [c])^2\right] \sqrt{e^{-i d x}\left(2\left(1+e^{2 i d x}\right) \cos [c]+2 i\left(-1+e^{2 i d x}\right) \sin [c]\right)} \right. \right. \\ & \quad \left. \left. \sqrt{1+e^{2 i d x} \cos [2 c]+i e^{2 i d x} \sin [2 c]} \right) / \left( 3 i d\left(1+e^{2 i d x}\right) \cos [c]-3 d\left(-1+e^{2 i d x}\right) \sin [c]\right) - \right. \\ & \quad \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x}(\cos [c]+i \sin [c])^2\right] \sqrt{e^{-i d x}\left(2\left(1+e^{2 i d x}\right) \cos [c]+2 i\left(-1+e^{2 i d x}\right) \sin [c]\right)} \right. \right. \\ & \quad \left. \left. \sqrt{1+e^{2 i d x} \cos [2 c]+i e^{2 i d x} \sin [2 c]} \right) / \left( -i d\left(1+e^{2 i d x}\right) \cos [c]+d\left(-1+e^{2 i d x}\right) \sin [c]\right) \right) - \\ & \left( 4 \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\ & \quad \left. \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\ & \left( 3 d(a+a \cos [c+d x])^2 \sqrt{1+\operatorname{Cot}[c]^2} \right) + \frac{1}{(a+a \cos [c+d x])^2} \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^4 \\ & \quad \sqrt{\cos [c+d x]} \\ & \left( \frac{4 \operatorname{Csc}[c]}{d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right] \sin \left[\frac{d x}{2}\right]}{d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^3 \sin \left[\frac{d x}{2}\right]}{3 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right) \end{aligned}$$

- **Problem 186: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\cos [c+d x]}(a+a \cos [c+d x])^2} d x$$

Optimal (type 4, 109 leaves, 5 steps):



$$\frac{\text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} + \frac{2 \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 a^2 d} - \frac{\sqrt{\text{Cos}[c+dx]} \text{Sin}[c+dx]}{a^2 d (1 + \text{Cos}[c+dx])} - \frac{\sqrt{\text{Cos}[c+dx]} \text{Sin}[c+dx]}{3 d (a + a \text{Cos}[c+dx])^2}$$

Result (type 5, 304 leaves):

$$\frac{1}{3 a^2 (1 + \text{Cos}[c+dx])^2} \text{Cos}\left[\frac{1}{2}(c+dx)\right]^4 \left( \left( 4 i \sqrt{2} e^{-i(c+dx)} \left( 3 (1 + e^{2i(c+dx)}) + 3 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - 2 e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \right) / \left( d (-1 + e^{2ic}) \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \right) - 1 / (2 d) \sqrt{\text{Cos}[c+dx]} \left( 7 \text{Cos}\left[\frac{1}{2}(c-dx)\right] + 2 \text{Cos}\left[\frac{1}{2}(3c+dx)\right] + 3 \text{Cos}\left[\frac{1}{2}(c+3dx)\right] \right) \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^3 \right)$$

■ **Problem 187: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\text{Cos}[c+dx]^{3/2} (a + a \text{Cos}[c+dx])^2} dx$$

Optimal (type 4, 136 leaves, 6 steps):

$$-\frac{4 \text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} - \frac{5 \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 a^2 d} + \frac{4 \text{Sin}[c+dx]}{a^2 d \sqrt{\text{Cos}[c+dx]}} - \frac{5 \text{Sin}[c+dx]}{3 a^2 d \sqrt{\text{Cos}[c+dx]} (1 + \text{Cos}[c+dx])} - \frac{\text{Sin}[c+dx]}{3 d \sqrt{\text{Cos}[c+dx]} (a + a \text{Cos}[c+dx])^2}$$

Result (type 5, 334 leaves):

$$\frac{1}{3 a^2 (1 + \text{Cos}[c+dx])^2} \text{Cos}\left[\frac{1}{2}(c+dx)\right]^4 \left( - \left( 4 i \sqrt{2} e^{-i(c+dx)} \left( 12 (1 + e^{2i(c+dx)}) + 12 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - 5 e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \right) / \left( d (-1 + e^{2ic}) \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \right) + 1 / (4 d \sqrt{\text{Cos}[c+dx]}) \left( 29 \text{Cos}\left[\frac{1}{2}(c-dx)\right] + 19 \text{Cos}\left[\frac{1}{2}(3c+dx)\right] + 31 \text{Cos}\left[\frac{1}{2}(c+3dx)\right] + 5 \text{Cos}\left[\frac{1}{2}(5c+3dx)\right] + 12 \text{Cos}\left[\frac{1}{2}(3c+5dx)\right] \right) \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^3 \right)$$

■ **Problem 188: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\text{Cos}[c+dx]^{5/2} (a + a \text{Cos}[c+dx])^2} dx$$

Optimal (type 4, 162 leaves, 7 steps) :

$$\frac{7 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} + \frac{10 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 a^2 d} + \frac{10 \operatorname{Sin}[c+dx]}{3 a^2 d \operatorname{Cos}[c+dx]^{3/2}} - \frac{7 \operatorname{Sin}[c+dx]}{a^2 d \sqrt{\operatorname{Cos}[c+dx]}} - \frac{7 \operatorname{Sin}[c+dx]}{3 a^2 d \operatorname{Cos}[c+dx]^{3/2} (1+\operatorname{Cos}[c+dx])} - \frac{\operatorname{Sin}[c+dx]}{3 d \operatorname{Cos}[c+dx]^{3/2} (a+a \operatorname{Cos}[c+dx])^2}$$

Result (type 5, 425 leaves) :

$$\left( 4 i \sqrt{2} e^{-i(c+dx)} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left( 21 (1 + e^{2i(c+dx)}) + 21 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - 10 e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \right) / \left( 3 d (-1 + e^{2ic}) \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} (a + a \operatorname{Cos}[c+dx])^2 \right) + \frac{1}{(a + a \operatorname{Cos}[c+dx])^2} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Cos}[c+dx]} - \left( \frac{2 (4 + 3 \operatorname{Cos}[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{d} - \frac{12 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Sin}\left[\frac{dx}{2}\right]}{3 d} + \frac{8 \operatorname{Sec}[c] \operatorname{Sec}[c+dx] (\operatorname{Sin}[c] - 6 \operatorname{Sin}[dx])}{3 d} + \frac{8 \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 \operatorname{Sin}[dx]}{3 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right)$$

■ **Problem 189: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Cos}[c+dx]^{11/2}}{(a + a \operatorname{Cos}[c+dx])^3} dx$$

Optimal (type 4, 207 leaves, 8 steps) :

$$\frac{231 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{10 a^3 d} - \frac{21 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{2 a^3 d} - \frac{21 \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{2 a^3 d} + \frac{77 \operatorname{Cos}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{10 a^3 d} - \frac{\operatorname{Cos}[c+dx]^{9/2} \operatorname{Sin}[c+dx]}{5 d (a + a \operatorname{Cos}[c+dx])^3} - \frac{4 \operatorname{Cos}[c+dx]^{7/2} \operatorname{Sin}[c+dx]}{5 a d (a + a \operatorname{Cos}[c+dx])^2} - \frac{63 \operatorname{Cos}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{10 d (a^3 + a^3 \operatorname{Cos}[c+dx])}$$

Result (type 5, 388 leaves) :

$$\frac{1}{5 a^3 d (1 + \operatorname{Cos}[c + d x])^3} \\ 2 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^6 \left( \left( 42 i \sqrt{2} e^{-i(c+d x)} \left( 11 (1 + e^{2 i(c+d x)}) + 11 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + \right. \right. \right. \\ \left. \left. \left. 5 e^{i(c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right]\right) \right) \right) / \left( (-1 + e^{2 i c}) \sqrt{e^{-i(c+d x)} (1 + e^{2 i(c+d x)})} \right) - \\ \sqrt{\operatorname{Cos}[c + d x]} \left( 264 \operatorname{Cot}[c] + 198 \operatorname{Csc}[c] + \frac{1}{16} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^5 \left( 1210 \operatorname{Sin}\left[\frac{d x}{2}\right] - 770 \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 840 \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - 150 \operatorname{Sin}\left[ \right. \right. \right. \\ \left. \left. \left. 2 c + \frac{3 d x}{2}\right] + 238 \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] + 40 \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] + 5 \operatorname{Sin}\left[3 c + \frac{7 d x}{2}\right] + 5 \operatorname{Sin}\left[4 c + \frac{7 d x}{2}\right] - \operatorname{Sin}\left[4 c + \frac{9 d x}{2}\right] - \operatorname{Sin}\left[5 c + \frac{9 d x}{2}\right] \right) \right) \right)$$

- **Problem 190: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^{9/2}}{(a + a \operatorname{Cos}[c + d x])^3} dx$$

Optimal (type 4, 181 leaves, 7 steps):

$$-\frac{119 \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{10 a^3 d} + \frac{11 \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{2 a^3 d} + \frac{11 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{2 a^3 d} - \\ \frac{\operatorname{Cos}[c + d x]^{7/2} \operatorname{Sin}[c + d x]}{5 d (a + a \operatorname{Cos}[c + d x])^3} - \frac{2 \operatorname{Cos}[c + d x]^{5/2} \operatorname{Sin}[c + d x]}{3 a d (a + a \operatorname{Cos}[c + d x])^2} - \frac{119 \operatorname{Cos}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{30 d (a^3 + a^3 \operatorname{Cos}[c + d x])}$$

Result (type 5, 369 leaves):

$$\frac{1}{5 a^3 (1 + \operatorname{Cos}[c + d x])^3} \\ \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^6 \left( - \left( 4 i \sqrt{2} e^{-i(c+d x)} \left( 119 (1 + e^{2 i(c+d x)}) + 119 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + \right. \right. \right. \\ \left. \left. \left. 55 e^{i(c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right]\right) \right) \right) / \left( d (-1 + e^{2 i c}) \sqrt{e^{-i(c+d x)} (1 + e^{2 i(c+d x)})} \right) + \\ \frac{1}{12 d} \sqrt{\operatorname{Cos}[c + d x]} \left( 1961 \operatorname{Cos}\left[\frac{1}{2}(c - d x)\right] + 1609 \operatorname{Cos}\left[\frac{1}{2}(3 c + d x)\right] + 1165 \operatorname{Cos}\left[\frac{1}{2}(c + 3 d x)\right] + 620 \operatorname{Cos}\left[\frac{1}{2}(5 c + 3 d x)\right] + \right. \\ \left. 292 \operatorname{Cos}\left[\frac{1}{2}(3 c + 5 d x)\right] + 65 \operatorname{Cos}\left[\frac{1}{2}(7 c + 5 d x)\right] + 5 \operatorname{Cos}\left[\frac{1}{2}(5 c + 7 d x)\right] - 5 \operatorname{Cos}\left[\frac{1}{2}(9 c + 7 d x)\right] \right) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^5$$

- **Problem 191: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^{7/2}}{(a + a \operatorname{Cos}[c + d x])^3} dx$$

Optimal (type 4, 155 leaves, 6 steps):

$$\frac{49 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{10 a^3 d} - \frac{13 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{6 a^3 d} -$$

$$\frac{\operatorname{Cos}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{5 d (a+a \operatorname{Cos}[c+dx])^3} - \frac{8 \operatorname{Cos}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{15 a d (a+a \operatorname{Cos}[c+dx])^2} - \frac{13 \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{6 d (a^3+a^3 \operatorname{Cos}[c+dx])}$$

Result (type 5, 435 leaves):

$$\left( 4 i \sqrt{2} e^{-i(c+dx)} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \left( 147 (1+e^{2i(c+dx)}) + 147 (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + \right. \right.$$

$$\left. \left. 65 e^{i(c+dx)} (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \right) /$$

$$\left( 15 d (-1+e^{2ic}) \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} (a+a \operatorname{Cos}[c+dx])^3 \right) + \frac{1}{(a+a \operatorname{Cos}[c+dx])^3}$$

$$\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Cos}[c+dx]} \left( -\frac{4(29+20 \operatorname{Cos}[c]) \operatorname{Csc}[c]}{5 d} - \frac{116 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{5 d} + \right.$$

$$\left. \frac{56 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Sin}\left[\frac{dx}{2}\right]}{15 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Sin}\left[\frac{dx}{2}\right]}{5 d} + \frac{56 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right)$$

■ **Problem 192: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^{5/2}}{(a+a \operatorname{Cos}[c+dx])^3} dx$$

Optimal (type 4, 155 leaves, 6 steps):

$$-\frac{9 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{10 a^3 d} + \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{2 a^3 d} -$$

$$\frac{\operatorname{Cos}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{5 d (a+a \operatorname{Cos}[c+dx])^3} - \frac{2 \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{5 a d (a+a \operatorname{Cos}[c+dx])^2} + \frac{9 \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{10 d (a^3+a^3 \operatorname{Cos}[c+dx])}$$

Result (type 5, 705 leaves):

$$\begin{aligned}
& - \frac{1}{10 (a + a \cos[c + dx])^3} 9 i \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\
& \quad \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) - \\
& \left( 2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left( d (a + a \cos[c + dx])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]} \\
& \left( \frac{36 \operatorname{Csc}[c]}{5 d} + \frac{36 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{5 d} - \frac{12 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{5 d} + \right. \\
& \quad \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{5 d} - \frac{12 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right)
\end{aligned}$$

■ **Problem 193: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{3/2}}{(a + a \cos[c + dx])^3} dx$$

Optimal (type 4, 155 leaves, 6 steps):

$$- \frac{\operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10 a^3 d} + \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6 a^3 d} - \frac{\sqrt{\cos[c + dx]} \sin[c + dx]}{5 d (a + a \cos[c + dx])^3} + \frac{4 \sqrt{\cos[c + dx]} \sin[c + dx]}{15 a d (a + a \cos[c + dx])^2} + \frac{\sqrt{\cos[c + dx]} \sin[c + dx]}{10 d (a^3 + a^3 \cos[c + dx])}$$

Result (type 5, 334 leaves):

$$\frac{1}{15 a^3 (1 + \operatorname{Cos}[c + d x])^3} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^6 \left( - \left( 4 i \sqrt{2} e^{-i(c+d x)} \left( 3 (1 + e^{2 i(c+d x)}) + 3 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + 5 e^{i(c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \right) \right) / \left( d (-1 + e^{2 i c}) \sqrt{e^{-i(c+d x)} (1 + e^{2 i(c+d x)})} \right) + 1 / (8 d) \sqrt{\operatorname{Cos}[c + d x]} \left( 14 \operatorname{Cos}\left[\frac{1}{2}(c - d x)\right] + 16 \operatorname{Cos}\left[\frac{1}{2}(3 c + d x)\right] + 20 \operatorname{Cos}\left[\frac{1}{2}(c + 3 d x)\right] - 5 \operatorname{Cos}\left[\frac{1}{2}(5 c + 3 d x)\right] + 3 \operatorname{Cos}\left[\frac{1}{2}(3 c + 5 d x)\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^5$$

- **Problem 194: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Cos}[c + d x]}}{(a + a \operatorname{Cos}[c + d x])^3} dx$$

Optimal (type 4, 155 leaves, 6 steps):

$$\frac{\operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{10 a^3 d} + \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{6 a^3 d} + \frac{\sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{5 d (a + a \operatorname{Cos}[c + d x])^3} + \frac{\sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{15 a d (a + a \operatorname{Cos}[c + d x])^2} - \frac{\sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{10 d (a^3 + a^3 \operatorname{Cos}[c + d x])}$$

Result (type 5, 334 leaves):

$$\frac{1}{15 a^3 (1 + \operatorname{Cos}[c + d x])^3} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^6 \left( \left( 4 i \sqrt{2} e^{-i(c+d x)} \left( 3 (1 + e^{2 i(c+d x)}) + 3 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] - 5 e^{i(c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \right) \right) / \left( d (-1 + e^{2 i c}) \sqrt{e^{-i(c+d x)} (1 + e^{2 i(c+d x)})} \right) - 1 / (8 d) \sqrt{\operatorname{Cos}[c + d x]} \left( 4 \operatorname{Cos}\left[\frac{1}{2}(c - d x)\right] + 26 \operatorname{Cos}\left[\frac{1}{2}(3 c + d x)\right] + 10 \operatorname{Cos}\left[\frac{1}{2}(c + 3 d x)\right] + 5 \operatorname{Cos}\left[\frac{1}{2}(5 c + 3 d x)\right] + 3 \operatorname{Cos}\left[\frac{1}{2}(3 c + 5 d x)\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^5$$

- **Problem 195: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Cos}[c + d x])^3} dx$$

Optimal (type 4, 155 leaves, 6 steps):

$$\frac{9 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{10 a^3 d} + \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{2 a^3 d} -$$

$$\frac{\sqrt{\cos[c+dx]} \sin[c+dx]}{5 d (a+a \cos[c+dx])^3} - \frac{2 \sqrt{\cos[c+dx]} \sin[c+dx]}{5 a d (a+a \cos[c+dx])^2} - \frac{9 \sqrt{\cos[c+dx]} \sin[c+dx]}{10 d (a^3+a^3 \cos[c+dx])}$$

Result (type 5, 705 leaves):

$$\frac{1}{10 (a+a \cos[c+dx])^3} 9 i \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right.$$

$$\left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) -$$

$$\left( 2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right.$$

$$\left. \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \right.$$

$$\left( d (a+a \cos[c+dx])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a+a \cos[c+dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]}$$

$$\left( -\frac{36 \operatorname{Csc}[c]}{5 d} - \frac{36 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{5 d} - \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{5 d} - \right.$$

$$\left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{5 d} - \frac{8 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right)$$

■ **Problem 196: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\cos[c+dx]^{3/2} (a+a \cos[c+dx])^3} dx$$

Optimal (type 4, 181 leaves, 7 steps):

$$\frac{49 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{10 a^3 d} - \frac{13 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{6 a^3 d} + \frac{49 \operatorname{Sin}[c+dx]}{10 a^3 d \sqrt{\operatorname{Cos}[c+dx]}} -$$

$$\frac{\operatorname{Sin}[c+dx]}{5 d \sqrt{\operatorname{Cos}[c+dx]} (a+a \operatorname{Cos}[c+dx])^3} - \frac{8 \operatorname{Sin}[c+dx]}{15 a d \sqrt{\operatorname{Cos}[c+dx]} (a+a \operatorname{Cos}[c+dx])^2} - \frac{13 \operatorname{Sin}[c+dx]}{6 d \sqrt{\operatorname{Cos}[c+dx]} (a^3+a^3 \operatorname{Cos}[c+dx])}$$

Result (type 5, 364 leaves):

$$\frac{1}{15 a^3 (1+\operatorname{Cos}[c+dx])^3}$$

$$\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^6 \left( - \left( 4 i \sqrt{2} e^{-i(c+dx)} \left( 147 (1+e^{2i(c+dx)}) + 147 (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - \right. \right. \right.$$

$$\left. \left. 65 e^{i(c+dx)} (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \right) / \left( d (-1+e^{2ic}) \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} \right) +$$

$$\frac{1}{16 d \sqrt{\operatorname{Cos}[c+dx]}} \left( 1284 \operatorname{Cos}\left[\frac{1}{2}(c-dx)\right] + 921 \operatorname{Cos}\left[\frac{1}{2}(3c+dx)\right] + 1243 \operatorname{Cos}\left[\frac{1}{2}(c+3dx)\right] + 374 \operatorname{Cos}\left[\frac{1}{2}(5c+3dx)\right] + \right.$$

$$\left. 670 \operatorname{Cos}\left[\frac{1}{2}(3c+5dx)\right] + 65 \operatorname{Cos}\left[\frac{1}{2}(7c+5dx)\right] + 147 \operatorname{Cos}\left[\frac{1}{2}(5c+7dx)\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5$$

■ **Problem 197: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\operatorname{Cos}[c+dx]^{5/2} (a+a \operatorname{Cos}[c+dx])^3} dx$$

Optimal (type 4, 207 leaves, 8 steps):

$$\frac{119 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{10 a^3 d} + \frac{11 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{2 a^3 d} + \frac{11 \operatorname{Sin}[c+dx]}{2 a^3 d \operatorname{Cos}[c+dx]^{3/2}} - \frac{119 \operatorname{Sin}[c+dx]}{10 a^3 d \sqrt{\operatorname{Cos}[c+dx]}} -$$

$$\frac{\operatorname{Sin}[c+dx]}{5 d \operatorname{Cos}[c+dx]^{3/2} (a+a \operatorname{Cos}[c+dx])^3} - \frac{2 \operatorname{Sin}[c+dx]}{3 a d \operatorname{Cos}[c+dx]^{3/2} (a+a \operatorname{Cos}[c+dx])^2} - \frac{119 \operatorname{Sin}[c+dx]}{30 d \operatorname{Cos}[c+dx]^{3/2} (a^3+a^3 \operatorname{Cos}[c+dx])}$$

Result (type 5, 394 leaves):



$$\frac{1}{5 a^3 (1 + \cos [c + d x])^3} \cos \left[ \frac{1}{2} (c + d x) \right]^6$$

$$\left( \left( 4 i \sqrt{2} e^{-i (c+d x)} \left( 119 (1 + e^{2 i (c+d x)}) + 119 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c+d x)} \right] - 55 e^{i (c+d x)} (-1 + e^{2 i c}) \right. \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c+d x)} \right] \right) \right) / \left( d (-1 + e^{2 i c}) \sqrt{e^{-i (c+d x)} (1 + e^{2 i (c+d x)})} \right) - \frac{1}{96 d \cos [c + d x]^{3/2}}$$

$$\left( 5134 \cos \left[ \frac{1}{2} (c - d x) \right] + 4148 \cos \left[ \frac{1}{2} (3 c + d x) \right] + 4664 \cos \left[ \frac{1}{2} (c + 3 d x) \right] + 2476 \cos \left[ \frac{1}{2} (5 c + 3 d x) \right] + 3340 \cos \left[ \frac{1}{2} (3 c + 5 d x) \right] + \right.$$

$$\left. 944 \cos \left[ \frac{1}{2} (7 c + 5 d x) \right] + 1620 \cos \left[ \frac{1}{2} (5 c + 7 d x) \right] + 165 \cos \left[ \frac{1}{2} (9 c + 7 d x) \right] + 357 \cos \left[ \frac{1}{2} (7 c + 9 d x) \right] \right) \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right]^5$$

- **Problem 198: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^{5/2} \sqrt{a + a \cos [c + d x]} dx$$

Optimal (type 3, 154 leaves, 5 steps):

$$\frac{5 \sqrt{a} \operatorname{ArcSin} \left[ \frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}} \right]}{8 d} + \frac{5 a \sqrt{\cos [c + d x]} \sin [c + d x]}{8 d \sqrt{a + a \cos [c + d x]}} + \frac{5 a \cos [c + d x]^{3/2} \sin [c + d x]}{12 d \sqrt{a + a \cos [c + d x]}} + \frac{a \cos [c + d x]^{5/2} \sin [c + d x]}{3 d \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 437 leaves):

$$\frac{1}{48 d \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}} \sqrt{\cos [c + d x]} \sqrt{a (1 + \cos [c + d x])}$$

$$\operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right] \left( -15 i \cos \left[ \frac{d x}{2} \right] \operatorname{Log} \left[ 2 \left( e^{i d x} \cos \left[ \frac{c}{2} \right] + i e^{i d x} \sin \left[ \frac{c}{2} \right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) \right] + \right.$$

$$15 i \operatorname{ArcTanh} \left[ \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right] \left( \cos \left[ \frac{d x}{2} \right] + i \sin \left[ \frac{d x}{2} \right] \right) +$$

$$15 \operatorname{Log} \left[ 2 \left( e^{i d x} \cos \left[ \frac{c}{2} \right] + i e^{i d x} \sin \left[ \frac{c}{2} \right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) \right] \sin \left[ \frac{d x}{2} \right] +$$

$$28 \sqrt{2} \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \sin \left[ \frac{1}{2} (c + d x) \right] +$$

$$6 \sqrt{2} \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \sin \left[ \frac{3}{2} (c + d x) \right] + 4 \sqrt{2} \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \sin \left[ \frac{5}{2} (c + d x) \right]$$

- **Problem 199: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^{3/2} \sqrt{a + a \cos [c + d x]} dx$$

Optimal (type 3, 116 leaves, 4 steps):

$$\frac{3 \sqrt{a} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{4 d} + \frac{3 a \sqrt{\cos[c+dx]} \sin[c+dx]}{4 d \sqrt{a+a \cos[c+dx]}} + \frac{a \cos[c+dx]^{3/2} \sin[c+dx]}{2 d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 396 leaves):

$$\frac{1}{8 d \sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}} \sqrt{\cos[c+dx]} \sqrt{a(1+\cos[c+dx])} \\ \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(-3 i \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \cos\left[\frac{c}{2}\right]+i e^{i d x} \sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}\right)\right]\right) + \\ 3 i \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right]+i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right]+i \sin\left[\frac{dx}{2}\right]\right) + \\ 3 \operatorname{Log}\left[2\left(e^{i d x} \cos\left[\frac{c}{2}\right]+i e^{i d x} \sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] + \\ 4 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx]+i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + 2 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx]+i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right]$$

■ **Problem 200: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]} dx$$

Optimal (type 3, 72 leaves, 3 steps):

$$\frac{\sqrt{a} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} + \frac{a \sqrt{\cos[c+dx]} \sin[c+dx]}{d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 354 leaves):

$$\frac{1}{2 d \sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}} \sqrt{\cos[c+dx]} \sqrt{a(1+\cos[c+dx])} \\ \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(-i \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \cos\left[\frac{c}{2}\right]+i e^{i d x} \sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}\right)\right]\right) + \\ i \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right]+i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right]+i \sin\left[\frac{dx}{2}\right]\right) + \\ \operatorname{Log}\left[2\left(e^{i d x} \cos\left[\frac{c}{2}\right]+i e^{i d x} \sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] + \\ 2 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx]+i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right]$$

- **Problem 201: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \cos[c + dx]}}{\sqrt{\cos[c + dx]}} dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$\frac{2 \sqrt{a} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d}$$

Result (type 3, 246 leaves):

$$\begin{aligned} & \left( i e^{\frac{idx}{2}} \sqrt{a(1+\cos[c+dx])} \left( \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2idx})\cos[c] + i(-1+e^{2idx})\sin[c]}\right] - \right. \right. \\ & \quad \left. \left. \operatorname{Log}\left[2\left(e^{idx}\cos\left[\frac{c}{2}\right] + i e^{idx}\sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx})\cos[c] + i(-1+e^{2idx})\sin[c]}\right)\right]\right] \right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \\ & \quad \left. \sqrt{e^{-idx}\left((1+e^{2idx})\cos[c] + i(-1+e^{2idx})\sin[c]\right)} \right) / \left( d \sqrt{2(1+e^{2idx})\cos[c] + 2i(-1+e^{2idx})\sin[c]} \right) \end{aligned}$$

- **Problem 206: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^{3/2} (a + a \cos[c + dx])^{3/2} dx$$

Optimal (type 3, 160 leaves, 6 steps):

$$\frac{11 a^{3/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{8 d} + \frac{11 a^2 \sqrt{\cos[c+dx]} \sin[c+dx]}{8 d \sqrt{a+a \cos[c+dx]}} + \frac{11 a^2 \cos[c+dx]^{3/2} \sin[c+dx]}{12 d \sqrt{a+a \cos[c+dx]}} + \frac{a^2 \cos[c+dx]^{5/2} \sin[c+dx]}{3 d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 438 leaves):

$$\begin{aligned} & \frac{1}{48 d \sqrt{(1+e^{2idx})\cos[c] + i(-1+e^{2idx})\sin[c]}} a \sqrt{\cos[c+dx]} \sqrt{a(1+\cos[c+dx])} \\ & \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left( -33 i \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2\left(e^{idx}\cos\left[\frac{c}{2}\right] + i e^{idx}\sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx})\cos[c] + i(-1+e^{2idx})\sin[c]}\right)\right] \right) + \\ & \quad 33 i \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2idx})\cos[c] + i(-1+e^{2idx})\sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) + \\ & \quad 33 \operatorname{Log}\left[2\left(e^{idx}\cos\left[\frac{c}{2}\right] + i e^{idx}\sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx})\cos[c] + i(-1+e^{2idx})\sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] + \\ & \quad 52 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + \\ & \quad 18 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] + 4 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{5}{2}(c+dx)\right] \end{aligned}$$

- **Problem 207: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^{3/2} dx$$

Optimal (type 3, 120 leaves, 5 steps):

$$\frac{7 a^{3/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{4 d} + \frac{7 a^2 \sqrt{\cos[c+dx]} \sin[c+dx]}{4 d \sqrt{a+a \cos[c+dx]}} + \frac{a^2 \cos[c+dx]^{3/2} \sin[c+dx]}{2 d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 397 leaves):

$$\frac{1}{8 d \sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}} a \sqrt{\cos[c+dx]} \sqrt{a(1+\cos[c+dx])} \\ \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(-7 i \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \cos\left[\frac{c}{2}\right]+i e^{i d x} \sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}\right)\right]\right) + \\ 7 i \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right]+i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}\right)\left(\cos\left[\frac{dx}{2}\right]+i \sin\left[\frac{dx}{2}\right]\right)\right] + \\ 7 \operatorname{Log}\left[2\left(e^{i d x} \cos\left[\frac{c}{2}\right]+i e^{i d x} \sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] + \\ 12 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx]+i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + 2 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx]+i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right]$$

- **Problem 208: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \cos[c+dx])^{3/2}}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 3, 75 leaves, 4 steps):

$$\frac{3 a^{3/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} + \frac{a^2 \sqrt{\cos[c+dx]} \sin[c+dx]}{d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 356 leaves):

$$\frac{1}{2 d \sqrt{(1+e^{2 i d x}) \operatorname{Cos}[c]+i(-1+e^{2 i d x}) \operatorname{Sin}[c]}} a \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a(1+\operatorname{Cos}[c+d x])} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \left(-3 i \operatorname{Cos}\left[\frac{d x}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \operatorname{Cos}\left[\frac{c}{2}\right]+i e^{i d x} \operatorname{Sin}\left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \operatorname{Cos}[c]+i(-1+e^{2 i d x}) \operatorname{Sin}[c]}\right)\right]\right) + 3 i \operatorname{ArcTanh}\left[\left(\operatorname{Cos}\left[\frac{c}{2}\right]+i \operatorname{Sin}\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i d x}) \operatorname{Cos}[c]+i(-1+e^{2 i d x}) \operatorname{Sin}[c]}\right) \left(\operatorname{Cos}\left[\frac{d x}{2}\right]+i \operatorname{Sin}\left[\frac{d x}{2}\right]\right)\right] + 3 \operatorname{Log}\left[2\left(e^{i d x} \operatorname{Cos}\left[\frac{c}{2}\right]+i e^{i d x} \operatorname{Sin}\left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \operatorname{Cos}[c]+i(-1+e^{2 i d x}) \operatorname{Sin}[c]}\right)\right] \operatorname{Sin}\left[\frac{d x}{2}\right] + 2 \sqrt{2} \sqrt{\operatorname{Cos}[c+d x]} (\operatorname{Cos}[d x]+i \operatorname{Sin}[d x]) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]$$

- **Problem 209: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \operatorname{Cos}[c+d x])^{3/2}}{\operatorname{Cos}[c+d x]^{3/2}} dx$$

Optimal (type 3, 76 leaves, 4 steps):

$$\frac{2 a^{3/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+d x]}{\sqrt{a+a \operatorname{Cos}[c+d x]}}\right]}{d} + \frac{2 a^2 \operatorname{Sin}[c+d x]}{d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+a \operatorname{Cos}[c+d x]}}$$

Result (type 3, 292 leaves):

$$\frac{1}{\sqrt{2} d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Cos}[c+d x]} (\operatorname{Cos}[d x]+i \operatorname{Sin}[d x])} a \sqrt{a(1+\operatorname{Cos}[c+d x])} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \left(\operatorname{Cos}\left[\frac{d x}{2}\right]+i \operatorname{Sin}\left[\frac{d x}{2}\right]\right) \left(i \operatorname{ArcTanh}\left[\left(\operatorname{Cos}\left[\frac{c}{2}\right]+i \operatorname{Sin}\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i d x}) \operatorname{Cos}[c]+i(-1+e^{2 i d x}) \operatorname{Sin}[c]}\right] \operatorname{Cos}[c+d x] - i \operatorname{Cos}[c+d x] \operatorname{Log}\left[2\left(e^{i d x} \operatorname{Cos}\left[\frac{c}{2}\right]+i e^{i d x} \operatorname{Sin}\left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \operatorname{Cos}[c]+i(-1+e^{2 i d x}) \operatorname{Sin}[c]}\right)\right] + 2 \sqrt{2} \left(\operatorname{Cos}\left[\frac{d x}{2}\right]-i \operatorname{Sin}\left[\frac{d x}{2}\right]\right) \sqrt{\operatorname{Cos}[c+d x]} (\operatorname{Cos}[d x]+i \operatorname{Sin}[d x]) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)$$

- **Problem 213: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+d x]^{3/2} (a+a \operatorname{Cos}[c+d x])^{5/2} dx$$

Optimal (type 3, 200 leaves, 6 steps):

$$\frac{163 a^{5/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{64 d} + \frac{163 a^3 \sqrt{\cos[c+dx]} \sin[c+dx]}{64 d \sqrt{a+a \cos[c+dx]}} +$$

$$\frac{163 a^3 \cos[c+dx]^{3/2} \sin[c+dx]}{96 d \sqrt{a+a \cos[c+dx]}} + \frac{17 a^3 \cos[c+dx]^{5/2} \sin[c+dx]}{24 d \sqrt{a+a \cos[c+dx]}} + \frac{a^2 \cos[c+dx]^{5/2} \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{4 d}$$

Result (type 3, 481 leaves) :

$$\frac{1}{384 d \sqrt{(1+e^{2ix}) \cos[c] + i(-1+e^{2ix}) \sin[c]}} a^2 \sqrt{\cos[c+dx]} \sqrt{a(1+\cos[c+dx])}$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left( -489 i \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2 \left( e^{ix} \cos\left[\frac{c}{2}\right] + i e^{ix} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2ix}) \cos[c] + i(-1+e^{2ix}) \sin[c]} \right) \right] \right) +$$

$$489 i \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2ix}) \cos[c] + i(-1+e^{2ix}) \sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) +$$

$$489 \operatorname{Log}\left[2 \left( e^{ix} \cos\left[\frac{c}{2}\right] + i e^{ix} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2ix}) \cos[c] + i(-1+e^{2ix}) \sin[c]} \right) \right] \sin\left[\frac{dx}{2}\right] +$$

$$800 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + 270 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] +$$

$$80 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{5}{2}(c+dx)\right] + 12 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{7}{2}(c+dx)\right]$$

■ **Problem 214: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]} (a+a \cos[c+dx])^{5/2} dx$$

Optimal (type 3, 160 leaves, 5 steps) :

$$\frac{25 a^{5/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{8 d} + \frac{25 a^3 \sqrt{\cos[c+dx]} \sin[c+dx]}{8 d \sqrt{a+a \cos[c+dx]}} +$$

$$\frac{13 a^3 \cos[c+dx]^{3/2} \sin[c+dx]}{12 d \sqrt{a+a \cos[c+dx]}} + \frac{a^2 \cos[c+dx]^{3/2} \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{3 d}$$

Result (type 3, 440 leaves) :

$$\frac{1}{48 d \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}} a^2 \sqrt{\cos [c + d x]} \sqrt{a (1 + \cos [c + d x])}$$

$$\operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] \left( -75 i \cos\left[\frac{d x}{2}\right] \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) \right] \right) +$$

$$75 i \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right] \left(\cos\left[\frac{d x}{2}\right] + i \sin\left[\frac{d x}{2}\right]\right) +$$

$$75 \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) \right] \sin\left[\frac{d x}{2}\right] +$$

$$124 \sqrt{2} \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin\left[\frac{1}{2}(c + d x)\right] +$$

$$30 \sqrt{2} \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin\left[\frac{3}{2}(c + d x)\right] + 4 \sqrt{2} \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin\left[\frac{5}{2}(c + d x)\right]$$

■ **Problem 215: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^{5/2}}{\sqrt{\cos [c + d x]}} dx$$

Optimal (type 3, 120 leaves, 4 steps):

$$\frac{19 a^{5/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}}\right]}{4 d} + \frac{9 a^3 \sqrt{\cos [c + d x]} \sin [c + d x]}{4 d \sqrt{a + a \cos [c + d x]}} + \frac{a^2 \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{2 d}$$

Result (type 3, 399 leaves):

$$\frac{1}{8 d \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}} a^2 \sqrt{\cos [c + d x]} \sqrt{a (1 + \cos [c + d x])}$$

$$\operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] \left( -19 i \cos\left[\frac{d x}{2}\right] \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) \right] \right) +$$

$$19 i \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right] \left(\cos\left[\frac{d x}{2}\right] + i \sin\left[\frac{d x}{2}\right]\right) +$$

$$19 \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) \right] \sin\left[\frac{d x}{2}\right] +$$

$$20 \sqrt{2} \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin\left[\frac{1}{2}(c + d x)\right] + 2 \sqrt{2} \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin\left[\frac{3}{2}(c + d x)\right]$$

■ **Problem 216: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^{5/2}}{\cos [c + d x]^{3/2}} dx$$

Optimal (type 3, 114 leaves, 4 steps) :

$$\frac{5 a^{5/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{d}-\frac{a^3 \sqrt{\cos [c+d x]} \sin [c+d x]}{d \sqrt{a+a \cos [c+d x]}}+\frac{2 a^2 \sqrt{a+a \cos [c+d x]} \sin [c+d x]}{d \sqrt{\cos [c+d x]}}$$

Result (type 3, 570 leaves) :

$$\frac{1}{4 \sqrt{2} d \sqrt{\cos [c+d x]} \sqrt{\cos [c+d x]} (\cos [d x]+i \sin [d x])} a^2 \sqrt{a(1+\cos [c+d x])} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \\ -\left(-5 i \cos \left[c+\frac{d x}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right)\right]\right)- \\ 5 i \cos \left[c+\frac{3 d x}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right)\right]+ \\ 10 i \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right] \cos [c+d x] \left(\cos \left[\frac{d x}{2}\right]+i \sin \left[\frac{d x}{2}\right]\right)- \\ 5 \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right)\right] \sin \left[c+\frac{d x}{2}\right]+ \\ 6 \sqrt{2} \sqrt{\cos [c+d x]} (\cos [d x]+i \sin [d x]) \sin \left[\frac{1}{2}(c+d x)\right]+2 \sqrt{2} \sqrt{\cos [c+d x]} (\cos [d x]+i \sin [d x]) \sin \left[\frac{3}{2}(c+d x)\right]+ \\ 5 \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right)\right] \sin \left[c+\frac{3 d x}{2}\right]$$

■ **Problem 217: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \cos [c+d x])^{5/2}}{\cos [c+d x]^{5/2}} d x$$

Optimal (type 3, 118 leaves, 4 steps) :

$$\frac{2 a^{5/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{d}+\frac{14 a^3 \sin [c+d x]}{3 d \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}+\frac{2 a^2 \sqrt{a+a \cos [c+d x]} \sin [c+d x]}{3 d \cos [c+d x]^{3/2}}$$

Result (type 3, 850 leaves) :



$$\begin{aligned}
& \frac{1}{4} (a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \\
& \left( \frac{1}{2} i \sin\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]}\right)\right] \right) \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]} \right) - \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]}\right] \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]} \right) \right) + \\
& \frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]}\right)\right] \right) \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]} \right) + \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]}\right] \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]} \right) \right) \right) + \\
& \sqrt{\cos[c + dx]} (a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \frac{4 \operatorname{Sec}[c + dx] \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{3 d} + \frac{\operatorname{Sec}[c + dx]^2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{6 d} \right)
\end{aligned}$$

■ **Problem 222: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \cos[e + fx]}}{\sqrt{\cos[e + fx]}} dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$\frac{2 \sqrt{a} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[e + fx]}{\sqrt{a + a \cos[e + fx]}}\right]}{f}$$

Result (type 3, 246 leaves):

$$\begin{aligned}
& \left( i e^{\frac{ifx}{2}} \sqrt{a (1 + \cos[e + fx])} \left( \operatorname{ArcTanh}\left[\left(\cos\left[\frac{e}{2}\right] + i \sin\left[\frac{e}{2}\right]\right) \sqrt{(1 + e^{2ifx}) \cos[e] + i (-1 + e^{2ifx}) \sin[e]}\right] - \right. \right. \\
& \quad \left. \left. \operatorname{Log}\left[2 \left( e^{ifx} \cos\left[\frac{e}{2}\right] + i e^{ifx} \sin\left[\frac{e}{2}\right] + \sqrt{(1 + e^{2ifx}) \cos[e] + i (-1 + e^{2ifx}) \sin[e]}\right)\right] \right) \operatorname{Sec}\left[\frac{1}{2} (e + fx)\right] \right. \\
& \quad \left. \sqrt{e^{-ifx} \left( (1 + e^{2ifx}) \cos[e] + i (-1 + e^{2ifx}) \sin[e] \right)} \right) \right) / \left( f \sqrt{2 (1 + e^{2ifx}) \cos[e] + 2 i (-1 + e^{2ifx}) \sin[e]} \right)
\end{aligned}$$

- **Problem 223: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a - a \cos[e + f x]}}{\sqrt{-\cos[e + f x]}} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{2 \sqrt{a} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[e + f x]}{\sqrt{a - a \cos[e + f x]}}\right]}{f}$$

Result (type 3, 214 leaves):

$$\begin{aligned} & \left( \sqrt{-\cos[e + f x]} \sqrt{a - a \cos[e + f x]} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right] \left( \operatorname{ArcTanh}\left[\left(\cos\left[\frac{e}{2}\right] + i \sin\left[\frac{e}{2}\right]\right) \sqrt{(1 + e^{2i f x}) \cos[e] + i(-1 + e^{2i f x}) \sin[e]}\right] + \right. \right. \\ & \quad \left. \left. \operatorname{Log}\left[2 \left(e^{i f x} \cos\left[\frac{e}{2}\right] + i e^{i f x} \sin\left[\frac{e}{2}\right] + \sqrt{(1 + e^{2i f x}) \cos[e] + i(-1 + e^{2i f x}) \sin[e]}\right)\right] \right) \right) \\ & \left( \cos\left[\frac{f x}{2}\right] + i \sin\left[\frac{f x}{2}\right] \right) \Big/ \left( \sqrt{2} f \sqrt{\cos[e + f x]} (\cos[f x] + i \sin[f x]) \right) \end{aligned}$$

- **Problem 224: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c + d x]^{5/2}}{\sqrt{a + a \cos[c + d x]}} dx$$

Optimal (type 3, 171 leaves, 7 steps):

$$\frac{7 \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c + d x]}{\sqrt{a + a \cos[c + d x]}}\right]}{4 \sqrt{a} d} - \frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c + d x]}{\sqrt{2} \sqrt{\cos[c + d x]} \sqrt{a + a \cos[c + d x]}}\right]}{\sqrt{a} d} - \frac{\sqrt{\cos[c + d x]} \sin[c + d x]}{4 d \sqrt{a + a \cos[c + d x]}} + \frac{\cos[c + d x]^{3/2} \sin[c + d x]}{2 d \sqrt{a + a \cos[c + d x]}}$$

Result (type 3, 251 leaves):

$$\begin{aligned} & \frac{1}{8 \sqrt{a} (1 + \cos[c + d x])} \\ & \cos\left[\frac{1}{2}(c + d x)\right] \left( 1 \Big/ \left( d \sqrt{1 + e^{2i(c + d x)}} \right) \sqrt{2} e^{\frac{1}{2}i(c + d x)} \sqrt{e^{-i(c + d x)} (1 + e^{2i(c + d x)})} \left( 7 d x - 7 i \operatorname{ArcSinh}\left[e^{i(c + d x)}\right] + 8 i \sqrt{2} \operatorname{Log}\left[1 + e^{i(c + d x)}\right] + \right. \right. \\ & \quad \left. \left. 7 i \operatorname{Log}\left[1 + \sqrt{1 + e^{2i(c + d x)}}\right] - 8 i \sqrt{2} \operatorname{Log}\left[1 - e^{i(c + d x)} + \sqrt{2} \sqrt{1 + e^{2i(c + d x)}}\right] \right) + \frac{4 \sqrt{\cos[c + d x]} \left(-2 \sin\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{3}{2}(c + d x)\right]\right)}{d} \right) \end{aligned}$$

- **Problem 225: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c + d x]^{3/2}}{\sqrt{a + a \cos[c + d x]}} dx$$

Optimal (type 3, 128 leaves, 6 steps):

$$-\frac{\text{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} + \frac{\sqrt{2} \text{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} + \frac{\sqrt{\cos[c+dx]} \sin[c+dx]}{d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 233 leaves):

$$\frac{1}{2 \sqrt{a} (1 + \cos[c + dx])}$$

$$\cos\left[\frac{1}{2} (c + dx)\right] \left( -1 / \left( d \sqrt{1 + e^{2i(c+dx)}} \right) i \sqrt{2} e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( -i dx - \text{ArcSinh}\left[e^{i(c+dx)}\right] + 2 \sqrt{2} \text{Log}\left[1 + e^{i(c+dx)}\right] + \right. \right.$$

$$\left. \left. \text{Log}\left[1 + \sqrt{1 + e^{2i(c+dx)}}\right] - 2 \sqrt{2} \text{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] \right) + \frac{4 \sqrt{\cos[c+dx]} \sin\left[\frac{1}{2} (c + dx)\right]}{d} \right)$$

- **Problem 226: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]}}{\sqrt{a+a \cos[c+dx]}} dx$$

Optimal (type 3, 95 leaves, 5 steps):

$$\frac{2 \text{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} - \frac{\sqrt{2} \text{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d}$$

Result (type 3, 197 leaves):

$$\left( (1 + e^{i(c+dx)}) \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \right.$$

$$\left. \left( dx - i \text{ArcSinh}\left[e^{i(c+dx)}\right] + i \sqrt{2} \text{Log}\left[1 + e^{i(c+dx)}\right] + i \text{Log}\left[1 + \sqrt{1 + e^{2i(c+dx)}}\right] - i \sqrt{2} \text{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] \right) \right) /$$

$$\left( \sqrt{2} d \sqrt{1 + e^{2i(c+dx)}} \sqrt{a (1 + \cos[c + dx])} \right)$$

- **Problem 227: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}} dx$$

Optimal (type 3, 56 leaves, 2 steps):

$$\frac{\sqrt{2} \text{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d}$$

Result (type 3, 136 leaves):

$$\frac{i \left( 1 + e^{i(c+dx)} \right) \sqrt{e^{-i(c+dx)} \left( 1 + e^{2i(c+dx)} \right) \left( \text{Log} \left[ 1 + e^{i(c+dx)} \right] - \text{Log} \left[ 1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}} \right] \right)}}{d \sqrt{1 + e^{2i(c+dx)}} \sqrt{a(1 + \text{Cos}[c+dx])}}$$

■ **Problem 228: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\text{Cos}[c+dx]^{3/2} \sqrt{a+a \text{Cos}[c+dx]}} dx$$

Optimal (type 3, 93 leaves, 4 steps):

$$-\frac{\sqrt{2} \text{ArcTan} \left[ \frac{\sqrt{a} \text{Sin}[c+dx]}{\sqrt{2} \sqrt{\text{Cos}[c+dx]} \sqrt{a+a \text{Cos}[c+dx]}} \right]}{\sqrt{a} d} + \frac{2 \text{Sin}[c+dx]}{d \sqrt{\text{Cos}[c+dx]} \sqrt{a+a \text{Cos}[c+dx]}}$$

Result (type 3, 146 leaves):

$$\left( \text{Cos} \left[ \frac{1}{2} (c+dx) \right] \left( i \sqrt{2} e^{-\frac{1}{2} i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} \left( \text{Log} \left[ 1 + e^{i(c+dx)} \right] - \text{Log} \left[ 1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}} \right] \right) + 4 \text{Sin} \left[ \frac{1}{2} (c+dx) \right] \right) \right) / \left( d \sqrt{\text{Cos}[c+dx]} \sqrt{a(1 + \text{Cos}[c+dx])} \right)$$

■ **Problem 229: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\text{Cos}[c+dx]^{5/2} \sqrt{a+a \text{Cos}[c+dx]}} dx$$

Optimal (type 3, 131 leaves, 5 steps):

$$\frac{\sqrt{2} \text{ArcTan} \left[ \frac{\sqrt{a} \text{Sin}[c+dx]}{\sqrt{2} \sqrt{\text{Cos}[c+dx]} \sqrt{a+a \text{Cos}[c+dx]}} \right]}{\sqrt{a} d} + \frac{2 \text{Sin}[c+dx]}{3 d \text{Cos}[c+dx]^{3/2} \sqrt{a+a \text{Cos}[c+dx]}} - \frac{2 \text{Sin}[c+dx]}{3 d \sqrt{\text{Cos}[c+dx]} \sqrt{a+a \text{Cos}[c+dx]}}$$

Result (type 3, 177 leaves):

$$\frac{1}{\sqrt{a(1 + \text{Cos}[c+dx])}} \text{Cos} \left[ \frac{1}{2} (c+dx) \right] \left( -\frac{2 i e^{\frac{1}{2} i(c+dx)} \sqrt{e^{-i(c+dx)} \left( 1 + e^{2i(c+dx)} \right) \left( \text{Log} \left[ 1 + e^{i(c+dx)} \right] - \text{Log} \left[ 1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}} \right] \right)}}{d \sqrt{1 + e^{2i(c+dx)}}} + \frac{8 \text{Sin} \left[ \frac{1}{2} (c+dx) \right]^3}{3 d \text{Cos}[c+dx]^{3/2}} \right)$$

■ **Problem 230: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\text{Cos}[c+dx]^{7/2} \sqrt{a+a \text{Cos}[c+dx]}} dx$$

Optimal (type 3, 169 leaves, 6 steps):

$$\begin{aligned}
& - \frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} + \frac{2 \sin[c+dx]}{5 d \cos[c+dx]^{5/2} \sqrt{a+a \cos[c+dx]}} - \\
& \frac{2 \sin[c+dx]}{15 d \cos[c+dx]^{3/2} \sqrt{a+a \cos[c+dx]}} + \frac{26 \sin[c+dx]}{15 d \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}
\end{aligned}$$

Result (type 3, 195 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{a} (1 + \cos[c+dx])} \cos\left[\frac{1}{2} (c+dx)\right] \left( \frac{2 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1 + e^{2 i (c+dx)})} \left( \log[1 + e^{i (c+dx)}] - \log[1 - e^{i (c+dx)}] + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}} \right)}{d \sqrt{1 + e^{2 i (c+dx)}}} + \right. \\
& \left. \frac{4 (3 - \cos[c+dx] + 13 \cos[c+dx]^2) \sin\left[\frac{1}{2} (c+dx)\right]}{15 d \cos[c+dx]^{5/2}} \right)
\end{aligned}$$

■ **Problem 231: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^{5/2}}{\sqrt{1 + \cos[c+dx]}} dx$$

Optimal (type 3, 126 leaves, 7 steps):

$$\begin{aligned}
& - \frac{\sqrt{2} \operatorname{ArcSin}\left[\frac{\sin[c+dx]}{1 + \cos[c+dx]}\right]}{d} + \frac{7 \operatorname{ArcSin}\left[\frac{\sin[c+dx]}{\sqrt{1 + \cos[c+dx]}}\right]}{4 d} - \frac{\sqrt{\cos[c+dx]} \sin[c+dx]}{4 d \sqrt{1 + \cos[c+dx]}} + \frac{\cos[c+dx]^{3/2} \sin[c+dx]}{2 d \sqrt{1 + \cos[c+dx]}}
\end{aligned}$$

Result (type 3, 249 leaves):

$$\begin{aligned}
& \frac{1}{8 \sqrt{1 + \cos[c+dx]}} \\
& \cos\left[\frac{1}{2} (c+dx)\right] \left( 1 / \left( d \sqrt{1 + e^{2 i (c+dx)}} \right) \sqrt{2} e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1 + e^{2 i (c+dx)})} \left( 7 dx - 7 i \operatorname{ArcSinh}\left[e^{i (c+dx)}\right] + 8 i \sqrt{2} \log[1 + e^{i (c+dx)}] + \right. \right. \\
& \left. \left. 7 i \log\left[1 + \sqrt{1 + e^{2 i (c+dx)}}\right] - 8 i \sqrt{2} \log\left[1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}\right] \right) + \frac{4 \sqrt{\cos[c+dx]} (-2 \sin\left[\frac{1}{2} (c+dx)\right] + \sin\left[\frac{3}{2} (c+dx)\right])}{d} \right)
\end{aligned}$$

■ **Problem 232: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{3/2}}{\sqrt{1 + \cos[c+dx]}} dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$\frac{\sqrt{2} \operatorname{ArcSin}\left[\frac{\sin[c+dx]}{1+\cos[c+dx]}\right]}{d} - \frac{\operatorname{ArcSin}\left[\frac{\sin[c+dx]}{\sqrt{1+\cos[c+dx]}}\right]}{d} + \frac{\sqrt{\cos[c+dx]} \sin[c+dx]}{d \sqrt{1+\cos[c+dx]}}$$

Result (type 3, 231 leaves):

$$\frac{1}{2 \sqrt{1+\cos[c+dx]}} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \left( -1 / \left( d \sqrt{1+e^{2i(c+dx)}} \right) i \sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})} \left( -i dx - \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] + 2 \sqrt{2} \operatorname{Log}\left[1+e^{i(c+dx)}\right] + \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] - 2 \sqrt{2} \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] \right) + \frac{4 \sqrt{\cos[c+dx]} \sin\left[\frac{1}{2}(c+dx)\right]}{d} \right)$$

- **Problem 233: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]}}{\sqrt{1+\cos[c+dx]}} dx$$

Optimal (type 3, 54 leaves, 5 steps):

$$-\frac{\sqrt{2} \operatorname{ArcSin}\left[\frac{\sin[c+dx]}{1+\cos[c+dx]}\right]}{d} + \frac{2 \operatorname{ArcSin}\left[\frac{\sin[c+dx]}{\sqrt{1+\cos[c+dx]}}\right]}{d}$$

Result (type 3, 170 leaves):

$$\frac{1}{d \sqrt{1+e^{2i(c+dx)}}} (1+e^{i(c+dx)}) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \left( dx - i \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] + i \sqrt{2} \operatorname{Log}\left[1+e^{i(c+dx)}\right] + i \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] - i \sqrt{2} \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] \right)$$

- **Problem 234: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\cos[c+dx]} \sqrt{1+\cos[c+dx]}} dx$$

Optimal (type 3, 27 leaves, 2 steps):

$$\frac{\sqrt{2} \operatorname{ArcSin}\left[\frac{\sin[c+dx]}{1+\cos[c+dx]}\right]}{d}$$

Result (type 3, 134 leaves):

$$\frac{i \left( 1 + e^{i(c+dx)} \right) \sqrt{e^{-i(c+dx)} \left( 1 + e^{2i(c+dx)} \right)} \left( \text{Log} \left[ 1 + e^{i(c+dx)} \right] - \text{Log} \left[ 1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}} \right] \right)}{d \sqrt{1 + e^{2i(c+dx)}} \sqrt{1 + \text{Cos}[c + dx]}}$$

- **Problem 235: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\text{Cos}[c + dx]^{3/2} \sqrt{1 + \text{Cos}[c + dx]}} dx$$

Optimal (type 3, 62 leaves, 3 steps):

$$-\frac{\sqrt{2} \text{ArcSin} \left[ \frac{\text{Sin}[c+dx]}{1+\text{Cos}[c+dx]} \right]}{d} + \frac{2 \text{Sin}[c + dx]}{d \sqrt{\text{Cos}[c + dx]} \sqrt{1 + \text{Cos}[c + dx]}}$$

Result (type 3, 144 leaves):

$$\left( \text{Cos} \left[ \frac{1}{2} (c + dx) \right] \left( i \sqrt{2} e^{-\frac{1}{2} i (c+dx)} \sqrt{1 + e^{2i(c+dx)}} \left( \text{Log} \left[ 1 + e^{i(c+dx)} \right] - \text{Log} \left[ 1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}} \right] \right) + 4 \text{Sin} \left[ \frac{1}{2} (c + dx) \right] \right) \right) / \left( d \sqrt{\text{Cos}[c + dx]} \sqrt{1 + \text{Cos}[c + dx]} \right)$$

- **Problem 236: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\text{Cos}[c + dx]^{5/2} \sqrt{1 + \text{Cos}[c + dx]}} dx$$

Optimal (type 3, 98 leaves, 5 steps):

$$\frac{\sqrt{2} \text{ArcSin} \left[ \frac{\text{Sin}[c+dx]}{1+\text{Cos}[c+dx]} \right]}{d} + \frac{2 \text{Sin}[c + dx]}{3 d \text{Cos}[c + dx]^{3/2} \sqrt{1 + \text{Cos}[c + dx]}} - \frac{2 \text{Sin}[c + dx]}{3 d \sqrt{\text{Cos}[c + dx]} \sqrt{1 + \text{Cos}[c + dx]}}$$

Result (type 3, 175 leaves):

$$\frac{1}{\sqrt{1 + \text{Cos}[c + dx]}} \text{Cos} \left[ \frac{1}{2} (c + dx) \right] \left( -\frac{2 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} \left( 1 + e^{2i(c+dx)} \right)} \left( \text{Log} \left[ 1 + e^{i(c+dx)} \right] - \text{Log} \left[ 1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}} \right] \right)}{d \sqrt{1 + e^{2i(c+dx)}}} + \frac{8 \text{Sin} \left[ \frac{1}{2} (c + dx) \right]^3}{3 d \text{Cos}[c + dx]^{3/2}} \right)$$

- **Problem 237: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\text{Cos}[c + dx]^{7/2} \sqrt{1 + \text{Cos}[c + dx]}} dx$$

Optimal (type 3, 134 leaves, 6 steps):

$$-\frac{\sqrt{2} \operatorname{ArcSin}\left[\frac{\sin[c+dx]}{1+\cos[c+dx]}\right]}{d} + \frac{2 \sin[c+dx]}{5d \cos[c+dx]^{5/2} \sqrt{1+\cos[c+dx]}} - \frac{2 \sin[c+dx]}{15d \cos[c+dx]^{3/2} \sqrt{1+\cos[c+dx]}} + \frac{26 \sin[c+dx]}{15d \sqrt{\cos[c+dx]} \sqrt{1+\cos[c+dx]}}$$

Result (type 3, 193 leaves):

$$\frac{1}{\sqrt{1+\cos[c+dx]}} \cos\left[\frac{1}{2}(c+dx)\right] \left( \frac{2i e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})} \left( \operatorname{Log}[1+e^{i(c+dx)}] - \operatorname{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right)}{d \sqrt{1+e^{2i(c+dx)}}} + \frac{4(3 - \cos[c+dx] + 13 \cos[c+dx]^2) \sin\left[\frac{1}{2}(c+dx)\right]}{15d \cos[c+dx]^{5/2}} \right)$$

■ **Problem 238: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^{5/2}}{(a+a\cos[c+dx])^{3/2}} dx$$

Optimal (type 3, 174 leaves, 7 steps):

$$-\frac{3 \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{a^{3/2}d} + \frac{9 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right]}{2\sqrt{2} a^{3/2}d} - \frac{\cos[c+dx]^{3/2} \sin[c+dx]}{2d(a+a\cos[c+dx])^{3/2}} + \frac{3\sqrt{\cos[c+dx]} \sin[c+dx]}{2ad\sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 262 leaves):

$$\frac{1}{2(a(1+\cos[c+dx]))^{3/2}} \cos\left[\frac{1}{2}(c+dx)\right]^3 \left( -1 / \left( d \sqrt{1+e^{2i(c+dx)}} \right) 3i \sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})} \right. \\ \left. \left( -2i dx - 2 \operatorname{ArcSinh}[e^{i(c+dx)}] + 3\sqrt{2} \operatorname{Log}[1+e^{i(c+dx)}] + 2 \operatorname{Log}[1+\sqrt{1+e^{2i(c+dx)}}] - 3\sqrt{2} \operatorname{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) + \\ \frac{2\sqrt{\cos[c+dx]} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left( 2 \sin\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{3}{2}(c+dx)\right] \right)}{d}$$

■ **Problem 239: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{3/2}}{(a+a\cos[c+dx])^{3/2}} dx$$

Optimal (type 3, 134 leaves, 6 steps):

$$\frac{2 \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{a^{3/2}d} - \frac{5 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right]}{2\sqrt{2} a^{3/2}d} - \frac{\sqrt{\cos[c+dx]} \sin[c+dx]}{2d(a+a\cos[c+dx])^{3/2}}$$

Result (type 3, 312 leaves):



$$\left( e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1 + e^{2 i (c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \right. \\ \left. \left( 4 dx - 4 i \operatorname{ArcSinh}\left[e^{i (c+dx)}\right] + 5 i \sqrt{2} \operatorname{Log}\left[1 + e^{i (c+dx)}\right] + 4 i \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c+dx)}}\right] - 5 i \sqrt{2} \operatorname{Log}\left[1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}\right]\right) \right) / \\ \left( \sqrt{2} d \sqrt{1 + e^{2 i (c+dx)}} (a (1 + \cos[c + dx]))^{3/2} \right) + \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sqrt{\cos[c + dx]} \left( -\frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{d} - \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{d} \right)}{(a (1 + \cos[c + dx]))^{3/2}}$$

■ **Problem 240: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c + dx]}}{(a + a \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 97 leaves, 4 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} + \frac{\sqrt{\cos[c + dx]} \sin[c + dx]}{2 d (a + a \cos[c + dx])^{3/2}}$$

Result (type 3, 248 leaves):

$$- \left( i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1 + e^{2 i (c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left( \operatorname{Log}\left[1 + e^{i (c+dx)}\right] - \operatorname{Log}\left[1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}\right] \right) \right) / \\ \left( d \sqrt{1 + e^{2 i (c+dx)}} (a (1 + \cos[c + dx]))^{3/2} \right) + \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sqrt{\cos[c + dx]} \left( \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{d} + \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{d} \right)}{(a (1 + \cos[c + dx]))^{3/2}}$$

■ **Problem 241: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\cos[c + dx]} (a + a \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 97 leaves, 4 steps):

$$\frac{3 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{\sqrt{\cos[c + dx]} \sin[c + dx]}{2 d (a + a \cos[c + dx])^{3/2}}$$

Result (type 3, 250 leaves):

$$- \left( 3 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1 + e^{2 i (c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left( \operatorname{Log}\left[1 + e^{i (c+dx)}\right] - \operatorname{Log}\left[1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}\right] \right) \right) / \\ \left( d \sqrt{1 + e^{2 i (c+dx)}} (a (1 + \cos[c + dx]))^{3/2} \right) + \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sqrt{\cos[c + dx]} \left( -\frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{d} - \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{d} \right)}{(a (1 + \cos[c + dx]))^{3/2}}$$

■ **Problem 242: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\cos[c+dx]^{3/2} (a+a\cos[c+dx])^{3/2}} dx$$

Optimal (type 3, 137 leaves, 5 steps):

$$-\frac{7 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{\sin[c+dx]}{2 d \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^{3/2}} + \frac{5 \sin[c+dx]}{2 a d \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 272 leaves):

$$\left( 7 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left( \operatorname{Log}[1+e^{i(c+dx)}] - \operatorname{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) /$$

$$\left( d \sqrt{1+e^{2i(c+dx)}} (a(1+\cos[c+dx]))^{3/2} \right) + \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sqrt{\cos[c+dx]} \left( \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sin}\left[\frac{dx}{2}\right]}{d} + \frac{8 \operatorname{Sec}[c+dx] \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]}{d} + \frac{\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Tan}\left[\frac{c}{2}\right]}{d} \right)}{(a(1+\cos[c+dx]))^{3/2}}$$

■ **Problem 243: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\cos[c+dx]^{5/2} (a+a\cos[c+dx])^{3/2}} dx$$

Optimal (type 3, 177 leaves, 6 steps):

$$\frac{11 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{\sin[c+dx]}{2 d \cos[c+dx]^{3/2} (a+a\cos[c+dx])^{3/2}} +$$

$$\frac{7 \sin[c+dx]}{6 a d \cos[c+dx]^{3/2} \sqrt{a+a\cos[c+dx]}} - \frac{19 \sin[c+dx]}{6 a d \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 304 leaves):

$$-\left( 11 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left( \operatorname{Log}[1+e^{i(c+dx)}] - \operatorname{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) /$$

$$\left( d \sqrt{1+e^{2i(c+dx)}} (a(1+\cos[c+dx]))^{3/2} \right) + \frac{1}{(a(1+\cos[c+dx]))^{3/2}}$$

$$\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sqrt{\cos[c+dx]} \left( -\frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sin}\left[\frac{dx}{2}\right]}{d} - \frac{32 \operatorname{Sec}[c+dx] \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]}{3 d} + \frac{8 \operatorname{Sec}[c+dx]^2 \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]}{3 d} - \frac{\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Tan}\left[\frac{c}{2}\right]}{d} \right)$$

■ **Problem 244: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^{7/2}}{(a+a\cos[c+dx])^{5/2}} dx$$

Optimal (type 3, 214 leaves, 8 steps) :

$$-\frac{5 \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{5/2} d} + \frac{115 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} -$$

$$\frac{\cos[c+dx]^{5/2} \sin[c+dx]}{4 d (a+a \cos[c+dx])^{5/2}} - \frac{15 \cos[c+dx]^{3/2} \sin[c+dx]}{16 a d (a+a \cos[c+dx])^{3/2}} + \frac{35 \sqrt{\cos[c+dx]} \sin[c+dx]}{16 a^2 d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 414 leaves) :

$$-\left(5 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1+e^{2 i (c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5\right.$$

$$\left. \left(-16 i dx - 16 \operatorname{ArcSinh}\left[e^{i (c+dx)}\right] + 23 \sqrt{2} \operatorname{Log}\left[1+e^{i (c+dx)}\right] + 16 \operatorname{Log}\left[1+\sqrt{1+e^{2 i (c+dx)}}\right] - 23 \sqrt{2} \operatorname{Log}\left[1-e^{i (c+dx)} + \sqrt{2} \sqrt{1+e^{2 i (c+dx)}}\right]\right) \right) /$$

$$\left(4 \sqrt{2} d \sqrt{1+e^{2 i (c+dx)}} (a(1+\cos[c+dx]))^{5/2}\right) + \frac{1}{(a(1+\cos[c+dx]))^{5/2}}$$

$$\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\cos[c+dx]} \left(\frac{8 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{d} + \frac{8 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{d} + \frac{23 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{4 d} - \right.$$

$$\left. \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{2 d} + \frac{23 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{4 d} - \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{2 d}\right)$$

- **Problem 245: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{5/2}}{(a+a \cos[c+dx])^{5/2}} dx$$

Optimal (type 3, 174 leaves, 7 steps) :

$$\frac{2 \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{5/2} d} - \frac{43 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{\cos[c+dx]^{3/2} \sin[c+dx]}{4 d (a+a \cos[c+dx])^{5/2}} - \frac{11 \sqrt{\cos[c+dx]} \sin[c+dx]}{16 a d (a+a \cos[c+dx])^{3/2}}$$

Result (type 3, 382 leaves) :

$$\left( e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1 + e^{2 i (c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \right. \\ \left. \left( 32 dx - 32 i \operatorname{ArcSinh}\left[e^{i (c+dx)}\right] + 43 i \sqrt{2} \operatorname{Log}\left[1 + e^{i (c+dx)}\right] + 32 i \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c+dx)}}\right] - 43 i \sqrt{2} \operatorname{Log}\left[1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}\right]\right) \right) / \\ \left( 4 \sqrt{2} d \sqrt{1 + e^{2 i (c+dx)}} (a (1 + \cos[c + dx]))^{5/2} \right) + \frac{1}{(a (1 + \cos[c + dx]))^{5/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\cos[c + dx]} \\ \left( -\frac{15 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sin}\left[\frac{dx}{2}\right]}{4 d} + \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sin}\left[\frac{dx}{2}\right]}{2 d} - \frac{15 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Tan}\left[\frac{c}{2}\right]}{4 d} + \frac{\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Tan}\left[\frac{c}{2}\right]}{2 d} \right)$$

- **Problem 246: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{3/2}}{(a + a \cos[c + dx])^{5/2}} dx$$

Optimal (type 3, 137 leaves, 5 steps):

$$\frac{3 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{\sqrt{\cos[c + dx]} \operatorname{Sin}[c + dx]}{4 d (a + a \cos[c + dx])^{5/2}} + \frac{7 \sqrt{\cos[c + dx]} \operatorname{Sin}[c + dx]}{16 a d (a + a \cos[c + dx])^{3/2}}$$

Result (type 3, 319 leaves):

$$-\left( 3 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1 + e^{2 i (c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \operatorname{Log}\left[1 + e^{i (c+dx)}\right] - \operatorname{Log}\left[1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}\right] \right) \right) / \\ \left( 4 d \sqrt{1 + e^{2 i (c+dx)}} (a (1 + \cos[c + dx]))^{5/2} \right) + \frac{1}{(a (1 + \cos[c + dx]))^{5/2}} \\ \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\cos[c + dx]} \left( \frac{7 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sin}\left[\frac{dx}{2}\right]}{4 d} - \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sin}\left[\frac{dx}{2}\right]}{2 d} + \frac{7 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Tan}\left[\frac{c}{2}\right]}{4 d} - \frac{\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Tan}\left[\frac{c}{2}\right]}{2 d} \right)$$

- **Problem 247: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c + dx]}}{(a + a \cos[c + dx])^{5/2}} dx$$

Optimal (type 3, 137 leaves, 5 steps):

$$\frac{5 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} + \frac{\sqrt{\cos[c + dx]} \operatorname{Sin}[c + dx]}{4 d (a + a \cos[c + dx])^{5/2}} + \frac{\sqrt{\cos[c + dx]} \operatorname{Sin}[c + dx]}{16 a d (a + a \cos[c + dx])^{3/2}}$$

Result (type 3, 319 leaves):

$$- \left( 5 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1 + e^{2 i (c+dx)})} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \left( \log [1 + e^{i (c+dx)}] - \log [1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}] \right) \right) /$$

$$\left( 4 d \sqrt{1 + e^{2 i (c+dx)}} (a (1 + \cos [c + dx]))^{5/2} \right) + \frac{1}{(a (1 + \cos [c + dx]))^{5/2}}$$

$$\cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sqrt{\cos [c + dx]} \left( \frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \sin \left[ \frac{dx}{2} \right]}{4 d} + \frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \sin \left[ \frac{dx}{2} \right]}{2 d} + \frac{\sec \left[ \frac{c}{2} + \frac{dx}{2} \right] \tan \left[ \frac{c}{2} \right]}{4 d} + \frac{\sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \tan \left[ \frac{c}{2} \right]}{2 d} \right)$$

- **Problem 248: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\cos [c + dx]} (a + a \cos [c + dx])^{5/2}} dx$$

Optimal (type 3, 137 leaves, 5 steps):

$$\frac{19 \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin [c+dx]}{\sqrt{2} \sqrt{\cos [c+dx]} \sqrt{a+a \cos [c+dx]}} \right]}{16 \sqrt{2} a^{5/2} d} - \frac{\sqrt{\cos [c + dx]} \sin [c + dx]}{4 d (a + a \cos [c + dx])^{5/2}} - \frac{9 \sqrt{\cos [c + dx]} \sin [c + dx]}{16 a d (a + a \cos [c + dx])^{3/2}}$$

Result (type 3, 319 leaves):

$$- \left( 19 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1 + e^{2 i (c+dx)})} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \left( \log [1 + e^{i (c+dx)}] - \log [1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}] \right) \right) /$$

$$\left( 4 d \sqrt{1 + e^{2 i (c+dx)}} (a (1 + \cos [c + dx]))^{5/2} \right) + \frac{1}{(a (1 + \cos [c + dx]))^{5/2}}$$

$$\cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sqrt{\cos [c + dx]} \left( - \frac{9 \sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \sin \left[ \frac{dx}{2} \right]}{4 d} - \frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \sin \left[ \frac{dx}{2} \right]}{2 d} - \frac{9 \sec \left[ \frac{c}{2} + \frac{dx}{2} \right] \tan \left[ \frac{c}{2} \right]}{4 d} - \frac{\sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \tan \left[ \frac{c}{2} \right]}{2 d} \right)$$

- **Problem 249: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\cos [c + dx]^{3/2} (a + a \cos [c + dx])^{5/2}} dx$$

Optimal (type 3, 177 leaves, 6 steps):

$$- \frac{75 \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin [c+dx]}{\sqrt{2} \sqrt{\cos [c+dx]} \sqrt{a+a \cos [c+dx]}} \right]}{16 \sqrt{2} a^{5/2} d} - \frac{\sin [c + dx]}{4 d \sqrt{\cos [c + dx]} (a + a \cos [c + dx])^{5/2}} -$$

$$\frac{13 \sin [c + dx]}{16 a d \sqrt{\cos [c + dx]} (a + a \cos [c + dx])^{3/2}} + \frac{49 \sin [c + dx]}{16 a^2 d \sqrt{\cos [c + dx]} \sqrt{a + a \cos [c + dx]}}$$

Result (type 3, 343 leaves):

$$\left( 75 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1 + e^{2 i (c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \log[1 + e^{i (c+dx)}] - \log[1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}] \right) \right) /$$

$$\left( 4 d \sqrt{1 + e^{2 i (c+dx)}} (a (1 + \cos[c + dx]))^{5/2} \right) + \frac{1}{(a (1 + \cos[c + dx]))^{5/2}}$$

$$\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\cos[c + dx]} \left( \frac{17 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{4 d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{2 d} + \right.$$

$$\left. \frac{16 \sec[c + dx] \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{d} + \frac{17 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{4 d} + \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{2 d} \right)$$

■ **Problem 250: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\cos[c + dx]^{5/2} (a + a \cos[c + dx])^{5/2}} dx$$

Optimal (type 3, 217 leaves, 7 steps):

$$\frac{163 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{\sin[c + dx]}{4 d \cos[c + dx]^{3/2} (a + a \cos[c + dx])^{5/2}} -$$

$$\frac{17 \sin[c + dx]}{16 a d \cos[c + dx]^{3/2} (a + a \cos[c + dx])^{3/2}} + \frac{95 \sin[c + dx]}{48 a^2 d \cos[c + dx]^{3/2} \sqrt{a + a \cos[c + dx]}} - \frac{299 \sin[c + dx]}{48 a^2 d \sqrt{\cos[c + dx]} \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 373 leaves):

$$-\left( 163 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1 + e^{2 i (c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \log[1 + e^{i (c+dx)}] - \log[1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}] \right) \right) /$$

$$\left( 4 d \sqrt{1 + e^{2 i (c+dx)}} (a (1 + \cos[c + dx]))^{5/2} \right) + \frac{1}{(a (1 + \cos[c + dx]))^{5/2}}$$

$$\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\cos[c + dx]} \left( -\frac{25 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{4 d} - \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{2 d} - \right.$$

$$\left. \frac{112 \sec[c + dx] \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{3 d} + \frac{16 \sec[c + dx]^2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{3 d} - \frac{25 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{4 d} - \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{2 d} \right)$$

■ **Problem 251: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c + dx]^{9/2}}{(a + a \cos[c + dx])^{7/2}} dx$$

Optimal (type 3, 254 leaves, 9 steps):

$$\begin{aligned}
& - \frac{7 \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{7/2} d} + \frac{637 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{64 \sqrt{2} a^{7/2} d} - \frac{\cos[c+dx]^{7/2} \sin[c+dx]}{6 d (a+a \cos[c+dx])^{7/2}} \\
& - \frac{7 \cos[c+dx]^{5/2} \sin[c+dx]}{16 a d (a+a \cos[c+dx])^{5/2}} - \frac{259 \cos[c+dx]^{3/2} \sin[c+dx]}{192 a^2 d (a+a \cos[c+dx])^{3/2}} + \frac{189 \sqrt{\cos[c+dx]} \sin[c+dx]}{64 a^3 d \sqrt{a+a \cos[c+dx]}}
\end{aligned}$$

Result (type 3, 477 leaves):

$$\begin{aligned}
& - \left( 7 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1+e^{2 i (c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \right. \\
& \quad \left. \left( -64 i dx - 64 \operatorname{ArcSinh}\left[e^{i (c+dx)}\right] + 91 \sqrt{2} \operatorname{Log}\left[1+e^{i (c+dx)}\right] + 64 \operatorname{Log}\left[1+\sqrt{1+e^{2 i (c+dx)}}\right] - 91 \sqrt{2} \operatorname{Log}\left[1-e^{i (c+dx)}+\sqrt{2} \sqrt{1+e^{2 i (c+dx)}}\right]\right) \right) / \\
& \quad \left( 8 \sqrt{2} d \sqrt{1+e^{2 i (c+dx)}} (a (1+\cos[c+dx]))^{7/2} \right) + \frac{1}{(a (1+\cos[c+dx]))^{7/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\cos[c+dx]} \\
& \quad \left( \frac{16 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{d} + \frac{16 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{d} + \frac{523 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{24 d} - \frac{15 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{4 d} \right. \\
& \quad \left. + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{dx}{2}\right]}{3 d} + \frac{523 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{24 d} - \frac{15 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{4 d} + \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \tan\left[\frac{c}{2}\right]}{3 d} \right)
\end{aligned}$$

■ **Problem 252: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{7/2}}{(a+a \cos[c+dx])^{7/2}} dx$$

Optimal (type 3, 214 leaves, 8 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{7/2} d} - \frac{177 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{64 \sqrt{2} a^{7/2} d} \\
& - \frac{\cos[c+dx]^{5/2} \sin[c+dx]}{6 d (a+a \cos[c+dx])^{7/2}} - \frac{17 \cos[c+dx]^{3/2} \sin[c+dx]}{48 a d (a+a \cos[c+dx])^{5/2}} - \frac{49 \sqrt{\cos[c+dx]} \sin[c+dx]}{64 a^2 d (a+a \cos[c+dx])^{3/2}}
\end{aligned}$$

Result (type 3, 445 leaves):

$$\left( e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left( 128 dx - 128 i \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] + 177 i \sqrt{2} \operatorname{Log}\left[1 + e^{i(c+dx)}\right] + 128 i \operatorname{Log}\left[1 + \sqrt{1 + e^{2i(c+dx)}}\right] - 177 i \sqrt{2} \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] \right) \right) / \left( 8 \sqrt{2} d \sqrt{1 + e^{2i(c+dx)}} (a(1 + \cos[c+dx]))^{7/2} \right) + \frac{1}{(a(1 + \cos[c+dx]))^{7/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\cos[c+dx]} \left( -\frac{247 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sin}\left[\frac{dx}{2}\right]}{24 d} + \frac{11 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sin}\left[\frac{dx}{2}\right]}{4 d} - \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sin}\left[\frac{dx}{2}\right]}{3 d} - \frac{247 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Tan}\left[\frac{c}{2}\right]}{24 d} + \frac{11 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Tan}\left[\frac{c}{2}\right]}{4 d} - \frac{\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right)$$

■ **Problem 253: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{5/2}}{(a+a\cos[c+dx])^{7/2}} dx$$

Optimal (type 3, 177 leaves, 6 steps):

$$\frac{5 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right]}{64 \sqrt{2} a^{7/2} d} - \frac{\cos[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{6 d (a+a\cos[c+dx])^{7/2}} - \frac{13 \sqrt{\cos[c+dx]} \operatorname{Sin}[c+dx]}{48 a d (a+a\cos[c+dx])^{5/2}} + \frac{67 \sqrt{\cos[c+dx]} \operatorname{Sin}[c+dx]}{192 a^2 d (a+a\cos[c+dx])^{3/2}}$$

Result (type 3, 382 leaves):

$$-\left( 5 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left( \operatorname{Log}\left[1 + e^{i(c+dx)}\right] - \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] \right) \right) / \left( 8 d \sqrt{1 + e^{2i(c+dx)}} (a(1 + \cos[c+dx]))^{7/2} \right) + \frac{1}{(a(1 + \cos[c+dx]))^{7/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\cos[c+dx]} \left( \frac{67 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sin}\left[\frac{dx}{2}\right]}{24 d} - \frac{7 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sin}\left[\frac{dx}{2}\right]}{4 d} + \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sin}\left[\frac{dx}{2}\right]}{3 d} + \frac{67 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Tan}\left[\frac{c}{2}\right]}{24 d} - \frac{7 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Tan}\left[\frac{c}{2}\right]}{4 d} + \frac{\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right)$$

■ **Problem 254: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{3/2}}{(a+a\cos[c+dx])^{7/2}} dx$$

Optimal (type 3, 177 leaves, 6 steps):

$$\frac{7 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right]}{64 \sqrt{2} a^{7/2} d} - \frac{\sqrt{\cos[c+dx]} \operatorname{Sin}[c+dx]}{6 d (a+a\cos[c+dx])^{7/2}} + \frac{3 \sqrt{\cos[c+dx]} \operatorname{Sin}[c+dx]}{16 a d (a+a\cos[c+dx])^{5/2}} + \frac{17 \sqrt{\cos[c+dx]} \operatorname{Sin}[c+dx]}{192 a^2 d (a+a\cos[c+dx])^{3/2}}$$



Result (type 3, 382 leaves) :

$$\begin{aligned}
 & - \left( 7 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1 + e^{2 i (c+dx)})} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^7 \left( \log [1 + e^{i (c+dx)}] - \log [1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}] \right) \right) / \\
 & \left( 8 d \sqrt{1 + e^{2 i (c+dx)}} (a (1 + \cos [c + dx]))^{7/2} \right) + \frac{1}{(a (1 + \cos [c + dx]))^{7/2}} \\
 & \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^7 \sqrt{\cos [c + dx]} \left( \frac{17 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{24 d} + \frac{3 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{4 d} - \right. \\
 & \left. \frac{\operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{3 d} + \frac{17 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Tan} \left[ \frac{c}{2} \right]}{24 d} + \frac{3 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \operatorname{Tan} \left[ \frac{c}{2} \right]}{4 d} - \frac{\operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \operatorname{Tan} \left[ \frac{c}{2} \right]}{3 d} \right)
 \end{aligned}$$

- **Problem 255: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos [c + dx]}}{(a + a \cos [c + dx])^{7/2}} dx$$

Optimal (type 3, 177 leaves, 6 steps) :

$$\frac{13 \operatorname{ArcTan} \left[ \frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{\cos [c+dx]} \sqrt{a+a \cos [c+dx]}} \right]}{64 \sqrt{2} a^{7/2} d} + \frac{\sqrt{\cos [c + dx]} \operatorname{Sin}[c + dx]}{6 d (a + a \cos [c + dx])^{7/2}} + \frac{\sqrt{\cos [c + dx]} \operatorname{Sin}[c + dx]}{16 a d (a + a \cos [c + dx])^{5/2}} - \frac{5 \sqrt{\cos [c + dx]} \operatorname{Sin}[c + dx]}{192 a^2 d (a + a \cos [c + dx])^{3/2}}$$

Result (type 3, 382 leaves) :

$$\begin{aligned}
 & - \left( 13 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1 + e^{2 i (c+dx)})} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^7 \left( \log [1 + e^{i (c+dx)}] - \log [1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}] \right) \right) / \\
 & \left( 8 d \sqrt{1 + e^{2 i (c+dx)}} (a (1 + \cos [c + dx]))^{7/2} \right) + \frac{1}{(a (1 + \cos [c + dx]))^{7/2}} \\
 & \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^7 \sqrt{\cos [c + dx]} \left( - \frac{5 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{24 d} + \frac{\operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{4 d} + \right. \\
 & \left. \frac{\operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{3 d} - \frac{5 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Tan} \left[ \frac{c}{2} \right]}{24 d} + \frac{\operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \operatorname{Tan} \left[ \frac{c}{2} \right]}{4 d} + \frac{\operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \operatorname{Tan} \left[ \frac{c}{2} \right]}{3 d} \right)
 \end{aligned}$$

- **Problem 256: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\cos [c + dx]} (a + a \cos [c + dx])^{7/2}} dx$$

Optimal (type 3, 177 leaves, 6 steps) :

$$\frac{63 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{64 \sqrt{2} a^{7/2} d} - \frac{\sqrt{\cos[c+dx]} \sin[c+dx]}{6 d (a+a \cos[c+dx])^{7/2}} - \frac{5 \sqrt{\cos[c+dx]} \sin[c+dx]}{16 a d (a+a \cos[c+dx])^{5/2}} - \frac{103 \sqrt{\cos[c+dx]} \sin[c+dx]}{192 a^2 d (a+a \cos[c+dx])^{3/2}}$$

Result (type 3, 382 leaves):

$$\begin{aligned} & - \left( 63 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left( \operatorname{Log}[1+e^{i(c+dx)}] - \operatorname{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\ & \left( 8 d \sqrt{1+e^{2i(c+dx)}} (a(1+\cos[c+dx]))^{7/2} \right) + \frac{1}{(a(1+\cos[c+dx]))^{7/2}} \\ & \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\cos[c+dx]} \left( -\frac{103 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{24 d} - \frac{5 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{4 d} - \right. \\ & \left. \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{dx}{2}\right]}{3 d} - \frac{103 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Tan}\left[\frac{c}{2}\right]}{24 d} - \frac{5 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Tan}\left[\frac{c}{2}\right]}{4 d} - \frac{\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right) \end{aligned}$$

■ **Problem 257: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\cos[c+dx]^{3/2} (a+a \cos[c+dx])^{7/2}} dx$$

Optimal (type 3, 217 leaves, 7 steps):

$$\begin{aligned} & \frac{363 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{64 \sqrt{2} a^{7/2} d} - \frac{\sin[c+dx]}{6 d \sqrt{\cos[c+dx]} (a+a \cos[c+dx])^{7/2}} - \\ & \frac{19 \sin[c+dx]}{48 a d \sqrt{\cos[c+dx]} (a+a \cos[c+dx])^{5/2}} - \frac{199 \sin[c+dx]}{192 a^2 d \sqrt{\cos[c+dx]} (a+a \cos[c+dx])^{3/2}} + \frac{691 \sin[c+dx]}{192 a^3 d \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}} \end{aligned}$$

Result (type 3, 406 leaves):

$$\begin{aligned} & \left( 363 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left( \operatorname{Log}[1+e^{i(c+dx)}] - \operatorname{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\ & \left( 8 d \sqrt{1+e^{2i(c+dx)}} (a(1+\cos[c+dx]))^{7/2} \right) + \frac{1}{(a(1+\cos[c+dx]))^{7/2}} \\ & \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\cos[c+dx]} \left( \frac{307 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{24 d} + \frac{9 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{4 d} + \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{dx}{2}\right]}{3 d} + \right. \\ & \left. \frac{32 \operatorname{Sec}[c+dx] \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{d} + \frac{307 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Tan}\left[\frac{c}{2}\right]}{24 d} + \frac{9 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Tan}\left[\frac{c}{2}\right]}{4 d} + \frac{\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right) \end{aligned}$$

■ **Problem 258: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\cos[c+dx]^{5/2} (a+a\cos[c+dx])^{7/2}} dx$$

Optimal (type 3, 257 leaves, 8 steps):

$$\frac{1015 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right]}{64 \sqrt{2} a^{7/2} d} - \frac{\sin[c+dx]}{6 d \cos[c+dx]^{3/2} (a+a\cos[c+dx])^{7/2}} - \frac{23 \sin[c+dx]}{48 a d \cos[c+dx]^{3/2} (a+a\cos[c+dx])^{5/2}} - \frac{109 \sin[c+dx]}{64 a^2 d \cos[c+dx]^{3/2} (a+a\cos[c+dx])^{3/2}} + \frac{193 \sin[c+dx]}{64 a^3 d \cos[c+dx]^{3/2} \sqrt{a+a\cos[c+dx]}} - \frac{629 \sin[c+dx]}{64 a^3 d \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 436 leaves):

$$-\left(1015 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left(\operatorname{Log}[1+e^{i(c+dx)}] - \operatorname{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}]\right)\right) / \left(8 d \sqrt{1+e^{2i(c+dx)}} (a(1+\cos[c+dx]))^{7/2}\right) + \frac{1}{(a(1+\cos[c+dx]))^{7/2}}$$

$$\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\cos[c+dx]} \left(-\frac{607 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{24 d} - \frac{13 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{4 d} - \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{dx}{2}\right]}{3 d} - \frac{320 \operatorname{Sec}[c+dx] \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{3 d} + \frac{32 \operatorname{Sec}[c+dx]^2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{3 d} - \frac{607 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Tan}\left[\frac{c}{2}\right]}{24 d} - \frac{13 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Tan}\left[\frac{c}{2}\right]}{4 d} - \frac{\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d}\right)$$

■ **Problem 259: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{7/2}}{(a+a\cos[c+dx])^{9/2}} dx$$

Optimal (type 3, 217 leaves, 7 steps):

$$\frac{35 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right]}{1024 \sqrt{2} a^{9/2} d} - \frac{\cos[c+dx]^{5/2} \sin[c+dx]}{8 d (a+a\cos[c+dx])^{9/2}} - \frac{19 \cos[c+dx]^{3/2} \sin[c+dx]}{96 a d (a+a\cos[c+dx])^{7/2}} - \frac{187 \sqrt{\cos[c+dx]} \sin[c+dx]}{768 a^2 d (a+a\cos[c+dx])^{5/2}} + \frac{853 \sqrt{\cos[c+dx]} \sin[c+dx]}{3072 a^3 d (a+a\cos[c+dx])^{3/2}}$$

Result (type 3, 445 leaves):

$$\begin{aligned}
& - \left( 35 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1 + e^{2 i (c+dx)})} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^9 \left( \log [1 + e^{i (c+dx)}] - \log [1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}] \right) \right) / \\
& \left( 64 d \sqrt{1 + e^{2 i (c+dx)}} (a (1 + \cos [c + dx]))^{9/2} \right) + \frac{1}{(a (1 + \cos [c + dx]))^{9/2}} \\
& \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^9 \sqrt{\cos [c + dx]} \left( \frac{853 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{192 d} - \frac{145 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{32 d} + \frac{43 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{24 d} - \right. \\
& \left. \frac{\operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{4 d} + \frac{853 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Tan} \left[ \frac{c}{2} \right]}{192 d} - \frac{145 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \operatorname{Tan} \left[ \frac{c}{2} \right]}{32 d} + \frac{43 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \operatorname{Tan} \left[ \frac{c}{2} \right]}{24 d} - \frac{\operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^7 \operatorname{Tan} \left[ \frac{c}{2} \right]}{4 d} \right)
\end{aligned}$$

- **Problem 260: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + dx]^{5/2}}{(a + a \cos [c + dx])^{9/2}} dx$$

Optimal (type 3, 217 leaves, 7 steps):

$$\begin{aligned}
& \frac{45 \operatorname{ArcTan} \left[ \frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{\cos [c+dx]} \sqrt{a+a \cos [c+dx]}} \right]}{1024 \sqrt{2} a^{9/2} d} - \frac{\cos [c + dx]^{3/2} \operatorname{Sin}[c + dx]}{8 d (a + a \cos [c + dx])^{9/2}} - \\
& \frac{5 \sqrt{\cos [c + dx]} \operatorname{Sin}[c + dx]}{32 a d (a + a \cos [c + dx])^{7/2}} + \frac{33 \sqrt{\cos [c + dx]} \operatorname{Sin}[c + dx]}{256 a^2 d (a + a \cos [c + dx])^{5/2}} + \frac{73 \sqrt{\cos [c + dx]} \operatorname{Sin}[c + dx]}{1024 a^3 d (a + a \cos [c + dx])^{3/2}}
\end{aligned}$$

Result (type 3, 445 leaves):

$$\begin{aligned}
& - \left( 45 i e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1 + e^{2 i (c+dx)})} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^9 \left( \log [1 + e^{i (c+dx)}] - \log [1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}] \right) \right) / \\
& \left( 64 d \sqrt{1 + e^{2 i (c+dx)}} (a (1 + \cos [c + dx]))^{9/2} \right) + \frac{1}{(a (1 + \cos [c + dx]))^{9/2}} \\
& \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^9 \sqrt{\cos [c + dx]} \left( \frac{73 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{64 d} + \frac{33 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{32 d} - \frac{9 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{8 d} + \right. \\
& \left. \frac{\operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8 \operatorname{Sin} \left[ \frac{dx}{2} \right]}{4 d} + \frac{73 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Tan} \left[ \frac{c}{2} \right]}{64 d} + \frac{33 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \operatorname{Tan} \left[ \frac{c}{2} \right]}{32 d} - \frac{9 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \operatorname{Tan} \left[ \frac{c}{2} \right]}{8 d} + \frac{\operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^7 \operatorname{Tan} \left[ \frac{c}{2} \right]}{4 d} \right)
\end{aligned}$$

- **Problem 263: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos [c + dx]^{3/2} \sqrt{a - a \cos [c + dx]} dx$$

Optimal (type 3, 129 leaves, 4 steps):

$$-\frac{3\sqrt{a}\operatorname{ArcTanh}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{\cos[c+dx]}\sqrt{a-a\cos[c+dx]}}\right]}{4d} + \frac{3a\sqrt{\cos[c+dx]}\sin[c+dx]}{4d\sqrt{a-a\cos[c+dx]}} - \frac{a\cos[c+dx]^{3/2}\sin[c+dx]}{2d\sqrt{a-a\cos[c+dx]}}$$

Result (type 3, 395 leaves):

$$-\frac{1}{8d\sqrt{(1+e^{2ix})\cos[c]+i(-1+e^{2ix})\sin[c]}}\sqrt{\cos[c+dx]}\sqrt{a-a\cos[c+dx]} \\ + \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]\left(3\cos\left[\frac{dx}{2}\right]\operatorname{Log}\left[2\left(e^{ix}\cos\left[\frac{c}{2}\right]+ie^{ix}\sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2ix})\cos[c]+i(-1+e^{2ix})\sin[c]}\right)\right]\right) + \\ 3\operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right]+i\sin\left[\frac{c}{2}\right]\right)\sqrt{(1+e^{2ix})\cos[c]+i(-1+e^{2ix})\sin[c]}\right]\left(\cos\left[\frac{dx}{2}\right]+i\sin\left[\frac{dx}{2}\right]\right) + \\ 3i\operatorname{Log}\left[2\left(e^{ix}\cos\left[\frac{c}{2}\right]+ie^{ix}\sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2ix})\cos[c]+i(-1+e^{2ix})\sin[c]}\right)\right]\sin\left[\frac{dx}{2}\right] - \\ 4\sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{\cos[c+dx](\cos[dx]+i\sin[dx])} + 2\sqrt{2}\cos\left[\frac{3}{2}(c+dx)\right]\sqrt{\cos[c+dx](\cos[dx]+i\sin[dx])}$$

■ **Problem 264: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]}\sqrt{a-a\cos[c+dx]}\,dx$$

Optimal (type 3, 85 leaves, 3 steps):

$$\frac{\sqrt{a}\operatorname{ArcTanh}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{\cos[c+dx]}\sqrt{a-a\cos[c+dx]}}\right]}{d} - \frac{a\sqrt{\cos[c+dx]}\sin[c+dx]}{d\sqrt{a-a\cos[c+dx]}}$$

Result (type 3, 352 leaves):

$$\frac{1}{2d\sqrt{(1+e^{2ix})\cos[c]+i(-1+e^{2ix})\sin[c]}}\sqrt{\cos[c+dx]}\sqrt{a-a\cos[c+dx]} \\ + \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]\left(\cos\left[\frac{dx}{2}\right]\operatorname{Log}\left[2\left(e^{ix}\cos\left[\frac{c}{2}\right]+ie^{ix}\sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2ix})\cos[c]+i(-1+e^{2ix})\sin[c]}\right)\right]\right) + \\ \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right]+i\sin\left[\frac{c}{2}\right]\right)\sqrt{(1+e^{2ix})\cos[c]+i(-1+e^{2ix})\sin[c]}\right]\left(\cos\left[\frac{dx}{2}\right]+i\sin\left[\frac{dx}{2}\right]\right) + \\ i\operatorname{Log}\left[2\left(e^{ix}\cos\left[\frac{c}{2}\right]+ie^{ix}\sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2ix})\cos[c]+i(-1+e^{2ix})\sin[c]}\right)\right]\sin\left[\frac{dx}{2}\right] - \\ 2\sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{\cos[c+dx](\cos[dx]+i\sin[dx])}$$

- **Problem 265: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a - a \cos[c + dx]}}{\sqrt{\cos[c + dx]}} dx$$

Optimal (type 3, 48 leaves, 2 steps):

$$\frac{2\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{\cos[c+dx]} \sqrt{a-a\cos[c+dx]}}\right]}{d}$$

Result (type 3, 243 leaves):

$$\begin{aligned} & - \left( e^{\frac{idx}{2}} \sqrt{a - a \cos[c + dx]} \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right] \left( \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]}\right] + \right. \right. \\ & \quad \left. \left. \operatorname{Log}\left[2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]}\right) \right] \right) \right) \\ & \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c] \right)} \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]} \right) \end{aligned}$$

- **Problem 269: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{1 - \cos[c + dx]} \cos[c + dx]^{3/2} dx$$

Optimal (type 3, 114 leaves, 4 steps):

$$\frac{3 \operatorname{ArcTanh}\left[\frac{\sin[c+dx]}{\sqrt{1-\cos[c+dx]} \sqrt{\cos[c+dx]}}\right]}{4d} + \frac{3\sqrt{\cos[c+dx]} \sin[c+dx]}{4d\sqrt{1-\cos[c+dx]}} - \frac{\cos[c+dx]^{3/2} \sin[c+dx]}{2d\sqrt{1-\cos[c+dx]}}$$

Result (type 3, 390 leaves):

$$\begin{aligned} & - \frac{1}{8d\sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]}} \sqrt{-(-1 + \cos[c + dx]) \cos[c + dx]} \\ & \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right] \left( 3 \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]}\right) \right] + \right. \\ & \quad 3 \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) + \\ & \quad \left. 3i \operatorname{Log}\left[2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]}\right) \right] \sin\left[\frac{dx}{2}\right] - \right. \\ & \quad \left. 4\sqrt{2} \cos\left[\frac{1}{2}(c + dx)\right] \sqrt{\cos[c + dx] (\cos[dx] + i \sin[dx])} + 2\sqrt{2} \cos\left[\frac{3}{2}(c + dx)\right] \sqrt{\cos[c + dx] (\cos[dx] + i \sin[dx])} \right) \end{aligned}$$

- **Problem 270: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{1 - \cos[c + dx]} \sqrt{\cos[c + dx]}}{dx} dx$$

Optimal (type 3, 72 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sin[c+dx]}{\sqrt{1-\cos[c+dx]}\sqrt{\cos[c+dx]}}\right]}{d} - \frac{\sqrt{\cos[c+dx]}\sin[c+dx]}{d\sqrt{1-\cos[c+dx]}}$$

Result (type 3, 340 leaves):

$$\frac{1}{2 d \sqrt{\cos[c + dx]} (\cos[dx] + i \sin[dx])} \text{Csc}\left[\frac{1}{2} (c + dx)\right] \left( \cos\left[\frac{dx}{2}\right] \text{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c]} \right)\right] + \text{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) + i \text{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c]} \right)\right] \sin\left[\frac{dx}{2}\right] - 2 \sqrt{2} \cos\left[\frac{1}{2} (c + dx)\right] \sqrt{\cos[c + dx]} (\cos[dx] + i \sin[dx]) \right) \sqrt{\cos[c + dx] \sin\left[\frac{1}{2} (c + dx)\right]^2}$$

- **Problem 271: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{1 - \cos[c + dx]}}{\sqrt{\cos[c + dx]}} dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$\frac{2 \text{ArcTanh}\left[\frac{\sin[c+dx]}{\sqrt{1-\cos[c+dx]}\sqrt{\cos[c+dx]}}\right]}{d}$$

Result (type 3, 242 leaves):

$$- \left( e^{\frac{i dx}{2}} \sqrt{1 - \cos[c + dx]} \text{Csc}\left[\frac{1}{2} (c + dx)\right] \left( \text{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c]}\right] + \text{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c]} \right)\right] \right) \sqrt{e^{-i dx} \left( (1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c]} \right)$$

■ **Problem 275: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c+d x]^{5 / 2}}{\sqrt{a-a \cos [c+d x]}} d x$$

Optimal (type 3, 185 leaves, 7 steps):

$$\frac{7 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{\cos [c+d x]} \sqrt{a-a \cos [c+d x]}}\right]}{4 \sqrt{a} d}-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a-a \cos [c+d x]}}\right]}{\sqrt{a} d}+\frac{\sqrt{\cos [c+d x]} \sin [c+d x]}{4 d \sqrt{a-a \cos [c+d x]}}+\frac{\cos [c+d x]^{3 / 2} \sin [c+d x]}{2 d \sqrt{a-a \cos [c+d x]}}$$

Result (type 3, 246 leaves):

$$\frac{1}{8 \sqrt{a-a \cos [c+d x]}}\left(\frac{4 \sqrt{\cos [c+d x]} \left(2 \cos \left[\frac{1}{2}(c+d x)\right]+\cos \left[\frac{3}{2}(c+d x)\right]\right)}{d}+1 / \left(d \sqrt{1+e^{2 i(c+d x)}}\right) \sqrt{2} e^{\frac{1}{2} i(c+d x)} \sqrt{e^{-i(c+d x)}\left(1+e^{2 i(c+d x)}\right)}\left(-7 i d x+\right.\right. \\ \left.\left.7 \operatorname{ArcSinh}\left[e^{i(c+d x)}\right]+8 \sqrt{2} \operatorname{Log}\left[1-e^{i(c+d x)}\right]+7 \operatorname{Log}\left[1+\sqrt{1+e^{2 i(c+d x)}}\right]-8 \sqrt{2} \operatorname{Log}\left[1+e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]\right)\right) \sin \left[\frac{1}{2}(c+d x)\right]$$

■ **Problem 276: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c+d x]^{3 / 2}}{\sqrt{a-a \cos [c+d x]}} d x$$

Optimal (type 3, 141 leaves, 6 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{\cos [c+d x]} \sqrt{a-a \cos [c+d x]}}\right]}{\sqrt{a} d}-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a-a \cos [c+d x]}}\right]}{\sqrt{a} d}+\frac{\sqrt{\cos [c+d x]} \sin [c+d x]}{d \sqrt{a-a \cos [c+d x]}}$$

Result (type 3, 229 leaves):

$$\frac{1}{2 \sqrt{a-a \cos [c+d x]}}\left(\frac{4 \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{\cos [c+d x]}}{d}+1 / \left(d \sqrt{1+e^{2 i(c+d x)}}\right) \sqrt{2} e^{\frac{1}{2} i(c+d x)} \sqrt{e^{-i(c+d x)}\left(1+e^{2 i(c+d x)}\right)}\left(-i d x+\operatorname{ArcSinh}\left[e^{i(c+d x)}\right]+\right.\right. \\ \left.\left.2 \sqrt{2} \operatorname{Log}\left[1-e^{i(c+d x)}\right]+\operatorname{Log}\left[1+\sqrt{1+e^{2 i(c+d x)}}\right]-2 \sqrt{2} \operatorname{Log}\left[1+e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]\right)\right) \sin \left[\frac{1}{2}(c+d x)\right]$$



- **Problem 277: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{\cos[c+dx]}}{\sqrt{a-a\cos[c+dx]}} dx$$

Optimal (type 3, 107 leaves, 5 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{\cos[c+dx]} \sqrt{a-a\cos[c+dx]}}\right]}{\sqrt{a} d} - \frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a-a\cos[c+dx]}}\right]}{\sqrt{a} d}$$

Result (type 3, 191 leaves):

$$-\left(\frac{i(-1+e^{i(c+dx)})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}}{\left(-i dx + \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] + \sqrt{2} \operatorname{Log}\left[1-e^{i(c+dx)}\right] + \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] - \sqrt{2} \operatorname{Log}\left[1+e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}\right]\right)}\right) / \left(\sqrt{2} d \sqrt{1+e^{2i(c+dx)}} \sqrt{a-a\cos[c+dx]}\right)$$

- **Problem 278: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\cos[c+dx]} \sqrt{a-a\cos[c+dx]}} dx$$

Optimal (type 3, 58 leaves, 2 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a-a\cos[c+dx]}}\right]}{\sqrt{a} d}$$

Result (type 3, 137 leaves):

$$-\frac{i(-1+e^{i(c+dx)})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})} \left(\operatorname{Log}\left[1-e^{i(c+dx)}\right] - \operatorname{Log}\left[1+e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}\right]\right)}{d \sqrt{1+e^{2i(c+dx)}} \sqrt{a-a\cos[c+dx]}}$$

- **Problem 279: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\cos[c+dx]^{3/2} \sqrt{a-a\cos[c+dx]}} dx$$

Optimal (type 3, 95 leaves, 4 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a-a\cos[c+dx]}}\right]}{\sqrt{a} d} + \frac{2 \sin[c+dx]}{d \sqrt{\cos[c+dx]} \sqrt{a-a\cos[c+dx]}}$$

Result (type 3, 194 leaves):

$$\left( e^{-\frac{1}{2}i(c+dx)} \left( 2(1+e^{i(c+dx)})\sqrt{1+e^{2i(c+dx)}} + \sqrt{2}(1+e^{2i(c+dx)})\operatorname{Log}[1-e^{i(c+dx)}] - \sqrt{2}(1+e^{2i(c+dx)})\operatorname{Log}[1+e^{i(c+dx)} + \sqrt{2}\sqrt{1+e^{2i(c+dx)}}] \right) \right. \\ \left. \sin\left[\frac{1}{2}(c+dx)\right] \right) / \left( d\sqrt{1+e^{2i(c+dx)}}\sqrt{\cos[c+dx]}\sqrt{a-a\cos[c+dx]} \right)$$

■ **Problem 280: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\cos[c+dx]^{5/2}\sqrt{a-a\cos[c+dx]}} dx$$

Optimal (type 3, 135 leaves, 5 steps):

$$-\frac{\sqrt{2}\operatorname{ArcTanh}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{2}\sqrt{\cos[c+dx]}\sqrt{a-a\cos[c+dx]}}\right]}{\sqrt{a}d} + \frac{2\sin[c+dx]}{3d\cos[c+dx]^{3/2}\sqrt{a-a\cos[c+dx]}} + \frac{2\sin[c+dx]}{3d\sqrt{\cos[c+dx]}\sqrt{a-a\cos[c+dx]}}$$

Result (type 3, 185 leaves):

$$\left( e^{-\frac{3}{2}i(c+dx)} \left( 2(1+e^{i(c+dx)})^3\sqrt{1+e^{2i(c+dx)}} + 3\sqrt{2}(1+e^{2i(c+dx)})^2 \left( \operatorname{Log}[1-e^{i(c+dx)}] - \operatorname{Log}[1+e^{i(c+dx)} + \sqrt{2}\sqrt{1+e^{2i(c+dx)}}] \right) \right) \right. \\ \left. \sin\left[\frac{1}{2}(c+dx)\right] \right) / \left( 6d\sqrt{1+e^{2i(c+dx)}}\cos[c+dx]^{3/2}\sqrt{a-a\cos[c+dx]} \right)$$

■ **Problem 281: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\cos[c+dx]^{7/2}\sqrt{a-a\cos[c+dx]}} dx$$

Optimal (type 3, 173 leaves, 6 steps):

$$-\frac{\sqrt{2}\operatorname{ArcTanh}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{2}\sqrt{\cos[c+dx]}\sqrt{a-a\cos[c+dx]}}\right]}{\sqrt{a}d} + \frac{2\sin[c+dx]}{5d\cos[c+dx]^{5/2}\sqrt{a-a\cos[c+dx]}} + \\ \frac{2\sin[c+dx]}{15d\cos[c+dx]^{3/2}\sqrt{a-a\cos[c+dx]}} + \frac{26\sin[c+dx]}{15d\sqrt{\cos[c+dx]}\sqrt{a-a\cos[c+dx]}}$$

Result (type 3, 192 leaves):

$$\frac{1}{\sqrt{a-a\cos[c+dx]}} \left( \frac{4\cos\left[\frac{1}{2}(c+dx)\right](3+\cos[c+dx]+13\cos[c+dx]^2)}{15d\cos[c+dx]^{5/2}} + \right. \\ \left. \frac{2e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})} \left( \operatorname{Log}[1-e^{i(c+dx)}] - \operatorname{Log}[1+e^{i(c+dx)} + \sqrt{2}\sqrt{1+e^{2i(c+dx)}}] \right) \right)}{d\sqrt{1+e^{2i(c+dx)}}} \sin\left[\frac{1}{2}(c+dx)\right]$$

■ **Problem 282: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c+d x]^{5 / 2}}{\sqrt{1-\cos [c+d x]}} d x$$

Optimal (type 3, 161 leaves, 7 steps):

$$\frac{7 \operatorname{ArcTanh}\left[\frac{\sin [c+d x]}{\sqrt{1-\cos [c+d x]} \sqrt{\cos [c+d x]}}\right]}{4 d}-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sin [c+d x]}{\sqrt{2} \sqrt{1-\cos [c+d x]} \sqrt{\cos [c+d x]}}\right]}{d}+\frac{\sqrt{\cos [c+d x]} \sin [c+d x]}{4 d \sqrt{1-\cos [c+d x]}}+\frac{\cos [c+d x]^{3 / 2} \sin [c+d x]}{2 d \sqrt{1-\cos [c+d x]}}$$

Result (type 3, 245 leaves):

$$\frac{1}{8 \sqrt{1-\cos [c+d x]}}\left(\frac{4 \sqrt{\cos [c+d x]} \left(2 \cos \left[\frac{1}{2}(c+d x)\right]+\cos \left[\frac{3}{2}(c+d x)\right]\right)}{d}+1 / \left(d \sqrt{1+e^{2 i(c+d x)}}\right) \sqrt{2} e^{\frac{1}{2} i(c+d x)} \sqrt{e^{-i(c+d x)}\left(1+e^{2 i(c+d x)}\right)}\left(-7 i d x+7 \operatorname{ArcSinh}\left[e^{i(c+d x)}\right]+8 \sqrt{2} \operatorname{Log}\left[1-e^{i(c+d x)}\right]+7 \operatorname{Log}\left[1+\sqrt{1+e^{2 i(c+d x)}}\right]-8 \sqrt{2} \operatorname{Log}\left[1+e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]\right)\right) \sin \left[\frac{1}{2}(c+d x)\right]$$

■ **Problem 283: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c+d x]^{3 / 2}}{\sqrt{1-\cos [c+d x]}} d x$$

Optimal (type 3, 118 leaves, 6 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sin [c+d x]}{\sqrt{1-\cos [c+d x]} \sqrt{\cos [c+d x]}}\right]}{d}-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sin [c+d x]}{\sqrt{2} \sqrt{1-\cos [c+d x]} \sqrt{\cos [c+d x]}}\right]}{d}+\frac{\sqrt{\cos [c+d x]} \sin [c+d x]}{d \sqrt{1-\cos [c+d x]}}$$

Result (type 3, 228 leaves):

$$\frac{1}{2 \sqrt{1-\cos [c+d x]}}\left(\frac{4 \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{\cos [c+d x]}}{d}+1 / \left(d \sqrt{1+e^{2 i(c+d x)}}\right) \sqrt{2} e^{\frac{1}{2} i(c+d x)} \sqrt{e^{-i(c+d x)}\left(1+e^{2 i(c+d x)}\right)}\left(-i d x+\operatorname{ArcSinh}\left[e^{i(c+d x)}\right]+2 \sqrt{2} \operatorname{Log}\left[1-e^{i(c+d x)}\right]+\operatorname{Log}\left[1+\sqrt{1+e^{2 i(c+d x)}}\right]-2 \sqrt{2} \operatorname{Log}\left[1+e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]\right)\right) \sin \left[\frac{1}{2}(c+d x)\right]$$

- **Problem 284: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos [c+d x]}}{\sqrt{1-\cos [c+d x]}} d x$$

Optimal (type 3, 85 leaves, 5 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sin [c+d x]}{\sqrt{1-\cos [c+d x]} \sqrt{\cos [c+d x]}}\right]}{d} - \frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sin [c+d x]}{\sqrt{2} \sqrt{1-\cos [c+d x]} \sqrt{\cos [c+d x]}}\right]}{d}$$

Result (type 3, 190 leaves):

$$-\left(\frac{i\left(-1+e^{i(c+d x)}\right) \sqrt{e^{-i(c+d x)}\left(1+e^{2 i(c+d x)}\right)}\left(-i d x+\operatorname{ArcSinh}\left[e^{i(c+d x)}\right]+\sqrt{2} \operatorname{Log}\left[1-e^{i(c+d x)}\right]+\operatorname{Log}\left[1+\sqrt{1+e^{2 i(c+d x)}}\right]-\sqrt{2} \operatorname{Log}\left[1+e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]\right)}{\left(\sqrt{2} d \sqrt{1+e^{2 i(c+d x)}} \sqrt{1-\cos [c+d x]}\right)}\right)$$

- **Problem 285: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{1-\cos [c+d x]} \sqrt{\cos [c+d x]}} d x$$

Optimal (type 3, 47 leaves, 2 steps):

$$\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sin [c+d x]}{\sqrt{2} \sqrt{1-\cos [c+d x]} \sqrt{\cos [c+d x]}}\right]}{d}$$

Result (type 3, 129 leaves):

$$\frac{i e^{-i(c+d x)}\left(-1+e^{i(c+d x)}\right) \sqrt{1+e^{2 i(c+d x)}}\left(\operatorname{Log}\left[1-e^{i(c+d x)}\right]-\operatorname{Log}\left[1+e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]\right)}{\sqrt{2} d \sqrt{-(-1+\cos [c+d x]) \cos [c+d x]}}$$

- **Problem 286: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{1-\cos [c+d x]} \cos [c+d x]^{3 / 2}} d x$$

Optimal (type 3, 83 leaves, 3 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sin [c+d x]}{\sqrt{2} \sqrt{1-\cos [c+d x]} \sqrt{\cos [c+d x]}}\right]}{d} + \frac{2 \sin [c+d x]}{d \sqrt{1-\cos [c+d x]} \sqrt{\cos [c+d x]}}$$

Result (type 3, 189 leaves):

$$\left( e^{-\frac{1}{2}i(c+dx)} \left( 2(1+e^{i(c+dx)})\sqrt{1+e^{2i(c+dx)}} + \sqrt{2}(1+e^{2i(c+dx)})\operatorname{Log}[1-e^{i(c+dx)}] - \sqrt{2}(1+e^{2i(c+dx)})\operatorname{Log}\left[1+e^{i(c+dx)} + \sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] \right) \right. \\ \left. \sin\left[\frac{1}{2}(c+dx)\right] \right) / \left( d\sqrt{1+e^{2i(c+dx)}}\sqrt{-(-1+\cos[c+dx])\cos[c+dx]} \right)$$

■ **Problem 287: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{1-\cos[c+dx]}\cos[c+dx]^{5/2}} dx$$

Optimal (type 3, 122 leaves, 5 steps):

$$-\frac{\sqrt{2}\operatorname{ArcTanh}\left[\frac{\sin[c+dx]}{\sqrt{2}\sqrt{1-\cos[c+dx]}\sqrt{\cos[c+dx]}}\right]}{d} + \frac{2\sin[c+dx]}{3d\sqrt{1-\cos[c+dx]}\cos[c+dx]^{3/2}} + \frac{2\sin[c+dx]}{3d\sqrt{1-\cos[c+dx]}\sqrt{\cos[c+dx]}}$$

Result (type 3, 184 leaves):

$$\left( e^{-\frac{3}{2}i(c+dx)} \left( 2(1+e^{i(c+dx)})^3\sqrt{1+e^{2i(c+dx)}} + 3\sqrt{2}(1+e^{2i(c+dx)})^2 \left( \operatorname{Log}[1-e^{i(c+dx)}] - \operatorname{Log}\left[1+e^{i(c+dx)} + \sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] \right) \right) \right) \\ \sin\left[\frac{1}{2}(c+dx)\right] \right) / \left( 6d\sqrt{1+e^{2i(c+dx)}}\sqrt{1-\cos[c+dx]}\cos[c+dx]^{3/2} \right)$$

■ **Problem 288: Attempted integration timed out after 120 seconds.**

$$\int \cos[c+dx]^{4/3}(a+a\cos[c+dx])^{1/3} dx$$

Optimal (type 6, 78 leaves, 3 steps):

$$\frac{2^{5/6}\operatorname{AppellF1}\left[\frac{1}{2}, -\frac{4}{3}, \frac{1}{6}, \frac{3}{2}, 1-\cos[c+dx], \frac{1}{2}(1-\cos[c+dx])\right](a+a\cos[c+dx])^{1/3}\sin[c+dx]}{d(1+\cos[c+dx])^{5/6}}$$

Result (type 1, 1 leaves):

???

■ **Problem 289: Unable to integrate problem.**

$$\int \cos[c+dx]^{4/3}(a+a\cos[c+dx])^{2/3} dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$\frac{2 \times 2^{1/6}\operatorname{AppellF1}\left[\frac{1}{2}, -\frac{4}{3}, -\frac{1}{6}, \frac{3}{2}, 1-\cos[c+dx], \frac{1}{2}(1-\cos[c+dx])\right](a+a\cos[c+dx])^{2/3}\sin[c+dx]}{d(1+\cos[c+dx])^{7/6}}$$

Result (type 8, 27 leaves):

$$\int \cos[c+dx]^{4/3}(a+a\cos[c+dx])^{2/3} dx$$

■ **Problem 290: Unable to integrate problem.**

$$\int \cos [c + d x]^{5/3} (a + a \cos [c + d x])^{2/3} dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$\frac{2 \times 2^{1/6} \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{5}{3}, -\frac{1}{6}, \frac{3}{2}, 1 - \cos [c + d x], \frac{1}{2} (1 - \cos [c + d x])\right] (a + a \cos [c + d x])^{2/3} \sin [c + d x]}{d (1 + \cos [c + d x])^{7/6}}$$

Result (type 8, 27 leaves):

$$\int \cos [c + d x]^{5/3} (a + a \cos [c + d x])^{2/3} dx$$

■ **Problem 291: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c + d x]) \sec [c + d x]^{7/2} dx$$

Optimal (type 4, 151 leaves, 9 steps):

$$-\frac{6 a \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} + \frac{2 a \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\sec [c + d x]}}{3 d} +$$

$$\frac{6 a \sqrt{\sec [c + d x]} \sin [c + d x]}{5 d} + \frac{2 a \sec [c + d x]^{3/2} \sin [c + d x]}{3 d} + \frac{2 a \sec [c + d x]^{5/2} \sin [c + d x]}{5 d}$$

Result (type 5, 268 leaves):

$$\frac{1}{15 (d - d e^{2 i c})} a (1 + \cos [c + d x]) \sec \left[ \frac{1}{2} (c + d x) \right]^2$$

$$\left( i \sqrt{2} e^{-i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \left( 9 (1 + e^{2 i (c + d x)}) + 9 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] + \right. \right.$$

$$\left. \left. 5 e^{i (c + d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c + d x)}\right] \right) + \right.$$

$$\left. (1 - e^{2 i c}) \sqrt{\sec [c + d x]} (9 \cos [d x] \operatorname{Csc}[c] + (5 + 3 \sec [c + d x]) \tan [c + d x]) \right)$$

■ **Problem 292: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x]) \sec [c + d x]^{5/2} dx$$

Optimal (type 4, 123 leaves, 8 steps):

$$-\frac{2a\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{d} + \frac{2a\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{3d} + \frac{2a\sqrt{\sec[c+dx]}\sin[c+dx]}{d} + \frac{2a\sec[c+dx]^{3/2}\sin[c+dx]}{3d}$$

Result (type 5, 255 leaves):

$$\frac{1}{3(d-de^{2ic})} a(1+\cos[c+dx])\sec\left[\frac{1}{2}(c+dx)\right]^2 \left( i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3(1+e^{2i(c+dx)})+3(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]+e^{i(c+dx)}\right. \right. \\ \left. \left. (-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right]\right)-(-1+e^{2ic})\sqrt{\sec[c+dx]}(3\cos[dx]\operatorname{Csc}[c]+\tan[c+dx]) \right)$$

■ **Problem 293: Result unnecessarily involves higher level functions.**

$$\int (a+a\cos[c+dx])\sec[c+dx]^{3/2}dx$$

Optimal (type 4, 97 leaves, 7 steps):

$$-\frac{2a\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{d} + \frac{2a\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{d} + \frac{2a\sqrt{\sec[c+dx]}\sin[c+dx]}{d}$$

Result (type 5, 124 leaves):

$$-\frac{1}{d}2iae^{-i(c+dx)} \left( -1+\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]+e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \sqrt{\sec[c+dx]}$$

■ **Problem 294: Result unnecessarily involves higher level functions.**

$$\int (a+a\cos[c+dx])\sqrt{\sec[c+dx]}dx$$

Optimal (type 4, 75 leaves, 6 steps):

$$\frac{2 a \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{d} + \frac{2 a \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{d}$$

Result (type 5, 141 leaves):

$$-\left(2 i a\left(1+e^{2 i(c+d x)}-2 \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]+2 e^{i(c+d x)} \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right]\right)\right) / \left(d\left(1+e^{2 i(c+d x)}\right) \sqrt{\sec [c+d x]}\right)$$

■ **Problem 295: Result unnecessarily involves higher level functions.**

$$\int \frac{a+a \cos [c+d x]}{\sqrt{\sec [c+d x]}} d x$$

Optimal (type 4, 101 leaves, 7 steps):

$$\frac{2 a \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{d} + \frac{2 a \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 d} + \frac{2 a \sin [c+d x]}{3 d \sqrt{\sec [c+d x]}}$$

Result (type 5, 140 leaves):

$$\frac{1}{3 d \sqrt{\sec [c+d x]}} a e^{-2 i c}(-i \cos [2 c]+\sin [2 c])\left(6-\frac{12 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]}{\sqrt{1+e^{2 i(c+d x)}}}+2 \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \sec [c+d x]+2 i \sin [c+d x]\right)$$

■ **Problem 296: Result unnecessarily involves higher level functions.**

$$\int \frac{a+a \cos [c+d x]}{\sec [c+d x]^{3 / 2}} d x$$

Optimal (type 4, 127 leaves, 8 steps):

$$\frac{6 a \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} + \frac{2 a \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 d} + \frac{2 a \sin [c+d x]}{5 d \sec [c+d x]^{3 / 2}} + \frac{2 a \sin [c+d x]}{3 d \sqrt{\sec [c+d x]}}$$

Result (type 5, 224 leaves):



$$\begin{aligned}
& -\frac{1}{120 d} i a e^{-3 i (c+d x)} (1 + \operatorname{Cos}[c+d x]) \left( -3 - 10 e^{i (c+d x)} + 33 e^{2 i (c+d x)} + \right. \\
& \quad 39 e^{4 i (c+d x)} + 10 e^{5 i (c+d x)} + 3 e^{6 i (c+d x)} - 72 e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c+d x)} \right] + \\
& \quad \left. 40 e^{3 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c+d x)} \right] \right) \operatorname{Sec} \left[ \frac{1}{2} (c+d x) \right]^2 \sqrt{\operatorname{Sec}[c+d x]}
\end{aligned}$$

■ **Problem 297: Result unnecessarily involves higher level functions.**

$$\int \frac{a + a \operatorname{Cos}[c+d x]}{\operatorname{Sec}[c+d x]^{5/2}} dx$$

Optimal (type 4, 151 leaves, 9 steps):

$$\begin{aligned}
& \frac{6 a \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\operatorname{Sec}[c+d x]}}{5 d} + \\
& \frac{10 a \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\operatorname{Sec}[c+d x]}}{21 d} + \frac{2 a \operatorname{Sin}[c+d x]}{7 d \operatorname{Sec}[c+d x]^{5/2}} + \frac{2 a \operatorname{Sin}[c+d x]}{5 d \operatorname{Sec}[c+d x]^{3/2}} + \frac{10 a \operatorname{Sin}[c+d x]}{21 d \sqrt{\operatorname{Sec}[c+d x]}}
\end{aligned}$$

Result (type 5, 198 leaves):

$$\begin{aligned}
& \frac{1}{420 d} a e^{-4 i (c+d x)} \sqrt{\operatorname{Sec}[c+d x]} (\operatorname{Cos}[4 (c+d x)] + i \operatorname{Sin}[4 (c+d x)]) \\
& \quad \left( -504 i \operatorname{Cos}[c+d x] + 504 i e^{-i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c+d x)} \right] - 200 i \sqrt{1 + e^{2 i (c+d x)}} \right. \\
& \quad \left. \operatorname{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c+d x)} \right] + 42 \operatorname{Sin}[c+d x] + 130 \operatorname{Sin}[2 (c+d x)] + 42 \operatorname{Sin}[3 (c+d x)] + 15 \operatorname{Sin}[4 (c+d x)] \right)
\end{aligned}$$

■ **Problem 298: Result unnecessarily involves higher level functions.**

$$\int (a + a \operatorname{Cos}[c+d x])^2 \operatorname{Sec}[c+d x]^{7/2} dx$$

Optimal (type 4, 161 leaves, 9 steps):

$$\begin{aligned}
& -\frac{16 a^2 \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\operatorname{Sec}[c+d x]}}{5 d} + \frac{4 a^2 \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\operatorname{Sec}[c+d x]}}{3 d} + \\
& \frac{16 a^2 \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{5 d} + \frac{4 a^2 \operatorname{Sec}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{3 d} + \frac{2 a^2 \operatorname{Sec}[c+d x]^{5/2} \operatorname{Sin}[c+d x]}{5 d}
\end{aligned}$$

Result (type 5, 261 leaves):

$$\frac{1}{30 d} a^2 (1 + \cos [c + d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^4$$

$$\left( -1 / (-1 + e^{2 i c}) 2 i \sqrt{2} e^{-i(c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1 + e^{2 i(c+d x)}}} \left( 12 (1 + e^{2 i(c+d x)}) + 12 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right]\right) + \right.$$

$$\left. -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + 5 e^{i(c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \right) +$$

$$\sqrt{\operatorname{Sec}[c + d x]} (24 \cos [d x] \operatorname{Csc}[c] + (10 + 3 \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x])$$

■ **Problem 299: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c + d x])^2 \operatorname{Sec}[c + d x]^{5/2} dx$$

Optimal (type 4, 131 leaves, 8 steps):

$$-\frac{4 a^2 \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{d} +$$

$$\frac{8 a^2 \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \frac{4 a^2 \sqrt{\operatorname{Sec}[c + d x]} \sin [c + d x]}{d} + \frac{2 a^2 \operatorname{Sec}[c + d x]^{3/2} \sin [c + d x]}{3 d}$$

Result (type 5, 250 leaves):

$$\frac{1}{6 d} a^2 (1 + \cos [c + d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^4$$

$$\left( -1 / (-1 + e^{2 i c}) 2 i \sqrt{2} e^{-i(c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1 + e^{2 i(c+d x)}}} \left( 3 (1 + e^{2 i(c+d x)}) + 3 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + \right. \right.$$

$$\left. \left. 2 e^{i(c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right]\right) + \sqrt{\operatorname{Sec}[c + d x]} (6 \cos [d x] \operatorname{Csc}[c] + \operatorname{Tan}[c + d x]) \right)$$

■ **Problem 301: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c + d x])^2 \sqrt{\operatorname{Sec}[c + d x]} dx$$

Optimal (type 4, 107 leaves, 7 steps):

$$\frac{4 a^2 \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{d} + \frac{8 a^2 \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \frac{2 a^2 \sin [c + d x]}{3 d \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 5, 127 leaves) :

$$\frac{1}{3 d \sqrt{\operatorname{Sec}[c+d x]}} a^2 \left( \frac{24 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]}{\sqrt{1+e^{2 i(c+d x)}}} + \right. \\ \left. 2 \left( -6 i - 4 i \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \operatorname{Sec}[c+d x] + \operatorname{Sin}[c+d x] \right) \right)$$

■ **Problem 302: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Cos}[c+d x])^2}{\sqrt{\operatorname{Sec}[c+d x]}} dx$$

Optimal (type 4, 135 leaves, 8 steps) :

$$\frac{16 a^2 \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\operatorname{Sec}[c+d x]}}{5 d} + \\ \frac{4 a^2 \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\operatorname{Sec}[c+d x]}}{3 d} + \frac{2 a^2 \operatorname{Sin}[c+d x]}{5 d \operatorname{Sec}[c+d x]^{3/2}} + \frac{4 a^2 \operatorname{Sin}[c+d x]}{3 d \sqrt{\operatorname{Sec}[c+d x]}}$$

Result (type 5, 136 leaves) :

$$\frac{1}{30 d \sqrt{\operatorname{Sec}[c+d x]}} a^2 \left( -96 i + \frac{192 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]}{\sqrt{1+e^{2 i(c+d x)}}} - \right. \\ \left. 40 i \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \operatorname{Sec}[c+d x] + 40 \operatorname{Sin}[c+d x] + 6 \operatorname{Sin}[2(c+d x)] \right)$$

■ **Problem 303: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Cos}[c+d x])^2}{\operatorname{Sec}[c+d x]^{3/2}} dx$$

Optimal (type 4, 161 leaves, 9 steps) :

$$\frac{12 a^2 \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\operatorname{Sec}[c+d x]}}{5 d} + \\ \frac{8 a^2 \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\operatorname{Sec}[c+d x]}}{7 d} + \frac{2 a^2 \operatorname{Sin}[c+d x]}{7 d \operatorname{Sec}[c+d x]^{5/2}} + \frac{4 a^2 \operatorname{Sin}[c+d x]}{5 d \operatorname{Sec}[c+d x]^{3/2}} + \frac{8 a^2 \operatorname{Sin}[c+d x]}{7 d \sqrt{\operatorname{Sec}[c+d x]}}$$

Result (type 5, 149 leaves) :

$$\frac{1}{140 d \sqrt{\sec [c+d x]}} a^2 \left( \frac{672 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]}{\sqrt{1+e^{2 i(c+d x)}}} + 2 \left( -168 i - 80 i \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \sec [c+d x] + 85 \sin [c+d x] + 28 \sin [2(c+d x)] + 5 \sin [3(c+d x)] \right) \right)$$

■ **Problem 304: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c+d x])^3 \sec [c+d x]^{9/2} dx$$

Optimal (type 4, 187 leaves, 17 steps):

$$-\frac{28 a^3 \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} + \frac{52 a^3 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{21 d} + \frac{28 a^3 \sqrt{\sec [c+d x]} \sin [c+d x]}{5 d} + \frac{52 a^3 \sec [c+d x]^{3/2} \sin [c+d x]}{21 d} + \frac{6 a^3 \sec [c+d x]^{5/2} \sin [c+d x]}{5 d} + \frac{2 a^3 \sec [c+d x]^{7/2} \sin [c+d x]}{7 d}$$

Result (type 5, 279 leaves):

$$\frac{1}{420 d} a^3 (1 + \cos [c+d x])^3 \sec \left[ \frac{1}{2}(c+d x) \right]^6 \left( -\frac{1}{-1+e^{2 i c}} 2 i \sqrt{2} e^{-i(c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}} \left( 147 (1+e^{2 i(c+d x)}) + 147 (-1+e^{2 i c}) \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + 65 e^{i(c+d x)} (-1+e^{2 i c}) \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \right) + \sqrt{\sec [c+d x]} (294 \cos [d x] \operatorname{Csc}[c] + (80 + 63 \cos [c+d x] + 65 \cos [2(c+d x)]) \sec [c+d x]^2 \tan [c+d x]) \right)$$

■ **Problem 305: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c+d x])^3 \sec [c+d x]^{7/2} dx$$

Optimal (type 4, 157 leaves, 15 steps):

$$\begin{aligned}
& - \frac{36 a^3 \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} + \frac{4 a^3 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{d} \\
& + \frac{36 a^3 \sqrt{\sec [c+d x]} \sin [c+d x]}{5 d} + \frac{2 a^3 \sec [c+d x]^{3 / 2} \sin [c+d x]}{d} + \frac{2 a^3 \sec [c+d x]^{5 / 2} \sin [c+d x]}{5 d}
\end{aligned}$$

Result (type 5, 259 leaves):

$$\begin{aligned}
& \frac{1}{20 d} a^3 (1+\cos [c+d x])^3 \sec \left[\frac{1}{2}(c+d x)\right]^6 \\
& \left( -1 / \left(-1+e^{2 i c}\right) 2 i \sqrt{2} e^{-i(c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}} \left(9\left(1+e^{2 i(c+d x)}\right)+9\left(-1+e^{2 i c}\right) \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]\right) \right. \\
& \quad \left. + 5 e^{i(c+d x)}\left(-1+e^{2 i c}\right) \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right]\right) + \\
& \quad \left. \sqrt{\sec [c+d x]}(18 \cos [d x] \csc [c]+(5+\sec [c+d x]) \tan [c+d x])\right)
\end{aligned}$$

■ **Problem 306: Result unnecessarily involves higher level functions.**

$$\int (a+a \cos [c+d x])^3 \sec [c+d x]^{5 / 2} d x$$

Optimal (type 4, 131 leaves, 13 steps):

$$\begin{aligned}
& - \frac{4 a^3 \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{d} + \\
& \frac{20 a^3 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 d} + \frac{6 a^3 \sqrt{\sec [c+d x]} \sin [c+d x]}{d} + \frac{2 a^3 \sec [c+d x]^{3 / 2} \sin [c+d x]}{3 d}
\end{aligned}$$

Result (type 5, 157 leaves):

$$\begin{aligned}
& - \frac{1}{3 d} i a^3 \sec [c+d x]^{3 / 2} \left( -6-6 \cos [2(c+d x)]+6 e^{-2 i(c+d x)}\left(1+e^{2 i(c+d x)}\right)^{3 / 2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] \right. \\
& \quad \left. + 20 \sqrt{1+e^{2 i(c+d x)}} \cos [c+d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right]+2 i \sin [c+d x]+9 i \sin [2(c+d x)]\right)
\end{aligned}$$

■ **Problem 307: Result unnecessarily involves higher level functions.**

$$\int (a+a \cos [c+d x])^3 \sec [c+d x]^{3 / 2} d x$$

Optimal (type 4, 131 leaves, 13 steps):

$$\frac{4 a^3 \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{d} +$$

$$\frac{20 a^3 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 d} + \frac{2 a^3 \sin [c+d x]}{3 d \sqrt{\sec [c+d x]}} + \frac{2 a^3 \sqrt{\sec [c+d x]} \sin [c+d x]}{d}$$

Result (type 5, 135 leaves):

$$\frac{1}{3 d \sqrt{\sec [c+d x]}} a^3 \left( \frac{24 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]}{\sqrt{1+e^{2 i(c+d x)}}} + \right.$$

$$\left. 2 \left( -6 i - 10 i \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \sec [c+d x] + \sin [c+d x] + 3 \tan [c+d x] \right) \right)$$

■ **Problem 308: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c+d x])^3 \sqrt{\sec [c+d x]} dx$$

Optimal (type 4, 131 leaves, 13 steps):

$$\frac{36 a^3 \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} +$$

$$\frac{4 a^3 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{d} + \frac{2 a^3 \sin [c+d x]}{5 d \sec [c+d x]^{3/2}} + \frac{2 a^3 \sin [c+d x]}{d \sqrt{\sec [c+d x]}}$$

Result (type 5, 137 leaves):

$$\frac{1}{10 d \sqrt{\sec [c+d x]}} a^3 \left( \frac{144 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]}{\sqrt{1+e^{2 i(c+d x)}}} + \right.$$

$$\left. 2 \left( -36 i - 20 i \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \sec [c+d x] + 10 \sin [c+d x] + \sin [2(c+d x)] \right) \right)$$

■ **Problem 309: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \cos [c+d x])^3}{\sqrt{\sec [c+d x]}} dx$$

Optimal (type 4, 161 leaves, 15 steps):

$$\frac{28 a^3 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{5d} +$$

$$\frac{52 a^3 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{21d} + \frac{2 a^3 \sin[c+dx]}{7d \sec[c+dx]^{5/2}} + \frac{6 a^3 \sin[c+dx]}{5d \sec[c+dx]^{3/2}} + \frac{52 a^3 \sin[c+dx]}{21d \sqrt{\sec[c+dx]}}$$

Result (type 5, 146 leaves):

$$\frac{1}{420 d \sqrt{\sec[c+dx]}} a^3 \left( -2352 i + \frac{4704 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]}{\sqrt{1+e^{2i(c+dx)}}} - 1040 i \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \sec[c+dx] + \right.$$

$$\left. 1070 \sin[c+dx] + 252 \sin[2(c+dx)] + 30 \sin[3(c+dx)] \right)$$

■ **Problem 310: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \cos[c+dx])^3}{\sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 187 leaves, 17 steps):

$$\frac{68 a^3 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{15d} + \frac{44 a^3 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{21d} +$$

$$\frac{2 a^3 \sin[c+dx]}{9d \sec[c+dx]^{7/2}} + \frac{6 a^3 \sin[c+dx]}{7d \sec[c+dx]^{5/2}} + \frac{68 a^3 \sin[c+dx]}{45d \sec[c+dx]^{3/2}} + \frac{44 a^3 \sin[c+dx]}{21d \sqrt{\sec[c+dx]}}$$

Result (type 5, 156 leaves):

$$\frac{1}{2520 d \sqrt{\sec[c+dx]}} a^3 \left( -11424 i + \frac{22848 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]}{\sqrt{1+e^{2i(c+dx)}}} - 5280 i \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \sec[c+dx] + \right.$$

$$\left. 5820 \sin[c+dx] + 2044 \sin[2(c+dx)] + 540 \sin[3(c+dx)] + 70 \sin[4(c+dx)] \right)$$

■ **Problem 311: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos[c+dx])^4 \sec[c+dx]^{9/2} dx$$

Optimal (type 4, 187 leaves, 19 steps):

$$\begin{aligned}
 & - \frac{64 a^4 \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} + \frac{136 a^4 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{21 d} + \\
 & \frac{64 a^4 \sqrt{\sec [c+d x]} \sin [c+d x]}{5 d} + \frac{94 a^4 \sec [c+d x]^{3 / 2} \sin [c+d x]}{21 d} + \frac{8 a^4 \sec [c+d x]^{5 / 2} \sin [c+d x]}{5 d} + \frac{2 a^4 \sec [c+d x]^{7 / 2} \sin [c+d x]}{7 d}
 \end{aligned}$$

Result (type 5, 271 leaves):

$$\begin{aligned}
 & \frac{1}{840 d} a^4 (1 + \cos [c+d x])^4 \sec \left[\frac{1}{2}(c+d x)\right]^8 \\
 & \left( - \frac{1}{-1 + e^{2 i c}} 4 i \sqrt{2} e^{-i(c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1 + e^{2 i(c+d x)}}} \left( 168 (1 + e^{2 i(c+d x)}) + 168 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] \right) + \right. \\
 & \quad \left. 85 e^{i(c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \right) + \\
 & \quad \left. \sqrt{\sec [c+d x]} (672 \cos [d x] \operatorname{Csc}[c] + (235 + 84 \sec [c+d x] + 15 \sec [c+d x]^2) \tan [c+d x]) \right)
 \end{aligned}$$

■ **Problem 312: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c+d x])^4 \sec [c+d x]^{7 / 2} dx$$

Optimal (type 4, 161 leaves, 17 steps):

$$\begin{aligned}
 & - \frac{56 a^4 \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} + \frac{32 a^4 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 d} + \\
 & \frac{66 a^4 \sqrt{\sec [c+d x]} \sin [c+d x]}{5 d} + \frac{8 a^4 \sec [c+d x]^{3 / 2} \sin [c+d x]}{3 d} + \frac{2 a^4 \sec [c+d x]^{5 / 2} \sin [c+d x]}{5 d}
 \end{aligned}$$

Result (type 5, 278 leaves):



$$\frac{1}{240 d} a^4 (1 + \operatorname{Cos}[c + d x])^4 \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^8$$

$$\left( -\frac{1}{-1 + e^{2 i c}} 8 i \sqrt{2} e^{-i(c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1 + e^{2 i(c+d x)}}} \left( 21 (1 + e^{2 i(c+d x)}) + 21 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] \right) + \right.$$

$$\left. 20 e^{i(c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \right) +$$

$$\left. \sqrt{\operatorname{Sec}[c + d x]} (-3 (-61 + 5 \operatorname{Cos}[2 c]) \operatorname{Cos}[d x] \operatorname{Csc}[c] + 30 \operatorname{Cos}[c] \operatorname{Sin}[d x] + 2 (20 + 3 \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]) \right)$$

■ **Problem 314: Result unnecessarily involves higher level functions.**

$$\int (a + a \operatorname{Cos}[c + d x])^4 \operatorname{Sec}[c + d x]^{3/2} dx$$

Optimal (type 4, 159 leaves, 16 steps):

$$\frac{56 a^4 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 d} +$$

$$\frac{32 a^4 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \frac{2 a^4 \operatorname{Sin}[c + d x]}{5 d \operatorname{Sec}[c + d x]^{3/2}} + \frac{8 a^4 \operatorname{Sin}[c + d x]}{3 d \sqrt{\operatorname{Sec}[c + d x]}} + \frac{2 a^4 \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{d}$$

Result (type 5, 150 leaves):

$$\frac{1}{30 d \sqrt{\operatorname{Sec}[c + d x]}}$$

$$a^4 \left( -336 i + \frac{672 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]}{\sqrt{1 + e^{2 i(c+d x)}}} - 320 i \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \operatorname{Sec}[c + d x] + \right.$$

$$\left. 80 \operatorname{Sin}[c + d x] + 3 \operatorname{Sec}[c + d x] \operatorname{Sin}[3(c + d x)] + 63 \operatorname{Tan}[c + d x] \right)$$

■ **Problem 315: Result unnecessarily involves higher level functions.**

$$\int (a + a \operatorname{Cos}[c + d x])^4 \sqrt{\operatorname{Sec}[c + d x]} dx$$

Optimal (type 4, 161 leaves, 17 steps):

$$\frac{64 a^4 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{5d} +$$

$$\frac{136 a^4 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{21d} + \frac{2 a^4 \sin[c+dx]}{7d \sec[c+dx]^{5/2}} + \frac{8 a^4 \sin[c+dx]}{5d \sec[c+dx]^{3/2}} + \frac{94 a^4 \sin[c+dx]}{21d \sqrt{\sec[c+dx]}}$$

Result (type 5, 146 leaves):

$$\frac{1}{420d \sqrt{\sec[c+dx]}} a^4 \left( -5376i + \frac{10752i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]}{\sqrt{1+e^{2i(c+dx)}}} - \right.$$

$$\left. 2720i \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \sec[c+dx] + 1910 \sin[c+dx] + 336 \sin[2(c+dx)] + 30 \sin[3(c+dx)] \right)$$

■ **Problem 316: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \cos[c+dx])^4}{\sqrt{\sec[c+dx]}} dx$$

Optimal (type 4, 187 leaves, 19 steps):

$$\frac{152 a^4 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{15d} + \frac{32 a^4 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{7d} +$$

$$\frac{2 a^4 \sin[c+dx]}{9d \sec[c+dx]^{7/2}} + \frac{8 a^4 \sin[c+dx]}{7d \sec[c+dx]^{5/2}} + \frac{122 a^4 \sin[c+dx]}{45d \sec[c+dx]^{3/2}} + \frac{32 a^4 \sin[c+dx]}{7d \sqrt{\sec[c+dx]}}$$

Result (type 5, 156 leaves):

$$\frac{1}{2520d \sqrt{\sec[c+dx]}} a^4 \left( -25536i + \frac{51072i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]}{\sqrt{1+e^{2i(c+dx)}}} - 11520i \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right.$$

$$\left. \sec[c+dx] + 12240 \sin[c+dx] + 3556 \sin[2(c+dx)] + 720 \sin[3(c+dx)] + 70 \sin[4(c+dx)] \right)$$

■ **Problem 317: Result unnecessarily involves higher level functions.**

$$\int \frac{\sec[c+dx]^{5/2}}{a + a \cos[c+dx]} dx$$

Optimal (type 4, 164 leaves, 9 steps):

$$\frac{3 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{ad} + \frac{5 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3ad} -$$

$$\frac{3 \sqrt{\sec[c+dx]} \sin[c+dx]}{ad} + \frac{5 \sec[c+dx]^{3/2} \sin[c+dx]}{3ad} - \frac{\sec[c+dx]^{5/2} \sin[c+dx]}{d(a+a \sec[c+dx])}$$

Result (type 5, 285 leaves):

$$\frac{1}{3ad(1+\cos[c+dx])} \cos\left[\frac{1}{2}(c+dx)\right]^2$$

$$\left( \frac{1}{-1+e^{2ic}} 2i\sqrt{2} e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \left( 9(1+e^{2i(c+dx)}) + 9(-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - \right. \right.$$

$$\left. \left. 5e^{i(c+dx)}(-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) - \right.$$

$$\left. \left. \sqrt{\sec[c+dx]} \left( 18 \cos[dx] \operatorname{Csc}[c] + \sec[c+dx] \left( -5 \sec\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] + \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) \right)$$

■ **Problem 318: Result unnecessarily involves higher level functions.**

$$\int \frac{\sec[c+dx]^{3/2}}{a+a \cos[c+dx]} dx$$

Optimal (type 4, 136 leaves, 8 steps):

$$-\frac{3 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{ad} -$$

$$\frac{\sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{ad} + \frac{3 \sqrt{\sec[c+dx]} \sin[c+dx]}{ad} - \frac{\sec[c+dx]^{3/2} \sin[c+dx]}{d(a+a \sec[c+dx])}$$

Result (type 5, 256 leaves):

$$\frac{1}{a(1+\cos[c+dx])} \cos\left[\frac{1}{2}(c+dx)\right]^2 \left( -1/(d(-1+e^{2ic})) 2i\sqrt{2} e^{-i(c+dx)} \right.$$

$$\left. \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \left( 3(1+e^{2i(c+dx)}) + 3(-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - \right. \right.$$

$$\left. \left. e^{i(c+dx)}(-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) + \frac{\sqrt{\sec[c+dx]} (6 \cos[dx] \operatorname{Csc}[c] - 2 \tan\left[\frac{1}{2}(c+dx)\right])}{d} \right)$$

■ **Problem 319: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{\sec[c+dx]}}{a+a\cos[c+dx]} dx$$

Optimal (type 4, 110 leaves, 7 steps):

$$\frac{\sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{ad} + \frac{\sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{ad} - \frac{\sqrt{\sec[c+dx]} \sin[c+dx]}{d(a+a\sec[c+dx])}$$

Result (type 5, 180 leaves):

$$-\frac{1}{ad(1+e^{i(c+dx)})^3} 4i \cos\left[\frac{1}{2}(c+dx)\right]^2 \left(1+e^{2i(c+dx)} - (1+e^{i(c+dx)}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) + e^{i(c+dx)} (1+e^{i(c+dx)}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \sqrt{\sec[c+dx]}$$

■ **Problem 320: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+a\cos[c+dx]) \sqrt{\sec[c+dx]}} dx$$

Optimal (type 4, 110 leaves, 7 steps):

$$-\frac{\sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{ad} + \frac{\sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{ad} + \frac{\sqrt{\sec[c+dx]} \sin[c+dx]}{d(a+a\sec[c+dx])}$$

Result (type 5, 181 leaves):

$$-\frac{1}{ad(1+e^{i(c+dx)})^3} 4i \cos\left[\frac{1}{2}(c+dx)\right]^2 \left(-1-e^{2i(c+dx)} + (1+e^{i(c+dx)}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) + e^{i(c+dx)} (1+e^{i(c+dx)}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \sqrt{\sec[c+dx]}$$

■ **Problem 321: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+a\cos[c+dx]) \sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 112 leaves, 7 steps):

$$\frac{3\sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{ad} - \frac{\sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{ad} - \frac{\sqrt{\sec[c+dx]} \sin[c+dx]}{d(a+a\sec[c+dx])}$$

Result (type 5, 311 leaves):

$$\frac{1}{a (1 + \cos [c + d x])}$$

$$\cos \left[ \frac{1}{2} (c + d x) \right]^2 \left( \frac{1}{d (-1 + e^{2 i c})} 2 i \sqrt{2} e^{-i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \left( 3 (1 + e^{2 i (c + d x)}) + 3 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ \right. \right. \right.$$

$$\left. \left. \left. -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] + e^{i (c + d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c + d x)} \right] \right) - \right.$$

$$\left. \frac{1}{2 d} \left( \cos \left[ \frac{1}{2} (c - d x) \right] + 2 \cos \left[ \frac{1}{2} (3 c + d x) \right] + 2 \cos \left[ \frac{1}{2} (c + 3 d x) \right] + \cos \left[ \frac{1}{2} (5 c + 3 d x) \right] \right) \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right] \sqrt{\operatorname{Sec} [c + d x]} \right)$$

■ **Problem 322: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \cos [c + d x]) \operatorname{Sec} [c + d x]^{5/2}} dx$$

Optimal (type 4, 140 leaves, 8 steps):

$$-\frac{3 \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{a d} +$$

$$\frac{5 \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{3 a d} + \frac{5 \sin [c + d x]}{3 a d \sqrt{\operatorname{Sec} [c + d x]}} - \frac{\sin [c + d x]}{d \sqrt{\operatorname{Sec} [c + d x]} (a + a \operatorname{Sec} [c + d x])}$$

Result (type 5, 374 leaves):

$$-\left( 2 i \sqrt{2} e^{-i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \left( 9 (1 + e^{2 i (c + d x)}) + 9 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] + \right. \right.$$

$$\left. \left. 5 e^{i (c + d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c + d x)} \right] \right) \right) / (3 d (-1 + e^{2 i c}) (a + a \cos [c + d x])) +$$

$$\frac{1}{a + a \cos [c + d x]} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \sqrt{\operatorname{Sec} [c + d x]} \left( \frac{(2 + \cos [2 c]) \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{d} + \frac{2 \cos [2 d x] \sin [2 c]}{3 d} - \right.$$

$$\left. \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] \sin \left[ \frac{d x}{2} \right]}{d} - \frac{4 \cos [c] \sin [d x]}{d} + \frac{2 \cos [2 c] \sin [2 d x]}{3 d} - \frac{2 \operatorname{Tan} \left[ \frac{c}{2} \right]}{d} \right)$$

- **Problem 323: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \cos[c + dx]) \sec[c + dx]^{7/2}} dx$$

Optimal (type 4, 168 leaves, 9 steps):

$$\frac{21 \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]} - 5 \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{5 a d} + \frac{7 \sin[c + dx]}{5 a d \sec[c + dx]^{3/2}} - \frac{5 \sin[c + dx]}{3 a d \sqrt{\sec[c + dx]}} - \frac{\sin[c + dx]}{d \sec[c + dx]^{3/2} (a + a \sec[c + dx])}$$

Result (type 5, 341 leaves):

$$\frac{1}{60 a d (1 + \cos[c + dx])} \cos\left[\frac{1}{2}(c + dx)\right]^2 \left( \frac{1}{-1 + e^{2ic}} 8 i \sqrt{2} e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \left( 63 (1 + e^{2i(c+dx)}) + 63 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) + 25 e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) - \sqrt{\sec[c + dx]} \left( 18 (17 + 11 \cos[2c]) \cos[dx] \operatorname{Csc}[c] + 4 \left( 10 \cos[2dx] \sin[2c] - 3 \cos[3dx] \sin[3c] - 30 \sec\left[\frac{c}{2}\right] \sec\left[\frac{1}{2}(c + dx)\right] \sin\left[\frac{dx}{2}\right] - 99 \cos[c] \sin[dx] + 10 \cos[2c] \sin[2dx] - 3 \cos[3c] \sin[3dx] - 30 \tan\left[\frac{c}{2}\right] \right) \right)$$

- **Problem 324: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^{5/2}}{(a + a \cos[c + dx])^2} dx$$

Optimal (type 4, 202 leaves, 10 steps):

$$\frac{7 \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]} + 10 \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{a^2 d} - \frac{7 \sqrt{\sec[c + dx]} \sin[c + dx]}{a^2 d} + \frac{10 \sec[c + dx]^{3/2} \sin[c + dx]}{3 a^2 d} - \frac{7 \sec[c + dx]^{5/2} \sin[c + dx]}{3 a^2 d (1 + \sec[c + dx])} - \frac{\sec[c + dx]^{7/2} \sin[c + dx]}{3 d (a + a \sec[c + dx])^2}$$

Result (type 5, 443 leaves):

$$\frac{1}{d (a + a \cos [c + d x])^2} 7 \sqrt{2} e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right]$$

$$\left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] +$$

$$\frac{20 \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\operatorname{Sec} [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^2} + \frac{1}{(a + a \cos [c + d x])^2}$$

$$\cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\operatorname{Sec} [c + d x]} \left( -\frac{14 \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{d} + \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} [c] (-3 \sin \left[ \frac{c}{2} \right] + 5 \sin \left[ \frac{3 c}{2} \right])}{3 d} \right) +$$

$$\left( \frac{32 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] \sin \left[ \frac{d x}{2} \right]}{3 d} + \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \sin \left[ \frac{d x}{2} \right]}{3 d} + \frac{8 \operatorname{Sec} [c] \operatorname{Sec} [c + d x] \sin [d x]}{3 d} + \frac{2 \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Tan} \left[ \frac{c}{2} \right]}{3 d} \right)$$

■ **Problem 325: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sec} [c + d x]^{3/2}}{(a + a \cos [c + d x])^2} dx$$

Optimal (type 4, 176 leaves, 9 steps):

$$-\frac{4 \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{a^2 d} - \frac{5 \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{3 a^2 d} +$$

$$\frac{4 \sqrt{\operatorname{Sec} [c + d x]} \sin [c + d x]}{a^2 d} - \frac{5 \operatorname{Sec} [c + d x]^{3/2} \sin [c + d x]}{3 a^2 d (1 + \operatorname{Sec} [c + d x])} - \frac{\operatorname{Sec} [c + d x]^{5/2} \sin [c + d x]}{3 d (a + a \operatorname{Sec} [c + d x])^2}$$

Result (type 5, 259 leaves):

$$-\frac{1}{6 a^2 d (1 + \cos [c + d x])^2}$$

$$e^{-i (2 c + d x)} \cos \left[ \frac{1}{2} (c + d x) \right] \sqrt{\operatorname{Sec} [c + d x]} \left( 12 i e^{-2 i (c + d x)} (1 + e^{i (c + d x)})^3 \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] + \right.$$

$$40 \cos \left[ \frac{1}{2} (c + d x) \right]^3 \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \left( \cos \left[ \frac{1}{2} (c + d x) \right] + i \sin \left[ \frac{1}{2} (c + d x) \right] \right) -$$

$$\left. i (29 + 50 \cos [c + d x] + 17 \cos [2 (c + d x)] - 12 i \sin [c + d x] - 7 i \sin [2 (c + d x)]) \right) \left( \cos \left[ \frac{1}{2} (3 c + d x) \right] + i \sin \left[ \frac{1}{2} (3 c + d x) \right] \right)$$

■ **Problem 326: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{\operatorname{Sec} [c + d x]}}{(a + a \cos [c + d x])^2} dx$$

Optimal (type 4, 149 leaves, 8 steps) :

$$\frac{\sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a^2 d} + \frac{2 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3 a^2 d} - \frac{\sqrt{\sec[c+dx]} \sin[c+dx]}{a^2 d (1 + \sec[c+dx])} - \frac{\sec[c+dx]^{3/2} \sin[c+dx]}{3 d (a + a \sec[c+dx])^2}$$

Result (type 5, 249 leaves) :

$$\frac{1}{6 a^2 d (1 + \cos[c+dx])^2} e^{-i(2c+dx)} \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{\sec[c+dx]} \left( 3 i e^{-2i(c+dx)} (1 + e^{i(c+dx)})^3 \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 16 \cos\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \left( \cos\left[\frac{1}{2}(c+dx)\right] + i \sin\left[\frac{1}{2}(c+dx)\right] \right) - i (5 + 14 \cos[c+dx] + 5 \cos[2(c+dx)] - i \sin[2(c+dx)]) \right) \left( \cos\left[\frac{1}{2}(3c+dx)\right] + i \sin\left[\frac{1}{2}(3c+dx)\right] \right)$$

■ **Problem 328: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + a \cos[c+dx])^2 \sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 149 leaves, 8 steps) :

$$-\frac{\sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a^2 d} + \frac{2 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3 a^2 d} + \frac{\sqrt{\sec[c+dx]} \sin[c+dx]}{a^2 d (1 + \sec[c+dx])} - \frac{\sec[c+dx]^{3/2} \sin[c+dx]}{3 d (a + a \sec[c+dx])^2}$$

Result (type 5, 247 leaves) :

$$\frac{1}{6 a^2 d (1 + \cos[c+dx])^2} e^{-i(2c+dx)} \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{\sec[c+dx]} \left( 16 \cos\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \left( \cos\left[\frac{1}{2}(c+dx)\right] + i \sin\left[\frac{1}{2}(c+dx)\right] \right) - i \left( -7 - 10 \cos[c+dx] - 7 \cos[2(c+dx)] + 3 e^{-2i(c+dx)} (1 + e^{i(c+dx)})^3 \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - i \sin[2(c+dx)] \right) \right) \left( \cos\left[\frac{1}{2}(3c+dx)\right] + i \sin\left[\frac{1}{2}(3c+dx)\right] \right)$$



■ **Problem 329: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + a \cos[c + dx])^2 \sec[c + dx]^{5/2}} dx$$

Optimal (type 4, 152 leaves, 8 steps):

$$\frac{4 \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{a^2 d} - \frac{5 \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{3 a^2 d} - \frac{5 \sqrt{\sec[c + dx]} \sin[c + dx]}{3 a^2 d (1 + \sec[c + dx])} - \frac{\sqrt{\sec[c + dx]} \sin[c + dx]}{3 d (a + a \sec[c + dx])^2}$$

Result (type 5, 231 leaves):

$$-\frac{1}{12 a^2 d (1 + \cos[c + dx])^2} i e^{-3 i (c + dx)} (1 + e^{i (c + dx)}) \left( 9 + 20 e^{i (c + dx)} + 25 e^{2 i (c + dx)} + 23 e^{3 i (c + dx)} + 16 e^{4 i (c + dx)} + 3 e^{5 i (c + dx)} - 5 i e^{i (c + dx)} (1 + e^{i (c + dx)})^3 \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] - 12 (1 + e^{i (c + dx)})^3 \sqrt{1 + e^{2 i (c + dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + dx)}\right] \right) \sqrt{\sec[c + dx]}$$

■ **Problem 330: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + a \cos[c + dx])^2 \sec[c + dx]^{7/2}} dx$$

Optimal (type 4, 178 leaves, 9 steps):

$$-\frac{7 \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{a^2 d} + \frac{10 \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{3 a^2 d} + \frac{10 \sin[c + dx]}{3 a^2 d \sqrt{\sec[c + dx]}} - \frac{7 \sin[c + dx]}{3 a^2 d \sqrt{\sec[c + dx]} (1 + \sec[c + dx])} - \frac{\sin[c + dx]}{3 d \sqrt{\sec[c + dx]} (a + a \sec[c + dx])^2}$$

Result (type 5, 270 leaves):

$$\frac{1}{3 a^2 (1 + \cos[c + dx])^2} \cos\left[\frac{1}{2}(c + dx)\right]^4 \left( \frac{40 \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{d} + \frac{1}{d (1 + e^{i (c + dx)})^3} 2 i e^{-2 i (c + dx)} \left( 1 + 33 e^{i (c + dx)} + 73 e^{2 i (c + dx)} + 87 e^{3 i (c + dx)} + 81 e^{4 i (c + dx)} + 53 e^{5 i (c + dx)} + 9 e^{6 i (c + dx)} - e^{7 i (c + dx)} - 42 e^{i (c + dx)} (1 + e^{i (c + dx)})^3 \sqrt{1 + e^{2 i (c + dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + dx)}\right] \right) \sqrt{\sec[c + dx]} \right)$$

■ **Problem 331: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + a \cos[c + dx])^2 \sec[c + dx]^{9/2}} dx$$

Optimal (type 4, 200 leaves, 10 steps):

$$\frac{56 \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]} - 5 \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{5 a^2 d} + \frac{56 \sin[c + dx]}{15 a^2 d \sec[c + dx]^{3/2}} - \frac{5 \sin[c + dx]}{a^2 d \sqrt{\sec[c + dx]}} - \frac{3 \sin[c + dx]}{a^2 d \sec[c + dx]^{3/2} (1 + \sec[c + dx])} - \frac{\sin[c + dx]}{3 d \sec[c + dx]^{3/2} (a + a \sec[c + dx])^2}$$

Result (type 5, 298 leaves):

$$-\frac{1}{15 a^2 d (1 + e^{i(c+dx)})^7} + 4 i e^{-i(c+dx)} \cos\left[\frac{1}{2}(c + dx)\right]^4 \left( -3 + 11 e^{i(c+dx)} + 504 e^{2i(c+dx)} + 1156 e^{3i(c+dx)} + 1378 e^{4i(c+dx)} + 1310 e^{5i(c+dx)} + 860 e^{6i(c+dx)} + 168 e^{7i(c+dx)} - 11 e^{8i(c+dx)} + 3 e^{9i(c+dx)} - 300 i e^{3i(c+dx)} (1 + e^{i(c+dx)})^3 \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] - 672 e^{2i(c+dx)} (1 + e^{i(c+dx)})^3 \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sqrt{\sec[c + dx]}$$

■ **Problem 332: Result unnecessarily involves higher level functions.**

$$\int \frac{\sec[c + dx]^{3/2}}{(a + a \cos[c + dx])^3} dx$$

Optimal (type 4, 221 leaves, 10 steps):

$$-\frac{49 \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]} - 13 \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{10 a^3 d} + \frac{49 \sqrt{\sec[c + dx]} \sin[c + dx]}{10 a^3 d} - \frac{\sec[c + dx]^{7/2} \sin[c + dx]}{5 d (a + a \sec[c + dx])^3} - \frac{8 \sec[c + dx]^{5/2} \sin[c + dx]}{15 a d (a + a \sec[c + dx])^2} - \frac{13 \sec[c + dx]^{3/2} \sin[c + dx]}{6 d (a^3 + a^3 \sec[c + dx])}$$

Result (type 5, 363 leaves):

$$\frac{1}{15 a^3 d (1 + \cos [c + d x])^3} 2 \cos \left[ \frac{1}{2} (c + d x) \right]^6$$

$$\left( -\frac{1}{-1 + e^{2 i c}} 2 i \sqrt{2} e^{-i (c+d x)} \sqrt{\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}}} \left( 147 (1 + e^{2 i (c+d x)}) + 147 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c+d x)} \right] - \right. \right.$$

$$\left. 65 e^{i (c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c+d x)} \right] \right) +$$

$$\frac{1}{32} \left( 1284 \cos \left[ \frac{1}{2} (c - d x) \right] + 921 \cos \left[ \frac{1}{2} (3 c + d x) \right] + 1243 \cos \left[ \frac{1}{2} (c + 3 d x) \right] + 374 \cos \left[ \frac{1}{2} (5 c + 3 d x) \right] + 670 \cos \left[ \frac{1}{2} (3 c + 5 d x) \right] + \right.$$

$$\left. 65 \cos \left[ \frac{1}{2} (7 c + 5 d x) \right] + 147 \cos \left[ \frac{1}{2} (5 c + 7 d x) \right] \right) \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right]^5 \sqrt{\operatorname{Sec} [c + d x]}$$

■ **Problem 333: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{\operatorname{Sec} [c + d x]}}{(a + a \cos [c + d x])^3} dx$$

Optimal (type 4, 195 leaves, 9 steps):

$$\frac{9 \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{10 a^3 d} + \frac{\sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{2 a^3 d} -$$

$$\frac{\operatorname{Sec} [c + d x]^{5/2} \operatorname{Sin} [c + d x]}{5 d (a + a \operatorname{Sec} [c + d x])^3} - \frac{2 \operatorname{Sec} [c + d x]^{3/2} \operatorname{Sin} [c + d x]}{5 a d (a + a \operatorname{Sec} [c + d x])^2} - \frac{9 \sqrt{\operatorname{Sec} [c + d x]} \operatorname{Sin} [c + d x]}{10 d (a^3 + a^3 \operatorname{Sec} [c + d x])}$$

Result (type 5, 281 leaves):

$$\frac{1}{40 a^3 d (1 + \cos [c + d x])^3}$$

$$e^{-i (2 c+d x)} \cos \left[ \frac{1}{2} (c + d x) \right] \sqrt{\operatorname{Sec} [c + d x]} \left( 9 i e^{-3 i (c+d x)} (1 + e^{i (c+d x)})^5 \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c+d x)} \right] + \right.$$

$$160 \cos \left[ \frac{1}{2} (c + d x) \right]^5 \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \left( \cos \left[ \frac{1}{2} (c + d x) \right] + i \sin \left[ \frac{1}{2} (c + d x) \right] \right) -$$

$$\left. 2 i (34 + 69 \cos [c + d x] + 34 \cos [2 (c + d x)] + 7 \cos [3 (c + d x)] - 2 i \sin [c + d x] - 6 i \sin [2 (c + d x)] - 2 i \sin [3 (c + d x)]) \right)$$

$$\left( \cos \left[ \frac{1}{2} (3 c + d x) \right] + i \sin \left[ \frac{1}{2} (3 c + d x) \right] \right)$$

■ **Problem 334: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + a \operatorname{Cos}[c + d x])^3 \sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 195 leaves, 9 steps):

$$\frac{\sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{10 a^3 d} + \frac{\sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{6 a^3 d} - \frac{\operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{5 d (a + a \operatorname{Sec}[c + d x])^3} - \frac{4 \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{15 a d (a + a \operatorname{Sec}[c + d x])^2} + \frac{\sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{6 d (a^3 + a^3 \operatorname{Sec}[c + d x])}$$

Result (type 5, 363 leaves):

$$\frac{1}{15 a^3 d (1 + \operatorname{Cos}[c + d x])^3} 2 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^6 \left( \frac{1}{-1 + e^{2 i c}} 2 i \sqrt{2} e^{-i(c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1 + e^{2 i(c+d x)}}} \left( 3 (1 + e^{2 i(c+d x)}) + 3 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] - 5 e^{i(c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \right) - \frac{1}{32} \left( 36 \operatorname{Cos}\left[\frac{1}{2}(c - d x)\right] + 9 \operatorname{Cos}\left[\frac{1}{2}(3 c + d x)\right] + 7 \operatorname{Cos}\left[\frac{1}{2}(c + 3 d x)\right] + 26 \operatorname{Cos}\left[\frac{1}{2}(5 c + 3 d x)\right] + 10 \operatorname{Cos}\left[\frac{1}{2}(3 c + 5 d x)\right] + 5 \operatorname{Cos}\left[\frac{1}{2}(7 c + 5 d x)\right] + 3 \operatorname{Cos}\left[\frac{1}{2}(5 c + 7 d x)\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^5 \sqrt{\operatorname{Sec}[c + d x]} \right)$$

■ **Problem 335: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + a \operatorname{Cos}[c + d x])^3 \operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 4, 195 leaves, 9 steps):

$$- \frac{\sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{10 a^3 d} + \frac{\sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{6 a^3 d} + \frac{\operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{5 d (a + a \operatorname{Sec}[c + d x])^3} - \frac{\sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{15 a d (a + a \operatorname{Sec}[c + d x])^2} + \frac{\sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{6 d (a^3 + a^3 \operatorname{Sec}[c + d x])}$$

Result (type 5, 363 leaves):

$$\frac{1}{15 a^3 d (1 + \cos [c + d x])^3} 2 \cos \left[ \frac{1}{2} (c + d x) \right]^6$$

$$\left( -\frac{1}{-1 + e^{2 i c}} 2 i \sqrt{2} e^{-i (c+d x)} \sqrt{\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}}} \left( 3 (1 + e^{2 i (c+d x)}) + 3 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c+d x)} \right] + \right. \right.$$

$$\left. 5 e^{i (c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c+d x)} \right] \right) +$$

$$\frac{1}{32} \left( 36 \cos \left[ \frac{1}{2} (c - d x) \right] + 9 \cos \left[ \frac{1}{2} (3 c + d x) \right] + 17 \cos \left[ \frac{1}{2} (c + 3 d x) \right] + 16 \cos \left[ \frac{1}{2} (5 c + 3 d x) \right] + 20 \cos \left[ \frac{1}{2} (3 c + 5 d x) \right] - \right.$$

$$\left. 5 \cos \left[ \frac{1}{2} (7 c + 5 d x) \right] + 3 \cos \left[ \frac{1}{2} (5 c + 7 d x) \right] \right) \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right]^5 \sqrt{\operatorname{Sec} [c + d x]}$$

■ **Problem 336: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + a \cos [c + d x])^3 \operatorname{Sec} [c + d x]^{5/2}} dx$$

Optimal (type 4, 195 leaves, 9 steps):

$$-\frac{9 \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{10 a^3 d} + \frac{\sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{2 a^3 d}$$

$$\frac{\operatorname{Sec} [c + d x]^{3/2} \operatorname{Sin} [c + d x]}{5 d (a + a \operatorname{Sec} [c + d x])^3} + \frac{2 \sqrt{\operatorname{Sec} [c + d x]} \operatorname{Sin} [c + d x]}{5 a d (a + a \operatorname{Sec} [c + d x])^2} + \frac{\sqrt{\operatorname{Sec} [c + d x]} \operatorname{Sin} [c + d x]}{2 d (a^3 + a^3 \operatorname{Sec} [c + d x])}$$

Result (type 5, 281 leaves):

$$\frac{1}{40 a^3 d (1 + \cos [c + d x])^3}$$

$$e^{-i (2 c+d x)} \cos \left[ \frac{1}{2} (c + d x) \right] \sqrt{\operatorname{Sec} [c + d x]} \left( -9 i e^{-3 i (c+d x)} (1 + e^{i (c+d x)})^5 \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c+d x)} \right] + \right.$$

$$160 \cos \left[ \frac{1}{2} (c + d x) \right]^5 \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \left( \cos \left[ \frac{1}{2} (c + d x) \right] + i \sin \left[ \frac{1}{2} (c + d x) \right] \right) +$$

$$\left. 2 i (34 + 64 \cos [c + d x] + 34 \cos [2 (c + d x)] + 12 \cos [3 (c + d x)] + 3 i \sin [c + d x] + 4 i \sin [2 (c + d x)] + 3 i \sin [3 (c + d x)]) \right)$$

$$\left( \cos \left[ \frac{1}{2} (3 c + d x) \right] + i \sin \left[ \frac{1}{2} (3 c + d x) \right] \right)$$

■ **Problem 337: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + a \cos[c + dx])^3 \sec[c + dx]^{7/2}} dx$$

Optimal (type 4, 195 leaves, 9 steps):

$$\frac{49 \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{10 a^3 d} - \frac{13 \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{6 a^3 d} - \frac{\sqrt{\sec[c + dx]} \sin[c + dx]}{5 d (a + a \sec[c + dx])^3} - \frac{8 \sqrt{\sec[c + dx]} \sin[c + dx]}{15 a d (a + a \sec[c + dx])^2} - \frac{13 \sqrt{\sec[c + dx]} \sin[c + dx]}{6 d (a^3 + a^3 \sec[c + dx])}$$

Result (type 5, 378 leaves):

$$\frac{1}{15 a^3 d (1 + \cos[c + dx])^3} 2 \cos\left[\frac{1}{2}(c + dx)\right]^6 \left( \frac{1}{-1 + e^{2ic}} 2i \sqrt{2} e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \left( 147 (1 + e^{2i(c+dx)}) + 147 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 65 e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) - \frac{1}{32} \left( 1134 \cos\left[\frac{1}{2}(c - dx)\right] + 1071 \cos\left[\frac{1}{2}(3c + dx)\right] + 923 \cos\left[\frac{1}{2}(c + 3dx)\right] + 694 \cos\left[\frac{1}{2}(5c + 3dx)\right] + 470 \cos\left[\frac{1}{2}(3c + 5dx)\right] + 265 \cos\left[\frac{1}{2}(7c + 5dx)\right] + 117 \cos\left[\frac{1}{2}(5c + 7dx)\right] + 30 \cos\left[\frac{1}{2}(9c + 7dx)\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 \sqrt{\sec[c + dx]} \right)$$

■ **Problem 338: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \cos[c + dx])^3 \sec[c + dx]^{9/2}} dx$$

Optimal (type 4, 221 leaves, 10 steps):

$$-\frac{119 \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{10 a^3 d} + \frac{11 \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{2 a^3 d} + \frac{11 \sin[c + dx]}{2 a^3 d \sqrt{\sec[c + dx]}} - \frac{\sin[c + dx]}{5 d \sqrt{\sec[c + dx]} (a + a \sec[c + dx])^3} - \frac{2 \sin[c + dx]}{3 a d \sqrt{\sec[c + dx]} (a + a \sec[c + dx])^2} - \frac{119 \sin[c + dx]}{30 d \sqrt{\sec[c + dx]} (a^3 + a^3 \sec[c + dx])}$$

Result (type 5, 521 leaves):

$$\begin{aligned}
& - \frac{1}{5 d (a + a \cos [c + d x])^3} 119 \sqrt{2} e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \\
& \operatorname{Csc} \left[ \frac{c}{2} \right] \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] + \\
& \frac{22 \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\operatorname{Sec} [c + d x]} \sin [c]}{d (a + a \cos [c + d x])^3} + \frac{1}{(a + a \cos [c + d x])^3} \\
& \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\operatorname{Sec} [c + d x]} \left( \frac{2 (89 + 30 \cos [2 c]) \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{5 d} + \frac{8 \cos [2 d x] \sin [2 c]}{3 d} - \right. \\
& \frac{172 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] \sin \left[ \frac{d x}{2} \right]}{3 d} + \frac{88 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \sin \left[ \frac{d x}{2} \right]}{15 d} - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sin \left[ \frac{d x}{2} \right]}{5 d} - \\
& \left. \frac{48 \cos [c] \sin [d x]}{d} + \frac{8 \cos [2 c] \sin [2 d x]}{3 d} - \frac{172 \tan \left[ \frac{c}{2} \right]}{3 d} + \frac{88 \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \tan \left[ \frac{c}{2} \right]}{15 d} - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \tan \left[ \frac{c}{2} \right]}{5 d} \right)
\end{aligned}$$

- **Problem 343: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos [c + d x]} \sqrt{\operatorname{Sec} [c + d x]} dx$$

Optimal (type 3, 57 leaves, 3 steps):

$$\frac{2 \sqrt{a} \operatorname{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\operatorname{Sec} [c + d x]}}{d}$$

Result (type 3, 216 leaves):

$$\begin{aligned}
& \left( i \sqrt{a (1 + \cos [c + d x])} \left( \operatorname{ArcTanh} \left[ \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right] - \right. \right. \\
& \left. \left. \operatorname{Log} \left[ 2 \left( e^{i d x} \cos \left[ \frac{c}{2} \right] + i e^{i d x} \sin \left[ \frac{c}{2} \right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) \right] \right) \right) \\
& \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right] \left( \cos \left[ \frac{d x}{2} \right] + i \sin \left[ \frac{d x}{2} \right] \right) / \left( \sqrt{2} d \sqrt{\operatorname{Sec} [c + d x]} \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \right)
\end{aligned}$$

- **Problem 344: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \cos [c + d x]}}{\sqrt{\operatorname{Sec} [c + d x]}} dx$$

Optimal (type 3, 92 leaves, 4 steps):

$$\frac{\sqrt{a} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} + \frac{a \sin[c+dx]}{d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 349 leaves):

$$\frac{1}{2 \sqrt{2} d \sqrt{\sec[c+dx]} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx])} \sqrt{a(1+\cos[c+dx])} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left( -i \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right)\right] \right) +$$

$$i \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) +$$

$$\operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right)\right] \sin\left[\frac{dx}{2}\right] +$$

$$2 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right]$$

- **Problem 345: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+a \cos[c+dx]}}{\operatorname{Sec}[c+dx]^{3/2}} dx$$

Optimal (type 3, 136 leaves, 5 steps):

$$\frac{3 \sqrt{a} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{4 d} + \frac{a \sin[c+dx]}{2 d \sqrt{a+a \cos[c+dx]} \operatorname{Sec}[c+dx]^{3/2}} + \frac{3 a \sin[c+dx]}{4 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 391 leaves):

$$\frac{1}{8 \sqrt{2} d \sqrt{\sec[c+dx]} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx])} \sqrt{a(1+\cos[c+dx])} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left( -3 i \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right)\right] \right) +$$

$$3 i \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) +$$

$$3 \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right)\right] \sin\left[\frac{dx}{2}\right] +$$

$$4 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + 2 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right]$$



- **Problem 349: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{3/2} \sec [c + d x]^{3/2} dx$$

Optimal (type 3, 96 leaves, 5 steps):

$$\frac{2 a^{3/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{d} + \frac{2 a^2 \sqrt{\sec [c+d x]} \sin [c+d x]}{d \sqrt{a+a \cos [c+d x]}}$$

Result (type 3, 297 leaves):

$$\frac{1}{d \sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}} a \sqrt{a(1+\cos [c+d x])} \sec \left[\frac{1}{2}(c+d x)\right] \sqrt{\sec [c+d x]} \\ \left(\cos \left[\frac{d x}{2}\right]+i \sin \left[\frac{d x}{2}\right]\right) \left(i \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right]\right) \cos [c+d x]- \\ i \cos [c+d x] \log \left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right]+ \\ 2 \sqrt{2}\left(\cos \left[\frac{d x}{2}\right]-i \sin \left[\frac{d x}{2}\right]\right) \sqrt{\cos [c+d x](\cos [d x]+i \sin [d x])} \sin \left[\frac{1}{2}(c+d x)\right]$$

- **Problem 350: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{3/2} \sqrt{\sec [c + d x]} dx$$

Optimal (type 3, 95 leaves, 5 steps):

$$\frac{3 a^{3/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{d} + \frac{a^2 \sin [c+d x]}{d \sqrt{a+a \cos [c+d x]} \sqrt{\sec [c+d x]}}$$

Result (type 3, 351 leaves):

$$\frac{1}{2 \sqrt{2} d \sqrt{\sec [c+d x]} \sqrt{\cos [c+d x](\cos [d x]+i \sin [d x])}} \\ a \sqrt{a(1+\cos [c+d x])} \sec \left[\frac{1}{2}(c+d x)\right] \left(-3 i \cos \left[\frac{d x}{2}\right] \log \left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right]\right) + \\ 3 i \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right] \left(\cos \left[\frac{d x}{2}\right]+i \sin \left[\frac{d x}{2}\right]\right) + \\ 3 \log \left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right] \sin \left[\frac{d x}{2}\right] + \\ 2 \sqrt{2} \sqrt{\cos [c+d x](\cos [d x]+i \sin [d x])} \sin \left[\frac{1}{2}(c+d x)\right]$$

- **Problem 351: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx])^{3/2}}{\sqrt{\sec[c + dx]}} dx$$

Optimal (type 3, 140 leaves, 6 steps):

$$\frac{7 a^{3/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{4 d} + \frac{a^2 \sin[c+dx]}{2 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2}} + \frac{7 a^2 \sin[c+dx]}{4 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 392 leaves):

$$\frac{1}{8 \sqrt{2} d \sqrt{\sec[c+dx]} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx])} a \sqrt{a (1 + \cos[c+dx])} \sec\left[\frac{1}{2} (c+dx)\right] \left(-7 i \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2 \left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c]}\right)\right] + 7 i \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) + 7 \operatorname{Log}\left[2 \left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] + 12 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2} (c+dx)\right] + 2 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2} (c+dx)\right]\right)$$

- **Problem 352: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx])^{3/2}}{\sec[c + dx]^{3/2}} dx$$

Optimal (type 3, 180 leaves, 7 steps):

$$\frac{11 a^{3/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{8 d} + \frac{a^2 \sin[c+dx]}{3 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{5/2}} + \frac{11 a^2 \sin[c+dx]}{12 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2}} + \frac{11 a^2 \sin[c+dx]}{8 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 433 leaves):

$$\begin{aligned}
& \frac{1}{48 \sqrt{2} d \sqrt{\sec[c+dx]} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx])} \\
& a \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{1}{2}(c+dx)\right] \left( -33 i \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right) \right] + \right. \\
& \quad \left. 33 i \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) + \right. \\
& \quad \left. 33 \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right) \right] \sin\left[\frac{dx}{2}\right] + \right. \\
& \quad \left. 52 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \quad \left. 18 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] + 4 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{5}{2}(c+dx)\right] \right)
\end{aligned}$$

- **Problem 356: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c+dx])^{5/2} \sec[c+dx]^{5/2} dx$$

Optimal (type 3, 138 leaves, 5 steps):

$$\frac{2 a^{5/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} + \frac{14 a^3 \sqrt{\sec[c+dx]} \sin[c+dx]}{3 d \sqrt{a+a \cos[c+dx]}} + \frac{2 a^2 \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{3 d}$$

Result (type 3, 882 leaves):

$$\begin{aligned}
& \frac{1}{4} \sqrt{\cos[c+dx]} (a(1+\cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c+dx]} \\
& \left( \frac{1}{2} i \sin\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]} \right) - \right. \\
& \quad \left. \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]} \right) \right) + \\
& \quad \frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]} \right) + \right. \\
& \quad \left. \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]} \right) \right) \right) + \\
& (a(1+\cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c+dx]} \left( \frac{4 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{3d} + \frac{4 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{3d} + \right. \\
& \quad \left. \frac{\sec[c+dx] \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{6d} \right)
\end{aligned}$$

■ **Problem 357: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c+dx])^{5/2} \sec[c+dx]^{3/2} dx$$

Optimal (type 3, 134 leaves, 5 steps):

$$\frac{5 a^{5/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} - \frac{a^3 \sin[c+dx]}{d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}} + \frac{2 a^2 \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]} \sin[c+dx]}{d}$$

Result (type 3, 575 leaves):

$$\frac{1}{4 d \sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}} a^2 \sqrt{a(1+\cos [c+d x])} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]$$

$$\sqrt{\operatorname{Sec}[c+d x]} \left(-5 i \cos \left[c+\frac{d x}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right]-\right.$$

$$\left.5 i \cos \left[c+\frac{3 d x}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right]\right)+$$

$$10 i \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right] \cos [c+d x] \left(\cos \left[\frac{d x}{2}\right]+i \sin \left[\frac{d x}{2}\right]\right)-$$

$$5 \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right] \sin \left[c+\frac{d x}{2}\right]+$$

$$6 \sqrt{2} \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])} \sin \left[\frac{1}{2}(c+d x)\right]+2 \sqrt{2} \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])} \sin \left[\frac{3}{2}(c+d x)\right]+$$

$$5 \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right] \sin \left[c+\frac{3 d x}{2}\right]$$

- **Problem 358: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a+a \cos [c+d x])^{5 / 2} \sqrt{\operatorname{Sec}[c+d x]} d x$$

Optimal (type 3, 140 leaves, 5 steps):

$$\frac{19 a^{5 / 2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{4 d} + \frac{9 a^3 \sin [c+d x]}{4 d \sqrt{a+a \cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{a^2 \sqrt{a+a \cos [c+d x]} \sin [c+d x]}{2 d \sqrt{\operatorname{Sec}[c+d x]}}$$

Result (type 3, 394 leaves):

$$\frac{1}{8 \sqrt{2} d \sqrt{\operatorname{Sec}[c+d x]} \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])}}$$

$$a^2 \sqrt{a(1+\cos [c+d x])} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \left(-19 i \cos \left[\frac{d x}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right]\right)+$$

$$19 i \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right] \left(\cos \left[\frac{d x}{2}\right]+i \sin \left[\frac{d x}{2}\right]\right)+$$

$$19 \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right] \sin \left[\frac{d x}{2}\right]+$$

$$20 \sqrt{2} \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])} \sin \left[\frac{1}{2}(c+d x)\right]+2 \sqrt{2} \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])} \sin \left[\frac{3}{2}(c+d x)\right]$$

- **Problem 359: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx])^{5/2}}{\sqrt{\sec[c + dx]}} dx$$

Optimal (type 3, 180 leaves, 6 steps):

$$\frac{25 a^{5/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{8 d} +$$

$$\frac{13 a^3 \sin[c+dx]}{12 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2}} + \frac{a^2 \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{3 d \sec[c+dx]^{3/2}} + \frac{25 a^3 \sin[c+dx]}{8 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 435 leaves):

$$\frac{1}{48 \sqrt{2} d \sqrt{\sec[c+dx]} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx])}$$

$$a^2 \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{1}{2}(c+dx)\right] \left(-75 i \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2\left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2 i dx}) \cos[c] + i(-1+e^{2 i dx}) \sin[c]}\right)\right]\right) +$$

$$75 i \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i dx}) \cos[c] + i(-1+e^{2 i dx}) \sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) +$$

$$75 \operatorname{Log}\left[2\left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2 i dx}) \cos[c] + i(-1+e^{2 i dx}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] +$$

$$124 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] +$$

$$30 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] + 4 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{5}{2}(c+dx)\right]$$

- **Problem 360: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx])^{5/2}}{\sec[c + dx]^{3/2}} dx$$

Optimal (type 3, 220 leaves, 7 steps):

$$\frac{163 a^{5/2} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{64 d} + \frac{17 a^3 \sin[c+dx]}{24 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{5/2}} +$$

$$\frac{a^2 \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{4 d \sec[c+dx]^{5/2}} + \frac{163 a^3 \sin[c+dx]}{96 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2}} + \frac{163 a^3 \sin[c+dx]}{64 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 476 leaves):

$$\frac{1}{384 \sqrt{2} d \sqrt{\sec[c+dx]} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx])} a^2 \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{1}{2}(c+dx)\right]$$

$$\left(-489 i \cos\left[\frac{dx}{2}\right] \log\left[2\left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]}\right)\right]\right) +$$

$$489 i \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) +$$

$$489 \log\left[2\left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] +$$

$$800 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + 270 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] +$$

$$80 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{5}{2}(c+dx)\right] + 12 \sqrt{2} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{7}{2}(c+dx)\right]$$

■ **Problem 361: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c+dx]^{7/2}}{\sqrt{1+\cos[c+dx]}} dx$$

Optimal (type 3, 154 leaves, 7 steps):

$$-\frac{\sqrt{2} \operatorname{ArcSin}\left[\frac{\sin[c+dx]}{1+\cos[c+dx]}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} +$$

$$\frac{26 \sqrt{\sec[c+dx]} \sin[c+dx]}{15 d \sqrt{1+\cos[c+dx]}} - \frac{2 \sec[c+dx]^{3/2} \sin[c+dx]}{15 d \sqrt{1+\cos[c+dx]}} + \frac{2 \sec[c+dx]^{5/2} \sin[c+dx]}{5 d \sqrt{1+\cos[c+dx]}}$$

Result (type 3, 260 leaves):

$$\left(i e^{-i(c+dx)} \cos\left[\frac{1}{2}(c+dx)\right] \left(26 - 30 e^{i(c+dx)} + 80 e^{2i(c+dx)} - 80 e^{3i(c+dx)} + 30 e^{4i(c+dx)} - 26 e^{5i(c+dx)} +\right.\right.$$

$$\left.15 \sqrt{2} (1+e^{2i(c+dx)})^{5/2} \log[1+e^{i(c+dx)}] - 15 \sqrt{2} (1+e^{2i(c+dx)})^{5/2} \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}]\right)$$

$$\sqrt{\sec[c+dx]} \left(\cos\left[\frac{1}{2}(c+dx)\right] + i \sin\left[\frac{1}{2}(c+dx)\right]\right) \Big/ \left(15 d (1+e^{2i(c+dx)})^2 \sqrt{1+\cos[c+dx]}\right)$$

■ **Problem 362: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c+dx]^{5/2}}{\sqrt{1+\cos[c+dx]}} dx$$

Optimal (type 3, 118 leaves, 6 steps):

$$\frac{\sqrt{2} \operatorname{ArcSin}\left[\frac{\sin[c+dx]}{1+\cos[c+dx]}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} - \frac{2 \sqrt{\sec[c+dx]} \sin[c+dx]}{3 d \sqrt{1+\cos[c+dx]}} + \frac{2 \sec[c+dx]^{3/2} \sin[c+dx]}{3 d \sqrt{1+\cos[c+dx]}}$$

Result (type 3, 211 leaves) :

$$\left( i e^{-i(c+dx)} \cos\left[\frac{1}{2}(c+dx)\right] \right. \\ \left. \left( 2(-1 + e^{i(c+dx)})^3 - 3\sqrt{2}(1 + e^{2i(c+dx)})^{3/2} \operatorname{Log}[1 + e^{i(c+dx)}] + 3\sqrt{2}(1 + e^{2i(c+dx)})^{3/2} \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2}\sqrt{1 + e^{2i(c+dx)}}] \right) \right. \\ \left. \sqrt{\operatorname{Sec}[c+dx]} \left( \cos\left[\frac{1}{2}(c+dx)\right] + i \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) / \left( 3d(1 + e^{2i(c+dx)})\sqrt{1 + \cos[c+dx]} \right)$$

- **Problem 363: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^{3/2}}{\sqrt{1 + \cos[c+dx]}} dx$$

Optimal (type 3, 82 leaves, 4 steps) :

$$-\frac{\sqrt{2} \operatorname{ArcSin}\left[\frac{\sin[c+dx]}{1+\cos[c+dx]}\right] \sqrt{\cos[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{d} + \frac{2\sqrt{\operatorname{Sec}[c+dx]} \sin[c+dx]}{d\sqrt{1 + \cos[c+dx]}}$$

Result (type 3, 170 leaves) :

$$\frac{1}{2d\sqrt{1 + \cos[c+dx]}} i e^{-i(c+dx)} (1 + e^{i(c+dx)}) \\ \left( 2 - 2e^{i(c+dx)} + \sqrt{2}\sqrt{1 + e^{2i(c+dx)}} \operatorname{Log}[1 + e^{i(c+dx)}] - \sqrt{2}\sqrt{1 + e^{2i(c+dx)}} \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2}\sqrt{1 + e^{2i(c+dx)}}] \right) \sqrt{\operatorname{Sec}[c+dx]}$$

- **Problem 364: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Sec}[c+dx]}}{\sqrt{1 + \cos[c+dx]}} dx$$

Optimal (type 3, 47 leaves, 3 steps) :

$$\frac{\sqrt{2} \operatorname{ArcSin}\left[\frac{\sin[c+dx]}{1+\cos[c+dx]}\right] \sqrt{\cos[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{d}$$

Result (type 3, 146 leaves) :

$$-\frac{1}{d\sqrt{1 + \cos[c+dx]}} 2i e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \sqrt{1 + e^{2i(c+dx)}} \cos\left[\frac{1}{2}(c+dx)\right] \left( \operatorname{Log}[1 + e^{i(c+dx)}] - \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2}\sqrt{1 + e^{2i(c+dx)}}] \right)$$

- **Problem 365: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{1 + \cos[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}} dx$$

Optimal (type 3, 94 leaves, 6 steps) :



$$\frac{\sqrt{2} \operatorname{ArcSin}\left[\frac{\sin[c+dx]}{1+\cos[c+dx]}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} + \frac{2 \operatorname{ArcSin}\left[\frac{\sin[c+dx]}{\sqrt{1+\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d}$$

Result (type 3, 207 leaves):

$$\frac{1}{d \sqrt{1+\cos[c+dx]}} \sqrt{2} e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{1}{2}(c+dx)\right] \\ \left( dx - i \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] + i \sqrt{2} \operatorname{Log}\left[1+e^{i(c+dx)}\right] + i \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] - i \sqrt{2} \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] \right)$$

■ **Problem 366: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{1+\cos[c+dx]} \sec[c+dx]^{3/2}} dx$$

Optimal (type 3, 125 leaves, 7 steps):

$$\frac{\sqrt{2} \operatorname{ArcSin}\left[\frac{\sin[c+dx]}{1+\cos[c+dx]}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} - \frac{\operatorname{ArcSin}\left[\frac{\sin[c+dx]}{\sqrt{1+\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} + \frac{\sin[c+dx]}{d \sqrt{1+\cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 244 leaves):

$$\frac{1}{2d \sqrt{1+\cos[c+dx]}} \\ i \cos\left[\frac{1}{2}(c+dx)\right] \left( \sqrt{2} e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left( i dx + \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] - 2 \sqrt{2} \operatorname{Log}\left[1+e^{i(c+dx)}\right] - \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] \right) + \right. \\ \left. 2 \sqrt{2} \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] \right) + 2i \sqrt{\sec[c+dx]} \left( \sin\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{3}{2}(c+dx)\right] \right)$$

■ **Problem 367: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c+dx]^{7/2}}{\sqrt{a+a \cos[c+dx]}} dx$$

Optimal (type 3, 189 leaves, 7 steps):

$$\frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a} d} +$$

$$\frac{26 \sqrt{\sec[c+dx]} \sin[c+dx]}{15 d \sqrt{a+a \cos[c+dx]}} - \frac{2 \sec[c+dx]^{3/2} \sin[c+dx]}{15 d \sqrt{a+a \cos[c+dx]}} + \frac{2 \sec[c+dx]^{5/2} \sin[c+dx]}{5 d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 262 leaves):

$$\left( i e^{-i(c+dx)} \cos\left[\frac{1}{2}(c+dx)\right] \left( 26 - 30 e^{i(c+dx)} + 80 e^{2i(c+dx)} - 80 e^{3i(c+dx)} + 30 e^{4i(c+dx)} - 26 e^{5i(c+dx)} + \right. \right. \\ \left. \left. 15 \sqrt{2} (1 + e^{2i(c+dx)})^{5/2} \operatorname{Log}[1 + e^{i(c+dx)}] - 15 \sqrt{2} (1 + e^{2i(c+dx)})^{5/2} \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) \right) \\ \sqrt{\sec[c+dx]} \left( \cos\left[\frac{1}{2}(c+dx)\right] + i \sin\left[\frac{1}{2}(c+dx)\right] \right) \Big/ \left( 15 d (1 + e^{2i(c+dx)})^2 \sqrt{a(1 + \cos[c+dx])} \right)$$

■ **Problem 368: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c+dx]^{5/2}}{\sqrt{a+a \cos[c+dx]}} dx$$

Optimal (type 3, 151 leaves, 6 steps):

$$\frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a} d} - \frac{2 \sqrt{\sec[c+dx]} \sin[c+dx]}{3 d \sqrt{a+a \cos[c+dx]}} + \frac{2 \sec[c+dx]^{3/2} \sin[c+dx]}{3 d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 213 leaves):

$$\left( i e^{-i(c+dx)} \cos\left[\frac{1}{2}(c+dx)\right] \right. \\ \left. \left( 2 (-1 + e^{i(c+dx)})^3 - 3 \sqrt{2} (1 + e^{2i(c+dx)})^{3/2} \operatorname{Log}[1 + e^{i(c+dx)}] + 3 \sqrt{2} (1 + e^{2i(c+dx)})^{3/2} \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) \right) \\ \sqrt{\sec[c+dx]} \left( \cos\left[\frac{1}{2}(c+dx)\right] + i \sin\left[\frac{1}{2}(c+dx)\right] \right) \Big/ \left( 3 d (1 + e^{2i(c+dx)}) \sqrt{a(1 + \cos[c+dx])} \right)$$

■ **Problem 369: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c+dx]^{3/2}}{\sqrt{a+a \cos[c+dx]}} dx$$

Optimal (type 3, 113 leaves, 5 steps):

$$\frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a} d} + \frac{2 \sqrt{\sec[c+dx]} \sin[c+dx]}{d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 172 leaves) :

$$\frac{1}{2 d \sqrt{a} (1 + \cos [c + d x])} i e^{-i (c+d x)} (1 + e^{i (c+d x)})$$

$$\left( 2 - 2 e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log} [1 + e^{i (c+d x)}] - \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log} [1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}] \right) \sqrt{\operatorname{Sec} [c + d x]}$$

- **Problem 370: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Sec} [c + d x]}}{\sqrt{a + a \cos [c + d x]}} dx$$

Optimal (type 3, 56 leaves, 3 steps) :

$$\frac{\sqrt{2} \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sqrt{\operatorname{Sec} [c+d x]} \sin [c+d x]}{\sqrt{2} \sqrt{a+a \cos [c+d x]}} \right]}{\sqrt{a} d}$$

Result (type 3, 148 leaves) :

$$-\frac{1}{d \sqrt{a} (1 + \cos [c + d x])} 2 i e^{-\frac{1}{2} i (c+d x)} \sqrt{\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}}} \sqrt{1 + e^{2 i (c+d x)}} \cos \left[ \frac{1}{2} (c + d x) \right] \left( \operatorname{Log} [1 + e^{i (c+d x)}] - \operatorname{Log} [1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}] \right)$$

- **Problem 371: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a + a \cos [c + d x]} \sqrt{\operatorname{Sec} [c + d x]}} dx$$

Optimal (type 3, 105 leaves, 6 steps) :

$$\frac{2 \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sqrt{\operatorname{Sec} [c+d x]} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}} \right]}{\sqrt{a} d} - \frac{\sqrt{2} \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sqrt{\operatorname{Sec} [c+d x]} \sin [c+d x]}{\sqrt{2} \sqrt{a+a \cos [c+d x]}} \right]}{\sqrt{a} d}$$

Result (type 3, 209 leaves) :

$$\frac{1}{d \sqrt{a} (1 + \cos [c + d x])} \sqrt{2} e^{-\frac{1}{2} i (c+d x)} \sqrt{\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}}} \sqrt{1 + e^{2 i (c+d x)}} \cos \left[ \frac{1}{2} (c + d x) \right]$$

$$\left( d x - i \operatorname{ArcSinh} [e^{i (c+d x)}] + i \sqrt{2} \operatorname{Log} [1 + e^{i (c+d x)}] + i \operatorname{Log} [1 + \sqrt{1 + e^{2 i (c+d x)}}] - i \sqrt{2} \operatorname{Log} [1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}] \right)$$

- **Problem 372: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a + a \cos [c + d x]} \operatorname{Sec} [c + d x]^{3/2}} dx$$

Optimal (type 3, 168 leaves, 7 steps) :

$$\frac{\text{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a} d} + \frac{\sqrt{2} \text{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a} d} + \frac{\sin[c+dx]}{d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 246 leaves):

$$\frac{1}{2 d \sqrt{a} (1 + \cos[c + d x])} + i \cos\left[\frac{1}{2} (c + d x)\right] \left( \sqrt{2} e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2 i (c+dx)}}} \sqrt{1 + e^{2 i (c+dx)}} \left( i d x + \text{ArcSinh}\left[e^{i (c+dx)}\right] - 2 \sqrt{2} \text{Log}\left[1 + e^{i (c+dx)}\right] - \text{Log}\left[1 + \sqrt{1 + e^{2 i (c+dx)}}\right] + 2 \sqrt{2} \text{Log}\left[1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}\right] \right) + 2 i \sqrt{\sec[c + d x]} \left( \sin\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{3}{2} (c + d x)\right] \right) \right)$$

■ **Problem 373: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c + d x]^{5/2}}{(a + a \cos[c + d x])^{3/2}} dx$$

Optimal (type 3, 197 leaves, 7 steps):

$$\frac{11 \text{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{2 \sqrt{2} a^{3/2} d} - \frac{19 \sqrt{\sec[c+dx]} \sin[c+dx]}{6 a d \sqrt{a+a \cos[c+dx]}} - \frac{\sec[c+dx]^{3/2} \sin[c+dx]}{2 d (a+a \cos[c+dx])^{3/2}} + \frac{7 \sec[c+dx]^{3/2} \sin[c+dx]}{6 a d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 316 leaves):

$$\begin{aligned}
& - \frac{1}{d (a (1 + \cos [c + dx]))^{3/2}} \\
& 11 i e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2 i (c+dx)}}} \sqrt{1 + e^{2 i (c+dx)}} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \left( \log [1 + e^{i (c+dx)}] - \log [1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}] \right) + \\
& \frac{1}{(a (1 + \cos [c + dx]))^{3/2}} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sqrt{\sec [c + dx]} \\
& \left( - \frac{38 \cos \left[ \frac{dx}{2} \right] \sin \left[ \frac{c}{2} \right]}{3 d} - \frac{38 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right]}{3 d} + \frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \sin \left[ \frac{dx}{2} \right]}{d} + \frac{8 \sec [c + dx] \sin \left[ \frac{c}{2} + \frac{dx}{2} \right]}{3 d} + \frac{\sec \left[ \frac{c}{2} + \frac{dx}{2} \right] \tan \left[ \frac{c}{2} \right]}{d} \right)
\end{aligned}$$

■ **Problem 374: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec [c + dx]^{3/2}}{(a + a \cos [c + dx])^{3/2}} dx$$

Optimal (type 3, 157 leaves, 6 steps):

$$- \frac{7 \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin [c+dx]}{\sqrt{2} \sqrt{\cos [c+dx]} \sqrt{a+a \cos [c+dx]}} \right] \sqrt{\cos [c+dx]} \sqrt{\sec [c+dx]}}{2 \sqrt{2} a^{3/2} d} - \frac{\sqrt{\sec [c+dx]} \sin [c+dx]}{2 d (a + a \cos [c + dx])^{3/2}} + \frac{5 \sqrt{\sec [c + dx]} \sin [c + dx]}{2 a d \sqrt{a + a \cos [c + dx]}}$$

Result (type 3, 288 leaves):

$$\begin{aligned}
& \frac{1}{d (a (1 + \cos [c + dx]))^{3/2}} \\
& 7 i e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2 i (c+dx)}}} \sqrt{1 + e^{2 i (c+dx)}} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \left( \log [1 + e^{i (c+dx)}] - \log [1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}] \right) + \frac{1}{(a (1 + \cos [c + dx]))^{3/2}} \\
& \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sqrt{\sec [c + dx]} \left( \frac{10 \cos \left[ \frac{dx}{2} \right] \sin \left[ \frac{c}{2} \right]}{d} + \frac{10 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right]}{d} - \frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \sin \left[ \frac{dx}{2} \right]}{d} - \frac{\sec \left[ \frac{c}{2} + \frac{dx}{2} \right] \tan \left[ \frac{c}{2} \right]}{d} \right)
\end{aligned}$$

■ **Problem 375: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec [c + dx]}}{(a + a \cos [c + dx])^{3/2}} dx$$

Optimal (type 3, 117 leaves, 5 steps):

$$\frac{3 \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin [c+dx]}{\sqrt{2} \sqrt{\cos [c+dx]} \sqrt{a+a \cos [c+dx]}} \right] \sqrt{\cos [c + dx]} \sqrt{\sec [c + dx]}}{2 \sqrt{2} a^{3/2} d} - \frac{\sin [c + dx]}{2 d (a + a \cos [c + dx])^{3/2} \sqrt{\sec [c + dx]}}$$

Result (type 3, 277 leaves):

$$\frac{1}{2 d (a (1 + \cos [c + d x]))^{3/2} \sqrt{\sec [c + d x]}} e^{-\frac{1}{2} i (c + d x)} \cos \left[ \frac{1}{2} (c + d x) \right]$$

$$\left( 2 e^{\frac{1}{2} i (c + d x)} \sec \left[ \frac{c}{2} \right] \sec [c + d x] \sin \left[ \frac{d x}{2} \right] + 2 e^{\frac{1}{2} i (c + d x)} \cos \left[ \frac{1}{2} (c + d x) \right] \sec [c + d x] \tan \left[ \frac{c}{2} \right] - e^{-i (c + d x)} \cos \left[ \frac{1}{2} (c + d x) \right]^2 \right.$$

$$\left. \left( -2 + 2 e^{i (c + d x)} - 3 \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Log} [1 + e^{i (c + d x)}] + 3 \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Log} [1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}] \right) (-i + \tan [c + d x]) \right)$$

- **Problem 376: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \cos [c + d x])^{3/2} \sqrt{\sec [c + d x]}} dx$$

Optimal (type 3, 117 leaves, 5 steps):

$$\frac{\operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{2} \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{2 \sqrt{2} a^{3/2} d} + \frac{\sin [c + d x]}{2 d (a + a \cos [c + d x])^{3/2} \sqrt{\sec [c + d x]}}$$

Result (type 3, 288 leaves):

$$-\frac{1}{d (a (1 + \cos [c + d x]))^{3/2}} i e^{-\frac{1}{2} i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \sqrt{1 + e^{2 i (c + d x)}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \left( \operatorname{Log} [1 + e^{i (c + d x)}] - \operatorname{Log} [1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}] \right) +$$

$$\frac{\cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \sqrt{\sec [c + d x]} \left( \frac{2 \cos \left[ \frac{d x}{2} \right] \sin \left[ \frac{c}{2} \right]}{d} + \frac{2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right]}{d} - \frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \sin \left[ \frac{d x}{2} \right]}{d} - \frac{\sec \left[ \frac{c}{2} + \frac{d x}{2} \right] \tan \left[ \frac{c}{2} \right]}{d} \right)}{(a (1 + \cos [c + d x]))^{3/2}}$$

- **Problem 377: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \cos [c + d x])^{3/2} \sec [c + d x]^{3/2}} dx$$

Optimal (type 3, 174 leaves, 7 steps):

$$\frac{2 \operatorname{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{a^{3/2} d} -$$

$$\frac{5 \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{2} \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{2 \sqrt{2} a^{3/2} d} - \frac{\sin [c + d x]}{2 d (a + a \cos [c + d x])^{3/2} \sqrt{\sec [c + d x]}}$$

Result (type 3, 263 leaves):

$$\left( \cos\left[\frac{1}{2}(c+dx)\right]^3 \left( \sqrt{2} e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \right. \right. \\ \left. \left. \left( 4dx - 4i \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] + 5i\sqrt{2} \operatorname{Log}\left[1+e^{i(c+dx)}\right] + 4i \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] - 5i\sqrt{2} \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right]\right) \right) + \right. \\ \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\sec[c+dx]} \left( \sin\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{3}{2}(c+dx)\right] \right) \right) \Bigg/ (2d(a(1+\cos[c+dx]))^{3/2})$$

■ **Problem 378: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a+a\cos[c+dx])^{3/2} \sec[c+dx]^{5/2}} dx$$

Optimal (type 3, 214 leaves, 8 steps):

$$-\frac{3 \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} + 9 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{a^{3/2} d} + \frac{9 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{2\sqrt{2} a^{3/2} d} - \\ \frac{\sin[c+dx]}{2d(a+a\cos[c+dx])^{3/2} \sec[c+dx]^{3/2}} + \frac{3 \sin[c+dx]}{2ad\sqrt{a+a\cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 347 leaves):

$$-\left( 3i e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \right. \\ \left. \left( -2idx - 2 \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] + 3\sqrt{2} \operatorname{Log}\left[1+e^{i(c+dx)}\right] + 2 \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] - 3\sqrt{2} \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right]\right) \right) \Bigg/ \\ \left( \sqrt{2} d (a(1+\cos[c+dx]))^{3/2} \right) + \frac{1}{(a(1+\cos[c+dx]))^{3/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sqrt{\sec[c+dx]} \\ \left( \frac{2 \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{d} - \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{d} + \frac{2 \cos\left[\frac{3c}{2}\right] \sin\left[\frac{3dx}{2}\right]}{d} - \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{d} \right)$$

■ **Problem 379: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c+dx]^{5/2}}{(a+a\cos[c+dx])^{5/2}} dx$$

Optimal (type 3, 237 leaves, 8 steps):

$$\frac{163 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} - \frac{299 \sqrt{\sec[c+dx]} \sin[c+dx]}{48 a^2 d \sqrt{a+a \cos[c+dx]}}}{16 \sqrt{2} a^{5/2} d} - \frac{\sec[c+dx]^{3/2} \sin[c+dx]}{4 d (a+a \cos[c+dx])^{5/2}} - \frac{17 \sec[c+dx]^{3/2} \sin[c+dx]}{16 a d (a+a \cos[c+dx])^{3/2}} + \frac{95 \sec[c+dx]^{3/2} \sin[c+dx]}{48 a^2 d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 387 leaves):

$$\begin{aligned} & - \left( 163 i e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \operatorname{Log}\left[1+e^{i(c+dx)}\right] - \operatorname{Log}\left[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}\right] \right) \right) / \\ & \left( 4 d (a (1 + \cos[c+dx]))^{5/2} + \frac{1}{(a (1 + \cos[c+dx]))^{5/2}} \right. \\ & \left. \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c+dx]} \left( -\frac{299 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{6 d} - \frac{299 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{6 d} + \frac{21 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{4 d} + \right. \right. \\ & \left. \left. \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{2 d} + \frac{16 \sec[c+dx] \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{3 d} + \frac{21 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{4 d} + \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{2 d} \right) \right) \end{aligned}$$

■ **Problem 380: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c+dx]^{3/2}}{(a+a \cos[c+dx])^{5/2}} dx$$

Optimal (type 3, 197 leaves, 7 steps):

$$\frac{75 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} - \frac{13 \sqrt{\sec[c+dx]} \sin[c+dx]}{16 a d (a+a \cos[c+dx])^{3/2}} + \frac{49 \sqrt{\sec[c+dx]} \sin[c+dx]}{16 a^2 d \sqrt{a+a \cos[c+dx]}}}{16 \sqrt{2} a^{5/2} d} - \frac{\sqrt{\sec[c+dx]} \sin[c+dx]}{4 d (a+a \cos[c+dx])^{5/2}}$$

Result (type 3, 361 leaves):



$$\left( 75 i e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \text{Log}[1+e^{i(c+dx)}] - \text{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) /$$

$$(4 d (a (1 + \cos[c + dx]))^{5/2}) + \frac{1}{(a (1 + \cos[c + dx]))^{5/2}}$$

$$\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c + dx]} \left( \frac{49 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{2 d} + \frac{49 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{2 d} - \frac{13 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{4 d} - \right.$$

$$\left. \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{2 d} - \frac{13 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{4 d} - \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{2 d} \right)$$

- **Problem 381: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec[c + dx]}}{(a + a \cos[c + dx])^{5/2}} dx$$

Optimal (type 3, 157 leaves, 6 steps):

$$\frac{19 \text{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c + dx]} \sqrt{\sec[c + dx]}}{16 \sqrt{2} a^{5/2} d} - \frac{\sin[c + dx]}{4 d (a + a \cos[c + dx])^{5/2} \sqrt{\sec[c + dx]}} - \frac{9 \sin[c + dx]}{16 a d (a + a \cos[c + dx])^{3/2} \sqrt{\sec[c + dx]}}$$

Result (type 3, 361 leaves):

$$- \left( 19 i e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \text{Log}[1+e^{i(c+dx)}] - \text{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) /$$

$$(4 d (a (1 + \cos[c + dx]))^{5/2}) + \frac{1}{(a (1 + \cos[c + dx]))^{5/2}}$$

$$\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c + dx]} \left( -\frac{9 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{2 d} - \frac{9 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{2 d} + \frac{5 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{4 d} + \right.$$

$$\left. \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{2 d} + \frac{5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{4 d} + \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{2 d} \right)$$

- **Problem 382: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \cos[c + dx])^{5/2} \sqrt{\sec[c + dx]}} dx$$

Optimal (type 3, 157 leaves, 6 steps):

$$\frac{5 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{16 \sqrt{2} a^{5/2} d} + \frac{\sin[c+dx]}{4 d (a + a \cos[c + dx])^{5/2} \sqrt{\sec[c + dx]}} + \frac{\sin[c + dx]}{16 a d (a + a \cos[c + dx])^{3/2} \sqrt{\sec[c + dx]}}$$

Result (type 3, 361 leaves):

$$-\left(5 i e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left(\operatorname{Log}\left[1+e^{i(c+dx)}\right] - \operatorname{Log}\left[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}\right]\right)\right) / \left(4 d (a (1 + \cos[c + dx]))^{5/2}\right) + \frac{1}{(a (1 + \cos[c + dx]))^{5/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c + dx]} \left(\frac{\cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{2 d} + \frac{\cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{2 d} + \frac{3 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{4 d} - \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{2 d} + \frac{3 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{4 d} - \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{2 d}\right)$$

- **Problem 383: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \cos[c + dx])^{5/2} \sec[c + dx]^{3/2}} dx$$

Optimal (type 3, 157 leaves, 6 steps):

$$\frac{3 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{16 \sqrt{2} a^{5/2} d} - \frac{\sin[c+dx]}{4 d (a + a \cos[c + dx])^{5/2} \sqrt{\sec[c + dx]}} + \frac{7 \sin[c + dx]}{16 a d (a + a \cos[c + dx])^{3/2} \sqrt{\sec[c + dx]}}$$

Result (type 3, 361 leaves):

$$\begin{aligned}
& - \left( 3 i e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\
& \left( 4 d (a (1 + \cos[c + dx]))^{5/2} \right) + \frac{1}{(a (1 + \cos[c + dx]))^{5/2}} \\
& \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c + dx]} \left( \frac{7 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{2 d} + \frac{7 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{2 d} - \frac{11 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{4 d} + \right. \\
& \left. \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{2 d} - \frac{11 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{4 d} + \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{2 d} \right)
\end{aligned}$$

■ **Problem 384: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \cos[c + dx])^{5/2} \sec[c + dx]^{5/2}} dx$$

Optimal (type 3, 214 leaves, 8 steps):

$$\frac{2 \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} - 43 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{a^{5/2} d} - \frac{11 \sin[c+dx]}{16 \sqrt{2} a^{5/2} d}$$

Result (type 3, 424 leaves):

$$\begin{aligned}
& \left( e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \right. \\
& \left. \left( 32 dx - 32 i \operatorname{ArcSinh}[e^{i(c+dx)}] + 43 i \sqrt{2} \log[1+e^{i(c+dx)}] + 32 i \log[1+\sqrt{1+e^{2i(c+dx)}}] - 43 i \sqrt{2} \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\
& \left( 4 \sqrt{2} d (a (1 + \cos[c + dx]))^{5/2} \right) + \frac{1}{(a (1 + \cos[c + dx]))^{5/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c + dx]} \left( -\frac{15 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{2 d} - \frac{15 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{2 d} + \right. \\
& \left. \frac{19 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{4 d} - \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{2 d} + \frac{19 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{4 d} - \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{2 d} \right)
\end{aligned}$$

■ **Problem 385: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \cos[c + dx])^{5/2} \sec[c + dx]^{7/2}} dx$$

Optimal (type 3, 254 leaves, 9 steps):

$$\frac{5 \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} + \frac{115 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{a^{5/2} d} - \frac{16 \sqrt{2} a^{5/2} d}{16 a^2 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}} - \frac{\operatorname{Sin}[c+dx]}{4 d (a+a \cos[c+dx])^{5/2} \sec[c+dx]^{5/2}} - \frac{15 \operatorname{Sin}[c+dx]}{16 a d (a+a \cos[c+dx])^{3/2} \sec[c+dx]^{3/2}} + \frac{35 \operatorname{Sin}[c+dx]}{16 a^2 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 289 leaves):

$$\frac{1}{8 d (a (1 + \cos[c + dx]))^{5/2}} \cos\left[\frac{1}{2} (c + dx)\right]^5 \left( -5 i \sqrt{2} e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2 i (c+dx)}}} \sqrt{1 + e^{2 i (c+dx)}} \right. \\ \left. \left( -16 i dx - 16 \operatorname{ArcSinh}\left[e^{i (c+dx)}\right] + 23 \sqrt{2} \operatorname{Log}\left[1 + e^{i (c+dx)}\right] + 16 \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c+dx)}}\right] - 23 \sqrt{2} \operatorname{Log}\left[1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}\right] \right) \right) + \\ \frac{1}{4} \sec\left[\frac{1}{2} (c + dx)\right]^4 \sqrt{\sec[c + dx]} \left( 16 \operatorname{Sin}\left[\frac{1}{2} (c + dx)\right] + 39 \operatorname{Sin}\left[\frac{3}{2} (c + dx)\right] + 47 \operatorname{Sin}\left[\frac{5}{2} (c + dx)\right] + 8 \operatorname{Sin}\left[\frac{7}{2} (c + dx)\right] \right)$$

■ **Problem 386: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c + dx]^{5/2}}{(a + a \cos[c + dx])^{7/2}} dx$$

Optimal (type 3, 277 leaves, 9 steps):

$$\frac{1015 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} - \frac{629 \sqrt{\sec[c+dx]} \operatorname{Sin}[c+dx]}{64 \sqrt{2} a^{7/2} d}}{64 \sqrt{2} a^{7/2} d} - \frac{629 \sqrt{\sec[c+dx]} \operatorname{Sin}[c+dx]}{64 a^3 d \sqrt{a+a \cos[c+dx]}} - \frac{\sec[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{6 d (a+a \cos[c+dx])^{7/2}} - \frac{23 \sec[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{48 a d (a+a \cos[c+dx])^{5/2}} - \frac{109 \sec[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{64 a^2 d (a+a \cos[c+dx])^{3/2}} + \frac{193 \sec[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{64 a^3 d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 450 leaves):

$$\begin{aligned}
& - \left( 1015 i e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\
& (8 d (a (1 + \cos[c + dx]))^{7/2}) + \frac{1}{(a (1 + \cos[c + dx]))^{7/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\sec[c + dx]} \\
& \left( -\frac{629 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{4 d} - \frac{629 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{4 d} + \frac{451 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{24 d} + \frac{31 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{12 d} + \right. \\
& \left. \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{dx}{2}\right]}{3 d} + \frac{32 \sec[c + dx] \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{3 d} + \frac{451 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{24 d} + \frac{31 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{12 d} + \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \tan\left[\frac{c}{2}\right]}{3 d} \right)
\end{aligned}$$

■ **Problem 387: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c + dx]^{3/2}}{(a + a \cos[c + dx])^{7/2}} dx$$

Optimal (type 3, 237 leaves, 8 steps):

$$\begin{aligned}
& \frac{363 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} - \frac{\sqrt{\sec[c+dx]} \sin[c+dx]}{6 d (a + a \cos[c + dx])^{7/2}}}{64 \sqrt{2} a^{7/2} d} \\
& \frac{19 \sqrt{\sec[c+dx]} \sin[c+dx]}{48 a d (a + a \cos[c + dx])^{5/2}} - \frac{199 \sqrt{\sec[c+dx]} \sin[c+dx]}{192 a^2 d (a + a \cos[c + dx])^{3/2}} + \frac{691 \sqrt{\sec[c+dx]} \sin[c+dx]}{192 a^3 d \sqrt{a + a \cos[c + dx]}}
\end{aligned}$$

Result (type 3, 424 leaves):

$$\begin{aligned}
& \left( 363 i e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\
& (8 d (a (1 + \cos[c + dx]))^{7/2}) + \frac{1}{(a (1 + \cos[c + dx]))^{7/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\sec[c + dx]} \\
& \left( \frac{691 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{12 d} + \frac{691 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{12 d} - \frac{199 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{24 d} - \frac{19 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{12 d} - \right. \\
& \left. \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{dx}{2}\right]}{3 d} - \frac{199 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{24 d} - \frac{19 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{12 d} - \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \tan\left[\frac{c}{2}\right]}{3 d} \right)
\end{aligned}$$

- **Problem 388: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec[c+dx]}}{(a+a\cos[c+dx])^{7/2}} dx$$

Optimal (type 3, 197 leaves, 7 steps):

$$\frac{63 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} - \frac{\sin[c+dx]}{6d(a+a\cos[c+dx])^{7/2} \sqrt{\sec[c+dx]}}}{64\sqrt{2} a^{7/2} d} - \frac{5 \sin[c+dx]}{16ad(a+a\cos[c+dx])^{5/2} \sqrt{\sec[c+dx]}} - \frac{103 \sin[c+dx]}{192a^2 d(a+a\cos[c+dx])^{3/2} \sqrt{\sec[c+dx]}}$$

Result (type 3, 424 leaves):

$$\begin{aligned} & - \left( 63 i e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left( \operatorname{Log}\left[1+e^{i(c+dx)}\right] - \operatorname{Log}\left[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}\right] \right) \right) / \\ & \left( (8d(a(1+\cos[c+dx]))^{7/2}) + \frac{1}{(a(1+\cos[c+dx]))^{7/2}} \right. \\ & \left. \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\sec[c+dx]} \left( -\frac{103 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{12d} - \frac{103 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{12d} + \frac{43 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{24d} + \right. \right. \\ & \left. \left. \frac{7 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{12d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{dx}{2}\right]}{3d} + \frac{43 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{24d} + \frac{7 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{12d} + \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \tan\left[\frac{c}{2}\right]}{3d} \right) \right) \end{aligned}$$

- **Problem 389: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+a\cos[c+dx])^{7/2} \sqrt{\sec[c+dx]}} dx$$

Optimal (type 3, 197 leaves, 7 steps):

$$\frac{13 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} + \frac{\sin[c+dx]}{6d(a+a\cos[c+dx])^{7/2} \sqrt{\sec[c+dx]}}}{64\sqrt{2} a^{7/2} d} - \frac{\sin[c+dx]}{16ad(a+a\cos[c+dx])^{5/2} \sqrt{\sec[c+dx]}} - \frac{5 \sin[c+dx]}{192a^2 d(a+a\cos[c+dx])^{3/2} \sqrt{\sec[c+dx]}}$$

Result (type 3, 424 leaves):

$$\begin{aligned}
& - \left( 13 i e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\
& \quad (8 d (a (1 + \cos[c + dx]))^{7/2}) + \frac{1}{(a (1 + \cos[c + dx]))^{7/2}} \\
& \quad \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\sec[c + dx]} \left( -\frac{5 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{12 d} - \frac{5 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{12 d} + \frac{17 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{24 d} + \frac{5 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{12 d} - \right. \\
& \quad \left. \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{dx}{2}\right]}{3 d} + \frac{17 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{24 d} + \frac{5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{12 d} - \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \tan\left[\frac{c}{2}\right]}{3 d} \right)
\end{aligned}$$

- **Problem 390: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \cos[c + dx])^{7/2} \sec[c + dx]^{3/2}} dx$$

Optimal (type 3, 197 leaves, 7 steps):

$$\begin{aligned}
& \frac{7 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} - \frac{\sin[c+dx]}{6 d (a + a \cos[c + dx])^{7/2} \sqrt{\sec[c + dx]}}}{64 \sqrt{2} a^{7/2} d} + \\
& \frac{3 \sin[c + dx]}{16 a d (a + a \cos[c + dx])^{5/2} \sqrt{\sec[c + dx]}} + \frac{17 \sin[c + dx]}{192 a^2 d (a + a \cos[c + dx])^{3/2} \sqrt{\sec[c + dx]}}
\end{aligned}$$

Result (type 3, 424 leaves):

$$\begin{aligned}
& - \left( 7 i e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\
& \quad (8 d (a (1 + \cos[c + dx]))^{7/2}) + \frac{1}{(a (1 + \cos[c + dx]))^{7/2}} \\
& \quad \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\sec[c + dx]} \left( \frac{17 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{12 d} + \frac{17 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{12 d} + \frac{19 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{24 d} - \frac{17 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{12 d} + \right. \\
& \quad \left. \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{dx}{2}\right]}{3 d} + \frac{19 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{24 d} - \frac{17 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{12 d} + \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \tan\left[\frac{c}{2}\right]}{3 d} \right)
\end{aligned}$$

- **Problem 391: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \cos[c + dx])^{7/2} \sec[c + dx]^{5/2}} dx$$

Optimal (type 3, 197 leaves, 7 steps):

$$\frac{5 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} - \frac{\sin[c+dx]}{6d (a+a \cos[c+dx])^{7/2} \sec[c+dx]^{3/2}}}{64 \sqrt{2} a^{7/2} d} + \frac{13 \sin[c+dx]}{48 a d (a+a \cos[c+dx])^{5/2} \sqrt{\sec[c+dx]}} + \frac{67 \sin[c+dx]}{192 a^2 d (a+a \cos[c+dx])^{3/2} \sqrt{\sec[c+dx]}}$$

Result (type 3, 424 leaves):

$$\begin{aligned} & - \left( 5 i e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left( \operatorname{Log}\left[1+e^{i(c+dx)}\right] - \operatorname{Log}\left[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}\right] \right) \right) / \\ & \left( 8 d (a (1 + \cos[c + dx]))^{7/2} + \frac{1}{(a (1 + \cos[c + dx]))^{7/2}} \right. \\ & \left. \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\sec[c + dx]} \left( \frac{67 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{12 d} + \frac{67 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{12 d} - \frac{151 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{24 d} + \frac{29 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{12 d} \right. \right. \\ & \left. \left. - \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{dx}{2}\right]}{3 d} - \frac{151 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{24 d} + \frac{29 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{12 d} - \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \tan\left[\frac{c}{2}\right]}{3 d} \right) \right) \end{aligned}$$

- **Problem 392: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \cos[c + dx])^{7/2} \sec[c + dx]^{7/2}} dx$$

Optimal (type 3, 254 leaves, 9 steps):

$$\frac{2 \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} - \frac{177 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{a^{7/2} d} - \frac{64 \sqrt{2} a^{7/2} d}{6d (a+a \cos[c+dx])^{7/2} \sec[c+dx]^{5/2}} - \frac{17 \sin[c+dx]}{48 a d (a+a \cos[c+dx])^{5/2} \sec[c+dx]^{3/2}} - \frac{49 \sin[c+dx]}{64 a^2 d (a+a \cos[c+dx])^{3/2} \sqrt{\sec[c+dx]}}$$

Result (type 3, 487 leaves):



$$\left( e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left(128 dx - 128 i \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] + \right. \right. \\ \left. \left. 177 i \sqrt{2} \operatorname{Log}\left[1+e^{i(c+dx)}\right] + 128 i \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] - 177 i \sqrt{2} \operatorname{Log}\left[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}\right]\right) \right) / \\ \left( 8 \sqrt{2} d (a(1+\cos[c+dx]))^{7/2} \right) + \frac{1}{(a(1+\cos[c+dx]))^{7/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\sec[c+dx]} \\ \left( -\frac{247 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{12 d} - \frac{247 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{12 d} + \frac{379 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{24 d} - \frac{41 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{12 d} + \right. \\ \left. \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{dx}{2}\right]}{3 d} + \frac{379 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{24 d} - \frac{41 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{12 d} + \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \tan\left[\frac{c}{2}\right]}{3 d} \right)$$

■ **Problem 393: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a+a\cos[c+dx])^{7/2} \sec[c+dx]^{9/2}} dx$$

Optimal (type 3, 294 leaves, 10 steps):

$$-\frac{7 \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{a^{7/2} d} + \frac{637 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{64 \sqrt{2} a^{7/2} d} - \\ \frac{\sin[c+dx]}{6 d (a+a\cos[c+dx])^{7/2} \sec[c+dx]^{7/2}} - \frac{7 \sin[c+dx]}{16 a d (a+a\cos[c+dx])^{5/2} \sec[c+dx]^{5/2}} - \\ \frac{259 \sin[c+dx]}{192 a^2 d (a+a\cos[c+dx])^{3/2} \sec[c+dx]^{3/2}} + \frac{189 \sin[c+dx]}{64 a^3 d \sqrt{a+a\cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 519 leaves):

$$\begin{aligned}
& - \left( 7 i e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \right. \\
& \quad \left. \left( -64 i dx - 64 \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] + 91 \sqrt{2} \operatorname{Log}\left[1+e^{i(c+dx)}\right] + 64 \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] - 91 \sqrt{2} \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right]\right) \right) / \\
& \quad \left( 8 \sqrt{2} d (a(1+\cos[c+dx]))^{7/2} + \frac{1}{(a(1+\cos[c+dx]))^{7/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\sec[c+dx]} \right. \\
& \quad \left( \frac{427 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{12 d} + \frac{8 \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{d} + \frac{427 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{12 d} - \frac{703 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{24 d} + \frac{53 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{12 d} \right. \\
& \quad \left. \left. \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{dx}{2}\right]}{3 d} + \frac{8 \cos\left[\frac{3c}{2}\right] \sin\left[\frac{3dx}{2}\right]}{d} - \frac{703 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{24 d} + \frac{53 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{12 d} - \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \tan\left[\frac{c}{2}\right]}{3 d} \right) \right)
\end{aligned}$$

- **Problem 394: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+a\cos[c+dx])^{9/2} \sec[c+dx]^{5/2}} dx$$

Optimal (type 3, 237 leaves, 8 steps):

$$\begin{aligned}
& \frac{45 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{1024 \sqrt{2} a^{9/2} d} - \frac{\sin[c+dx]}{8 d (a+a\cos[c+dx])^{9/2} \sec[c+dx]^{3/2}} \\
& \frac{5 \sin[c+dx]}{32 a d (a+a\cos[c+dx])^{7/2} \sqrt{\sec[c+dx]}} + \frac{33 \sin[c+dx]}{256 a^2 d (a+a\cos[c+dx])^{5/2} \sqrt{\sec[c+dx]}} + \frac{73 \sin[c+dx]}{1024 a^3 d (a+a\cos[c+dx])^{3/2} \sqrt{\sec[c+dx]}}
\end{aligned}$$

Result (type 3, 487 leaves):

$$\begin{aligned}
& - \left( 45 i e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^9 \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\
& (64 d (a (1 + \cos[c + dx]))^{9/2}) + \frac{1}{(a (1 + \cos[c + dx]))^{9/2}} \\
& \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^9 \sqrt{\sec[c + dx]} \left( \frac{73 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{32 d} + \frac{73 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{32 d} + \frac{59 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{64 d} - \right. \\
& \frac{105 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{32 d} + \frac{13 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{dx}{2}\right]}{8 d} - \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \sin\left[\frac{dx}{2}\right]}{4 d} + \\
& \left. \frac{59 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{64 d} - \frac{105 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{32 d} + \frac{13 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \tan\left[\frac{c}{2}\right]}{8 d} - \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \tan\left[\frac{c}{2}\right]}{4 d} \right)
\end{aligned}$$

- **Problem 395: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \cos[c + dx])^{9/2} \sec[c + dx]^{7/2}} dx$$

Optimal (type 3, 237 leaves, 8 steps):

$$\begin{aligned}
& \frac{35 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{1024 \sqrt{2} a^{9/2} d} - \frac{\sin[c+dx]}{8 d (a + a \cos[c + dx])^{9/2} \sec[c + dx]^{5/2}} - \\
& \frac{19 \sin[c + dx]}{96 a d (a + a \cos[c + dx])^{7/2} \sec[c + dx]^{3/2}} - \frac{187 \sin[c + dx]}{768 a^2 d (a + a \cos[c + dx])^{5/2} \sqrt{\sec[c + dx]}} + \frac{853 \sin[c + dx]}{3072 a^3 d (a + a \cos[c + dx])^{3/2} \sqrt{\sec[c + dx]}}
\end{aligned}$$

Result (type 3, 487 leaves):

$$\begin{aligned}
& - \left( 35 i e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^9 \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\
& \left( (64 d (a (1 + \cos[c + dx]))^{9/2}) + \frac{1}{(a (1 + \cos[c + dx]))^{9/2}} \right. \\
& \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^9 \sqrt{\sec[c + dx]} \left( \frac{853 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{96 d} + \frac{853 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{96 d} - \frac{2593 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{192 d} + \right. \\
& \frac{779 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{96 d} - \frac{55 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{dx}{2}\right]}{24 d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \sin\left[\frac{dx}{2}\right]}{4 d} - \\
& \left. \frac{2593 \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{192 d} + \frac{779 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{96 d} - \frac{55 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \tan\left[\frac{c}{2}\right]}{24 d} + \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \tan\left[\frac{c}{2}\right]}{4 d} \right)
\end{aligned}$$

■ **Problem 397: Unable to integrate problem.**

$$\int \cos[c + dx]^m (a + a \cos[c + dx])^4 dx$$

Optimal (type 5, 302 leaves, 7 steps):

$$\begin{aligned}
& \frac{a^4 (55 + 29m + 4m^2) \cos[c + dx]^{1+m} \sin[c + dx]}{d (2+m) (3+m) (4+m)} + \\
& \frac{\cos[c + dx]^{1+m} (a^2 + a^2 \cos[c + dx])^2 \sin[c + dx]}{d (4+m)} + \frac{2 (5+m) \cos[c + dx]^{1+m} (a^4 + a^4 \cos[c + dx]) \sin[c + dx]}{d (3+m) (4+m)} - \\
& \frac{a^4 (35 + 40m + 8m^2) \cos[c + dx]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[c + dx]^2\right] \sin[c + dx]}{d (1+m) (2+m) (4+m) \sqrt{\sin[c + dx]^2}} - \\
& \frac{4 a^4 (5 + 2m) \cos[c + dx]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos[c + dx]^2\right] \sin[c + dx]}{d (2+m) (3+m) \sqrt{\sin[c + dx]^2}}
\end{aligned}$$

Result (type 8, 23 leaves):

$$\int \cos[c + dx]^m (a + a \cos[c + dx])^4 dx$$

■ **Problem 398: Unable to integrate problem.**

$$\int \cos[c + dx]^m (a + a \cos[c + dx])^3 dx$$

Optimal (type 5, 232 leaves, 6 steps):

$$\frac{a^3 (7 + 2m) \cos[c + dx]^{1+m} \sin[c + dx]}{d(2+m)(3+m)} + \frac{\cos[c + dx]^{1+m} (a^3 + a^3 \cos[c + dx]) \sin[c + dx]}{d(3+m)}$$

$$\frac{a^3 (5 + 4m) \cos[c + dx]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[c + dx]^2\right] \sin[c + dx]}{d(1+m)(2+m) \sqrt{\sin[c + dx]^2}}$$

$$\frac{a^3 (11 + 4m) \cos[c + dx]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos[c + dx]^2\right] \sin[c + dx]}{d(2+m)(3+m) \sqrt{\sin[c + dx]^2}}$$

Result (type 8, 23 leaves):

$$\int \cos[c + dx]^m (a + a \cos[c + dx])^3 dx$$

■ **Problem 399: Unable to integrate problem.**

$$\int \cos[c + dx]^m (a + a \cos[c + dx])^2 dx$$

Optimal (type 5, 173 leaves, 4 steps):

$$\frac{a^2 \cos[c + dx]^{1+m} \sin[c + dx]}{d(2+m)} - \frac{a^2 (3 + 2m) \cos[c + dx]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[c + dx]^2\right] \sin[c + dx]}{d(1+m)(2+m) \sqrt{\sin[c + dx]^2}}$$

$$\frac{2a^2 \cos[c + dx]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos[c + dx]^2\right] \sin[c + dx]}{d(2+m) \sqrt{\sin[c + dx]^2}}$$

Result (type 8, 23 leaves):

$$\int \cos[c + dx]^m (a + a \cos[c + dx])^2 dx$$

■ **Problem 400: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos[c + dx]^m (a + a \cos[c + dx]) dx$$

Optimal (type 5, 131 leaves, 3 steps):

$$\frac{a \cos[c + dx]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[c + dx]^2\right] \sin[c + dx]}{d(1+m) \sqrt{\sin[c + dx]^2}}$$

$$\frac{a \cos[c + dx]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos[c + dx]^2\right] \sin[c + dx]}{d(2+m) \sqrt{\sin[c + dx]^2}}$$

Result (type 5, 215 leaves):

$$\frac{1}{d(-1+m)m(1+m)} i^{2^{-1-m}} a \left(1 + e^{2i(c+dx)}\right)^{-1-m} \left(e^{-i(c+dx)} \left(1 + e^{2i(c+dx)}\right)\right)^{1+m} \left( (-1+m)m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-1-m), -m, \frac{1-m}{2}, -e^{2i(c+dx)}\right] + e^{i(c+dx)}(1+m) \left( e^{i(c+dx)} m \operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, -m, \frac{3-m}{2}, -e^{2i(c+dx)}\right] + 2(-1+m) \operatorname{Hypergeometric2F1}\left[-m, -\frac{m}{2}, 1-\frac{m}{2}, -e^{2i(c+dx)}\right] \right) \right)$$

■ **Problem 401: Unable to integrate problem.**

$$\int \frac{\cos[c+dx]^m}{a+a\cos[c+dx]} dx$$

Optimal (type 5, 156 leaves, 4 steps):

$$\frac{\cos[c+dx]^m \sin[c+dx]}{d(a+a\cos[c+dx])} - \frac{\cos[c+dx]^m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{m}{2}, \frac{2+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]}{ad\sqrt{\sin[c+dx]^2}} + \frac{m\cos[c+dx]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]}{ad(1+m)\sqrt{\sin[c+dx]^2}}$$

Result (type 8, 23 leaves):

$$\int \frac{\cos[c+dx]^m}{a+a\cos[c+dx]} dx$$

■ **Problem 402: Unable to integrate problem.**

$$\int \frac{\cos[c+dx]^m}{(a+a\cos[c+dx])^2} dx$$

Optimal (type 5, 229 leaves, 5 steps):

$$-\frac{2(1-m)\cos[c+dx]^{1+m}\sin[c+dx]}{3a^2d(1+\cos[c+dx])} - \frac{\cos[c+dx]^{1+m}\sin[c+dx]}{3d(a+a\cos[c+dx])^2} + \frac{(1-2m)m\cos[c+dx]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]}{3a^2d(1+m)\sqrt{\sin[c+dx]^2}} - \frac{2(1-m)(1+m)\cos[c+dx]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]}{3a^2d(2+m)\sqrt{\sin[c+dx]^2}}$$

Result (type 8, 23 leaves):

$$\int \frac{\cos[c+dx]^m}{(a+a\cos[c+dx])^2} dx$$

■ **Problem 411: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x]) \sec [c + d x] dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$b x + \frac{a \operatorname{ArcTanh}[\sin [c + d x]]}{d}$$

Result (type 3, 73 leaves):

$$b x - \frac{a \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d}$$

■ **Problem 412: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x]) \sec [c + d x]^2 dx$$

Optimal (type 3, 24 leaves, 4 steps):

$$\frac{b \operatorname{ArcTanh}[\sin [c + d x]]}{d} + \frac{a \tan [c + d x]}{d}$$

Result (type 3, 81 leaves):

$$- \frac{b \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{b \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a \tan [c + d x]}{d}$$

■ **Problem 415: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x]) \sec [c + d x]^5 dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$\frac{3 a \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} + \frac{b \tan [c + d x]}{d} + \frac{3 a \sec [c + d x] \tan [c + d x]}{8 d} + \frac{a \sec [c + d x]^3 \tan [c + d x]}{4 d} + \frac{b \tan [c + d x]^3}{3 d}$$

Result (type 3, 227 leaves):

$$\begin{aligned} & - \frac{3 a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{3 a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \\ & \frac{16 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^4}{3 a} + \frac{16 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^2}{3 a} - \frac{16 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^4}{a} - \\ & \frac{2 b \tan [c + d x]}{3 d} + \frac{b \sec [c + d x]^2 \tan [c + d x]}{3 d} \end{aligned}$$

■ **Problem 416: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x]) \sec [c + d x]^6 dx$$

Optimal (type 3, 101 leaves, 6 steps):

$$\frac{3 b \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} + \frac{a \tan [c + d x]}{d} + \frac{3 b \sec [c + d x] \tan [c + d x]}{8 d} + \frac{b \sec [c + d x]^3 \tan [c + d x]}{4 d} + \frac{2 a \tan [c + d x]^3}{3 d} + \frac{a \tan [c + d x]^5}{5 d}$$

Result (type 3, 249 leaves):

$$\begin{aligned} & -\frac{3 b \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{3 b \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \\ & \frac{b}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^4} + \frac{3 b}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{b}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^4} - \\ & \frac{3 b}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{8 a \tan [c+d x]}{15 d} + \frac{4 a \sec [c+d x]^2 \tan [c+d x]}{15 d} + \frac{a \sec [c+d x]^4 \tan [c+d x]}{5 d} \end{aligned}$$

■ **Problem 422: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^2 \sec [c + d x] dx$$

Optimal (type 3, 33 leaves, 3 steps):

$$2 a b x + \frac{a^2 \operatorname{ArcTanh}[\sin [c + d x]]}{d} + \frac{b^2 \sin [c + d x]}{d}$$

Result (type 3, 105 leaves):

$$2 a b x - \frac{a^2 \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right]}{d} + \frac{a^2 \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right]}{d} + \frac{b^2 \cos [d x] \sin [c]}{d} + \frac{b^2 \cos [c] \sin [d x]}{d}$$

■ **Problem 423: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^2 \sec [c + d x]^2 dx$$

Optimal (type 3, 33 leaves, 4 steps):

$$b^2 x + \frac{2 a b \operatorname{ArcTanh}[\sin [c + d x]]}{d} + \frac{a^2 \tan [c + d x]}{d}$$

Result (type 3, 77 leaves):

$$\frac{1}{d}\left(b\left(b c + b d x - 2 a \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]\right) + 2 a \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]\right) + a^2 \tan [c + d x]$$



■ **Problem 424: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^2 \sec [c + d x]^3 dx$$

Optimal (type 3, 59 leaves, 5 steps):

$$\frac{(a^2 + 2 b^2) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{2 a b \tan [c + d x]}{d} + \frac{a^2 \sec [c + d x] \tan [c + d x]}{2 d}$$

Result (type 3, 164 leaves):

$$\frac{1}{4 d} \left( -2 (a^2 + 2 b^2) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \right. \\ \left. 2 a^2 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] + 4 b^2 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \right. \\ \left. \frac{a^2}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \frac{a^2}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + 8 a b \tan [c + d x] \right)$$

■ **Problem 426: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^2 \sec [c + d x]^5 dx$$

Optimal (type 3, 110 leaves, 6 steps):

$$\frac{(3 a^2 + 4 b^2) \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} + \frac{2 a b \tan [c + d x]}{d} + \frac{(3 a^2 + 4 b^2) \sec [c + d x] \tan [c + d x]}{8 d} + \frac{a^2 \sec [c + d x]^3 \tan [c + d x]}{4 d} + \frac{2 a b \tan [c + d x]^3}{3 d}$$

Result (type 3, 375 leaves):

$$- \frac{3 a^2 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{8 d} - \frac{b^2 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{2 d} + \\ \frac{3 a^2 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{8 d} + \frac{b^2 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{2 d} + \frac{a^2}{16 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^4} + \\ \frac{3 a^2}{16 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{b^2}{4 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \frac{a^2}{16 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^4} - \\ \frac{3 a^2}{16 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \frac{b^2}{4 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{4 a b \tan [c + d x]}{3 d} + \frac{2 a b \sec [c + d x]^2 \tan [c + d x]}{3 d}$$

■ **Problem 427: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^2 \sec [c + d x]^6 dx$$

Optimal (type 3, 135 leaves, 7 steps) :

$$\frac{3 a b \operatorname{ArcTanh}[\sin [c+d x]]}{4 d} + \frac{\left(4 a^2+5 b^2\right) \tan [c+d x]}{5 d} + \frac{3 a b \operatorname{Sec}[c+d x] \tan [c+d x]}{4 d} +$$

$$\frac{a b \operatorname{Sec}[c+d x]^3 \tan [c+d x]}{2 d} + \frac{a^2 \operatorname{Sec}[c+d x]^4 \tan [c+d x]}{5 d} + \frac{\left(4 a^2+5 b^2\right) \tan [c+d x]^3}{15 d}$$

Result (type 3, 301 leaves) :

$$-\frac{3 a b \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]}{4 d} + \frac{3 a b \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]}{4 d} + \frac{a b}{8 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^4} +$$

$$\frac{3 a b}{8 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{a b}{8 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^4} - \frac{3 a b}{8 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} +$$

$$\frac{8 a^2 \tan [c+d x]}{15 d} + \frac{2 b^2 \tan [c+d x]}{3 d} + \frac{4 a^2 \operatorname{Sec}[c+d x]^2 \tan [c+d x]}{15 d} + \frac{b^2 \operatorname{Sec}[c+d x]^2 \tan [c+d x]}{3 d} + \frac{a^2 \operatorname{Sec}[c+d x]^4 \tan [c+d x]}{5 d}$$

■ **Problem 434: Result more than twice size of optimal antiderivative.**

$$\int (a+b \cos [c+d x])^3 \operatorname{Sec}[c+d x]^3 dx$$

Optimal (type 3, 79 leaves, 4 steps) :

$$b^3 x + \frac{a\left(a^2+6 b^2\right) \operatorname{ArcTanh}[\sin [c+d x]]}{2 d} + \frac{5 a^2 b \tan [c+d x]}{2 d} + \frac{a^2(a+b \cos [c+d x]) \operatorname{Sec}[c+d x] \tan [c+d x]}{2 d}$$

Result (type 3, 256 leaves) :

$$\frac{1}{4 d} \operatorname{Sec}[c+d x]^2\left(2 b^3 c+2 b^3 d x-a^3 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]-6 a b^2 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]+a^3 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]+6 a b^2 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]+\cos [2(c+d x)]\right. \\ \left.\left(2 b^3(c+d x)-a\left(a^2+6 b^2\right) \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]+a\left(a^2+6 b^2\right) \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]\right)+2 a^3 \sin [c+d x]+6 a^2 b \sin [2(c+d x)]\right)$$

■ **Problem 435: Result more than twice size of optimal antiderivative.**

$$\int (a+b \cos [c+d x])^3 \operatorname{Sec}[c+d x]^4 dx$$

Optimal (type 3, 109 leaves, 6 steps) :

$$\frac{b\left(3 a^2+2 b^2\right) \operatorname{ArcTanh}[\sin [c+d x]]}{2 d} + \frac{a\left(2 a^2+9 b^2\right) \tan [c+d x]}{3 d} + \frac{7 a^2 b \operatorname{Sec}[c+d x] \tan [c+d x]}{6 d} + \frac{a^2(a+b \cos [c+d x]) \operatorname{Sec}[c+d x]^2 \tan [c+d x]}{3 d}$$

Result (type 3, 383 leaves) :

$$\frac{(-3 a^2 b - 2 b^3) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{2 d} + \frac{(3 a^2 b + 2 b^3) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{2 d} +$$

$$\frac{a^3 + 9 a^2 b}{12 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{a^3 \sin\left[\frac{1}{2}(c+dx)\right]}{6 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{a^3 \sin\left[\frac{1}{2}(c+dx)\right]}{6 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} +$$

$$\frac{-a^3 - 9 a^2 b}{12 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{2 a^3 \sin\left[\frac{1}{2}(c+dx)\right] + 9 a b^2 \sin\left[\frac{1}{2}(c+dx)\right]}{3 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)} + \frac{2 a^3 \sin\left[\frac{1}{2}(c+dx)\right] + 9 a b^2 \sin\left[\frac{1}{2}(c+dx)\right]}{3 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}$$

■ **Problem 436: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + dx])^3 \sec[c + dx]^5 dx$$

Optimal (type 3, 133 leaves, 7 steps) :

$$\frac{3 a (a^2 + 4 b^2) \operatorname{ArcTanh}\left[\sin[c + dx]\right]}{8 d} + \frac{b (2 a^2 + b^2) \tan[c + dx]}{d} +$$

$$\frac{3 a (a^2 + 4 b^2) \sec[c + dx] \tan[c + dx]}{8 d} + \frac{3 a^2 b \sec[c + dx]^2 \tan[c + dx]}{4 d} + \frac{a^2 (a + b \cos[c + dx]) \sec[c + dx]^3 \tan[c + dx]}{4 d}$$

Result (type 3, 455 leaves) :

$$-\frac{3 (a^3 + 4 a b^2) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8 d} + \frac{3 (a^3 + 4 a b^2) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8 d} +$$

$$\frac{a^3}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{3 a^3 + 4 a^2 b + 12 a b^2}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} +$$

$$\frac{a^2 b \sin\left[\frac{1}{2}(c+dx)\right]}{2 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{a^3}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{a^2 b \sin\left[\frac{1}{2}(c+dx)\right]}{2 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} +$$

$$\frac{-3 a^3 - 4 a^2 b - 12 a b^2}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{2 a^2 b \sin\left[\frac{1}{2}(c+dx)\right] + b^3 \sin\left[\frac{1}{2}(c+dx)\right]}{d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)} + \frac{2 a^2 b \sin\left[\frac{1}{2}(c+dx)\right] + b^3 \sin\left[\frac{1}{2}(c+dx)\right]}{d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}$$

■ **Problem 437: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + dx])^3 \sec[c + dx]^6 dx$$

Optimal (type 3, 169 leaves, 7 steps) :

$$\frac{b(9a^2 + 4b^2) \operatorname{ArcTanh}[\sin[c + dx]]}{8d} + \frac{a(4a^2 + 15b^2) \tan[c + dx]}{5d} + \frac{b(9a^2 + 4b^2) \sec[c + dx] \tan[c + dx]}{8d} +$$

$$\frac{11a^2 b \sec[c + dx]^3 \tan[c + dx]}{20d} + \frac{a^2(a + b \cos[c + dx]) \sec[c + dx]^4 \tan[c + dx]}{5d} + \frac{a(4a^2 + 15b^2) \tan[c + dx]^3}{15d}$$

Result (type 3, 619 leaves):

$$\frac{(-9a^2 b - 4b^3) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d} + \frac{(9a^2 b + 4b^3) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d} +$$

$$\frac{2a^3 + 15a^2 b}{80d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{19a^3 + 135a^2 b + 60ab^2 + 60b^3}{240d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{a^3 \sin\left[\frac{1}{2}(c + dx)\right]}{20d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^5} +$$

$$\frac{a^3 \sin\left[\frac{1}{2}(c + dx)\right]}{20d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^5} + \frac{-2a^3 - 15a^2 b}{80d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{-19a^3 - 135a^2 b - 60ab^2 - 60b^3}{240d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} +$$

$$\frac{2(4a^3 \sin\left[\frac{1}{2}(c + dx)\right] + 15ab^2 \sin\left[\frac{1}{2}(c + dx)\right])}{15d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \frac{2(4a^3 \sin\left[\frac{1}{2}(c + dx)\right] + 15ab^2 \sin\left[\frac{1}{2}(c + dx)\right])}{15d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)} +$$

$$\frac{19a^3 \sin\left[\frac{1}{2}(c + dx)\right] + 60ab^2 \sin\left[\frac{1}{2}(c + dx)\right]}{120d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{19a^3 \sin\left[\frac{1}{2}(c + dx)\right] + 60ab^2 \sin\left[\frac{1}{2}(c + dx)\right]}{120d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3}$$

■ **Problem 445: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + dx])^4 \sec[c + dx]^4 dx$$

Optimal (type 3, 115 leaves, 5 steps):

$$b^4 x + \frac{2ab(a^2 + 2b^2) \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{a^2(2a^2 + 17b^2) \tan[c + dx]}{3d} +$$

$$\frac{4a^3 b \sec[c + dx] \tan[c + dx]}{3d} + \frac{a^2(a + b \cos[c + dx])^2 \sec[c + dx]^2 \tan[c + dx]}{3d}$$

Result (type 3, 246 leaves):

$$\frac{1}{12d} \operatorname{Sec}[c+dx]^3 \left( 9b \operatorname{Cos}[c+dx] \left( b^3(c+dx) - 2a(a^2+2b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 2a(a^2+2b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) + 3b \operatorname{Cos}[3(c+dx)] \left( b^3(c+dx) - 2a(a^2+2b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 2a(a^2+2b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) + 4a^2(2a^2+9b^2+6ab \operatorname{Cos}[c+dx] + (a^2+9b^2) \operatorname{Cos}[2(c+dx)]) \operatorname{Sin}[c+dx] \right)$$

■ **Problem 446: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Cos}[c+dx])^4 \operatorname{Sec}[c+dx]^5 dx$$

Optimal (type 3, 154 leaves, 7 steps):

$$\frac{(3a^4 + 24a^2b^2 + 8b^4) \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{8d} + \frac{4ab(2a^2 + 3b^2) \operatorname{Tan}[c+dx]}{3d} + \frac{a^2(3a^2 + 22b^2) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{8d} + \frac{5a^3b \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{6d} + \frac{a^2(a+b \operatorname{Cos}[c+dx])^2 \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx]}{4d}$$

Result (type 3, 487 leaves):

$$\frac{(-3a^4 - 24a^2b^2 - 8b^4) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{(3a^4 + 24a^2b^2 + 8b^4) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{a^4}{16d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{9a^4 + 16a^3b + 72a^2b^2}{48d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{2a^3b \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{3d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{a^4}{16d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{2a^3b \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{3d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{-9a^4 - 16a^3b - 72a^2b^2}{48d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{4(2a^3b \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 3ab^3 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])}{3d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)} + \frac{4(2a^3b \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 3ab^3 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])}{3d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)}$$

■ **Problem 447: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Cos}[c+dx])^4 \operatorname{Sec}[c+dx]^6 dx$$

Optimal (type 3, 188 leaves, 8 steps):

$$\frac{a b (3 a^2 + 4 b^2) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{(8 a^4 + 60 a^2 b^2 + 15 b^4) \operatorname{Tan}[c + d x]}{15 d} + \frac{a b (3 a^2 + 4 b^2) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 d} +$$

$$\frac{a^2 (4 a^2 + 27 b^2) \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{15 d} + \frac{3 a^3 b \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{5 d} + \frac{a^2 (a + b \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^4 \operatorname{Tan}[c + d x]}{5 d}$$

Result (type 3, 663 leaves):

$$\frac{(-3 a^3 b - 4 a b^3) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} + \frac{(3 a^3 b + 4 a b^3) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} +$$

$$\frac{a^4 + 10 a^3 b}{40 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{19 a^4 + 180 a^3 b + 120 a^2 b^2 + 240 a b^3}{240 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{a^4 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{20 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^5} +$$

$$\frac{a^4 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{20 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^5} + \frac{-a^4 - 10 a^3 b}{40 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{-19 a^4 - 180 a^3 b - 120 a^2 b^2 - 240 a b^3}{240 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} +$$

$$\frac{19 a^4 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 120 a^2 b^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{120 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^3} + \frac{19 a^4 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 120 a^2 b^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{120 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^3} +$$

$$\frac{8 a^4 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 60 a^2 b^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 15 b^4 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{15 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)} + \frac{8 a^4 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 60 a^2 b^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 15 b^4 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{15 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)}$$

■ **Problem 459: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[c + d x]^5}{(a + b \operatorname{Cos}[c + d x])^2} dx$$

Optimal (type 3, 266 leaves, 7 steps):

$$-\frac{a (4 a^2 + b^2) x}{b^5} + \frac{2 a^4 (4 a^2 - 5 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{a+b}}\right]}{(a-b)^{3/2} b^5 (a+b)^{3/2} d} + \frac{(12 a^4 - 7 a^2 b^2 - 2 b^4) \operatorname{Sin}[c + d x]}{3 b^4 (a^2 - b^2) d} -$$

$$\frac{a (2 a^2 - b^2) \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{b^3 (a^2 - b^2) d} + \frac{(4 a^2 - b^2) \operatorname{Cos}[c + d x]^2 \operatorname{Sin}[c + d x]}{3 b^2 (a^2 - b^2) d} - \frac{a^2 \operatorname{Cos}[c + d x]^3 \operatorname{Sin}[c + d x]}{b (a^2 - b^2) d (a + b \operatorname{Cos}[c + d x])}$$

Result (type 3, 176 leaves):

$$\frac{1}{12 b^5 d} \left( -12 a (2 a - i b) (2 a + i b) (c + d x) + \frac{24 a^4 (4 a^2 - 5 b^2) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a^2+b^2}}\right]}{(-a^2+b^2)^{3/2}} + \right.$$

$$\left. 9 b (4 a^2 + b^2) \operatorname{Sin}[c + d x] + \frac{12 a^5 b \operatorname{Sin}[c + d x]}{(a-b)(a+b)(a+b \operatorname{Cos}[c + d x])} - 6 a b^2 \operatorname{Sin}[2(c + d x)] + b^3 \operatorname{Sin}[3(c + d x)] \right)$$

- **Problem 491: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \operatorname{Cos}[c + d x]} \operatorname{Sec}[c + d x]^2 dx$$

Optimal (type 4, 197 leaves, 9 steps):

$$-\frac{\sqrt{a + b \operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{d \sqrt{\frac{a+b \operatorname{Cos}[c+dx]}{a+b}}} + \frac{a \sqrt{\frac{a+b \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{d \sqrt{a + b \operatorname{Cos}[c + d x]}} +$$

$$\frac{b \sqrt{\frac{a+b \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{d \sqrt{a + b \operatorname{Cos}[c + d x]}} + \frac{\sqrt{a + b \operatorname{Cos}[c + d x]} \operatorname{Tan}[c + d x]}{d}$$

Result (type 4, 418 leaves):

$$\begin{aligned}
& -\frac{1}{4d}b \left( \frac{2\sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} - \right. \\
& \left. \left( 2i\sqrt{\frac{b-b\cos[c+dx]}{a+b}} \sqrt{-\frac{b+b\cos[c+dx]}{a-b}} \cos[2(c+dx)] \left( 2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + \right. \right. \right. \\
& \left. \left. b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] - \right. \right. \right. \\
& \left. \left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \right) \sin[c+dx] \right) / \\
& \left( a\sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]}^2 \sqrt{-\frac{a^2-b^2-2a(a+b\cos[c+dx])+(a+b\cos[c+dx])^2}{b^2}} \right. \\
& \left. \left. \left( 2a^2-b^2-4a(a+b\cos[c+dx])+2(a+b\cos[c+dx])^2 \right) \right) \right) + \frac{\sqrt{a+b\cos[c+dx]} \tan[c+dx]}{d}
\end{aligned}$$

■ **Problem 492: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+b\cos[c+dx]} \sec[c+dx]^3 dx$$

Optimal (type 4, 262 leaves, 10 steps):

$$\begin{aligned}
& -\frac{b\sqrt{a+b\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{4ad\sqrt{\frac{a+b\cos[c+dx]}{a+b}}} + \frac{3b\sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{4d\sqrt{a+b\cos[c+dx]}} + \\
& \frac{(4a^2-b^2)\sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{4ad\sqrt{a+b\cos[c+dx]}} + \frac{b\sqrt{a+b\cos[c+dx]} \tan[c+dx]}{4ad} + \frac{\sqrt{a+b\cos[c+dx]} \sec[c+dx] \tan[c+dx]}{2d}
\end{aligned}$$

Result (type 4, 515 leaves):



$$\begin{aligned}
& \frac{1}{16 a d} \left( \frac{8 a b \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2\left(8 a^2-3 b^2\right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \left( 2 i b^2 \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left( 2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \right. \\
& \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} \left( 2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2 \right) \right) \right) + \\
& \frac{\sqrt{a+b \cos [c+d x]}\left(\frac{b \tan [c+d x]}{4 a}+\frac{1}{2} \sec [c+d x] \tan [c+d x]\right)}{d}
\end{aligned}$$

- **Problem 498: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a+b \cos [c+d x])^{3 / 2} \sec [c+d x]^2 d x$$

Optimal (type 4, 209 leaves, 9 steps):

$$\begin{aligned}
& -\frac{a \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{d \sqrt{\frac{a+b \cos [c+d x]}{a+b}}} + \frac{\left(a^2+2 b^2\right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{d \sqrt{a+b \cos [c+d x]}} + \\
& \frac{3 a b \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{d \sqrt{a+b \cos [c+d x]}} + \frac{a \sqrt{a+b \cos [c+d x]} \tan [c+d x]}{d}
\end{aligned}$$

Result (type 4, 472 leaves):

$$\begin{aligned}
& \frac{1}{4d} b \left( \frac{8b \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b \cos[c+dx]}} + \right. \\
& \frac{10a \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b \cos[c+dx]}} + \left( 2i \sqrt{\frac{b-b \cos[c+dx]}{a+b}} \sqrt{\frac{b+b \cos[c+dx]}{a-b}} \cos[2(c+dx)] \right. \\
& \left. \left( 2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin[c+dx] \right) / \\
& \left( \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]}^2 \sqrt{-\frac{a^2-b^2-2a(a+b \cos[c+dx])+(a+b \cos[c+dx])^2}{b^2}} \right. \\
& \left. \left. \left( 2a^2-b^2-4a(a+b \cos[c+dx])+2(a+b \cos[c+dx])^2 \right) \right) \right) + \frac{a \sqrt{a+b \cos[c+dx]} \tan[c+dx]}{d}
\end{aligned}$$

■ **Problem 499: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos[c+dx])^{3/2} \sec[c+dx]^3 dx$$

Optimal (type 4, 255 leaves, 10 steps):

$$\begin{aligned}
& -\frac{5b \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{4d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \frac{7ab \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{4d \sqrt{a+b \cos[c+dx]}} + \\
& \frac{(4a^2+3b^2) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{4d \sqrt{a+b \cos[c+dx]}} + \frac{5b \sqrt{a+b \cos[c+dx]} \tan[c+dx]}{4d} + \frac{a \sqrt{a+b \cos[c+dx]} \sec[c+dx] \tan[c+dx]}{2d}
\end{aligned}$$

Result (type 4, 508 leaves) :

$$\frac{1}{16 d} \left( \frac{8 a b \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right.$$

$$\frac{2\left(8 a^2+b^2\right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \left(10 i b^2 \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)]\right.$$

$$\left. \left(2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b\left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \sin [c+d x]\right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}\right)^2$$

$$\left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}}\left(2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2\right)\right)\right)$$

$$\frac{\sqrt{a+b \cos [c+d x]}\left(\frac{5}{4} b \tan [c+d x]+\frac{1}{2} a \sec [c+d x] \tan [c+d x]\right)}{d}$$

■ **Problem 504: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos [c+d x])^{5/2} \sec [c+d x] dx$$

Optimal (type 4, 222 leaves, 9 steps) :

$$\frac{14 a b \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{3 d \sqrt{\frac{a+b \cos [c+d x]}{a+b}}} + \frac{2 b\left(2 a^2+b^2\right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{3 d \sqrt{a+b \cos [c+d x]}} +$$

$$\frac{2 a^3 \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{d \sqrt{a+b \cos [c+d x]}} + \frac{2 b^2 \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{3 d}$$

Result (type 4, 379 leaves) :

$$\frac{1}{6d} \left( \frac{4b(9a^2 + b^2) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b \cos[c+dx]}} + \right.$$

$$\frac{2a(6a^2 + 7b^2) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b \cos[c+dx]}} + \frac{1}{\sqrt{-\frac{1}{a+b}}} 14i \sqrt{\frac{b(-1 + \cos[c+dx])}{a+b}} \sqrt{\frac{b(1 + \cos[c+dx])}{-a+b}} \operatorname{Csc}[c+dx]$$

$$\left. \left( -2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( -2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \right. \right. \right.$$

$$\left. \left. \frac{a+b}{a-b}\right] + b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] \right) \right) + 4b^2 \sqrt{a+b \cos[c+dx]} \operatorname{Sin}[c+dx]$$

■ **Problem 505: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos[c+dx])^{5/2} \operatorname{Sec}[c+dx]^2 dx$$

Optimal (type 4, 222 leaves, 9 steps) :

$$-\frac{(a^2 - 2b^2) \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \frac{a(a^2 + 4b^2) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{d \sqrt{a+b \cos[c+dx]}} +$$

$$\frac{5a^2 b \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{d \sqrt{a+b \cos[c+dx]}} + \frac{a^2 \sqrt{a+b \cos[c+dx]} \operatorname{Tan}[c+dx]}{d}$$

Result (type 4, 390 leaves) :

$$\frac{1}{4d}$$

$$\left( \frac{24 a b^2 \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{2 b\left(9 a^2+2 b^2\right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{1}{a b \sqrt{-\frac{1}{a+b}}} 2 i\left(a^2-2 b^2\right) \sqrt{-\frac{b(-1+\cos [c+d x])}{a+b}} \sqrt{-\frac{b(1+\cos [c+d x])}{a-b}} \operatorname{Csc}[c+d x] \left(2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b\left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right)\right) + 4 a^2 \sqrt{a+b \cos [c+d x]} \operatorname{Tan}[c+d x] \right)$$

■ **Problem 506: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos [c+d x])^{5/2} \operatorname{Sec}[c+d x]^3 dx$$

Optimal (type 4, 270 leaves, 10 steps):

$$\frac{9 a b \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{4 d \sqrt{\frac{a+b \cos [c+d x]}{a+b}}} + \frac{b\left(11 a^2+8 b^2\right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{4 d \sqrt{a+b \cos [c+d x]}} + \frac{a\left(4 a^2+15 b^2\right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{4 d \sqrt{a+b \cos [c+d x]}} + \frac{9 a b \sqrt{a+b \cos [c+d x]} \operatorname{Tan}[c+d x]}{4 d} + \frac{a^2 \sqrt{a+b \cos [c+d x]} \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{2 d}$$

Result (type 4, 395 leaves):

$$\frac{1}{8d} \left( \frac{4b(a^2 + 4b^2) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b \cos[c+dx]}} + \right.$$

$$\frac{a(8a^2 + 21b^2) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b \cos[c+dx]}} - \frac{1}{\sqrt{-\frac{1}{a+b}}} 9i \sqrt{\frac{b(-1 + \cos[c+dx])}{a+b}} \sqrt{\frac{b(1 + \cos[c+dx])}{-a+b}}$$

$$\operatorname{Csc}[c+dx] \left( -2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( -2a \operatorname{EllipticF}\left[ \right. \right.$$

$$\left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] \right) \right) +$$

$$\left. 2a \sqrt{a+b \cos[c+dx]} (2a + 9b \cos[c+dx]) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx] \right)$$

■ **Problem 507: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos[c+dx])^{5/2} \operatorname{Sec}[c+dx]^4 dx$$

Optimal (type 4, 323 leaves, 11 steps):

$$-\frac{(16a^2 + 33b^2) \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{24d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \frac{a(16a^2 + 59b^2) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{24d \sqrt{a+b \cos[c+dx]}} +$$

$$\frac{5b(4a^2 + b^2) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{8d \sqrt{a+b \cos[c+dx]}} + \frac{(16a^2 + 33b^2) \sqrt{a+b \cos[c+dx]} \operatorname{Tan}[c+dx]}{24d} +$$

$$\frac{13ab \sqrt{a+b \cos[c+dx]} \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{12d} + \frac{a^2 \sqrt{a+b \cos[c+dx]} \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3d}$$

Result (type 4, 563 leaves) :

$$\begin{aligned}
 & -\frac{1}{96 d} \\
 & b \left( -\frac{104 a b \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{2(-104 a^2+3 b^2) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \left(2 i\left(16 a^2+33 b^2\right)\right. \right. \\
 & \left. \left. \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \left(2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + \right. \right. \right. \\
 & \left. \left. \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, \right. \right. \right. \right. \\
 & \left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \sin [c+d x]\right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2}\right. \right. \\
 & \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}}\left(2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2\right)\right)\right) + \\
 & \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left(\frac{1}{24} \sec [c+d x]\left(16 a^2 \sin [c+d x]+33 b^2 \sin [c+d x]\right)+\frac{13}{12} a b \sec [c+d x] \tan [c+d x]+ \right. \\
 & \left. \frac{1}{3} a^2 \sec [c+d x]^2 \tan [c+d x]\right)
 \end{aligned}$$

■ **Problem 514: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{3+4 \cos [c+d x]} \sec [c+d x]^2 dx$$

Optimal (type 4, 95 leaves, 6 steps) :

$$-\frac{\sqrt{7} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{8}{7}\right]}{d} + \frac{3 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{8}{7}\right]}{\sqrt{7} d} + \frac{4 \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{8}{7}\right]}{\sqrt{7} d} + \frac{\sqrt{3+4 \cos [c+d x]} \tan [c+d x]}{d}$$

Result (type 4, 157 leaves) :

$$\frac{1}{21 d} \left( 6 \sqrt{7} \operatorname{EllipticPi} \left[ 2, \frac{1}{2} (c + d x), \frac{8}{7} \right] + \right. \\ \left. 1 / \left( \sqrt{\sin [c + d x]^2} \right) i \sqrt{7} \left( 21 \operatorname{EllipticE} \left[ i \operatorname{ArcSinh} \left[ \sqrt{3 + 4 \cos [c + d x]} \right], -\frac{1}{7} \right] - 12 \operatorname{EllipticF} \left[ i \operatorname{ArcSinh} \left[ \sqrt{3 + 4 \cos [c + d x]} \right], -\frac{1}{7} \right] - \right. \right. \\ \left. \left. 8 \operatorname{EllipticPi} \left[ -\frac{1}{3}, i \operatorname{ArcSinh} \left[ \sqrt{3 + 4 \cos [c + d x]} \right], -\frac{1}{7} \right] \right) \sin [c + d x] + 21 \sqrt{3 + 4 \cos [c + d x]} \tan [c + d x] \right)$$

■ **Problem 515: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{3 + 4 \cos [c + d x]} \operatorname{Sec} [c + d x]^3 dx$$

Optimal (type 4, 135 leaves, 7 steps):

$$-\frac{\sqrt{7} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), \frac{8}{7} \right]}{3 d} + \frac{3 \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), \frac{8}{7} \right]}{\sqrt{7} d} + \\ \frac{5 \operatorname{EllipticPi} \left[ 2, \frac{1}{2} (c + d x), \frac{8}{7} \right]}{3 \sqrt{7} d} + \frac{\sqrt{3 + 4 \cos [c + d x]} \tan [c + d x]}{3 d} + \frac{\sqrt{3 + 4 \cos [c + d x]} \operatorname{Sec} [c + d x] \tan [c + d x]}{2 d}$$

Result (type 4, 194 leaves):

$$\frac{1}{6 d} \left( \frac{12 \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), \frac{8}{7} \right]}{\sqrt{7}} + \frac{6 \operatorname{EllipticPi} \left[ 2, \frac{1}{2} (c + d x), \frac{8}{7} \right]}{\sqrt{7}} + \right. \\ \left. 1 / \left( 3 \sqrt{7} \sqrt{\sin [c + d x]^2} \right) 2 i \left( 21 \operatorname{EllipticE} \left[ i \operatorname{ArcSinh} \left[ \sqrt{3 + 4 \cos [c + d x]} \right], -\frac{1}{7} \right] - 12 \operatorname{EllipticF} \left[ i \operatorname{ArcSinh} \left[ \sqrt{3 + 4 \cos [c + d x]} \right], -\frac{1}{7} \right] - \right. \right. \\ \left. \left. 8 \operatorname{EllipticPi} \left[ -\frac{1}{3}, i \operatorname{ArcSinh} \left[ \sqrt{3 + 4 \cos [c + d x]} \right], -\frac{1}{7} \right] \right) \sin [c + d x] + (3 + 2 \cos [c + d x]) \sqrt{3 + 4 \cos [c + d x]} \operatorname{Sec} [c + d x] \tan [c + d x] \right)$$

■ **Problem 521: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{3 - 4 \cos [c + d x]} \operatorname{Sec} [c + d x]^2 dx$$

Optimal (type 4, 98 leaves, 6 steps):

$$-\frac{\sqrt{7} \operatorname{EllipticE} \left[ \frac{1}{2} (c + \pi + d x), \frac{8}{7} \right]}{d} + \frac{3 \operatorname{EllipticF} \left[ \frac{1}{2} (c + \pi + d x), \frac{8}{7} \right]}{\sqrt{7} d} + \frac{4 \operatorname{EllipticPi} \left[ 2, \frac{1}{2} (c + \pi + d x), \frac{8}{7} \right]}{\sqrt{7} d} + \frac{\sqrt{3 - 4 \cos [c + d x]} \tan [c + d x]}{d}$$

Result (type 4, 178 leaves):



$$\frac{1}{21 d} \left( -\frac{42 \sqrt{-3+4 \operatorname{Cos}[c+d x]} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), 8\right]}{\sqrt{3-4 \operatorname{Cos}[c+d x]}} - \right. \\ \left. 1 / \left( \sqrt{\operatorname{Sin}[c+d x]^2} \right) i \sqrt{7} \left( 21 \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{3-4 \operatorname{Cos}[c+d x]}\right], -\frac{1}{7}\right] - 12 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{3-4 \operatorname{Cos}[c+d x]}\right], -\frac{1}{7}\right] - \right. \right. \\ \left. \left. 8 \operatorname{EllipticPi}\left[-\frac{1}{3}, i \operatorname{ArcSinh}\left[\sqrt{3-4 \operatorname{Cos}[c+d x]}\right], -\frac{1}{7}\right]\right) \operatorname{Sin}[c+d x] + 21 \sqrt{3-4 \operatorname{Cos}[c+d x]} \operatorname{Tan}[c+d x] \right)$$

■ **Problem 522: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{3-4 \operatorname{Cos}[c+d x]} \operatorname{Sec}[c+d x]^3 dx$$

Optimal (type 4, 138 leaves, 7 steps):

$$\frac{\sqrt{7} \operatorname{EllipticE}\left[\frac{1}{2}(c+\pi+d x), \frac{8}{7}\right]}{3 d} - \frac{3 \operatorname{EllipticF}\left[\frac{1}{2}(c+\pi+d x), \frac{8}{7}\right]}{\sqrt{7} d} - \\ \frac{5 \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+\pi+d x), \frac{8}{7}\right]}{3 \sqrt{7} d} - \frac{\sqrt{3-4 \operatorname{Cos}[c+d x]} \operatorname{Tan}[c+d x]}{3 d} + \frac{\sqrt{3-4 \operatorname{Cos}[c+d x]} \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{2 d}$$

Result (type 4, 237 leaves):

$$\frac{1}{6 d} \left( -\frac{12 \sqrt{-3+4 \operatorname{Cos}[c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 8\right]}{\sqrt{3-4 \operatorname{Cos}[c+d x]}} + \frac{6 \sqrt{-3+4 \operatorname{Cos}[c+d x]} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), 8\right]}{\sqrt{3-4 \operatorname{Cos}[c+d x]}} + \right. \\ \left. 1 / \left( 3 \sqrt{7} \sqrt{\operatorname{Sin}[c+d x]^2} \right) 2 i \left( 21 \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{3-4 \operatorname{Cos}[c+d x]}\right], -\frac{1}{7}\right] - 12 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{3-4 \operatorname{Cos}[c+d x]}\right], -\frac{1}{7}\right] - \right. \right. \\ \left. \left. 8 \operatorname{EllipticPi}\left[-\frac{1}{3}, i \operatorname{ArcSinh}\left[\sqrt{3-4 \operatorname{Cos}[c+d x]}\right], -\frac{1}{7}\right]\right) \operatorname{Sin}[c+d x] - \sqrt{3-4 \operatorname{Cos}[c+d x]} (-3+2 \operatorname{Cos}[c+d x]) \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x] \right)$$

■ **Problem 528: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+d x]^2}{\sqrt{a+b \operatorname{Cos}[c+d x]}} dx$$

Optimal (type 4, 206 leaves, 9 steps):

$$\begin{aligned}
& - \frac{\sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] + \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{a d \sqrt{\frac{a+b \cos [c+d x]}{a+b}}} + \frac{\sqrt{a+b \cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{d \sqrt{a+b \cos [c+d x]}} \\
& \frac{b \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{a d \sqrt{a+b \cos [c+d x]}} + \frac{\sqrt{a+b \cos [c+d x]} \operatorname{Tan}[c+d x]}{a d}
\end{aligned}$$

Result (type 4, 424 leaves):

$$\begin{aligned}
& - \frac{1}{4 a d} \\
& b \left( \frac{6 \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \left( 2 i \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \left( 2 a(a-b) \operatorname{EllipticE}\left[ \right. \right. \right. \right. \\
& \quad \left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] - \right. \right. \right. \\
& \quad \left. \left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right)\right) \sin [c+d x] \right) / \\
& \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} \right. \\
& \quad \left. \left( 2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2 \right) \right) + \frac{\sqrt{a+b \cos [c+d x]} \operatorname{Tan}[c+d x]}{a d}
\end{aligned}$$

■ **Problem 529: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c+d x]^3}{\sqrt{a+b \cos [c+d x]}} dx$$

Optimal (type 4, 268 leaves, 10 steps):

$$\frac{3b\sqrt{a+b\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right] - b\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{4a^2d\sqrt{\frac{a+b\cos[c+dx]}{a+b}}} + \frac{4ad\sqrt{a+b\cos[c+dx]}}{4a^2d} + \frac{(4a^2+3b^2)\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{4a^2d\sqrt{a+b\cos[c+dx]}} - \frac{3b\sqrt{a+b\cos[c+dx]}\tan[c+dx]}{4a^2d} + \frac{\sqrt{a+b\cos[c+dx]}\operatorname{Sec}[c+dx]\tan[c+dx]}{2ad}$$

Result (type 4, 518 leaves):

$$\frac{1}{16a^2d} \left( \frac{8ab\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \frac{2(8a^2+9b^2)\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} - \left( 6ib^2\sqrt{\frac{b-b\cos[c+dx]}{a+b}} \right. \right. \\ \left. \left. \sqrt{-\frac{b+b\cos[c+dx]}{a-b}}\cos[2(c+dx)] \left( 2a(a-b)\operatorname{EllipticE}\left[ i\operatorname{ArcSinh}\left[ \sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos[c+dx]} \right], \frac{a+b}{a-b} \right] + b \left( 2a\operatorname{EllipticF}\left[ \right. \right. \right. \right. \\ \left. \left. \left. i\operatorname{ArcSinh}\left[ \sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos[c+dx]} \right], \frac{a+b}{a-b} \right] - b\operatorname{EllipticPi}\left[ \frac{a+b}{a}, i\operatorname{ArcSinh}\left[ \sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos[c+dx]} \right], \frac{a+b}{a-b} \right] \right) \right) \right) \\ \left. \sin[c+dx] \right) / \left( a\sqrt{-\frac{1}{a+b}}\sqrt{1-\cos[c+dx]}^2 \sqrt{-\frac{a^2-b^2-2a(a+b\cos[c+dx])+(a+b\cos[c+dx])^2}{b^2}} \right. \\ \left. (2a^2-b^2-4a(a+b\cos[c+dx])+2(a+b\cos[c+dx])^2) \right) + \frac{\sqrt{a+b\cos[c+dx]}\left(-\frac{3b\tan[c+dx]}{4a^2} + \frac{\operatorname{Sec}[c+dx]\tan[c+dx]}{2a}\right)}{d}$$

■ **Problem 535: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]}{(a+b\cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 176 leaves, 7 steps):

$$-\frac{2 b \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{a\left(a^2-b^2\right) d \sqrt{\frac{a+b \cos [c+d x]}{a+b}}}+\frac{2 \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{a d \sqrt{a+b \cos [c+d x]}}+\frac{2 b^2 \sin [c+d x]}{a\left(a^2-b^2\right) d \sqrt{a+b \cos [c+d x]}}$$

Result (type 4, 520 leaves):

$$\frac{2 b^2 \sin [c+d x]}{a\left(a^2-b^2\right) d \sqrt{a+b \cos [c+d x]}}-\frac{1}{2 a(-a+b)(a+b) d}$$

$$\left(-\frac{4 a b \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}}+\frac{2\left(2 a^2-3 b^2\right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}}+\right.$$

$$\left.\left(2 i b^2 \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)]\left(2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]+b\left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]-b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \sin [c+d x]\right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2}\right.\right.$$

$$\left.\left.\sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}}\left(2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2\right)\right)\right)$$

■ **Problem 536: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec [c+d x]^2}{(a+b \cos [c+d x])^{3 / 2}} d x$$

Optimal (type 4, 277 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(a^2 - 3b^2) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] + \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{a^2 (a^2 - b^2) d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \frac{\sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{a d \sqrt{a + b \cos[c + dx]}} \\
& \frac{3b \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{a^2 d \sqrt{a + b \cos[c + dx]}} + \frac{b (a^2 - 3b^2) \sin[c + dx]}{a^2 (a^2 - b^2) d \sqrt{a + b \cos[c + dx]}} + \frac{\tan[c + dx]}{a d \sqrt{a + b \cos[c + dx]}}
\end{aligned}$$

Result (type 4, 551 leaves) :

$$\begin{aligned}
& - \frac{1}{4 a^2 (a - b) (a + b) d} \\
& b \left( \frac{8 a b \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} + \frac{2 (7 a^2 - 9 b^2) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} - \left( 2 i (a^2 - 3 b^2) \right. \right. \\
& \left. \left. \frac{\sqrt{\frac{b - b \cos[c + dx]}{a + b}} \sqrt{\frac{b + b \cos[c + dx]}{a - b}} \cos[2 (c + dx)] \left( 2 a (a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a + b}{a - b}\right] + \right. \right. \\
& \left. \left. b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a + b}{a - b}\right] - b \operatorname{EllipticPi}\left[\frac{a + b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos[c + dx]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a + b}{a - b}\right] \right) \sin[c + dx] \right) / \left( a \sqrt{-\frac{1}{a + b}} \sqrt{1 - \cos[c + dx]}^2 \sqrt{-\frac{a^2 - b^2 - 2 a (a + b \cos[c + dx]) + (a + b \cos[c + dx])^2}{b^2}} \right. \\
& \left. \left. \left. (2 a^2 - b^2 - 4 a (a + b \cos[c + dx]) + 2 (a + b \cos[c + dx])^2) \right) \right) + \frac{\sqrt{a + b \cos[c + dx]} \left( -\frac{2 b^3 \sin[c + dx]}{a^2 (a^2 - b^2) (a + b \cos[c + dx])} + \frac{\tan[c + dx]}{a^2} \right)}{d}
\end{aligned}$$

■ **Problem 537: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c + dx]^3}{(a + b \cos[c + dx])^{3/2}} dx$$

Optimal (type 4, 345 leaves, 11 steps) :

$$\frac{b (7 a^2 - 15 b^2) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4 a^3 (a^2 - b^2) d \sqrt{\frac{a + b \cos[c + dx]}{a+b}}} -$$

$$\frac{5 b \sqrt{\frac{a + b \cos[c + dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] + (4 a^2 + 15 b^2) \sqrt{\frac{a + b \cos[c + dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4 a^2 d \sqrt{a + b \cos[c + dx]} + 4 a^3 d \sqrt{a + b \cos[c + dx]}}$$

$$\frac{b^2 (7 a^2 - 15 b^2) \sin[c + dx]}{4 a^3 (a^2 - b^2) d \sqrt{a + b \cos[c + dx]}} - \frac{5 b \tan[c + dx]}{4 a^2 d \sqrt{a + b \cos[c + dx]}} + \frac{\sec[c + dx] \tan[c + dx]}{2 a d \sqrt{a + b \cos[c + dx]}}$$

Result (type 4, 597 leaves):

$$-\frac{1}{16 a^3 (-a + b) (a + b) d}$$

$$\left( \frac{2 (4 a^3 b - 20 a b^3) \sqrt{\frac{a + b \cos[c + dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} + \frac{2 (8 a^4 + 29 a^2 b^2 - 45 b^4) \sqrt{\frac{a + b \cos[c + dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} \right) -$$

$$\left( 2 i (7 a^2 b^2 - 15 b^4) \sqrt{\frac{b - b \cos[c + dx]}{a + b}} \sqrt{-\frac{b + b \cos[c + dx]}{a - b}} \cos[2(c + dx)] \left( 2 a (a - b) \right. \right.$$

$$\left. \operatorname{EllipticE}\left[ i \operatorname{ArcSinh}\left[ \sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]} \right], \frac{a+b}{a-b} \right] + b \left( 2 a \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[ \sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]} \right], \frac{a+b}{a-b} \right] - \right. \right.$$

$$\left. \left. b \operatorname{EllipticPi}\left[ \frac{a+b}{a}, i \operatorname{ArcSinh}\left[ \sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]} \right], \frac{a+b}{a-b} \right] \right) \sin[c + dx] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1 - \cos[c + dx]}^2 \right.$$

$$\left. \left. \sqrt{-\frac{a^2 - b^2 - 2 a (a + b \cos[c + dx]) + (a + b \cos[c + dx])^2}{b^2}} (2 a^2 - b^2 - 4 a (a + b \cos[c + dx]) + 2 (a + b \cos[c + dx])^2) \right) \right) +$$

$$\frac{\sqrt{a + b \cos[c + dx]} \left( \frac{2 b^4 \sin[c + dx]}{a^3 (a^2 - b^2) (a + b \cos[c + dx])} - \frac{7 b \tan[c + dx]}{4 a^3} + \frac{\sec[c + dx] \tan[c + dx]}{2 a^2} \right)}{d}$$

- **Problem 544: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c + d x]}{(a + b \operatorname{Cos}[c + d x])^{5/2}} dx$$

Optimal (type 4, 320 leaves, 10 steps):

$$\begin{aligned} & - \frac{2 b (7 a^2 - 3 b^2) \sqrt{a + b \operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right] + 2 b \sqrt{\frac{a + b \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{3 a^2 (a^2 - b^2)^2 d \sqrt{\frac{a + b \operatorname{Cos}[c + d x]}{a + b}}} + \frac{2 b \sqrt{\frac{a + b \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{3 a (a^2 - b^2) d \sqrt{a + b \operatorname{Cos}[c + d x]}} + \\ & \frac{2 \sqrt{\frac{a + b \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{a^2 d \sqrt{a + b \operatorname{Cos}[c + d x]}} + \frac{2 b^2 \operatorname{Sin}[c + d x]}{3 a (a^2 - b^2) d (a + b \operatorname{Cos}[c + d x])^{3/2}} + \frac{2 b^2 (7 a^2 - 3 b^2) \operatorname{Sin}[c + d x]}{3 a^2 (a^2 - b^2)^2 d \sqrt{a + b \operatorname{Cos}[c + d x]}} \end{aligned}$$

Result (type 4, 622 leaves):

$$\begin{aligned}
& \frac{1}{6 a^2 (a-b)^2 (a+b)^2 d} \\
& \left( \frac{2 (-12 a^3 b + 4 a b^3) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] + 2 (6 a^4 - 19 a^2 b^2 + 9 b^4) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} \right) + \frac{2 i (-7 a^2 b^2 + 3 b^4) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)]}{\sqrt{a+b \cos [c+d x]}} \\
& \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] \right. \right. \\
& \left. \left. - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \left/ \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \right. \\
& \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) + \\
& \frac{\sqrt{a+b \cos [c+d x]} \left( \frac{2 b^2 \sin [c+d x]}{3 a(a^2-b^2)(a+b \cos [c+d x])^2} + \frac{2(7 a^2 b^2 \sin [c+d x]-3 b^4 \sin [c+d x])}{3 a^2(a^2-b^2)^2(a+b \cos [c+d x])} \right)}{d}
\end{aligned}$$

■ **Problem 545: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec [c+d x]^2}{(a+b \cos [c+d x])^{5/2}} dx$$

Optimal (type 4, 380 leaves, 11 steps):



$$\begin{aligned}
& - \frac{(3a^4 - 26a^2b^2 + 15b^4) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{3a^3(a^2 - b^2)^2 d \sqrt{\frac{a + b \cos[c + dx]}{a+b}}} + \\
& \frac{(3a^2 - 5b^2) \sqrt{\frac{a + b \cos[c + dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] - 5b \sqrt{\frac{a + b \cos[c + dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{3a^2(a^2 - b^2) d \sqrt{a + b \cos[c + dx]} - a^3 d \sqrt{a + b \cos[c + dx]}} + \\
& \frac{b(3a^2 - 5b^2) \sin[c + dx]}{3a^2(a^2 - b^2) d (a + b \cos[c + dx])^{3/2}} + \frac{b(3a^4 - 26a^2b^2 + 15b^4) \sin[c + dx]}{3a^3(a^2 - b^2)^2 d \sqrt{a + b \cos[c + dx]}} + \frac{\tan[c + dx]}{a d (a + b \cos[c + dx])^{3/2}}
\end{aligned}$$

Result (type 4, 638 leaves):

$$\begin{aligned}
& - \frac{1}{12 a^3 (-a+b)^2 (a+b)^2 d} b \left( \frac{2 (-36 a^3 b + 20 a b^3) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b \cos[c+dx]}} + \right. \\
& \frac{2 (33 a^4 - 86 a^2 b^2 + 45 b^4) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b \cos[c+dx]}} - \\
& \left. \left( 2 i (3 a^4 - 26 a^2 b^2 + 15 b^4) \sqrt{\frac{b-b \cos[c+dx]}{a+b}} \sqrt{-\frac{b+b \cos[c+dx]}{a-b}} \cos[2 (c+dx)] \right. \right. \\
& \left. \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \right) \sin[c+dx] \right) / \\
& \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]} \sqrt{-\frac{a^2-b^2-2a(a+b \cos[c+dx])+(a+b \cos[c+dx])^2}{b^2}} \right. \\
& \left. \left. \left( 2 a^2 - b^2 - 4 a (a+b \cos[c+dx]) + 2 (a+b \cos[c+dx])^2 \right) \right) \right) + \\
& \frac{\sqrt{a+b \cos[c+dx]} \left( -\frac{2 b^3 \sin[c+dx]}{3 a^2 (a^2-b^2) (a+b \cos[c+dx])^2} - \frac{4 (5 a^2 b^3 \sin[c+dx] - 3 b^5 \sin[c+dx])}{3 a^3 (a^2-b^2)^2 (a+b \cos[c+dx])} + \frac{\tan[c+dx]}{a^3} \right)}{d}
\end{aligned}$$

■ **Problem 552: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c+dx]^2}{\sqrt{3+4 \cos[c+dx]}} dx$$

Optimal (type 4, 101 leaves, 6 steps):

$$-\frac{\sqrt{7} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{8}{7}\right]}{3d} + \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{8}{7}\right]}{\sqrt{7}d} - \frac{4 \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{8}{7}\right]}{3\sqrt{7}d} + \frac{\sqrt{3+4\cos[c+dx]} \operatorname{Tan}[c+dx]}{3d}$$

Result (type 4, 158 leaves):

$$\frac{1}{3d} \left( -\frac{6 \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{8}{7}\right]}{\sqrt{7}} + \right. \\ \left. 1 / \left( 3\sqrt{7} \sqrt{\sin[c+dx]^2} \right) i \left( 21 \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{3+4\cos[c+dx]}\right], -\frac{1}{7}\right] - 12 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{3+4\cos[c+dx]}\right], -\frac{1}{7}\right] - \right. \right. \\ \left. \left. 8 \operatorname{EllipticPi}\left[-\frac{1}{3}, i \operatorname{ArcSinh}\left[\sqrt{3+4\cos[c+dx]}\right], -\frac{1}{7}\right] \right) \sin[c+dx] + \sqrt{3+4\cos[c+dx]} \operatorname{Tan}[c+dx] \right)$$

■ **Problem 553: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c+dx]^3}{\sqrt{3+4\cos[c+dx]}} dx$$

Optimal (type 4, 137 leaves, 7 steps):

$$\frac{\sqrt{7} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{8}{7}\right]}{3d} - \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{8}{7}\right]}{3\sqrt{7}d} + \\ \frac{\sqrt{7} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{8}{7}\right]}{3d} - \frac{\sqrt{3+4\cos[c+dx]} \operatorname{Tan}[c+dx]}{3d} + \frac{\sqrt{3+4\cos[c+dx]} \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{6d}$$

Result (type 4, 195 leaves):

$$\frac{1}{6d} \left( \frac{4 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{8}{7}\right]}{\sqrt{7}} + \frac{18 \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{8}{7}\right]}{\sqrt{7}} - \right. \\ \left. 1 / \left( 3\sqrt{7} \sqrt{\sin[c+dx]^2} \right) 2i \left( 21 \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{3+4\cos[c+dx]}\right], -\frac{1}{7}\right] - 12 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{3+4\cos[c+dx]}\right], -\frac{1}{7}\right] - \right. \right. \\ \left. \left. 8 \operatorname{EllipticPi}\left[-\frac{1}{3}, i \operatorname{ArcSinh}\left[\sqrt{3+4\cos[c+dx]}\right], -\frac{1}{7}\right] \right) \sin[c+dx] - (-1+2\cos[c+dx]) \sqrt{3+4\cos[c+dx]} \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx] \right)$$

■ **Problem 559: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c+dx]^2}{\sqrt{3-4\cos[c+dx]}} dx$$

Optimal (type 4, 104 leaves, 6 steps):

$$-\frac{\sqrt{7} \operatorname{EllipticE}\left[\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right]}{3d} + \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right]}{\sqrt{7}d} - \frac{4 \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+\pi+dx), \frac{8}{7}\right]}{3\sqrt{7}d} + \frac{\sqrt{3-4\cos[c+dx]} \operatorname{Tan}[c+dx]}{3d}$$

Result (type 4, 179 leaves):

$$\frac{1}{3d} \left( \frac{6\sqrt{-3+4\cos[c+dx]} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), 8\right]}{\sqrt{3-4\cos[c+dx]}} - \frac{1}{\left(3\sqrt{7}\sqrt{\sin[c+dx]^2}\right) i} \left( 21 \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{3-4\cos[c+dx]}\right], -\frac{1}{7}\right] - 12 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{3-4\cos[c+dx]}\right], -\frac{1}{7}\right] - 8 \operatorname{EllipticPi}\left[-\frac{1}{3}, i \operatorname{ArcSinh}\left[\sqrt{3-4\cos[c+dx]}\right], -\frac{1}{7}\right] \right) \sin[c+dx] + \sqrt{3-4\cos[c+dx]} \operatorname{Tan}[c+dx] \right)$$

■ **Problem 560: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c+dx]^3}{\sqrt{3-4\cos[c+dx]}} dx$$

Optimal (type 4, 140 leaves, 7 steps):

$$-\frac{\sqrt{7} \operatorname{EllipticE}\left[\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right]}{3d} + \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right]}{3\sqrt{7}d} - \frac{\sqrt{7} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+\pi+dx), \frac{8}{7}\right]}{3d} + \frac{\sqrt{3-4\cos[c+dx]} \operatorname{Tan}[c+dx]}{3d} + \frac{\sqrt{3-4\cos[c+dx]} \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{6d}$$

Result (type 4, 236 leaves):

$$\frac{1}{6d} \left( -\frac{4\sqrt{-3+4\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 8\right]}{\sqrt{3-4\cos[c+dx]}} + \frac{18\sqrt{-3+4\cos[c+dx]} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), 8\right]}{\sqrt{3-4\cos[c+dx]}} - \frac{1}{\left(3\sqrt{7}\sqrt{\sin[c+dx]^2}\right) 2i} \left( 21 \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{3-4\cos[c+dx]}\right], -\frac{1}{7}\right] - 12 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{3-4\cos[c+dx]}\right], -\frac{1}{7}\right] - 8 \operatorname{EllipticPi}\left[-\frac{1}{3}, i \operatorname{ArcSinh}\left[\sqrt{3-4\cos[c+dx]}\right], -\frac{1}{7}\right] \right) \sin[c+dx] + \sqrt{3-4\cos[c+dx]} (1+2\cos[c+dx]) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx] \right)$$

■ **Problem 586: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\cos[c+dx]^{3/2} (a+b\cos[c+dx])} dx$$

Optimal (type 4, 77 leaves, 5 steps):

$$-\frac{2 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{ad} - \frac{2b \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a(a+b)d} + \frac{2 \operatorname{Sin}[c+dx]}{ad \sqrt{\operatorname{Cos}[c+dx]}}$$

Result (type 4, 199 leaves):

$$-\frac{1}{2ad} \left( \frac{6b \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a+b} + \frac{2a \left( 2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] - \frac{2a \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a+b} \right)}{b} - \frac{4 \operatorname{Sin}[c+dx]}{\sqrt{\operatorname{Cos}[c+dx]}} + 1 \left/ \left( ab \sqrt{\operatorname{Sin}[c+dx]^2} \right) 2 \left( -2ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] + 2a(a+b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] + (2a^2 - b^2) \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] \right) \operatorname{Sin}[c+dx] \right)$$

■ **Problem 587: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Cos}[c+dx]^{5/2} (a+b \operatorname{Cos}[c+dx])} dx$$

Optimal (type 4, 128 leaves, 7 steps):

$$\frac{2b \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} + \frac{2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3ad} + \frac{2b^2 \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a^2(a+b)d} + \frac{2 \operatorname{Sin}[c+dx]}{3ad \operatorname{Cos}[c+dx]^{3/2}} - \frac{2b \operatorname{Sin}[c+dx]}{a^2 d \sqrt{\operatorname{Cos}[c+dx]}}$$

Result (type 4, 258 leaves):

$$\frac{1}{6a^2 d} \left( \frac{2(2a^2 + 9b^2) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a+b} + 8a \left( 2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] - \frac{2a \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a+b} \right) + \left( 6 \operatorname{Cos}[2(c+dx)] \left( -2ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] + 2a(a+b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] + (2a^2 - b^2) \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] \right) \operatorname{Sin}[c+dx] \right) \left/ \left( a \sqrt{1 - \operatorname{Cos}[c+dx]^2} (-1 + 2 \operatorname{Cos}[c+dx]^2) \right) \right) + \frac{\sqrt{\operatorname{Cos}[c+dx]} \left( -\frac{2b \operatorname{Tan}[c+dx]}{a^2} + \frac{2 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{3a} \right)}{d}$$

■ **Problem 603: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+b \operatorname{Cos}[c+dx]} dx$$

Optimal (type 4, 438 leaves, 7 steps) :

$$\begin{aligned}
 & -\frac{1}{4bd} (a-b) \sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{4bd} \\
 & \sqrt{a+b} (a+2b) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{4b^2d} \\
 & \sqrt{a+b} (a^2-4b^2) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \\
 & \frac{a \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{4bd \sqrt{\operatorname{Cos}[c+dx]}} + \frac{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{2d}
 \end{aligned}$$

Result (type 4, 1152 leaves) :

$$\begin{aligned}
 & \frac{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{2d} + \\
 & \frac{1}{8d} \left( - \left( 12a^2 \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
 & \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \\
 & 16ab \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
 & \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) -
 \end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \Bigg) + \\
2a & \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
\frac{1}{b} 2a & \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}}$$

■ **Problem 604: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} dx$$

Optimal (type 4, 371 leaves, 7 steps):

$$\begin{aligned} & -\frac{1}{ad} (a-b) \sqrt{a+b} \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\ & \frac{\sqrt{a+b} \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{d} - \frac{1}{bd} \\ & a \sqrt{a+b} \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\ & \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}} \end{aligned}$$

Result (type 4, 2437 leaves):

$$\begin{aligned} & \left( \sqrt{\cos[c+dx]} (1+\cos[c+dx])^{3/2} \sec\left[\frac{1}{2}(c+dx)\right]^2 \right. \\ & \left( 2(a+b) \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - 4a \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \right. \\ & \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - 4a \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\ & \left. \left. b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sec\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] + 2a \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] - b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) / \end{aligned}$$



$$\begin{aligned}
& \left( 4 d \left( \frac{1}{8 (a + b \cos [c + d x])^{3/2}} b (1 + \cos [c + d x])^{3/2} \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \sin [c + d x] \left( 2 (a + b) \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \right. \right. \right. \\
& \quad \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] \right], \frac{-a + b}{a + b} \right] - 4 a \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] \right], \frac{-a + b}{a + b} \right] - \\
& \quad 4 a \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticPi} \left[ -1, -\operatorname{ArcSin} \left[ \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] \right], \frac{-a + b}{a + b} \right] + b \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \\
& \quad \left. \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right] \sin \left[ \frac{3}{2} (c + d x) \right] + 2 a \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] - b \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] \right) - \\
& \quad \frac{1}{8 \sqrt{a + b \cos [c + d x]}} 3 \sqrt{1 + \cos [c + d x]} \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \sin [c + d x] \left( 2 (a + b) \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \right. \\
& \quad \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] \right], \frac{-a + b}{a + b} \right] - 4 a \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] \right], \frac{-a + b}{a + b} \right] - \\
& \quad 4 a \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticPi} \left[ -1, -\operatorname{ArcSin} \left[ \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] \right], \frac{-a + b}{a + b} \right] + b \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \\
& \quad \left. \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right] \sin \left[ \frac{3}{2} (c + d x) \right] + 2 a \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] - b \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] \right) + \\
& \quad \frac{1}{4 \sqrt{a + b \cos [c + d x]}} (1 + \cos [c + d x])^{3/2} \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] \left( 2 (a + b) \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \right. \\
& \quad \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] \right], \frac{-a + b}{a + b} \right] - 4 a \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] \right], \frac{-a + b}{a + b} \right] - \\
& \quad 4 a \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticPi} \left[ -1, -\operatorname{ArcSin} \left[ \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] \right], \frac{-a + b}{a + b} \right] + b \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}}
\end{aligned}$$

$$\begin{aligned}
& \left. \sec\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] + 2a \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] - b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] \right) + \\
& \frac{1}{4\sqrt{a+b\cos[c+dx]}} (1+\cos[c+dx])^{3/2} \sec\left[\frac{1}{2}(c+dx)\right]^2 \left( \frac{3}{2} b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \cos\left[\frac{3}{2}(c+dx)\right] \sec\left[\frac{1}{2}(c+dx)\right] + \right. \\
& a \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sec\left[\frac{1}{2}(c+dx)\right]^2 - \frac{1}{2} b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sec\left[\frac{1}{2}(c+dx)\right]^2 + \\
& \left. (a+b) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \left(-\frac{b\sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(a+b\cos[c+dx])\sin[c+dx]}{(a+b)(1+\cos[c+dx])^2}\right) \right. \\
& \left. \frac{1}{\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} - \right. \\
& \left. \frac{2a \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \left(-\frac{b\sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(a+b\cos[c+dx])\sin[c+dx]}{(a+b)(1+\cos[c+dx])^2}\right)}{\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} - \frac{1}{\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} \right. \\
& \left. 2a \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \left(-\frac{b\sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(a+b\cos[c+dx])\sin[c+dx]}{(a+b)(1+\cos[c+dx])^2}\right) + \right. \\
& \left. b \sec\left[\frac{1}{2}(c+dx)\right] \left(\frac{\cos[c+dx]\sin[c+dx]}{(1+\cos[c+dx])^2} - \frac{\sin[c+dx]}{1+\cos[c+dx]}\right) \sin\left[\frac{3}{2}(c+dx)\right] + \frac{a \left(\frac{\cos[c+dx]\sin[c+dx]}{(1+\cos[c+dx])^2} - \frac{\sin[c+dx]}{1+\cos[c+dx]}\right) \tan\left[\frac{1}{2}(c+dx)\right]}{2\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}}} - \right. \\
& \left. \frac{b \left(\frac{\cos[c+dx]\sin[c+dx]}{(1+\cos[c+dx])^2} - \frac{\sin[c+dx]}{1+\cos[c+dx]}\right) \tan\left[\frac{1}{2}(c+dx)\right]}{2\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}}} + \frac{1}{2} b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sec\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] \tan\left[\frac{1}{2}(c+dx)\right] - \right. \\
& \left. \frac{2a \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sec\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{1-\frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} + \frac{2a \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sec\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]\right)^2} \sqrt{1-\frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \right.
\end{aligned}$$

$$\left. \left. \left. \left. \frac{(a+b) \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}{\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right] \right) \right) \right) \right)$$

■ **Problem 607: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \cos[c+dx]} \, dx}{\cos[c+dx]^{5/2}}$$

Optimal (type 4, 271 leaves, 4 steps):

$$\begin{aligned} & \frac{1}{3 a^2 d} 2 (a-b) b \sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{3 a d} \\ & 2 (a-b) \sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \\ & \frac{2 \sqrt{a+b \cos[c+dx]} \operatorname{Sin}[c+dx]}{3 d \cos[c+dx]^{3/2}} \end{aligned}$$

Result (type 4, 2854 leaves):

$$\begin{aligned} & \frac{1}{3 d} 4 a \left( \left( \sqrt{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ & \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], \frac{2 a}{a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\ & \left. \left( b \sqrt{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticPi}\left[\frac{a}{a+b}, \text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], \frac{2a}{a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& \left( b\sqrt{\cos[c+dx]}(1+\cos[c+dx])^{3/2} \sec\left[\frac{1}{2}(c+dx)\right]^2 \left( 2(a+b)\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticE}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - \right. \right. \\
& 4a\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticF}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - \\
& 4a\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \\
& \left. \left. b\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sec\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] + 2a\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] - b\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) / \\
& \left( 6ad \left( \frac{1}{8(a+b\cos[c+dx])^{3/2}} b(1+\cos[c+dx])^{3/2} \sec\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] \left( 2(a+b)\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \right. \right. \right. \\
& \left. \left. \left. \text{EllipticE}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - 4a\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticF}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - \right. \right. \right. \\
& \left. \left. 4a\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + b\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \right. \right. \\
& \left. \left. \left. \sec\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] + 2a\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] - b\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) - \right. \\
& \left. \frac{1}{8\sqrt{a+b\cos[c+dx]}} 3\sqrt{1+\cos[c+dx]} \sec\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] \left( 2(a+b)\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - 4a \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - \\
& 4a \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \\
& \text{Sec}\left[\frac{1}{2}(c+dx)\right] \text{Sin}\left[\frac{3}{2}(c+dx)\right] + 2a \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{Tan}\left[\frac{1}{2}(c+dx)\right] - b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{Tan}\left[\frac{1}{2}(c+dx)\right] \Bigg) + \\
& \frac{1}{4\sqrt{a+b\cos[c+dx]}} (1+\cos[c+dx])^{3/2} \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \text{Tan}\left[\frac{1}{2}(c+dx)\right] \left( 2(a+b) \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \right. \\
& \left. \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - 4a \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - \right. \\
& \left. 4a \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \right. \\
& \left. \text{Sec}\left[\frac{1}{2}(c+dx)\right] \text{Sin}\left[\frac{3}{2}(c+dx)\right] + 2a \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{Tan}\left[\frac{1}{2}(c+dx)\right] - b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{Tan}\left[\frac{1}{2}(c+dx)\right] \right) + \\
& \frac{1}{4\sqrt{a+b\cos[c+dx]}} (1+\cos[c+dx])^{3/2} \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left( \frac{3}{2}b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \cos\left[\frac{3}{2}(c+dx)\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \left. a \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 - \frac{1}{2}b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 + \frac{1}{\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} \right. \\
& \left. (a+b) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \left( -\frac{b\sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(a+b\cos[c+dx])\sin[c+dx]}{(a+b)(1+\cos[c+dx])^2} \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{2 a \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]\left(-\frac{b \operatorname{Sin}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}+\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Sin}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])^2}\right)}{\sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}}}}-\frac{1}{\sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}}} \\
& + \frac{2 a \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]\left(-\frac{b \operatorname{Sin}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}+\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Sin}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])^2}\right)}{\sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}}} \\
& + \frac{b \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]\left(\frac{\operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{(1+\operatorname{Cos}[c+d x])^2}-\frac{\operatorname{Sin}[c+d x]}{1+\operatorname{Cos}[c+d x]}\right) \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right]+a\left(\frac{\operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{(1+\operatorname{Cos}[c+d x])^2}-\frac{\operatorname{Sin}[c+d x]}{1+\operatorname{Cos}[c+d x]}\right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{2 \sqrt{\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}}+\frac{\sqrt{\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}}{\sqrt{\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}} \\
& + \frac{b\left(\frac{\operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{(1+\operatorname{Cos}[c+d x])^2}-\frac{\operatorname{Sin}[c+d x]}{1+\operatorname{Cos}[c+d x]}\right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{2 \sqrt{\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}}+\frac{1}{2} b \sqrt{\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]- \\
& \frac{2 a \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{1-\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}+\frac{2 a \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{1-\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}+ \\
& \frac{(a+b) \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}{\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \Bigg) \Bigg) \Bigg) + \\
& \frac{\sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]}\left(\frac{2 b \operatorname{Tan}[c+d x]}{3 a}+\frac{2}{3} \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]\right)}{d}
\end{aligned}$$

- **Problem 608: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\operatorname{Cos}[c+d x]^{7/2}} dx$$

Optimal (type 4, 329 leaves, 5 steps):

$$\frac{1}{15 a^3 d} 2 (a-b) \sqrt{a+b} (9 a^2 - 2 b^2) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{15 a^2 d}$$

$$2 (a-b) \sqrt{a+b} (9 a+2 b) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{2 \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{5 d \operatorname{Cos}[c+d x]^{5/2}} + \frac{2 b \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{15 a d \operatorname{Cos}[c+d x]^{3/2}}$$

Result (type 4, 1253 leaves):

$$-\frac{1}{15 a^2 d}$$

$$\left( - \left( 4 a (2 a^2 b - 2 b^3) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - \right.$$

$$4 a (9 a^3 - 2 a b^2) \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - \right.$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& 2(9a^2b - 2b^3) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \csc[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /
\end{aligned}$$



$$\left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) +$$

$$\frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2 \sec[c+dx] (9a^2 \sin[c+dx] - 2b^2 \sin[c+dx])}{15a^2} + \right.$$

$$\frac{2b \sec[c+dx] \tan[c+dx]}{15a} +$$

$$\frac{2}{5} \sec[c+dx]^2$$

$$\left. \tan[c+dx] \right)$$

- **Problem 609: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \cos[c+dx]}}{\cos[c+dx]^{9/2}} dx$$

Optimal (type 4, 389 leaves, 6 steps):

$$\frac{1}{105 a^4 d} 2 (a-b) b \sqrt{a+b} (19 a^2 + 8 b^2) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{105 a^3 d} 2 (a-b) \sqrt{a+b} (25 a^2 + 6 a b + 8 b^2) \cot[c+dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2 \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{7 d \cos[c+dx]^{7/2}} + \frac{2 b \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{35 a d \cos[c+dx]^{5/2}} + \frac{2 (25 a^2 - 4 b^2) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{105 a^2 d \cos[c+dx]^{3/2}}$$

Result (type 4, 1304 leaves):

$$\frac{1}{105 a^3 d}$$

$$\begin{aligned}
& \left( - \left( 4 a (25 a^4 - 17 a^2 b^2 - 8 b^4) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a (-19 a^3 b - 8 a b^3) \right. \\
& \quad \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \quad \left. 2 (-19 a^2 b^2 - 8 b^4) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx] \right. \right. \\
& \quad \left. \left. + b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} - 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \quad \left. \left. \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right) + \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \\
& \quad \left( \frac{2 \operatorname{Sec}[c+dx]^2 (25 a^2 \operatorname{Sin}[c+dx] - 4 b^2 \operatorname{Sin}[c+dx])}{105 a^2} + \frac{2 \operatorname{Sec}[c+dx] (19 a^2 b \operatorname{Sin}[c+dx] + 8 b^3 \operatorname{Sin}[c+dx])}{105 a^3} + \right. \\
& \quad \left. \frac{2 b \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{35 a} + \frac{2}{7} \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx] \right)
\end{aligned}$$

- **Problem 610: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^{3/2} (a+b \operatorname{Cos}[c+dx])^{3/2} dx$$

Optimal (type 4, 508 leaves, 8 steps):

$$\begin{aligned}
& -\frac{1}{24abd} (a-b) \sqrt{a+b} (3a^2 + 16b^2) \operatorname{Cot}[c+dx] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Cos}[c+dx]}{\sqrt{a+b}\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{24bd} \\
& \sqrt{a+b} (a+2b) (3a+8b) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Cos}[c+dx]}{\sqrt{a+b}\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{8b^2d} \\
& a\sqrt{a+b} (a^2 - 12b^2) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Cos}[c+dx]}{\sqrt{a+b}\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \\
& \frac{(3a^2 + 16b^2) \sqrt{a+b}\operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{24bd\sqrt{\operatorname{Cos}[c+dx]}} + \frac{a\sqrt{\operatorname{Cos}[c+dx]}\sqrt{a+b}\operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{4d} + \\
& \frac{\sqrt{\operatorname{Cos}[c+dx]} (a+b\operatorname{Cos}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{3d}
\end{aligned}$$

Result (type 4, 1189 leaves):

$$\begin{aligned}
& \frac{1}{48d} \\
& \left( -\left( 4a(17a^2 + 16b^2) \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\operatorname{Cos}[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\operatorname{Cos}[c+dx]}\sqrt{a+b\operatorname{Cos}[c+dx]} \right) - \right. \\
& \left. 208a^2b \left( \left( \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\operatorname{Cos}[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) + \\
& 2(3a^2+16b^2) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b}2a \left( \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right. \right. \\
& \left. \left. \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right) \right) \right)
\end{aligned}$$

$$\left. \left. \left. \left. \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left(b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}\right) \right) \right) \right) + \left. \left. \left. \left. \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right) \right) + \frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left(\frac{7}{12} a \sin[c+dx] + \frac{1}{6} b \sin[2(c+dx)]\right)}{d} \right)$$

■ **Problem 611: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{3/2} dx$$

Optimal (type 4, 433 leaves, 8 steps):

$$\begin{aligned} & -\frac{1}{4d} 5(a-b) \sqrt{a+b} \cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{4d} \\ & \sqrt{a+b} (5a+2b) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{4bd} \\ & \sqrt{a+b} (3a^2+4b^2) \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\ & \frac{3a \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{4d \sqrt{\cos[c+dx]}} + \frac{(a+b \cos[c+dx])^{3/2} \sin[c+dx]}{2d \sqrt{\cos[c+dx]}} \end{aligned}$$

Result (type 4, 1165 leaves):

$$\begin{aligned} & \frac{b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2d} + \\ & \frac{1}{8d} \left( - \left( \left( 28a^2b \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right) \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& 4a(8a^2+4b^2) \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right. \\
& \left. \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) + \\
& 10ab \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\text{Sec}[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\text{Sec}[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\text{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b}2a \left( \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right.
\end{aligned}$$

$$\left. \begin{aligned} & \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \Bigg/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\ & \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\ & \left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \Bigg/ \right. \\ & \left. \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right) \end{aligned} \right)$$

■ **Problem 613: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{3/2}}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 337 leaves, 5 steps):

$$\begin{aligned} & \frac{1}{d} (a-b) \sqrt{a+b} \cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{d} \\ & 2(a-2b) \sqrt{a+b} \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\ & \frac{1}{d} 2b \sqrt{a+b} \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} \end{aligned}$$

Result (type 4, 1162 leaves):



$$\begin{aligned}
& - \left( 4 a^2 b \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \\
& \frac{1}{d} 4 a (a^2 - b^2) \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \frac{2 a \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{d \sqrt{\operatorname{Cos}[c+dx]}} - \frac{1}{d} 2 a b \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\ \left. \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right)$$

- **Problem 614: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{3/2}}{\operatorname{Cos}[c+dx]^{5/2}} dx$$

Optimal (type 4, 277 leaves, 4 steps):

$$\frac{1}{3 a d} 8 (a-b) b \sqrt{a+b} \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{3 a d}$$

$$2(a-3 b)(a-b) \sqrt{a+b} \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{2 a \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 d \operatorname{Cos}[c+d x]^{3/2}}$$

Result (type 4, 1183 leaves):

$$\frac{1}{3 d} \left( \left( 4 a (a^2 - b^2) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) +$$

$$16 a^2 b \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) -$$

$$\left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right)$$

$$\begin{aligned}
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4 \right/ \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) \right\} - \\
& 8 b^2 \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \sec [c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \sec [c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \sec [c+d x]}{a+b}} \right. \\
& \left. \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right. \\
& \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4 \right/ \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) \right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\
& \left. \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4 \right/ \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) \right) \right\} + \\
& \left. \left. \left. \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) \right) + \frac{\sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left(\frac{8}{3} b \tan [c+d x] + \frac{2}{3} a \sec [c+d x] \tan [c+d x]\right)}{d}
\end{aligned}$$

■ **Problem 615: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \cos[c + dx])^{3/2}}{\cos[c + dx]^{7/2}} dx$$

Optimal (type 4, 325 leaves, 5 steps):

$$\begin{aligned} & \frac{1}{5 a^2 d} 2 (a - b) \sqrt{a + b} (3 a^2 + b^2) \cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \\ & \frac{1}{5 a d} 2 (a - b) (3 a - b) \sqrt{a + b} \cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \\ & \frac{2 a \sqrt{a + b \cos[c + dx]} \sin[c + dx]}{5 d \cos[c + dx]^{5/2}} + \frac{4 b \sqrt{a + b \cos[c + dx]} \sin[c + dx]}{5 d \cos[c + dx]^{3/2}} \end{aligned}$$

Result (type 4, 1245 leaves):

$$\begin{aligned} & -\frac{1}{5 a d} \left( \left( -4 a (-a^2 b + b^3) \sqrt{\frac{(a + b) \cot\left[\frac{1}{2}(c + dx)\right]^2}{-a + b}} \sqrt{-\frac{(a + b) \cos[c + dx] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \operatorname{Csc}[c + dx] \right. \right. \\ & \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a + b}\right] \sin\left[\frac{1}{2}(c + dx)\right]^4 \right) / \left( (a + b) \sqrt{\cos[c + dx]} \sqrt{a + b \cos[c + dx]} \right) - 4 a \right. \\ & \left. (3 a^3 + a b^2) \left( \left( \sqrt{\frac{(a + b) \cot\left[\frac{1}{2}(c + dx)\right]^2}{-a + b}} \sqrt{-\frac{(a + b) \cos[c + dx] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \operatorname{Csc}[c + dx] \right. \right. \right. \\ & \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a + b}\right] \sin\left[\frac{1}{2}(c + dx)\right]^4 \right) / \left( (a + b) \sqrt{\cos[c + dx]} \sqrt{a + b \cos[c + dx]} \right) - \right. \end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& 2(3a^2b+b^3) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} + \right. \\
& \left. \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \right. \\
& \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \left. \csc[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) +$$

$$\frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2 \sec[c+dx] (3a^2 \sin[c+dx] + b^2 \sin[c+dx])}{5a} + \right.$$

$$\frac{4}{5} b \sec[c+dx]$$

$$\tan[c+dx] + \frac{2}{5} a \sec[c+dx]^2$$

$$\left. \tan[c+dx] \right)$$

- **Problem 616: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{3/2}}{\cos[c+dx]^{9/2}} dx$$

Optimal (type 4, 387 leaves, 6 steps):

$$\frac{1}{105 a^3 d} 4 (a-b) b \sqrt{a+b} (41 a^2 - 3 b^2) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{105 a^2 d} 2 (a-b) \sqrt{a+b} (25 a^2 - 57 a b - 6 b^2) \cot[c+dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2 a \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{7 d \cos[c+dx]^{7/2}} + \frac{16 b \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{35 d \cos[c+dx]^{5/2}} + \frac{2 (25 a^2 + 3 b^2) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{105 a d \cos[c+dx]^{3/2}}$$

Result (type 4, 1302 leaves):

$$\frac{1}{105 a^2 d}$$

$$\begin{aligned}
& \left( - \left( 4 a (25 a^4 - 31 a^2 b^2 + 6 b^4) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a (-82 a^3 b + 6 a b^3) \right. \\
& \quad \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \quad \left. 2 (-82 a^2 b^2 + 6 b^4) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx] \right. \right. \\
& \quad \left. \left. + b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) \right)
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \quad \left. \left. \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right) + \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \\
& \quad \left( \frac{2 \operatorname{Sec}[c+dx]^2 (25 a^2 \operatorname{Sin}[c+dx] + 3 b^2 \operatorname{Sin}[c+dx])}{105 a} + \frac{4 \operatorname{Sec}[c+dx] (41 a^2 b \operatorname{Sin}[c+dx] - 3 b^3 \operatorname{Sin}[c+dx])}{105 a^2} + \right. \\
& \quad \left. \frac{16}{35} b \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] + \right. \\
& \quad \left. \frac{2}{7} a \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx] \right)
\end{aligned}$$

- **Problem 617: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{3/2}}{\operatorname{Cos}[c+dx]^{11/2}} dx$$

Optimal (type 4, 454 leaves, 7 steps):

$$\frac{1}{315 a^4 d} 2 (a-b) \sqrt{a+b} (147 a^4 + 33 a^2 b^2 + 8 b^4) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+dx]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{315 a^3 d} 2 (a-b) \sqrt{a+b} (147 a^3 - 39 a^2 b - 6 a b^2 - 8 b^3) \operatorname{Cot}[c+dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+dx]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{2 a \sqrt{a+b} \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{9 d \operatorname{Cos}[c+dx]^{9/2}} +$$

$$\frac{20 b \sqrt{a+b} \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{63 d \operatorname{Cos}[c+dx]^{7/2}} + \frac{2 (49 a^2 + 3 b^2) \sqrt{a+b} \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{315 a d \operatorname{Cos}[c+dx]^{5/2}} + \frac{8 b (22 a^2 - b^2) \sqrt{a+b} \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{315 a^2 d \operatorname{Cos}[c+dx]^{3/2}}$$

Result (type 4, 1368 leaves):

$$-\frac{1}{315 a^3 d} \left( - \left( 4 a (-39 a^4 b + 31 a^2 b^3 + 8 b^5) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a (147 a^5 + 33 a^3 b^2 + 8 a b^4)$$

$$\left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) -$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \\
& 2(147a^4b + 33a^2b^3 + 8b^5) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right. + \\
& \left. \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right. \\
& \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left. \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) +$$

$$\frac{1}{d} \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left( \frac{2 \sec [c+d x]^3 (49 a^2 \sin [c+d x] + 3 b^2 \sin [c+d x])}{315 a} +
\right.$$

$$\frac{8 \sec [c+d x]^2 (22 a^2 b \sin [c+d x] - b^3 \sin [c+d x])}{315 a^2} +$$

$$\frac{2 \sec [c+d x] (147 a^4 \sin [c+d x] + 33 a^2 b^2 \sin [c+d x] + 8 b^4 \sin [c+d x])}{315 a^3} +$$

$$\frac{20}{63} b$$

$$\sec [c+d x]^3$$

$$\tan [c+d x] + \frac{2}{9} a$$

$$\left. \sec [c+d x]^4 \tan [c+d x] \right)$$

- **Problem 618: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos [c+d x]} (a+b \cos [c+d x])^{5/2} dx$$

Optimal (type 4, 506 leaves, 8 steps):

$$\begin{aligned}
& -\frac{1}{24ad} (a-b) \sqrt{a+b} (33a^2 + 16b^2) \cot[c+dx] \\
& \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{24d} \\
& \sqrt{a+b} (33a^2 + 26ab + 16b^2) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\
& \frac{1}{8bd} 5a\sqrt{a+b} (a^2 + 4b^2) \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(33a^2 + 16b^2)\sqrt{a+b}\cos[c+dx] \sin[c+dx]}{24d\sqrt{\cos[c+dx]}} + \\
& \frac{13ab\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx] \sin[c+dx]}{12d} + \frac{b^2\cos[c+dx]^{3/2}\sqrt{a+b}\cos[c+dx] \sin[c+dx]}{3d}
\end{aligned}$$

Result (type 4, 1203 leaves):

$$\begin{aligned}
& \frac{1}{48d} \\
& \left( - \left( 4a(59a^2b + 16b^3) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+d \right. \right. \\
& \left. \left. x\right] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& 4a(48a^3 + 76ab^2) \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+ \right. \right.
\end{aligned}$$

$$\begin{aligned}
& dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right] / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left(\sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x]\right. \\
& \left.\operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right] / \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right)\right) + \\
& 2\left(33 a^2 b+16 b^3\right)\left(\frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Sec}[c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Sec}[c+d x]}{a+b}}}\right) + \\
& \frac{1}{b} 2 a\left(\left(a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x]\right.\right. \\
& \left.\left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right] / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \\
& \left.\left(a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x]\right)\right)
\end{aligned}$$

$$\left. \left. \left. \left. \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left(b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}\right) \right) \right) \right) + \left. \left. \left. \left. \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right) \right) + \frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left(\frac{13}{12} a b \sin[c+dx] + \frac{1}{6} b^2 \sin[2(c+dx)]\right)}{d} \right)$$

■ **Problem 619: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{5/2}}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 443 leaves, 7 steps):

$$\begin{aligned} & -\frac{1}{4d} 9(a-b)b\sqrt{a+b} \cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{4d} \\ & \sqrt{a+b} (8a^2+9ab+2b^2) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{4d} \\ & \sqrt{a+b} (15a^2+4b^2) \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\ & \frac{9ab\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{4d \sqrt{\cos[c+dx]}} + \frac{b^2 \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2d} \end{aligned}$$

Result (type 4, 1179 leaves):

$$\begin{aligned} & \frac{b^2 \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2d} + \\ & \frac{1}{8d} \left( - \left( 4a(8a^3+11ab^2) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc\left[\frac{1}{2}(c+dx)\right] \right) \right) \end{aligned}$$

$$\begin{aligned}
& (c + dx) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \Big/ \left((a+b) \sqrt{\cos [c+dx]} \sqrt{a+b \cos [c+dx]}\right) - \\
& 4a(24a^2b + 4b^3) \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+ \right. \right. \\
& \left. \left. dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \Big/ \left((a+b) \sqrt{\cos [c+dx]} \sqrt{a+b \cos [c+dx]}\right) - \right. \\
& \left. \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \Big/ \left(b \sqrt{\cos [c+dx]} \sqrt{a+b \cos [c+dx]}\right) \right) \right) + \\
& 18ab^2 \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos [c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos [c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \cos [c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.
\end{aligned}$$



$$\left. \left. \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) \right) \right) \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) -$$

$$\left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right.$$

$$\left. \left. \left. \left. \left. \csc[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) \right) \right) \right) /$$

$$\left. \left. \left. \left. \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right) \right) \right) \right)$$

■ **Problem 620: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{5/2}}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 445 leaves, 7 steps):

$$\frac{1}{ad} (a-b) \sqrt{a+b} (2a^2-b^2) \cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{d}$$

$$\sqrt{a+b} (2a^2-6ab-b^2) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{d} 5ab \sqrt{a+b} \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2a^2 \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}} - \frac{(2a^2-b^2) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}}$$

Result (type 4, 1185 leaves) :

$$\frac{2 a^2 \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{d \sqrt{\cos [c+d x]}} + \frac{1}{2 d}$$

$$\left( \left( 4 a (-4 a^2 b - b^3) \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \csc [c+d x] \right. \right.$$

$$\left. \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right) / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + 4 a \right.$$

$$(2 a^3 - 6 a b^2) \left( \left( \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \csc [c+d x] \right. \right.$$

$$\left. \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right) / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \right.$$

$$\left( \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \csc [c+d x] \right.$$

$$\left. \left. \text{EllipticPi} \left[ -\frac{a}{b}, \text{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right) / \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \right.$$

$$\begin{aligned}
& 2 (2 a^2 b - b^3) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}}}\right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \left. \csc[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \left. \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right)
\end{aligned}$$

■ **Problem 622: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{5/2}}{\cos[c+dx]^{7/2}} dx$$

Optimal (type 4, 338 leaves, 5 steps):

$$\frac{1}{15 a d}$$

$$2 (a - b) \sqrt{a + b} (9 a^2 + 23 b^2) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Cos}[c + d x]}}{\sqrt{a + b} \sqrt{\operatorname{Cos}[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} -$$

$$\frac{1}{15 a d} 2 (a - b) \sqrt{a + b} (9 a^2 - 8 a b + 15 b^2) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Cos}[c + d x]}}{\sqrt{a + b} \sqrt{\operatorname{Cos}[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}}$$

$$\sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} + \frac{2 a^2 \sqrt{a + b \operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{5 d \operatorname{Cos}[c + d x]^{5/2}} + \frac{22 a b \sqrt{a + b \operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{15 d \operatorname{Cos}[c + d x]^{3/2}}$$

Result (type 4, 1248 leaves):

$$\frac{1}{15 d}$$

$$\left( \left( 4 a (-8 a^2 b + 8 b^3) \sqrt{\frac{(a + b) \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]^2}{-a + b}} \sqrt{-\frac{(a + b) \operatorname{Cos}[c + d x] \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}} \sqrt{\frac{(a + b \operatorname{Cos}[c + d x]) \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}} \operatorname{Csc}[c + d x] \right. \right.$$

$$\left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a + b \operatorname{Cos}[c + d x]) \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2 a}{-a + b}\right] \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]^4 \right) / \left( (a + b) \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + b \operatorname{Cos}[c + d x]} \right) + 4 a$$

$$(9 a^3 + 23 a b^2) \left( \left( \sqrt{\frac{(a + b) \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]^2}{-a + b}} \sqrt{-\frac{(a + b) \operatorname{Cos}[c + d x] \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}} \sqrt{\frac{(a + b \operatorname{Cos}[c + d x]) \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}} \operatorname{Csc}[c + d x] \right. \right.$$

$$\left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a + b \operatorname{Cos}[c + d x]) \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2 a}{-a + b}\right] \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]^4 \right) / \left( (a + b) \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + b \operatorname{Cos}[c + d x]} \right) -$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& 2(9a^2b + 23b^3) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /
\end{aligned}$$

$$\left( \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \frac{1}{d}$$

$$\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2}{15} \sec[c+dx] (9a^2 \sin[c+dx] + 23b^2 \sin[c+dx]) + \frac{22}{15} ab \sec[c+dx] \tan[c+dx] + \frac{2}{5} a^2 \sec[c+dx]^2 \tan[c+dx] \right)$$

■ **Problem 623: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{5/2}}{\cos[c+dx]^{9/2}} dx$$

Optimal (type 4, 387 leaves, 6 steps):

$$\frac{1}{21 a^2 d} {}_2F_1\left(\frac{a-b}{a+b}, \frac{a+b}{a-b}, \frac{29 a^2 + 3 b^2}{21 a d} \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right], -\frac{a+b}{a-b}\right)$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{21 a d} {}_2F_1\left(\frac{a-b}{a+b}, \frac{a+b}{a-b}, \frac{5 a^2 - 24 a b + 3 b^2}{21 a d} \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right], -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{2 a^2 \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{7 d \cos[c+dx]^{7/2}} + \frac{6 a b \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{7 d \cos[c+dx]^{5/2}} + \frac{2 (5 a^2 + 9 b^2) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{21 d \cos[c+dx]^{3/2}}$$

Result (type 4, 1302 leaves):

$$\frac{1}{21 a d} \left( - \left( 4 a (5 a^4 - 2 a^2 b^2 - 3 b^4) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right] \right) \right)$$

$$\begin{aligned}
& c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \Bigg/ \left((a+b) \sqrt{\cos [c+dx]} \sqrt{a+b \cos [c+dx]}\right) - \\
& 4a(-29a^3b - 3ab^3) \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+ \right. \right. \\
& \left. \left. dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \Bigg/ \left((a+b) \sqrt{\cos [c+dx]} \sqrt{a+b \cos [c+dx]}\right) - \right. \\
& \left. \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \Bigg/ \left(b \sqrt{\cos [c+dx]} \sqrt{a+b \cos [c+dx]}\right) \right) \right) + \\
& 2(-29a^2b^2 - 3b^4) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos [c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos [c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \cos [c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2a \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.
\end{aligned}$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) -$$

$$\left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right.$$

$$\left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) +$$

$$\left. \left. \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right) + \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}$$

$$\left( \frac{2}{21} \sec[c+dx]^2 (5a^2 \sin[c+dx] + 9b^2 \sin[c+dx]) + \frac{2 \sec[c+dx] (29a^2 b \sin[c+dx] + 3b^3 \sin[c+dx])}{21a} +$$

$$\frac{6}{7} a b \sec[c+dx]^2 \tan[c+dx] +$$

$$\frac{2}{7} a^2 \sec[c+dx]^3 \tan[c+dx] \right)$$

- **Problem 624: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{5/2}}{\cos[c+dx]^{11/2}} dx$$

Optimal (type 4, 454 leaves, 7 steps):



$$\frac{1}{315 a^3 d} 2 (a-b) \sqrt{a+b} (147 a^4 + 279 a^2 b^2 - 10 b^4) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{315 a^2 d}$$

$$2 (a-b) \sqrt{a+b} (147 a^3 - 114 a^2 b + 165 a b^2 + 10 b^3) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{2 a^2 \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{9 d \operatorname{Cos}[c+d x]^{9/2}} + \frac{38 a b \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{63 d \operatorname{Cos}[c+d x]^{7/2}} +$$

$$\frac{2 (49 a^2 + 75 b^2) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{315 d \operatorname{Cos}[c+d x]^{5/2}} + \frac{2 b (163 a^2 + 5 b^2) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{315 a d \operatorname{Cos}[c+d x]^{3/2}}$$

Result (type 4, 1368 leaves):

$$-\frac{1}{315 a^2 d} \left( - \left( 4 a (-114 a^4 b + 124 a^2 b^3 - 10 b^5) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\ \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \right. \\ \left. \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (147 a^5 + 279 a^3 b^2 - 10 a b^4) \right. \\ \left. \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right. \\ \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - \right. \right.$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \\
& 2(147a^4b + 279a^2b^3 - 10b^5) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) +$$

$$\frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2}{315} \sec[c+dx]^3 (49 a^2 \sin[c+dx] + 75 b^2 \sin[c+dx]) + \right.$$

$$\frac{2 \sec[c+dx]^2 (163 a^2 b \sin[c+dx] + 5 b^3 \sin[c+dx])}{315 a} +$$

$$\frac{2 \sec[c+dx] (147 a^4 \sin[c+dx] + 279 a^2 b^2 \sin[c+dx] - 10 b^4 \sin[c+dx])}{315 a^2} +$$

$$\frac{38}{63} a b$$

$$\left. \left( \sec[c+dx]^3 \tan[c+dx] + \frac{2}{9} \right. \right.$$

$$\left. \left. a^2 \sec[c+dx]^4 \tan[c+dx] \right) \right)$$

- **Problem 625: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{5/2}}{\cos[c+dx]^{13/2}} dx$$

Optimal (type 4, 522 leaves, 8 steps):

$$\frac{1}{693 a^4 d} 2 (a-b) b \sqrt{a+b} (741 a^4 + 51 a^2 b^2 + 8 b^4) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{693 a^3 d} 2 (a-b) \sqrt{a+b} (135 a^4 - 606 a^3 b + 57 a^2 b^2 + 6 a b^3 + 8 b^4)$$

$$\cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2 a^2 \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{11 d \cos[c+dx]^{11/2}} + \frac{46 a b \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{99 d \cos[c+dx]^{9/2}} + \frac{2 (81 a^2 + 113 b^2) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{693 d \cos[c+dx]^{7/2}} +$$

$$\frac{2 b (229 a^2 + 3 b^2) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{693 a d \cos[c+dx]^{5/2}} + \frac{2 (135 a^4 + 205 a^2 b^2 - 4 b^4) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{693 a^2 d \cos[c+dx]^{3/2}}$$

Result (type 4, 1431 leaves):

$$\begin{aligned}
& \frac{1}{693 a^3 d} \left( - \left( 4 a (135 a^6 - 78 a^4 b^2 - 49 a^2 b^4 - 8 b^6) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a (-741 a^5 b - 51 a^3 b^3 - 8 a b^5) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2 (-741 a^4 b^2 - 51 a^2 b^4 - 8 b^6) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right.$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\ \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \left. \right) +$$

$$\frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2}{693} \operatorname{Sec}[c+dx]^4 (81 a^2 \operatorname{Sin}[c+dx] + 113 b^2 \operatorname{Sin}[c+dx]) + \right. \\ \frac{2 \operatorname{Sec}[c+dx]^3 (229 a^2 b \operatorname{Sin}[c+dx] + 3 b^3 \operatorname{Sin}[c+dx])}{693 a} + \\ \frac{2 \operatorname{Sec}[c+dx]^2 (135 a^4 \operatorname{Sin}[c+dx] + 205 a^2 b^2 \operatorname{Sin}[c+dx] - 4 b^4 \operatorname{Sin}[c+dx])}{693 a^2} + \\ \left. \frac{2 \operatorname{Sec}[c+dx] (741 a^4 b \operatorname{Sin}[c+dx] + 51 a^2 b^3 \operatorname{Sin}[c+dx] + 8 b^5 \operatorname{Sin}[c+dx])}{693 a^3} + \right. \\ \left. \frac{46}{99} a b \operatorname{Sec}[c+dx]^4 \operatorname{Tan}[c+dx] + \frac{2}{11} a^2 \operatorname{Sec}[c+dx]^5 \operatorname{Tan}[c+dx] \right)$$

■ **Problem 626:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cos [c+d x]^{3 / 2}}{\sqrt{a+b \cos [c+d x]}} d x$$

Optimal (type 4, 379 leaves, 8 steps):

$$-\frac{1}{a b d}(a-b) \sqrt{a+b} \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} +$$

$$\frac{\sqrt{a+b} \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}}{b d} + \frac{1}{b^2 d}$$

$$a \sqrt{a+b} \cot [c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} +$$

$$\frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b d \sqrt{\cos [c+d x]}}$$

Result (type 4, 479 leaves):

$$\frac{1}{2 b \sqrt{\frac{a-b}{a+b}} d \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{a+b \cos [c+d x]}}$$

$$\sqrt{\cos [c+d x]} \left( 2 i (a-b) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] - \right.$$

$$4 i a \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] +$$

$$4 i a \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] + b \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}$$

$$\left. \sec \left[\frac{1}{2}(c+d x)\right] \sin \left[\frac{3}{2}(c+d x)\right] + 2 a \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[\frac{1}{2}(c+d x)\right] - b \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[\frac{1}{2}(c+d x)\right] \right)$$

■ **Problem 629: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\cos [c+d x]^{3 / 2} \sqrt{a+b \cos [c+d x]}} d x$$

Optimal (type 4, 224 leaves, 3 steps) :

$$\frac{1}{a^2 d} 2 (a-b) \sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} -$$

$$\frac{2 \sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}}}{a d}$$

Result (type 4, 894 leaves) :

$$\frac{1}{d} 4 a \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) -$$

$$\left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right.$$

$$\left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) +$$

$$\frac{2 \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{a d \sqrt{\operatorname{Cos}[c+dx]}} - \frac{1}{a \sqrt{\frac{a-b}{a+b}} d \sqrt{\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{a+b \operatorname{Cos}[c+dx]}}$$

$$\sqrt{\operatorname{Cos}[c+dx]} \left( 2 i (a-b) \sqrt{\frac{a+b \operatorname{Cos}[c+dx]}{(a+b)(1+\operatorname{Cos}[c+dx])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] -$$

$$\begin{aligned}
& 4 i a \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] + \\
& 4 i a \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] + \\
& b \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right] + \\
& 2 a \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[\frac{1}{2}(c+d x)\right] - b \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[\frac{1}{2}(c+d x)\right]
\end{aligned}$$

■ **Problem 630: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\cos [c+d x]^{5/2} \sqrt{a+b \cos [c+d x]}} dx$$

Optimal (type 4, 274 leaves, 4 steps):

$$\begin{aligned}
& -\frac{1}{3 a^3 d} 4(a-b) b \sqrt{a+b} \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \frac{1}{3 a^2 d} \\
& 2 \sqrt{a+b}(a+2 b) \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \\
& \frac{2 \sqrt{a+b \cos [c+d x]} \operatorname{Sin}[c+d x]}{3 a d \cos [c+d x]^{3/2}}
\end{aligned}$$

Result (type 4, 1191 leaves):

$$\frac{1}{3 a^2 d} \left( - \left( 4 a (a^2 + 2 b^2) \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right)
\right)$$



$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& 8a^2b \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right. \\
& \left. \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) + \\
& 4b^2 \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\text{Sec}[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\text{Sec}[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\text{Sec}[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b}2a \left( \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& \left. \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( -\frac{4b \tan[c+dx]}{3a^2} + \frac{2 \sec[c+dx] \tan[c+dx]}{3a} \right)}{d}
\end{aligned}$$

■ **Problem 631: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{5/2}}{(a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 465 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(3a^2 - b^2) \operatorname{Cot}[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}}}{a b^2 \sqrt{a+b} d} + \\
& \frac{(3a+b) \operatorname{Cot}[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}}}{b^2 \sqrt{a+b} d} + \frac{1}{b^3 d} \\
& 3a\sqrt{a+b} \operatorname{Cot}[c + dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \\
& \frac{2a^2\sqrt{\cos[c+dx]}\sin[c+dx]}{b(a^2 - b^2)d\sqrt{a+b}\cos[c+dx]} + \frac{(3a^2 - b^2)\sqrt{a+b}\cos[c+dx]\sin[c+dx]}{b^2(a^2 - b^2)d\sqrt{\cos[c+dx]}}
\end{aligned}$$

Result (type 4, 1201 leaves):

$$\frac{2a^2\sqrt{\cos[c+dx]}\sin[c+dx]}{b(-a^2 + b^2)d\sqrt{a+b}\cos[c+dx]} + \frac{1}{2(a-b)b(a+b)d}$$

$$\begin{aligned}
& \left( - \left( 4a(a^2 - b^2) \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx] \right) - \right. \\
& \left. 8a^2b \left( \left( \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx] \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& 2(3a^2 - b^2) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /
\end{aligned}$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}}$$

■ **Problem 632: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{3/2}}{(a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 387 leaves, 6 steps):

$$\frac{2 \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{b \sqrt{a+b} d}$$

$$\frac{2 \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{b \sqrt{a+b} d} - \frac{1}{b^2 d}$$

$$\frac{2 \sqrt{a+b} \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{b (a^2 - b^2) d \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}}$$

Result (type 4, 985 leaves):

$$\frac{2 a \sqrt{\cos[c+dx]} \sin[c+dx]}{(a^2 - b^2) d \sqrt{a+b \cos[c+dx]}} - \frac{1}{(a-b)(a+b) d}$$

$$\left( -4 a b \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right.$$

$$\left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \right.$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& 2a \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}}$$

■ **Problem 634: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 267 leaves, 4 steps):

$$\frac{2 b \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{a^2 \sqrt{a+b} d} +$$

$$\frac{2 \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{a \sqrt{a+b} d} - \frac{2 b \sin[c+dx]}{(a^2 - b^2) d \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}}$$

Result (type 4, 2867 leaves):

$$\frac{1}{(a^2 - b^2) d} 4 a \left( \left( \sqrt{\cot\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], \frac{2 a}{a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.$$

$$\left. \left( b \sqrt{\cot\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right) \right)$$

$$\begin{aligned}
& \left. \text{EllipticPi}\left[\frac{a}{a+b}, \text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], \frac{2a}{a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) + \\
& \frac{2b^2\sqrt{\cos[c+dx]}\sin[c+dx]}{a(a^2-b^2)d\sqrt{a+b\cos[c+dx]}} - \left( b\sqrt{\cos[c+dx]}(1+\cos[c+dx])^{3/2}\sec\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \left. \left( 2(a+b)\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticE}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - 4a\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - 4a\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right. \right. \\
& \left. \left. b\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sec\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] + 2a\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] - b\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) / \\
& \left( 2a(a^2-b^2)d \left( \frac{1}{8(a+b\cos[c+dx])^{3/2}} b(1+\cos[c+dx])^{3/2}\sec\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] \left( 2(a+b)\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \right. \right. \right. \\
& \left. \left. \text{EllipticE}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - 4a\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticF}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - \right. \right. \\
& \left. \left. 4a\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + b\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \right. \right. \\
& \left. \left. \sec\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] + 2a\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] - b\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) - \\
& \frac{1}{8\sqrt{a+b\cos[c+dx]}} 3\sqrt{1+\cos[c+dx]}\sec\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] \left( 2(a+b)\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \right)
\end{aligned}$$



$$\begin{aligned}
& \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - 4a \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - \\
& 4a \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \\
& \text{Sec}\left[\frac{1}{2}(c+dx)\right] \text{Sin}\left[\frac{3}{2}(c+dx)\right] + 2a \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{Tan}\left[\frac{1}{2}(c+dx)\right] - b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{Tan}\left[\frac{1}{2}(c+dx)\right] \Bigg) + \\
& \frac{1}{4\sqrt{a+b\cos[c+dx]}} (1+\cos[c+dx])^{3/2} \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \text{Tan}\left[\frac{1}{2}(c+dx)\right] \left( 2(a+b) \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \right. \\
& \left. \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - 4a \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - \right. \\
& \left. 4a \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \right. \\
& \left. \text{Sec}\left[\frac{1}{2}(c+dx)\right] \text{Sin}\left[\frac{3}{2}(c+dx)\right] + 2a \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{Tan}\left[\frac{1}{2}(c+dx)\right] - b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{Tan}\left[\frac{1}{2}(c+dx)\right] \right) + \\
& \frac{1}{4\sqrt{a+b\cos[c+dx]}} (1+\cos[c+dx])^{3/2} \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left( \frac{3}{2}b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \cos\left[\frac{3}{2}(c+dx)\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \left. a \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 - \frac{1}{2}b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 + \frac{1}{\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} \right. \\
& \left. (a+b) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \left( -\frac{b\sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(a+b\cos[c+dx])\sin[c+dx]}{(a+b)(1+\cos[c+dx])^2} \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{2 a \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]\left(-\frac{b \operatorname{Sin}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}+\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Sin}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])^2}\right)-\frac{1}{\sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}}}}{\sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}}}} \\
& + 2 a \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]\left(-\frac{b \operatorname{Sin}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}+\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Sin}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])^2}\right)+ \\
& \frac{b \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]\left(\frac{\operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{(1+\operatorname{Cos}[c+d x])^2}-\frac{\operatorname{Sin}[c+d x]}{1+\operatorname{Cos}[c+d x]}\right) \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right]+a\left(\frac{\operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{(1+\operatorname{Cos}[c+d x])^2}-\frac{\operatorname{Sin}[c+d x]}{1+\operatorname{Cos}[c+d x]}\right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{2 \sqrt{\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}}+\sqrt{\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}} \\
& - \frac{b\left(\frac{\operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{(1+\operatorname{Cos}[c+d x])^2}-\frac{\operatorname{Sin}[c+d x]}{1+\operatorname{Cos}[c+d x]}\right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{2 \sqrt{\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}}+\frac{1}{2} b \sqrt{\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}- \\
& \frac{2 a \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{1-\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}+\frac{2 a \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{1-\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}+ \\
& \frac{(a+b) \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}{\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right) \right) \right) \right)
\end{aligned}$$

- **Problem 635: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Cos}[c+d x]^{3/2}(a+b \operatorname{Cos}[c+d x])^{3/2}} dx$$

Optimal (type 4, 285 leaves, 4 steps):

$$\frac{2(a^2 - 2b^2) \cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{a^3 \sqrt{a+b} d} -$$

$$\frac{2(a+2b) \cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{a^2 \sqrt{a+b} d} +$$

$$\frac{2b^2 \sin[c + dx]}{a(a^2 - b^2) d \sqrt{\cos[c + dx]} \sqrt{a + b \cos[c + dx]}}$$

Result (type 4, 1233 leaves):

$$\frac{1}{a^2(-a+b)(a+b)d}$$

$$\left( - \left( 4a(2a^2b - 2b^3) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) -$$

$$4a(a^3 - 2ab^2) \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) -$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \Bigg) + \\
& 2(a^2 b - 2b^3) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \Bigg) +
\end{aligned}$$

$$\left. \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) + \frac{\sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left( -\frac{2 b^3 \sin [c+d x]}{a^2 (a^2-b^2) (a+b \cos [c+d x])} + \frac{2 \tan [c+d x]}{a^2} \right)}{d}$$

- **Problem 636: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\cos [c+d x]^{5/2} (a+b \cos [c+d x])^{3/2}} dx$$

Optimal (type 4, 357 leaves, 5 steps):

$$\begin{aligned} & -\frac{1}{3 a^4 \sqrt{a+b} d} 2 b (5 a^2-8 b^2) \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}+ \\ & \frac{1}{3 a^3 \sqrt{a+b} d} 2(a+2 b)(a+4 b) \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}+ \\ & \frac{2 b^2 \sin [c+d x]}{a\left(a^2-b^2\right) d \cos [c+d x]^{3/2} \sqrt{a+b \cos [c+d x]}}+\frac{2\left(a^2-4 b^2\right) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{3 a^2\left(a^2-b^2\right) d \cos [c+d x]^{3/2}} \end{aligned}$$

Result (type 4, 1269 leaves):

$$\begin{aligned} & \frac{1}{3 a^3(a-b)(a+b) d} \\ & \left( -\left( 4 a\left(a^4+7 a^2 b^2-8 b^4\right) \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\ & \left. \left. \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right],-\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right) / \right. \\ & \left. \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right)-4 a\left(5 a^3 b-8 a b^3\right) \right) \end{aligned}$$

$$\begin{aligned}
& \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + \\
& 2(5a^2b^2 - 8b^4) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\ \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \\ \left. \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) + \frac{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2b^4 \operatorname{Sin}[c+dx]}{a^3 (a^2-b^2) (a+b \operatorname{Cos}[c+dx])} - \frac{10b \operatorname{Tan}[c+dx]}{3a^3} + \frac{2 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{3a^2} \right)}{d}$$

■ **Problem 637: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Cos}[c+dx]^{7/2} (a+b \operatorname{Cos}[c+dx])^{3/2}} dx$$

Optimal (type 4, 433 leaves, 6 steps):

$$\frac{1}{5a^5 \sqrt{a+b} d} 2 (3a^4 + 8a^2 b^2 - 16b^4) \operatorname{Cot}[c+dx] \\ \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{5a^4 \sqrt{a+b} d} \\ 2(3a+4b)(a^2+4b^2) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \\ \frac{2b^2 \operatorname{Sin}[c+dx]}{a(a^2-b^2) d \operatorname{Cos}[c+dx]^{5/2} \sqrt{a+b \operatorname{Cos}[c+dx]}} + \frac{2(a^2-6b^2) \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{5a^2(a^2-b^2) d \operatorname{Cos}[c+dx]^{5/2}} - \frac{2b(3a^2-8b^2) \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{5a^3(a^2-b^2) d \operatorname{Cos}[c+dx]^{3/2}}$$

Result (type 4, 1314 leaves):

$$\frac{1}{5a^4(-a+b)(a+b)d}$$

$$\begin{aligned}
& (a^2 + 4b^2) \left( - \left[ 4a(4a^2b - 4b^3) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right] / \right. \\
& \quad \left. \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4a(3a^3 - 4ab^2) \right. \\
& \quad \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right] / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right] / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \quad \left. 2(3a^2b - 4b^3) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx] \right. \right. \\
& \quad \left. \left. b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) \right) +
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \left. \right) + \frac{1}{d} \\
& \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( -\frac{2b^5 \operatorname{Sin}[c+dx]}{a^4 (a^2 - b^2) (a+b \operatorname{Cos}[c+dx])} + \frac{2 \operatorname{Sec}[c+dx] (3a^2 \operatorname{Sin}[c+dx] + 11b^2 \operatorname{Sin}[c+dx])}{5a^4} - \right. \\
& \quad \left. \frac{6b \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{5a^3} + \frac{2 \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{5a^2} \right)
\end{aligned}$$

- **Problem 638: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^{5/2}}{(a+b \operatorname{Cos}[c+dx])^{5/2}} dx$$

Optimal (type 4, 497 leaves, 7 steps):

$$\frac{2(3a^2 - 7b^2) \cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{3(a-b)b^2(a+b)^{3/2}d} - \frac{1}{3(a-b)b^2(a+b)^{3/2}d}$$

$$\frac{2(3a^2 + ab - 6b^2) \cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{b^3 d} - \frac{1}{b^3 d} \frac{2\sqrt{a+b} \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{2a^2\sqrt{\cos[c+dx]}\sin[c+dx]} - \frac{2a^2(3a^2 - 7b^2)\sin[c+dx]}{3b^2(a^2 - b^2)^2 d \sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx]}$$

Result (type 4, 1282 leaves):

$$\frac{\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx]}{d} \left( \frac{2a^2\sin[c+dx]}{3b(-a^2+b^2)(a+b)\cos[c+dx]^2} + \frac{2(3a^3\sin[c+dx] - 7ab^2\sin[c+dx])}{3b(-a^2+b^2)^2(a+b)\cos[c+dx]} \right) - \frac{1}{3(a-b)^2b(a+b)^2d}$$

$$\left( - \left( 4a(a^3 - ab^2) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx] \right) - 4a \right.$$

$$\left. (-a^2b - 3b^3) \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx] \right) - \right.$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& 2(3a^3 - 7ab^2) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right. + \\
& \left. \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right. \\
& \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}}$$

■ **Problem 639: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^{3/2}}{(a+b \cos [c+d x])^{5/2}} dx$$

Optimal (type 4, 342 leaves, 5 steps):

$$\frac{8 b \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}}{3 a(a-b)(a+b)^{3/2} d} +$$

$$\frac{2(a-3 b) \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}}{3 a(a-b)(a+b)^{3/2} d} +$$

$$\frac{2 a \sqrt{\cos [c+d x]} \sin [c+d x]}{3\left(a^2-b^2\right) d(a+b \cos [c+d x])^{3/2}} - \frac{8 a b \sin [c+d x]}{3\left(a^2-b^2\right)^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}}$$

Result (type 4, 1237 leaves):

$$\frac{\sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}}{d} \left( \frac{2 a \sin [c+d x]}{3\left(a^2-b^2\right)(a+b \cos [c+d x])^2} + \frac{8 b^2 \sin [c+d x]}{3\left(a^2-b^2\right)^2(a+b \cos [c+d x])} \right) + \frac{1}{3(a-b)^2(a+b)^2 d}$$

$$\left( - \left( 4 a\left(a^2-b^2\right) \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) +$$

$$\begin{aligned}
& 16 a^2 b \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) - \\
& 8 b^2 \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\ \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\ \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}}$$

- **Problem 640: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Cos}[c+dx]}}{(a+b \operatorname{Cos}[c+dx])^{5/2}} dx$$

Optimal (type 4, 359 leaves, 5 steps):

$$\frac{2(3a^2+b^2) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}}}{3a^2(a-b)(a+b)^{3/2}d} + \\ \frac{2(3a-b) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}}}{3a(a-b)(a+b)^{3/2}d} - \\ \frac{2b \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{3(a^2-b^2)d(a+b \operatorname{Cos}[c+dx])^{3/2}} + \frac{2(3a^2+b^2) \operatorname{Sin}[c+dx]}{3(a^2-b^2)^2 d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]}}$$

Result (type 4, 1273 leaves):

$$\frac{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( -\frac{2b \operatorname{Sin}[c+dx]}{3(a^2-b^2)(a+b \operatorname{Cos}[c+dx])^2} - \frac{2(3a^2b \operatorname{Sin}[c+dx]+b^3 \operatorname{Sin}[c+dx])}{3a(a^2-b^2)^2(a+b \operatorname{Cos}[c+dx])} \right)}{d} + \frac{1}{3a(a-b)^2(a+b)^2d}$$

$$\begin{aligned}
& \left( - \left( 4 a (-a^2 b + b^3) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad 4 a (3 a^3 + a b^2) \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \quad 2 (3 a^2 b + b^3) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) +
\end{aligned}$$

$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\ \left. \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right)$$

- **Problem 641: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\operatorname{Cos}[c+dx]} (a+b \operatorname{Cos}[c+dx])^{5/2}} dx$$

Optimal (type 4, 381 leaves, 5 steps):



$$\frac{4 b (3 a^2 - b^2) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}}}{3 a^3 (a-b) (a+b)^{3/2} d} + \frac{1}{3 a^2 (a-b) (a+b)^{3/2} d}$$

$$+ \frac{2 (3 a^2 - 3 a b - 2 b^2) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}}}{3 a (a^2 - b^2) d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]}} + \frac{2 b^2 \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 a (a^2 - b^2) d (a+b \operatorname{Cos}[c+d x])^{3/2}} - \frac{4 b (3 a^2 - b^2) \operatorname{Sin}[c+d x]}{3 a (a^2 - b^2)^2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]}}$$

Result (type 4, 1296 leaves):

$$\frac{\sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \left( \frac{2 b^2 \operatorname{Sin}[c+d x]}{3 a (a^2 - b^2) (a+b \operatorname{Cos}[c+d x])^2} + \frac{4 (3 a^2 b^2 \operatorname{Sin}[c+d x] - b^4 \operatorname{Sin}[c+d x])}{3 a^2 (a^2 - b^2)^2 (a+b \operatorname{Cos}[c+d x])} \right)}{d} +$$

$$\frac{1}{3 a^2 (a-b)^2 (a+b)^2 d} \left( - \left( 4 a (3 a^4 - 5 a^2 b^2 + 2 b^4) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2} (c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]^4 \right) / \right.$$

$$\left. \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (-6 a^3 b + 2 a b^3) \right.$$

$$\left. \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2} (c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) -$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& 2(-6a^2b^2 + 2b^4) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /
\end{aligned}$$

$$\left( (b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right)$$

- **Problem 642: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\cos[c+dx]^{3/2} (a+b \cos[c+dx])^{5/2}} dx$$

Optimal (type 4, 398 leaves, 5 steps):

$$\frac{1}{3 a^4 (a-b) (a+b)^{3/2} d} 2 (3 a^4 - 15 a^2 b^2 + 8 b^4) \operatorname{Cot}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{3 a^3 (a-b) (a+b)^{3/2} d}$$

$$2 (3 a^3 + 9 a^2 b - 6 a b^2 - 8 b^3) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2 b^2 \sin[c+dx]}{3 a (a^2 - b^2) d \sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{3/2}} + \frac{8 b^2 (2 a^2 - b^2) \sin[c+dx]}{3 a^2 (a^2 - b^2)^2 d \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}}$$

Result (type 4, 1321 leaves):

$$-\frac{1}{3 a^3 (a-b)^2 (a+b)^2 d}$$

$$\left( - \left( 4 a (9 a^4 b - 17 a^2 b^3 + 8 b^5) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right]}, -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.$$

$$\left. \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4 a (3 a^5 - 15 a^3 b^2 + 8 a b^4) \right)$$

$$\begin{aligned}
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left. \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2(3a^4b - 15a^2b^3 + 8b^5) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
\left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
\left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \left. \right) + \frac{1}{d} \\
\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( -\frac{2b^3 \operatorname{Sin}[c+dx]}{3a^2(a^2-b^2)(a+b \operatorname{Cos}[c+dx])^2} - \frac{2(9a^2b^3 \operatorname{Sin}[c+dx] - 5b^5 \operatorname{Sin}[c+dx])}{3a^3(a^2-b^2)^2(a+b \operatorname{Cos}[c+dx])} + \right. \\
\left. \frac{2 \operatorname{Tan}[c+dx]}{a^3} \right)$$

- **Problem 643: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Cos}[c+dx]^{5/2} (a+b \operatorname{Cos}[c+dx])^{5/2}} dx$$

Optimal (type 4, 473 leaves, 6 steps):

$$\begin{aligned}
& - \frac{1}{3 a^5 (a-b) (a+b)^{3/2} d} \\
& 8 b (2 a^4 - 7 a^2 b^2 + 4 b^4) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \\
& \frac{1}{3 a^4 (a-b) (a+b)^{3/2} d} 2 (a^4 + 9 a^3 b + 16 a^2 b^2 - 12 a b^3 - 16 b^4) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{2 b^2 \operatorname{Sin}[c+d x]}{3 a (a^2 - b^2) d \operatorname{Cos}[c+d x]^{3/2} (a+b \operatorname{Cos}[c+d x])^{3/2}} + \\
& \frac{4 b^2 (5 a^2 - 3 b^2) \operatorname{Sin}[c+d x]}{3 a^2 (a^2 - b^2)^2 d \operatorname{Cos}[c+d x]^{3/2} \sqrt{a+b \operatorname{Cos}[c+d x]}} + \frac{2 (a^4 - 13 a^2 b^2 + 8 b^4) \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 a^3 (a^2 - b^2)^2 d \operatorname{Cos}[c+d x]^{3/2}}
\end{aligned}$$

Result (type 4, 1351 leaves):

$$\begin{aligned}
& \frac{1}{3 a^4 (a-b)^2 (a+b)^2 d} \left( \left( 4 a (a^6 + 15 a^4 b^2 - 32 a^2 b^4 + 16 b^6) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \right. \\
& \left. \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (8 a^5 b - 28 a^3 b^3 + 16 a b^5) \right. \\
& \left. \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \\
& 2(8a^4b^2 - 28a^2b^4 + 16b^6) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) /
\end{aligned}$$

$$\left( \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) + \frac{1}{d}$$

$$\sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left( \frac{2 b^4 \sin [c+d x]}{3 a^3 (a^2-b^2)(a+b \cos [c+d x])^2} + \frac{8(3 a^2 b^4 \sin [c+d x]-2 b^6 \sin [c+d x])}{3 a^4 (a^2-b^2)^2 (a+b \cos [c+d x])} - \frac{16 b \tan [c+d x]}{3 a^4} + \frac{2 \sec [c+d x] \tan [c+d x]}{3 a^3} \right)$$

■ **Problem 644: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\cos [c+d x]} \sqrt{2+3 \cos [c+d x]}} dx$$

Optimal (type 4, 32 leaves, 1 step):

$$\frac{2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sin [c+d x]}{1+\cos [c+d x]}\right], \frac{1}{5}\right]}{\sqrt{5} d}$$

Result (type 4, 131 leaves):

$$\left( 2 \sqrt{\cos [c+d x]} \sqrt{2+3 \cos [c+d x]} \sqrt{\cot \left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{2} \sqrt{(2+3 \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}\right], -4\right] \right) /$$

$$\left( d \sqrt{\frac{-2-3 \cos [c+d x]}{-1+\cos [c+d x]}} \sqrt{\frac{\cos [c+d x]}{-1+\cos [c+d x]}} \right)$$

■ **Problem 645: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\cos [c+d x]} \sqrt{-2+3 \cos [c+d x]}} dx$$

Optimal (type 4, 25 leaves, 1 step):

$$\frac{2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sin [c+d x]}{1+\cos [c+d x]}\right], 5\right]}{d}$$

Result (type 4, 156 leaves):



$$\left( 4 \sqrt{\cot\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{-(-2+3\cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2 \csc[c+dx]} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{2}\sqrt{-(-2+3\cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}\right], \frac{4}{5}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( \sqrt{5} d \sqrt{\cos[c+dx]} \sqrt{-2+3\cos[c+dx]} \right)$$

■ **Problem 646: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{2-3\cos[c+dx]} \sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 56 leaves, 2 steps):

$$\frac{2\sqrt{-\cos[c+dx]} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sin[c+dx]}{1-\cos[c+dx]}\right], \frac{1}{5}\right]}{\sqrt{5} d \sqrt{\cos[c+dx]}}$$

Result (type 4, 143 leaves):

$$\left( 4 \sqrt{\cot\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{(2-3\cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2 \csc[c+dx]} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{2}\sqrt{\cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}\right], -4\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( d \sqrt{2-3\cos[c+dx]} \sqrt{\cos[c+dx]} \right)$$

■ **Problem 647: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{-2-3\cos[c+dx]} \sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 49 leaves, 2 steps):

$$\frac{2\sqrt{-\cos[c+dx]} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sin[c+dx]}{1-\cos[c+dx]}\right], 5\right]}{d \sqrt{\cos[c+dx]}}$$

Result (type 4, 153 leaves):

$$\left( 4 \sqrt{\cot\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{-\cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{(2+3\cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Csc}[c+dx] \right. \\ \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{5}{2}} \sqrt{\frac{\cos[c+dx]}{-1+\cos[c+dx]}}\right], \frac{4}{5}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( \sqrt{5} d \sqrt{-2-3\cos[c+dx]} \sqrt{\cos[c+dx]} \right)$$

■ **Problem 648: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\cos[c+dx]} \sqrt{3+2\cos[c+dx]}} dx$$

Optimal (type 4, 58 leaves, 1 step):

$$\frac{2 \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{3+2\cos[c+dx]}}{\sqrt{5}\sqrt{\cos[c+dx]}}\right], -5\right] \sqrt{-\tan[c+dx]^2}}{d}$$

Result (type 4, 140 leaves):

$$\left( 4 \sqrt{\cos[c+dx]} \sqrt{3+2\cos[c+dx]} \sqrt{-\cot\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(3+2\cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{\sqrt{6}}}\right], 6\right] \right) / \\ \left( d \sqrt{-\cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{(3+2\cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2} \right)$$

■ **Problem 649: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{3-2\cos[c+dx]} \sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 60 leaves, 1 step):

$$\frac{2 \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{3-2\cos[c+dx]}}{\sqrt{\cos[c+dx]}}\right], -\frac{1}{5}\right] \sqrt{-\tan[c+dx]^2}}{\sqrt{5} d}$$

Result (type 4, 144 leaves):

$$\left( 4 \sqrt{\cot\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{(3-2\cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{-\cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2} \right. \\ \left. \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{\cos[c+dx]}{-1+\cos[c+dx]}}}{\sqrt{3}}\right], 6\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( d \sqrt{3-2\cos[c+dx]} \sqrt{\cos[c+dx]} \right)$$

■ **Problem 652: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{-\cos[c+dx]} \sqrt{2+3\cos[c+dx]}} dx$$

Optimal (type 4, 54 leaves, 2 steps):

$$\frac{2 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sin[c+dx]}{1+\cos[c+dx]}\right], \frac{1}{5}\right]}{\sqrt{5} d \sqrt{-\cos[c+dx]}}$$

Result (type 4, 150 leaves):

$$-\left( 4 \sqrt{\cot\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{-\cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{(2+3\cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Csc}[c+dx]} \right. \\ \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{2} \sqrt{(2+3\cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}\right], -4\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( d \sqrt{-\cos[c+dx]} \sqrt{2+3\cos[c+dx]} \right)$$

■ **Problem 653: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{-\cos[c+dx]} \sqrt{-2+3\cos[c+dx]}} dx$$

Optimal (type 4, 47 leaves, 2 steps):

$$\frac{2 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sin[c+dx]}{1+\cos[c+dx]}\right], 5\right]}{d \sqrt{-\cos[c+dx]}}$$

Result (type 4, 158 leaves):

$$\left( 4 \sqrt{\cot\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{-(-2+3\cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2} \csc[c+dx] \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{2}\sqrt{-(-2+3\cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}\right], \frac{4}{5}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( \sqrt{5} d \sqrt{-\cos[c+dx]} \sqrt{-2+3\cos[c+dx]} \right)$$

■ **Problem 654: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{2-3\cos[c+dx]} \sqrt{-\cos[c+dx]}} dx$$

Optimal (type 4, 34 leaves, 1 step):

$$\frac{2 \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sin[c+dx]}{1-\cos[c+dx]}\right], \frac{1}{5}\right]}{\sqrt{5} d}$$

Result (type 4, 145 leaves):

$$\left( 4 \sqrt{\cot\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{(2-3\cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2} \csc[c+dx] \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{2}\sqrt{\cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}\right], -4\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( d \sqrt{2-3\cos[c+dx]} \sqrt{-\cos[c+dx]} \right)$$

■ **Problem 655: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{-2-3\cos[c+dx]} \sqrt{-\cos[c+dx]}} dx$$

Optimal (type 4, 27 leaves, 1 step):

$$\frac{2 \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sin[c+dx]}{1-\cos[c+dx]}\right], 5\right]}{d}$$

Result (type 4, 155 leaves):

$$\left( 4 \sqrt{\cot\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{-\cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{(2+3\cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2} \csc[c+dx] \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{5}{2}} \sqrt{\frac{\cos[c+dx]}{-1+\cos[c+dx]}}\right], \frac{4}{5}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( \sqrt{5} d \sqrt{-2-3\cos[c+dx]} \sqrt{-\cos[c+dx]} \right)$$

■ **Problem 658: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{-\cos[c+dx]} \sqrt{-3+2\cos[c+dx]}} dx$$

Optimal (type 4, 62 leaves, 1 step):

$$\frac{2 \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-3+2\cos[c+dx]}}{\sqrt{-\cos[c+dx]}}\right], -\frac{1}{5}\right] \sqrt{-\tan[c+dx]^2}}{\sqrt{5} d}$$

Result (type 4, 160 leaves):

$$\left( 4 \sqrt{-\cot\left[\frac{1}{2}(c+dx)\right]^2} \cot[c+dx] \sqrt{-\cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{-(-3+2\cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2} \right. \\ \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{-3+2\cos[c+dx]}{-1+\cos[c+dx]}}}{\sqrt{3}}\right], \frac{6}{5}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( \sqrt{5} d (-\cos[c+dx])^{3/2} \sqrt{-3+2\cos[c+dx]} \right)$$

■ **Problem 659: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{-3-2\cos[c+dx]} \sqrt{-\cos[c+dx]}} dx$$

Optimal (type 4, 60 leaves, 1 step):

$$\frac{2 \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-3-2\cos[c+dx]}}{\sqrt{5} \sqrt{-\cos[c+dx]}}\right], -5\right] \sqrt{-\tan[c+dx]^2}}{d}$$

Result (type 4, 155 leaves):

$$\left( 4 \sqrt{\cot\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{-\cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{(3+2\cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Csc}[c+dx] \right. \\ \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{5}{3}} \sqrt{\frac{\cos[c+dx]}{-1+\cos[c+dx]}}\right], \frac{6}{5}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( \sqrt{5} d \sqrt{-3-2\cos[c+dx]} \sqrt{-\cos[c+dx]} \right)$$

■ **Problem 660: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]}}{\sqrt{2+3\cos[c+dx]}} dx$$

Optimal (type 4, 77 leaves, 1 step):

$$\frac{4 \cot[c+dx] \operatorname{EllipticPi}\left[\frac{5}{3}, \operatorname{ArcSin}\left[\frac{\sqrt{2+3\cos[c+dx]}}{\sqrt{5}\sqrt{\cos[c+dx]}}\right], 5\right] \sqrt{-1-\sec[c+dx]} \sqrt{1-\sec[c+dx]}}{3d}$$

Result (type 4, 175 leaves):

$$\left( 2 \sqrt{\cos[c+dx]} \sqrt{2+3\cos[c+dx]} \sqrt{\cot\left[\frac{1}{2}(c+dx)\right]^2 \csc[c+dx]} \left( 3 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{2}\sqrt{(2+3\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}\right], -4\right] - 5 \operatorname{EllipticPi}\left[-\frac{2}{3}, \operatorname{ArcSin}\left[\frac{1}{2}\sqrt{(2+3\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}\right], -4\right] \right) \right) / \left( 3d \sqrt{\frac{-2-3\cos[c+dx]}{-1+\cos[c+dx]}} \sqrt{\frac{\cos[c+dx]}{-1+\cos[c+dx]}} \right)$$

■ **Problem 662: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]}}{\sqrt{2-3\cos[c+dx]}} dx$$

Optimal (type 4, 99 leaves, 2 steps):

$$-\frac{1}{3\sqrt{5}d\sqrt{-\cos[c+dx]}} 4 \cos[c+dx]^{3/2} \csc[c+dx] \operatorname{EllipticPi}\left[\frac{1}{3}, \operatorname{ArcSin}\left[\frac{\sqrt{2-3\cos[c+dx]}}{\sqrt{-\cos[c+dx]}}\right], \frac{1}{5}\right] \sqrt{-1+\sec[c+dx]} \sqrt{1+\sec[c+dx]}$$

Result (type 4, 201 leaves):

$$\left( 4 \sqrt{\cot\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{-(-2+3\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2 \csc[c+dx]} \left( 3 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{2}\sqrt{(2-3\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}\right], \frac{4}{5}\right] - \operatorname{EllipticPi}\left[\frac{2}{3}, \operatorname{ArcSin}\left[\frac{1}{2}\sqrt{(2-3\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}\right], \frac{4}{5}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( 3\sqrt{5}d\sqrt{2-3\cos[c+dx]} \sqrt{\cos[c+dx]} \right)$$

■ **Problem 664: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]}}{\sqrt{3+2\cos[c+dx]}} dx$$

Optimal (type 4, 73 leaves, 1 step):

$$\frac{3 \cot[c+dx] \operatorname{EllipticPi}\left[\frac{5}{2}, \operatorname{ArcSin}\left[\frac{\sqrt{3+2\cos[c+dx]}}{\sqrt{5}\sqrt{\cos[c+dx]}}\right], -5\right] \sqrt{1-\sec[c+dx]} \sqrt{1+\sec[c+dx]}}{d}$$

Result (type 4, 184 leaves):

$$\left( 2 \sqrt{\cos[c+dx]} \sqrt{3+2\cos[c+dx]} \sqrt{-\cot\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Csc}[c+dx] \left( 2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(3+2\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{\sqrt{6}}}\right], 6\right] - \right. \right. \\ \left. \left. 5 \operatorname{EllipticPi}\left[-\frac{3}{2}, \operatorname{ArcSin}\left[\frac{\sqrt{(3+2\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{\sqrt{6}}}\right], 6\right] \right) \right) / \\ \left( d \sqrt{-\cos[c+dx]} \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{(3+2\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2} \right)$$

■ **Problem 665: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]}}{\sqrt{3-2\cos[c+dx]}} dx$$

Optimal (type 4, 75 leaves, 1 step):

$$\frac{3 \cot[c+dx] \operatorname{EllipticPi}\left[-\frac{1}{2}, \operatorname{ArcSin}\left[\frac{\sqrt{3-2\cos[c+dx]}}{\sqrt{\cos[c+dx]}}\right], -\frac{1}{5}\right] \sqrt{1-\sec[c+dx]} \sqrt{1+\sec[c+dx]}}{\sqrt{5} d}$$

Result (type 4, 185 leaves):

$$\left( \sqrt{\cos[c+dx]} \sqrt{\frac{-3+2\cos[c+dx]}{-1+\cos[c+dx]}} \sqrt{-\cot\left[\frac{1}{2}(c+dx)\right]^2} \right.$$

$$\left. \left( 2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{-3+2\cos[c+dx]}{-1+\cos[c+dx]}}}{\sqrt{3}}}\right], \frac{6}{5}\right] + \operatorname{EllipticPi}\left[\frac{3}{2}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{-3+2\cos[c+dx]}{-1+\cos[c+dx]}}}{\sqrt{3}}}\right], \frac{6}{5}\right] \tan\left[\frac{1}{2}(c+dx)\right] \right) /$$

$$\left( \sqrt{5} d \sqrt{3-2\cos[c+dx]} \sqrt{\frac{\cos[c+dx]}{-1+\cos[c+dx]}} \right)$$

■ **Problem 666: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{\cos[c+dx]}}{\sqrt{-3+2\cos[c+dx]}} dx$$

Optimal (type 4, 99 leaves, 2 steps):

$$\frac{1}{\sqrt{5} d \sqrt{-\cos[c+dx]}} {}_3F_2\left[\cos[c+dx]^{3/2} \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{1}{2}, \operatorname{ArcSin}\left[\frac{\sqrt{-3+2\cos[c+dx]}}{\sqrt{-\cos[c+dx]}}\right], -\frac{1}{5}\right] \sqrt{1-\operatorname{Sec}[c+dx]} \sqrt{1+\operatorname{Sec}[c+dx]}\right]$$

Result (type 4, 135 leaves):

$$- \left( 2 i \sqrt{-3+2\cos[c+dx]} \sqrt{\frac{\cos[c+dx]}{5+5\cos[c+dx]}} \right.$$

$$\left. \left( \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{5} \tan\left[\frac{1}{2}(c+dx)\right]\right]\right], -\frac{1}{5}\right] - 2 \operatorname{EllipticPi}\left[\frac{1}{5}, i \operatorname{ArcSinh}\left[\sqrt{5} \tan\left[\frac{1}{2}(c+dx)\right]\right]\right], -\frac{1}{5}\right] \right) / \left( \sqrt{\cos[c+dx]} \sqrt{\frac{3-2\cos[c+dx]}{1+\cos[c+dx]}} \right)$$

■ **Problem 670: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{-\cos[c+dx]}}{\sqrt{2-3\cos[c+dx]}} dx$$

Optimal (type 4, 77 leaves, 1 step):



$$\frac{4 \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{1}{3}, \operatorname{ArcSin}\left[\frac{\sqrt{2-3 \operatorname{Cos}[c+d x]}}{\sqrt{-\operatorname{Cos}[c+d x]}}\right], \frac{1}{5}\right] \sqrt{-1+\operatorname{Sec}[c+d x]} \sqrt{1+\operatorname{Sec}[c+d x]}}{3 \sqrt{5} d}$$

Result (type 4, 203 leaves):

$$\left( 4 \sqrt{\operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Cot}[c+d x] \sqrt{\operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2} \right. \\ \left. \sqrt{-(-2+3 \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2} \left( 3 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{2} \sqrt{(2-3 \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}\right], \frac{4}{5}\right] - \operatorname{EllipticPi}\left[\frac{2}{3}, \right. \right. \right. \\ \left. \left. \operatorname{ArcSin}\left[\frac{1}{2} \sqrt{(2-3 \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}\right], \frac{4}{5}\right] \right) \sin\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( 3 \sqrt{5} d \sqrt{2-3 \operatorname{Cos}[c+d x]} (-\operatorname{Cos}[c+d x])^{3/2} \right)$$

■ **Problem 671: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{-\operatorname{Cos}[c+d x]}}{\sqrt{-2-3 \operatorname{Cos}[c+d x]}} dx$$

Optimal (type 4, 79 leaves, 1 step):

$$\frac{4 \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{5}{3}, \operatorname{ArcSin}\left[\frac{\sqrt{-2-3 \operatorname{Cos}[c+d x]}}{\sqrt{5} \sqrt{-\operatorname{Cos}[c+d x]}}\right], 5\right] \sqrt{-1-\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]}}{3 d}$$

Result (type 4, 194 leaves):

$$\left( 4 \sqrt{\operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{-\operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{(2+3 \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2} \right. \\ \left. \operatorname{Csc}[c+d x] \left( 3 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{2} \sqrt{(2+3 \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}\right], -4\right] - 5 \right. \right. \\ \left. \left. \operatorname{EllipticPi}\left[-\frac{2}{3}, \operatorname{ArcSin}\left[\frac{1}{2} \sqrt{(2+3 \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}\right], -4\right] \right) \sin\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( 3 d \sqrt{-2-3 \operatorname{Cos}[c+d x]} \sqrt{-\operatorname{Cos}[c+d x]} \right)$$

■ **Problem 672: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{-\operatorname{Cos}[c+d x]}}{\sqrt{3+2 \operatorname{Cos}[c+d x]}} dx$$

Optimal (type 4, 95 leaves, 2 steps):

$$-\frac{1}{d} \sqrt{-\cos[c+dx]} \sqrt{\cos[c+dx]} \csc[c+dx] \operatorname{EllipticPi}\left[\frac{5}{2}, \operatorname{ArcSin}\left[\frac{\sqrt{3+2\cos[c+dx]}}{\sqrt{5}\sqrt{\cos[c+dx]}}\right], -5\right] \sqrt{1-\sec[c+dx]} \sqrt{1+\sec[c+dx]}$$

Result (type 4, 198 leaves):

$$\left( 2 \sqrt{-\cot\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{-\cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2} \right. \\ \left. \sqrt{(3+2\cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2} \csc[c+dx] \left( 2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(3+2\cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}}{\sqrt{6}}\right], 6\right] - \right. \right. \\ \left. \left. 5 \operatorname{EllipticPi}\left[-\frac{3}{2}, \operatorname{ArcSin}\left[\frac{\sqrt{(3+2\cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}}{\sqrt{6}}\right], 6\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( d \sqrt{-\cos[c+dx]} \sqrt{3+2\cos[c+dx]} \right)$$

■ **Problem 673: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{-\cos[c+dx]}}{\sqrt{3-2\cos[c+dx]}} dx$$

Optimal (type 4, 97 leaves, 2 steps):

$$\frac{1}{\sqrt{5}d} \sqrt{-\cos[c+dx]} \sqrt{\cos[c+dx]} \csc[c+dx] \operatorname{EllipticPi}\left[-\frac{1}{2}, \operatorname{ArcSin}\left[\frac{\sqrt{3-2\cos[c+dx]}}{\sqrt{\cos[c+dx]}}\right], -\frac{1}{5}\right] \sqrt{1-\sec[c+dx]} \sqrt{1+\sec[c+dx]}$$

Result (type 4, 202 leaves):

$$\left( 2 \sqrt{-\cot\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{-\cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{-(-3+2\cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2} \right. \\ \left. \operatorname{Csc}[c+dx] \left( 2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-3+2\cos[c+dx]}}{\sqrt{-1+\cos[c+dx]}}\right], \frac{6}{5}\right] + \operatorname{EllipticPi}\left[\frac{3}{2}, \operatorname{ArcSin}\left[\frac{\sqrt{-3+2\cos[c+dx]}}{\sqrt{-1+\cos[c+dx]}}\right], \frac{6}{5}\right] \right) \right. \\ \left. \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( \sqrt{5} d \sqrt{3-2\cos[c+dx]} \sqrt{-\cos[c+dx]} \right)$$

- **Problem 674: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{-\cos[c+dx]}}{\sqrt{-3+2\cos[c+dx]}} dx$$

Optimal (type 4, 77 leaves, 1 step):

$$\frac{3 \cot[c+dx] \operatorname{EllipticPi}\left[-\frac{1}{2}, \operatorname{ArcSin}\left[\frac{\sqrt{-3+2\cos[c+dx]}}{\sqrt{-\cos[c+dx]}}\right], -\frac{1}{5}\right] \sqrt{1-\sec[c+dx]} \sqrt{1+\sec[c+dx]}}{\sqrt{5} d}$$

Result (type 4, 140 leaves):

$$\left( 2 i \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{-3+2\cos[c+dx]} \right. \\ \left. \left( \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{5} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{1}{5}\right] - 2 \operatorname{EllipticPi}\left[\frac{1}{5}, i \operatorname{ArcSinh}\left[\sqrt{5} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{1}{5}\right] \right) \right) / \\ \left( \sqrt{5} d \sqrt{-\cos[c+dx]} \sqrt{\frac{3-2\cos[c+dx]}{1+\cos[c+dx]}} \right)$$

- **Problem 675: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{-\cos[c+dx]}}{\sqrt{-3-2\cos[c+dx]}} dx$$

Optimal (type 4, 75 leaves, 1 step):

$$\frac{3 \cot [c+d x] \operatorname{EllipticPi}\left[\frac{5}{2}, \operatorname{ArcSin}\left[\frac{\sqrt{-3-2 \cos [c+d x]}}{\sqrt{5} \sqrt{-\cos [c+d x]}}\right], -5\right] \sqrt{1-\sec [c+d x]} \sqrt{1+\sec [c+d x]}}{d}$$

Result (type 4, 198 leaves):

$$\left( 2 \sqrt{-\cot \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{-\cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2} \right. \\ \left. \sqrt{(3+2 \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Csc}[c+d x] \left( 2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(3+2 \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{\sqrt{6}}}\right], 6\right] - \right. \right. \\ \left. \left. 5 \operatorname{EllipticPi}\left[-\frac{3}{2}, \operatorname{ArcSin}\left[\frac{\sqrt{(3+2 \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{\sqrt{6}}}\right], 6\right] \right) \sin \left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( d \sqrt{-3-2 \cos [c+d x]} \sqrt{-\cos [c+d x]} \right)$$

■ **Problem 676: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^{2/3}}{a+b \cos [c+d x]} dx$$

Optimal (type 6, 176 leaves, 5 steps):

$$\frac{b \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, \sin [c+d x]^2, -\frac{b^2 \sin [c+d x]^2}{a^2-b^2}\right] \cos [c+d x]^{2/3} \sin [c+d x]}{(a^2-b^2) d (\cos [c+d x]^2)^{1/3}} + \\ \frac{a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, \sin [c+d x]^2, -\frac{b^2 \sin [c+d x]^2}{a^2-b^2}\right] (\cos [c+d x]^2)^{1/6} \sin [c+d x]}{(a^2-b^2) d \cos [c+d x]^{1/3}}$$

Result (type 6, 4685 leaves):

$$\left( 9 (a^2-b^2) \sin [c+d x] \right. \\ \left. \left( - \left( a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\tan [c+d x]^2, -\frac{a^2 \tan [c+d x]^2}{a^2-b^2}\right] \sqrt{1+\tan [c+d x]^2} \right) / \left( -9 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \right. \right. \right.$$

$$\begin{aligned}
& -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}] + 2 \left( 3 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \right. \\
& \left. (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) \tan[c+dx]^2 + \\
& \left( b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) / \left( -9 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \right. \\
& \left. \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \right. \right. \\
& \left. \left. 5 (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{11}{6}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) \tan[c+dx]^2 \right) / \\
& \left( d \cos[c+dx]^{1/3} (a+b \cos[c+dx]) (1+\tan[c+dx]^2)^{5/6} (-b^2+a^2(1+\tan[c+dx]^2)) \right) \\
& \left( -\frac{1}{(1+\tan[c+dx]^2)^{5/6} (-b^2+a^2(1+\tan[c+dx]^2))^2} \right. \\
& 18 a^2 (a^2-b^2) \sec[c+dx]^2 \tan[c+dx]^2 \left( -\left( a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \sqrt{1+\tan[c+dx]^2} \right) / \right. \\
& \left( -9 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + 2 \left( 3 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) \tan[c+dx]^2 + \right. \\
& \left. \left( b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) / \left( -9 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\tan[c+dx]^2, \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \right. \right. \\
& \left. \left. 5 (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{11}{6}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) \tan[c+dx]^2 \right) \right) - \\
& \frac{1}{(1+\tan[c+dx]^2)^{11/6} (-b^2+a^2(1+\tan[c+dx]^2))} 15 (a^2-b^2) \sec[c+dx]^2 \tan[c+dx]^2 \\
& \left( -\left( a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \sqrt{1+\tan[c+dx]^2} \right) / \right. \\
& \left( -9 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + 2 \left( 3 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} + (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Tan}[c+dx]^2 \Bigg)^2 + \\
& \left( b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \right) / \left( -9 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \\
& \left. \left. -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + 5 (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{11}{6}, \right. \right. \right. \\
& \left. \left. \left. 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \right) \operatorname{Tan}[c+dx]^2 \right) \Bigg) + \frac{1}{(1+\operatorname{Tan}[c+dx]^2)^{5/6} (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2))} \\
& 9 (a^2-b^2) \operatorname{Sec}[c+dx]^2 \left( -\left( a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \sqrt{1+\operatorname{Tan}[c+dx]^2} \right) / \right. \\
& \left( -9 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + 2 \left( 3 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \right) \operatorname{Tan}[c+dx]^2 \right) + \\
& \left( b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \right) / \left( -9 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \\
& \left. \left. -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + 5 (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{11}{6}, \right. \right. \right. \\
& \left. \left. \left. 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \right) \operatorname{Tan}[c+dx]^2 \right) \Bigg) + \frac{1}{(1+\operatorname{Tan}[c+dx]^2)^{5/6} (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2))} \\
& 9 (a^2-b^2) \operatorname{Tan}[c+dx] \left( -\left( a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) / \right. \\
& \left( \sqrt{1+\operatorname{Tan}[c+dx]^2} \left( -9 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + 2 \left( 3 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \right) \operatorname{Tan}[c+dx]^2 \right) \Bigg) - \\
& \left( a \left( -\frac{2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3 (a^2-b^2)} - \frac{2}{9} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \right. \right. \right. \\
& \left. \left. \left. \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \sqrt{1+\operatorname{Tan}[c+dx]^2} \right) / \Bigg)
\end{aligned}$$

$$\begin{aligned}
& \left( -9 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + 2 \left( 3 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) + \\
& \left( b \left( -\frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 (a^2 - b^2)} - \frac{5}{9} \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{11}{6}, \right. \right. \right. \\
& \quad \left. \left. \left. 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \right) / \\
& \left( -9 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \left( 6 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + 5 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{11}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) + \\
& \left( a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sqrt{1 + \tan[c + dx]^2} \left( 4 \left( 3 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \\
& \quad \left. 9 (a^2 - b^2) \left( -\frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 (a^2 - b^2)} - \right. \right. \\
& \quad \left. \left. \frac{2}{9} \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \right) + \\
& 2 \tan[c + dx]^2 \left( 3 a^2 \left( -\frac{1}{5 (a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{3}, 3, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \quad \left. \left. \frac{2}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{4}{3}, 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \right) + \\
& (a^2 - b^2) \left( -\frac{1}{5 (a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{4}{3}, 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \\
& \quad \left. \frac{8}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{7}{3}, 1, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( -9 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + 2 \left( 3 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right)^2 - \\
& \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \left( 2 \left( 6 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right. \right. \right. \\
& \quad \left. \left. \left. 5 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{11}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \sec[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \quad \left. \left. 9 (a^2 - b^2) \left( -\frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sec[c + dx]^2 \tan[c + dx]}{3 (a^2 - b^2)} - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{5}{9} \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{11}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sec[c + dx]^2 \tan[c + dx] \right) \right) + \right. \\
& \quad \tan[c + dx]^2 \left( 6 a^2 \left( -\frac{1}{5 (a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{5}{6}, 3, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sec[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{11}{6}, 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sec[c + dx]^2 \tan[c + dx] \right) \right) + \\
& \quad \left. 5 (a^2 - b^2) \left( -\frac{1}{5 (a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{11}{6}, 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sec[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \quad \left. \left. \frac{11}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{17}{6}, 1, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sec[c + dx]^2 \tan[c + dx] \right) \right) \right) \right) \Big/ \\
& \left( -9 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \left( 6 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + 5 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{11}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right)^2 \Big) \Big) \Big)
\end{aligned}$$

■ **Problem 677: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{1/3}}{a + b \cos[c + dx]} dx$$

Optimal (type 6, 176 leaves, 5 steps):



$$\begin{aligned}
& - \frac{b \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, \sin[c+dx]^2, -\frac{b^2 \sin[c+dx]^2}{a^2-b^2}\right] \cos[c+dx]^{1/3} \sin[c+dx]}{(a^2-b^2) d (\cos[c+dx]^2)^{1/6}} + \\
& \frac{a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \sin[c+dx]^2, -\frac{b^2 \sin[c+dx]^2}{a^2-b^2}\right] (\cos[c+dx]^2)^{1/3} \sin[c+dx]}{(a^2-b^2) d \cos[c+dx]^{2/3}}
\end{aligned}$$

Result (type 6, 4684 leaves):

$$\begin{aligned}
& \left( 9 (a^2 - b^2) \sin[c+dx] \right. \\
& \left( - \left( a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \sqrt{1+\tan[c+dx]^2} \right) / \left( -9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, \right. \right. \right. \\
& \left. \left. -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \right. \right. \\
& \left. \left. (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) \tan[c+dx]^2 \right) + \\
& \left( b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) / \left( -9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \right. \\
& \left. 2 \left( 3 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \right. \right. \\
& \left. \left. 2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) \tan[c+dx]^2 \right) \right) / \\
& \left( d \cos[c+dx]^{2/3} (a + b \cos[c+dx]) (1 + \tan[c+dx]^2)^{2/3} (-b^2 + a^2 (1 + \tan[c+dx]^2)) \right) \\
& \left( - \frac{1}{(1 + \tan[c+dx]^2)^{2/3} (-b^2 + a^2 (1 + \tan[c+dx]^2))^2} \right. \\
& 18 a^2 (a^2 - b^2) \sec[c+dx]^2 \tan[c+dx]^2 \left( - \left( a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \sqrt{1+\tan[c+dx]^2} \right) / \right. \\
& \left( -9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \\
& \left. \left. -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) \tan[c+dx]^2 \right) + \\
& \left( b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) / \left( -9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\tan[c+dx]^2, \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \left( \sqrt{1 + \tan[c + dx]^2} \left( -9 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \left( 6 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) - \\
& \left( a \left( -\frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 (a^2 - b^2)} - \frac{1}{9} \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{7}{6}, 1, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \sqrt{1 + \tan[c + dx]^2} \right) / \\
& \left( -9 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \left( 6 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) + \\
& \left( b \left( -\frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 (a^2 - b^2)} - \frac{4}{9} \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{3}, \right. \right. \right. \\
& \quad \left. \left. \left. 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \right) / \\
& \left( -9 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + 2 \left( 3 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + 2 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) + \\
& \left( a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sqrt{1 + \tan[c + dx]^2} \left( 2 \left( 6 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \\
& \quad \left. 9 (a^2 - b^2) \left( -\frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 (a^2 - b^2)} - \right. \right. \\
& \quad \left. \left. \frac{1}{9} \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \tan[c+dx]^2 \left( 6a^2 \left( -\frac{1}{5(a^2-b^2)} 12a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{6}, 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \quad \left. \frac{1}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{7}{6}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \\
& \quad (a^2-b^2) \left( -\frac{1}{5(a^2-b^2)} 6a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{7}{6}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \\
& \quad \left. \frac{7}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{13}{6}, 1, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \Big) \Big) \Big) \Big) / \\
& \left( -9(a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] + \left( 6a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \\
& \quad \left. \left. -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] + (a^2-b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \right) \tan[c+dx]^2 \Big)^2 - \\
& \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \left( 4 \left( 3a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. 2(a^2-b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \\
& \quad \left. 9(a^2-b^2) \left( -\frac{2a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx]}{3(a^2-b^2)} - \right. \right. \\
& \quad \left. \left. \frac{4}{9} \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \right) + \\
& \quad 2 \tan[c+dx]^2 \left( 3a^2 \left( -\frac{1}{5(a^2-b^2)} 12a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{2}{3}, 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \quad \left. \frac{4}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{5}{3}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \\
& \quad 2(a^2-b^2) \left( -\frac{1}{5(a^2-b^2)} 6a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{5}{3}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \\
& \quad \left. \left. 2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{8}{3}, 1, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \right) \Big) \Big) \Big) \Big) /
\end{aligned}$$

$$\left( -9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + 2 \left( 3 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \right. \right. \\ \left. \left. - \frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right) + 2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \right) \tan[c + dx]^2 \Bigg) \Bigg) \Bigg)$$

■ **Problem 678: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\cos[c + dx]^{1/3} (a + b \cos[c + dx])} dx$$

Optimal (type 6, 176 leaves, 5 steps):

$$- \frac{b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, \sin[c + dx]^2, -\frac{b^2 \sin[c + dx]^2}{a^2 - b^2}\right] (\cos[c + dx]^2)^{1/6} \sin[c + dx]}{(a^2 - b^2) d \cos[c + dx]^{1/3}} + \\ \frac{a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \sin[c + dx]^2, -\frac{b^2 \sin[c + dx]^2}{a^2 - b^2}\right] (\cos[c + dx]^2)^{2/3} \sin[c + dx]}{(a^2 - b^2) d \cos[c + dx]^{4/3}}$$

Result (type 6, 4676 leaves):

$$\left( 9 (a^2 - b^2) \sin[c + dx] \right. \\ \left( - \left( a \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \sqrt{1 + \tan[c + dx]^2} \right) / \left( -9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, \right. \right. \right. \\ \left. \left. -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + \right. \right. \\ \left. \left. (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \right) \tan[c + dx]^2 \right) + \\ \left( b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \right) / \left( -9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + \right. \\ \left. 2 \left( 3 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + \right. \right. \\ \left. \left. (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \right) \tan[c + dx]^2 \right) \Bigg) \Bigg) / \\ \left( d \cos[c + dx]^{4/3} (a + b \cos[c + dx]) (1 + \tan[c + dx]^2)^{1/3} (-b^2 + a^2 (1 + \tan[c + dx]^2)) \right)$$

$$\begin{aligned}
& \left( -\frac{1}{(1 + \tan[c + dx]^2)^{1/3} (-b^2 + a^2 (1 + \tan[c + dx]^2))^2} \right. \\
& 18 a^2 (a^2 - b^2) \operatorname{Sec}[c + dx]^2 \tan[c + dx]^2 \left( -\left( a \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \sqrt{1 + \tan[c + dx]^2} \right) / \right. \\
& \left( -9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \right) \tan[c + dx]^2 \right) + \\
& \left( b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \right) / \left( -9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\tan[c + dx]^2, \right. \right. \\
& \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + 2 \left( 3 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + \right. \right. \\
& \left. \left. (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \right) \tan[c + dx]^2 \right) \left. \right) - \\
& \frac{1}{(1 + \tan[c + dx]^2)^{4/3} (-b^2 + a^2 (1 + \tan[c + dx]^2))} 6 (a^2 - b^2) \operatorname{Sec}[c + dx]^2 \tan[c + dx]^2 \\
& \left( -\left( a \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \sqrt{1 + \tan[c + dx]^2} \right) / \right. \\
& \left( -9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \right) \tan[c + dx]^2 \right) + \\
& \left( b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \right) / \left( -9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\tan[c + dx]^2, \right. \right. \\
& \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + 2 \left( 3 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, \right. \right. \\
& \left. \left. 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \right) \tan[c + dx]^2 \right) \left. \right) + \frac{1}{(1 + \tan[c + dx]^2)^{1/3} (-b^2 + a^2 (1 + \tan[c + dx]^2))} \\
& 9 (a^2 - b^2) \operatorname{Sec}[c + dx]^2 \left( -\left( a \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \sqrt{1 + \tan[c + dx]^2} \right) / \right. \\
& \left( -9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Tan}[c+dx]^2 \Big) + \\
& \left( b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \right) / \left( -9(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \\
& \left. \left. -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + 2 \left( 3 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, \right. \right. \right. \\
& \left. \left. \left. 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Tan}[c+dx]^2 \right) \right) + \frac{1}{(1+\operatorname{Tan}[c+dx]^2)^{1/3} (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2))} \\
& 9(a^2-b^2) \operatorname{Tan}[c+dx] \left( - \left( a \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) / \left( \sqrt{1+\operatorname{Tan}[c+dx]^2} \right)^2 \right. \\
& \left. \left( -9(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{6}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \right) \operatorname{Tan}[c+dx]^2 \right) \Big) - \\
& \left( a \left( -\frac{2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{6}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3(a^2-b^2)} + \frac{1}{9} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, \right. \right. \right. \\
& \left. \left. \left. 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \sqrt{1+\operatorname{Tan}[c+dx]^2} \right) / \\
& \left( -9(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{6}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \right) \operatorname{Tan}[c+dx]^2 \right) + \\
& \left( b \left( -\frac{2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3(a^2-b^2)} - \frac{2}{9} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, \right. \right. \right. \\
& \left. \left. \left. 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right) / \\
& \left( -9(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + 2 \left( 3 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} + (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \tan^2[c+dx] \Bigg) + \\
& \left( a \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \sqrt{1+\tan^2[c+dx]} \left( 2 \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{6}, 2, \frac{5}{2}, -\tan^2[c+dx], \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, 1, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \right) \sec^2[c+dx] \tan^2[c+dx] - \right. \\
& \quad \left. 9 (a^2-b^2) \left( -\frac{1}{3(a^2-b^2)} 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{6}, 2, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \sec^2[c+dx] \tan^2[c+dx] + \right. \right. \\
& \quad \left. \left. \frac{1}{9} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, 1, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \sec^2[c+dx] \tan^2[c+dx] \right) + \right. \\
& \quad \tan^2[c+dx]^2 \left( 6 a^2 \left( -\frac{1}{5(a^2-b^2)} 12 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{6}, 3, \frac{7}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \sec^2[c+dx] \tan^2[c+dx] + \right. \right. \\
& \quad \left. \left. \frac{1}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{6}, 2, \frac{7}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \sec^2[c+dx] \tan^2[c+dx] \right) + \right. \\
& \quad \left. (-a^2+b^2) \left( -\frac{1}{5(a^2-b^2)} 6 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{6}, 2, \frac{7}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \sec^2[c+dx] \tan^2[c+dx] - \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{5}{2}, \frac{11}{6}, 1, \frac{7}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \sec^2[c+dx] \tan^2[c+dx] \right) \right) \Bigg) \Bigg) / \\
& \left( -9 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] + \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{6}, 2, \frac{5}{2}, -\tan^2[c+dx], \right. \right. \right. \\
& \quad \left. \left. -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, 1, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \right) \tan^2[c+dx] \Bigg)^2 - \\
& \left( b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \left( 4 \left( 3 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] + \right. \right. \right. \\
& \quad \left. \left. (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \right) \sec^2[c+dx] \tan^2[c+dx] - \right. \\
& \quad \left. 9 (a^2-b^2) \left( -\frac{2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \sec^2[c+dx] \tan^2[c+dx]}{3(a^2-b^2)} - \right. \right. \\
& \quad \left. \left. \frac{2}{9} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \sec^2[c+dx] \tan^2[c+dx] \right) + \right.
\end{aligned}$$



$$\begin{aligned}
& 2 \tan[c + dx]^2 \left( 3 a^2 \left( -\frac{1}{5(a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{3}, 3, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sec[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \left. \frac{2}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{4}{3}, 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sec[c + dx]^2 \tan[c + dx] \right) + \\
& (a^2 - b^2) \left( -\frac{1}{5(a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{4}{3}, 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sec[c + dx]^2 \tan[c + dx] - \right. \\
& \left. \frac{8}{5} \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{7}{3}, 1, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sec[c + dx]^2 \tan[c + dx] \right) \Bigg) \Bigg) / \\
& \left( -9(a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + 2 \left( 3 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \\
& \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \tan[c + dx]^2 \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 679: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\cos[c + dx]^{2/3} (a + b \cos[c + dx])} dx$$

Optimal (type 6, 176 leaves, 5 steps):

$$\begin{aligned}
& \frac{b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \sin[c + dx]^2, -\frac{b^2 \sin[c + dx]^2}{a^2 - b^2} \right] (\cos[c + dx]^2)^{1/3} \sin[c + dx]}{(a^2 - b^2) d \cos[c + dx]^{2/3}} + \\
& \frac{a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, \sin[c + dx]^2, -\frac{b^2 \sin[c + dx]^2}{a^2 - b^2} \right] (\cos[c + dx]^2)^{5/6} \sin[c + dx]}{(a^2 - b^2) d \cos[c + dx]^{5/3}}
\end{aligned}$$

Result (type 6, 4679 leaves):

$$\begin{aligned}
& \left( 9(a^2 - b^2) \sin[c + dx] \right. \\
& \left. \left( -\left( a \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sqrt{1 + \tan[c + dx]^2} \right) / \left( -9(a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, \right. \right. \right. \right. \\
& \left. \left. -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + 2 \left( 3 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{3}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \tan[c + dx]^2 \right) \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) / \left( -9 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \\
& \left. \left( 6 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + (a^2-b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \right) \\
& \left. \operatorname{Tan}[c+dx]^2 \right) \Bigg) / \\
& \left( d \operatorname{Cos}[c+dx]^{5/3} (a+b \operatorname{Cos}[c+dx]) (1+\operatorname{Tan}[c+dx]^2)^{1/6} (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2)) \right) \\
& \left( -\frac{1}{(1+\operatorname{Tan}[c+dx]^2)^{1/6} (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2))^2} \right. \\
& 18 a^2 (a^2-b^2) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]^2 \left( -\left( a \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \sqrt{1+\operatorname{Tan}[c+dx]^2} \right) / \right. \\
& \left( -9 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + 2 \left( 3 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{3}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) + \\
& \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) / \left( -9 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \\
& \left. \left. -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \left( 6 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \left. \left. (a^2-b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) \Bigg) - \\
& \frac{1}{(1+\operatorname{Tan}[c+dx]^2)^{7/6} (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2))} 3 (a^2-b^2) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]^2 \\
& \left( -\left( a \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \sqrt{1+\operatorname{Tan}[c+dx]^2} \right) / \right. \\
& \left( -9 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + 2 \left( 3 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{3}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) + \\
& \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) / \left( -9 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \left. + \left( 6 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] + (a^2-b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{7}{6}, \right. \right. \right. \\
& \left. \left. \left. 1, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] \right) \tan^2[c+dx] \right) + \frac{1}{(1+\tan^2[c+dx])^{1/6} (-b^2+a^2(1+\tan^2[c+dx]))} \\
9 (a^2-b^2) \operatorname{Sec}[c+dx]^2 & \left( - \left( a \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] \sqrt{1+\tan^2[c+dx]} \right) / \right. \\
& \left( -9 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] + 2 \left( 3 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{3}, 2, \frac{5}{2}, -\tan^2[c+dx], \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] \right) \tan^2[c+dx] \right) + \\
& \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] \right) / \left( -9 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan^2[c+dx], \right. \right. \\
& \left. \left. -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] + \left( 6 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] + (a^2-b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{7}{6}, \right. \right. \right. \\
& \left. \left. \left. 1, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] \right) \tan^2[c+dx] \right) + \frac{1}{(1+\tan^2[c+dx])^{1/6} (-b^2+a^2(1+\tan^2[c+dx]))} \\
9 (a^2-b^2) \tan[c+dx] & \left( - \left( a \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) / \left( \sqrt{1+\tan^2[c+dx]} \right) \right. \\
& \left( -9 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] + 2 \left( 3 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{3}, 2, \frac{5}{2}, -\tan^2[c+dx], \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] \right) \tan^2[c+dx] \right) - \\
& \left( a \left( -\frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{3}, 2, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx]}{3 (a^2-b^2)} + \frac{2}{9} \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2}{3}, \right. \right. \right. \\
& \left. \left. \left. 1, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \sqrt{1+\tan^2[c+dx]} \right) / \\
& \left( -9 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] + 2 \left( 3 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{3}, 2, \frac{5}{2}, -\tan^2[c+dx], \right. \right. \right. \\
& \left. \left. \left. -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] \right) \tan^2[c+dx] \right) +
\end{aligned}$$

$$\begin{aligned}
& \left( b \left( -\frac{2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] \operatorname{Sec}[c+d x]^2 \tan[c+d x]}{3\left(a^2-b^2\right)} - \frac{1}{9} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{6}, \right. \right. \\
& \quad \left. \left. 1, \frac{5}{2}, -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] \operatorname{Sec}[c+d x]^2 \tan[c+d x]\right) \Bigg) / \\
& \left( -9\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] + \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c+d x]^2, \right. \right. \right. \\
& \quad \left. \left. -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] + \left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] \right) \tan[c+d x]^2 \Bigg) + \\
& \left( a \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] \sqrt{1+\tan[c+d x]^2} \left( 4 \left( 3 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{3}, 2, \frac{5}{2}, -\tan[c+d x]^2, \right. \right. \right. \right. \\
& \quad \left. \left. -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] + \left(-a^2+b^2\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] \right) \operatorname{Sec}[c+d x]^2 \tan[c+d x] - \right. \\
& \quad \left. 9\left(a^2-b^2\right) \left( -\frac{1}{3\left(a^2-b^2\right)} 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{3}, 2, \frac{5}{2}, -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] \operatorname{Sec}[c+d x]^2 \tan[c+d x] + \right. \right. \\
& \quad \left. \left. \frac{2}{9} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] \operatorname{Sec}[c+d x]^2 \tan[c+d x] \right) + \right. \\
& \quad \left. 2 \tan[c+d x]^2 \left( 3 a^2 \left( -\frac{1}{5\left(a^2-b^2\right)} 12 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{3}, 3, \frac{7}{2}, -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] \operatorname{Sec}[c+d x]^2 \tan[c+d x] + \right. \right. \\
& \quad \left. \left. \frac{2}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2}{3}, 2, \frac{7}{2}, -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] \operatorname{Sec}[c+d x]^2 \tan[c+d x] \right) + \right. \\
& \quad \left. \left(-a^2+b^2\right) \left( -\frac{1}{5\left(a^2-b^2\right)} 6 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2}{3}, 2, \frac{7}{2}, -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] \operatorname{Sec}[c+d x]^2 \tan[c+d x] - \right. \right. \\
& \quad \left. \left. \frac{4}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{3}, 1, \frac{7}{2}, -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] \operatorname{Sec}[c+d x]^2 \tan[c+d x] \right) \right) \Bigg) / \\
& \left( -9\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] + 2 \left( 3 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{3}, 2, \frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] + \left(-a^2+b^2\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] \right) \tan[c+d x]^2 \Bigg)^2 - \\
& \left( b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] \left( 2 \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c+d x]^2, -\frac{a^2 \tan[c+d x]^2}{a^2-b^2}\right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \\
& 9 (a^2 - b^2) \left( -\frac{2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 (a^2 - b^2)} - \right. \\
& \left. \frac{1}{9} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) + \\
& \tan[c + dx]^2 \left( 6 a^2 \left( -\frac{1}{5 (a^2 - b^2)} 12 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{6}, 3, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \left. \left. \frac{1}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{6}, 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) + \right. \\
& \left. (a^2 - b^2) \left( -\frac{1}{5 (a^2 - b^2)} 6 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{6}, 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \left. \left. \frac{7}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{13}{6}, 1, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( -9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + \left( 6 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \\
& \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \tan[c + dx]^2 \right) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 681: Attempted integration timed out after 120 seconds.**

$$\int \frac{\cos[c + dx]^{5/3}}{\sqrt{a + b \cos[c + dx]}} dx$$

Optimal (type 9, 27 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\cos[c + dx]^{5/3}}{\sqrt{a + b \cos[c + dx]}}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 687: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{\cos[c + dx]^{4/3} \sqrt{a + b \cos[c + dx]}} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

$$\text{Unintegrable}\left[\frac{1}{\text{Cos}[c + d x]^{4/3} \sqrt{a + b \text{Cos}[c + d x]}}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 689: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{\text{Cos}[c + d x]^{7/3} \sqrt{a + b \text{Cos}[c + d x]}} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

$$\text{Unintegrable}\left[\frac{1}{\text{Cos}[c + d x]^{7/3} \sqrt{a + b \text{Cos}[c + d x]}}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 719: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^{3/2}}{(a + b \text{Cos}[c + d x])^2} dx$$

Optimal (type 4, 277 leaves, 11 steps) :

$$\begin{aligned} & - \frac{(2 a^2 - 3 b^2) \sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{a^2 (a^2 - b^2) d} + \\ & \frac{b \sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{a (a^2 - b^2) d} - \frac{b (5 a^2 - 3 b^2) \sqrt{\text{Cos}[c + d x]} \text{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2} (c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{a^2 (a - b) (a + b)^2 d} + \\ & \frac{(2 a^2 - 3 b^2) \sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{a^2 (a^2 - b^2) d} + \frac{b^2 \text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{a (a^2 - b^2) d (b + a \text{Sec}[c + d x])} \end{aligned}$$

Result (type 4, 635 leaves) :

$$\frac{\sqrt{\sec[c+dx]} \left( \frac{(2a^2-3b^2)\sin[c+dx]}{a^2(a^2-b^2)} + \frac{b^2\sin[c+dx]}{a(a^2-b^2)(a+b\cos[c+dx])} \right)}{d} - \frac{1}{4a^2(a-b)(a+b)d}$$

$$\left( - \left( 2(4a^3-8ab^2)\cos[c+dx]^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] (b+a\sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / \right.$$

$$\left. (b(a+b\cos[c+dx])(1-\cos[c+dx]^2)) + \right.$$

$$\left( 2(10a^2b-9b^3)\cos[c+dx]^2 \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right) \right.$$

$$\left. (b+a\sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / (a(a+b\cos[c+dx])(1-\cos[c+dx]^2)) +$$

$$\left( (2a^2b-3b^3)\cos[2(c+dx)](b+a\sec[c+dx]) \left( -4ab+4ab\sec[c+dx]^2-4ab\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right. \right.$$

$$\left. \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + 2(2a-b)b\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + \right.$$

$$4a^2\operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} -$$

$$2b^2\operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \left. \right) \sin[c+dx] \Big/$$

$$\left( ab^2(a+b\cos[c+dx])(1-\cos[c+dx]^2)\sqrt{\sec[c+dx]}(2-\sec[c+dx]^2) \right)$$

■ **Problem 720: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec[c+dx]}}{(a+b\cos[c+dx])^2} dx$$

Optimal (type 4, 217 leaves, 10 steps):

$$-\frac{b\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{a(a^2-b^2)d} - \frac{\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{(a^2-b^2)d} +$$

$$\frac{(3a^2-b^2)\sqrt{\cos[c+dx]}\operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{a(a-b)(a+b)^2d} + \frac{b^2\sqrt{\sec[c+dx]}\sin[c+dx]}{a(a^2-b^2)d(b+a\sec[c+dx])}$$

Result (type 4, 590 leaves):

$$\frac{\sqrt{\sec[c+dx]} \left( \frac{b \sin[c+dx]}{a(a^2-b^2)} + \frac{b \sin[c+dx]}{(-a^2+b^2)(a+b \cos[c+dx])} \right)}{d} +$$

$$\frac{1}{4a(-a+b)(a+b)d} \left( - \left( 8a \cos[c+dx]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] (b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / \right.$$

$$\left. \left( (a+b \cos[c+dx]) (1-\cos[c+dx]^2) \right) + \right.$$

$$\left. \left( 2(-4a^2+3b^2) \cos[c+dx]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \right) \right.$$

$$\left. (b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / \left( a(a+b \cos[c+dx]) (1-\cos[c+dx]^2) \right) +$$

$$\left( \cos[2(c+dx)] (b+a \sec[c+dx]) \left( -4ab+4ab \sec[c+dx]^2 - 4ab \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \right.$$

$$\left. \sqrt{1-\sec[c+dx]^2} + 2(2a-b)b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + \right.$$

$$\left. 4a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} - \right.$$

$$\left. 2b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \right) \sin[c+dx] \Big/$$

$$\left( a(a+b \cos[c+dx]) (1-\cos[c+dx]^2) \sqrt{\sec[c+dx]} (2-\sec[c+dx]^2) \right)$$

■ **Problem 721: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \cos[c+dx])^2 \sqrt{\sec[c+dx]}} dx$$

Optimal (type 4, 208 leaves, 10 steps):

$$\frac{\sqrt{\cos[c+dx]} \operatorname{EllipticE} \left[ \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{(a^2-b^2)d} + \frac{a \sqrt{\cos[c+dx]} \operatorname{EllipticF} \left[ \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{b(a^2-b^2)d} -$$

$$\frac{(a^2+b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticPi} \left[ \frac{2b}{a+b}, \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{(a-b)b(a+b)^2d} - \frac{b \sqrt{\sec[c+dx]} \sin[c+dx]}{(a^2-b^2)d(b+a \sec[c+dx])}$$

Result (type 4, 580 leaves):



$$\begin{aligned}
& \frac{\sqrt{\sec[c+dx]} \left( -\frac{\sin[c+dx]}{a^2-b^2} + \frac{a \sin[c+dx]}{(a^2-b^2)(a+b \cos[c+dx])} \right)}{d} + \\
& \frac{1}{4(a-b)(a+b)d} \left( -\left( 8a \cos[c+dx]^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] (b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / \right. \\
& \quad \left. (b(a+b \cos[c+dx])(1-\cos[c+dx]^2)) - \right. \\
& \quad \left. \left( 2b \cos[c+dx]^2 \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right) \right. \right. \\
& \quad \left. \left. (b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / (a(a+b \cos[c+dx])(1-\cos[c+dx]^2)) + \right. \\
& \quad \left. \left( \cos[2(c+dx)] (b+a \sec[c+dx]) \left( -4ab + 4ab \sec[c+dx]^2 - 4ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \right. \right. \right. \\
& \quad \left. \left. \sqrt{1-\sec[c+dx]^2} + 2(2a-b)b \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + \right. \right. \\
& \quad \left. \left. 4a^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} - \right. \right. \\
& \quad \left. \left. 2b^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \right) \sin[c+dx] \right) / \right. \\
& \quad \left. \left. (ab(a+b \cos[c+dx])(1-\cos[c+dx]^2) \sqrt{\sec[c+dx]} (2-\sec[c+dx]^2)) \right) \right)
\end{aligned}$$

■ **Problem 722: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \cos[c+dx])^2 \sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 223 leaves, 10 steps):

$$\begin{aligned}
& -\frac{a \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]} + (a^2-2b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{b(a^2-b^2)d} + \frac{(a^2-2b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{b^2(a^2-b^2)d} - \\
& \frac{a(a^2-3b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{(a-b)b^2(a+b)^2d} + \frac{a \sqrt{\sec[c+dx]} \sin[c+dx]}{(a^2-b^2)d(b+a \sec[c+dx])}
\end{aligned}$$

Result (type 4, 577 leaves):

$$\frac{\sqrt{\sec[c+dx]} \left( \frac{a \sin[c+dx]}{b(a^2-b^2)} + \frac{a^2 \sin[c+dx]}{b(-a^2+b^2)(a+b \cos[c+dx])} \right)}{d} +$$

$$\frac{1}{4(-a+b)(a+b)d} \left( - \left( 8 \cos[c+dx]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] (b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / \right.$$

$$\left. \left( (a+b \cos[c+dx]) (1-\cos[c+dx]^2) \right) - \right.$$

$$\left. \left( 2 \cos[c+dx]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \right) \right.$$

$$\left. (b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / \left( (a+b \cos[c+dx]) (1-\cos[c+dx]^2) \right) +$$

$$\left( \cos[2(c+dx)] (b+a \sec[c+dx]) \left( -4ab + 4ab \sec[c+dx]^2 - 4ab \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \right.$$

$$\left. \sqrt{1-\sec[c+dx]^2} + 2(2a-b)b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + \right.$$

$$4a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} -$$

$$\left. 2b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \right) \sin[c+dx] \Big/$$

$$\left( b^2 (a+b \cos[c+dx]) (1-\cos[c+dx]^2) \sqrt{\sec[c+dx]} (2-\sec[c+dx]^2) \right)$$

■ **Problem 723: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \cos[c+dx])^2 \sec[c+dx]^{5/2}} dx$$

Optimal (type 4, 245 leaves, 10 steps):

$$\frac{(3a^2 - 2b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticE} \left[ \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]} - a(3a^2 - 4b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticF} \left[ \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{b^2(a^2 - b^2)d} +$$

$$\frac{a^2(3a^2 - 5b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticPi} \left[ \frac{2b}{a+b}, \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{(a-b)b^3(a+b)^2d} - \frac{a^2 \sqrt{\sec[c+dx]} \sin[c+dx]}{b(a^2 - b^2)d(b+a \sec[c+dx])}$$

Result (type 4, 611 leaves):

$$\frac{\sqrt{\sec[c+dx]} \left( -\frac{a^2 \sin[c+dx]}{b^2(a^2-b^2)} - \frac{a^3 \sin[c+dx]}{b^2(-a^2+b^2)(a+b \cos[c+dx])} \right)}{d} +$$

$$\frac{1}{4(a-b)b(a+b)d} \left( -\left( 8a \cos[c+dx]^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] (b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) \right) /$$

$$\left( (a+b \cos[c+dx]) (1-\cos[c+dx]^2) \right) +$$

$$\left( 2(a^2-2b^2) \cos[c+dx]^2 \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right) \right)$$

$$(b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \Big/ (a(a+b \cos[c+dx]) (1-\cos[c+dx]^2)) +$$

$$\left( (3a^2-2b^2) \cos[2(c+dx)] (b+a \sec[c+dx]) \left( -4ab+4ab \sec[c+dx]^2-4ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right) \right)$$

$$\sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + 2(2a-b)b \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} +$$

$$4a^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} -$$

$$2b^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \Big) \sin[c+dx] \Big/$$

$$\left( ab^2(a+b \cos[c+dx]) (1-\cos[c+dx]^2) \sqrt{\sec[c+dx]} (2-\sec[c+dx]^2) \right)$$

■ **Problem 726: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec[c+dx]}}{(a+b \cos[c+dx])^3} dx$$

Optimal (type 4, 321 leaves, 11 steps):

$$\frac{3b(3a^2-b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]} - (7a^2-b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{4a^2(a^2-b^2)^2 d} + \frac{(7a^2-b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{4a(a^2-b^2)^2 d} +$$

$$\frac{3(5a^4-2a^2b^2+b^4) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{4a^2(a-b)^2(a+b)^3 d} +$$

$$\frac{b^2 \sec[c+dx]^{3/2} \sin[c+dx]}{2a(a^2-b^2)d(b+a \sec[c+dx])^2} + \frac{3b^2(3a^2-b^2) \sqrt{\sec[c+dx]} \sin[c+dx]}{4a^2(a^2-b^2)^2 d(b+a \sec[c+dx])}$$

Result (type 4, 700 leaves):

$$\frac{1}{16 a^2 (a-b)^2 (a+b)^2 d} \left( - \left( 2 \left( -32 a^3 b + 8 a b^3 \right) \cos [c+d x]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] (b+a \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / \right. \\ \left. (b(a+b \cos [c+d x]) (1-\cos [c+d x]^2)) + \right. \\ \left. \left( 2 \left( 16 a^4 - 19 a^2 b^2 + 9 b^4 \right) \cos [c+d x]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right) \right. \right. \\ \left. \left. (b+a \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / (a(a+b \cos [c+d x]) (1-\cos [c+d x]^2)) + \right. \\ \left. \left( \left( -9 a^2 b^2 + 3 b^4 \right) \cos [2(c+d x)] (b+a \sec [c+d x]) \left( -4 a b + 4 a b \sec [c+d x]^2 - 4 a b \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right. \right. \right. \\ \left. \left. \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + 2(2 a-b) b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + \right. \right. \\ \left. \left. 4 a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} - \right. \right. \\ \left. \left. 2 b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} \right) \sin [c+d x] \right) / \\ \left. \left( a b^2 (a+b \cos [c+d x]) (1-\cos [c+d x]^2) \sqrt{\sec [c+d x]} (2-\sec [c+d x]^2) \right) \right) + \\ \frac{\sqrt{\sec [c+d x]} \left( \frac{3 b \left( 3 a^2 - b^2 \right) \sin [c+d x]}{4 a^2 \left( a^2 - b^2 \right)^2} - \frac{b \sin [c+d x]}{2 \left( a^2 - b^2 \right) (a+b \cos [c+d x])^2} + \frac{-7 a^2 b \sin [c+d x] + b^3 \sin [c+d x]}{4 a \left( a^2 - b^2 \right)^2 (a+b \cos [c+d x])} \right)}{d}$$

■ **Problem 727: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \cos [c+d x])^3 \sqrt{\sec [c+d x]}} dx$$

Optimal (type 4, 317 leaves, 11 steps):

$$\frac{(5 a^2 + b^2) \sqrt{\cos [c+d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]} - 3 (a^2 + b^2) \sqrt{\cos [c+d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{4 a \left( a^2 - b^2 \right)^2 d} + \frac{3 (a^2 + b^2) \sqrt{\cos [c+d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{4 b \left( a^2 - b^2 \right)^2 d} - \\ \frac{(3 a^4 + 10 a^2 b^2 - b^4) \sqrt{\cos [c+d x]} \operatorname{EllipticPi} \left[ \frac{2 b}{a+b}, \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{4 a (a-b)^2 b (a+b)^3 d} + \\ \frac{b^2 \sqrt{\sec [c+d x]} \sin [c+d x]}{2 a \left( a^2 - b^2 \right) d (b+a \sec [c+d x])^2} - \frac{b \left( 7 a^2 - b^2 \right) \sqrt{\sec [c+d x]} \sin [c+d x]}{4 a \left( a^2 - b^2 \right)^2 d (b+a \sec [c+d x])}$$

Result (type 4, 680 leaves):

$$\frac{1}{16 a (a-b)^2 (a+b)^2 d} \left( - \left( 2 (16 a^3 + 8 a b^2) \cos [c+d x]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] (b+a \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / \right. \\ \left. (b(a+b \cos [c+d x]) (1-\cos [c+d x]^2)) + \right. \\ \left. \left( 2 (-9 a^2 b + 3 b^3) \cos [c+d x]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right) \right. \right. \\ \left. \left. (b+a \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / \left( a(a+b \cos [c+d x]) (1-\cos [c+d x]^2) \right) + \right. \\ \left. \left( (5 a^2 b + b^3) \cos [2(c+d x)] (b+a \sec [c+d x]) \left( -4 a b + 4 a b \sec [c+d x]^2 - 4 a b \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right. \right. \right. \\ \left. \left. \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + 2(2 a-b) b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + \right. \right. \\ \left. \left. 4 a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} - \right. \right. \\ \left. \left. 2 b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} \right) \sin [c+d x] \right) / \right. \\ \left. \left( a b^2 (a+b \cos [c+d x]) (1-\cos [c+d x]^2) \sqrt{\sec [c+d x]} (2-\sec [c+d x]^2) \right) \right) + \\ \frac{\sqrt{\sec [c+d x]} \left( -\frac{(5 a^2+b^2) \sin [c+d x]}{4 a (a^2-b^2)^2} + \frac{a \sin [c+d x]}{2 (a^2-b^2) (a+b \cos [c+d x])^2} + \frac{3 (a^2 \sin [c+d x]+b^2 \sin [c+d x])}{4 (a^2-b^2)^2 (a+b \cos [c+d x])} \right)}{d}$$

■ **Problem 728: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \cos [c+d x])^3 \sec [c+d x]^{3/2}} dx$$

Optimal (type 4, 302 leaves, 11 steps):

$$\frac{(a^2+5 b^2) \sqrt{\cos [c+d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]} + a (a^2-7 b^2) \sqrt{\cos [c+d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{4 b (a^2-b^2)^2 d} + \frac{a (a^2-7 b^2) \sqrt{\cos [c+d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{4 b^2 (a^2-b^2)^2 d} - \\ \frac{(a^4-10 a^2 b^2-3 b^4) \sqrt{\cos [c+d x]} \operatorname{EllipticPi} \left[ \frac{2 b}{a+b}, \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{4 (a-b)^2 b^2 (a+b)^3 d} - \\ \frac{b \sqrt{\sec [c+d x]} \sin [c+d x]}{2 (a^2-b^2) d (b+a \sec [c+d x])^2} + \frac{3 (a^2+b^2) \sqrt{\sec [c+d x]} \sin [c+d x]}{4 (a^2-b^2)^2 d (b+a \sec [c+d x])}$$

Result (type 4, 671 leaves):

$$\begin{aligned}
& - \frac{1}{16 (a-b)^2 (a+b)^2 d} \left( - \left( 48 a \cos [c+d x]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\operatorname{Sec}[c+d x]} \right], -1 \right] (b+a \operatorname{Sec}[c+d x]) \sqrt{1-\operatorname{Sec}[c+d x]^2} \sin [c+d x] \right) \right. \\
& \quad \left. (a+b \cos [c+d x]) (1-\cos [c+d x]^2) \right) + \\
& \quad \left( 2 (-5 a^2 - b^2) \cos [c+d x]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\operatorname{Sec}[c+d x]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\operatorname{Sec}[c+d x]} \right], -1 \right] \right) \right. \\
& \quad \left. (b+a \operatorname{Sec}[c+d x]) \sqrt{1-\operatorname{Sec}[c+d x]^2} \sin [c+d x] \right) \Big/ (a(a+b \cos [c+d x]) (1-\cos [c+d x]^2)) + \\
& \quad \left( (a^2+5 b^2) \cos [2(c+d x)] (b+a \operatorname{Sec}[c+d x]) \left( -4 a b + 4 a b \operatorname{Sec}[c+d x]^2 - 4 a b \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\operatorname{Sec}[c+d x]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} + 2(2 a-b) b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\operatorname{Sec}[c+d x]} \right], -1 \right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} + \right. \right. \\
& \quad \left. \left. 4 a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\operatorname{Sec}[c+d x]} \right], -1 \right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} - \right. \right. \\
& \quad \left. \left. 2 b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\operatorname{Sec}[c+d x]} \right], -1 \right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} \right) \sin [c+d x] \right) \Big/ \\
& \quad \left( a b^2 (a+b \cos [c+d x]) (1-\cos [c+d x]^2) \sqrt{\operatorname{Sec}[c+d x]} (2-\operatorname{Sec}[c+d x]^2) \right) \Big) + \\
& \quad \frac{\sqrt{\operatorname{Sec}[c+d x]} \left( \frac{(a^2+5 b^2) \sin [c+d x]}{4 b (-a^2+b^2)^2} + \frac{a^2 \sin [c+d x]}{2 b (-a^2+b^2) (a+b \cos [c+d x])^2} + \frac{a^3 \sin [c+d x] - 7 a b^2 \sin [c+d x]}{4 b (-a^2+b^2)^2 (a+b \cos [c+d x])} \right)}{d}
\end{aligned}$$

■ **Problem 729: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \cos [c+d x])^3 \operatorname{Sec}[c+d x]^{5/2}} dx$$

Optimal (type 4, 319 leaves, 11 steps):

$$\begin{aligned}
& - \frac{3 a (a^2 - 3 b^2) \sqrt{\cos [c+d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\operatorname{Sec}[c+d x]}}{4 b^2 (a^2 - b^2)^2 d} + \\
& \frac{(3 a^4 - 5 a^2 b^2 + 8 b^4) \sqrt{\cos [c+d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\operatorname{Sec}[c+d x]}}{4 b^3 (a^2 - b^2)^2 d} - \\
& \frac{3 a (a^4 - 2 a^2 b^2 + 5 b^4) \sqrt{\cos [c+d x]} \operatorname{EllipticPi} \left[ \frac{2 b}{a+b}, \frac{1}{2} (c+d x), 2 \right] \sqrt{\operatorname{Sec}[c+d x]}}{4 (a-b)^2 b^3 (a+b)^3 d} + \\
& \frac{a \sqrt{\operatorname{Sec}[c+d x]} \sin [c+d x]}{2 (a^2 - b^2) d (b+a \operatorname{Sec}[c+d x])^2} + \frac{a (a^2 - 7 b^2) \sqrt{\operatorname{Sec}[c+d x]} \sin [c+d x]}{4 b (a^2 - b^2)^2 d (b+a \operatorname{Sec}[c+d x])}
\end{aligned}$$

Result (type 4, 694 leaves):

$$\begin{aligned}
& - \frac{1}{16 (a-b)^2 b (a+b)^2 d} \\
& \left( - \left( 2 (-8 a^2 b - 16 b^3) \cos[c+dx]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] (b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) \right) / \\
& \left( b (a+b \cos[c+dx]) (1-\cos[c+dx]^2) + \right. \\
& \left. \left( 2 (a^3+5 a b^2) \cos[c+dx]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \right) \right. \right. \\
& \left. \left. (b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) \right) / \left( a (a+b \cos[c+dx]) (1-\cos[c+dx]^2) \right) + \\
& \left( (3 a^3 - 9 a b^2) \cos[2(c+dx)] (b+a \sec[c+dx]) \left( -4 a b + 4 a b \sec[c+dx]^2 - 4 a b \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \right. \right. \\
& \left. \left. \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + 2 (2 a - b) b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + \right. \right. \\
& \left. \left. 4 a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} - \right. \right. \\
& \left. \left. 2 b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \right) \sin[c+dx] \right) / \\
& \left( a b^2 (a+b \cos[c+dx]) (1-\cos[c+dx]^2) \sqrt{\sec[c+dx]} (2-\sec[c+dx]^2) \right) + \\
& \frac{\sqrt{\sec[c+dx]} \left( \frac{3 a (a^2-3 b^2) \sin[c+dx]}{4 b^2 (a^2-b^2)^2} - \frac{a^3 \sin[c+dx]}{2 b^2 (-a^2+b^2) (a+b \cos[c+dx])^2} + \frac{-5 a^4 \sin[c+dx]+11 a^2 b^2 \sin[c+dx]}{4 b^2 (-a^2+b^2)^2 (a+b \cos[c+dx])} \right)}{d}
\end{aligned}$$

■ **Problem 730: Unable to integrate problem.**

$$\int \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{7/2} dx$$

Optimal (type 4, 369 leaves, 6 steps):

$$\begin{aligned}
& \frac{1}{15 a^3 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (9 a^2 - 2 b^2) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \\
& \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{15 a^2 d \sqrt{\sec[c+dx]}} \\
& 2 (a-b) \sqrt{a+b} (9 a+2 b) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \\
& \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{2 b \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{15 a d} + \frac{2 \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{5/2} \sin[c+dx]}{5 d}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a + b \cos[c + dx]} \sec[c + dx]^{7/2} dx$$

■ **Problem 731: Unable to integrate problem.**

$$\int \sqrt{a + b \cos[c + dx]} \sec[c + dx]^{5/2} dx$$

Optimal (type 4, 311 leaves, 5 steps):

$$\frac{1}{3 a^2 d \sqrt{\sec[c + dx]}} 2 (a - b) b \sqrt{a + b} \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \frac{1}{3 a d \sqrt{\sec[c + dx]}}$$

$$2 (a - b) \sqrt{a + b} \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} +$$

$$\frac{2 \sqrt{a + b \cos[c + dx]} \sec[c + dx]^{3/2} \sin[c + dx]}{3 d}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a + b \cos[c + dx]} \sec[c + dx]^{5/2} dx$$

■ **Problem 733: Unable to integrate problem.**

$$\int \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} dx$$

Optimal (type 4, 155 leaves, 2 steps):

$$-\frac{1}{\sqrt{a + b} d} 2 \sqrt{\cos[c + dx]} \sqrt{\frac{a(1 - \cos[c + dx])}{a + b \cos[c + dx]}} \sqrt{\frac{a(1 + \cos[c + dx])}{a + b \cos[c + dx]}}$$

$$(a + b \cos[c + dx]) \operatorname{Csc}[c + dx] \operatorname{EllipticPi}\left[\frac{b}{a + b}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b} \sqrt{\cos[c + dx]}}{\sqrt{a + b \cos[c + dx]}}\right], -\frac{a - b}{a + b}\right] \sqrt{\sec[c + dx]}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} dx$$

■ **Problem 735: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + b \cos[c + dx]}}{\sec[c + dx]^{3/2}} dx$$



Optimal (type 4, 498 leaves, 8 steps) :

$$\begin{aligned}
 & -\frac{1}{4 b d \sqrt{\operatorname{Sec}[c+d x]}} (a-b) \sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \\
 & \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{4 b d \sqrt{\operatorname{Sec}[c+d x]}} \\
 & \sqrt{a+b} (a+2 b) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \\
 & \frac{1}{4 b^2 d \sqrt{\operatorname{Sec}[c+d x]}} \sqrt{a+b} (a^2-4 b^2) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
 & \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{\sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{2 d \sqrt{\operatorname{Sec}[c+d x]}} + \frac{a \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 b d}
 \end{aligned}$$

Result (type 4, 1113 leaves) :

$$\begin{aligned}
 & \frac{\sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[2(c+d x)]}{4 d} + \\
 & \left( -a^2 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a b \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 2 a b \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + a^2 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \right. \\
 & a b \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 2 i a^2 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
 & \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 8 i b^2 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] \\
 & \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
 & 2 i a^2 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}
 \end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+8 i b^2 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}- \\
& i a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+2 i\left(a^2+a b-2 b^2\right) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \Bigg) / \\
& \left(4 b \sqrt{\frac{a-b}{a+b}} d \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right)\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right)^{3 / 2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}\right)
\end{aligned}$$

■ **Problem 736: Unable to integrate problem.**

$$\int (a+b \operatorname{Cos}[c+d x])^{3 / 2} \operatorname{Sec}[c+d x]^{9 / 2} d x$$

Optimal (type 4, 427 leaves, 7 steps):

$$\frac{1}{105 a^3 d \sqrt{\sec [c+d x]}} 4 (a-b) b \sqrt{a+b} \left(41 a^2-3 b^2\right) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}+\frac{1}{105 a^2 d \sqrt{\sec [c+d x]}}$$

$$2(a-b) \sqrt{a+b} \left(25 a^2-57 a b-6 b^2\right) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}+\frac{2\left(25 a^2+3 b^2\right) \sqrt{a+b \cos [c+d x]} \sec [c+d x]^{3 / 2} \sin [c+d x]}{105 a d}+$$

$$\frac{16 b \sqrt{a+b \cos [c+d x]} \sec [c+d x]^{5 / 2} \sin [c+d x]}{35 d}+\frac{2 a \sqrt{a+b \cos [c+d x]} \sec [c+d x]^{7 / 2} \sin [c+d x]}{7 d}$$

Result (type 8, 27 leaves):

$$\int (a+b \cos [c+d x])^{3 / 2} \sec [c+d x]^{9 / 2} d x$$

■ **Problem 737: Unable to integrate problem.**

$$\int (a+b \cos [c+d x])^{3 / 2} \sec [c+d x]^{7 / 2} d x$$

Optimal (type 4, 365 leaves, 6 steps):

$$\frac{1}{5 a^2 d \sqrt{\sec [c+d x]}} 2(a-b) \sqrt{a+b} \left(3 a^2+b^2\right) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}-\frac{1}{5 a d \sqrt{\sec [c+d x]}} 2(a-b)(3 a-b) \sqrt{a+b} \sqrt{\cos [c+d x]}$$

$$\operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}+$$

$$\frac{4 b \sqrt{a+b \cos [c+d x]} \sec [c+d x]^{3 / 2} \sin [c+d x]}{5 d}+\frac{2 a \sqrt{a+b \cos [c+d x]} \sec [c+d x]^{5 / 2} \sin [c+d x]}{5 d}$$

Result (type 8, 27 leaves):

$$\int (a+b \cos [c+d x])^{3 / 2} \sec [c+d x]^{7 / 2} d x$$

■ **Problem 738: Unable to integrate problem.**

$$\int (a + b \cos [c + d x])^{3/2} \sec [c + d x]^{5/2} dx$$

Optimal (type 4, 317 leaves, 5 steps):

$$\frac{1}{3 a d \sqrt{\sec [c + d x]}}$$

$$8 (a - b) b \sqrt{a + b} \sqrt{\cos [c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \sec [c + d x])}{a + b}} \sqrt{\frac{a (1 + \sec [c + d x])}{a - b}} +$$

$$\frac{1}{3 a d \sqrt{\sec [c + d x]}} 2 (a - 3 b) (a - b) \sqrt{a + b} \sqrt{\cos [c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}}\right], -\frac{a + b}{a - b}\right]$$

$$\sqrt{\frac{a (1 - \sec [c + d x])}{a + b}} \sqrt{\frac{a (1 + \sec [c + d x])}{a - b}} + \frac{2 a \sqrt{a + b \cos [c + d x]} \sec [c + d x]^{3/2} \sin [c + d x]}{3 d}$$

Result (type 8, 27 leaves):

$$\int (a + b \cos [c + d x])^{3/2} \sec [c + d x]^{5/2} dx$$

■ **Problem 743: Attempted integration timed out after 120 seconds.**

$$\int (a + b \cos [c + d x])^{5/2} \sec [c + d x]^{11/2} dx$$

Optimal (type 4, 494 leaves, 8 steps):

$$\frac{1}{315 a^3 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (147 a^4 + 279 a^2 b^2 - 10 b^4) \sqrt{\cos[c+dx]}$$

$$\operatorname{Csc}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{315 a^2 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (147 a^3 - 114 a^2 b + 165 a b^2 + 10 b^3) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2 b (163 a^2 + 5 b^2) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{315 a d} + \frac{2 (49 a^2 + 75 b^2) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{5/2} \sin[c+dx]}{315 d} +$$

$$\frac{38 a b \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{7/2} \sin[c+dx]}{63 d} + \frac{2 a^2 \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{9/2} \sin[c+dx]}{9 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 744: Unable to integrate problem.**

$$\int (a+b \cos[c+dx])^{5/2} \sec[c+dx]^{9/2} dx$$

Optimal (type 4, 427 leaves, 7 steps):

$$\frac{1}{21 a^2 d \sqrt{\sec[c+dx]}} 2 (a-b) b \sqrt{a+b} (29 a^2 + 3 b^2) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{21 a d \sqrt{\sec[c+dx]}}$$

$$2 (a-b) \sqrt{a+b} (5 a^2 - 24 a b + 3 b^2) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{2 (5 a^2 + 9 b^2) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{21 d} +$$

$$\frac{6 a b \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{5/2} \sin[c+dx]}{7 d} + \frac{2 a^2 \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{7/2} \sin[c+dx]}{7 d}$$

Result (type 8, 27 leaves):

$$\int (a + b \cos [c + d x])^{5/2} \sec [c + d x]^{9/2} dx$$

■ **Problem 745: Attempted integration timed out after 120 seconds.**

$$\int (a + b \cos [c + d x])^{5/2} \sec [c + d x]^{7/2} dx$$

Optimal (type 4, 378 leaves, 6 steps) :

$$\frac{1}{15 a d \sqrt{\sec [c + d x]}} 2 (a - b) \sqrt{a + b} (9 a^2 + 23 b^2) \sqrt{\cos [c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}}\right], -\frac{a + b}{a - b}\right]$$

$$\sqrt{\frac{a (1 - \sec [c + d x])}{a + b}} \sqrt{\frac{a (1 + \sec [c + d x])}{a - b}} - \frac{1}{15 a d \sqrt{\sec [c + d x]}} 2 (a - b) \sqrt{a + b} (9 a^2 - 8 a b + 15 b^2) \sqrt{\cos [c + d x]}$$

$$\operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \sec [c + d x])}{a + b}} \sqrt{\frac{a (1 + \sec [c + d x])}{a - b}} +$$

$$\frac{22 a b \sqrt{a + b \cos [c + d x]} \sec [c + d x]^{3/2} \sin [c + d x]}{15 d} + \frac{2 a^2 \sqrt{a + b \cos [c + d x]} \sec [c + d x]^{5/2} \sin [c + d x]}{5 d}$$

Result (type 1, 1 leaves) :

???

■ **Problem 748: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^{5/2} \sqrt{\sec [c + d x]} dx$$

Optimal (type 4, 503 leaves, 8 steps) :

$$\begin{aligned}
& - \frac{1}{4 d \sqrt{\operatorname{Sec}[c+d x]}} 9 (a-b) b \sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \\
& \frac{1}{4 d \sqrt{\operatorname{Sec}[c+d x]}} \sqrt{a+b} (8 a^2+9 a b+2 b^2) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{4 d \sqrt{\operatorname{Sec}[c+d x]}} \sqrt{a+b} (15 a^2+4 b^2) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \\
& \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \\
& \frac{b^2 \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{2 d \sqrt{\operatorname{Sec}[c+d x]}} + \frac{9 a b \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d}
\end{aligned}$$

Result (type 4, 3693 leaves):

$$\begin{aligned}
& \frac{b^2 \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[2(c+d x)]}{4 d} + \\
& \left( \left( \frac{3 a^2 b}{\sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{b^3}{2 \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{a^3 \sqrt{\operatorname{Sec}[c+d x]}}{\sqrt{a+b \operatorname{Cos}[c+d x]}} + \right. \right. \\
& \left. \left. \frac{11 a b^2 \sqrt{\operatorname{Sec}[c+d x]}}{8 \sqrt{a+b \operatorname{Cos}[c+d x]}} + \frac{9 a b^2 \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{8 \sqrt{a+b \operatorname{Cos}[c+d x]}} \right) \right. \\
& \left. \left( -18 a b (a+b) \sqrt{\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] - \right. \right. \\
& \left. \left. 4(4 a^3-12 a^2 b+a b^2-2 b^3) \sqrt{\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] + \right. \right. \\
& \left. \left. 4 b(15 a^2+4 b^2) \sqrt{\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 9 a b \cos [c+d x] (a+b \cos [c+d x]) \sec \left[ \frac{1}{2} (c+d x) \right]^2 \tan \left[ \frac{1}{2} (c+d x) \right] \Bigg) \Bigg) / \\
& \left( 4 d \sqrt{a+b \cos [c+d x]} \sqrt{\sec \left[ \frac{1}{2} (c+d x) \right]^2} \sqrt{\cos \left[ \frac{1}{2} (c+d x) \right]^2 \sec [c+d x]} \left( -1 + \tan \left[ \frac{1}{2} (c+d x) \right]^2 \right) \right. \\
& \left. - \left( \left( \sqrt{\sec \left[ \frac{1}{2} (c+d x) \right]^2} \tan \left[ \frac{1}{2} (c+d x) \right] \left( -18 a b (a+b) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \right. \right. \right. \right. \\
& \left. \left. \left. \text{EllipticE} \left[ \text{ArcSin} \left[ \tan \left[ \frac{1}{2} (c+d x) \right] \right], \frac{-a+b}{a+b} \right] - 4 \left( 4 a^3 - 12 a^2 b + a b^2 - 2 b^3 \right) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \right. \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \tan \left[ \frac{1}{2} (c+d x) \right] \right], \frac{-a+b}{a+b} \right] + 4 b \left( 15 a^2 + 4 b^2 \right) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \text{EllipticPi} \left[ \right. \right. \right. \\
& \left. \left. \left. -1, -\text{ArcSin} \left[ \tan \left[ \frac{1}{2} (c+d x) \right] \right], \frac{-a+b}{a+b} \right] - 9 a b \cos [c+d x] (a+b \cos [c+d x]) \sec \left[ \frac{1}{2} (c+d x) \right]^2 \tan \left[ \frac{1}{2} (c+d x) \right] \right) \right) \Bigg) \Bigg) / \\
& \left( 4 \sqrt{a+b \cos [c+d x]} \sqrt{\cos \left[ \frac{1}{2} (c+d x) \right]^2 \sec [c+d x]} \left( -1 + \tan \left[ \frac{1}{2} (c+d x) \right]^2 \right) \right) \Bigg) + \\
& \left( b \sin [c+d x] \left( -18 a b (a+b) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \text{EllipticE} \left[ \text{ArcSin} \left[ \tan \left[ \frac{1}{2} (c+d x) \right] \right], \frac{-a+b}{a+b} \right] - \right. \right. \\
& \left. \left. 4 \left( 4 a^3 - 12 a^2 b + a b^2 - 2 b^3 \right) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \text{EllipticF} \left[ \text{ArcSin} \left[ \tan \left[ \frac{1}{2} (c+d x) \right] \right], \frac{-a+b}{a+b} \right] + \right. \right. \\
& \left. \left. 4 b \left( 15 a^2 + 4 b^2 \right) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \text{EllipticPi} \left[ -1, -\text{ArcSin} \left[ \tan \left[ \frac{1}{2} (c+d x) \right] \right], \frac{-a+b}{a+b} \right] - \right. \right. \\
& \left. \left. \left. \right) \right) \right)
\end{aligned}$$



$$\begin{aligned}
& \left. \left. \left. 9 a b \cos [c+d x] (a+b \cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)\right)\right) / \\
& \left( 8 (a+b \cos [c+d x])^{3/2} \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]} \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right)\right) - \\
& \left( \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \left(-18 a b (a+b) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] - \right. \right. \\
& \left. 4\left(4 a^3-12 a^2 b+a b^2-2 b^3\right) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& \left. 4 b\left(15 a^2+4 b^2\right) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] - \right. \\
& \left. \left. \left. 9 a b \cos [c+d x] (a+b \cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)\right)\right) / \\
& \left( 8 \sqrt{a+b \cos [c+d x]} \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]} \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right)\right) + \\
& \left( -\frac{9}{2} a b \cos [c+d x] (a+b \cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 - \frac{1}{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}} \right. \\
& \left. 9 a b (a+b) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2} - \frac{\sin [c+d x]}{1+\cos [c+d x]}\right) - \right. \\
& \left. \frac{1}{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}} 2\left(4 a^3-12 a^2 b+a b^2-2 b^3\right) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} - \frac{\sin[c+dx]}{1+\cos[c+dx]} \right) + \frac{1}{\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}}} 2b(15a^2+4b^2) \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \\
& \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \left( \frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} - \frac{\sin[c+dx]}{1+\cos[c+dx]} \right) - \\
& \frac{1}{\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} 9ab(a+b) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \left( -\frac{b\sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(a+b\cos[c+dx])\sin[c+dx]}{(a+b)(1+\cos[c+dx])^2} \right) - \frac{1}{\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} 2(4a^3-12a^2b+ab^2-2b^3) \\
& \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \left( -\frac{b\sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(a+b\cos[c+dx])\sin[c+dx]}{(a+b)(1+\cos[c+dx])^2} \right) + \\
& \frac{1}{\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} 2b(15a^2+4b^2) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \left( -\frac{b\sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(a+b\cos[c+dx])\sin[c+dx]}{(a+b)(1+\cos[c+dx])^2} \right) + 9ab^2\cos[c+dx] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] \text{Tan}\left[\frac{1}{2}(c+dx)\right] + \\
& 9ab(a+b\cos[c+dx]) \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] \text{Tan}\left[\frac{1}{2}(c+dx)\right] - 9ab\cos[c+dx](a+b\cos[c+dx]) \\
& \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - \frac{2(4a^3-12a^2b+ab^2-2b^3) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{1-\frac{(-a+b)\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} - \\
& \frac{2b(15a^2+4b^2) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{1-\frac{(-a+b)\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} -
\end{aligned}$$

$$\left. \frac{9 a b (a+b) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\frac{(-a+b) \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}{\sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}} \right. /$$

$$\left( 4 \sqrt{a+b \cos [c+d x]} \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]} \left(-1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right) \right) -$$

$$\left( \left( -18 a b (a+b) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] - \right. \right.$$

$$4\left(4 a^3-12 a^2 b+a b^2-2 b^3\right) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] +$$

$$4 b\left(15 a^2+4 b^2\right) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] -$$

$$9 a b \cos [c+d x](a+b \cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \tan \left[\frac{1}{2}(c+d x)\right] \left. \right)$$

$$\left( -\cos \left[\frac{1}{2}(c+d x)\right] \operatorname{Sec}[c+d x] \sin \left[\frac{1}{2}(c+d x)\right] + \cos \left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x] \tan [c+d x] \right) /$$

$$\left( 8 \sqrt{a+b \cos [c+d x]} \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2} \left(\cos \left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]\right)^{3 / 2} \left(-1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right) \right)$$

■ **Problem 750: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos [c+d x])^{5 / 2}}{\operatorname{Sec}[c+d x]^{3 / 2}} d x$$

Optimal (type 4, 638 leaves, 10 steps):

$$\begin{aligned}
& - \frac{1}{192 b d \sqrt{\sec[c+d x]}} (a-b) \sqrt{a+b} (15 a^2 + 284 b^2) \sqrt{\cos[c+d x]} \operatorname{Csc}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+d x]}}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} + \frac{1}{192 b d \sqrt{\sec[c+d x]}} \\
& \sqrt{a+b} (15 a^3 + 118 a^2 b + 284 a b^2 + 72 b^3) \sqrt{\cos[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+d x]}}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} + \frac{1}{64 b^2 d \sqrt{\sec[c+d x]}} \sqrt{a+b} (5 a^4 - 120 a^2 b^2 - 48 b^4) \sqrt{\cos[c+d x]} \\
& \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+d x]}}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} + \\
& \frac{b^2 \sqrt{a+b \cos[c+d x]} \sin[c+d x]}{4 d \sec[c+d x]^{5/2}} + \frac{17 a b \sqrt{a+b \cos[c+d x]} \sin[c+d x]}{24 d \sec[c+d x]^{3/2}} + \frac{(59 a^2 + 36 b^2) \sqrt{a+b \cos[c+d x]} \sin[c+d x]}{96 d \sqrt{\sec[c+d x]}} + \\
& \frac{a(15 a^2 + 284 b^2) \sqrt{a+b \cos[c+d x]} \sqrt{\sec[c+d x]} \sin[c+d x]}{192 b d}
\end{aligned}$$

Result (type 4, 1642 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{a+b \cos[c+d x]} \sqrt{\sec[c+d x]} \left( \frac{17}{96} a b \sin[c+d x] + \frac{1}{192} (59 a^2 + 48 b^2) \sin[2(c+d x)] + \frac{17}{96} a b \sin[3(c+d x)] + \frac{1}{32} b^2 \sin[4(c+d x)] \right) + \\
& \left( -15 a^4 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 15 a^3 b \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 284 a^2 b^2 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \right. \\
& 284 a b^3 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 30 a^3 b \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 568 a b^3 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + \\
& 15 a^4 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 15 a^3 b \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 284 a^2 b^2 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \\
& \left. 284 a b^3 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 30 i a^4 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+720 i a^2 b^2 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right],-\frac{a+b}{a-b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+ \\
288 i b^4 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right],-\frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}-30 i a^4 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right],-\frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+ \\
720 i a^2 b^2 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right],-\frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+288 i b^4 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right],-\frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}- \\
i a\left(15 a^3-15 a^2 b+284 a b^2-284 b^3\right) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right],-\frac{a+b}{a-b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+ \\
2 i\left(15 a^4+59 a^3 b-38 a^2 b^2+36 a b^3-72 b^4\right) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right],-\frac{a+b}{a-b}\right]
\end{aligned}$$

$$\sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c + dx)\right]^2\right) \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}}$$

$$\left(192 b \sqrt{\frac{a - b}{a + b}} d \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \left(-1 + \tan\left[\frac{1}{2}(c + dx)\right]^2\right) \left(1 + \tan\left[\frac{1}{2}(c + dx)\right]^2\right)^{3/2} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}}\right)$$

■ **Problem 751: Unable to integrate problem.**

$$\int \frac{\sec[c + dx]^{5/2}}{\sqrt{a + b \cos[c + dx]}} dx$$

Optimal (type 4, 314 leaves, 5 steps):

$$-\frac{1}{3 a^3 d \sqrt{\sec[c + dx]}} 4 (a - b) b \sqrt{a + b} \sqrt{\cos[c + dx]} \csc[c + dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \frac{1}{3 a^2 d \sqrt{\sec[c + dx]}}$$

$$2 \sqrt{a + b} (a + 2 b) \sqrt{\cos[c + dx]} \csc[c + dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} +$$

$$\frac{2 \sqrt{a + b \cos[c + dx]} \sec[c + dx]^{3/2} \sin[c + dx]}{3 a d}$$

Result (type 8, 27 leaves):

$$\int \frac{\sec[c + dx]^{5/2}}{\sqrt{a + b \cos[c + dx]}} dx$$

■ **Problem 752: Unable to integrate problem.**

$$\int \frac{\sec[c + dx]^{3/2}}{\sqrt{a + b \cos[c + dx]}} dx$$

Optimal (type 4, 264 leaves, 4 steps):

$$\frac{1}{a^2 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{a d \sqrt{\sec[c+dx]}}$$

$$2 \sqrt{a+b} \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}$$

Result (type 8, 27 leaves):

$$\int \frac{\sec[c+dx]^{3/2}}{\sqrt{a+b \cos[c+dx]}} dx$$

■ **Problem 754: Unable to integrate problem.**

$$\int \frac{1}{\sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]}} dx$$

Optimal (type 4, 136 leaves, 2 steps):

$$-\frac{1}{b d \sqrt{\sec[c+dx]}}$$

$$2 \sqrt{a+b} \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{\sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]}} dx$$

■ **Problem 755: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 474 leaves, 9 steps):

$$\begin{aligned}
& - \frac{1}{a b d \sqrt{\operatorname{Sec}[c+d x]}} \\
& (a-b) \sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \\
& \frac{1}{b d \sqrt{\operatorname{Sec}[c+d x]}} \sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{b^2 d \sqrt{\operatorname{Sec}[c+d x]}} \\
& a \sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \\
& \frac{\operatorname{Sin}[c+d x]}{d \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{a \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{b d \sqrt{a+b \operatorname{Cos}[c+d x]}}
\end{aligned}$$

Result (type 4, 759 leaves):



$$\begin{aligned}
& \frac{1}{b \sqrt{\frac{a-b}{a+b}} d \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^{3/2} \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left( a \sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} + b \sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} + \right. \\
& a \sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} - b \sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} + \\
& 2 i a \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 2 i a \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& i (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& \left. 2 i a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right)
\end{aligned}$$

■ **Problem 756: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{a+b \cos[c+dx]} \sec[c+dx]^{5/2}} dx$$

Optimal (type 4, 505 leaves, 8 steps):

$$\frac{1}{4 b^2 d \sqrt{\sec [c+d x]}} 3 (a-b) \sqrt{a+b} \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \frac{1}{4 b^2 d \sqrt{\sec [c+d x]}}$$

$$(3 a-2 b) \sqrt{a+b} \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} -$$

$$\frac{1}{4 b^3 d \sqrt{\sec [c+d x]}} \sqrt{a+b} (3 a^2+4 b^2) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{2 b d \sqrt{\sec [c+d x]}} - \frac{3 a \sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]} \sin [c+d x]}{4 b^2 d}$$

Result (type 4, 1153 leaves):

$$\frac{\sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]} \sin [2(c+d x)]}{4 b d} -$$

$$\left(\sqrt{\frac{a+b+a \tan \left[\frac{1}{2}(c+d x)\right]^2-b \tan \left[\frac{1}{2}(c+d x)\right]^2}{1+\tan \left[\frac{1}{2}(c+d x)\right]^2}}\left(3 a^2 \sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]+3 a b \sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]-\right.\right.$$

$$\left.6 a b \sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]^3-3 a^2 \sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]^5+3 a b \sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]^5+6 i a^2 \operatorname{EllipticPi}\left[\frac{a+b}{a-b},\right.\right.$$

$$\left. i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \tan \left[\frac{1}{2}(c+d x)\right]^2-b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+\right.$$

$$\left.8 i b^2 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}\right.$$

$$\left.\sqrt{\frac{a+b+a \tan \left[\frac{1}{2}(c+d x)\right]^2-b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+6 i a^2 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right]\right.$$

$$\left.\tan \left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \tan \left[\frac{1}{2}(c+d x)\right]^2-b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+\right.$$

$$\begin{aligned}
& 8 i b^2 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2-b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 3 i a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2-b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 i(3 a^2-a b+2 b^2) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2-b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(4 b^2 \sqrt{\frac{a-b}{a+b}} d\left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}}\left(b\left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)-a\left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right)
\end{aligned}$$

■ **Problem 757: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[c+dx]^{5/2}}{(a+b \operatorname{Cos}[c+dx])^{3/2}} dx$$

Optimal (type 4, 397 leaves, 6 steps):

$$\begin{aligned}
& - \left( 2 b (5 a^2 - 8 b^2) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} \right) / \left( 3 a^4 \sqrt{a+b} d \sqrt{\operatorname{Sec}[c+d x]} \right) + \\
& \left( 2 (a+2 b)(a+4 b) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} \right) / \left( 3 a^3 \sqrt{a+b} d \sqrt{\operatorname{Sec}[c+d x]} \right) + \\
& \frac{2 b^2 \operatorname{Sec}[c+d x]^{3/2} \sin [c+d x]}{a\left(a^2-b^2\right) d \sqrt{a+b \cos [c+d x]}} + \frac{2\left(a^2-4 b^2\right) \sqrt{a+b \cos [c+d x]} \operatorname{Sec}[c+d x]^{3/2} \sin [c+d x]}{3 a^2\left(a^2-b^2\right) d}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\operatorname{Sec}[c+d x]^{5/2}}{(a+b \cos [c+d x])^{3/2}} dx$$

■ **Problem 758: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[c+d x]^{3/2}}{(a+b \cos [c+d x])^{3/2}} dx$$

Optimal (type 4, 325 leaves, 5 steps):

$$\begin{aligned}
& \left( 2 \left(a^2-2 b^2\right) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} \right) / \\
& \quad \left( a^3 \sqrt{a+b} d \sqrt{\operatorname{Sec}[c+d x]} \right) - \\
& \left( 2 (a+2 b) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} \right) / \\
& \quad \left( a^2 \sqrt{a+b} d \sqrt{\operatorname{Sec}[c+d x]} \right) + \frac{2 b^2 \sqrt{\operatorname{Sec}[c+d x]} \sin [c+d x]}{a\left(a^2-b^2\right) d \sqrt{a+b \cos [c+d x]}}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\operatorname{Sec}[c + d x]^{3/2}}{(a + b \operatorname{Cos}[c + d x])^{3/2}} dx$$

- **Problem 759: Unable to integrate problem.**

$$\int \frac{\sqrt{\operatorname{Sec}[c + d x]}}{(a + b \operatorname{Cos}[c + d x])^{3/2}} dx$$

Optimal (type 4, 307 leaves, 5 steps):

$$\left( 2 b \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Cos}[c + d x]}}{\sqrt{a + b} \sqrt{\operatorname{Cos}[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} \right) /$$

$$\left( a^2 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]} \right) + \frac{1}{a \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]}} 2 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Csc}[c + d x]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Cos}[c + d x]}}{\sqrt{a + b} \sqrt{\operatorname{Cos}[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} - \frac{2 b \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{(a^2 - b^2) d \sqrt{a + b \operatorname{Cos}[c + d x]}}$$

Result (type 8, 27 leaves):

$$\int \frac{\sqrt{\operatorname{Sec}[c + d x]}}{(a + b \operatorname{Cos}[c + d x])^{3/2}} dx$$

- **Problem 761: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \operatorname{Cos}[c + d x])^{3/2} \operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 4, 447 leaves, 7 steps):

$$\frac{1}{b\sqrt{a+b}d\sqrt{\sec[c+dx]}} 2\sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{b\sqrt{a+b}d\sqrt{\sec[c+dx]}}$$

$$2\sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{b^2d\sqrt{\sec[c+dx]}} 2\sqrt{a+b}\sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{2a^2\sqrt{\sec[c+dx]}\sin[c+dx]}{b(a^2-b^2)d\sqrt{a+b\cos[c+dx]}}$$

Result (type 4, 1175 leaves):

$$\frac{\sqrt{a+b\cos[c+dx]}\sqrt{\sec[c+dx]}\left(\frac{2a\sin[c+dx]}{b(a^2-b^2)} + \frac{2a^2\sin[c+dx]}{b(-a^2+b^2)(a+b\cos[c+dx])}\right)}{d} -$$

$$\left(2\left(-a^2\sqrt{\frac{a-b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right] - ab\sqrt{\frac{a-b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right] + 2ab\sqrt{\frac{a-b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]^3 + a^2\sqrt{\frac{a-b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]^5 -\right.\right.$$

$$\left. ab\sqrt{\frac{a-b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]^5 - 2ia^2\operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i\operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right]\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}\right.$$

$$\left.\sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2-b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2ib^2\operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i\operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right]\right.$$

$$\left.\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}\sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2-b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -\right.$$

$$\left. 2ia^2\operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i\operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right]\tan\left[\frac{1}{2}(c+dx)\right]^2\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}\right.$$

$$\left.\sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2-b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2ib^2\operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i\operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right]\right)$$

$$\begin{aligned}
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& i a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + i (2a^2 - ab - b^2) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \\
& \left. \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) \Bigg) \Bigg) / \\
& \left( b \sqrt{\frac{a-b}{a+b}} (a^2 - b^2) d \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \right. \\
& \left. \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right)
\end{aligned}$$

■ **Problem 763: Unable to integrate problem.**

$$\int \frac{\sec[c+dx]^{5/2}}{(a+b \cos[c+dx])^{5/2}} dx$$

Optimal (type 4, 513 leaves, 7 steps):

$$\begin{aligned}
& - \left[ 8 b (2 a^4 - 7 a^2 b^2 + 4 b^4) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} \right] / \left(3 a^5 (a-b)(a+b)^{3/2} d \sqrt{\operatorname{Sec}[c+d x]}\right) + \\
& \left( 2 \left(a^4 + 9 a^3 b + 16 a^2 b^2 - 12 a b^3 - 16 b^4\right) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} \right] / \left(3 a^4 (a-b)(a+b)^{3/2} d \sqrt{\operatorname{Sec}[c+d x]}\right) + \\
& \frac{2 b^2 \operatorname{Sec}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{3 a\left(a^2-b^2\right) d(a+b \cos [c+d x])^{3/2}} + \frac{4 b^2\left(5 a^2-3 b^2\right) \operatorname{Sec}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{3 a^2\left(a^2-b^2\right)^2 d \sqrt{a+b \cos [c+d x]}} + \\
& \frac{2\left(a^4-13 a^2 b^2+8 b^4\right) \sqrt{a+b \cos [c+d x]} \operatorname{Sec}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{3 a^3\left(a^2-b^2\right)^2 d}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\operatorname{Sec}[c+d x]^{5/2}}{(a+b \cos [c+d x])^{5/2}} dx$$

■ **Problem 764: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[c+d x]^{3/2}}{(a+b \cos [c+d x])^{5/2}} dx$$

Optimal (type 4, 438 leaves, 6 steps):



$$\left( 2 (3 a^4 - 15 a^2 b^2 + 8 b^4) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \right. \\ \left. \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} \right) / \left( 3 a^4 (a-b) (a+b)^{3/2} d \sqrt{\operatorname{Sec}[c+d x]} \right) - \\ \left( 2 (3 a^3 + 9 a^2 b - 6 a b^2 - 8 b^3) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \right. \\ \left. \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} \right) / \left( 3 a^3 (a-b) (a+b)^{3/2} d \sqrt{\operatorname{Sec}[c+d x]} \right) + \\ \frac{2 b^2 \sqrt{\operatorname{Sec}[c+d x]} \sin [c+d x]}{3 a \left(a^2-b^2\right) d (a+b \cos [c+d x])^{3/2}} + \frac{8 b^2 \left(2 a^2-b^2\right) \sqrt{\operatorname{Sec}[c+d x]} \sin [c+d x]}{3 a^2 \left(a^2-b^2\right)^2 d \sqrt{a+b \cos [c+d x]}}$$

Result (type 8, 27 leaves):

$$\int \frac{\operatorname{Sec}[c+d x]^{3/2}}{(a+b \cos [c+d x])^{5/2}} dx$$

■ **Problem 765: Unable to integrate problem.**

$$\int \frac{\sqrt{\operatorname{Sec}[c+d x]}}{(a+b \cos [c+d x])^{5/2}} dx$$

Optimal (type 4, 421 leaves, 6 steps):

$$\left( 4 b \left(3 a^2-b^2\right) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} \right) / \\ \left( 3 a^3 (a-b) (a+b)^{3/2} d \sqrt{\operatorname{Sec}[c+d x]} \right) + \left( 2 \left(3 a^2-3 a b-2 b^2\right) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \right. \\ \left. \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} \right) / \left( 3 a^2 (a-b) (a+b)^{3/2} d \sqrt{\operatorname{Sec}[c+d x]} \right) + \\ \frac{2 b^2 \sin [c+d x]}{3 a \left(a^2-b^2\right) d (a+b \cos [c+d x])^{3/2} \sqrt{\operatorname{Sec}[c+d x]}} - \frac{4 b \left(3 a^2-b^2\right) \sqrt{\operatorname{Sec}[c+d x]} \sin [c+d x]}{3 a \left(a^2-b^2\right)^2 d \sqrt{a+b \cos [c+d x]}}$$

Result (type 8, 27 leaves) :

$$\int \frac{\sqrt{\sec[c+dx]}}{(a+b\cos[c+dx])^{5/2}} dx$$

■ **Problem 766: Unable to integrate problem.**

$$\int \frac{1}{(a+b\cos[c+dx])^{5/2} \sqrt{\sec[c+dx]}} dx$$

Optimal (type 4, 399 leaves, 6 steps) :

$$\begin{aligned} & - \left( 2 (3a^2 + b^2) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} \right) / \\ & \quad \left( 3a^2 (a-b) (a+b)^{3/2} d \sqrt{\sec[c+dx]} \right) + \\ & \left( 2 (3a-b) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} \right) / \\ & \quad \left( 3a(a-b) (a+b)^{3/2} d \sqrt{\sec[c+dx]} \right) - \frac{2b \sin[c+dx]}{3(a^2 - b^2) d (a+b\cos[c+dx])^{3/2} \sqrt{\sec[c+dx]}} + \frac{2(3a^2 + b^2) \sqrt{\sec[c+dx]} \sin[c+dx]}{3(a^2 - b^2)^2 d \sqrt{a+b\cos[c+dx]}} \end{aligned}$$

Result (type 8, 27 leaves) :

$$\int \frac{1}{(a+b\cos[c+dx])^{5/2} \sqrt{\sec[c+dx]}} dx$$

■ **Problem 767: Unable to integrate problem.**

$$\int \frac{1}{(a+b\cos[c+dx])^{5/2} \sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 382 leaves, 6 steps) :

$$\left( 8 b \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}\right) /$$

$$\left( 3 a(a-b)(a+b)^{3 / 2} d \sqrt{\sec [c+d x]}\right) +$$

$$\left( 2(a-3 b) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}\right) /$$

$$\left( 3 a(a-b)(a+b)^{3 / 2} d \sqrt{\sec [c+d x]}\right) + \frac{2 a \sin [c+d x]}{3\left(a^2-b^2\right) d(a+b \cos [c+d x])^{3 / 2} \sqrt{\sec [c+d x]}} - \frac{8 a b \sqrt{\sec [c+d x]} \sin [c+d x]}{3\left(a^2-b^2\right)^2 d \sqrt{a+b \cos [c+d x]}}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{(a+b \cos [c+d x])^{5 / 2} \sec [c+d x]^{3 / 2}} d x$$

■ **Problem 768: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \cos [c+d x])^{5 / 2} \sec [c+d x]^{5 / 2}} d x$$

Optimal (type 4, 557 leaves, 8 steps):

$$\left( 2\left(3 a^2-7 b^2\right) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}\right) /$$

$$\left( 3(a-b) b^2(a+b)^{3 / 2} d \sqrt{\sec [c+d x]}\right) -$$

$$\left( 2\left(3 a^2+a b-6 b^2\right) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}}\right.$$

$$\left. \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}\right) / \left( 3(a-b) b^2(a+b)^{3 / 2} d \sqrt{\sec [c+d x]}\right) - \frac{1}{b^3 d \sqrt{\sec [c+d x]}}$$

$$2 \sqrt{a+b} \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} -$$

$$\frac{2 a^2 \sin [c+d x]}{3 b\left(a^2-b^2\right) d(a+b \cos [c+d x])^{3 / 2} \sqrt{\sec [c+d x]}} - \frac{2 a^2\left(3 a^2-7 b^2\right) \sqrt{\sec [c+d x]} \sin [c+d x]}{3 b^2\left(a^2-b^2\right)^2 d \sqrt{a+b \cos [c+d x]}}$$

Result (type 4, 1716 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]} \left( \frac{2 a\left(3 a^2-7 b^2\right) \sin [c+d x]}{3 b^2\left(a^2-b^2\right)^2}-\frac{2 a^3 \sin [c+d x]}{3 b^2\left(-a^2+b^2\right)\left(a+b \cos [c+d x]\right)^2}-\frac{8\left(a^4 \sin [c+d x]-2 a^2 b^2 \sin [c+d x]\right)}{3 b^2\left(-a^2+b^2\right)^2\left(a+b \cos [c+d x]\right)}\right)+ \\
& \left(2\left(3 a^4 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+3 a^3 b \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]-7 a^2 b^2 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]-\right.\right. \\
& 7 a b^3 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]-6 a^3 b \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3+14 a b^3 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3- \\
& 3 a^4 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+3 a^3 b \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+7 a^2 b^2 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5- \\
& 7 a b^3 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+6 i a^4 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}-12 i a^2 b^2 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+ \\
& 6 i b^4 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+6 i a^4 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}- \\
& 12 i a^2 b^2 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+6 i b^4 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right]
\end{aligned}$$

$$\begin{aligned}
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& i a (3a^3 - 3a^2b - 7ab^2 + 7b^3) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& i (6a^4 - 2a^3b - 13a^2b^2 + 6ab^3 + 3b^4) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \\
& \left. \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) / \\
& \left(3b^2 \sqrt{\frac{a-b}{a+b}} (a^2 - b^2)^2 d \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^4\right)\right)
\end{aligned}$$

■ **Problem 773: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^m}{a+b \cos[c+dx]} dx$$

Optimal (type 6, 190 leaves, 5 steps):

$$\begin{aligned}
& \frac{a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1-m}{2}, 1, \frac{3}{2}, \sin[c+dx]^2, -\frac{b^2 \sin[c+dx]^2}{a^2-b^2}\right] \cos[c+dx]^{-1+m} (\cos[c+dx]^2)^{\frac{1-m}{2}} \sin[c+dx]}{(a^2 - b^2) d} - \\
& \frac{b \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, \sin[c+dx]^2, -\frac{b^2 \sin[c+dx]^2}{a^2-b^2}\right] \cos[c+dx]^m (\cos[c+dx]^2)^{-m/2} \sin[c+dx]}{(a^2 - b^2) d}
\end{aligned}$$

Result (type 6, 6355 leaves):

$$\begin{aligned}
& \left(3(a^2 - b^2) \cos[c+dx]^{-1+m} \sin[c+dx] (1 + \tan[c+dx]^2)^{-1-\frac{m}{2}} \right. \\
& \left. \left(b + a \sqrt{1 + \tan[c+dx]^2}\right) \left(-\left(a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] \sqrt{1 + \tan[c+dx]^2}\right)\right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + dx]^2 \right) + \\
& \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1+m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] \right) / \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1+m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + dx]^2, \right. \right. \\
& \quad \left. \left. -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + dx]^2 \right) \Big) / \\
& \left( d (a + b \operatorname{Cos}[c + dx]) \left( a + \frac{b}{\sqrt{1 + \operatorname{Tan}[c + dx]^2}} \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c + dx]^2)) \right) \\
& \left( -\frac{1}{\left( a + \frac{b}{\sqrt{1 + \operatorname{Tan}[c + dx]^2}} \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c + dx]^2))^2} - 6 a^2 (a^2 - b^2) \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]^2 (1 + \operatorname{Tan}[c + dx]^2)^{-1 - \frac{m}{2}} \right. \\
& \quad \left( b + a \sqrt{1 + \operatorname{Tan}[c + dx]^2} \right) \left( -\left( a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] \sqrt{1 + \operatorname{Tan}[c + dx]^2} \right) / \right. \\
& \quad \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + dx]^2 \right) + \\
& \quad \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1+m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] \right) / \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1+m}{2}, 1, \frac{3}{2}, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + dx]^2 \right) \Big) + \\
& \frac{1}{\left( a + \frac{b}{\sqrt{1 + \operatorname{Tan}[c + dx]^2}} \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c + dx]^2))} - 3 a (a^2 - b^2) \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]^2 (1 + \operatorname{Tan}[c + dx]^2)^{-\frac{3}{2} - \frac{m}{2}}
\end{aligned}$$

$$\begin{aligned}
& \left( - \left( a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \sqrt{1+\operatorname{Tan}[c+dx]^2} \right) / \right. \\
& \quad \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) + \\
& \quad \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1+m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) / \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1+m}{2}, 1, \frac{3}{2}, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) \Bigg) + \\
& \frac{1}{\left( a + \frac{b}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^2 (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2))} - 3 b (a^2-b^2) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]^2 (1+\operatorname{Tan}[c+dx]^2)^{-\frac{5}{2}-\frac{m}{2}} \\
& \quad \left( b + a \sqrt{1+\operatorname{Tan}[c+dx]^2} \right) \left( - \left( a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \sqrt{1+\operatorname{Tan}[c+dx]^2} \right) / \right. \\
& \quad \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) + \\
& \quad \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1+m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) / \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1+m}{2}, 1, \frac{3}{2}, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) \Bigg) + \\
& \frac{1}{\left( a + \frac{b}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^2 (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2))} - 6 (a^2-b^2) \left( -1 - \frac{m}{2} \right) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]^2 (1+\operatorname{Tan}[c+dx]^2)^{-2-\frac{m}{2}} \\
& \quad \left( b + a \sqrt{1+\operatorname{Tan}[c+dx]^2} \right) \left( - \left( a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \sqrt{1+\operatorname{Tan}[c+dx]^2} \right) / \right.
\end{aligned}$$

$$\begin{aligned}
& \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + d x]^2 \right) + \\
& \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1+m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) / \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1+m}{2}, 1, \frac{3}{2}, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + d x]^2 \right) \Bigg) + \\
& \frac{1}{\left( a + \frac{b}{\sqrt{1 + \operatorname{Tan}[c + d x]^2}} \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c + d x]^2))} - 3 (a^2 - b^2) \operatorname{Sec}[c + d x]^2 (1 + \operatorname{Tan}[c + d x]^2)^{-1 - \frac{m}{2}} \left( b + a \sqrt{1 + \operatorname{Tan}[c + d x]^2} \right) \\
& \left( - \left( a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \sqrt{1 + \operatorname{Tan}[c + d x]^2} \right) / \right. \\
& \quad \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + d x]^2 \right) + \\
& \quad \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1+m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) / \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1+m}{2}, 1, \frac{3}{2}, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + d x]^2 \right) \Bigg) + \\
& \frac{1}{\left( a + \frac{b}{\sqrt{1 + \operatorname{Tan}[c + d x]^2}} \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c + d x]^2))} - 3 (a^2 - b^2) \operatorname{Tan}[c + d x] (1 + \operatorname{Tan}[c + d x]^2)^{-1 - \frac{m}{2}} \left( b + a \sqrt{1 + \operatorname{Tan}[c + d x]^2} \right) \\
& \left( - \left( a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] \right) / \left( \sqrt{1 + \operatorname{Tan}[c + d x]^2} \right. \right. \\
& \quad \left. \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \right. \right. \right. \right.
\end{aligned}$$



$$\begin{aligned}
& - \frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \Big] + (a^2 - b^2) \operatorname{mAppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan^2[c + dx], -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] \tan^2[c + dx] \Big) \Big) - \\
& \left( a \left( -\frac{1}{3} \operatorname{mAppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\tan^2[c + dx], -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] \sec^2[c + dx] \tan^2[c + dx] - \right. \right. \\
& \quad \left. \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan^2[c + dx], -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] \sec^2[c + dx] \tan^2[c + dx]}{3 (a^2 - b^2)} \right) \sqrt{1 + \tan^2[c + dx]} \right) \Big) / \\
& \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan^2[c + dx], -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan^2[c + dx], \right. \right. \right. \\
& \quad \left. \left. -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] + (a^2 - b^2) \operatorname{mAppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan^2[c + dx], -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] \right) \tan^2[c + dx] \Big) + \\
& \left( b \left( -\frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\tan^2[c + dx], -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] \sec^2[c + dx] \tan^2[c + dx]}{3 (a^2 - b^2)} - \frac{1}{3} (1+m) \operatorname{AppellF1} \left[ \right. \right. \\
& \quad \left. \left. \frac{3}{2}, 1 + \frac{1+m}{2}, 1, \frac{5}{2}, -\tan^2[c + dx], -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] \sec^2[c + dx] \tan^2[c + dx] \right) \Big) \Big) / \\
& \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1+m}{2}, 1, \frac{3}{2}, -\tan^2[c + dx], -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\tan^2[c + dx], \right. \right. \right. \\
& \quad \left. \left. -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] + (a^2 - b^2) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3+m}{2}, 1, \frac{5}{2}, -\tan^2[c + dx], -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] \right) \tan^2[c + dx] \Big) + \\
& \left( a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan^2[c + dx], -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] \sqrt{1 + \tan^2[c + dx]} \right) \left( 2 \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan^2[c + dx], \right. \right. \right. \\
& \quad \left. \left. -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] + (a^2 - b^2) \operatorname{mAppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan^2[c + dx], -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] \right) \sec^2[c + dx] \tan^2[c + dx] - \\
& 3 (a^2 - b^2) \left( -\frac{1}{3} \operatorname{mAppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\tan^2[c + dx], -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] \sec^2[c + dx] \tan^2[c + dx] - \right. \\
& \quad \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan^2[c + dx], -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] \sec^2[c + dx] \tan^2[c + dx]}{3 (a^2 - b^2)} \right) + \\
& \tan^2[c + dx] \left( 2 a^2 \left( -\frac{3}{5} \operatorname{mAppellF1} \left[ \frac{5}{2}, 1 + \frac{m}{2}, 2, \frac{7}{2}, -\tan^2[c + dx], -\frac{a^2 \tan^2[c + dx]}{a^2 - b^2} \right] \sec^2[c + dx] \tan^2[c + dx] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{5(a^2 - b^2)} 12 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{m}{2}, 3, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \Bigg) + \\
& (a^2 - b^2) m \left( -\frac{1}{5(a^2 - b^2)} 6 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2+m}{2}, 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \\
& \left. \frac{3}{5} (2+m) \operatorname{AppellF1}\left[\frac{5}{2}, 1 + \frac{2+m}{2}, 1, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \Bigg) \Bigg) \Bigg) / \\
& \left( -3(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \\
& \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + (a^2 - b^2) m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \right) \tan[c + dx]^2 \right)^2 - \\
& \left( b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1+m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \left( 2 \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \right. \\
& \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + (a^2 - b^2) (1+m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \right) \operatorname{Sec}[c + dx]^2 \tan[ \\
& c + dx] - 3(a^2 - b^2) \left( -\frac{1}{3(a^2 - b^2)} 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \\
& \left. \frac{1}{3} (1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 1 + \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \Bigg) + \\
& \tan[c + dx]^2 \left( 2 a^2 \left( -\frac{1}{5(a^2 - b^2)} 12 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1+m}{2}, 3, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \left. \frac{3}{5} (1+m) \operatorname{AppellF1}\left[\frac{5}{2}, 1 + \frac{1+m}{2}, 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \Bigg) + \\
& (a^2 - b^2) (1+m) \left( -\frac{1}{5(a^2 - b^2)} 6 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3+m}{2}, 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \\
& \left. \frac{3}{5} (3+m) \operatorname{AppellF1}\left[\frac{5}{2}, 1 + \frac{3+m}{2}, 1, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \Bigg) \Bigg) \Bigg) / \\
& \left( -3(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1+m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2}\right] + \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \left. \left( -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right) \right) \tan[c + dx]^2 - \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c + dx]^2, \frac{a^2 \tan[c + dx]^2}{-a^2 + b^2} \right] \right) / \\
& \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) \right) / \\
& \left( d (a + b \cos[c + dx])^2 \left( -3 (a^2 - b^2) m \sec[c + dx]^2 \tan[c + dx]^2 (1 + \tan[c + dx]^2)^{-1 - \frac{m}{2}} \right. \right. \\
& \left. \left. \left( -\operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] / \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \right. \right. \\
& \left. \left. \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \left. \left. 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) (-b^2 + a^2 (1 + \tan[c + dx]^2)) \right) + \\
& \frac{1}{(b^2 - a^2 (1 + \tan[c + dx]^2))^2} 2 b \left( \left( a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 2, \frac{3}{2}, -\tan[c + dx]^2, \frac{a^2 \tan[c + dx]^2}{-a^2 + b^2} \right] \sqrt{1 + \tan[c + dx]^2} \right) / \right. \\
& \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 2, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \\
& \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 + m), 3, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) (-1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, \right. \right. \\
& \left. \left. -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 - \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c + dx]^2, \frac{a^2 \tan[c + dx]^2}{-a^2 + b^2} \right] \right) / \\
& \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) \right) + \\
& 3 (a^2 - b^2) \sec[c + dx]^2 (1 + \tan[c + dx]^2)^{-m/2} \left( -\operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] / \right. \\
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] \operatorname{Tan}[c + dx]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c + dx]^2)) \Bigg) + \\
& \frac{1}{(b^2 - a^2 (1 + \operatorname{Tan}[c + dx]^2))^2} 2 b \left( \left( a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 2, \frac{3}{2}, -\operatorname{Tan}[c + dx]^2, \frac{a^2 \operatorname{Tan}[c + dx]^2}{-a^2 + b^2} \right] \sqrt{1 + \operatorname{Tan}[c + dx]^2} \right) / \right. \\
& \quad \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 2, \frac{3}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 + m), 3, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) (-1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1 + m}{2}, 2, \frac{5}{2}, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + dx]^2 - \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[c + dx]^2, \frac{a^2 \operatorname{Tan}[c + dx]^2}{-a^2 + b^2} \right] \right) / \Bigg) \\
& \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] + \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, \right. \right. \right. \\
& \quad \left. \left. -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2 + m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + dx]^2 \Bigg) \Bigg) + \\
& 3 (a^2 - b^2) \operatorname{Tan}[c + dx] (1 + \operatorname{Tan}[c + dx]^2)^{-m/2} \left( \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx] \right) / \right. \\
& \quad \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + dx]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c + dx]^2))^2 - \\
& \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx] - \right. \\
& \quad \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{3 (a^2 - b^2)} \right) / \Bigg) \\
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + dx]^2, -\frac{a^2 \operatorname{Tan}[c + dx]^2}{a^2 - b^2} \right] + \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] + 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \left. \right) \tan[c+dx]^2 \Bigg) \\
& \left. (-b^2 + a^2 (1 + \tan[c+dx]^2)) \right) + \frac{1}{(b^2 - a^2 (1 + \tan[c+dx]^2))^3} 8 a^2 b \operatorname{Sec}[c+dx]^2 \tan[c+dx] \\
& \left( \left( a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2+b^2} \right] \sqrt{1 + \tan[c+dx]^2} \right) / \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), \right. \right. \right. \\
& \left. \left. \left. 2, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] + \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \right) \tan[c+dx]^2 \right) - \right. \\
& \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2+b^2} \right] \right) / \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c+dx]^2, \right. \right. \\
& \left. \left. -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] + \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \right) \tan[c+dx]^2 \right) \Bigg) + \\
& \left( \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \left( 2 \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] + 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \\
& \left. 3 (a^2 - b^2) \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \left. \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx]}{3 (a^2 - b^2)} \right) \right) + \tan[c+dx]^2 \\
& \left( (a^2 - b^2) m \left( -\frac{1}{5 (a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \left. \left. \frac{6}{5} \left( 1 + \frac{m}{2} \right) \operatorname{AppellF1} \left[ \frac{5}{2}, 2 + \frac{m}{2}, 1, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \right. \\
& \left. 2 a^2 \left( -\frac{3}{5} m \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \frac{1}{5 (a^2 - b^2)} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 12 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{m}{2}, 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \Big) \Big) \Big) \Big) / \\
& \left( \left( -3 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \left( (a^2-b^2) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+\frac{m}{2}, 1, \frac{5}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) \tan[c+dx]^2 \right)^2 \\
& \left( -b^2 + a^2 (1 + \tan[c+dx]^2) \right) \Big) + \frac{1}{(b^2 - a^2 (1 + \tan[c+dx]^2))^2} 2 b \left( \left( a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\tan[c+dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{a^2 \tan[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) / \left( \sqrt{1 + \tan[c+dx]^2} \left( -3 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \left( 4 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \right. \right. \right. \\
& \quad \left. \left. \left. (a^2-b^2) (-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) \tan[c+dx]^2 \right) \right) + \\
& \left( a \left( -\frac{1}{3} (-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 1+\frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] + 1 / (3 (-a^2+b^2)) \right. \right. \\
& \quad \left. \left. 4 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \sqrt{1 + \tan[c+dx]^2} \right) / \\
& \left( -3 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \left( 4 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} (-1+m), 3, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + (a^2-b^2) (-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) \right) \\
& \tan[c+dx]^2 \Big) - \left( b \left( -\frac{1}{3} m \operatorname{AppellF1}\left[\frac{3}{2}, 1+\frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] + \right. \right. \\
& \quad \left. \left. 4 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \right) \Big) \Big) / \\
& \left( -3 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \left( 4 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + (a^2-b^2) m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) \tan[c+dx]^2 \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2+b^2} \right] \sqrt{1+\operatorname{Tan}[c+dx]^2} \right. \\
& \left. \left( 2 \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + (a^2-b^2) (-1+m) \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - 3 (a^2-b^2) \right. \\
& \quad \left. \left( -\frac{1}{3} (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1+\frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \frac{1}{3 (a^2-b^2)} \right. \right. \\
& \quad \left. \left. 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) + \operatorname{Tan}[c+dx]^2 \right. \\
& \quad \left. \left( 4 a^2 \left( -\frac{3}{5} (-1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1+\frac{1}{2} (-1+m), 3, \frac{7}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \right. \right. \right. \\
& \quad \left. \left. \frac{1}{5 (a^2-b^2)} 18 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2} (-1+m), 4, \frac{7}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) + \right. \\
& \quad \left. (a^2-b^2) (-1+m) \left( -\frac{1}{5 (a^2-b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1+m}{2}, 3, \frac{7}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+ \right. \right. \\
& \quad \left. \left. dx] - \frac{3}{5} (1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1+\frac{1+m}{2}, 2, \frac{7}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right) \right) \Bigg) / \\
& \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \right. \right. \right. \\
& \quad \left. \left. \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \\
& \quad \left. (a^2-b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \Bigg)^2 + \\
& \left( b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2+b^2} \right] \left( 2 \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \right. \\
& \quad \left. 3 (a^2-b^2) \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+\frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \right. \right. \\
& \quad \left. \left. \frac{1}{3 (a^2-b^2)} 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right) + \right.
\end{aligned}$$



$$\begin{aligned} & \tan[c+dx]^2 \left( 4 a^2 \left( -\frac{3}{5} m \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{m}{2}, 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] - \right. \right. \\ & \quad \left. \frac{1}{5(a^2 - b^2)} 18 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{m}{2}, 4, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] \right) + \\ & \quad \left( (a^2 - b^2) m \left( -\frac{1}{5(a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{2+m}{2}, 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] - \right. \right. \\ & \quad \left. \left. \frac{3}{5} (2+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{2+m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] \right) \right) / \\ & \quad \left( -3(a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \\ & \quad \left. \left. -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \tan[c+dx]^2 \right) \right) \end{aligned}$$

- **Problem 778: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{1 - \cos[x]}}{\sqrt{a - \cos[x]}} dx$$

Optimal (type 3, 26 leaves, 2 steps):

$$-2 \operatorname{ArcTan} \left[ \frac{\sin[x]}{\sqrt{1 - \cos[x]} \sqrt{a - \cos[x]}} \right]$$

Result (type 3, 47 leaves):

$$i \sqrt{2 - 2 \cos[x]} \operatorname{Csc} \left[ \frac{x}{2} \right] \operatorname{Log} \left[ i \sqrt{2} \cos \left[ \frac{x}{2} \right] + \sqrt{a - \cos[x]} \right]$$

- **Problem 788: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c+dx])^{1/3} (A + B \cos[c+dx]) dx$$

Optimal (type 5, 102 leaves, 3 steps):

$$\frac{3 B (a + a \cos[c+dx])^{1/3} \sin[c+dx]}{4 d} + \frac{(4 A + B) (a + a \cos[c+dx])^{1/3} \operatorname{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2} (1 - \cos[c+dx]) \right] \sin[c+dx]}{2 \times 2^{1/6} d (1 + \cos[c+dx])^{5/6}}$$

Result (type 5, 213 leaves):

$$\frac{1}{32d} 3 (a (1 + \cos[c + dx]))^{1/3} \left( -8 (4A + B) \cot\left[\frac{c}{2}\right] + 8B \cos[dx] \sin[c] + \left( 2 (4A + B) \operatorname{Csc}\left[\frac{c}{4}\right] \right. \right. \\ \left. \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{i dx} (\cos[c] + i \sin[c])\right] + e^{i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -e^{i dx} (\cos[c] + i \sin[c])\right] \right) \right) \right) \\ \operatorname{Sec}\left[\frac{c}{4}\right] (1 + e^{i dx} \cos[c] + i e^{i dx} \sin[c])^{1/3} \Big/ \left( (1 + e^{i dx}) \cos\left[\frac{c}{2}\right] + i (-1 + e^{i dx}) \sin\left[\frac{c}{2}\right] + 8B \cos[c] \sin[dx] \right)$$

■ **Problem 790: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos[c + dx]}{(a + a \cos[c + dx])^{2/3}} dx$$

Optimal (type 5, 105 leaves, 3 steps):

$$\frac{3(A - B) \sin[c + dx]}{d(a + a \cos[c + dx])^{2/3}} - \frac{2^{5/6}(A - 2B)(a + a \cos[c + dx])^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1 - \cos[c + dx])\right] \sin[c + dx]}{ad(1 + \cos[c + dx])^{5/6}}$$

Result (type 5, 197 leaves):

$$\left( 3 \cos\left[\frac{1}{2}(c + dx)\right] \left( -4 \left( (-2A + 3B) \cos\left[\frac{dx}{2}\right] + B \cos\left[c + \frac{dx}{2}\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] - (A - 2B) e^{-\frac{1}{2}i dx} \operatorname{Csc}\left[\frac{c}{4}\right] \right. \right. \\ \left. \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{i dx} (\cos[c] + i \sin[c])\right] + e^{i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -e^{i dx} (\cos[c] + i \sin[c])\right] \right) \right) \right) \\ \operatorname{Sec}\left[\frac{c}{4}\right] (1 + e^{i dx} \cos[c] + i e^{i dx} \sin[c])^{1/3} \Big/ (4d(a(1 + \cos[c + dx]))^{2/3})$$

■ **Problem 922: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(b \cos[c + dx])^n (A + B \cos[c + dx])}{\sqrt{\cos[c + dx]}} dx$$

Optimal (type 5, 163 leaves, 4 steps):

$$- \left( 2A \sqrt{\cos[c + dx]} (b \cos[c + dx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(1 + 2n), \frac{1}{4}(5 + 2n), \cos[c + dx]^2\right] \sin[c + dx] \right) \Big/ \left( d(1 + 2n) \sqrt{\sin[c + dx]^2} \right) - \\ \left( 2B \cos[c + dx]^{3/2} (b \cos[c + dx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(3 + 2n), \frac{1}{4}(7 + 2n), \cos[c + dx]^2\right] \sin[c + dx] \right) \Big/ \left( d(3 + 2n) \sqrt{\sin[c + dx]^2} \right)$$

Result (type 6, 4951 leaves):

$$\left( 2 \left( \cos\left[\frac{1}{2}(c + dx)\right]^2 \right)^{\frac{3}{2}+n} \cos[c + dx]^{-n} (b \cos[c + dx])^n \left( \cos[c + dx] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \right)^{-\frac{1}{2}+n} \right. \\ \left. \left( \frac{1}{2} B \cos[c + dx]^{\frac{1}{2}+n} + A \cos[c + dx]^{\frac{3}{2}+n} + \frac{1}{2} B \cos[c + dx]^{\frac{1}{2}+n} \cos[2(c + dx)] + \frac{1}{2} i B \cos[c + dx]^{\frac{1}{2}+n} \sin[2(c + dx)] + \right. \right.$$

$$\begin{aligned}
& \text{Sec}[c+dx] \left( -\frac{1}{2} i B \text{Cos}[c+dx]^{\frac{1}{2}+n} \text{Cos}[2(c+dx)] \text{Sin}[c+dx] + A \text{Cos}[c+dx]^{\frac{1}{2}+n} \text{Sin}[c+dx]^2 + \right. \\
& \quad \left. \text{Sin}[c+dx] \left( -\frac{1}{2} i B \text{Cos}[c+dx]^{\frac{1}{2}+n} + \frac{1}{2} B \text{Cos}[c+dx]^{\frac{1}{2}+n} \text{Sin}[2(c+dx)] \right) \right) \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right] \left( \left( 9(A+B) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) / \right. \\
& \quad \left( 3 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \left( -(3+2n) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-n, \frac{5}{2}+n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + (1-2n) \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
& \quad \left( 5(-A+B) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) / \\
& \quad \left( -5 \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \left( (3+2n) \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}-n, \frac{5}{2}+n, \frac{7}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + (-1+2n) \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{7}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \left. \right) / \\
& \left( 3d \left( \frac{1}{3} \left( \text{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \right)^{\frac{1}{2}+n} \left( \text{Cos}[c+dx] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right)^{-\frac{1}{2}+n} \left( \left( 9(A+B) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) / \left( 3 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \quad \left( -(3+2n) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-n, \frac{5}{2}+n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \quad \left. \left. (1-2n) \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
& \quad \left( 5(-A+B) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) / \left( -5 \right. \\
& \quad \left. \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \left( (3+2n) \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}-n, \frac{5}{2}+n, \frac{7}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + (-1+2n) \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{7}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \left. \right) - \\
& \frac{2}{3} \left( \frac{3}{2}+n \right) \left( \text{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \right)^{\frac{1}{2}+n} \left( \text{Cos}[c+dx] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right)^{-\frac{1}{2}+n} \text{Sin}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \quad \left( \left( 9(A+B) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) / \left( 3 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{3}{2}, \right. \right. \right.
\end{aligned}$$







$$\frac{5}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \\ \left. (-1 + 2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2} - n, \frac{3}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right]\right] \tan\left[\frac{1}{2}(c + dx)\right]^2\right)^2\right)\right)$$

■ **Problem 923: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(b \cos[c + dx])^n (A + B \cos[c + dx])}{\cos[c + dx]^{3/2}} dx$$

Optimal (type 5, 163 leaves, 4 steps):

$$\frac{2 A (b \cos[c + dx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(-1 + 2n), \frac{1}{4}(3 + 2n), \cos[c + dx]^2\right] \sin[c + dx]}{d(1 - 2n) \sqrt{\cos[c + dx]} \sqrt{\sin[c + dx]^2}} - \\ \left(2 B \sqrt{\cos[c + dx]} (b \cos[c + dx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(1 + 2n), \frac{1}{4}(5 + 2n), \cos[c + dx]^2\right] \sin[c + dx]\right) / \left(d(1 + 2n) \sqrt{\sin[c + dx]^2}\right)$$

Result (type 6, 4842 leaves):

$$\left(6 \sqrt{\cos[c + dx]} (b \cos[c + dx])^n \right. \\ \left. \left( A \cos[c + dx]^{\frac{1}{2}+n} + \sec[c + dx] \left( \frac{1}{2} B \cos[c + dx]^{\frac{1}{2}+n} + \frac{1}{2} B \cos[c + dx]^{\frac{1}{2}+n} \cos[2(c + dx)] + \frac{1}{2} i B \cos[c + dx]^{\frac{1}{2}+n} \sin[2(c + dx)] \right) \right) + \right. \\ \left. \sec[c + dx]^2 \left( -\frac{1}{2} i B \cos[c + dx]^{\frac{1}{2}+n} \cos[2(c + dx)] \sin[c + dx] + A \cos[c + dx]^{\frac{1}{2}+n} \sin[c + dx]^2 + \right. \right. \\ \left. \left. \sin[c + dx] \left( -\frac{1}{2} i B \cos[c + dx]^{\frac{1}{2}+n} + \frac{1}{2} B \cos[c + dx]^{\frac{1}{2}+n} \sin[2(c + dx)] \right) \right) \right) \tan\left[\frac{1}{2}(c + dx)\right] \\ \left. \left( \left( (A - B) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} - n, \frac{1}{2} + n, \frac{3}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \left( -1 + \tan\left[\frac{1}{2}(c + dx)\right]^2 \right) \right) \right) / \right. \\ \left. \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} - n, \frac{1}{2} + n, \frac{3}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] - \left( (1 + 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} - n, \frac{3}{2} + n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, \right. \right. \right. \right. \\ \left. \left. \left. -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + (-1 + 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2} - n, \frac{1}{2} + n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c + dx)\right]^2 \right) + \right. \\ \left. \left( 2 A \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2} - n, \frac{1}{2} + n, \frac{3}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) / \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2} - n, \frac{1}{2} + n, \frac{3}{2}, \right. \right. \right. \\ \left. \left. \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] - \left( (1 + 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2} - n, \frac{3}{2} + n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) + \right. \\ \left. \left. (-3 + 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2} - n, \frac{1}{2} + n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c + dx)\right]^2 \right) \right) \right) / \right)$$









$$\begin{aligned}
& \frac{1}{3} \left( \frac{3}{2} - n \right) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{2} - n, \frac{1}{2} + n, \frac{5}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right] \Bigg) - \\
& \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \left( (1 + 2n) \left( -\frac{3}{5} \left( \frac{3}{2} + n \right) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{3}{2} - n, \frac{5}{2} + n, \frac{7}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \right. \right. \\
& \quad \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right] + \frac{3}{5} \left( \frac{3}{2} - n \right) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{5}{2} - n, \frac{3}{2} + n, \frac{7}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \\
& \quad \left. \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right] \right) + (-3 + 2n) \left( -\frac{3}{5} \left( \frac{1}{2} + n \right) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{5}{2} - n, \frac{3}{2} + n, \frac{7}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \right. \\
& \quad \operatorname{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right] + \frac{3}{5} \left( \frac{5}{2} - n \right) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{7}{2} - n, \frac{1}{2} + n, \frac{7}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \\
& \quad \left. \operatorname{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right] \right) \Bigg) \Bigg) / \left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{3}{2} - n, \frac{1}{2} + n, \frac{3}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] - \left( (1 + 2n) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2} - n, \frac{3}{2} + n, \frac{5}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (-3 + 2n) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{2} - n, \frac{1}{2} + n, \frac{5}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 924: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(b \cos [c + dx])^n (A + B \cos [c + dx])}{\cos [c + dx]^{5/2}} dx$$

Optimal (type 5, 163 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 A (b \cos [c + dx])^n \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{4} (-3 + 2n), \frac{1}{4} (1 + 2n), \cos [c + dx]^2 \right] \sin [c + dx]}{d (3 - 2n) \cos [c + dx]^{3/2} \sqrt{\sin [c + dx]^2}} + \\
& \frac{2 B (b \cos [c + dx])^n \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{4} (-1 + 2n), \frac{1}{4} (3 + 2n), \cos [c + dx]^2 \right] \sin [c + dx]}{d (1 - 2n) \sqrt{\cos [c + dx]} \sqrt{\sin [c + dx]^2}}
\end{aligned}$$

Result (type 6, 4948 leaves):

$$\begin{aligned}
& \left( 2 \cos [c + dx]^{-n} (b \cos [c + dx])^n \right. \\
& \quad \left( A \cos [c + dx]^{-\frac{1}{2}+n} + \operatorname{Sec} [c + dx]^2 \left( \frac{1}{2} B \cos [c + dx]^{\frac{1}{2}+n} + \frac{1}{2} B \cos [c + dx]^{\frac{1}{2}+n} \cos [2 (c + dx)] + \frac{1}{2} i B \cos [c + dx]^{\frac{1}{2}+n} \sin [2 (c + dx)] \right) \right. \\
& \quad \left. \left. \operatorname{Sec} [c + dx]^3 \left( -\frac{1}{2} i B \cos [c + dx]^{\frac{1}{2}+n} \cos [2 (c + dx)] \sin [c + dx] + A \cos [c + dx]^{\frac{1}{2}+n} \sin [c + dx]^2 + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \sin[c+dx] \left( -\frac{1}{2} i B \cos[c+dx]^{\frac{1}{2}+n} + \frac{1}{2} B \cos[c+dx]^{\frac{1}{2}+n} \sin[2(c+dx)] \right) \tan\left[\frac{1}{2}(c+dx)\right] \left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{-\frac{5}{2}+n} \\
& \left( \frac{1}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{-\frac{1}{2}+n} \left( \left( 9(A+B) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) / \right. \\
& \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \left( (1-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (5-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
& \left( 5(-A+B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) / \\
& \left( -5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \left( (-1+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}-n, \frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (-5+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \left. \right) / \\
& \left( 3d \left( -\frac{2}{3} \left( -\frac{5}{2}+n \right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^2 \left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{-\frac{7}{2}+n} \left( \frac{1}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{-\frac{1}{2}+n} \right. \right. \\
& \left( \left( 9(A+B) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) / \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{3}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \left( (1-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) + \right. \right. \\
& \left. \left. (5-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
& \left( 5(-A+B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) / \\
& \left( -5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \left( (-1+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}-n, \frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (-5+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \right. \right. \\
& \left. \left. \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \left. \right) + \frac{1}{3} \sec\left[\frac{1}{2}(c+dx)\right]^2 \left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{-\frac{5}{2}+n}
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{1}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2} \right)^{-\frac{1}{2}+n} \left( \left( 9(A+B) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) / \right. \\
& \quad \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \\
& \quad \left. \left( (1-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. (5-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c + dx)\right]^2 \right) + \\
& \quad \left. \left( 5(-A+B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \tan\left[\frac{1}{2}(c + dx)\right]^2 \right) / \right. \\
& \quad \left. \left( -5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. \left( (-1+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}-n, \frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \right. \right. \\
& \quad \left. \left. \left. (-5+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c + dx)\right]^2 \right) \right) - \\
& \quad \frac{2}{3} \left( -\frac{1}{2}+n \right) \sec\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right]^2 \left( 1 - \tan\left[\frac{1}{2}(c + dx)\right]^2 \right)^{-\frac{5}{2}+n} \left( \frac{1}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2} \right)^{\frac{1}{2}+n} \\
& \quad \left( \left( 9(A+B) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) / \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{3}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \left( (1-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. (5-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c + dx)\right]^2 \right) + \\
& \quad \left( 5(-A+B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \tan\left[\frac{1}{2}(c + dx)\right]^2 \right) / \\
& \quad \left( -5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \\
& \quad \left. \left( (-1+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}-n, \frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. \left. (-5+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c + dx)\right]^2 \right) \right) +
\end{aligned}$$





$$\frac{\frac{5}{7} \left( \frac{7}{2} - n \right) \text{AppellF1} \left[ \frac{7}{2}, \frac{9}{2} - n, -\frac{1}{2} + n, \frac{9}{2}, \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \text{Tan} \left[ \frac{1}{2} (c + d x) \right] \right)}{\left( -5 \text{AppellF1} \left[ \frac{3}{2}, \frac{5}{2} - n, -\frac{1}{2} + n, \frac{5}{2}, \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right] + \left( (-1 + 2 n) \text{AppellF1} \left[ \frac{5}{2}, \frac{5}{2} - n, \frac{1}{2} + n, \frac{7}{2}, \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right] + (-5 + 2 n) \text{AppellF1} \left[ \frac{5}{2}, \frac{7}{2} - n, -\frac{1}{2} + n, \frac{7}{2}, \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right] \right) \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right)}{\left. \right)} \right)$$

■ **Problem 925: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(b \cos [c + d x])^n (A + B \cos [c + d x])}{\cos [c + d x]^{7/2}} dx$$

Optimal (type 5, 163 leaves, 4 steps):

$$\frac{2 A (b \cos [c + d x])^n \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{4} (-5 + 2 n), \frac{1}{4} (-1 + 2 n), \cos [c + d x]^2 \right] \sin [c + d x]}{d (5 - 2 n) \cos [c + d x]^{5/2} \sqrt{\sin [c + d x]^2}} +$$

$$\frac{2 B (b \cos [c + d x])^n \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{4} (-3 + 2 n), \frac{1}{4} (1 + 2 n), \cos [c + d x]^2 \right] \sin [c + d x]}{d (3 - 2 n) \cos [c + d x]^{3/2} \sqrt{\sin [c + d x]^2}}$$

Result (type 6, 4948 leaves):

$$\left( 2 \cos [c + d x]^{-n} (b \cos [c + d x])^n \right.$$

$$\left. \left( A \cos [c + d x]^{-\frac{3}{2}+n} + \text{Sec} [c + d x]^3 \left( \frac{1}{2} B \cos [c + d x]^{\frac{1}{2}+n} + \frac{1}{2} B \cos [c + d x]^{\frac{1}{2}+n} \cos [2 (c + d x)] + \frac{1}{2} i B \cos [c + d x]^{\frac{1}{2}+n} \sin [2 (c + d x)] \right) \right) \right.$$

$$\left. \text{Sec} [c + d x]^4 \left( -\frac{1}{2} i B \cos [c + d x]^{\frac{1}{2}+n} \cos [2 (c + d x)] \sin [c + d x] + A \cos [c + d x]^{\frac{1}{2}+n} \sin [c + d x]^2 + \right. \right.$$

$$\left. \left. \sin [c + d x] \left( -\frac{1}{2} i B \cos [c + d x]^{\frac{1}{2}+n} + \frac{1}{2} B \cos [c + d x]^{\frac{1}{2}+n} \sin [2 (c + d x)] \right) \right) \right) \text{Tan} \left[ \frac{1}{2} (c + d x) \right] \left( 1 - \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right)^{-\frac{7}{2}+n}$$

$$\left( \frac{1}{1 + \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2} \right)^{-\frac{3}{2}+n} \left( \left( 9 (A + B) \text{AppellF1} \left[ \frac{1}{2}, \frac{7}{2} - n, -\frac{3}{2} + n, \frac{3}{2}, \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right] \right) \right) /$$

$$\left( 3 \text{AppellF1} \left[ \frac{1}{2}, \frac{7}{2} - n, -\frac{3}{2} + n, \frac{3}{2}, \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right] + \left( (3 - 2 n) \text{AppellF1} \left[ \frac{3}{2}, \frac{7}{2} - n, -\frac{1}{2} + n, \frac{5}{2}, \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2, \right. \right. \right.$$



$$\begin{aligned}
& -\tan\left[\frac{1}{2}(c+dx)\right]^2 + (7-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}-n, -\frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \Big) + \\
& \left( 5(-A+B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) / \\
& \left( -5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \left. \left( (-3+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \left. \left. (-7+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2}-n, -\frac{3}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \Big) / \\
& \left( 3d \left( -\frac{2}{3} \left( -\frac{7}{2}+n \right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^2 \left( 1 - \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^{-\frac{9}{2}+n} \left( \frac{1}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{-\frac{3}{2}+n} \right. \right. \\
& \left. \left( \left( 9(A+B) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}-n, -\frac{3}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) / \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}-n, -\frac{3}{2}+n, \frac{3}{2}, \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \left( (3-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \left. \left. (7-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}-n, -\frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
& \left( 5(-A+B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) / \\
& \left( -5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \left. \left( (-3+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (-7+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2}-n, -\frac{3}{2}+n, \right. \right. \right. \\
& \left. \left. \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \frac{1}{3} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left( 1 - \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^{-\frac{7}{2}+n} \right. \\
& \left. \left( \frac{1}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{\frac{3}{2}+n} \right) \left( \left( 9(A+B) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}-n, -\frac{3}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) / \right. \\
& \left. \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}-n, -\frac{3}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \left. \left. \left( (3-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (7-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}-n, -\frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \Bigg) + \\
& \left(5(-A+B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \Bigg) / \right. \\
& \left(-5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \left. \left((-3+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \left. \left. (-7+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2}-n, -\frac{3}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \Bigg) - \right. \\
& \left. \frac{2}{3} \left(-\frac{3}{2}+n\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^2 \left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{-\frac{7}{2}+n} \left(\frac{1}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}\right)^{-\frac{1}{2}+n} \right. \\
& \left. \left( \left(9(A+B) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}-n, -\frac{3}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) / \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}-n, -\frac{3}{2}+n, \frac{3}{2}, \right. \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \left(3-2n\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) + \right. \\
& \left. \left. (7-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}-n, -\frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \right. \\
& \left(5(-A+B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \Bigg) / \right. \\
& \left(-5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \left. \left((-3+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \left. \left. (-7+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2}-n, -\frac{3}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \Bigg) + \right. \\
& \left. \frac{2}{3} \tan\left[\frac{1}{2}(c+dx)\right] \left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{-\frac{7}{2}+n} \left(\frac{1}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}\right)^{-\frac{3}{2}+n} \right. \\
& \left. \left( \left(9(A+B) \left(-\frac{1}{3} \left(-\frac{3}{2}+n\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \right. \right. \right. \\
& \left. \left. \frac{1}{3} \left(\frac{7}{2}-n\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}-n, -\frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \Bigg) / \right.
\end{aligned}$$



$$\begin{aligned}
& \left( \tan\left[\frac{1}{2}(c+dx)\right] \right)^2 \left( (7-2n) \left( -\frac{3}{5} \left( -\frac{3}{2} + n \right) \text{AppellF1}\left[\frac{5}{2}, \frac{9}{2} - n, -\frac{1}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \right. \\
& \left. \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{3}{5} \left( \frac{9}{2} - n \right) \text{AppellF1}\left[\frac{5}{2}, \frac{11}{2} - n, -\frac{3}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \right. \\
& \left. \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) \Big/ \left( 3 \text{AppellF1}\left[\frac{1}{2}, \frac{7}{2} - n, -\frac{3}{2} + n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \left. \left( (3-2n) \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2} - n, -\frac{1}{2} + n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (7-2n) \text{AppellF1}\left[\frac{3}{2}, \right. \right. \right. \\
& \quad \left. \left. \frac{9}{2} - n, -\frac{3}{2} + n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 - \\
& \left( 5(-A+B) \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2} - n, -\frac{3}{2} + n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \left. \left( \left( (-3+2n) \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2} - n, -\frac{1}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \right. \\
& \quad \left. \left. (-7+2n) \text{AppellF1}\left[\frac{5}{2}, \frac{9}{2} - n, -\frac{3}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - \right. \\
& \quad 5 \left( -\frac{3}{5} \left( -\frac{3}{2} + n \right) \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2} - n, -\frac{1}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \quad \left. \frac{3}{5} \left( \frac{7}{2} - n \right) \text{AppellF1}\left[\frac{5}{2}, \frac{9}{2} - n, -\frac{3}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) + \\
& \quad \tan\left[\frac{1}{2}(c+dx)\right]^2 \left( (-3+2n) \left( -\frac{5}{7} \left( -\frac{1}{2} + n \right) \text{AppellF1}\left[\frac{7}{2}, \frac{7}{2} - n, \frac{1}{2} + n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{5}{7} \left( \frac{7}{2} - n \right) \text{AppellF1}\left[\frac{7}{2}, \frac{9}{2} - n, -\frac{1}{2} + n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \right. \\
& \quad \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) + (-7+2n) \left( -\frac{5}{7} \left( -\frac{3}{2} + n \right) \text{AppellF1}\left[\frac{7}{2}, \frac{9}{2} - n, -\frac{1}{2} + n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{5}{7} \left( \frac{9}{2} - n \right) \text{AppellF1}\left[\frac{7}{2}, \frac{11}{2} - n, -\frac{3}{2} + n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \Big/ \left( -5 \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2} - n, -\frac{3}{2} + n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \left( (-3+2n) \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2} - n, -\frac{1}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. (-7+2n) \text{AppellF1}\left[\frac{5}{2}, \frac{9}{2} - n, -\frac{3}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \Big) \Big) \Big)
\end{aligned}$$

■ Problem 926: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(b \cos [c + d x])^n (A + B \cos [c + d x])}{\cos [c + d x]^{9/2}} dx$$

Optimal (type 5, 163 leaves, 4 steps):

$$\frac{2 A (b \cos [c + d x])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(-7 + 2 n), \frac{1}{4}(-3 + 2 n), \cos [c + d x]^2\right] \sin [c + d x]}{d (7 - 2 n) \cos [c + d x]^{7/2} \sqrt{\sin [c + d x]^2}} +$$

$$\frac{2 B (b \cos [c + d x])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(-5 + 2 n), \frac{1}{4}(-1 + 2 n), \cos [c + d x]^2\right] \sin [c + d x]}{d (5 - 2 n) \cos [c + d x]^{5/2} \sqrt{\sin [c + d x]^2}}$$

Result (type 6, 4948 leaves):

$$\left( 2 \cos [c + d x]^{-n} (b \cos [c + d x])^n \right.$$

$$\left. \left( A \cos [c + d x]^{-\frac{5}{2}+n} + \sec [c + d x]^4 \left( \frac{1}{2} B \cos [c + d x]^{\frac{1}{2}+n} + \frac{1}{2} B \cos [c + d x]^{\frac{1}{2}+n} \cos [2 (c + d x)] + \frac{1}{2} i B \cos [c + d x]^{\frac{1}{2}+n} \sin [2 (c + d x)] \right) \right) + \right.$$

$$\left. \sec [c + d x]^5 \left( -\frac{1}{2} i B \cos [c + d x]^{\frac{1}{2}+n} \cos [2 (c + d x)] \sin [c + d x] + A \cos [c + d x]^{\frac{1}{2}+n} \sin [c + d x]^2 + \right. \right.$$

$$\left. \left. \sin [c + d x] \left( -\frac{1}{2} i B \cos [c + d x]^{\frac{1}{2}+n} + \frac{1}{2} B \cos [c + d x]^{\frac{1}{2}+n} \sin [2 (c + d x)] \right) \right) \right) \tan \left[ \frac{1}{2} (c + d x) \right] \left( 1 - \tan \left[ \frac{1}{2} (c + d x) \right]^2 \right)^{-\frac{9}{2}+n}$$

$$\left( \frac{1}{1 + \tan \left[ \frac{1}{2} (c + d x) \right]^2} \right)^{-\frac{5}{2}+n} \left( \left( 9 (A + B) \operatorname{AppellF1}\left[ \frac{1}{2}, \frac{9}{2} - n, -\frac{5}{2} + n, \frac{3}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] \right) / \right.$$

$$\left( 3 \operatorname{AppellF1}\left[ \frac{1}{2}, \frac{9}{2} - n, -\frac{5}{2} + n, \frac{3}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] + \left( (5 - 2 n) \operatorname{AppellF1}\left[ \frac{3}{2}, \frac{9}{2} - n, -\frac{3}{2} + n, \frac{5}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, \right. \right. \right.$$

$$\left. \left. -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] + (9 - 2 n) \operatorname{AppellF1}\left[ \frac{3}{2}, \frac{11}{2} - n, -\frac{5}{2} + n, \frac{5}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (c + d x) \right]^2 \right) +$$

$$\left( 5 (-A + B) \operatorname{AppellF1}\left[ \frac{3}{2}, \frac{9}{2} - n, -\frac{5}{2} + n, \frac{5}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] \tan \left[ \frac{1}{2} (c + d x) \right]^2 \right) /$$

$$\left( -5 \operatorname{AppellF1}\left[ \frac{3}{2}, \frac{9}{2} - n, -\frac{5}{2} + n, \frac{5}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] + \right.$$

$$\left. \left( (-5 + 2 n) \operatorname{AppellF1}\left[ \frac{5}{2}, \frac{9}{2} - n, -\frac{3}{2} + n, \frac{7}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] + \right.$$



$$\begin{aligned}
& (-9 + 2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{11}{2} - n, -\frac{5}{2} + n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \Big) - \\
& \frac{2}{3} \left(-\frac{5}{2} + n\right) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \left(1 - \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right)^{-\frac{9}{2}+n} \left(\frac{1}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2}\right)^{\frac{3}{2}+n} \\
& \left( \left( 9(A + B) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{9}{2} - n, -\frac{5}{2} + n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) / \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{9}{2} - n, -\frac{5}{2} + n, \frac{3}{2}, \right. \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) + \left( (5 - 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2} - n, -\frac{3}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \right) + \\
& \quad \left( (9 - 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{11}{2} - n, -\frac{5}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \Big) + \\
& \left( 5(-A + B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2} - n, -\frac{5}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \right) / \\
& \left( -5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2} - n, -\frac{5}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) + \\
& \left( (-5 + 2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2} - n, -\frac{3}{2} + n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) + \\
& \quad \left( (-9 + 2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{11}{2} - n, -\frac{5}{2} + n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \Big) \Big) + \\
& \frac{2}{3} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \left(1 - \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right)^{-\frac{9}{2}+n} \left(\frac{1}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2}\right)^{-\frac{5}{2}+n} \\
& \left( \left( 9(A + B) \left(-\frac{1}{3} \left(-\frac{5}{2} + n\right)\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2} - n, -\frac{3}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \right) + \right. \\
& \quad \left. \frac{1}{3} \left(\frac{9}{2} - n\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{11}{2} - n, -\frac{5}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \right) \Big) / \\
& \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{9}{2} - n, -\frac{5}{2} + n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) + \\
& \left( (5 - 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2} - n, -\frac{3}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) + \\
& \quad \left( (9 - 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{11}{2} - n, -\frac{5}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \Big) + \\
& \left( 5(-A + B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2} - n, -\frac{5}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \right) \Big) /
\end{aligned}$$





$$\begin{aligned}
& (9 - 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{11}{2} - n, -\frac{5}{2} + n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \tan\left[\frac{1}{2}(c + dx)\right]^2 - \\
& \left(5(-A + B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2} - n, -\frac{5}{2} + n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \tan\left[\frac{1}{2}(c + dx)\right]^2\right. \\
& \left.\left(\left((-5 + 2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2} - n, -\frac{3}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right.\right.\right. \\
& \quad \left.(-9 + 2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{11}{2} - n, -\frac{5}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right] - \right. \\
& \quad \left.5\left(-\frac{3}{5}\left(-\frac{5}{2} + n\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2} - n, -\frac{3}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right] + \right. \right. \\
& \quad \left. \left.\frac{3}{5}\left(\frac{9}{2} - n\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{11}{2} - n, -\frac{5}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right]\right)\right) + \\
& \tan\left[\frac{1}{2}(c + dx)\right]^2 \left(\left(-5 + 2n\right)\left(-\frac{5}{7}\left(-\frac{3}{2} + n\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{9}{2} - n, -\frac{1}{2} + n, \frac{9}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right. \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right] + \frac{5}{7}\left(\frac{9}{2} - n\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{11}{2} - n, -\frac{3}{2} + n, \frac{9}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right]\right) + (-9 + 2n)\left(-\frac{5}{7}\left(-\frac{5}{2} + n\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{11}{2} - n, -\frac{3}{2} + n, \right. \right. \\
& \quad \left. \left.\frac{9}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right] + \frac{5}{7}\left(\frac{11}{2} - n\right) \right. \\
& \quad \left. \operatorname{AppellF1}\left[\frac{7}{2}, \frac{13}{2} - n, -\frac{5}{2} + n, \frac{9}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right]\right)\right) \Big/ \\
& \left(-5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2} - n, -\frac{5}{2} + n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \left((-5 + 2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2} - n, \right. \right. \right. \\
& \quad \left. \left. -\frac{3}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \\
& \quad \left. \left. (-9 + 2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{11}{2} - n, -\frac{5}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \tan\left[\frac{1}{2}(c + dx)\right]^2\right)\right) \Big) \Big)
\end{aligned}$$

■ **Problem 930: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^m (A + B \cos[c + dx])}{(b \cos[c + dx])^{1/3}} dx$$

Optimal (type 5, 167 leaves, 4 steps):

$$\frac{3 A \operatorname{Cos}[c+d x]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6}(2+3 m), \frac{1}{6}(8+3 m), \operatorname{Cos}[c+d x]^2\right] \operatorname{Sin}[c+d x]}{d(2+3 m)(b \operatorname{Cos}[c+d x])^{1/3} \sqrt{\operatorname{Sin}[c+d x]^2}}$$

$$\frac{3 B \operatorname{Cos}[c+d x]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6}(5+3 m), \frac{1}{6}(11+3 m), \operatorname{Cos}[c+d x]^2\right] \operatorname{Sin}[c+d x]}{d(5+3 m)(b \operatorname{Cos}[c+d x])^{1/3} \sqrt{\operatorname{Sin}[c+d x]^2}}$$

$$d(5+3 m)(b \operatorname{Cos}[c+d x])^{1/3} \sqrt{\operatorname{Sin}[c+d x]^2}$$

Result (type 6, 4959 leaves):

$$\begin{aligned} & \left( 2 \left( \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2 \right)^{\frac{5}{3}+m} \operatorname{Cos}[c+d x]^{1/3} \left( \operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \right)^{-\frac{1}{3}+m} \right. \\ & \left. \left( \frac{1}{2} B \operatorname{Cos}[c+d x]^{\frac{2}{3}+m} + A \operatorname{Cos}[c+d x]^{\frac{5}{3}+m} + \frac{1}{2} B \operatorname{Cos}[c+d x]^{\frac{2}{3}+m} \operatorname{Cos}[2(c+d x)] + \frac{1}{2} i B \operatorname{Cos}[c+d x]^{\frac{2}{3}+m} \operatorname{Sin}[2(c+d x)] + \right. \right. \\ & \left. \left. \operatorname{Sec}[c+d x] \left( -\frac{1}{2} i B \operatorname{Cos}[c+d x]^{\frac{2}{3}+m} \operatorname{Cos}[2(c+d x)] \operatorname{Sin}[c+d x] + A \operatorname{Cos}[c+d x]^{\frac{2}{3}+m} \operatorname{Sin}[c+d x]^2 + \right. \right. \right. \\ & \left. \left. \left. \operatorname{Sin}[c+d x] \left( -\frac{1}{2} i B \operatorname{Cos}[c+d x]^{\frac{2}{3}+m} + \frac{1}{2} B \operatorname{Cos}[c+d x]^{\frac{2}{3}+m} \operatorname{Sin}[2(c+d x)] \right) \right) \right) \right) \\ & \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \left( \left( 9(A+B) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \right) / \right. \\ & \left( 9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] + 2 \left( -(5+3 m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \right. \\ & \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] + (1-3 m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right) + \\ & \left( 5(-A+B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right) / \\ & \left( -15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] + 2 \left( (5+3 m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \right. \\ & \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] + (-1+3 m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{4}{3}-m, \frac{5}{3}+m, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right) \right) / \\ & \left( d(b \operatorname{Cos}[c+d x])^{1/3} \left( \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2 \right)^{\frac{2}{3}+m} \left( \operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \right)^{-\frac{1}{3}+m} \right. \\ & \left. \left( \left( 9(A+B) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \right) / \left( 9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \right. \right. \\ & \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] + 2 \left( -(5+3 m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \right) \right) \right) \right) \end{aligned}$$

$$\begin{aligned}
& (1 - 3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3} - m, \frac{5}{3} + m, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \tan\left[\frac{1}{2}(c + dx)\right]^2 + \\
& \left(5(-A + B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3} - m, \frac{5}{3} + m, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \tan\left[\frac{1}{2}(c + dx)\right]^2\right) / \\
& \left(-15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3} - m, \frac{5}{3} + m, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \\
& \left. 2\left((5 + 3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{3} - m, \frac{8}{3} + m, \frac{7}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \right. \\
& \left. \left. (-1 + 3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{4}{3} - m, \frac{5}{3} + m, \frac{7}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c + dx)\right]^2\right) - \\
& 2\left(\frac{5}{3} + m\right) \left(\cos\left[\frac{1}{2}(c + dx)\right]^2\right)^{\frac{2}{3} + m} \left(\cos[c + dx] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2\right)^{-\frac{1}{3} + m} \sin\left[\frac{1}{2}(c + dx)\right]^2 \\
& \left(\left(9(A + B) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3} - m, \frac{5}{3} + m, \frac{3}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(c + dx)\right]^2\right) + 2\left(- (5 + 3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3} - m, \frac{8}{3} + m, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \right. \\
& \left. \left. (1 - 3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3} - m, \frac{5}{3} + m, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c + dx)\right]^2\right) + \\
& \left(5(-A + B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3} - m, \frac{5}{3} + m, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \tan\left[\frac{1}{2}(c + dx)\right]^2\right) / \\
& \left(-15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3} - m, \frac{5}{3} + m, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \\
& \left. 2\left((5 + 3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{3} - m, \frac{8}{3} + m, \frac{7}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \right. \\
& \left. \left. (-1 + 3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{4}{3} - m, \frac{5}{3} + m, \frac{7}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c + dx)\right]^2\right) + \\
& 2\left(-\frac{1}{3} + m\right) \left(\cos\left[\frac{1}{2}(c + dx)\right]^2\right)^{\frac{5}{3} + m} \left(\cos[c + dx] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2\right)^{-\frac{4}{3} + m} \tan\left[\frac{1}{2}(c + dx)\right] \\
& \left(-\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \sin[c + dx] + \cos[c + dx] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right]\right) \\
& \left(\left(9(A + B) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3} - m, \frac{5}{3} + m, \frac{3}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \right. \\
& \left. \left. 9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3} - m, \frac{5}{3} + m, \frac{3}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \right.
\end{aligned}$$





$$\begin{aligned}
& \frac{3}{5} \left( \frac{1}{3} - m \right) \text{AppellF1} \left[ \frac{5}{2}, \frac{4}{3} - m, \frac{5}{3} + m, \frac{7}{2}, \tan \left[ \frac{1}{2} (c + dx) \right]^2, -\tan \left[ \frac{1}{2} (c + dx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \tan \left[ \frac{1}{2} (c + dx) \right] \Bigg) + \\
& 2 \tan \left[ \frac{1}{2} (c + dx) \right]^2 \left( (5 + 3m) \left( -\frac{5}{7} \left( \frac{8}{3} + m \right) \text{AppellF1} \left[ \frac{7}{2}, \frac{1}{3} - m, \frac{11}{3} + m, \frac{9}{2}, \tan \left[ \frac{1}{2} (c + dx) \right]^2, -\tan \left[ \frac{1}{2} (c + dx) \right]^2 \right] \right. \right. \\
& \quad \text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \tan \left[ \frac{1}{2} (c + dx) \right] + \frac{5}{7} \left( \frac{1}{3} - m \right) \text{AppellF1} \left[ \frac{7}{2}, \frac{4}{3} - m, \frac{8}{3} + m, \frac{9}{2}, \right. \\
& \quad \left. \left. \tan \left[ \frac{1}{2} (c + dx) \right]^2, -\tan \left[ \frac{1}{2} (c + dx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \tan \left[ \frac{1}{2} (c + dx) \right] \right) + (-1 + 3m) \\
& \quad \left( -\frac{5}{7} \left( \frac{5}{3} + m \right) \text{AppellF1} \left[ \frac{7}{2}, \frac{4}{3} - m, \frac{8}{3} + m, \frac{9}{2}, \tan \left[ \frac{1}{2} (c + dx) \right]^2, -\tan \left[ \frac{1}{2} (c + dx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \tan \left[ \frac{1}{2} (c + dx) \right] + \right. \\
& \quad \left. \frac{5}{7} \left( \frac{4}{3} - m \right) \text{AppellF1} \left[ \frac{7}{2}, \frac{7}{3} - m, \frac{5}{3} + m, \frac{9}{2}, \tan \left[ \frac{1}{2} (c + dx) \right]^2, -\tan \left[ \frac{1}{2} (c + dx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \tan \left[ \frac{1}{2} (c + dx) \right] \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( -15 \text{AppellF1} \left[ \frac{3}{2}, \frac{1}{3} - m, \frac{5}{3} + m, \frac{5}{2}, \tan \left[ \frac{1}{2} (c + dx) \right]^2, -\tan \left[ \frac{1}{2} (c + dx) \right]^2 \right] + 2 \left( (5 + 3m) \text{AppellF1} \left[ \frac{5}{2}, \frac{1}{3} - m, \right. \right. \right. \\
& \quad \left. \left. \frac{8}{3} + m, \frac{7}{2}, \tan \left[ \frac{1}{2} (c + dx) \right]^2, -\tan \left[ \frac{1}{2} (c + dx) \right]^2 \right] + \right. \\
& \quad \left. \left. (-1 + 3m) \text{AppellF1} \left[ \frac{5}{2}, \frac{4}{3} - m, \frac{5}{3} + m, \frac{7}{2}, \tan \left[ \frac{1}{2} (c + dx) \right]^2, -\tan \left[ \frac{1}{2} (c + dx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (c + dx) \right]^2 \right) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 931: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + dx]^m (A + B \cos [c + dx])}{(b \cos [c + dx])^{2/3}} dx$$

Optimal (type 5, 167 leaves, 4 steps):

$$\frac{3 A \cos [c + dx]^{1+m} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{6} (1 + 3m), \frac{1}{6} (7 + 3m), \cos [c + dx]^2 \right] \sin [c + dx]}{d (1 + 3m) (b \cos [c + dx])^{2/3} \sqrt{\sin [c + dx]^2}} + \frac{3 B \cos [c + dx]^{2+m} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{6} (4 + 3m), \frac{1}{6} (10 + 3m), \cos [c + dx]^2 \right] \sin [c + dx]}{d (4 + 3m) (b \cos [c + dx])^{2/3} \sqrt{\sin [c + dx]^2}}$$

Result (type 6, 4951 leaves):

$$\left( 2 \left( \cos \left[ \frac{1}{2} (c + dx) \right]^2 \right)^{\frac{4}{3}+m} \cos [c + dx]^{2/3} \left( \cos [c + dx] \text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \right)^{-\frac{2}{3}+m} \right. \\
\left. \left( \frac{1}{2} B \cos [c + dx]^{\frac{1}{3}+m} + A \cos [c + dx]^{\frac{4}{3}+m} + \frac{1}{2} B \cos [c + dx]^{\frac{1}{3}+m} \cos [2 (c + dx)] + \frac{1}{2} B \cos [c + dx]^{\frac{1}{3}+m} \sin [2 (c + dx)] + \right. \right.$$



$$\begin{aligned}
& \left. \begin{aligned}
& \left( \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2 \right) - 2 \left( (4+3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}-m, \frac{7}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \left. (-2+3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{3}-m, \frac{4}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
& \left( 5(A-B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}-m, \frac{4}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) / \\
& \left( 15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}-m, \frac{4}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \\
& \left. 2 \left( (4+3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2}{3}-m, \frac{7}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \left. \left. (-2+3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{3}-m, \frac{4}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) + \\
& 2 \left( -\frac{2}{3}+m \right) \left( \cos\left[\frac{1}{2}(c+dx)\right]^2 \right)^{\frac{4}{3}+m} \left( \cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right)^{-\frac{5}{3}+m} \tan\left[\frac{1}{2}(c+dx)\right] \\
& \left( -\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] + \cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \\
& \left( \left( 9(A+B) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}-m, \frac{4}{3}+m, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) / \right. \\
& \left( 9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}-m, \frac{4}{3}+m, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \\
& \left. 2 \left( (4+3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}-m, \frac{7}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \left. \left. (-2+3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{3}-m, \frac{4}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
& \left( 5(A-B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}-m, \frac{4}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) / \\
& \left( 15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}-m, \frac{4}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \\
& \left. 2 \left( (4+3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2}{3}-m, \frac{7}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \left. \left. (-2+3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{3}-m, \frac{4}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) + \\
& 2 \left( \cos\left[\frac{1}{2}(c+dx)\right]^2 \right)^{\frac{4}{3}+m} \left( \cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right)^{-\frac{2}{3}+m} \tan\left[\frac{1}{2}(c+dx)\right]
\end{aligned}
\right.
\end{aligned}$$







$$\frac{7}{3} + m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] +$$

$$\left(-2+3m\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{3}-m, \frac{4}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2\right)\right)$$

■ **Problem 932: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^m (A+B\cos[c+dx])}{(b\cos[c+dx])^{4/3}} dx$$

Optimal (type 5, 171 leaves, 4 steps):

$$\frac{3A\cos[c+dx]^m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6}(-1+3m), \frac{1}{6}(5+3m), \cos[c+dx]^2\right] \sin[c+dx]}{bd(1-3m)(b\cos[c+dx])^{1/3}\sqrt{\sin[c+dx]^2}}$$

$$\frac{3B\cos[c+dx]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6}(2+3m), \frac{1}{6}(8+3m), \cos[c+dx]^2\right] \sin[c+dx]}{bd(2+3m)(b\cos[c+dx])^{1/3}\sqrt{\sin[c+dx]^2}}$$

Result (type 6, 4853 leaves):

$$\left(18\cos[c+dx]^{2+m}\right.$$

$$\left.\left(A\cos[c+dx]^{\frac{2}{3}+m} + \sec[c+dx] \left(\frac{1}{2}B\cos[c+dx]^{\frac{2}{3}+m} + \frac{1}{2}B\cos[c+dx]^{\frac{2}{3}+m}\cos[2(c+dx)] + \frac{1}{2}iB\cos[c+dx]^{\frac{2}{3}+m}\sin[2(c+dx)]\right) + \right.$$

$$\left.\sec[c+dx]^2 \left(-\frac{1}{2}iB\cos[c+dx]^{\frac{2}{3}+m}\cos[2(c+dx)]\sin[c+dx] + A\cos[c+dx]^{\frac{2}{3}+m}\sin[c+dx]^2 + \right.\right.$$

$$\left.\left.\sin[c+dx] \left(-\frac{1}{2}iB\cos[c+dx]^{\frac{2}{3}+m} + \frac{1}{2}B\cos[c+dx]^{\frac{2}{3}+m}\sin[2(c+dx)]\right)\right)\right) \tan\left[\frac{1}{2}(c+dx)\right]$$

$$\left.\left.\left(\left((A-B)\operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}-m, \frac{2}{3}+m, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right) / \right.$$

$$\left.\left(9\operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}-m, \frac{2}{3}+m, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + 2\left(-2+3m\right)\operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right.\right.\right.$$

$$\left.\left.-\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (1-3m)\operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}-m, \frac{2}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right) +$$

$$\left.\left(2A\operatorname{AppellF1}\left[\frac{1}{2}, \frac{4}{3}-m, \frac{2}{3}+m, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) / \left(9\operatorname{AppellF1}\left[\frac{1}{2}, \frac{4}{3}-m, \frac{2}{3}+m, \frac{3}{2}, \right.\right.\right.$$

$$\left.\left.\tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - 2\left((2+3m)\operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right.\right.$$

$$\left.\left.(-4+3m)\operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{3}-m, \frac{2}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right) /$$







$$\begin{aligned}
& \frac{1}{3} \left( \frac{4}{3} - m \right) \text{AppellF1} \left[ \frac{3}{2}, \frac{7}{3} - m, \frac{2}{3} + m, \frac{5}{2}, \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \text{Tan} \left[ \frac{1}{2} (c + dx) \right] \Big) - \\
& 2 \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \left( (2 + 3m) \left( -\frac{3}{5} \left( \frac{5}{3} + m \right) \text{AppellF1} \left[ \frac{5}{2}, \frac{4}{3} - m, \frac{8}{3} + m, \frac{7}{2}, \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \right. \right. \\
& \quad \text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \text{Tan} \left[ \frac{1}{2} (c + dx) \right] + \frac{3}{5} \left( \frac{4}{3} - m \right) \text{AppellF1} \left[ \frac{5}{2}, \frac{7}{3} - m, \frac{5}{3} + m, \frac{7}{2}, \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \\
& \quad \left. \left. \text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \text{Tan} \left[ \frac{1}{2} (c + dx) \right] \right) + (-4 + 3m) \left( -\frac{3}{5} \left( \frac{2}{3} + m \right) \text{AppellF1} \left[ \frac{5}{2}, \frac{7}{3} - m, \frac{5}{3} + m, \frac{7}{2}, \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \text{Tan} \left[ \frac{1}{2} (c + dx) \right] + \frac{3}{5} \left( \frac{7}{3} - m \right) \text{AppellF1} \left[ \frac{5}{2}, \frac{10}{3} - m, \frac{2}{3} + m, \frac{7}{2}, \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, \right. \right. \\
& \quad \left. \left. -\text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \text{Tan} \left[ \frac{1}{2} (c + dx) \right] \right) \Big) \Big) / \left( 9 \text{AppellF1} \left[ \frac{1}{2}, \frac{4}{3} - m, \frac{2}{3} + m, \frac{3}{2}, \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] - 2 \left( (2 + 3m) \text{AppellF1} \left[ \frac{3}{2}, \frac{4}{3} - m, \frac{5}{3} + m, \frac{5}{2}, \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (-4 + 3m) \text{AppellF1} \left[ \frac{3}{2}, \frac{7}{3} - m, \frac{2}{3} + m, \frac{5}{2}, \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \right) \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right) \Big) \Big)
\end{aligned}$$

## Test results for the 4 problems in "4.2.2.2 (g sin)^p (a+b cos)^m (c+d cos)^n.m"

- **Problem 1: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{d \cos[e + fx]} \sqrt{g \sin[e + fx]}}{a + b \cos[e + fx]} dx$$

Optimal (type 4, 509 leaves, 16 steps):

$$\begin{aligned}
& \frac{\sqrt{d} \sqrt{g} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{g \sin[e+fx]}}{\sqrt{g} \sqrt{d \cos[e+fx]}}\right]}{\sqrt{2} b f} + \frac{\sqrt{d} \sqrt{g} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{g \sin[e+fx]}}{\sqrt{g} \sqrt{d \cos[e+fx]}}\right]}{\sqrt{2} b f} + \\
& \frac{2 \sqrt{2} a d \sqrt{g} \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \sin[e+fx]}}{\sqrt{g} \sqrt{1+\cos[e+fx]}}\right], -1\right]}{b \sqrt{-a+b} \sqrt{a+b} f \sqrt{d \cos[e+fx]}} - \\
& \frac{2 \sqrt{2} a d \sqrt{g} \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \sin[e+fx]}}{\sqrt{g} \sqrt{1+\cos[e+fx]}}\right], -1\right]}{b \sqrt{-a+b} \sqrt{a+b} f \sqrt{d \cos[e+fx]}} + \\
& \frac{\sqrt{d} \sqrt{g} \operatorname{Log}\left[\sqrt{g} - \frac{\sqrt{2} \sqrt{d} \sqrt{g \sin[e+fx]}}{\sqrt{d \cos[e+fx]}} + \sqrt{g} \operatorname{Tan}[e+fx]\right]}{2 \sqrt{2} b f} - \frac{\sqrt{d} \sqrt{g} \operatorname{Log}\left[\sqrt{g} + \frac{\sqrt{2} \sqrt{d} \sqrt{g \sin[e+fx]}}{\sqrt{d \cos[e+fx]}} + \sqrt{g} \operatorname{Tan}[e+fx]\right]}{2 \sqrt{2} b f}
\end{aligned}$$

Result (type 4, 272 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{-a-b} \sqrt{a-b} b f \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}} \sqrt{g \sin[e+fx]}} \\
& 2 \sqrt{2} g \sqrt{d \cos[e+fx]} \left( -i \sqrt{-a-b} \sqrt{a-b} \operatorname{EllipticPi}\left[-i, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}\right], -1\right] + \right. \\
& \quad \left. i \sqrt{-a-b} \sqrt{a-b} \operatorname{EllipticPi}\left[i, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}\right], -1\right] + a \left( \operatorname{EllipticPi}\left[-\frac{\sqrt{a-b}}{\sqrt{-a-b}}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}\right], -1\right] - \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[\frac{\sqrt{a-b}}{\sqrt{-a-b}}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}\right], -1\right] \right) \right) \sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}
\end{aligned}$$

- **Problem 2: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{d \cos[e+fx]}}{(a+b \cos[e+fx]) \sqrt{g \sin[e+fx]}} dx$$

Optimal (type 4, 209 leaves, 4 steps):



$$\frac{2\sqrt{2}\sqrt{d}\operatorname{EllipticPi}\left[-\frac{a}{b-\sqrt{-a^2+b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d}\cos[e+fx]}{\sqrt{d}\sqrt{1+\sin[e+fx]}}\right], -1\right]\sqrt{\sin[e+fx]}}{\sqrt{-a^2+b^2}f\sqrt{g\sin[e+fx]}}$$

$$\frac{2\sqrt{2}\sqrt{d}\operatorname{EllipticPi}\left[-\frac{a}{b+\sqrt{-a^2+b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d}\cos[e+fx]}{\sqrt{d}\sqrt{1+\sin[e+fx]}}\right], -1\right]\sqrt{\sin[e+fx]}}{\sqrt{-a^2+b^2}f\sqrt{g\sin[e+fx]}}$$

Result (type 6, 594 leaves):

$$\frac{1}{f(a+b\cos[e+fx])\sqrt{g\sin[e+fx]}(1+\tan[e+fx]^2)^{3/2}} 2\sqrt{d}\cos[e+fx]\sec[e+fx]^2\sqrt{\tan[e+fx]}$$

$$\left(b+a\sqrt{1+\tan[e+fx]^2}\right)\left(\frac{1}{4\sqrt{2}(a^2-b^2)^{3/4}}\sqrt{a}\left(-2\operatorname{ArcTan}\left[1-\frac{\sqrt{2}\sqrt{a}\sqrt{\tan[e+fx]}}{(a^2-b^2)^{1/4}}\right]+2\operatorname{ArcTan}\left[1+\frac{\sqrt{2}\sqrt{a}\sqrt{\tan[e+fx]}}{(a^2-b^2)^{1/4}}\right]\right)-\right.$$

$$\left.\operatorname{Log}\left[\sqrt{a^2-b^2}-\sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\tan[e+fx]}+a\tan[e+fx]\right]+\operatorname{Log}\left[\sqrt{a^2-b^2}+\sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\tan[e+fx]}+a\tan[e+fx]\right]\right)+$$

$$\left(5b(a^2-b^2)\operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\tan[e+fx]^2, -\frac{a^2\tan[e+fx]^2}{a^2-b^2}\right]\sqrt{\tan[e+fx]}\right)/$$

$$\left(\sqrt{1+\tan[e+fx]^2}\left(-5(a^2-b^2)\operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\tan[e+fx]^2, -\frac{a^2\tan[e+fx]^2}{a^2-b^2}\right]+2\left(2a^2\operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, -\tan[e+fx]^2, -\frac{a^2\tan[e+fx]^2}{a^2-b^2}\right]+(a^2-b^2)\operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, -\tan[e+fx]^2, -\frac{a^2\tan[e+fx]^2}{a^2-b^2}\right]\right)\tan[e+fx]^2(-b^2+a^2(1+\tan[e+fx]^2))\right)\right)$$

- **Problem 3: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g\sin[e+fx]}}{\sqrt{d}\cos[e+fx](a+b\cos[e+fx])} dx$$

Optimal (type 4, 208 leaves, 5 steps):

$$\frac{2\sqrt{2}\sqrt{g}\sqrt{\cos[e+fx]}\operatorname{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g}\sin[e+fx]}{\sqrt{g}\sqrt{1+\cos[e+fx]}}\right], -1\right]}{\sqrt{-a+b}\sqrt{a+b}f\sqrt{d}\cos[e+fx]} +$$

$$\frac{2\sqrt{2}\sqrt{g}\sqrt{\cos[e+fx]}\operatorname{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g}\sin[e+fx]}{\sqrt{g}\sqrt{1+\cos[e+fx]}}\right], -1\right]}{\sqrt{-a+b}\sqrt{a+b}f\sqrt{d}\cos[e+fx]}$$

Result (type 6, 596 leaves):

$$\left( 2 \operatorname{Sec}[e+fx]^2 \sqrt{g \sin[e+fx]} \left( b+a \sqrt{1+\tan[e+fx]^2} \right) \right.$$

$$\left. \left( \frac{1}{4\sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}} \left( -2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{a}\sqrt{\tan[e+fx]}}{(a^2-b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{a}\sqrt{\tan[e+fx]}}{(a^2-b^2)^{1/4}}\right] \right) + \right.$$

$$\left. \left. \operatorname{Log}\left[\sqrt{a^2-b^2} - \sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\tan[e+fx]} + a \tan[e+fx]\right] - \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\tan[e+fx]} + a \tan[e+fx]\right] \right) \right)$$

$$\left( 7b(a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e+fx]^2, -\frac{a^2 \tan[e+fx]^2}{a^2-b^2}\right] \tan[e+fx]^{3/2} \right) /$$

$$\left( 3\sqrt{1+\tan[e+fx]^2} \left( -7(a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e+fx]^2, -\frac{a^2 \tan[e+fx]^2}{a^2-b^2}\right] + 2 \left( 2a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right.$$

$$\left. \left. \left. -\tan[e+fx]^2, -\frac{a^2 \tan[e+fx]^2}{a^2-b^2}\right] + (a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\tan[e+fx]^2, -\frac{a^2 \tan[e+fx]^2}{a^2-b^2}\right] \right) \tan[e+fx]^2 \right)$$

$$\left. \left. \left. \left. \left. (-b^2+a^2(1+\tan[e+fx]^2)) \right) \right) \right) \right) / \left( f\sqrt{d}\cos[e+fx](a+b\cos[e+fx])\sqrt{\tan[e+fx]}(1+\tan[e+fx]^2)^{3/2} \right)$$

■ **Problem 4: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{d}\cos[e+fx](a+b\cos[e+fx])\sqrt{g}\sin[e+fx]} dx$$

Optimal (type 4, 273 leaves, 7 steps):

$$\begin{aligned}
& \frac{2\sqrt{2} b \operatorname{EllipticPi}\left[-\frac{a}{b-\sqrt{-a^2+b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d} \operatorname{Cos}[e+fx]}{\sqrt{d} \sqrt{1+\operatorname{Sin}[e+fx]}}\right], -1\right] \sqrt{\operatorname{Sin}[e+fx]}}{a \sqrt{-a^2+b^2} \sqrt{d} f \sqrt{g \operatorname{Sin}[e+fx]}} + \\
& \frac{2\sqrt{2} b \operatorname{EllipticPi}\left[-\frac{a}{b+\sqrt{-a^2+b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d} \operatorname{Cos}[e+fx]}{\sqrt{d} \sqrt{1+\operatorname{Sin}[e+fx]}}\right], -1\right] \sqrt{\operatorname{Sin}[e+fx]}}{a \sqrt{-a^2+b^2} \sqrt{d} f \sqrt{g \operatorname{Sin}[e+fx]}} + \frac{\operatorname{EllipticF}\left[e-\frac{\pi}{4}+fx, 2\right] \sqrt{\operatorname{Sin}[2e+2fx]}}{a f \sqrt{d} \operatorname{Cos}[e+fx] \sqrt{g \operatorname{Sin}[e+fx]}}
\end{aligned}$$

Result (type 6, 5869 leaves):

$$\begin{aligned}
& \left( 4 (a+b) \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^3 \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] \right. \\
& \left( \left( 25 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \right) / \left( 5 (a+b) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left. \left. -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + 2 \left( -2 (a-b) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) + \right. \\
& \left. (a+b) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& \left( 9 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) / \\
& \left( 9 (a+b) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
& \left. 2 \left( -2 (a-b) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \right. \\
& \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) / \\
& \left( 5 f \sqrt{\operatorname{Cos}[e+fx]} \sqrt{d} \operatorname{Cos}[e+fx] (a+b \operatorname{Cos}[e+fx])^2 \sqrt{\operatorname{Sin}[e+fx]} \sqrt{g \operatorname{Sin}[e+fx]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{1}{5(a+b)\cos[e+fx]\sin[e+fx]^{3/2}} 2(a+b)\cos\left[\frac{1}{2}(e+fx)\right]^3 \sqrt{\cos[e+fx]} \sin\left[\frac{1}{2}(e+fx)\right] \right. \\
& \left. \left( \left( 25 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) / \left( 5(a+b) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) + 2 \left( -2(a-b) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) + \right. \right. \\
& \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) + \right. \\
& \left( 9 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) / \\
& \left( 9(a+b) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
& \left. 2 \left( -2(a-b) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + (a+b) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) + \frac{1}{5\sqrt{\cos[e+fx]}(a+b)\cos[e+fx]\sqrt{\sin[e+fx]}} \\
& 2(a+b)\cos\left[\frac{1}{2}(e+fx)\right]^4 \left( \left( 25 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) / \right. \\
& \left( 5(a+b) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
& \left. 2 \left( -2(a-b) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + (a+b) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \left( 9 \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right) / \\
& \left( 9 (a+b) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] + \right. \\
& \quad \left. 2 \left( -2 (a-b) \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] + (a+b) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right) \right) - \\
& \frac{1}{5 \sqrt{\operatorname{Cos}[e + f x]} (a+b \operatorname{Cos}[e + f x]) \sqrt{\operatorname{Sin}[e + f x]}} 6 (a+b) \operatorname{Cos} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Sin} \left[ \frac{1}{2} (e + f x) \right]^2 \\
& \left( \left( 25 \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \right) / \right. \\
& \quad \left( 5 (a+b) \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] + \right. \\
& \quad \left. 2 \left( -2 (a-b) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] + (a+b) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right) + \\
& \left( 9 \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right) / \\
& \left( 9 (a+b) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] + \right. \\
& \quad \left. 2 \left( -2 (a-b) \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] + (a+b) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right] \tan\left[\frac{1}{2}(e+fx)\right]^2\right) \right) + \\
& \frac{1}{5\sqrt{\cos[e+fx]}(a+b\cos[e+fx])^2} 4b(a+b)\cos\left[\frac{1}{2}(e+fx)\right]^3\sin\left[\frac{1}{2}(e+fx)\right]\sqrt{\sin[e+fx]} \\
& \left( \left( 25\text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \right) / \\
& \left( 5(a+b)\text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) + \\
& 2\left(-2(a-b)\text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + (a+b)\right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right] \tan\left[\frac{1}{2}(e+fx)\right]^2\right) \right) + \\
& \left( 9\text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) / \\
& \left( 9(a+b)\text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) + \\
& 2\left(-2(a-b)\text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + (a+b)\right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right] \tan\left[\frac{1}{2}(e+fx)\right]^2\right) \right) + \\
& \frac{1}{5\cos[e+fx]^{3/2}(a+b\cos[e+fx])} 2(a+b)\cos\left[\frac{1}{2}(e+fx)\right]^3\sin\left[\frac{1}{2}(e+fx)\right]\sqrt{\sin[e+fx]} \\
& \left( \left( 25\text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \right) / \\
& \left( 5(a+b)\text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) +
\end{aligned}$$

$$\begin{aligned}
& 2 \left( -2 (a-b) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + (a+b) \right. \\
& \quad \left. \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 + \\
& \left( 9 \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) / \\
& \left( 9 (a+b) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& \quad \left. 2 \left( -2 (a-b) \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + (a+b) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) + \\
& \frac{1}{5 \sqrt{\operatorname{Cos}[e+f x]} (a+b \operatorname{Cos}[e+f x]) \sqrt{\operatorname{Sin}[e+f x]}} 4 (a+b) \operatorname{Cos} \left[ \frac{1}{2} (e+f x) \right]^3 \operatorname{Sin} \left[ \frac{1}{2} (e+f x) \right] \\
& \left( \left( 25 \left( -1 / (5 (a+b)) (a-b) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right] + \right. \right. \right. \\
& \quad \left. \left. \frac{1}{10} \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right] \right) \right) / \\
& \left( 5 (a+b) \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& \quad \left. 2 \left( -2 (a-b) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + (a+b) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) +
\end{aligned}$$

$$\begin{aligned}
& \left( 9 \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] \right) / \\
& \left( 9 (a+b) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( -2 (a-b) \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] + (a+b) \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right) + \\
& \left( 9 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \left( -1 / (9 (a+b)) 5 (a-b) \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \right. \right. \\
& \left. \left. \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] + \frac{5}{18} \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \right. \right. \\
& \left. \left. \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] \right) \right) / \left( 9 (a+b) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( -2 (a-b) \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] + (a+b) \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{3}{2}, 1, \right. \right. \\
& \left. \left. \frac{13}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 - \left( 25 \operatorname{AppellF1} \left[ \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, \right. \right. \\
& \left. \left. -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \right) \left( 2 \left( -2 (a-b) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] + \right. \right. \\
& \left. \left. (a+b) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \right) \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] + 5 (a+b) \right) \\
& \left( -\frac{1}{5 (a+b)} (a-b) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] + \right.
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{10} \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \Bigg) + \\
& 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left( -2(a-b) \left( -\frac{1}{9(a+b)} 10(a-b) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 3, \frac{13}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{5}{18} \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 2, \frac{13}{4}, \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) + (a+b) \left( -\frac{1}{9(a+b)} \right. \\
& \quad \left. 5(a-b) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 2, \frac{13}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. \frac{5}{6} \operatorname{AppellF1}\left[\frac{9}{4}, \frac{5}{2}, 1, \frac{13}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( 5(a+b) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + 2 \left( -2(a-b) \operatorname{AppellF1}\left[ \right. \right. \right. \\
& \quad \left. \left. \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
& \quad \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 - \\
& \left( 9 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \quad \left. \left( 2 \left( -2(a-b) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \right. \right. \\
& \quad \left. \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + 9(a+b) \right. \right. \\
& \quad \left. \left. \left( -\frac{1}{9(a+b)} 5(a-b) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{5}{18} \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \Bigg) + \\
& 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left( -2(a-b) \left( -\frac{1}{13(a+b)} 18(a-b) \operatorname{AppellF1}\left[\frac{13}{4}, \frac{1}{2}, 3, \frac{17}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{9}{26} \operatorname{AppellF1}\left[\frac{13}{4}, \frac{3}{2}, 2, \frac{17}{4}, \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) + (a+b) \left( -\frac{1}{13(a+b)} \right. \\
& \quad \left. 9(a-b) \operatorname{AppellF1}\left[\frac{13}{4}, \frac{3}{2}, 2, \frac{17}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. \frac{27}{26} \operatorname{AppellF1}\left[\frac{13}{4}, \frac{5}{2}, 1, \frac{17}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( 9(a+b) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + 2 \left( -2(a-b) \operatorname{AppellF1}\left[ \right. \right. \right. \\
& \quad \left. \left. \left. \frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) + \right. \\
& \quad \left. (a+b) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

## Test results for the 1 problems in "4.2.2.3 (g cos)^p (a+b cos)^m (c+d cos)^n.m"

- Problem 1: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+a \operatorname{Cos}[e+fx])^2 \operatorname{Sec}[e+fx]^2}{-c+c \operatorname{Cos}[e+fx]} dx$$

Optimal (type 3, 65 leaves, 6 steps):

$$-\frac{3 a^2 \operatorname{ArcTanh}[\operatorname{Sin}[e+fx]]}{c f} + \frac{4 a^2 \operatorname{Sin}[e+fx]}{c f (1-\operatorname{Cos}[e+fx])} - \frac{a^2 \operatorname{Tan}[e+fx]}{c f}$$

Result (type 3, 194 leaves):

$$\frac{1}{c f (-1 + \cos[e + f x])} 2 a^2 \sin\left[\frac{1}{2} (e + f x)\right] \\ \left( 4 \operatorname{Csc}\left[\frac{e}{2}\right] \sin\left[\frac{f x}{2}\right] + \sin\left[\frac{1}{2} (e + f x)\right] \left( -3 \operatorname{Log}\left[\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right]\right] + 3 \operatorname{Log}\left[\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right]\right] \right) + \right. \\ \left. \sin[f x] / \left( \left( \cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right] \right) \left( \cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right] \right) \left( \cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right) \left( \cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \right) \right)$$

## Test results for the 644 problems in "4.2.3.1 (a+b cos)^m (c+d cos)^n (A+B cos).m"

- **Problem 5: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + d x]) (A + B \cos[c + d x]) \sec[c + d x] dx$$

Optimal (type 3, 32 leaves, 4 steps):

$$a (A + B) x + \frac{a A \operatorname{ArcTanh}[\sin[c + d x]]}{d} + \frac{a B \sin[c + d x]}{d}$$

Result (type 3, 104 leaves):

$$a A x + a B x - \frac{a A \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{d x}{2}\right] - \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a A \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{d x}{2}\right] + \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a B \cos[d x] \sin[c]}{d} + \frac{a B \cos[c] \sin[d x]}{d}$$

- **Problem 6: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + d x]) (A + B \cos[c + d x]) \sec[c + d x]^2 dx$$

Optimal (type 3, 32 leaves, 4 steps):

$$a B x + \frac{a (A + B) \operatorname{ArcTanh}[\sin[c + d x]]}{d} + \frac{a A \tan[c + d x]}{d}$$

Result (type 3, 159 leaves):

$$a B x - \frac{a A \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{d x}{2}\right] - \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} - \frac{a B \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{d x}{2}\right] - \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \\ \frac{a A \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{d x}{2}\right] + \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a B \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{d x}{2}\right] + \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a A \tan[c + d x]}{d}$$

- **Problem 7: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + d x]) (A + B \cos[c + d x]) \sec[c + d x]^3 dx$$

Optimal (type 3, 56 leaves, 6 steps):

$$\frac{a (A + 2 B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{a (A + B) \operatorname{Tan}[c + d x]}{d} + \frac{a A \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 d}$$

Result (type 3, 154 leaves):

$$\frac{1}{4 d} a \left( -2 (A + 2 B) \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] + \right. \\ \left. 2 A \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] + 4 B \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] + \right. \\ \left. \frac{A}{\left( \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \frac{A}{\left( \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right)^2} + 4 (A + B) \operatorname{Tan}[c + d x] \right)$$

■ **Problem 9: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Cos}[c + d x]) (A + B \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x]^5 dx$$

Optimal (type 3, 106 leaves, 7 steps):

$$\frac{a (3 A + 4 B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{a (A + B) \operatorname{Tan}[c + d x]}{d} + \\ \frac{a (3 A + 4 B) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{a A \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d} + \frac{a (A + B) \operatorname{Tan}[c + d x]^3}{3 d}$$

Result (type 3, 403 leaves):

$$- \frac{3 a A \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right]}{8 d} - \frac{a B \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right]}{2 d} + \frac{3 a A \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right]}{8 d} + \\ \frac{a B \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right]}{2 d} + \frac{a A}{16 d \left( \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right)^4} + \frac{3 a A}{16 d \left( \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \\ \frac{a B}{4 d \left( \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \frac{a A}{16 d \left( \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right)^4} - \frac{3 a A}{16 d \left( \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \\ \frac{a B}{4 d \left( \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{2 a A \operatorname{Tan}[c + d x]}{3 d} + \frac{2 a B \operatorname{Tan}[c + d x]}{3 d} + \frac{a A \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d} + \frac{a B \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d}$$

■ **Problem 16: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Cos}[c + d x])^2 (A + B \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x]^3 dx$$

Optimal (type 3, 88 leaves, 5 steps):

$$a^2 B x + \frac{a^2 (3 A + 4 B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{a^2 (3 A + 2 B) \operatorname{Tan}[c + d x]}{2 d} + \frac{A (a^2 + a^2 \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 d}$$

Result (type 3, 277 leaves):

$$\frac{1}{16} a^2 (1 + \operatorname{Cos}[c + d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^4$$

$$\left( 4 B x - \frac{2 (3 A + 4 B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} + \frac{2 (3 A + 4 B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} + \right.$$

$$\left. \frac{A}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{4 (2 A + B) \operatorname{Sin}\left[\frac{d x}{2}\right]}{d \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)} - \right.$$

$$\left. \frac{A}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{4 (2 A + B) \operatorname{Sin}\left[\frac{d x}{2}\right]}{d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)} \right)$$

■ **Problem 17: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Cos}[c + d x])^2 (A + B \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x]^4 dx$$

Optimal (type 3, 113 leaves, 7 steps):

$$\frac{a^2 (2 A + 3 B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{a^2 (5 A + 6 B) \operatorname{Tan}[c + d x]}{3 d} +$$

$$\frac{a^2 (4 A + 3 B) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{6 d} + \frac{A (a^2 + a^2 \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d}$$

Result (type 3, 753 leaves):

$$\begin{aligned}
& \frac{(-2A - 3B)(a + a \cos[c + dx])^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4}{8d} + \\
& \frac{(2A + 3B)(a + a \cos[c + dx])^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4}{8d} + \frac{A(a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{24d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \frac{(a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(7A \cos\left[\frac{c}{2}\right] + 3B \cos\left[\frac{c}{2}\right] - 5A \sin\left[\frac{c}{2}\right] - 3B \sin\left[\frac{c}{2}\right]\right)}{48d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{(a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(5A \sin\left[\frac{dx}{2}\right] + 6B \sin\left[\frac{dx}{2}\right]\right)}{12d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \frac{A(a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{24d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \frac{(a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(-7A \cos\left[\frac{c}{2}\right] - 3B \cos\left[\frac{c}{2}\right] - 5A \sin\left[\frac{c}{2}\right] - 3B \sin\left[\frac{c}{2}\right]\right)}{48d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{(a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(5A \sin\left[\frac{dx}{2}\right] + 6B \sin\left[\frac{dx}{2}\right]\right)}{12d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 23: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^3 (A + B \cos[c + dx]) \operatorname{Sec}[c + dx]^2 dx$$

Optimal (type 3, 110 leaves, 6 steps):

$$\begin{aligned}
& \frac{1}{2} a^3 (6A + 7B)x + \frac{a^3 (3A + B) \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{5a^3 B \sin[c + dx]}{2d} - \\
& \frac{(2A - B)(a^3 + a^3 \cos[c + dx]) \sin[c + dx]}{2d} + \frac{aA(a + a \cos[c + dx])^2 \tan[c + dx]}{d}
\end{aligned}$$

Result (type 3, 272 leaves):

$$\begin{aligned}
& \frac{1}{32} a^3 (1 + \cos[c + dx])^3 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^6 \\
& \left( 2(6A + 7B)x - \frac{4(3A + B) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{d} + \frac{4(3A + B) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{d} + \right. \\
& \frac{4(A + 3B) \cos[dx] \sin[c]}{d} + \frac{B \cos[2dx] \sin[2c]}{d} + \frac{4(A + 3B) \cos[c] \sin[dx]}{d} + \frac{B \cos[2c] \sin[2dx]}{d} + \\
& \left. \frac{4A \sin\left[\frac{dx}{2}\right]}{d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \frac{4A \sin\left[\frac{dx}{2}\right]}{d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)} \right)
\end{aligned}$$

■ **Problem 25: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^3 (A + B \cos [c + d x]) \sec [c + d x]^4 dx$$

Optimal (type 3, 125 leaves, 6 steps):

$$a^3 B x + \frac{a^3 (5 A + 7 B) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{5 a^3 (A + B) \tan [c + d x]}{2 d} +$$

$$\frac{(5 A + 3 B) (a^3 + a^3 \cos [c + d x]) \sec [c + d x] \tan [c + d x]}{6 d} + \frac{a A (a + a \cos [c + d x])^2 \sec [c + d x]^2 \tan [c + d x]}{3 d}$$

Result (type 3, 786 leaves):

$$\frac{1}{8} B x (a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 + \frac{(-5 A - 7 B) (a + a \cos [c + d x])^3 \operatorname{Log} \left[ \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6}{16 d} +$$

$$\frac{(5 A + 7 B) (a + a \cos [c + d x])^3 \operatorname{Log} \left[ \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6}{16 d} + \frac{A (a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sin \left[ \frac{d x}{2} \right]}{48 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^3} +$$

$$\frac{(a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \left( 10 A \cos \left[ \frac{c}{2} \right] + 3 B \cos \left[ \frac{c}{2} \right] - 8 A \sin \left[ \frac{c}{2} \right] - 3 B \sin \left[ \frac{c}{2} \right] \right)}{96 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^2} +$$

$$\frac{(a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \left( 11 A \sin \left[ \frac{d x}{2} \right] + 9 B \sin \left[ \frac{d x}{2} \right] \right)}{24 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)} + \frac{A (a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sin \left[ \frac{d x}{2} \right]}{48 d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^3} +$$

$$\frac{(a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \left( -10 A \cos \left[ \frac{c}{2} \right] - 3 B \cos \left[ \frac{c}{2} \right] - 8 A \sin \left[ \frac{c}{2} \right] - 3 B \sin \left[ \frac{c}{2} \right] \right)}{96 d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^2} +$$

$$\frac{(a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \left( 11 A \sin \left[ \frac{d x}{2} \right] + 9 B \sin \left[ \frac{d x}{2} \right] \right)}{24 d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)}$$

■ **Problem 32: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^4 (A + B \cos [c + d x]) \sec [c + d x]^2 dx$$

Optimal (type 3, 150 leaves, 7 steps):

$$\frac{1}{2} a^4 (13 A + 12 B) x + \frac{a^4 (4 A + B) \operatorname{ArcTanh}[\sin [c + d x]]}{d} + \frac{5 a^4 (A + 2 B) \sin [c + d x]}{2 d} -$$

$$\frac{(3 A - B) (a^2 + a^2 \cos [c + d x])^2 \sin [c + d x]}{3 d} - \frac{(3 A - 8 B) (a^4 + a^4 \cos [c + d x]) \sin [c + d x]}{6 d} + \frac{a A (a + a \cos [c + d x])^3 \tan [c + d x]}{d}$$

Result (type 3, 312 leaves):

$$\frac{1}{192} a^4 (1 + \cos [c + dx])^4 \operatorname{Sec} \left[ \frac{1}{2} (c + dx) \right]^8$$

$$\left( \begin{aligned} & 78 Ax + 72 Bx - \frac{12 (4A + B) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right]}{d} + \frac{12 (4A + B) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right]}{d} + \\ & \frac{3 (16A + 27B) \cos [dx] \sin [c]}{d} + \frac{3 (A + 4B) \cos [2dx] \sin [2c]}{d} + \frac{B \cos [3dx] \sin [3c]}{d} + \\ & \frac{3 (16A + 27B) \cos [c] \sin [dx]}{d} + \frac{3 (A + 4B) \cos [2c] \sin [2dx]}{d} + \frac{B \cos [3c] \sin [3dx]}{d} + \\ & \frac{12A \sin \left[ \frac{dx}{2} \right]}{d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right)} + \frac{12A \sin \left[ \frac{dx}{2} \right]}{d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right)} \end{aligned} \right)$$

■ **Problem 33: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + dx])^4 (A + B \cos [c + dx]) \operatorname{Sec} [c + dx]^3 dx$$

Optimal (type 3, 162 leaves, 7 steps):

$$\frac{1}{2} a^4 (8A + 13B)x + \frac{a^4 (13A + 8B) \operatorname{ArcTanh}[\sin [c + dx]]}{2d} - \frac{5a^4 (A - B) \sin [c + dx]}{2d} - \frac{(6A + B) (a^4 + a^4 \cos [c + dx]) \sin [c + dx]}{2d} +$$

$$\frac{(5A + 2B) (a^2 + a^2 \cos [c + dx])^2 \tan [c + dx]}{2d} + \frac{aA (a + a \cos [c + dx])^3 \operatorname{Sec} [c + dx] \tan [c + dx]}{2d}$$

Result (type 3, 688 leaves):

$$\frac{1}{32} (8A + 13B)x (a + a \cos [c + dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8 + \frac{(-13A - 8B) (a + a \cos [c + dx])^4 \operatorname{Log} \left[ \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8}{32d} +$$

$$\frac{(13A + 8B) (a + a \cos [c + dx])^4 \operatorname{Log} \left[ \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8}{32d} +$$

$$\frac{(A + 4B) \cos [dx] (a + a \cos [c + dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8 \sin [c]}{16d} + \frac{B \cos [2dx] (a + a \cos [c + dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8 \sin [2c]}{64d} +$$

$$\frac{(A + 4B) \cos [c] (a + a \cos [c + dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8 \sin [dx]}{16d} + \frac{B \cos [2c] (a + a \cos [c + dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8 \sin [2dx]}{64d} +$$

$$\frac{A (a + a \cos [c + dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8}{64d \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)^2} + \frac{(a + a \cos [c + dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8 (4A \sin \left[ \frac{dx}{2} \right] + B \sin \left[ \frac{dx}{2} \right])}{16d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)} -$$

$$\frac{A (a + a \cos [c + dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8}{64d \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)^2} + \frac{(a + a \cos [c + dx])^4 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^8 (4A \sin \left[ \frac{dx}{2} \right] + B \sin \left[ \frac{dx}{2} \right])}{16d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)}$$



■ **Problem 34: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^4 (A + B \cos [c + d x]) \sec [c + d x]^4 dx$$

Optimal (type 3, 165 leaves, 7 steps):

$$a^4 (A + 4 B) x + \frac{a^4 (12 A + 13 B) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} - \frac{5 a^4 (2 A + B) \sin [c + d x]}{2 d} + \frac{(11 A + 9 B) (a^4 + a^4 \cos [c + d x]) \tan [c + d x]}{3 d} + \frac{(2 A + B) (a^2 + a^2 \cos [c + d x])^2 \sec [c + d x] \tan [c + d x]}{2 d} + \frac{a A (a + a \cos [c + d x])^3 \sec [c + d x]^2 \tan [c + d x]}{3 d}$$

Result (type 3, 380 leaves):

$$a^4 \left( \frac{(A + 4 B) (c + d x)}{d} + \frac{(-12 A - 13 B) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{2 d} + \frac{(12 A + 13 B) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{2 d} + \frac{13 A + 3 B}{12 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{A \sin \left[ \frac{1}{2} (c + d x) \right]}{6 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^3} + \frac{A \sin \left[ \frac{1}{2} (c + d x) \right]}{6 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^3} + \frac{-13 A - 3 B}{12 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{4 \left( 5 A \sin \left[ \frac{1}{2} (c + d x) \right] + 3 B \sin \left[ \frac{1}{2} (c + d x) \right] \right)}{3 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)} + \frac{4 \left( 5 A \sin \left[ \frac{1}{2} (c + d x) \right] + 3 B \sin \left[ \frac{1}{2} (c + d x) \right] \right)}{3 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)} + \frac{B \sin [c + d x]}{d} \right)$$

■ **Problem 38: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^4 (A + B \cos [c + d x])}{a + a \cos [c + d x]} dx$$

Optimal (type 3, 153 leaves, 7 steps):

$$-\frac{3 (4 A - 5 B) x}{8 a} + \frac{4 (A - B) \sin [c + d x]}{a d} - \frac{3 (4 A - 5 B) \cos [c + d x] \sin [c + d x]}{8 a d} - \frac{(4 A - 5 B) \cos [c + d x]^3 \sin [c + d x]}{4 a d} + \frac{(A - B) \cos [c + d x]^4 \sin [c + d x]}{d (a + a \cos [c + d x])} - \frac{4 (A - B) \sin [c + d x]^3}{3 a d}$$

Result (type 3, 311 leaves):

$$\frac{1}{192 a d (1 + \operatorname{Cos}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( -72 (4 A - 5 B) d x \operatorname{Cos}\left[\frac{d x}{2}\right] - 72 (4 A - 5 B) d x \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 552 A \operatorname{Sin}\left[\frac{d x}{2}\right] - 552 B \operatorname{Sin}\left[\frac{d x}{2}\right] + 168 A \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 168 B \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 144 A \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - 120 B \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 144 A \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] - 120 B \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] - 16 A \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] + 40 B \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] - 16 A \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] + 40 B \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] + 8 A \operatorname{Sin}\left[3 c + \frac{7 d x}{2}\right] - 5 B \operatorname{Sin}\left[3 c + \frac{7 d x}{2}\right] + 8 A \operatorname{Sin}\left[4 c + \frac{7 d x}{2}\right] - 5 B \operatorname{Sin}\left[4 c + \frac{7 d x}{2}\right] + 3 B \operatorname{Sin}\left[4 c + \frac{9 d x}{2}\right] + 3 B \operatorname{Sin}\left[5 c + \frac{9 d x}{2}\right] \right)$$

■ **Problem 39: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^3 (A + B \operatorname{Cos}[c + d x])}{a + a \operatorname{Cos}[c + d x]} dx$$

Optimal (type 3, 122 leaves, 6 steps):

$$\frac{3(A-B)x}{2a} - \frac{(3A-4B)\operatorname{Sin}[c+dx]}{ad} + \frac{3(A-B)\operatorname{Cos}[c+dx]\operatorname{Sin}[c+dx]}{2ad} + \frac{(A-B)\operatorname{Cos}[c+dx]^3\operatorname{Sin}[c+dx]}{d(a+a\operatorname{Cos}[c+dx])} + \frac{(3A-4B)\operatorname{Sin}[c+dx]^3}{3ad}$$

Result (type 3, 249 leaves):

$$\frac{1}{24 a d (1 + \operatorname{Cos}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( 36 (A - B) d x \operatorname{Cos}\left[\frac{d x}{2}\right] + 36 (A - B) d x \operatorname{Cos}\left[c + \frac{d x}{2}\right] - 60 A \operatorname{Sin}\left[\frac{d x}{2}\right] + 69 B \operatorname{Sin}\left[\frac{d x}{2}\right] - 12 A \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 21 B \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 9 A \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 18 B \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - 9 A \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 18 B \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 3 A \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] - 2 B \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] + 3 A \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] - 2 B \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] + B \operatorname{Sin}\left[3 c + \frac{7 d x}{2}\right] + B \operatorname{Sin}\left[4 c + \frac{7 d x}{2}\right] \right)$$

■ **Problem 40: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^2 (A + B \operatorname{Cos}[c + d x])}{a + a \operatorname{Cos}[c + d x]} dx$$

Optimal (type 3, 90 leaves, 2 steps):

$$-\frac{(A-B)x}{a} + \frac{Bx}{2a} + \frac{(A-B)\operatorname{Sin}[c+dx]}{ad} + \frac{B\operatorname{Cos}[c+dx]\operatorname{Sin}[c+dx]}{2ad} + \frac{(A-B)\operatorname{Sin}[c+dx]}{ad(1+\operatorname{Cos}[c+dx])}$$

Result (type 3, 197 leaves):

$$\frac{1}{8 a d (1 + \operatorname{Cos}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( -4(2A - 3B) d x \operatorname{Cos}\left[\frac{d x}{2}\right] - 4(2A - 3B) d x \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 20A \operatorname{Sin}\left[\frac{d x}{2}\right] - 20B \operatorname{Sin}\left[\frac{d x}{2}\right] + 4A \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 4B \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 4A \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - 3B \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 4A \operatorname{Sin}\left[2c + \frac{3 d x}{2}\right] - 3B \operatorname{Sin}\left[2c + \frac{3 d x}{2}\right] + B \operatorname{Sin}\left[2c + \frac{5 d x}{2}\right] + B \operatorname{Sin}\left[3c + \frac{5 d x}{2}\right] \right)$$

- **Problem 41: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x] (A + B \operatorname{Cos}[c + d x])}{a + a \operatorname{Cos}[c + d x]} dx$$

Optimal (type 3, 54 leaves, 5 steps):

$$\frac{(A - B) x}{a} + \frac{B \operatorname{Sin}[c + d x]}{a d} - \frac{(A - B) \operatorname{Sin}[c + d x]}{a d (1 + \operatorname{Cos}[c + d x])}$$

Result (type 3, 126 leaves):

$$\frac{1}{2 a d (1 + \operatorname{Cos}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( 2(A - B) d x \operatorname{Cos}\left[\frac{d x}{2}\right] + 2(A - B) d x \operatorname{Cos}\left[c + \frac{d x}{2}\right] - 4A \operatorname{Sin}\left[\frac{d x}{2}\right] + 5B \operatorname{Sin}\left[\frac{d x}{2}\right] + B \operatorname{Sin}\left[c + \frac{d x}{2}\right] + B \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + B \operatorname{Sin}\left[2c + \frac{3 d x}{2}\right] \right)$$

- **Problem 42: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Cos}[c + d x]}{a + a \operatorname{Cos}[c + d x]} dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{B x}{a} + \frac{(A - B) \operatorname{Sin}[c + d x]}{d (a + a \operatorname{Cos}[c + d x])}$$

Result (type 3, 72 leaves):

$$\frac{\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( B d x \operatorname{Cos}\left[\frac{d x}{2}\right] + B d x \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 2(A - B) \operatorname{Sin}\left[\frac{d x}{2}\right] \right)}{a d (1 + \operatorname{Cos}[c + d x])}$$

- **Problem 43: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x]}{a + a \operatorname{Cos}[c + d x]} dx$$

Optimal (type 3, 44 leaves, 3 steps):

$$\frac{A \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{a d} - \frac{(A - B) \operatorname{Sin}[c + d x]}{d (a + a \operatorname{Cos}[c + d x])}$$

Result (type 3, 109 leaves):

$$\frac{1}{a d (1 + \cos [c + d x])} 2 \cos \left[ \frac{1}{2} (c + d x) \right] \left( A \cos \left[ \frac{1}{2} (c + d x) \right] \left( -\log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] \right) + (-A + B) \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] \right)$$

■ **Problem 44: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + d x]) \sec [c + d x]^2}{a + a \cos [c + d x]} dx$$

Optimal (type 3, 69 leaves, 5 steps) :

$$-\frac{(A - B) \operatorname{ArcTanh}[\sin [c + d x]]}{a d} + \frac{(2 A - B) \tan [c + d x]}{a d} - \frac{(A - B) \tan [c + d x]}{d (a + a \cos [c + d x])}$$

Result (type 3, 201 leaves) :

$$\frac{1}{a d (1 + \cos [c + d x])} 2 \cos \left[ \frac{1}{2} (c + d x) \right] \left( (A - B) \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + \cos \left[ \frac{1}{2} (c + d x) \right] \left( (A - B) \left( \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] - \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] \right) + (A \sin [d x]) / \left( \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) \right)$$

■ **Problem 45: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + d x]) \sec [c + d x]^3}{a + a \cos [c + d x]} dx$$

Optimal (type 3, 107 leaves, 6 steps) :

$$\frac{(3 A - 2 B) \operatorname{ArcTanh}[\sin [c + d x]]}{2 a d} - \frac{2 (A - B) \tan [c + d x]}{a d} + \frac{(3 A - 2 B) \sec [c + d x] \tan [c + d x]}{2 a d} - \frac{(A - B) \sec [c + d x] \tan [c + d x]}{d (a + a \cos [c + d x])}$$

Result (type 3, 289 leaves) :

$$\frac{1}{2 a d (1 + \cos [c + d x])} \cos \left[ \frac{1}{2} (c + d x) \right] \left( 4 (-A + B) \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + \right. \\ \left. \cos \left[ \frac{1}{2} (c + d x) \right] \left( (-6 A + 4 B) \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] + 6 A \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] - \right. \right. \\ \left. \left. 4 B \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \frac{A}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \frac{A}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} - (4 (A - B) \right. \right. \\ \left. \left. \sin [d x]) \right) / \left( \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) \right)$$

■ **Problem 46: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + d x]) \sec [c + d x]^4}{a + a \cos [c + d x]} dx$$

Optimal (type 3, 131 leaves, 6 steps):

$$-\frac{3 (A - B) \operatorname{ArcTanh}[\sin [c + d x]]}{2 a d} + \frac{(4 A - 3 B) \tan [c + d x]}{a d} - \\ \frac{3 (A - B) \sec [c + d x] \tan [c + d x]}{2 a d} - \frac{(A - B) \sec [c + d x]^2 \tan [c + d x]}{d (a + a \cos [c + d x])} + \frac{(4 A - 3 B) \tan [c + d x]^3}{3 a d}$$

Result (type 3, 569 leaves):

$$\frac{3 (A - B) \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \log \left[ \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right]}{d (a + a \cos [c + d x])} - \\ \frac{3 (A - B) \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \log \left[ \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right]}{d (a + a \cos [c + d x])} + \frac{1}{48 d (a + a \cos [c + d x])} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] \sec \left[ \frac{c}{2} \right] \sec [c] \sec [c + d x]^3 \\ \left( 6 A \sin \left[ \frac{d x}{2} \right] + 6 B \sin \left[ \frac{d x}{2} \right] + 39 A \sin \left[ \frac{3 d x}{2} \right] - 27 B \sin \left[ \frac{3 d x}{2} \right] - 24 A \sin \left[ c - \frac{d x}{2} \right] + 12 B \sin \left[ c - \frac{d x}{2} \right] - 6 A \sin \left[ c + \frac{d x}{2} \right] + \right. \\ \left. 6 B \sin \left[ c + \frac{d x}{2} \right] - 24 A \sin \left[ 2 c + \frac{d x}{2} \right] + 24 B \sin \left[ 2 c + \frac{d x}{2} \right] + 21 A \sin \left[ c + \frac{3 d x}{2} \right] - 9 B \sin \left[ c + \frac{3 d x}{2} \right] + 9 A \sin \left[ 2 c + \frac{3 d x}{2} \right] - \right. \\ \left. 9 B \sin \left[ 2 c + \frac{3 d x}{2} \right] - 9 A \sin \left[ 3 c + \frac{3 d x}{2} \right] + 9 B \sin \left[ 3 c + \frac{3 d x}{2} \right] + 7 A \sin \left[ c + \frac{5 d x}{2} \right] - 3 B \sin \left[ c + \frac{5 d x}{2} \right] + A \sin \left[ 2 c + \frac{5 d x}{2} \right] + \right. \\ \left. 3 B \sin \left[ 2 c + \frac{5 d x}{2} \right] - 3 A \sin \left[ 3 c + \frac{5 d x}{2} \right] + 3 B \sin \left[ 3 c + \frac{5 d x}{2} \right] - 9 A \sin \left[ 4 c + \frac{5 d x}{2} \right] + 9 B \sin \left[ 4 c + \frac{5 d x}{2} \right] + 16 A \sin \left[ 2 c + \frac{7 d x}{2} \right] - \right. \\ \left. 12 B \sin \left[ 2 c + \frac{7 d x}{2} \right] + 10 A \sin \left[ 3 c + \frac{7 d x}{2} \right] - 6 B \sin \left[ 3 c + \frac{7 d x}{2} \right] + 6 A \sin \left[ 4 c + \frac{7 d x}{2} \right] - 6 B \sin \left[ 4 c + \frac{7 d x}{2} \right] \right)$$

■ **Problem 47: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^4 (A + B \cos[c + dx])}{(a + a \cos[c + dx])^2} dx$$

Optimal (type 3, 170 leaves, 7 steps) :

$$\frac{(7A - 10B)x}{2a^2} - \frac{4(2A - 3B)\sin[c + dx]}{a^2 d} + \frac{(7A - 10B)\cos[c + dx]\sin[c + dx]}{2a^2 d} +$$

$$\frac{(7A - 10B)\cos[c + dx]^3 \sin[c + dx]}{3a^2 d (1 + \cos[c + dx])} + \frac{(A - B)\cos[c + dx]^4 \sin[c + dx]}{3d(a + a \cos[c + dx])^2} + \frac{4(2A - 3B)\sin[c + dx]^3}{3a^2 d}$$

Result (type 3, 369 leaves) :

$$\frac{1}{48a^2 d (1 + \cos[c + dx])^2}$$

$$\cos\left[\frac{1}{2}(c + dx)\right] \sec\left[\frac{c}{2}\right] \left( 36(7A - 10B) dx \cos\left[\frac{dx}{2}\right] + 36(7A - 10B) dx \cos\left[c + \frac{dx}{2}\right] + 84A dx \cos\left[c + \frac{3dx}{2}\right] - 120B dx \cos\left[c + \frac{3dx}{2}\right] + \right.$$

$$84A dx \cos\left[2c + \frac{3dx}{2}\right] - 120B dx \cos\left[2c + \frac{3dx}{2}\right] - 381A \sin\left[\frac{dx}{2}\right] + 516B \sin\left[\frac{dx}{2}\right] + 147A \sin\left[c + \frac{dx}{2}\right] -$$

$$156B \sin\left[c + \frac{dx}{2}\right] - 239A \sin\left[c + \frac{3dx}{2}\right] + 342B \sin\left[c + \frac{3dx}{2}\right] - 63A \sin\left[2c + \frac{3dx}{2}\right] + 118B \sin\left[2c + \frac{3dx}{2}\right] -$$

$$15A \sin\left[2c + \frac{5dx}{2}\right] + 30B \sin\left[2c + \frac{5dx}{2}\right] - 15A \sin\left[3c + \frac{5dx}{2}\right] + 30B \sin\left[3c + \frac{5dx}{2}\right] + 3A \sin\left[3c + \frac{7dx}{2}\right] -$$

$$\left. 3B \sin\left[3c + \frac{7dx}{2}\right] + 3A \sin\left[4c + \frac{7dx}{2}\right] - 3B \sin\left[4c + \frac{7dx}{2}\right] + B \sin\left[4c + \frac{9dx}{2}\right] + B \sin\left[5c + \frac{9dx}{2}\right] \right)$$

■ **Problem 48: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^3 (A + B \cos[c + dx])}{(a + a \cos[c + dx])^2} dx$$

Optimal (type 3, 147 leaves, 3 steps) :

$$-\frac{(4A - 7B)x}{2a^2} + \frac{2(5A - 8B)\sin[c + dx]}{3a^2 d} - \frac{(4A - 7B)\cos[c + dx]\sin[c + dx]}{2a^2 d} +$$

$$\frac{(5A - 8B)\cos[c + dx]^2 \sin[c + dx]}{3a^2 d (1 + \cos[c + dx])} + \frac{(A - B)\cos[c + dx]^3 \sin[c + dx]}{3d(a + a \cos[c + dx])^2}$$

Result (type 3, 315 leaves) :

$$\frac{1}{48 a^2 d (1 + \cos [c + d x])^2} \cos \left[ \frac{1}{2} (c + d x) \right] \sec \left[ \frac{c}{2} \right] \left( -36 (4 A - 7 B) d x \cos \left[ \frac{d x}{2} \right] - 36 (4 A - 7 B) d x \cos \left[ c + \frac{d x}{2} \right] - 48 A d x \cos \left[ c + \frac{3 d x}{2} \right] + 84 B d x \cos \left[ c + \frac{3 d x}{2} \right] - 48 A d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 84 B d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 264 A \sin \left[ \frac{d x}{2} \right] - 381 B \sin \left[ \frac{d x}{2} \right] - 120 A \sin \left[ c + \frac{d x}{2} \right] + 147 B \sin \left[ c + \frac{d x}{2} \right] + 164 A \sin \left[ c + \frac{3 d x}{2} \right] - 239 B \sin \left[ c + \frac{3 d x}{2} \right] + 36 A \sin \left[ 2 c + \frac{3 d x}{2} \right] - 63 B \sin \left[ 2 c + \frac{3 d x}{2} \right] + 12 A \sin \left[ 2 c + \frac{5 d x}{2} \right] - 15 B \sin \left[ 2 c + \frac{5 d x}{2} \right] + 12 A \sin \left[ 3 c + \frac{5 d x}{2} \right] - 15 B \sin \left[ 3 c + \frac{5 d x}{2} \right] + 3 B \sin \left[ 3 c + \frac{7 d x}{2} \right] + 3 B \sin \left[ 4 c + \frac{7 d x}{2} \right] \right)$$

■ **Problem 50: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x] (A + B \cos [c + d x])}{(a + a \cos [c + d x])^2} dx$$

Optimal (type 3, 70 leaves, 4 steps):

$$\frac{B x}{a^2} + \frac{(2 A - 5 B) \sin [c + d x]}{3 a^2 d (1 + \cos [c + d x])} - \frac{(A - B) \sin [c + d x]}{3 d (a + a \cos [c + d x])^2}$$

Result (type 3, 153 leaves):

$$\frac{1}{24 a^2 d} \sec \left[ \frac{c}{2} \right] \sec \left[ \frac{1}{2} (c + d x) \right]^3 \left( 9 B d x \cos \left[ \frac{d x}{2} \right] + 9 B d x \cos \left[ c + \frac{d x}{2} \right] + 3 B d x \cos \left[ c + \frac{3 d x}{2} \right] + 3 B d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 6 A \sin \left[ \frac{d x}{2} \right] - 18 B \sin \left[ \frac{d x}{2} \right] - 6 A \sin \left[ c + \frac{d x}{2} \right] + 12 B \sin \left[ c + \frac{d x}{2} \right] + 4 A \sin \left[ c + \frac{3 d x}{2} \right] - 10 B \sin \left[ c + \frac{3 d x}{2} \right] \right)$$

■ **Problem 52: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + d x]) \sec [c + d x]}{(a + a \cos [c + d x])^2} dx$$

Optimal (type 3, 79 leaves, 4 steps):

$$\frac{A \operatorname{ArcTanh}[\sin [c + d x]]}{a^2 d} - \frac{(4 A - B) \sin [c + d x]}{3 a^2 d (1 + \cos [c + d x])} - \frac{(A - B) \sin [c + d x]}{3 d (a + a \cos [c + d x])^2}$$

Result (type 3, 170 leaves):

$$-\frac{1}{3 a^2 d (1 + \cos [c + d x])^2} 2 \cos \left[ \frac{1}{2} (c + d x) \right] \left( 6 A \cos \left[ \frac{1}{2} (c + d x) \right]^3 \left( \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] - \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] \right) + (A - B) \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + 2 (4 A - B) \cos \left[ \frac{1}{2} (c + d x) \right]^2 \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + (A - B) \cos \left[ \frac{1}{2} (c + d x) \right] \tan \left[ \frac{c}{2} \right] \right)$$

■ **Problem 53: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]^2}{(a + a \cos[c + dx])^2} dx$$

Optimal (type 3, 107 leaves, 6 steps):

$$-\frac{(2A - B) \operatorname{ArcTanh}[\sin[c + dx]]}{a^2 d} + \frac{2(5A - 2B) \tan[c + dx]}{3a^2 d} - \frac{(2A - B) \tan[c + dx]}{a^2 d (1 + \cos[c + dx])} - \frac{(A - B) \tan[c + dx]}{3d (a + a \cos[c + dx])^2}$$

Result (type 3, 264 leaves):

$$\frac{1}{3a^2 d (1 + \cos[c + dx])^2} 2 \cos\left[\frac{1}{2}(c + dx)\right] \left( (A - B) \sec\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] + 2(7A - 4B) \cos\left[\frac{1}{2}(c + dx)\right]^2 \sec\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] + 6 \cos\left[\frac{1}{2}(c + dx)\right]^3 \right. \\ \left. \left( (2A - B) \left( \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + (A \sin[dx]) / \left( \left( \cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \right. \right. \right. \\ \left. \left. \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right) \left( \cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) \right) \right) + (A - B) \cos\left[\frac{1}{2}(c + dx)\right] \tan\left[\frac{c}{2}\right] \right)$$

■ **Problem 54: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]^3}{(a + a \cos[c + dx])^2} dx$$

Optimal (type 3, 152 leaves, 7 steps):

$$\frac{(7A - 4B) \operatorname{ArcTanh}[\sin[c + dx]]}{2a^2 d} - \frac{2(8A - 5B) \tan[c + dx]}{3a^2 d} + \\ \frac{(7A - 4B) \sec[c + dx] \tan[c + dx]}{2a^2 d} - \frac{(8A - 5B) \sec[c + dx] \tan[c + dx]}{3a^2 d (1 + \cos[c + dx])} - \frac{(A - B) \sec[c + dx] \tan[c + dx]}{3d (a + a \cos[c + dx])^2}$$

Result (type 3, 574 leaves):



$$\begin{aligned}
& - \frac{2 (7 A - 4 B) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + dx])^2} + \\
& \frac{2 (7 A - 4 B) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + dx])^2} + \frac{1}{48 d (a + a \operatorname{Cos}[c + dx])^2} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \\
& \left( 14 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 14 B \operatorname{Sin}\left[\frac{dx}{2}\right] - 97 A \operatorname{Sin}\left[\frac{3 dx}{2}\right] + 64 B \operatorname{Sin}\left[\frac{3 dx}{2}\right] + 126 A \operatorname{Sin}\left[c - \frac{dx}{2}\right] - 84 B \operatorname{Sin}\left[c - \frac{dx}{2}\right] - 42 A \operatorname{Sin}\left[c + \frac{dx}{2}\right] + \right. \\
& 42 B \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 98 A \operatorname{Sin}\left[2 c + \frac{dx}{2}\right] - 56 B \operatorname{Sin}\left[2 c + \frac{dx}{2}\right] + 3 A \operatorname{Sin}\left[c + \frac{3 dx}{2}\right] - 6 B \operatorname{Sin}\left[c + \frac{3 dx}{2}\right] - 37 A \operatorname{Sin}\left[2 c + \frac{3 dx}{2}\right] + \\
& 34 B \operatorname{Sin}\left[2 c + \frac{3 dx}{2}\right] + 63 A \operatorname{Sin}\left[3 c + \frac{3 dx}{2}\right] - 36 B \operatorname{Sin}\left[3 c + \frac{3 dx}{2}\right] - 75 A \operatorname{Sin}\left[c + \frac{5 dx}{2}\right] + 48 B \operatorname{Sin}\left[c + \frac{5 dx}{2}\right] - 15 A \operatorname{Sin}\left[2 c + \frac{5 dx}{2}\right] + \\
& 6 B \operatorname{Sin}\left[2 c + \frac{5 dx}{2}\right] - 39 A \operatorname{Sin}\left[3 c + \frac{5 dx}{2}\right] + 30 B \operatorname{Sin}\left[3 c + \frac{5 dx}{2}\right] + 21 A \operatorname{Sin}\left[4 c + \frac{5 dx}{2}\right] - 12 B \operatorname{Sin}\left[4 c + \frac{5 dx}{2}\right] - \\
& \left. 32 A \operatorname{Sin}\left[2 c + \frac{7 dx}{2}\right] + 20 B \operatorname{Sin}\left[2 c + \frac{7 dx}{2}\right] - 12 A \operatorname{Sin}\left[3 c + \frac{7 dx}{2}\right] + 6 B \operatorname{Sin}\left[3 c + \frac{7 dx}{2}\right] - 20 A \operatorname{Sin}\left[4 c + \frac{7 dx}{2}\right] + 14 B \operatorname{Sin}\left[4 c + \frac{7 dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 55: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \operatorname{Cos}[c + dx]) \operatorname{Sec}[c + dx]^4}{(a + a \operatorname{Cos}[c + dx])^2} dx$$

Optimal (type 3, 179 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(10 A - 7 B) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2 a^2 d} + \frac{4 (3 A - 2 B) \operatorname{Tan}[c + dx]}{a^2 d} - \frac{(10 A - 7 B) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2 a^2 d} - \\
& \frac{(10 A - 7 B) \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{3 a^2 d (1 + \operatorname{Cos}[c + dx])} - \frac{(A - B) \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{3 d (a + a \operatorname{Cos}[c + dx])^2} + \frac{4 (3 A - 2 B) \operatorname{Tan}[c + dx]^3}{3 a^2 d}
\end{aligned}$$

Result (type 3, 686 leaves):

$$\frac{2(10A - 7B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d(a + a \cos[c + dx])^2} -$$

$$\frac{2(10A - 7B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d(a + a \cos[c + dx])^2} + \frac{1}{96d(a + a \cos[c + dx])^2}$$

$$\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c] \sec[c + dx]^3 \left( -6A \sin\left[\frac{dx}{2}\right] + 45B \sin\left[\frac{dx}{2}\right] + 310A \sin\left[\frac{3dx}{2}\right] - 201B \sin\left[\frac{3dx}{2}\right] - 306A \sin\left[c - \frac{dx}{2}\right] + \right.$$

$$195B \sin\left[c - \frac{dx}{2}\right] + 42A \sin\left[c + \frac{dx}{2}\right] - 51B \sin\left[c + \frac{dx}{2}\right] - 270A \sin\left[2c + \frac{dx}{2}\right] + 189B \sin\left[2c + \frac{dx}{2}\right] + 50A \sin\left[c + \frac{3dx}{2}\right] -$$

$$B \sin\left[c + \frac{3dx}{2}\right] + 90A \sin\left[2c + \frac{3dx}{2}\right] - 81B \sin\left[2c + \frac{3dx}{2}\right] - 170A \sin\left[3c + \frac{3dx}{2}\right] + 119B \sin\left[3c + \frac{3dx}{2}\right] + 198A \sin\left[c + \frac{5dx}{2}\right] -$$

$$129B \sin\left[c + \frac{5dx}{2}\right] + 42A \sin\left[2c + \frac{5dx}{2}\right] - 9B \sin\left[2c + \frac{5dx}{2}\right] + 66A \sin\left[3c + \frac{5dx}{2}\right] - 57B \sin\left[3c + \frac{5dx}{2}\right] - 90A \sin\left[4c + \frac{5dx}{2}\right] +$$

$$63B \sin\left[4c + \frac{5dx}{2}\right] + 114A \sin\left[2c + \frac{7dx}{2}\right] - 75B \sin\left[2c + \frac{7dx}{2}\right] + 36A \sin\left[3c + \frac{7dx}{2}\right] - 15B \sin\left[3c + \frac{7dx}{2}\right] +$$

$$48A \sin\left[4c + \frac{7dx}{2}\right] - 39B \sin\left[4c + \frac{7dx}{2}\right] - 30A \sin\left[5c + \frac{7dx}{2}\right] + 21B \sin\left[5c + \frac{7dx}{2}\right] + 48A \sin\left[3c + \frac{9dx}{2}\right] -$$

$$\left. 32B \sin\left[3c + \frac{9dx}{2}\right] + 22A \sin\left[4c + \frac{9dx}{2}\right] - 12B \sin\left[4c + \frac{9dx}{2}\right] + 26A \sin\left[5c + \frac{9dx}{2}\right] - 20B \sin\left[5c + \frac{9dx}{2}\right] \right)$$

■ **Problem 56: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^5 (A + B \cos[c + dx])}{(a + a \cos[c + dx])^3} dx$$

Optimal (type 3, 218 leaves, 8 steps):

$$\frac{(13A - 23B)x}{2a^3} - \frac{4(19A - 34B) \sin[c + dx]}{5a^3 d} + \frac{(13A - 23B) \cos[c + dx] \sin[c + dx]}{2a^3 d} + \frac{(A - B) \cos[c + dx]^5 \sin[c + dx]}{5d(a + a \cos[c + dx])^3} +$$

$$\frac{(8A - 13B) \cos[c + dx]^4 \sin[c + dx]}{15ad(a + a \cos[c + dx])^2} + \frac{(13A - 23B) \cos[c + dx]^3 \sin[c + dx]}{3d(a^3 + a^3 \cos[c + dx])} + \frac{4(19A - 34B) \sin[c + dx]^3}{15a^3 d}$$

Result (type 3, 491 leaves):

1

$$480 a^3 d (1 + \cos[c + dx])^3$$

$$\begin{aligned} & \cos\left[\frac{1}{2}(c + dx)\right] \sec\left[\frac{c}{2}\right] \left( 600(13A - 23B) dx \cos\left[\frac{dx}{2}\right] + 600(13A - 23B) dx \cos\left[c + \frac{dx}{2}\right] + 3900A dx \cos\left[c + \frac{3dx}{2}\right] - 6900B dx \cos\left[c + \frac{3dx}{2}\right] + \right. \\ & 3900A dx \cos\left[2c + \frac{3dx}{2}\right] - 6900B dx \cos\left[2c + \frac{3dx}{2}\right] + 780A dx \cos\left[2c + \frac{5dx}{2}\right] - 1380B dx \cos\left[2c + \frac{5dx}{2}\right] + 780A dx \cos\left[3c + \frac{5dx}{2}\right] - \\ & 1380B dx \cos\left[3c + \frac{5dx}{2}\right] - 12760A \sin\left[\frac{dx}{2}\right] + 20410B \sin\left[\frac{dx}{2}\right] + 7560A \sin\left[c + \frac{dx}{2}\right] - 11110B \sin\left[c + \frac{dx}{2}\right] - 9230A \sin\left[c + \frac{3dx}{2}\right] + \\ & 15380B \sin\left[c + \frac{3dx}{2}\right] + 930A \sin\left[2c + \frac{3dx}{2}\right] - 380B \sin\left[2c + \frac{3dx}{2}\right] - 2782A \sin\left[2c + \frac{5dx}{2}\right] + 4777B \sin\left[2c + \frac{5dx}{2}\right] - \\ & 750A \sin\left[3c + \frac{5dx}{2}\right] + 1625B \sin\left[3c + \frac{5dx}{2}\right] - 105A \sin\left[3c + \frac{7dx}{2}\right] + 230B \sin\left[3c + \frac{7dx}{2}\right] - 105A \sin\left[4c + \frac{7dx}{2}\right] + 230B \sin\left[4c + \frac{7dx}{2}\right] + \\ & \left. 15A \sin\left[4c + \frac{9dx}{2}\right] - 20B \sin\left[4c + \frac{9dx}{2}\right] + 15A \sin\left[5c + \frac{9dx}{2}\right] - 20B \sin\left[5c + \frac{9dx}{2}\right] + 5B \sin\left[5c + \frac{11dx}{2}\right] + 5B \sin\left[6c + \frac{11dx}{2}\right] \right) \end{aligned}$$

■ **Problem 57: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^4 (A + B \cos[c + dx])}{(a + a \cos[c + dx])^3} dx$$

Optimal (type 3, 193 leaves, 4 steps):

$$\begin{aligned} & -\frac{(6A - 13B)x}{2a^3} + \frac{8(9A - 19B) \sin[c + dx]}{15a^3 d} - \frac{(6A - 13B) \cos[c + dx] \sin[c + dx]}{2a^3 d} + \\ & \frac{(A - B) \cos[c + dx]^4 \sin[c + dx]}{5d(a + a \cos[c + dx])^3} + \frac{(6A - 11B) \cos[c + dx]^3 \sin[c + dx]}{15ad(a + a \cos[c + dx])^2} + \frac{4(9A - 19B) \cos[c + dx]^2 \sin[c + dx]}{15d(a^3 + a^3 \cos[c + dx])} \end{aligned}$$

Result (type 3, 435 leaves):

$$\frac{1}{480 a^3 d (1 + \cos [c + dx])^3} \cos \left[ \frac{1}{2} (c + dx) \right] \sec \left[ \frac{c}{2} \right] \left( -600 (6A - 13B) dx \cos \left[ \frac{dx}{2} \right] - 600 (6A - 13B) dx \cos \left[ c + \frac{dx}{2} \right] - 1800 A dx \cos \left[ c + \frac{3dx}{2} \right] + 3900 B dx \cos \left[ c + \frac{3dx}{2} \right] - 1800 A dx \cos \left[ 2c + \frac{3dx}{2} \right] + 3900 B dx \cos \left[ 2c + \frac{3dx}{2} \right] - 360 A dx \cos \left[ 2c + \frac{5dx}{2} \right] + 780 B dx \cos \left[ 2c + \frac{5dx}{2} \right] - 360 A dx \cos \left[ 3c + \frac{5dx}{2} \right] + 780 B dx \cos \left[ 3c + \frac{5dx}{2} \right] + 7020 A \sin \left[ \frac{dx}{2} \right] - 12760 B \sin \left[ \frac{dx}{2} \right] - 4500 A \sin \left[ c + \frac{dx}{2} \right] + 7560 B \sin \left[ c + \frac{dx}{2} \right] + 4860 A \sin \left[ c + \frac{3dx}{2} \right] - 9230 B \sin \left[ c + \frac{3dx}{2} \right] - 900 A \sin \left[ 2c + \frac{3dx}{2} \right] + 930 B \sin \left[ 2c + \frac{3dx}{2} \right] + 1452 A \sin \left[ 2c + \frac{5dx}{2} \right] - 2782 B \sin \left[ 2c + \frac{5dx}{2} \right] + 300 A \sin \left[ 3c + \frac{5dx}{2} \right] - 750 B \sin \left[ 3c + \frac{5dx}{2} \right] + 60 A \sin \left[ 3c + \frac{7dx}{2} \right] - 105 B \sin \left[ 3c + \frac{7dx}{2} \right] + 60 A \sin \left[ 4c + \frac{7dx}{2} \right] - 105 B \sin \left[ 4c + \frac{7dx}{2} \right] + 15 B \sin \left[ 4c + \frac{9dx}{2} \right] + 15 B \sin \left[ 5c + \frac{9dx}{2} \right] \right)$$

■ **Problem 58: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + dx]^3 (A + B \cos [c + dx])}{(a + a \cos [c + dx])^3} dx$$

Optimal (type 3, 147 leaves, 7 steps):

$$\frac{(A - 3B)x}{a^3} - \frac{(7A - 27B) \sin [c + dx]}{15 a^3 d} + \frac{(A - B) \cos [c + dx]^3 \sin [c + dx]}{5 d (a + a \cos [c + dx])^3} + \frac{(4A - 9B) \cos [c + dx]^2 \sin [c + dx]}{15 a d (a + a \cos [c + dx])^2} - \frac{(A - 3B) \sin [c + dx]}{d (a^3 + a^3 \cos [c + dx])}$$

Result (type 3, 361 leaves):

$$\frac{1}{120 a^3 d (1 + \cos [c + dx])^3} \cos \left[ \frac{1}{2} (c + dx) \right] \sec \left[ \frac{c}{2} \right] \left( 300 (A - 3B) dx \cos \left[ \frac{dx}{2} \right] + 300 (A - 3B) dx \cos \left[ c + \frac{dx}{2} \right] + 150 A dx \cos \left[ c + \frac{3dx}{2} \right] - 450 B dx \cos \left[ c + \frac{3dx}{2} \right] + 150 A dx \cos \left[ 2c + \frac{3dx}{2} \right] - 450 B dx \cos \left[ 2c + \frac{3dx}{2} \right] + 30 A dx \cos \left[ 2c + \frac{5dx}{2} \right] - 90 B dx \cos \left[ 2c + \frac{5dx}{2} \right] + 30 A dx \cos \left[ 3c + \frac{5dx}{2} \right] - 90 B dx \cos \left[ 3c + \frac{5dx}{2} \right] - 740 A \sin \left[ \frac{dx}{2} \right] + 1755 B \sin \left[ \frac{dx}{2} \right] + 540 A \sin \left[ c + \frac{dx}{2} \right] - 1125 B \sin \left[ c + \frac{dx}{2} \right] - 460 A \sin \left[ c + \frac{3dx}{2} \right] + 1215 B \sin \left[ c + \frac{3dx}{2} \right] + 180 A \sin \left[ 2c + \frac{3dx}{2} \right] - 225 B \sin \left[ 2c + \frac{3dx}{2} \right] - 128 A \sin \left[ 2c + \frac{5dx}{2} \right] + 363 B \sin \left[ 2c + \frac{5dx}{2} \right] + 75 B \sin \left[ 3c + \frac{5dx}{2} \right] + 15 B \sin \left[ 3c + \frac{7dx}{2} \right] + 15 B \sin \left[ 4c + \frac{7dx}{2} \right] \right)$$

■ **Problem 59: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^2 (A + B \cos[c + dx])}{(a + a \cos[c + dx])^3} dx$$

Optimal (type 3, 116 leaves, 5 steps):

$$\frac{Bx}{a^3} + \frac{(A - B) \cos[c + dx]^2 \sin[c + dx]}{5d(a + a \cos[c + dx])^3} - \frac{(2A - 7B) \sin[c + dx]}{15ad(a + a \cos[c + dx])^2} + \frac{(4A - 29B) \sin[c + dx]}{15d(a^3 + a^3 \cos[c + dx])}$$

Result (type 3, 241 leaves):

$$\frac{1}{480a^3d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 \left( 150Bdx \cos\left[\frac{dx}{2}\right] + 150Bdx \cos\left[c + \frac{dx}{2}\right] + 75Bdx \cos\left[c + \frac{3dx}{2}\right] + 75Bdx \cos\left[2c + \frac{3dx}{2}\right] + 15Bdx \cos\left[2c + \frac{5dx}{2}\right] + 15Bdx \cos\left[3c + \frac{5dx}{2}\right] + 80A \sin\left[\frac{dx}{2}\right] - 370B \sin\left[\frac{dx}{2}\right] - 60A \sin\left[c + \frac{dx}{2}\right] + 270B \sin\left[c + \frac{dx}{2}\right] + 40A \sin\left[c + \frac{3dx}{2}\right] - 230B \sin\left[c + \frac{3dx}{2}\right] - 30A \sin\left[2c + \frac{3dx}{2}\right] + 90B \sin\left[2c + \frac{3dx}{2}\right] + 14A \sin\left[2c + \frac{5dx}{2}\right] - 64B \sin\left[2c + \frac{5dx}{2}\right] \right)$$

■ **Problem 63: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx]) \operatorname{Sec}[c + dx]^2}{(a + a \cos[c + dx])^3} dx$$

Optimal (type 3, 145 leaves, 7 steps):

$$-\frac{(3A - B) \operatorname{ArcTanh}[\sin[c + dx]]}{a^3d} + \frac{2(36A - 11B) \tan[c + dx]}{15a^3d} - \frac{(A - B) \tan[c + dx]}{5d(a + a \cos[c + dx])^3} - \frac{(9A - 4B) \tan[c + dx]}{15ad(a + a \cos[c + dx])^2} - \frac{(3A - B) \tan[c + dx]}{d(a^3 + a^3 \cos[c + dx])}$$

Result (type 3, 482 leaves):

$$\frac{1}{120 a^3 d (1 + \cos [c + d x])^3} \left( 960 (3 A - B) \cos \left[ \frac{1}{2} (c + d x) \right]^6 \left( \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] - \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) + \cos \left[ \frac{1}{2} (c + d x) \right] \sec \left[ \frac{c}{2} \right] \sec [c] \sec [c + d x] \left( -5 (51 A - 32 B) \sin \left[ \frac{d x}{2} \right] + (567 A - 167 B) \sin \left[ \frac{3 d x}{2} \right] - 600 A \sin \left[ c - \frac{d x}{2} \right] + 170 B \sin \left[ c - \frac{d x}{2} \right] + 375 A \sin \left[ c + \frac{d x}{2} \right] - 170 B \sin \left[ c + \frac{d x}{2} \right] - 480 A \sin \left[ 2 c + \frac{d x}{2} \right] + 160 B \sin \left[ 2 c + \frac{d x}{2} \right] - 60 A \sin \left[ c + \frac{3 d x}{2} \right] + 75 B \sin \left[ c + \frac{3 d x}{2} \right] + 402 A \sin \left[ 2 c + \frac{3 d x}{2} \right] - 167 B \sin \left[ 2 c + \frac{3 d x}{2} \right] - 225 A \sin \left[ 3 c + \frac{3 d x}{2} \right] + 75 B \sin \left[ 3 c + \frac{3 d x}{2} \right] + 315 A \sin \left[ c + \frac{5 d x}{2} \right] - 95 B \sin \left[ c + \frac{5 d x}{2} \right] + 30 A \sin \left[ 2 c + \frac{5 d x}{2} \right] + 15 B \sin \left[ 2 c + \frac{5 d x}{2} \right] + 240 A \sin \left[ 3 c + \frac{5 d x}{2} \right] - 95 B \sin \left[ 3 c + \frac{5 d x}{2} \right] - 45 A \sin \left[ 4 c + \frac{5 d x}{2} \right] + 15 B \sin \left[ 4 c + \frac{5 d x}{2} \right] + 72 A \sin \left[ 2 c + \frac{7 d x}{2} \right] - 22 B \sin \left[ 2 c + \frac{7 d x}{2} \right] + 15 A \sin \left[ 3 c + \frac{7 d x}{2} \right] + 57 A \sin \left[ 4 c + \frac{7 d x}{2} \right] - 22 B \sin \left[ 4 c + \frac{7 d x}{2} \right] \right)$$

■ **Problem 64: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + d x]) \sec [c + d x]^3}{(a + a \cos [c + d x])^3} dx$$

Optimal (type 3, 196 leaves, 8 steps):

$$\frac{(13 A - 6 B) \operatorname{ArcTanh}[\sin [c + d x]]}{2 a^3 d} - \frac{8 (19 A - 9 B) \tan [c + d x]}{15 a^3 d} + \frac{(13 A - 6 B) \sec [c + d x] \tan [c + d x]}{2 a^3 d} - \frac{(A - B) \sec [c + d x] \tan [c + d x]}{5 d (a + a \cos [c + d x])^3} - \frac{(11 A - 6 B) \sec [c + d x] \tan [c + d x]}{15 a d (a + a \cos [c + d x])^2} - \frac{4 (19 A - 9 B) \sec [c + d x] \tan [c + d x]}{15 d (a^3 + a^3 \cos [c + d x])}$$

Result (type 3, 686 leaves):

$$\begin{aligned}
& - \frac{4 (13 A - 6 B) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + dx])^3} + \\
& \frac{4 (13 A - 6 B) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + dx])^3} + \frac{1}{480 d (a + a \operatorname{Cos}[c + dx])^3} \\
& \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \left( 1235 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 870 B \operatorname{Sin}\left[\frac{dx}{2}\right] - 3805 A \operatorname{Sin}\left[\frac{3 dx}{2}\right] + 1830 B \operatorname{Sin}\left[\frac{3 dx}{2}\right] + 4329 A \operatorname{Sin}\left[c - \frac{dx}{2}\right] - \right. \\
& 2094 B \operatorname{Sin}\left[c - \frac{dx}{2}\right] - 1989 A \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 1314 B \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 3575 A \operatorname{Sin}\left[2c + \frac{dx}{2}\right] - 1650 B \operatorname{Sin}\left[2c + \frac{dx}{2}\right] + 475 A \operatorname{Sin}\left[c + \frac{3 dx}{2}\right] - \\
& 450 B \operatorname{Sin}\left[c + \frac{3 dx}{2}\right] - 2005 A \operatorname{Sin}\left[2c + \frac{3 dx}{2}\right] + 1230 B \operatorname{Sin}\left[2c + \frac{3 dx}{2}\right] + 2275 A \operatorname{Sin}\left[3c + \frac{3 dx}{2}\right] - 1050 B \operatorname{Sin}\left[3c + \frac{3 dx}{2}\right] - \\
& 2673 A \operatorname{Sin}\left[c + \frac{5 dx}{2}\right] + 1278 B \operatorname{Sin}\left[c + \frac{5 dx}{2}\right] - 105 A \operatorname{Sin}\left[2c + \frac{5 dx}{2}\right] - 90 B \operatorname{Sin}\left[2c + \frac{5 dx}{2}\right] - 1593 A \operatorname{Sin}\left[3c + \frac{5 dx}{2}\right] + \\
& 918 B \operatorname{Sin}\left[3c + \frac{5 dx}{2}\right] + 975 A \operatorname{Sin}\left[4c + \frac{5 dx}{2}\right] - 450 B \operatorname{Sin}\left[4c + \frac{5 dx}{2}\right] - 1325 A \operatorname{Sin}\left[2c + \frac{7 dx}{2}\right] + 630 B \operatorname{Sin}\left[2c + \frac{7 dx}{2}\right] - \\
& 255 A \operatorname{Sin}\left[3c + \frac{7 dx}{2}\right] + 60 B \operatorname{Sin}\left[3c + \frac{7 dx}{2}\right] - 875 A \operatorname{Sin}\left[4c + \frac{7 dx}{2}\right] + 480 B \operatorname{Sin}\left[4c + \frac{7 dx}{2}\right] + 195 A \operatorname{Sin}\left[5c + \frac{7 dx}{2}\right] - 90 B \operatorname{Sin}\left[5c + \frac{7 dx}{2}\right] - \\
& \left. 304 A \operatorname{Sin}\left[3c + \frac{9 dx}{2}\right] + 144 B \operatorname{Sin}\left[3c + \frac{9 dx}{2}\right] - 90 A \operatorname{Sin}\left[4c + \frac{9 dx}{2}\right] + 30 B \operatorname{Sin}\left[4c + \frac{9 dx}{2}\right] - 214 A \operatorname{Sin}\left[5c + \frac{9 dx}{2}\right] + 114 B \operatorname{Sin}\left[5c + \frac{9 dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 65: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + dx]^5 (A + B \operatorname{Cos}[c + dx])}{(a + a \operatorname{Cos}[c + dx])^4} dx$$

Optimal (type 3, 229 leaves, 5 steps):

$$\begin{aligned}
& - \frac{(8 A - 21 B) x}{2 a^4} + \frac{8 (83 A - 216 B) \operatorname{Sin}[c + dx]}{105 a^4 d} - \frac{(8 A - 21 B) \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]}{2 a^4 d} + \frac{(52 A - 129 B) \operatorname{Cos}[c + dx]^3 \operatorname{Sin}[c + dx]}{105 a^4 d (1 + \operatorname{Cos}[c + dx])^2} + \\
& \frac{4 (83 A - 216 B) \operatorname{Cos}[c + dx]^2 \operatorname{Sin}[c + dx]}{105 a^4 d (1 + \operatorname{Cos}[c + dx])} + \frac{(A - B) \operatorname{Cos}[c + dx]^5 \operatorname{Sin}[c + dx]}{7 d (a + a \operatorname{Cos}[c + dx])^4} + \frac{(A - 2 B) \operatorname{Cos}[c + dx]^4 \operatorname{Sin}[c + dx]}{5 a d (a + a \operatorname{Cos}[c + dx])^3}
\end{aligned}$$

Result (type 3, 555 leaves):

$$\frac{1}{6720 a^4 d (1 + \cos [c + d x])^4} \cos \left[ \frac{1}{2} (c + d x) \right] \sec \left[ \frac{c}{2} \right] \left( -14700 (8 A - 21 B) d x \cos \left[ \frac{d x}{2} \right] - 14700 (8 A - 21 B) d x \cos \left[ c + \frac{d x}{2} \right] - 70560 A d x \cos \left[ c + \frac{3 d x}{2} \right] + 185220 B d x \cos \left[ c + \frac{3 d x}{2} \right] - 70560 A d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 185220 B d x \cos \left[ 2 c + \frac{3 d x}{2} \right] - 23520 A d x \cos \left[ 2 c + \frac{5 d x}{2} \right] + 61740 B d x \cos \left[ 2 c + \frac{5 d x}{2} \right] - 23520 A d x \cos \left[ 3 c + \frac{5 d x}{2} \right] + 61740 B d x \cos \left[ 3 c + \frac{5 d x}{2} \right] - 3360 A d x \cos \left[ 3 c + \frac{7 d x}{2} \right] + 8820 B d x \cos \left[ 3 c + \frac{7 d x}{2} \right] - 3360 A d x \cos \left[ 4 c + \frac{7 d x}{2} \right] + 8820 B d x \cos \left[ 4 c + \frac{7 d x}{2} \right] + 243320 A \sin \left[ \frac{d x}{2} \right] - 539490 B \sin \left[ \frac{d x}{2} \right] - 184520 A \sin \left[ c + \frac{d x}{2} \right] + 386190 B \sin \left[ c + \frac{d x}{2} \right] + 184464 A \sin \left[ c + \frac{3 d x}{2} \right] - 422478 B \sin \left[ c + \frac{3 d x}{2} \right] - 72240 A \sin \left[ 2 c + \frac{3 d x}{2} \right] + 132930 B \sin \left[ 2 c + \frac{3 d x}{2} \right] + 77168 A \sin \left[ 2 c + \frac{5 d x}{2} \right] - 181461 B \sin \left[ 2 c + \frac{5 d x}{2} \right] - 8400 A \sin \left[ 3 c + \frac{5 d x}{2} \right] + 3675 B \sin \left[ 3 c + \frac{5 d x}{2} \right] + 15164 A \sin \left[ 3 c + \frac{7 d x}{2} \right] - 36003 B \sin \left[ 3 c + \frac{7 d x}{2} \right] + 2940 A \sin \left[ 4 c + \frac{7 d x}{2} \right] - 9555 B \sin \left[ 4 c + \frac{7 d x}{2} \right] + 420 A \sin \left[ 4 c + \frac{9 d x}{2} \right] - 945 B \sin \left[ 4 c + \frac{9 d x}{2} \right] + 420 A \sin \left[ 5 c + \frac{9 d x}{2} \right] - 945 B \sin \left[ 5 c + \frac{9 d x}{2} \right] + 105 B \sin \left[ 5 c + \frac{11 d x}{2} \right] + 105 B \sin \left[ 6 c + \frac{11 d x}{2} \right] \right)$$

■ **Problem 66: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^4 (A + B \cos [c + d x])}{(a + a \cos [c + d x])^4} dx$$

Optimal (type 3, 185 leaves, 8 steps):

$$\frac{(A - 4 B) x}{a^4} - \frac{(55 A - 244 B) \sin [c + d x]}{105 a^4 d} + \frac{(25 A - 88 B) \cos [c + d x]^2 \sin [c + d x]}{105 a^4 d (1 + \cos [c + d x])^2} - \frac{(A - 4 B) \sin [c + d x]}{a^4 d (1 + \cos [c + d x])} + \frac{(A - B) \cos [c + d x]^4 \sin [c + d x]}{7 d (a + a \cos [c + d x])^4} + \frac{(5 A - 12 B) \cos [c + d x]^3 \sin [c + d x]}{35 a d (a + a \cos [c + d x])^3}$$

Result (type 3, 481 leaves):



$$\frac{1}{1680 a^4 d (1 + \cos[c + dx])^4} \cos\left[\frac{1}{2}(c + dx)\right] \sec\left[\frac{c}{2}\right]$$

$$\left( 7350 (A - 4B) dx \cos\left[\frac{dx}{2}\right] + 7350 (A - 4B) dx \cos\left[c + \frac{dx}{2}\right] + 4410 A dx \cos\left[c + \frac{3dx}{2}\right] - 17640 B dx \cos\left[c + \frac{3dx}{2}\right] + 4410 A dx \cos\left[2c + \frac{3dx}{2}\right] - \right.$$

$$17640 B dx \cos\left[2c + \frac{3dx}{2}\right] + 1470 A dx \cos\left[2c + \frac{5dx}{2}\right] - 5880 B dx \cos\left[2c + \frac{5dx}{2}\right] + 1470 A dx \cos\left[3c + \frac{5dx}{2}\right] - 5880 B dx \cos\left[3c + \frac{5dx}{2}\right] + \left.$$

$$210 A dx \cos\left[3c + \frac{7dx}{2}\right] - 840 B dx \cos\left[3c + \frac{7dx}{2}\right] + 210 A dx \cos\left[4c + \frac{7dx}{2}\right] - 840 B dx \cos\left[4c + \frac{7dx}{2}\right] - 19880 A \sin\left[\frac{dx}{2}\right] + \left.$$

$$60830 B \sin\left[\frac{dx}{2}\right] + 16520 A \sin\left[c + \frac{dx}{2}\right] - 46130 B \sin\left[c + \frac{dx}{2}\right] - 14280 A \sin\left[c + \frac{3dx}{2}\right] + 46116 B \sin\left[c + \frac{3dx}{2}\right] + 7560 A \sin\left[2c + \frac{3dx}{2}\right] - \left.$$

$$18060 B \sin\left[2c + \frac{3dx}{2}\right] - 5600 A \sin\left[2c + \frac{5dx}{2}\right] + 19292 B \sin\left[2c + \frac{5dx}{2}\right] + 1680 A \sin\left[3c + \frac{5dx}{2}\right] - 2100 B \sin\left[3c + \frac{5dx}{2}\right] - \left.$$

$$1040 A \sin\left[3c + \frac{7dx}{2}\right] + 3791 B \sin\left[3c + \frac{7dx}{2}\right] + 735 B \sin\left[4c + \frac{7dx}{2}\right] + 105 B \sin\left[4c + \frac{9dx}{2}\right] + 105 B \sin\left[5c + \frac{9dx}{2}\right] \left. \right)$$

■ **Problem 67: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^3 (A + B \cos[c + dx])}{(a + a \cos[c + dx])^4} dx$$

Optimal (type 3, 154 leaves, 6 steps):

$$\frac{Bx}{a^4} - \frac{(6A - 55B) \sin[c + dx]}{105 a^4 d (1 + \cos[c + dx])^2} + \frac{(12A - 215B) \sin[c + dx]}{105 a^4 d (1 + \cos[c + dx])} + \frac{(A - B) \cos[c + dx]^3 \sin[c + dx]}{7d (a + a \cos[c + dx])^4} + \frac{(3A - 10B) \cos[c + dx]^2 \sin[c + dx]}{35 a d (a + a \cos[c + dx])^3}$$

Result (type 3, 329 leaves):

$$\frac{1}{13440 a^4 d} \sec\left[\frac{c}{2}\right] \sec\left[\frac{1}{2}(c + dx)\right]^7$$

$$\left( 3675 B dx \cos\left[\frac{dx}{2}\right] + 3675 B dx \cos\left[c + \frac{dx}{2}\right] + 2205 B dx \cos\left[c + \frac{3dx}{2}\right] + 2205 B dx \cos\left[2c + \frac{3dx}{2}\right] + 735 B dx \cos\left[2c + \frac{5dx}{2}\right] + \right.$$

$$735 B dx \cos\left[3c + \frac{5dx}{2}\right] + 105 B dx \cos\left[3c + \frac{7dx}{2}\right] + 105 B dx \cos\left[4c + \frac{7dx}{2}\right] + 1260 A \sin\left[\frac{dx}{2}\right] - 9940 B \sin\left[\frac{dx}{2}\right] - 1260 A \sin\left[c + \frac{dx}{2}\right] + \left.$$

$$8260 B \sin\left[c + \frac{dx}{2}\right] + 882 A \sin\left[c + \frac{3dx}{2}\right] - 7140 B \sin\left[c + \frac{3dx}{2}\right] - 630 A \sin\left[2c + \frac{3dx}{2}\right] + 3780 B \sin\left[2c + \frac{3dx}{2}\right] + 294 A \sin\left[2c + \frac{5dx}{2}\right] - \left.$$

$$2800 B \sin\left[2c + \frac{5dx}{2}\right] - 210 A \sin\left[3c + \frac{5dx}{2}\right] + 840 B \sin\left[3c + \frac{5dx}{2}\right] + 72 A \sin\left[3c + \frac{7dx}{2}\right] - 520 B \sin\left[3c + \frac{7dx}{2}\right] \left. \right)$$

■ **Problem 72: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]^2}{(a + a \cos[c + dx])^4} dx$$

Optimal (type 3, 175 leaves, 8 steps):

$$-\frac{(4A - B) \operatorname{ArcTanh}[\sin[c + dx]]}{a^4 d} + \frac{8(83A - 20B) \tan[c + dx]}{105 a^4 d} - \frac{(88A - 25B) \tan[c + dx]}{105 a^4 d (1 + \cos[c + dx])^2} - \frac{(4A - B) \tan[c + dx]}{a^4 d (1 + \cos[c + dx])} - \frac{(A - B) \tan[c + dx]}{7d(a + a \cos[c + dx])^4} - \frac{(12A - 5B) \tan[c + dx]}{35 a d (a + a \cos[c + dx])^3}$$

Result (type 3, 670 leaves):

$$\frac{16(4A - B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d(a + a \cos[c + dx])^4} - \frac{16(4A - B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d(a + a \cos[c + dx])^4} + \frac{1}{1680 d (a + a \cos[c + dx])^4} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + dx]$$

$$\left(-10780A \sin\left[\frac{dx}{2}\right] + 4165B \sin\left[\frac{dx}{2}\right] + 18788A \sin\left[\frac{3dx}{2}\right] - 4445B \sin\left[\frac{3dx}{2}\right] - 20524A \sin\left[c - \frac{dx}{2}\right] + 4795B \sin\left[c - \frac{dx}{2}\right] + 14644A \sin\left[c + \frac{dx}{2}\right] - 4795B \sin\left[c + \frac{dx}{2}\right] - 16660A \sin\left[2c + \frac{dx}{2}\right] + 4165B \sin\left[2c + \frac{dx}{2}\right] - 4690A \sin\left[c + \frac{3dx}{2}\right] + 2275B \sin\left[c + \frac{3dx}{2}\right] + 14378A \sin\left[2c + \frac{3dx}{2}\right] - 4445B \sin\left[2c + \frac{3dx}{2}\right] - 9100A \sin\left[3c + \frac{3dx}{2}\right] + 2275B \sin\left[3c + \frac{3dx}{2}\right] + 11668A \sin\left[c + \frac{5dx}{2}\right] - 2785B \sin\left[c + \frac{5dx}{2}\right] - 630A \sin\left[2c + \frac{5dx}{2}\right] + 735B \sin\left[2c + \frac{5dx}{2}\right] + 9358A \sin\left[3c + \frac{5dx}{2}\right] - 2785B \sin\left[3c + \frac{5dx}{2}\right] - 2940A \sin\left[4c + \frac{5dx}{2}\right] + 735B \sin\left[4c + \frac{5dx}{2}\right] + 4228A \sin\left[2c + \frac{7dx}{2}\right] - 1015B \sin\left[2c + \frac{7dx}{2}\right] + 315A \sin\left[3c + \frac{7dx}{2}\right] + 105B \sin\left[3c + \frac{7dx}{2}\right] + 3493A \sin\left[4c + \frac{7dx}{2}\right] - 1015B \sin\left[4c + \frac{7dx}{2}\right] - 420A \sin\left[5c + \frac{7dx}{2}\right] + 105B \sin\left[5c + \frac{7dx}{2}\right] + 664A \sin\left[3c + \frac{9dx}{2}\right] - 160B \sin\left[3c + \frac{9dx}{2}\right] + 105A \sin\left[4c + \frac{9dx}{2}\right] + 559A \sin\left[5c + \frac{9dx}{2}\right] - 160B \sin\left[5c + \frac{9dx}{2}\right]\right)$$

■ **Problem 73: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx]) \operatorname{Sec}[c + dx]^3}{(a + a \cos[c + dx])^4} dx$$

Optimal (type 3, 232 leaves, 9 steps):

$$\frac{(21A - 8B) \operatorname{ArcTanh}[\sin[c + dx]]}{2 a^4 d} - \frac{8(216A - 83B) \tan[c + dx]}{105 a^4 d} + \frac{(21A - 8B) \operatorname{Sec}[c + dx] \tan[c + dx]}{2 a^4 d} - \frac{(129A - 52B) \operatorname{Sec}[c + dx] \tan[c + dx]}{105 a^4 d (1 + \cos[c + dx])^2} - \frac{4(216A - 83B) \operatorname{Sec}[c + dx] \tan[c + dx]}{105 a^4 d (1 + \cos[c + dx])} - \frac{(A - B) \operatorname{Sec}[c + dx] \tan[c + dx]}{7d(a + a \cos[c + dx])^4} - \frac{(2A - B) \operatorname{Sec}[c + dx] \tan[c + dx]}{5 a d (a + a \cos[c + dx])^3}$$

Result (type 3, 798 leaves):

$$\begin{aligned}
& - \frac{8 (21 A - 8 B) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + dx])^4} + \\
& \frac{8 (21 A - 8 B) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + dx])^4} + \frac{1}{6720 d (a + a \operatorname{Cos}[c + dx])^4} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \\
& \left( 73206 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 38668 B \operatorname{Sin}\left[\frac{dx}{2}\right] - 166668 A \operatorname{Sin}\left[\frac{3 dx}{2}\right] + 64384 B \operatorname{Sin}\left[\frac{3 dx}{2}\right] + 183162 A \operatorname{Sin}\left[c - \frac{dx}{2}\right] - 70896 B \operatorname{Sin}\left[c - \frac{dx}{2}\right] - \right. \\
& 100842 A \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 50316 B \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 155526 A \operatorname{Sin}\left[2c + \frac{dx}{2}\right] - 59248 B \operatorname{Sin}\left[2c + \frac{dx}{2}\right] + 37380 A \operatorname{Sin}\left[c + \frac{3 dx}{2}\right] - \\
& 22820 B \operatorname{Sin}\left[c + \frac{3 dx}{2}\right] - 101148 A \operatorname{Sin}\left[2c + \frac{3 dx}{2}\right] + 48004 B \operatorname{Sin}\left[2c + \frac{3 dx}{2}\right] + 102900 A \operatorname{Sin}\left[3c + \frac{3 dx}{2}\right] - 39200 B \operatorname{Sin}\left[3c + \frac{3 dx}{2}\right] - \\
& 119364 A \operatorname{Sin}\left[c + \frac{5 dx}{2}\right] + 46032 B \operatorname{Sin}\left[c + \frac{5 dx}{2}\right] + 8820 A \operatorname{Sin}\left[2c + \frac{5 dx}{2}\right] - 8750 B \operatorname{Sin}\left[2c + \frac{5 dx}{2}\right] - 78204 A \operatorname{Sin}\left[3c + \frac{5 dx}{2}\right] + \\
& 35742 B \operatorname{Sin}\left[3c + \frac{5 dx}{2}\right] + 49980 A \operatorname{Sin}\left[4c + \frac{5 dx}{2}\right] - 19040 B \operatorname{Sin}\left[4c + \frac{5 dx}{2}\right] - 64053 A \operatorname{Sin}\left[2c + \frac{7 dx}{2}\right] + 24664 B \operatorname{Sin}\left[2c + \frac{7 dx}{2}\right] - \\
& 3885 A \operatorname{Sin}\left[3c + \frac{7 dx}{2}\right] - 1050 B \operatorname{Sin}\left[3c + \frac{7 dx}{2}\right] - 44733 A \operatorname{Sin}\left[4c + \frac{7 dx}{2}\right] + 19834 B \operatorname{Sin}\left[4c + \frac{7 dx}{2}\right] + 15435 A \operatorname{Sin}\left[5c + \frac{7 dx}{2}\right] - \\
& 5880 B \operatorname{Sin}\left[5c + \frac{7 dx}{2}\right] - 21987 A \operatorname{Sin}\left[3c + \frac{9 dx}{2}\right] + 8456 B \operatorname{Sin}\left[3c + \frac{9 dx}{2}\right] - 3675 A \operatorname{Sin}\left[4c + \frac{9 dx}{2}\right] + 630 B \operatorname{Sin}\left[4c + \frac{9 dx}{2}\right] - \\
& 16107 A \operatorname{Sin}\left[5c + \frac{9 dx}{2}\right] + 6986 B \operatorname{Sin}\left[5c + \frac{9 dx}{2}\right] + 2205 A \operatorname{Sin}\left[6c + \frac{9 dx}{2}\right] - 840 B \operatorname{Sin}\left[6c + \frac{9 dx}{2}\right] - 3456 A \operatorname{Sin}\left[4c + \frac{11 dx}{2}\right] + \\
& \left. 1328 B \operatorname{Sin}\left[4c + \frac{11 dx}{2}\right] - 840 A \operatorname{Sin}\left[5c + \frac{11 dx}{2}\right] + 210 B \operatorname{Sin}\left[5c + \frac{11 dx}{2}\right] - 2616 A \operatorname{Sin}\left[6c + \frac{11 dx}{2}\right] + 1118 B \operatorname{Sin}\left[6c + \frac{11 dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 78: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \operatorname{Cos}[c + dx]} (A + B \operatorname{Cos}[c + dx]) \operatorname{Sec}[c + dx] dx$$

Optimal (type 3, 66 leaves, 3 steps):

$$\frac{2 \sqrt{a} A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{d} + \frac{2 a B \operatorname{Sin}[c+dx]}{d \sqrt{a+a \operatorname{Cos}[c+dx]}}$$

Result (type 3, 1393 leaves):

$$\begin{aligned}
& - \left( \left( \left( \frac{1}{4} - \frac{i}{4} \right) A (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right. \\
& (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + \\
& \left. \left. 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \sqrt{a(1 + \operatorname{Cos}[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right) - \\
& \frac{i A \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{-\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{\sqrt{2} d} - \\
& \frac{i A \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{\sqrt{2} d} - \\
& \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{2\sqrt{2} d} - \\
& \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{2\sqrt{2} d} + \\
& \frac{2 B \cos \left[ \frac{dx}{2} \right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] \sin \left[ \frac{c}{2} \right]}{d} - \\
& \frac{2 i A \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Cot} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{d \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} + \\
& \frac{1}{d \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right)} \\
& \sqrt{2} A \sqrt{a(1 + \cos[c + dx])} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] \\
& \left( -dx \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right) \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} + \\
& \frac{2 B \cos \left[ \frac{c}{2} \right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] \sin \left[ \frac{dx}{2} \right]}{d}
\end{aligned}$$

■ **Problem 79: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos [c + d x]} (A + B \cos [c + d x]) \sec [c + d x]^2 dx$$

Optimal (type 3, 68 leaves, 3 steps):

$$\frac{\sqrt{a} (A + 2 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}}\right]}{d} + \frac{a A \tan [c + d x]}{d \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 470 leaves):

$$\begin{aligned} & \frac{1}{8 d} \sqrt{a (1 + \cos [c + d x])} \sec \left[ \frac{1}{2} (c + d x) \right] \\ & \left( \frac{2 \sqrt{2} (A + 2 B) \operatorname{Log}\left[\sqrt{2} + 2 \sin \left[ \frac{1}{2} (c + d x) \right]\right]}{-1 + \sqrt{2} \sin \left[ \frac{c}{2} \right]} + \frac{2 i (A + 2 B) \operatorname{ArcTan}\left[\frac{\cos \left[ \frac{1}{4} (c + d x) \right] - (-1 + \sqrt{2}) \sin \left[ \frac{1}{4} (c + d x) \right]}{(1 + \sqrt{2}) \cos \left[ \frac{1}{4} (c + d x) \right] - \sin \left[ \frac{1}{4} (c + d x) \right]}\right] (\sqrt{2} - 2 \sin \left[ \frac{c}{2} \right])}{-1 + \sqrt{2} \sin \left[ \frac{c}{2} \right]} + \right. \\ & \frac{2 i (A + 2 B) \operatorname{ArcTan}\left[\frac{\cos \left[ \frac{1}{4} (c + d x) \right] - (1 + \sqrt{2}) \sin \left[ \frac{1}{4} (c + d x) \right]}{(-1 + \sqrt{2}) \cos \left[ \frac{1}{4} (c + d x) \right] - \sin \left[ \frac{1}{4} (c + d x) \right]}\right] (\sqrt{2} - 2 \sin \left[ \frac{c}{2} \right])}{-1 + \sqrt{2} \sin \left[ \frac{c}{2} \right]} + \\ & \frac{(A + 2 B) \operatorname{Log}\left[2 - \sqrt{2} \cos \left[ \frac{1}{2} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + d x) \right]\right] (\sqrt{2} - 2 \sin \left[ \frac{c}{2} \right])}{-1 + \sqrt{2} \sin \left[ \frac{c}{2} \right]} + \\ & \frac{(A + 2 B) \operatorname{Log}\left[2 + \sqrt{2} \cos \left[ \frac{1}{2} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + d x) \right]\right] (\sqrt{2} - 2 \sin \left[ \frac{c}{2} \right])}{-1 + \sqrt{2} \sin \left[ \frac{c}{2} \right]} + \\ & \left. \frac{4 A}{\cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right]} - \frac{4 A}{\cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right]} \right) \end{aligned}$$

■ **Problem 80: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos [c + d x]} (A + B \cos [c + d x]) \sec [c + d x]^3 dx$$

Optimal (type 3, 117 leaves, 4 steps):

$$\frac{\sqrt{a} (3 A + 4 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}}\right]}{4 d} + \frac{a (3 A + 4 B) \tan [c + d x]}{4 d \sqrt{a + a \cos [c + d x]}} + \frac{a A \sec [c + d x] \tan [c + d x]}{2 d \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 567 leaves) :

$$\begin{aligned} & \frac{1}{32 d} \sqrt{a (1 + \cos [c + d x])} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \left( -2 i \sqrt{2} (3 A + 4 B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4} (c + d x)\right] - (-1 + \sqrt{2}) \sin\left[\frac{1}{4} (c + d x)\right]}{(1 + \sqrt{2}) \cos\left[\frac{1}{4} (c + d x)\right] - \sin\left[\frac{1}{4} (c + d x)\right]}\right] - \right. \\ & 2 i \sqrt{2} (3 A + 4 B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4} (c + d x)\right] - (1 + \sqrt{2}) \sin\left[\frac{1}{4} (c + d x)\right]}{(-1 + \sqrt{2}) \cos\left[\frac{1}{4} (c + d x)\right] - \sin\left[\frac{1}{4} (c + d x)\right]}\right] + 2 \sqrt{2} (3 A + 4 B) \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{1}{2} (c + d x)\right]\right] - \\ & \left. \sqrt{2} (3 A + 4 B) \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2} (c + d x)\right] - \sqrt{2} \sin\left[\frac{1}{2} (c + d x)\right]\right] - \sqrt{2} (3 A + 4 B) \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2} (c + d x)\right] - \sqrt{2} \sin\left[\frac{1}{2} (c + d x)\right]\right] \right) + \\ & \frac{8 A \sin\left[\frac{d x}{2}\right]}{\left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right)^2} + \frac{4 (3 A + 4 B) \cos\left[\frac{c}{2}\right] - 4 (A + 4 B) \sin\left[\frac{c}{2}\right]}{\left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right)} \\ & \left. \frac{8 A \sin\left[\frac{d x}{2}\right]}{\left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right)^2} - \frac{4 \left((3 A + 4 B) \cos\left[\frac{c}{2}\right] + (A + 4 B) \sin\left[\frac{c}{2}\right]\right)}{\left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right)} \right) \end{aligned}$$

■ **Problem 81: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos [c + d x]} (A + B \cos [c + d x]) \operatorname{Sec}[c + d x]^4 dx$$

Optimal (type 3, 160 leaves, 5 steps) :

$$\frac{\sqrt{a} (5 A + 6 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}}\right]}{8 d} + \frac{a (5 A + 6 B) \tan [c + d x]}{8 d \sqrt{a + a \cos [c + d x]}} + \frac{a (5 A + 6 B) \operatorname{Sec}[c + d x] \tan [c + d x]}{12 d \sqrt{a + a \cos [c + d x]}} + \frac{a A \operatorname{Sec}[c + d x]^2 \tan [c + d x]}{3 d \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 1108 leaves) :

$$\begin{aligned}
& \frac{i(-5A - 6B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{16\sqrt{2}d} + \\
& \frac{i(-5A - 6B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{16\sqrt{2}d} + \\
& \frac{(5A + 6B) \sqrt{a(1 + \cos[c + dx])} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{16\sqrt{2}d} + \\
& \frac{(-5A - 6B) \sqrt{a(1 + \cos[c + dx])} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{32\sqrt{2}d} + \\
& \frac{(-5A - 6B) \sqrt{a(1 + \cos[c + dx])} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{32\sqrt{2}d} + \\
& \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{12d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + 2B \sin\left[\frac{dx}{2}\right])}{8d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (5A \cos\left[\frac{c}{2}\right] + 6B \cos\left[\frac{c}{2}\right] - 3A \sin\left[\frac{c}{2}\right] - 2B \sin\left[\frac{c}{2}\right])}{16d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \\
& \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{12d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + 2B \sin\left[\frac{dx}{2}\right])}{8d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-5A \cos\left[\frac{c}{2}\right] - 6B \cos\left[\frac{c}{2}\right] - 3A \sin\left[\frac{c}{2}\right] - 2B \sin\left[\frac{c}{2}\right])}{16d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

- **Problem 86: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{3/2} (A + B \cos[c + dx]) \operatorname{Sec}[c + dx] dx$$

Optimal (type 3, 105 leaves, 4 steps):

$$\frac{2a^{3/2} A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} + \frac{2a^2 (3A + 4B) \sin[c + dx]}{3d \sqrt{a + a \cos[c + dx]}} + \frac{2aB \sqrt{a + a \cos[c + dx]} \sin[c + dx]}{3d}$$

Result (type 3, 1531 leaves):

$$\begin{aligned}
& - \left( \left( \left( \frac{1}{8} - \frac{i}{8} \right) A (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right. \right. \\
& \quad (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + \\
& \quad \left. \left. 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x (a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \right) / \\
& \quad \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right) - \\
& \quad \frac{i A \operatorname{ArcTan}\left[ \frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]} \right] (a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2\sqrt{2} d} - \\
& \quad \frac{i A \operatorname{ArcTan}\left[ \frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]} \right] (a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2\sqrt{2} d} - \\
& \quad \frac{A (a (1 + \cos[c + dx]))^{3/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{4\sqrt{2} d} - \\
& \quad \frac{A (a (1 + \cos[c + dx]))^{3/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{4\sqrt{2} d} + \\
& \quad \frac{(2A + 3B) \cos\left[\frac{dx}{2}\right] (a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{c}{2}\right]}{2d} - \\
& \quad \frac{i A \operatorname{ArcTan}\left[ \frac{2i \cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right] (a (1 + \cos[c + dx]))^{3/2} \operatorname{Cot}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{d \sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} + \\
& \quad \left( A (a (1 + \cos[c + dx]))^{3/2} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \right)
\end{aligned}$$



$$\left( -dx \cos\left[\frac{c}{2}\right] + 2 \log\left[\sqrt{2} + 2 \cos\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] \right) \sin\left[\frac{c}{2}\right] + \frac{4i\sqrt{2} \operatorname{ArcTan}\left[\frac{2i\cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2\sin\left[\frac{c}{2}\right])\tan\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}} \right) /$$

$$\left(\sqrt{2} d \left(4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2\right)\right) + \frac{B \cos\left[\frac{3dx}{2}\right] (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{3c}{2}\right]}{6d} +$$

$$\frac{(2A + 3B) \cos\left[\frac{c}{2}\right] (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{2d} +$$

$$\frac{B \cos\left[\frac{3c}{2}\right] (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{3dx}{2}\right]}{6d}$$

- **Problem 87: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{3/2} (A + B \cos[c + dx]) \sec[c + dx]^2 dx$$

Optimal (type 3, 103 leaves, 4 steps):

$$\frac{a^{3/2} (3A + 2B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{d} - \frac{a^2 (A - 2B) \sin[c + dx]}{d \sqrt{a + a \cos[c + dx]}} + \frac{a A \sqrt{a + a \cos[c + dx]} \tan[c + dx]}{d}$$

Result (type 3, 514 leaves):

$$\begin{aligned}
& \frac{1}{16d} (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{1}{2}(c + dx)\right]^3 \left( 2\sqrt{2}(3A + 2B) \operatorname{Log}\left[\sqrt{2} + 2\sin\left[\frac{1}{2}(c + dx)\right]\right] + \right. \\
& 16B \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right] + \frac{2i(3A + 2B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c + dx)\right] - (-1 + \sqrt{2})\sin\left[\frac{1}{4}(c + dx)\right]}{(1 + \sqrt{2})\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]}\right]}{-1 + \sqrt{2}\sin\left[\frac{c}{2}\right]} \left(\sqrt{2} - 2\sin\left[\frac{c}{2}\right]\right) + \\
& \frac{2i(3A + 2B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c + dx)\right] - (1 + \sqrt{2})\sin\left[\frac{1}{4}(c + dx)\right]}{(-1 + \sqrt{2})\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]}\right]}{-1 + \sqrt{2}\sin\left[\frac{c}{2}\right]} \left(\sqrt{2} - 2\sin\left[\frac{c}{2}\right]\right) + \\
& \frac{(3A + 2B) \operatorname{Log}\left[2 - \sqrt{2}\cos\left[\frac{1}{2}(c + dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c + dx)\right]\right]}{-1 + \sqrt{2}\sin\left[\frac{c}{2}\right]} \left(\sqrt{2} - 2\sin\left[\frac{c}{2}\right]\right) + \\
& \frac{(3A + 2B) \operatorname{Log}\left[2 + \sqrt{2}\cos\left[\frac{1}{2}(c + dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c + dx)\right]\right]}{-1 + \sqrt{2}\sin\left[\frac{c}{2}\right]} \left(\sqrt{2} - 2\sin\left[\frac{c}{2}\right]\right) + \\
& \left. 16B \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] + \frac{4A}{\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]} - \frac{4A}{\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]} \right)
\end{aligned}$$

- **Problem 88: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{3/2} (A + B \cos[c + dx]) \sec[c + dx]^3 dx$$

Optimal (type 3, 119 leaves, 4 steps):

$$\frac{a^{3/2} (7A + 12B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right]}{4d} + \frac{a^2 (5A + 4B) \tan[c + dx]}{4d \sqrt{a + a \cos[c + dx]}} + \frac{aA \sqrt{a + a \cos[c + dx]} \sec[c + dx] \tan[c + dx]}{2d}$$

Result (type 3, 573 leaves):

$$\frac{1}{64 d} (a (1 + \cos [c + d x]))^{3/2} \sec \left[ \frac{1}{2} (c + d x) \right]^3 \left( -2 i \sqrt{2} (7 A + 12 B) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c + d x) \right] - (-1 + \sqrt{2}) \sin \left[ \frac{1}{4} (c + d x) \right]}{(1 + \sqrt{2}) \cos \left[ \frac{1}{4} (c + d x) \right] - \sin \left[ \frac{1}{4} (c + d x) \right]} \right] - \right.$$

$$2 i \sqrt{2} (7 A + 12 B) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c + d x) \right] - (1 + \sqrt{2}) \sin \left[ \frac{1}{4} (c + d x) \right]}{(-1 + \sqrt{2}) \cos \left[ \frac{1}{4} (c + d x) \right] - \sin \left[ \frac{1}{4} (c + d x) \right]} \right] + 2 \sqrt{2} (7 A + 12 B) \operatorname{Log} \left[ \sqrt{2} + 2 \sin \left[ \frac{1}{2} (c + d x) \right] \right] -$$

$$\sqrt{2} (7 A + 12 B) \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{1}{2} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + d x) \right] \right] - \sqrt{2} (7 A + 12 B) \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{1}{2} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + d x) \right] \right] +$$

$$\frac{8 A \sin \left[ \frac{d x}{2} \right]}{(\cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right])^2} + \frac{4 (7 A + 4 B) \cos \left[ \frac{c}{2} \right] - 4 (5 A + 4 B) \sin \left[ \frac{c}{2} \right]}{(\cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right])} +$$

$$\frac{8 A \sin \left[ \frac{d x}{2} \right]}{(\cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right])^2} - \frac{4 ((7 A + 4 B) \cos \left[ \frac{c}{2} \right] + (5 A + 4 B) \sin \left[ \frac{c}{2} \right])}{(\cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right])} \right)$$

■ **Problem 89: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{3/2} (A + B \cos [c + d x]) \sec [c + d x]^4 dx$$

Optimal (type 3, 164 leaves, 5 steps):

$$\frac{a^{3/2} (11 A + 14 B) \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right]}{8 d} + \frac{a^2 (11 A + 14 B) \tan [c + d x]}{8 d \sqrt{a + a \cos [c + d x]}} +$$

$$\frac{a^2 (7 A + 6 B) \sec [c + d x] \tan [c + d x]}{12 d \sqrt{a + a \cos [c + d x]}} + \frac{a A \sqrt{a + a \cos [c + d x]} \sec [c + d x]^2 \tan [c + d x]}{3 d}$$

Result (type 3, 1132 leaves):

$$\begin{aligned}
& \frac{i(-11A - 14B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{32\sqrt{2}d} (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 + \\
& \frac{i(-11A - 14B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{32\sqrt{2}d} (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 + \\
& \frac{(11A + 14B) (a(1 + \cos[c + dx]))^{3/2} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{32\sqrt{2}d} + \\
& \frac{(-11A - 14B) (a(1 + \cos[c + dx]))^{3/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{64\sqrt{2}d} + \\
& \frac{(-11A - 14B) (a(1 + \cos[c + dx]))^{3/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{64\sqrt{2}d} + \\
& \frac{A (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{24d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \frac{(a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (3A \sin\left[\frac{dx}{2}\right] + 2B \sin\left[\frac{dx}{2}\right])}{16d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{(a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (11A \cos\left[\frac{c}{2}\right] + 14B \cos\left[\frac{c}{2}\right] - 5A \sin\left[\frac{c}{2}\right] - 10B \sin\left[\frac{c}{2}\right])}{32d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \\
& \frac{A (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{24d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \frac{(a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (3A \sin\left[\frac{dx}{2}\right] + 2B \sin\left[\frac{dx}{2}\right])}{16d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{(a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-11A \cos\left[\frac{c}{2}\right] - 14B \cos\left[\frac{c}{2}\right] - 5A \sin\left[\frac{c}{2}\right] - 10B \sin\left[\frac{c}{2}\right])}{32d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

- **Problem 90: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{3/2} (A + B \cos[c + dx]) \sec[c + dx]^5 dx$$

Optimal (type 3, 209 leaves, 6 steps):

$$\frac{a^{3/2} (75 A + 88 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{64 d} + \frac{a^2 (75 A + 88 B) \tan[c+dx]}{64 d \sqrt{a+a \cos[c+dx]}} + \frac{a^2 (75 A + 88 B) \sec[c+dx] \tan[c+dx]}{96 d \sqrt{a+a \cos[c+dx]}} +$$

$$\frac{a^2 (9 A + 8 B) \sec[c+dx]^2 \tan[c+dx]}{24 d \sqrt{a+a \cos[c+dx]}} + \frac{a A \sqrt{a+a \cos[c+dx]} \sec[c+dx]^3 \tan[c+dx]}{4 d}$$

Result (type 3, 1416 leaves):

$$\frac{i (-75 A - 88 B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]-\sqrt{2} \sin\left[\frac{c}{4}+\frac{dx}{4}\right]}{-\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sqrt{2} \cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]}\right] (a (1 + \cos[c+dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{256 \sqrt{2} d} +$$

$$\frac{i (-75 A - 88 B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sin\left[\frac{c}{4}+\frac{dx}{4}\right]-\sqrt{2} \sin\left[\frac{c}{4}+\frac{dx}{4}\right]}{\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sqrt{2} \cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]}\right] (a (1 + \cos[c+dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{256 \sqrt{2} d} +$$

$$\frac{(75 A + 88 B) (a (1 + \cos[c+dx]))^{3/2} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{256 \sqrt{2} d} +$$

$$\frac{(-75 A - 88 B) (a (1 + \cos[c+dx]))^{3/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{512 \sqrt{2} d} +$$

$$\frac{(-75 A - 88 B) (a (1 + \cos[c+dx]))^{3/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{512 \sqrt{2} d} +$$

$$\frac{A (a (1 + \cos[c+dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{32 d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^4} +$$

$$\frac{(a (1 + \cos[c+dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (15 A \cos\left[\frac{c}{2}\right] + 8 B \cos\left[\frac{c}{2}\right] - 9 A \sin\left[\frac{c}{2}\right] - 8 B \sin\left[\frac{c}{2}\right])}{192 d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^3} +$$

$$\frac{(a (1 + \cos[c+dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (19 A \sin\left[\frac{dx}{2}\right] + 24 B \sin\left[\frac{dx}{2}\right])}{128 d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2} +$$

$$\frac{(a (1 + \cos[c+dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (75 A \cos\left[\frac{c}{2}\right] + 88 B \cos\left[\frac{c}{2}\right] - 37 A \sin\left[\frac{c}{2}\right] - 40 B \sin\left[\frac{c}{2}\right])}{256 d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])} +$$

$$\frac{A (a (1 + \cos[c+dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{32 d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^4} +$$

$$\frac{(a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(-15A \cos\left[\frac{c}{2}\right] - 8B \cos\left[\frac{c}{2}\right] - 9A \sin\left[\frac{c}{2}\right] - 8B \sin\left[\frac{c}{2}\right]\right)}{192d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} +$$

$$\frac{(a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(19A \sin\left[\frac{dx}{2}\right] + 24B \sin\left[\frac{dx}{2}\right]\right)}{128d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} +$$

$$\frac{(a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(-75A \cos\left[\frac{c}{2}\right] - 88B \cos\left[\frac{c}{2}\right] - 37A \sin\left[\frac{c}{2}\right] - 40B \sin\left[\frac{c}{2}\right]\right)}{256d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}$$

■ **Problem 94: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{5/2} (A + B \cos[c + dx]) \operatorname{Sec}[c + dx] dx$$

Optimal (type 3, 142 leaves, 5 steps):

$$\frac{2a^{5/2} A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} + \frac{2a^3 (35A + 32B) \sin[c+dx]}{15d \sqrt{a+a \cos[c+dx]}} +$$

$$\frac{2a^2 (5A + 8B) \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{15d} + \frac{2aB (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{5d}$$

Result (type 3, 1640 leaves):

$$-\left(\left(\left(\frac{1}{16} - \frac{i}{16}\right) A (1 + e^{ic}) \left(\sqrt{2} - (1-i) e^{\frac{ic}{2}} + (16-16i) e^{\frac{3ic}{2}+idx} + (20+20i) \sqrt{2} e^{2ic+\frac{3idx}{2}} - (34-34i) e^{\frac{5ic}{2}+2idx} - (20+20i) \sqrt{2} e^{3ic+\frac{5idx}{2}} + (16-16i) e^{\frac{7ic}{2}+3idx} + (4+4i) \sqrt{2} e^{4ic+\frac{7idx}{2}} - (1-i) e^{\frac{9ic}{2}+4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4+4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)}\right) x (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \Big/ \left(\left((-1-i) + \sqrt{2} e^{\frac{ic}{2}}\right) (-1 + e^{ic}) \left(i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)}\right)^2\right) -$$

$$\frac{i A \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{4\sqrt{2}d} (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 -$$

$$\frac{i A \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{4\sqrt{2}d} (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 -$$

$$\frac{A (a(1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8\sqrt{2}d}$$

$$\frac{A (a (1 + \cos [c + dx]))^{5/2} \log \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{8 \sqrt{2} d} +$$

$$\frac{5 (A + B) \cos \left[ \frac{dx}{2} \right] (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sin \left[ \frac{c}{2} \right]}{4 d} -$$

$$i A \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i (-\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right]) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] (a (1 + \cos [c + dx]))^{5/2} \cot \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5$$

$$\frac{2 d \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}}{2 d \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} +$$

$$\left( A (a (1 + \cos [c + dx]))^{5/2} \operatorname{Csc} \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \right)$$

$$\left( -dx \cos \left[ \frac{c}{2} \right] + 2 \log \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i (-\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right]) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) /$$

$$\left( 2 \sqrt{2} d \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right) \right) + \frac{(2A + 5B) \cos \left[ \frac{3dx}{2} \right] (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sin \left[ \frac{3c}{2} \right]}{24 d} +$$

$$\frac{B \cos \left[ \frac{5dx}{2} \right] (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sin \left[ \frac{5c}{2} \right]}{40 d} +$$

$$\frac{5 (A + B) \cos \left[ \frac{c}{2} \right] (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sin \left[ \frac{dx}{2} \right]}{4 d} +$$

$$\frac{(2A + 5B) \cos \left[ \frac{3c}{2} \right] (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sin \left[ \frac{3dx}{2} \right]}{24 d} +$$

$$\frac{B \cos \left[ \frac{5c}{2} \right] (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sin \left[ \frac{5dx}{2} \right]}{40 d}$$

■ **Problem 95: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x]) \sec [c + d x]^2 dx$$

Optimal (type 3, 144 leaves, 5 steps):

$$\frac{a^{5/2} (5 A + 2 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{d} + \frac{a^3 (3 A + 14 B) \sin [c+d x]}{3 d \sqrt{a+a \cos [c+d x]}} - \frac{a^2 (3 A - 2 B) \sqrt{a+a \cos [c+d x]} \sin [c+d x]}{3 d} + \frac{a A (a+a \cos [c+d x])^{3/2} \tan [c+d x]}{d}$$

Result (type 3, 460 leaves):

$$\frac{1}{96 d} (a (1 + \cos [c + d x]))^{5/2} \sec \left[ \frac{1}{2} (c + d x) \right]^5 \left( -6 i \sqrt{2} (5 A + 2 B) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c + d x) \right] - (-1 + \sqrt{2}) \sin \left[ \frac{1}{4} (c + d x) \right]}{(1 + \sqrt{2}) \cos \left[ \frac{1}{4} (c + d x) \right] - \sin \left[ \frac{1}{4} (c + d x) \right]} \right] - 6 i \sqrt{2} (5 A + 2 B) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c + d x) \right] - (1 + \sqrt{2}) \sin \left[ \frac{1}{4} (c + d x) \right]}{(-1 + \sqrt{2}) \cos \left[ \frac{1}{4} (c + d x) \right] - \sin \left[ \frac{1}{4} (c + d x) \right]} \right] + 6 \sqrt{2} (5 A + 2 B) \operatorname{Log} \left[ \sqrt{2} + 2 \sin \left[ \frac{1}{2} (c + d x) \right] \right] - 3 \sqrt{2} (5 A + 2 B) \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{1}{2} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + d x) \right] \right] - 3 \sqrt{2} (5 A + 2 B) \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{1}{2} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + d x) \right] \right] + 24 (2 A + 5 B) \cos \left[ \frac{d x}{2} \right] \sin \left[ \frac{c}{2} \right] + 8 B \cos \left[ \frac{3 d x}{2} \right] \sin \left[ \frac{3 c}{2} \right] + 24 (2 A + 5 B) \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + 8 B \cos \left[ \frac{3 c}{2} \right] \sin \left[ \frac{3 d x}{2} \right] + \frac{12 A}{\cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right]} - \frac{12 A}{\cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right]} \right)$$

■ **Problem 96: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x]) \sec [c + d x]^3 dx$$

Optimal (type 3, 156 leaves, 5 steps):

$$\frac{a^{5/2} (19 A + 20 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{4 d} - \frac{a^3 (9 A - 4 B) \sin [c+d x]}{4 d \sqrt{a+a \cos [c+d x]}} + \frac{a^2 (7 A + 4 B) \sqrt{a+a \cos [c+d x]} \tan [c+d x]}{4 d} + \frac{a A (a+a \cos [c+d x])^{3/2} \sec [c+d x] \tan [c+d x]}{2 d}$$

Result (type 3, 605 leaves):



$$\begin{aligned} & \frac{1}{128 d} (a (1 + \cos [c + d x]))^{5/2} \sec \left[ \frac{1}{2} (c + d x) \right]^5 \left( -2 i \sqrt{2} (19 A + 20 B) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c + d x) \right] - (-1 + \sqrt{2}) \sin \left[ \frac{1}{4} (c + d x) \right]}{(1 + \sqrt{2}) \cos \left[ \frac{1}{4} (c + d x) \right] - \sin \left[ \frac{1}{4} (c + d x) \right]} \right] - \right. \\ & 2 i \sqrt{2} (19 A + 20 B) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c + d x) \right] - (1 + \sqrt{2}) \sin \left[ \frac{1}{4} (c + d x) \right]}{(-1 + \sqrt{2}) \cos \left[ \frac{1}{4} (c + d x) \right] - \sin \left[ \frac{1}{4} (c + d x) \right]} \right] + \\ & 2 \sqrt{2} (19 A + 20 B) \operatorname{Log} \left[ \sqrt{2} + 2 \sin \left[ \frac{1}{2} (c + d x) \right] \right] - \sqrt{2} (19 A + 20 B) \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{1}{2} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + d x) \right] \right] - \\ & \sqrt{2} (19 A + 20 B) \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{1}{2} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + d x) \right] \right] + 64 B \cos \left[ \frac{d x}{2} \right] \sin \left[ \frac{c}{2} \right] + 64 B \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + \\ & \frac{8 A \sin \left[ \frac{d x}{2} \right]}{(\cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right])^2} + \frac{4 (11 A + 4 B) \cos \left[ \frac{c}{2} \right] - 4 (9 A + 4 B) \sin \left[ \frac{c}{2} \right]}{(\cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right])} + \\ & \frac{8 A \sin \left[ \frac{d x}{2} \right]}{(\cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right])^2} - \frac{4 ((11 A + 4 B) \cos \left[ \frac{c}{2} \right] + (9 A + 4 B) \sin \left[ \frac{c}{2} \right])}{(\cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right])} \right) \end{aligned}$$

■ **Problem 97: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x]) \sec [c + d x]^4 dx$$

Optimal (type 3, 164 leaves, 5 steps):

$$\begin{aligned} & \frac{a^{5/2} (25 A + 38 B) \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right]}{8 d} + \frac{a^3 (49 A + 54 B) \tan [c + d x]}{24 d \sqrt{a + a \cos [c + d x]}} + \\ & \frac{a^2 (3 A + 2 B) \sqrt{a + a \cos [c + d x]} \sec [c + d x] \tan [c + d x]}{4 d} + \frac{a A (a + a \cos [c + d x])^{3/2} \sec [c + d x]^2 \tan [c + d x]}{3 d} \end{aligned}$$

Result (type 3, 1132 leaves):

$$\begin{aligned}
& \frac{i(-25A - 38B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{64\sqrt{2}d} (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 + \\
& \frac{i(-25A - 38B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{64\sqrt{2}d} (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 + \\
& \frac{(25A + 38B) (a(1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{64\sqrt{2}d} + \\
& \frac{(-25A - 38B) (a(1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{128\sqrt{2}d} + \\
& \frac{(-25A - 38B) (a(1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{128\sqrt{2}d} + \\
& \frac{A (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{48d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \frac{(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (5A \sin\left[\frac{dx}{2}\right] + 2B \sin\left[\frac{dx}{2}\right])}{32d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (25A \cos\left[\frac{c}{2}\right] + 22B \cos\left[\frac{c}{2}\right] - 15A \sin\left[\frac{c}{2}\right] - 18B \sin\left[\frac{c}{2}\right])}{64d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \\
& \frac{A (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{48d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \frac{(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (5A \sin\left[\frac{dx}{2}\right] + 2B \sin\left[\frac{dx}{2}\right])}{32d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-25A \cos\left[\frac{c}{2}\right] - 22B \cos\left[\frac{c}{2}\right] - 15A \sin\left[\frac{c}{2}\right] - 18B \sin\left[\frac{c}{2}\right])}{64d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 98: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{5/2} (A + B \cos[c + dx]) \sec[c + dx]^5 dx$$

Optimal (type 3, 209 leaves, 6 steps):

$$\begin{aligned}
& \frac{a^{5/2} (163A + 200B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{64d} + \frac{a^3 (163A + 200B) \operatorname{Tan}[c + dx]}{64d \sqrt{a + a \cos[c + dx]}} + \frac{a^3 (95A + 104B) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{96d \sqrt{a + a \cos[c + dx]}} + \\
& \frac{a^2 (11A + 8B) \sqrt{a + a \cos[c + dx]} \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{24d} + \frac{aA (a + a \cos[c + dx])^{3/2} \operatorname{Sec}[c + dx]^3 \operatorname{Tan}[c + dx]}{4d}
\end{aligned}$$

Result (type 3, 1416 leaves):

$$\begin{aligned}
 & \frac{i (-163 A - 200 B) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{-\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{512 \sqrt{2} d} + \\
 & \frac{i (-163 A - 200 B) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{512 \sqrt{2} d} + \\
 & \frac{(163 A + 200 B) (a (1 + \cos [c + dx]))^{5/2} \operatorname{Log} \left[ \sqrt{2} + 2 \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{512 \sqrt{2} d} + \\
 & \frac{(-163 A - 200 B) (a (1 + \cos [c + dx]))^{5/2} \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{1024 \sqrt{2} d} + \\
 & \frac{(-163 A - 200 B) (a (1 + \cos [c + dx]))^{5/2} \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{1024 \sqrt{2} d} + \\
 & \frac{A (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sin \left[ \frac{dx}{2} \right]}{64 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)^4} + \\
 & \frac{(a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \left( 23 A \cos \left[ \frac{c}{2} \right] + 8 B \cos \left[ \frac{c}{2} \right] - 17 A \sin \left[ \frac{c}{2} \right] - 8 B \sin \left[ \frac{c}{2} \right] \right)}{384 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)^3} + \\
 & \frac{(a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \left( 43 A \sin \left[ \frac{dx}{2} \right] + 40 B \sin \left[ \frac{dx}{2} \right] \right)}{256 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)^2} + \\
 & \frac{(a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \left( 163 A \cos \left[ \frac{c}{2} \right] + 200 B \cos \left[ \frac{c}{2} \right] - 77 A \sin \left[ \frac{c}{2} \right] - 120 B \sin \left[ \frac{c}{2} \right] \right)}{512 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)} + \\
 & \frac{A (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sin \left[ \frac{dx}{2} \right]}{64 d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)^4} + \\
 & \frac{(a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \left( -23 A \cos \left[ \frac{c}{2} \right] - 8 B \cos \left[ \frac{c}{2} \right] - 17 A \sin \left[ \frac{c}{2} \right] - 8 B \sin \left[ \frac{c}{2} \right] \right)}{384 d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)^3} +
 \end{aligned}$$

$$\frac{(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (43A \sin\left[\frac{dx}{2}\right] + 40B \sin\left[\frac{dx}{2}\right])}{256d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2} +$$

$$\frac{(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-163A \cos\left[\frac{c}{2}\right] - 200B \cos\left[\frac{c}{2}\right] - 77A \sin\left[\frac{c}{2}\right] - 120B \sin\left[\frac{c}{2}\right])}{512d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])}$$

- **Problem 99: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{5/2} (A + B \cos[c + dx]) \sec[c + dx]^6 dx$$

Optimal (type 3, 254 leaves, 7 steps):

$$\frac{a^{5/2} (283A + 326B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{128d} + \frac{a^3 (283A + 326B) \tan[c + dx]}{128d \sqrt{a + a \cos[c + dx]}} +$$

$$\frac{a^3 (283A + 326B) \sec[c + dx] \tan[c + dx]}{192d \sqrt{a + a \cos[c + dx]}} + \frac{a^3 (157A + 170B) \sec[c + dx]^2 \tan[c + dx]}{240d \sqrt{a + a \cos[c + dx]}} +$$

$$\frac{a^2 (13A + 10B) \sqrt{a + a \cos[c + dx]} \sec[c + dx]^3 \tan[c + dx]}{40d} + \frac{aA (a + a \cos[c + dx])^{3/2} \sec[c + dx]^4 \tan[c + dx]}{5d}$$

Result (type 3, 759 leaves):

$$\begin{aligned}
& \frac{i (-283 A - 326 B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{1024 \sqrt{2} d} (a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 + \\
& \frac{i (-283 A - 326 B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{1024 \sqrt{2} d} (a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 + \\
& \frac{(283 A + 326 B) (a (1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{1024 \sqrt{2} d} + \\
& \frac{(-283 A - 326 B) (a (1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{2048 \sqrt{2} d} + \\
& \frac{(-283 A - 326 B) (a (1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{2048 \sqrt{2} d} + \\
& \frac{1}{122880 d} (a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Sec}[c + dx]^5 \\
& \left(21610 A \sin\left[\frac{c}{2} + \frac{dx}{2}\right] + 20660 B \sin\left[\frac{c}{2} + \frac{dx}{2}\right] + 2080 A \sin\left[\frac{3c}{2} + \frac{3dx}{2}\right] - 3520 B \sin\left[\frac{3c}{2} + \frac{3dx}{2}\right] + 20376 A \sin\left[\frac{5c}{2} + \frac{5dx}{2}\right] + \right. \\
& \left. 20400 B \sin\left[\frac{5c}{2} + \frac{5dx}{2}\right] + 1415 A \sin\left[\frac{7c}{2} + \frac{7dx}{2}\right] + 1630 B \sin\left[\frac{7c}{2} + \frac{7dx}{2}\right] + 4245 A \sin\left[\frac{9c}{2} + \frac{9dx}{2}\right] + 4890 B \sin\left[\frac{9c}{2} + \frac{9dx}{2}\right]\right)
\end{aligned}$$

■ **Problem 104: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx]) \operatorname{Sec}[c + dx]}{\sqrt{a + a \cos[c + dx]}} dx$$

Optimal (type 3, 91 leaves, 5 steps):

$$\frac{2 A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} - \frac{\sqrt{2} (A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d}$$

Result (type 3, 1682 leaves):

$$\begin{aligned}
& - \left( \left( \frac{1}{2} - \frac{i}{2} \right) A (1 + e^{ic}) \right. \\
& \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + (16 - 16i) e^{\frac{7ic}{2} + 3idx} + \right. \\
& \left. (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16 \sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34 \sqrt{2} e^{2i(c+dx)} + 40i e^{\frac{5}{2}i(c+dx)} - \right.
\end{aligned}$$

$$\begin{aligned}
& \left. 16 \sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4+4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] (B+A \operatorname{Sec}[c+dx]) \Bigg/ \\
& \left( \left( (-1-i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1+e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \sqrt{a(1+\cos[c+dx])} (A+B \cos[c+dx]) \right) \Bigg) - \\
& \frac{i \sqrt{2} A \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]-\sqrt{2} \sin\left[\frac{c}{4}+\frac{dx}{4}\right]}{-\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sqrt{2} \cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] (B+A \operatorname{Sec}[c+dx])}{d \sqrt{a(1+\cos[c+dx])} (A+B \cos[c+dx])} - \\
& \frac{i \sqrt{2} A \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sin\left[\frac{c}{4}+\frac{dx}{4}\right]-\sqrt{2} \sin\left[\frac{c}{4}+\frac{dx}{4}\right]}{\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sqrt{2} \cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] (B+A \operatorname{Sec}[c+dx])}{d \sqrt{a(1+\cos[c+dx])} (A+B \cos[c+dx])} + \\
& \frac{2(A-B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (B+A \operatorname{Sec}[c+dx])}{d \sqrt{a(1+\cos[c+dx])} (A+B \cos[c+dx])} - \\
& \frac{2(A-B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (B+A \operatorname{Sec}[c+dx])}{d \sqrt{a(1+\cos[c+dx])} (A+B \cos[c+dx])} - \\
& \frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (B+A \operatorname{Sec}[c+dx])}{\sqrt{2} d \sqrt{a(1+\cos[c+dx])} (A+B \cos[c+dx])} - \\
& \frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (B+A \operatorname{Sec}[c+dx])}{\sqrt{2} d \sqrt{a(1+\cos[c+dx])} (A+B \cos[c+dx])} - \\
& \frac{4i A \operatorname{ArcTan}\left[\frac{2i \cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] \operatorname{Cot}\left[\frac{c}{2}\right] (B+A \operatorname{Sec}[c+dx])}{d \sqrt{a(1+\cos[c+dx])} (A+B \cos[c+dx])} + \\
& \sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2} \\
& \left( 2 \sqrt{2} A \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] (B+A \operatorname{Sec}[c+dx]) \right)
\end{aligned}$$

$$\left( -d x \cos\left[\frac{c}{2}\right] + 2 \log\left[\sqrt{2} + 2 \cos\left[\frac{d x}{2}\right]\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{d x}{2}\right] \right) \sin\left[\frac{c}{2}\right] + \frac{4 i \sqrt{2} \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right] - i \left(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]\right) \tan\left[\frac{d x}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right) /$$

$$\left( d \sqrt{a (1 + \cos[c + d x])} (A + B \cos[c + d x]) \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) \right)$$

- **Problem 105: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + d x]) \sec[c + d x]^2}{\sqrt{a + a \cos[c + d x]}} dx$$

Optimal (type 3, 119 leaves, 6 steps):

$$-\frac{(A - 2 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + d x]}{\sqrt{a + a \cos[c + d x]}}\right]}{\sqrt{a} d} + \frac{\sqrt{2} (A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + d x]}{\sqrt{2} \sqrt{a + a \cos[c + d x]}}\right]}{\sqrt{a} d} + \frac{A \tan[c + d x]}{d \sqrt{a + a \cos[c + d x]}}$$

Result (type 3, 532 leaves):

$$\frac{1}{4 d \sqrt{a} (1 + \operatorname{Cos}[c + d x])}$$

$$\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \left( -8 (A - B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]\right] + 8 (A - B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]\right] - \right.$$

$$2 \sqrt{2} (A - 2 B) \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \frac{2 i (A - 2 B) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - (-1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}{(1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}\right]}{-1 + \sqrt{2} \operatorname{Sin}\left[\frac{c}{2}\right]} \left(\sqrt{2} - 2 \operatorname{Sin}\left[\frac{c}{2}\right]\right) -$$

$$\frac{2 i (A - 2 B) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - (1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}{(-1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}\right]}{-1 + \sqrt{2} \operatorname{Sin}\left[\frac{c}{2}\right]} \left(\sqrt{2} - 2 \operatorname{Sin}\left[\frac{c}{2}\right]\right) -$$

$$\frac{(A - 2 B) \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{-1 + \sqrt{2} \operatorname{Sin}\left[\frac{c}{2}\right]} \left(\sqrt{2} - 2 \operatorname{Sin}\left[\frac{c}{2}\right]\right) -$$

$$\frac{(A - 2 B) \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{-1 + \sqrt{2} \operatorname{Sin}\left[\frac{c}{2}\right]} \left(\sqrt{2} - 2 \operatorname{Sin}\left[\frac{c}{2}\right]\right) +$$

$$\left. \frac{4 A}{\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]} - \frac{4 A}{\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]} \right)$$

- **Problem 106: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x]^3}{\sqrt{a + a \operatorname{Cos}[c + d x]}} dx$$

Optimal (type 3, 165 leaves, 7 steps):

$$\frac{(7 A - 4 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[c + d x]}{\sqrt{a + a \operatorname{Cos}[c + d x]}}\right]}{4 \sqrt{a} d} - \frac{\sqrt{2} (A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[c + d x]}{\sqrt{2} \sqrt{a + a \operatorname{Cos}[c + d x]}}\right]}{\sqrt{a} d} - \frac{(A - 4 B) \operatorname{Tan}[c + d x]}{4 d \sqrt{a + a \operatorname{Cos}[c + d x]}} + \frac{A \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 d \sqrt{a + a \operatorname{Cos}[c + d x]}}$$

Result (type 3, 724 leaves):



$$\frac{1}{16 d \sqrt{a} (1 + \cos[c + dx])}$$

$$\cos\left[\frac{1}{2}(c + dx)\right] \left( 32 (A - B) \log\left[\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]\right] - 32 (A - B) \log\left[\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right]\right] + \right.$$

$$\left. 2 \sqrt{2} (7A - 4B) \log\left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(c + dx)\right]\right] + \frac{2i(7A - 4B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c + dx)\right] - (-1 + \sqrt{2}) \sin\left[\frac{1}{4}(c + dx)\right]}{(1 + \sqrt{2}) \cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]}\right] (\sqrt{2} - 2 \sin\left[\frac{c}{2}\right])}{-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]} + \right.$$

$$\left. \frac{2i(7A - 4B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c + dx)\right] - (1 + \sqrt{2}) \sin\left[\frac{1}{4}(c + dx)\right]}{(-1 + \sqrt{2}) \cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]}\right] (\sqrt{2} - 2 \sin\left[\frac{c}{2}\right])}{-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]} + \right.$$

$$\left. \frac{(7A - 4B) \log\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c + dx)\right]\right] (\sqrt{2} - 2 \sin\left[\frac{c}{2}\right])}{-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]} + \right.$$

$$\left. \frac{(7A - 4B) \log\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c + dx)\right]\right] (\sqrt{2} - 2 \sin\left[\frac{c}{2}\right])}{-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]} + \right.$$

$$\left. \frac{8A \sin\left[\frac{dx}{2}\right]}{(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right])^2} - \frac{4((A - 4B) \cos\left[\frac{c}{2}\right] + (-3A + 4B) \sin\left[\frac{c}{2}\right])}{(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right])} + \right.$$

$$\left. \frac{8A \sin\left[\frac{dx}{2}\right]}{(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])^2} + \frac{4((A - 4B) \cos\left[\frac{c}{2}\right] + (3A - 4B) \sin\left[\frac{c}{2}\right])}{(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])} \right)$$

■ **Problem 107: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^4 (A + B \cos[c + dx])}{(a + a \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 261 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(15A - 19B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{2\sqrt{2} a^{3/2} d} + \frac{(A-B) \cos[c+dx]^4 \sin[c+dx]}{2d(a+a \cos[c+dx])^{3/2}} + \frac{(651A - 799B) \sin[c+dx]}{105ad \sqrt{a+a \cos[c+dx]}} + \\
& \frac{(63A - 67B) \cos[c+dx]^2 \sin[c+dx]}{70ad \sqrt{a+a \cos[c+dx]}} - \frac{(7A - 11B) \cos[c+dx]^3 \sin[c+dx]}{14ad \sqrt{a+a \cos[c+dx]}} - \frac{(273A - 397B) \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{210a^2 d}
\end{aligned}$$

Result (type 3, 713 leaves):

$$\begin{aligned}
& \frac{(15A - 19B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{d(a(1+\cos[c+dx]))^{3/2}} + \frac{(-15A + 19B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{d(a(1+\cos[c+dx]))^{3/2}} + \\
& \frac{5(4A - 5B) \cos\left[\frac{dx}{2}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{c}{2}\right]}{d(a(1+\cos[c+dx]))^{3/2}} - \frac{(6A - 11B) \cos\left[\frac{3dx}{2}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{3c}{2}\right]}{3d(a(1+\cos[c+dx]))^{3/2}} + \\
& \frac{(2A - 3B) \cos\left[\frac{5dx}{2}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{5c}{2}\right]}{5d(a(1+\cos[c+dx]))^{3/2}} + \frac{B \cos\left[\frac{7dx}{2}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{7c}{2}\right]}{7d(a(1+\cos[c+dx]))^{3/2}} + \frac{5(4A - 5B) \cos\left[\frac{c}{2}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{d(a(1+\cos[c+dx]))^{3/2}} - \\
& \frac{(6A - 11B) \cos\left[\frac{3c}{2}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{3dx}{2}\right]}{3d(a(1+\cos[c+dx]))^{3/2}} + \frac{(2A - 3B) \cos\left[\frac{5c}{2}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{5dx}{2}\right]}{5d(a(1+\cos[c+dx]))^{3/2}} + \frac{B \cos\left[\frac{7c}{2}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{7dx}{2}\right]}{7d(a(1+\cos[c+dx]))^{3/2}} + \\
& \frac{(A-B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2d(a(1+\cos[c+dx]))^{3/2} \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2} + \frac{(-A+B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2d(a(1+\cos[c+dx]))^{3/2} \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2}
\end{aligned}$$

■ **Problem 112: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A+B \cos[c+dx]) \operatorname{Sec}[c+dx]}{(a+a \cos[c+dx])^{3/2}} dx$$

Optimal (type 3, 127 leaves, 6 steps):

$$\frac{2A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{3/2} d} - \frac{(5A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{2\sqrt{2} a^{3/2} d} - \frac{(A-B) \sin[c+dx]}{2d(a+a \cos[c+dx])^{3/2}}$$

Result (type 3, 2128 leaves):

$$\begin{aligned}
& - \left( (1-i)A(1+e^{ic}) \right. \\
& \left. \left( \sqrt{2} - (1-i)e^{\frac{ic}{2}} + (16-16i)e^{\frac{3ic}{2}+idx} + (20+20i)\sqrt{2}e^{2ic+\frac{3idx}{2}} - (34-34i)e^{\frac{5ic}{2}+2idx} - (20+20i)\sqrt{2}e^{3ic+\frac{5idx}{2}} + (16-16i)e^{\frac{7ic}{2}+3idx} + \right. \right. \\
& \left. \left. (4+4i)\sqrt{2}e^{4ic+\frac{7idx}{2}} - (1-i)e^{\frac{9ic}{2}+4idx} + 8ie^{\frac{1}{2}i(c+dx)} - 16\sqrt{2}e^{i(c+dx)} - 40ie^{\frac{3}{2}i(c+dx)} + 34\sqrt{2}e^{2i(c+dx)} + 40ie^{\frac{5}{2}i(c+dx)} - \right. \right. \\
& \left. \left. 16\sqrt{2}e^{3i(c+dx)} - 8ie^{\frac{7}{2}i(c+dx)} + \sqrt{2}e^{4i(c+dx)} - (4+4i)\sqrt{2}e^{\frac{1}{2}i(2c+dx)} \right) x \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] (B+A \operatorname{Sec}[c+dx]) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right. \\
& \left. (a(1 + \cos[c + dx]))^{3/2} (A + B \cos[c + dx]) \right) - \\
& \frac{2i\sqrt{2} A \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{c+dx}{4}\right] - \sin\left[\frac{c+dx}{4}\right] - \sqrt{2} \sin\left[\frac{c+dx}{4}\right]}{-\cos\left[\frac{c+dx}{4}\right] + \sqrt{2} \cos\left[\frac{c+dx}{4}\right] - \sin\left[\frac{c+dx}{4}\right]} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx] (B + A \sec[c + dx])}{d(a(1 + \cos[c + dx]))^{3/2} (A + B \cos[c + dx])} + \\
& \frac{(5A - B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx] \operatorname{Log} \left[ \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] \right] (B + A \sec[c + dx])}{d(a(1 + \cos[c + dx]))^{3/2} (A + B \cos[c + dx])} + \\
& \frac{(-5A + B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx] \operatorname{Log} \left[ \cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] \right] (B + A \sec[c + dx])}{d(a(1 + \cos[c + dx]))^{3/2} (A + B \cos[c + dx])} - \\
& \frac{\sqrt{2} A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx] \operatorname{Log} \left[ 2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right] (B + A \sec[c + dx])}{d(a(1 + \cos[c + dx]))^{3/2} (A + B \cos[c + dx])} + \\
& \left( (1 - i) \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx] (B + A \sec[c + dx]) \right. \\
& \left. \left( (1 + i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1 - i) \sin\left[\frac{c}{4}\right] - i\sqrt{2} \sin\left[\frac{c}{4}\right] \right) \left( (-1 - i) A \cos\left[\frac{c}{4}\right] + \sqrt{2} A \cos\left[\frac{c}{4}\right] + (1 - i) A \sin\left[\frac{c}{4}\right] - i\sqrt{2} A \sin\left[\frac{c}{4}\right] \right) \right) / \\
& \left( \sqrt{2} d(a(1 + \cos[c + dx]))^{3/2} (A + B \cos[c + dx]) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right) - \\
& \left( \left( \frac{1}{2} + \frac{i}{2} \right) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx] \operatorname{Log} \left[ 2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right] (B + A \sec[c + dx]) \right. \\
& \left. \left( (1 + i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1 - i) \sin\left[\frac{c}{4}\right] - i\sqrt{2} \sin\left[\frac{c}{4}\right] \right) \left( (-1 - i) A \cos\left[\frac{c}{4}\right] + \sqrt{2} A \cos\left[\frac{c}{4}\right] + (1 - i) A \sin\left[\frac{c}{4}\right] - i\sqrt{2} A \sin\left[\frac{c}{4}\right] \right) \right) / \\
& \left( \sqrt{2} d(a(1 + \cos[c + dx]))^{3/2} (A + B \cos[c + dx]) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right) - \\
& \frac{8i A \operatorname{ArcTan} \left[ \frac{2i \cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx] \operatorname{Cot}\left[\frac{c}{2}\right] (B + A \sec[c + dx])}{d(a(1 + \cos[c + dx]))^{3/2} (A + B \cos[c + dx]) \sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} +
\end{aligned}$$

$$\left( 4 \sqrt{2} A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] (B + A \operatorname{Sec}[c+dx]) \right. \\
\left. - dx \cos\left[\frac{c}{2}\right] + 2 \operatorname{Log}\left[\sqrt{2} + 2 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + \frac{4 i \sqrt{2} \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right] - i (-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right) \\
\left( d (a (1 + \cos[c+dx]))^{3/2} (A + B \cos[c+dx]) \left(4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2\right) \right) + \\
\frac{(-A + B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] (B + A \operatorname{Sec}[c+dx])}{2 d (a (1 + \cos[c+dx]))^{3/2} (A + B \cos[c+dx]) \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2} + \\
\frac{(A - B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] (B + A \operatorname{Sec}[c+dx])}{2 d (a (1 + \cos[c+dx]))^{3/2} (A + B \cos[c+dx]) \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2}$$

■ **Problem 113: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c+dx]) \operatorname{Sec}[c+dx]^2}{(a + a \cos[c+dx])^{3/2}} dx$$

Optimal (type 3, 170 leaves, 7 steps):

$$-\frac{(3A - 2B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{3/2} d} + \frac{(9A - 5B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{(A - B) \tan[c+dx]}{2 d (a + a \cos[c+dx])^{3/2}} + \frac{(3A - B) \tan[c+dx]}{2 a d \sqrt{a + a \cos[c+dx]}}$$

Result (type 3, 1058 leaves):

$$\begin{aligned}
& \frac{(-9A + 5B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{d(a(1 + \cos[c + dx]))^{3/2}} + \frac{(9A - 5B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{d(a(1 + \cos[c + dx]))^{3/2}} - \\
& \frac{\sqrt{2}(3A - 2B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\sqrt{2} + 2\sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d(a(1 + \cos[c + dx]))^{3/2}} - \frac{i(3A - 2B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2}\sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2}\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (\sqrt{2} - 2\sin\left[\frac{c}{2}\right])}{d(a(1 + \cos[c + dx]))^{3/2} (-1 + \sqrt{2}\sin\left[\frac{c}{2}\right])} - \\
& \frac{i(3A - 2B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2}\sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2}\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (\sqrt{2} - 2\sin\left[\frac{c}{2}\right])}{d(a(1 + \cos[c + dx]))^{3/2} (-1 + \sqrt{2}\sin\left[\frac{c}{2}\right])} - \\
& \frac{(3A - 2B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[2 - \sqrt{2}\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2}\sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (\sqrt{2} - 2\sin\left[\frac{c}{2}\right])}{2d(a(1 + \cos[c + dx]))^{3/2} (-1 + \sqrt{2}\sin\left[\frac{c}{2}\right])} - \\
& \frac{(3A - 2B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[2 + \sqrt{2}\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2}\sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (\sqrt{2} - 2\sin\left[\frac{c}{2}\right])}{2d(a(1 + \cos[c + dx]))^{3/2} (-1 + \sqrt{2}\sin\left[\frac{c}{2}\right])} + \\
& \frac{(A - B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2d(a(1 + \cos[c + dx]))^{3/2} (\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right])^2} + \frac{(-A + B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2d(a(1 + \cos[c + dx]))^{3/2} (\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right])^2} + \\
& \frac{2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{d(a(1 + \cos[c + dx]))^{3/2} (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])} - \frac{2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{d(a(1 + \cos[c + dx]))^{3/2} (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])}
\end{aligned}$$

■ **Problem 114: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx]) \operatorname{Sec}[c + dx]^3}{(a + a \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 221 leaves, 8 steps):

$$\frac{(19A - 12B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right]}{4a^{3/2}d} - \frac{(13A - 9B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{2} \sqrt{a + a \cos[c + dx]}}\right]}{2\sqrt{2}a^{3/2}d} - \\
\frac{(7A - 6B) \operatorname{Tan}[c + dx]}{4ad\sqrt{a + a \cos[c + dx]}} - \frac{(A - B) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2d(a + a \cos[c + dx])^{3/2}} + \frac{(2A - B) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2ad\sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 1402 leaves):

$$\begin{aligned}
& \frac{(13A - 9B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{d(a(1 + \cos[c + dx]))^{3/2}} + \\
& \frac{(-13A + 9B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{d(a(1 + \cos[c + dx]))^{3/2}} + \frac{(19A - 12B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{2\sqrt{2} d(a(1 + \cos[c + dx]))^{3/2}} + \\
& \left( i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(19\sqrt{2}A - 12\sqrt{2}B - 38A \sin\left[\frac{c}{2}\right] + 24B \sin\left[\frac{c}{2}\right]\right) \right) / \\
& \left(4d(a(1 + \cos[c + dx]))^{3/2} \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right)\right) + \\
& \left( i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(19\sqrt{2}A - 12\sqrt{2}B - 38A \sin\left[\frac{c}{2}\right] + 24B \sin\left[\frac{c}{2}\right]\right) \right) / \\
& \left(4d(a(1 + \cos[c + dx]))^{3/2} \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right)\right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \left(19\sqrt{2}A - 12\sqrt{2}B - 38A \sin\left[\frac{c}{2}\right] + 24B \sin\left[\frac{c}{2}\right]\right) \right) / \\
& \left(8d(a(1 + \cos[c + dx]))^{3/2} \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right)\right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \left(19\sqrt{2}A - 12\sqrt{2}B - 38A \sin\left[\frac{c}{2}\right] + 24B \sin\left[\frac{c}{2}\right]\right) \right) / \\
& \left(8d(a(1 + \cos[c + dx]))^{3/2} \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right)\right) + \frac{(-A + B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2d(a(1 + \cos[c + dx]))^{3/2} \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2} + \\
& \frac{(A - B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2d(a(1 + \cos[c + dx]))^{3/2} \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2} + \frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{d(a(1 + \cos[c + dx]))^{3/2} \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(-5A \cos\left[\frac{c}{2}\right] + 4B \cos\left[\frac{c}{2}\right] + 7A \sin\left[\frac{c}{2}\right] - 4B \sin\left[\frac{c}{2}\right]\right)}{2d(a(1 + \cos[c + dx]))^{3/2} \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{d(a(1 + \cos[c + dx]))^{3/2} \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(5A \cos\left[\frac{c}{2}\right] - 4B \cos\left[\frac{c}{2}\right] + 7A \sin\left[\frac{c}{2}\right] - 4B \sin\left[\frac{c}{2}\right]\right)}{2d(a(1 + \cos[c + dx]))^{3/2} \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 120: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]}{(a + a \cos[c + dx])^{5/2}} dx$$

Optimal (type 3, 164 leaves, 7 steps):

$$\frac{2 A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{5/2} d} - \frac{(43 A - 3 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A - B) \sin[c + dx]}{4 d (a + a \cos[c + dx])^{5/2}} - \frac{(11 A - 3 B) \sin[c + dx]}{16 a d (a + a \cos[c + dx])^{3/2}}$$

Result (type 3, 2334 leaves):

$$\begin{aligned} & - \left( (2 - 2i) A (1 + e^{ic}) \right. \\ & \quad \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + (16 - 16i) e^{\frac{7ic}{2} + 3idx} + \right. \\ & \quad (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + 40i e^{\frac{5}{2}i(c+dx)} - \\ & \quad \left. 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx] (B + A \sec[c + dx]) \Big/ \\ & \quad \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right. \\ & \quad \left. (a (1 + \cos[c + dx]))^{5/2} (A + B \cos[c + dx]) \right) \Big) - \\ & \quad \frac{4i \sqrt{2} A \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx] (B + A \sec[c + dx])}{d (a (1 + \cos[c + dx]))^{5/2} (A + B \cos[c + dx])} + \\ & \quad \frac{(43 A - 3 B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx] \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (B + A \sec[c + dx])}{4 d (a (1 + \cos[c + dx]))^{5/2} (A + B \cos[c + dx])} + \\ & \quad \frac{(-43 A + 3 B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx] \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (B + A \sec[c + dx])}{4 d (a (1 + \cos[c + dx]))^{5/2} (A + B \cos[c + dx])} - \\ & \quad \frac{2\sqrt{2} A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx] \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (B + A \sec[c + dx])}{d (a (1 + \cos[c + dx]))^{5/2} (A + B \cos[c + dx])} + \\ & \quad \left( (1 - i) \sqrt{2} \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx] (B + A \sec[c + dx]) \right. \\ & \quad \left. \left( (1 + i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1 - i) \sin\left[\frac{c}{4}\right] - i \sqrt{2} \sin\left[\frac{c}{4}\right] \right) \left( (-1 - i) A \cos\left[\frac{c}{4}\right] + \sqrt{2} A \cos\left[\frac{c}{4}\right] + (1 - i) A \sin\left[\frac{c}{4}\right] - i \sqrt{2} A \sin\left[\frac{c}{4}\right] \right) \right) \Big/ \end{aligned}$$

$$\begin{aligned}
& \left( d (a (1 + \cos [c + dx]))^{5/2} (A + B \cos [c + dx]) \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \right) - \\
& \left( (1 + i) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \cos [c + dx] \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right) (B + A \operatorname{Sec} [c + dx]) \right. \\
& \quad \left. \left( (1 + i) \cos \left[ \frac{c}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} \right] - (1 - i) \sin \left[ \frac{c}{4} \right] - i \sqrt{2} \sin \left[ \frac{c}{4} \right] \right) \left( (-1 - i) A \cos \left[ \frac{c}{4} \right] + \sqrt{2} A \cos \left[ \frac{c}{4} \right] + (1 - i) A \sin \left[ \frac{c}{4} \right] - i \sqrt{2} A \sin \left[ \frac{c}{4} \right] \right) \right) / \\
& \left( \sqrt{2} d (a (1 + \cos [c + dx]))^{5/2} (A + B \cos [c + dx]) \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \right) - \\
& \frac{16 i A \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i (-\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right]) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \cos [c + dx] \operatorname{Cot} \left[ \frac{c}{2} \right] (B + A \operatorname{Sec} [c + dx])}{d (a (1 + \cos [c + dx]))^{5/2} (A + B \cos [c + dx]) \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} + \\
& \left( 8 \sqrt{2} A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \cos [c + dx] \operatorname{Csc} \left[ \frac{c}{2} \right] (B + A \operatorname{Sec} [c + dx]) \right) \\
& \left( -dx \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i (-\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right]) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) / \\
& \left( d (a (1 + \cos [c + dx]))^{5/2} (A + B \cos [c + dx]) \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right) \right) + \\
& \frac{(-A + B) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \cos [c + dx] (B + A \operatorname{Sec} [c + dx])}{8 d (a (1 + \cos [c + dx]))^{5/2} (A + B \cos [c + dx]) \left( \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] \right)^4} + \\
& \frac{(-11 A + 3 B) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \cos [c + dx] (B + A \operatorname{Sec} [c + dx])}{8 d (a (1 + \cos [c + dx]))^{5/2} (A + B \cos [c + dx]) \left( \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] \right)^2} + \\
& \frac{(A - B) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \cos [c + dx] (B + A \operatorname{Sec} [c + dx])}{8 d (a (1 + \cos [c + dx]))^{5/2} (A + B \cos [c + dx]) \left( \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] \right)^4} +
\end{aligned}$$



$$\frac{(11 A - 3 B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx] (B + A \sec[c + dx])}{8 d (a (1 + \cos[c + dx]))^{5/2} (A + B \cos[c + dx]) \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2}$$

- **Problem 121: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]^2}{(a + a \cos[c + dx])^{5/2}} dx$$

Optimal (type 3, 207 leaves, 8 steps):

$$-\frac{(5 A - 2 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{5/2} d} + \frac{(115 A - 43 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A - B) \tan[c + dx]}{4 d (a + a \cos[c + dx])^{5/2}} - \frac{(15 A - 7 B) \tan[c + dx]}{16 a d (a + a \cos[c + dx])^{3/2}} + \frac{(35 A - 11 B) \tan[c + dx]}{16 a^2 d \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 632 leaves):

$$\frac{1}{32 d (a (1 + \cos [c + d x]))^{5/2}}$$

$$\cos \left[ \frac{1}{2} (c + d x) \right] \left( 8 (-115 A + 43 B) \cos \left[ \frac{1}{2} (c + d x) \right]^4 \log \left[ \cos \left[ \frac{1}{4} (c + d x) \right] - \sin \left[ \frac{1}{4} (c + d x) \right] \right] + 8 (115 A - 43 B) \cos \left[ \frac{1}{2} (c + d x) \right]^4 \right.$$

$$\log \left[ \cos \left[ \frac{1}{4} (c + d x) \right] + \sin \left[ \frac{1}{4} (c + d x) \right] \right] - 64 \sqrt{2} (5 A - 2 B) \cos \left[ \frac{1}{2} (c + d x) \right]^4 \log \left[ \sqrt{2} + 2 \sin \left[ \frac{1}{2} (c + d x) \right] \right] -$$

$$\frac{64 i (5 A - 2 B) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c + d x) \right] - (-1 + \sqrt{2}) \sin \left[ \frac{1}{4} (c + d x) \right]}{(1 + \sqrt{2}) \cos \left[ \frac{1}{4} (c + d x) \right] - \sin \left[ \frac{1}{4} (c + d x) \right]} \right] \cos \left[ \frac{1}{2} (c + d x) \right]^4 \left( \sqrt{2} - 2 \sin \left[ \frac{c}{2} \right] \right)}{-1 + \sqrt{2} \sin \left[ \frac{c}{2} \right]} -$$

$$\frac{64 i (5 A - 2 B) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c + d x) \right] - (1 + \sqrt{2}) \sin \left[ \frac{1}{4} (c + d x) \right]}{(-1 + \sqrt{2}) \cos \left[ \frac{1}{4} (c + d x) \right] - \sin \left[ \frac{1}{4} (c + d x) \right]} \right] \cos \left[ \frac{1}{2} (c + d x) \right]^4 \left( \sqrt{2} - 2 \sin \left[ \frac{c}{2} \right] \right)}{-1 + \sqrt{2} \sin \left[ \frac{c}{2} \right]} - \frac{1}{-1 + \sqrt{2} \sin \left[ \frac{c}{2} \right]}$$

$$32 (5 A - 2 B) \cos \left[ \frac{1}{2} (c + d x) \right]^4 \log \left[ 2 - \sqrt{2} \cos \left[ \frac{1}{2} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + d x) \right] \right] \left( \sqrt{2} - 2 \sin \left[ \frac{c}{2} \right] \right) -$$

$$\frac{1}{-1 + \sqrt{2} \sin \left[ \frac{c}{2} \right]} 32 (5 A - 2 B) \cos \left[ \frac{1}{2} (c + d x) \right]^4 \log \left[ 2 + \sqrt{2} \cos \left[ \frac{1}{2} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + d x) \right] \right] \left( \sqrt{2} - 2 \sin \left[ \frac{c}{2} \right] \right) +$$

$$\left. 2 (67 A - 11 B + 10 (11 A - 3 B) \cos [c + d x] + (35 A - 11 B) \cos [2 (c + d x)]) \operatorname{Sec}[c + d x] \sin \left[ \frac{1}{2} (c + d x) \right] \right)$$

■ **Problem 122: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + d x]) \operatorname{Sec}[c + d x]^3}{(a + a \cos [c + d x])^{5/2}} dx$$

Optimal (type 3, 264 leaves, 9 steps):

$$\frac{(39 A - 20 B) \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right]}{4 a^{5/2} d} - \frac{(219 A - 115 B) \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{2} \sqrt{a + a \cos [c + d x]}} \right]}{16 \sqrt{2} a^{5/2} d} - \frac{7 (9 A - 5 B) \tan [c + d x]}{16 a^2 d \sqrt{a + a \cos [c + d x]}}$$

$$+ \frac{(A - B) \operatorname{Sec}[c + d x] \tan [c + d x]}{4 d (a + a \cos [c + d x])^{5/2}} - \frac{(19 A - 11 B) \operatorname{Sec}[c + d x] \tan [c + d x]}{16 a d (a + a \cos [c + d x])^{3/2}} + \frac{(31 A - 15 B) \operatorname{Sec}[c + d x] \tan [c + d x]}{16 a^2 d \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 966 leaves):

$$\begin{aligned}
& \frac{(219 A - 115 B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{4 d (a (1 + \cos[c + dx]))^{5/2}} + \frac{(-219 A + 115 B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{4 d (a (1 + \cos[c + dx]))^{5/2}} + \\
& \frac{(39 A - 20 B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{\sqrt{2} d (a (1 + \cos[c + dx]))^{5/2}} + \frac{i (39 A - 20 B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right)}{2 d (a (1 + \cos[c + dx]))^{5/2} \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right)} + \\
& \frac{i (39 A - 20 B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right)}{2 d (a (1 + \cos[c + dx]))^{5/2} \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right)} + \\
& \frac{(39 A - 20 B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right)}{4 d (a (1 + \cos[c + dx]))^{5/2} \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right)} + \\
& \frac{(39 A - 20 B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right)}{4 d (a (1 + \cos[c + dx]))^{5/2} \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right)} + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sec[c + dx]^2 \left(-47 A \sin\left[\frac{c}{2} + \frac{dx}{2}\right] + 51 B \sin\left[\frac{c}{2} + \frac{dx}{2}\right] - 79 A \sin\left[\frac{3c}{2} + \frac{3dx}{2}\right] + 59 B \sin\left[\frac{3c}{2} + \frac{3dx}{2}\right] - \right. \right. \\
& \left. \left. 127 A \sin\left[\frac{5c}{2} + \frac{5dx}{2}\right] + 75 B \sin\left[\frac{5c}{2} + \frac{5dx}{2}\right] - 63 A \sin\left[\frac{7c}{2} + \frac{7dx}{2}\right] + 35 B \sin\left[\frac{7c}{2} + \frac{7dx}{2}\right]\right) \right) / (64 d (a (1 + \cos[c + dx]))^{5/2})
\end{aligned}$$

- **Problem 123: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^{5/2} (a + a \cos[c + dx]) (A + B \cos[c + dx]) dx$$

Optimal (type 4, 159 leaves, 8 steps):

$$\begin{aligned}
& \frac{2 a (9 A + 7 B) \operatorname{EllipticE}\left[\frac{1}{2} (c + dx), 2\right]}{15 d} + \frac{10 a (A + B) \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), 2\right]}{21 d} + \frac{10 a (A + B) \sqrt{\cos[c + dx]} \sin[c + dx]}{21 d} + \\
& \frac{2 a (9 A + 7 B) \cos[c + dx]^{3/2} \sin[c + dx]}{45 d} + \frac{2 a (A + B) \cos[c + dx]^{5/2} \sin[c + dx]}{7 d} + \frac{2 a B \cos[c + dx]^{7/2} \sin[c + dx]}{9 d}
\end{aligned}$$

Result (type 5, 914 leaves):

$$a \left( \sqrt{\cos[c + dx]} (1 + \cos[c + dx]) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right)$$

$$\left( -\frac{(9A+7B)\cot[c]}{15d} + \frac{23(A+B)\cos[dx]\sin[c]}{84d} + \frac{(18A+19B)\cos[2dx]\sin[2c]}{180d} + \frac{(A+B)\cos[3dx]\sin[3c]}{28d} + \frac{B\cos[4dx]\sin[4c]}{72d} + \frac{23(A+B)\cos[c]\sin[dx]}{84d} + \frac{(18A+19B)\cos[2c]\sin[2dx]}{180d} + \frac{(A+B)\cos[3c]\sin[3dx]}{28d} + \frac{B\cos[4c]\sin[4dx]}{72d} \right) - \frac{1}{21d\sqrt{1+\cot[c]^2}}$$

$$5A(1+\cos[c+dx])\operatorname{Csc}[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]]$$

$$\sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{21d\sqrt{1+\cot[c]^2}} 5B(1+\cos[c+dx])\operatorname{Csc}[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]}$$

$$\sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{10d} 3A(1+\cos[c+dx])\operatorname{Csc}[c]\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -$$

$$\frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} - \frac{1}{30d} 7B(1+\cos[c+dx])\operatorname{Csc}[c]\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -$$

$$\left( \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right)$$

- **Problem 124: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{3/2} (a + a \cos[c + d x]) (A + B \cos[c + d x]) dx$$

Optimal (type 4, 132 leaves, 7 steps):

$$\frac{6 a (A + B) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{2 a (7 A + 5 B) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{21 d} +$$

$$\frac{2 a (7 A + 5 B) \sqrt{\cos[c + d x]} \sin[c + d x]}{21 d} + \frac{2 a (A + B) \cos[c + d x]^{3/2} \sin[c + d x]}{5 d} + \frac{2 a B \cos[c + d x]^{5/2} \sin[c + d x]}{7 d}$$

Result (type 5, 872 leaves):

$$a \left( \sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \left( -\frac{3 (A + B) \cot[c]}{5 d} + \frac{(28 A + 23 B) \cos[d x] \sin[c]}{84 d} + \frac{(A + B) \cos[2 d x] \sin[2 c]}{10 d} + \right. \right.$$

$$\left. \frac{B \cos[3 d x] \sin[3 c]}{28 d} + \frac{(28 A + 23 B) \cos[c] \sin[d x]}{84 d} + \frac{(A + B) \cos[2 c] \sin[2 d x]}{10 d} + \frac{B \cos[3 c] \sin[3 d x]}{28 d} \right) - \frac{1}{3 d \sqrt{1 + \cot[c]^2}}$$

$$A (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sec[d x - \text{ArcTan}[\cot[c]]]$$

$$\sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{21 d \sqrt{1 + \cot[c]^2}} 5 B (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]}$$

$$\sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \frac{1}{10 d} 3 A (1 + \cos[c + d x]) \text{Csc}[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\
\left. \left( \sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\
\left. \frac{\frac{\text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) - \frac{1}{10 d} 3 B (1 + \text{Cos}[c + d x]) \text{Csc}[c] \text{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \\
\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\
\left. \left( \sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\
\left. \frac{\frac{\text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) \right)$$

■ **Problem 125: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\text{Cos}[c + d x]} (a + a \text{Cos}[c + d x]) (A + B \text{Cos}[c + d x]) dx$$

Optimal (type 4, 101 leaves, 6 steps):

$$\frac{2 a (5 A + 3 B) \text{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right]}{5 d} + \frac{2 a (A + B) \text{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right]}{3 d} + \\
\frac{2 a (A + B) \sqrt{\text{Cos}[c + d x]} \text{Sin}[c + d x]}{3 d} + \frac{2 a B \text{Cos}[c + d x]^{3/2} \text{Sin}[c + d x]}{5 d}$$

Result (type 5, 830 leaves):

$$\begin{aligned}
& a \left( \sqrt{\cos[c+dx] (1+\cos[c+dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right. \\
& \left. \left( -\frac{(5A+3B)\cot[c]}{5d} + \frac{(A+B)\cos[dx]\sin[c]}{3d} + \frac{B\cos[2dx]\sin[2c]}{10d} + \frac{(A+B)\cos[c]\sin[dx]}{3d} + \frac{B\cos[2c]\sin[2dx]}{10d} \right) - \frac{1}{3d\sqrt{1+\cot[c]^2}} \right. \\
& A(1+\cos[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sec[dx - \operatorname{ArcTan}[\cot[c]]] \\
& \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \\
& \frac{1}{3d\sqrt{1+\cot[c]^2}} B(1+\cos[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{2d} A(1+\cos[c+dx]) \operatorname{Csc}[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) - \frac{1}{10d} 3B(1+\cos[c+dx]) \operatorname{Csc}[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /
\end{aligned}$$

$$\left( \frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) -$$

■ **Problem 126: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x]) (A + B \cos[c + d x])}{\sqrt{\cos[c + d x]}} dx$$

Optimal (type 4, 70 leaves, 5 steps):

$$\frac{2 a (A + B) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{d} + \frac{2 a (3 A + B) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} + \frac{2 a B \sqrt{\cos[c + d x]} \sin[c + d x]}{3 d}$$

Result (type 5, 784 leaves):

$$a \left( \sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \left( -\frac{(A + B) \cot[c]}{d} + \frac{B \cos[d x] \sin[c]}{3 d} + \frac{B \cos[c] \sin[d x]}{3 d} \right) - \frac{1}{d \sqrt{1 + \cot[c]^2}} \right.$$

$$A (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Sec}[d x - \text{ArcTan}[\cot[c]]]$$

$$\sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{3 d \sqrt{1 + \cot[c]^2}} B (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\text{Sec}[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$\sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \frac{1}{2 d} A (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$



$$\left( \frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} - \frac{1}{2d} B (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \right)$$

$$\left( \frac{\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right)}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) \left( \frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right)$$

■ **Problem 127: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x]) (A + B \cos[c + d x])}{\cos[c + d x]^{3/2}} dx$$

Optimal (type 4, 66 leaves, 5 steps):

$$-\frac{2 a (A - B) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{d} + \frac{2 a (A + B) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{d} + \frac{2 a A \sin[c + d x]}{d \sqrt{\cos[c + d x]}}$$

Result (type 5, 783 leaves):

$$a \left( \sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \left( -\frac{(-2 A + B + B \cos[2 c]) \csc[c] \sec[c]}{2 d} + \frac{A \sec[c] \sec[c + d x] \sin[d x]}{d} \right) - \frac{1}{d \sqrt{1 + \cot[c]^2}} \right)$$

$$A (1 + \cos[c + d x]) \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sec[d x - \text{ArcTan}[\cot[c]]]$$

$$\begin{aligned}
& \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} - \\
& \frac{1}{d \sqrt{1 + \text{Cot}[c]^2}} B (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\
& \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} + \frac{1}{2 d} A (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\
& \left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\
& \left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\
& \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]} \sqrt{1 + \text{Tan}[c]^2}}} \right) - \frac{1}{2 d} B (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\
& \left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\
& \left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\
& \left. \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]} \sqrt{1 + \text{Tan}[c]^2}}} \right) \right)
\end{aligned}$$

■ **Problem 128: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx]) (A + B \cos[c + dx])}{\cos[c + dx]^{5/2}} dx$$

Optimal (type 4, 95 leaves, 6 steps):

$$-\frac{2a(A+B) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{2a(A+3B) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} + \frac{2aA \sin[c+dx]}{3d \cos[c+dx]^{3/2}} + \frac{2a(A+B) \sin[c+dx]}{d \sqrt{\cos[c+dx]}}$$

Result (type 5, 813 leaves):

$$a \left( \sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right. \\ \left. \left( \frac{(A+B) \operatorname{Csc}[c] \operatorname{Sec}[c]}{d} + \frac{A \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 \sin[dx]}{3d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (A \sin[c] + 3A \sin[dx] + 3B \sin[dx])}{3d} \right) - \frac{1}{3d \sqrt{1 + \operatorname{Cot}[c]^2}} \right. \\ A (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\ \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\ \frac{1}{d \sqrt{1 + \operatorname{Cot}[c]^2}} B (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\ \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\ \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \frac{1}{2d} A (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\ \left. \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \right. \\ \left. \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \right.$$

$$\left( \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) + \frac{1}{2 d} B (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right.$$

$$\left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2}} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) \right)$$

■ **Problem 129: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x]) (A + B \cos[c + d x])}{\cos[c + d x]^{7/2}} dx$$

Optimal (type 4, 132 leaves, 7 steps):

$$-\frac{2 a (3 A + 5 B) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{2 a (A + B) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} +$$

$$\frac{2 a A \sin[c + d x]}{5 d \cos[c + d x]^{5/2}} + \frac{2 a (A + B) \sin[c + d x]}{3 d \cos[c + d x]^{3/2}} + \frac{2 a (3 A + 5 B) \sin[c + d x]}{5 d \sqrt{\cos[c + d x]}}$$

Result (type 5, 865 leaves):

$$a \left( \sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \right.$$

$$\left. \left( \frac{(3 A + 5 B) \text{Csc}[c] \text{Sec}[c]}{5 d} + \frac{A \text{Sec}[c] \text{Sec}[c + d x]^3 \sin[d x]}{5 d} + \frac{\text{Sec}[c] \text{Sec}[c + d x]^2 (3 A \sin[c] + 5 A \sin[d x] + 5 B \sin[d x])}{15 d} \right) + \right.$$

$$\begin{aligned}
& \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (5A \operatorname{Sin}[c] + 5B \operatorname{Sin}[c] + 9A \operatorname{Sin}[dx] + 15B \operatorname{Sin}[dx])}{15d} \right) - \frac{1}{3d \sqrt{1+\operatorname{Cot}[c]^2}} \\
& A (1+\operatorname{Cos}[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\
& \frac{1}{3d \sqrt{1+\operatorname{Cot}[c]^2}} B (1+\operatorname{Cos}[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \frac{1}{10d} 3A (1+\operatorname{Cos}[c+dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\
& \left( \sqrt{1 - \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \\
& \frac{\frac{\operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \operatorname{Cos}[c]^2 \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2}}{\sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}} \right) + \frac{1}{2d} B (1+\operatorname{Cos}[c+dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\
& \left( \sqrt{1 - \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) -
\end{aligned}$$

$$\left( \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right)$$

- **Problem 130: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{3/2} (a + a \cos[c + d x])^2 (A + B \cos[c + d x]) dx$$

Optimal (type 4, 194 leaves, 8 steps):

$$\frac{4 a^2 (9 A + 8 B) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{4 a^2 (6 A + 5 B) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \frac{4 a^2 (6 A + 5 B) \sqrt{\cos[c + d x]} \sin[c + d x]}{21 d} +$$

$$\frac{4 a^2 (9 A + 8 B) \cos[c + d x]^{3/2} \sin[c + d x]}{45 d} + \frac{2 a^2 (9 A + 11 B) \cos[c + d x]^{5/2} \sin[c + d x]}{63 d} + \frac{2 B \cos[c + d x]^{5/2} (a^2 + a^2 \cos[c + d x]) \sin[c + d x]}{9 d}$$

Result (type 5, 944 leaves):

$$\sqrt{\cos[c + d x]} (a + a \cos[c + d x])^2 \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( -\frac{(9 A + 8 B) \cot[c]}{15 d} + \frac{(51 A + 46 B) \cos[d x] \sin[c]}{168 d} + \frac{(36 A + 37 B) \cos[2 d x] \sin[2 c]}{360 d} + \frac{(A + 2 B) \cos[3 d x] \sin[3 c]}{56 d} + \frac{B \cos[4 d x] \sin[4 c]}{144 d} + \right.$$

$$\left. \frac{(51 A + 46 B) \cos[c] \sin[d x]}{168 d} + \frac{(36 A + 37 B) \cos[2 c] \sin[2 d x]}{360 d} + \frac{(A + 2 B) \cos[3 c] \sin[3 d x]}{56 d} + \frac{B \cos[4 c] \sin[4 d x]}{144 d} \right) - \frac{1}{7 d \sqrt{1 + \cot[c]^2}}$$

$$2 A (a + a \cos[c + d x])^2 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \text{Sec}[d x - \text{ArcTan}[\cot[c]]]$$

$$\sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{21 d \sqrt{1 + \cot[c]^2}} 5 B (a + a \cos[c + d x])^2 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right]$$

$$\text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \text{Sec}[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$\sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \frac{1}{10 d} 3 A (a + a \cos[c + d x])^2 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\tan[c]]] \right]^2 \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \\
\left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right. \\
\left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) - \frac{1}{15 d} 4 B (a + a \cos[c + d x])^2 \csc[c] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \\
\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\tan[c]]] \right]^2 \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \\
\left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right. \\
\left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right)$$

- **Problem 131: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c + d x]} (a + a \cos[c + d x])^2 (A + B \cos[c + d x]) dx$$

Optimal (type 4, 161 leaves, 7 steps):

$$\frac{4 a^2 (4 A + 3 B) \text{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right]}{5 d} + \frac{4 a^2 (7 A + 6 B) \text{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right]}{21 d} + \frac{4 a^2 (7 A + 6 B) \sqrt{\cos[c + d x]} \sin[c + d x]}{21 d} + \\
\frac{2 a^2 (7 A + 9 B) \cos[c + d x]^{3/2} \sin[c + d x]}{35 d} + \frac{2 B \cos[c + d x]^{3/2} (a^2 + a^2 \cos[c + d x]) \sin[c + d x]}{7 d}$$

Result (type 5, 898 leaves):

$$\begin{aligned}
& \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^2 \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \left( -\frac{(4A+3B)\cot[c]}{5d} + \frac{(56A+51B)\cos[dx]\sin[c]}{168d} + \frac{(A+2B)\cos[2dx]\sin[2c]}{20d} \right. \\
& \quad \left. + \frac{B\cos[3dx]\sin[3c]}{56d} + \frac{(56A+51B)\cos[c]\sin[dx]}{168d} + \frac{(A+2B)\cos[2c]\sin[2dx]}{20d} + \frac{B\cos[3c]\sin[3dx]}{56d} \right) - \\
& \frac{1}{3d\sqrt{1+\cot[c]^2}} A (a+a\cos[c+dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{7d\sqrt{1+\cot[c]^2}} \\
& 2B (a+a\cos[c+dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \\
& \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{5d} 2A (a+a\cos[c+dx])^2 \operatorname{Csc}[c] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) - \frac{1}{10d} 3B (a+a\cos[c+dx])^2 \operatorname{Csc}[c] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /
\end{aligned}$$



$$\left( \frac{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2}} - \frac{\frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2}} \right)$$

- **Problem 132: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^2 (A + B \cos [c + d x])}{\sqrt{\cos [c + d x]}} dx$$

Optimal (type 4, 126 leaves, 6 steps):

$$\frac{4 a^2 (5 A + 4 B) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{4 a^2 (2 A + B) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} + \frac{2 a^2 (5 A + 7 B) \sqrt{\cos [c + d x]} \sin [c + d x]}{15 d} + \frac{2 B \sqrt{\cos [c + d x]} (a^2 + a^2 \cos [c + d x]) \sin [c + d x]}{5 d}$$

Result (type 5, 852 leaves):

$$\sqrt{\cos [c + d x]} (a + a \cos [c + d x])^2 \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( -\frac{(5 A + 4 B) \cot [c]}{5 d} + \frac{(A + 2 B) \cos [d x] \sin [c]}{6 d} + \frac{B \cos [2 d x] \sin [2 c]}{20 d} + \frac{(A + 2 B) \cos [c] \sin [d x]}{6 d} + \frac{B \cos [2 c] \sin [2 d x]}{20 d} \right) - \frac{1}{3 d \sqrt{1 + \cot [c]^2}}$$

$$2 A (a + a \cos [c + d x])^2 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \text{ArcTan}[\cot [c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \text{Sec}[d x - \text{ArcTan}[\cot [c]]]$$

$$\sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\cot [c]]]} -$$

$$\frac{1}{3 d \sqrt{1 + \cot [c]^2}} B (a + a \cos [c + d x])^2 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \text{ArcTan}[\cot [c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\text{Sec}[d x - \text{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]]}$$

$$\sqrt{1 + \sin [d x - \text{ArcTan}[\cot [c]]]} - \frac{1}{2 d} A (a + a \cos [c + d x])^2 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\tan[c]]] \right]^2 \right) \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} - \frac{1}{5 d} 2 B (a + a \cos[c + d x])^2 \text{Csc}[c] \text{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\tan[c]]] \right]^2 \right) \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}}$$

■ **Problem 133: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^2 (A + B \cos[c + d x])}{\cos[c + d x]^{3/2}} dx$$

Optimal (type 4, 118 leaves, 6 steps):

$$\frac{4 a^2 B \text{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right]}{d} + \frac{4 a^2 (3 A + 2 B) \text{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right]}{3 d} - \frac{2 a^2 (3 A - B) \sqrt{\cos[c + d x]} \sin[c + d x]}{3 d} + \frac{2 A (a^2 + a^2 \cos[c + d x]) \sin[c + d x]}{d \sqrt{\cos[c + d x]}}$$

Result (type 5, 623 leaves):

$$\begin{aligned}
& \sqrt{\cos [c+d x]} (a+a \cos [c+d x])^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \\
& \left(-\frac{(-A+2 B+A \cos [2 c]+2 B \cos [2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{4 d}+\frac{B \cos [d x] \sin [c]}{6 d}+\frac{B \cos [c] \sin [d x]}{6 d}+\frac{A \operatorname{Sec}[c] \operatorname{Sec}[c+d x] \sin [d x]}{2 d}\right)- \\
& \frac{1}{d \sqrt{1+\cot [c]^2}} A(a+a \cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}-\frac{1}{3 d \sqrt{1+\cot [c]^2}} \\
& 2 B(a+a \cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \\
& \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}-\frac{1}{2 d} B(a+a \cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \\
& \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right)- \\
& \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}}
\end{aligned}$$

- **Problem 134: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \cos [c+d x])^2 (A+B \cos [c+d x])}{\cos [c+d x]^{5/2}} d x$$

Optimal (type 4, 120 leaves, 6 steps):

$$-\frac{4a^2 A \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{4a^2(2A+3B) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} + \frac{2a^2(5A+3B) \sin[c+dx]}{3d\sqrt{\cos[c+dx]}} + \frac{2A(a^2+a^2\cos[c+dx]) \sin[c+dx]}{3d\cos[c+dx]^{3/2}}$$

Result (type 5, 624 leaves):

$$\begin{aligned} & \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\ & \left( -\frac{(-4A-B+B\cos[2c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{4d} + \frac{A \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 \sin[dx]}{6d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (A \sin[c] + 6A \sin[dx] + 3B \sin[dx])}{6d} \right) - \\ & \frac{1}{3d\sqrt{1+\cot[c]^2}} 2A(a+a\cos[c+dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]\right]^2 \\ & \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\ & \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{d\sqrt{1+\cot[c]^2}} \\ & B(a+a\cos[c+dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\ & \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\ & \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} + \frac{1}{2d} A(a+a\cos[c+dx])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\ & \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]\right]^2 \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\ & \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\ & \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \end{aligned}$$

■ **Problem 135: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx])^2 (A + B \cos[c + dx])}{\cos[c + dx]^{7/2}} dx$$

Optimal (type 4, 159 leaves, 7 steps):

$$\begin{aligned} & - \frac{4 a^2 (4 A + 5 B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5 d} + \frac{4 a^2 (A + 2 B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3 d} \\ & + \frac{2 a^2 (7 A + 5 B) \sin[c + dx]}{15 d \cos[c + dx]^{3/2}} + \frac{4 a^2 (4 A + 5 B) \sin[c + dx]}{5 d \sqrt{\cos[c + dx]}} + \frac{2 A (a^2 + a^2 \cos[c + dx]) \sin[c + dx]}{5 d \cos[c + dx]^{5/2}} \end{aligned}$$

Result (type 5, 883 leaves):

$$\begin{aligned} & \sqrt{\cos[c + dx]} (a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\ & \left( \frac{(4 A + 5 B) \operatorname{Csc}[c] \operatorname{Sec}[c]}{5 d} + \frac{A \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^3 \sin[dx]}{10 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 (3 A \sin[c] + 10 A \sin[dx] + 5 B \sin[dx])}{30 d} \right. \\ & \left. + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + dx] (10 A \sin[c] + 5 B \sin[c] + 24 A \sin[dx] + 30 B \sin[dx])}{30 d} \right) - \frac{1}{3 d \sqrt{1 + \operatorname{Cot}[c]^2}} \\ & A (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\ & \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\ & \frac{1}{3 d \sqrt{1 + \operatorname{Cot}[c]^2}} 2 B (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\ & \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\ & \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \frac{1}{5 d} 2 A (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\ & \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\ & \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \end{aligned}$$

$$\left( \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) + \frac{1}{2d} B (a + a \cos[c + d x])^2 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]\right]^2 \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \sqrt{1 + \tan[c]^2} \right) -$$

$$\left( \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right)$$

■ **Problem 136: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^2 (A + B \cos[c + d x])}{\cos[c + d x]^{9/2}} dx$$

Optimal (type 4, 194 leaves, 8 steps):

$$-\frac{4 a^2 (3 A + 4 B) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^2 (6 A + 7 B) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} +$$

$$\frac{2 a^2 (9 A + 7 B) \sin[c + d x]}{35 d \cos[c + d x]^{5/2}} + \frac{4 a^2 (6 A + 7 B) \sin[c + d x]}{21 d \cos[c + d x]^{3/2}} + \frac{4 a^2 (3 A + 4 B) \sin[c + d x]}{5 d \sqrt{\cos[c + d x]}} + \frac{2 A (a^2 + a^2 \cos[c + d x]) \sin[c + d x]}{7 d \cos[c + d x]^{7/2}}$$

Result (type 5, 925 leaves):

$$\sqrt{\cos[c + d x]} (a + a \cos[c + d x])^2 \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( \frac{(3 A + 4 B) \csc[c] \sec[c]}{5 d} + \frac{A \sec[c] \sec[c + d x]^4 \sin[d x]}{14 d} + \frac{\sec[c] \sec[c + d x]^3 (5 A \sin[c] + 14 A \sin[d x] + 7 B \sin[d x])}{70 d} + \right.$$

$$\frac{\sec[c] \sec[c + d x]^2 (42 A \sin[c] + 21 B \sin[c] + 60 A \sin[d x] + 70 B \sin[d x])}{210 d} +$$

$$\left. \frac{\sec[c] \sec[c + d x] (30 A \sin[c] + 35 B \sin[c] + 63 A \sin[d x] + 84 B \sin[d x])}{105 d} \right) - \frac{1}{7 d \sqrt{1 + \cot[c]^2}}$$

$$\begin{aligned}
& 2 A (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\
& \frac{1}{3 d \sqrt{1 + \operatorname{Cot}[c]^2}} B (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \\
& \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \frac{1}{10 d} 3 A (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\
& \left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \\
& \frac{\frac{\sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{1}{5 d} 2 B (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\
& \left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \\
& \frac{\frac{\sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}}
\end{aligned}$$

**Problem 137: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^{3/2} (a + a \cos[c + dx])^3 (A + B \cos[c + dx]) dx$$

Optimal (type 4, 237 leaves, 9 steps):

$$\frac{4 a^3 (17 A + 15 B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{15 d} + \frac{4 a^3 (121 A + 105 B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{231 d} +$$

$$\frac{4 a^3 (121 A + 105 B) \sqrt{\cos[c + dx]} \sin[c + dx]}{231 d} + \frac{4 a^3 (17 A + 15 B) \cos[c + dx]^{3/2} \sin[c + dx]}{45 d} + \frac{20 a^3 (22 A + 21 B) \cos[c + dx]^{5/2} \sin[c + dx]}{693 d} +$$

$$\frac{2 a B \cos[c + dx]^{5/2} (a + a \cos[c + dx])^2 \sin[c + dx]}{11 d} + \frac{2 (11 A + 15 B) \cos[c + dx]^{5/2} (a^3 + a^3 \cos[c + dx]) \sin[c + dx]}{99 d}$$

Result (type 5, 990 leaves):

$$\sqrt{\cos[c + dx]} (a + a \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6$$

$$\left( -\frac{(17 A + 15 B) \cot[c]}{30 d} + \frac{(2134 A + 1953 B) \cos[dx] \sin[c]}{7392 d} + \frac{(73 A + 75 B) \cos[2 dx] \sin[2 c]}{720 d} + \frac{3 (44 A + 63 B) \cos[3 dx] \sin[3 c]}{4928 d} + \right.$$

$$\frac{(A + 3 B) \cos[4 dx] \sin[4 c]}{288 d} + \frac{B \cos[5 dx] \sin[5 c]}{704 d} + \frac{(2134 A + 1953 B) \cos[c] \sin[dx]}{7392 d} + \frac{(73 A + 75 B) \cos[2 c] \sin[2 dx]}{720 d} +$$

$$\left. \frac{3 (44 A + 63 B) \cos[3 c] \sin[3 dx]}{4928 d} + \frac{(A + 3 B) \cos[4 c] \sin[4 dx]}{288 d} + \frac{B \cos[5 c] \sin[5 dx]}{704 d} \right) - \frac{1}{42 d \sqrt{1 + \cot[c]^2}}$$

$$11 A (a + a \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]]$$

$$\sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{22 d \sqrt{1 + \cot[c]^2}} 5 B (a + a \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]}$$

$$\sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{60 d} 17 A (a + a \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /$$



$$\left( \frac{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}} \right) - \frac{1}{4 d} B (a + a \cos [c + d x])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \text{ArcTan}[\text{Tan}[c]]]^2\right\} \sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c] \right) / \right.$$

$$\left. \frac{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}} \right)$$

■ **Problem 138: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos [c + d x]} (a + a \cos [c + d x])^3 (A + B \cos [c + d x]) dx$$

Optimal (type 4, 204 leaves, 8 steps):

$$\frac{4 a^3 (21 A + 17 B) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{4 a^3 (13 A + 11 B) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} +$$

$$\frac{4 a^3 (13 A + 11 B) \sqrt{\cos [c + d x]} \sin [c + d x]}{21 d} + \frac{4 a^3 (24 A + 23 B) \cos [c + d x]^{3/2} \sin [c + d x]}{105 d} +$$

$$\frac{2 a B \cos [c + d x]^{3/2} (a + a \cos [c + d x])^2 \sin [c + d x]}{9 d} + \frac{2 (9 A + 13 B) \cos [c + d x]^{3/2} (a^3 + a^3 \cos [c + d x]) \sin [c + d x]}{63 d}$$

Result (type 5, 944 leaves):

$$\sqrt{\cos [c + d x]} (a + a \cos [c + d x])^3 \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \left( -\frac{(21 A + 17 B) \cot [c]}{30 d} + \frac{(107 A + 97 B) \cos [d x] \sin [c]}{336 d} + \right.$$

$$\left. \frac{(54 A + 73 B) \cos [2 d x] \sin [2 c]}{720 d} + \frac{(A + 3 B) \cos [3 d x] \sin [3 c]}{112 d} + \frac{B \cos [4 d x] \sin [4 c]}{288 d} + \frac{(107 A + 97 B) \cos [c] \sin [d x]}{336 d} + \right.$$

$$\begin{aligned}
& \left. \frac{(54 A + 73 B) \cos[2 c] \sin[2 d x]}{720 d} + \frac{(A + 3 B) \cos[3 c] \sin[3 d x]}{112 d} + \frac{B \cos[4 c] \sin[4 d x]}{288 d} \right) - \frac{1}{42 d \sqrt{1 + \cot[c]^2}} \\
13 A & (a + a \cos[c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot[c]]] \\
& \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\cot[c]]]} - \\
& \frac{1}{42 d \sqrt{1 + \cot[c]^2}} 11 B (a + a \cos[c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{20 d} 7 A (a + a \cos[c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[d x + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \frac{\frac{\sin[d x + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) - \frac{1}{60 d} 17 B (a + a \cos[c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[d x + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -
\end{aligned}$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}$$

- **Problem 139: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^3 (A + B \cos[c + d x])}{\sqrt{\cos[c + d x]}} dx$$

Optimal (type 4, 171 leaves, 7 steps):

$$\frac{4 a^3 (9 A + 7 B) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{4 a^3 (21 A + 13 B) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{21 d} + \frac{4 a^3 (42 A + 41 B) \sqrt{\cos[c + d x]} \sin[c + d x]}{105 d} + \frac{2 a B \sqrt{\cos[c + d x]} (a + a \cos[c + d x])^2 \sin[c + d x]}{7 d} + \frac{2 (7 A + 11 B) \sqrt{\cos[c + d x]} (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{35 d}$$

Result (type 5, 898 leaves):

$$\begin{aligned} & \sqrt{\cos[c + d x]} (a + a \cos[c + d x])^3 \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \left( -\frac{(9 A + 7 B) \cot[c]}{10 d} + \frac{(84 A + 107 B) \cos[d x] \sin[c]}{336 d} + \frac{(A + 3 B) \cos[2 d x] \sin[2 c]}{40 d} \right. \\ & \left. + \frac{B \cos[3 d x] \sin[3 c]}{112 d} + \frac{(84 A + 107 B) \cos[c] \sin[d x]}{336 d} + \frac{(A + 3 B) \cos[2 c] \sin[2 d x]}{40 d} + \frac{B \cos[3 c] \sin[3 d x]}{112 d} \right) - \\ & \frac{1}{2 d \sqrt{1 + \cot[c]^2}} A (a + a \cos[c + d x])^3 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \\ & \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \\ & \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \frac{1}{42 d \sqrt{1 + \cot[c]^2}} \\ & 13 B (a + a \cos[c + d x])^3 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\ & \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \\ & \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \frac{1}{20 d} 9 A (a + a \cos[c + d x])^3 \text{Csc}[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \end{aligned}$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\tan[c]]] \right]^2 \right) \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} - \frac{1}{20 d} 7 B (a + a \cos[c + d x])^3 \text{Csc}[c] \text{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^6$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\tan[c]]] \right]^2 \right) \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}}$$

■ **Problem 140: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^3 (A + B \cos[c + d x])}{\cos[c + d x]^{3/2}} dx$$

Optimal (type 4, 169 leaves, 7 steps):

$$\frac{4 a^3 (5 A + 9 B) \text{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right]}{5 d} + \frac{4 a^3 (5 A + 3 B) \text{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right]}{3 d} - \frac{4 a^3 (5 A - 6 B) \sqrt{\cos[c + d x]} \sin[c + d x]}{15 d} + \frac{2 a A (a + a \cos[c + d x])^2 \sin[c + d x]}{d \sqrt{\cos[c + d x]}} - \frac{2 (5 A - B) \sqrt{\cos[c + d x]} (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{5 d}$$

Result (type 5, 888 leaves):

$$\begin{aligned}
& \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^3 \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \left( -\frac{(5A+18B+15A\cos[2c]+18B\cos[2c])\csc[c]\sec[c]}{40d} + \frac{(A+3B)\cos[dx]\sin[c]}{12d} \right. \\
& \quad \left. + \frac{B\cos[2dx]\sin[2c]}{40d} + \frac{(A+3B)\cos[c]\sin[dx]}{12d} + \frac{A\sec[c]\sec[c+dx]\sin[dx]}{4d} + \frac{B\cos[2c]\sin[2dx]}{40d} \right) - \\
& \frac{1}{6d\sqrt{1+\cot[c]^2}} 5A (a+a\cos[c+dx])^3 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]\right]^2 \\
& \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \sec[dx-\operatorname{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]} - \frac{1}{2d\sqrt{1+\cot[c]^2}} \\
& B (a+a\cos[c+dx])^3 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]\right]^2 \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \\
& \sec[dx-\operatorname{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]} - \frac{1}{4d} A (a+a\cos[c+dx])^3 \csc[c] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx+\operatorname{ArcTan}[\tan[c]]]\right]^2 \sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1-\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2} \sqrt{1+\tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1+\tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx+\operatorname{ArcTan}[\tan[c]]] \sqrt{1+\tan[c]^2}}{\cos[c]^2+\sin[c]^2}}{\sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2}} \right) - \frac{1}{20d} 9B (a+a\cos[c+dx])^3 \csc[c] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx+\operatorname{ArcTan}[\tan[c]]]\right]^2 \sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /
\end{aligned}$$

$$\left( \frac{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2}} - \frac{\frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2}} \right)$$

■ **Problem 141: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^3 (A + B \cos [c + d x])}{\cos [c + d x]^{5/2}} dx$$

Optimal (type 4, 161 leaves, 7 steps):

$$-\frac{4 a^3 (A - B) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{d} + \frac{20 a^3 (A + B) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} - \frac{4 a^3 (4 A + B) \sqrt{\cos [c + d x]} \sin [c + d x]}{3 d} + \frac{2 a A (a + a \cos [c + d x])^2 \sin [c + d x]}{3 d \cos [c + d x]^{3/2}} + \frac{2 (7 A + 3 B) (a^3 + a^3 \cos [c + d x]) \sin [c + d x]}{3 d \sqrt{\cos [c + d x]}}$$

Result (type 5, 879 leaves):

$$\sqrt{\cos [c + d x]} (a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \left( -\frac{(-5 A + B + A \cos [2 c] + 3 B \cos [2 c]) \csc [c] \sec [c]}{8 d} + \frac{B \cos [d x] \sin [c]}{12 d} + \frac{B \cos [c] \sin [d x]}{12 d} + \frac{A \sec [c] \sec [c + d x]^2 \sin [d x]}{12 d} + \frac{\sec [c] \sec [c + d x] (A \sin [c] + 9 A \sin [d x] + 3 B \sin [d x])}{12 d} \right) - \frac{1}{6 d \sqrt{1 + \cot [c]^2}}$$

$$5 A (a + a \cos [c + d x])^3 \csc [c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \text{ArcTan}[\cot [c]]]^2\right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sec [d x - \text{ArcTan}[\cot [c]]]$$

$$\sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\cot [c]]]} - \frac{1}{6 d \sqrt{1 + \cot [c]^2}} 5 B (a + a \cos [c + d x])^3 \csc [c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \text{ArcTan}[\cot [c]]]^2\right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6$$

$$\sec [d x - \text{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]]} + \frac{1}{4 d} A (a + a \cos [c + d x])^3 \csc [c] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\tan[c]]] \right]^2 \right) \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) - \frac{1}{4 d} B (a + a \cos[c + d x])^3 \csc[c] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\tan[c]]] \right]^2 \right) \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right)$$

■ **Problem 142: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^3 (A + B \cos[c + d x])}{\cos[c + d x]^{7/2}} dx$$

Optimal (type 4, 171 leaves, 7 steps):

$$-\frac{4 a^3 (9 A + 5 B) \text{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right]}{5 d} + \frac{4 a^3 (3 A + 5 B) \text{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right]}{3 d} + \frac{4 a^3 (21 A + 20 B) \sin[c + d x]}{15 d \sqrt{\cos[c + d x]}} + \frac{2 a A (a + a \cos[c + d x])^2 \sin[c + d x]}{5 d \cos[c + d x]^{5/2}} + \frac{2 (9 A + 5 B) (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{15 d \cos[c + d x]^{3/2}}$$

Result (type 5, 890 leaves):

$$\begin{aligned}
& \sqrt{\cos [c+d x]} (a+a \cos [c+d x])^3 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6\left(-\frac{(-36 A-25 B+5 B \cos [2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{40 d}+\right. \\
& \quad \frac{A \operatorname{Sec}[c] \operatorname{Sec}[c+d x]^3 \sin [d x]}{20 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2(3 A \sin [c]+15 A \sin [d x]+5 B \sin [d x])}{60 d}+ \\
& \quad \left.\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x](15 A \sin [c]+5 B \sin [c]+54 A \sin [d x]+45 B \sin [d x])}{60 d}\right)-\frac{1}{2 d \sqrt{1+\cot [c]^2}} \\
& A(a+a \cos [c+d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \\
& \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}- \\
& \frac{1}{6 d \sqrt{1+\cot [c]^2}} 5 B(a+a \cos [c+d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 \\
& \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}+\frac{1}{20 d} 9 A(a+a \cos [c+d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \\
& \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right)- \\
& \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}}+\frac{1}{4 d} B(a+a \cos [c+d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) /
\end{aligned}$$



$$\left( \frac{\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} - \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right)$$

- **Problem 143: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^3 (A + B \cos [c + d x])}{\cos [c + d x]^{9/2}} dx$$

Optimal (type 4, 204 leaves, 8 steps):

$$-\frac{4 a^3 (7 A + 9 B) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^3 (13 A + 21 B) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \frac{4 a^3 (41 A + 42 B) \sin [c + d x]}{105 d \cos [c + d x]^{3/2}} + \frac{4 a^3 (7 A + 9 B) \sin [c + d x]}{5 d \sqrt{\cos [c + d x]}} + \frac{2 a A (a + a \cos [c + d x])^2 \sin [c + d x]}{7 d \cos [c + d x]^{7/2}} + \frac{2 (11 A + 7 B) (a^3 + a^3 \cos [c + d x]) \sin [c + d x]}{35 d \cos [c + d x]^{5/2}}$$

Result (type 5, 925 leaves):

$$\sqrt{\cos [c + d x]} (a + a \cos [c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \left( \frac{(7 A + 9 B) \operatorname{Csc}[c] \operatorname{Sec}[c]}{10 d} + \frac{A \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^4 \sin [d x]}{28 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 (5 A \sin [c] + 21 A \sin [d x] + 7 B \sin [d x])}{140 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (63 A \sin [c] + 21 B \sin [c] + 130 A \sin [d x] + 105 B \sin [d x])}{420 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x] (130 A \sin [c] + 105 B \sin [c] + 294 A \sin [d x] + 378 B \sin [d x])}{420 d} \right) - \frac{1}{42 d \sqrt{1 + \cot [c]^2}}$$

$$13 A (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \frac{1}{2 d \sqrt{1 + \cot [c]^2}} B (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} + \frac{1}{20 d} 7 A (a + a \cos[c + d x])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} + \frac{1}{20 d} 9 B (a + a \cos[c + d x])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}}$$

- **Problem 144: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^3 (A + B \cos[c + d x])}{\cos[c + d x]^{11/2}} dx$$

Optimal (type 4, 237 leaves, 9 steps):

$$\begin{aligned}
& - \frac{4 a^3 (17 A + 21 B) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{4 a^3 (11 A + 13 B) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \\
& \frac{4 a^3 (23 A + 24 B) \operatorname{Sin}[c + d x]}{105 d \operatorname{Cos}[c + d x]^{5/2}} + \frac{4 a^3 (11 A + 13 B) \operatorname{Sin}[c + d x]}{21 d \operatorname{Cos}[c + d x]^{3/2}} + \frac{4 a^3 (17 A + 21 B) \operatorname{Sin}[c + d x]}{15 d \sqrt{\operatorname{Cos}[c + d x]}} + \\
& \frac{2 a A (a + a \operatorname{Cos}[c + d x])^2 \operatorname{Sin}[c + d x]}{9 d \operatorname{Cos}[c + d x]^{9/2}} + \frac{2 (13 A + 9 B) (a^3 + a^3 \operatorname{Cos}[c + d x]) \operatorname{Sin}[c + d x]}{63 d \operatorname{Cos}[c + d x]^{7/2}}
\end{aligned}$$

Result (type 5, 967 leaves):

$$\begin{aligned}
& \sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \left( \frac{(17 A + 21 B) \operatorname{Csc}[c] \operatorname{Sec}[c]}{30 d} + \frac{A \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^5 \operatorname{Sin}[d x]}{36 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^4 (7 A \operatorname{Sin}[c] + 27 A \operatorname{Sin}[d x] + 9 B \operatorname{Sin}[d x])}{252 d} \right. \\
& \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x] (55 A \operatorname{Sin}[c] + 65 B \operatorname{Sin}[c] + 119 A \operatorname{Sin}[d x] + 147 B \operatorname{Sin}[d x])}{210 d} + \\
& \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 (135 A \operatorname{Sin}[c] + 45 B \operatorname{Sin}[c] + 238 A \operatorname{Sin}[d x] + 189 B \operatorname{Sin}[d x])}{1260 d} + \\
& \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (238 A \operatorname{Sin}[c] + 189 B \operatorname{Sin}[c] + 330 A \operatorname{Sin}[d x] + 390 B \operatorname{Sin}[d x])}{1260 d} \right) - \\
& \frac{1}{42 d \sqrt{1 + \operatorname{Cot}[c]^2}} 11 A (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \frac{1}{42 d \sqrt{1 + \operatorname{Cot}[c]^2}} \\
& 13 B (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \frac{1}{60 d} 17 A (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /
\end{aligned}$$

$$\left( \frac{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2}}{\frac{\frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}}} + \frac{1}{20 d} 7 B (a + a \cos [c + d x])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \right) -$$

$$\left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \text{ArcTan}[\text{Tan}[c]]]^2\right\] \sin [d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right.$$

$$\left. \left( \frac{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2}}{\frac{\frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}}} \right) -$$

■ **Problem 145: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^{5/2} (A + B \cos [c + d x])}{a + a \cos [c + d x]} dx$$

Optimal (type 4, 156 leaves, 6 steps):

$$-\frac{3(5A - 7B) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5ad} + \frac{5(A - B) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3ad} +$$

$$\frac{5(A - B) \sqrt{\cos [c + dx]} \sin [c + dx]}{3ad} - \frac{(5A - 7B) \cos [c + dx]^{3/2} \sin [c + dx]}{5ad} + \frac{(A - B) \cos [c + dx]^{5/2} \sin [c + dx]}{d(a + a \cos [c + dx])}$$

Result (type 5, 1182 leaves):

$$-\frac{1}{4(a + a \cos [c + dx])} 3 i A \cos \left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right) \right.$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(3 i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] - 3 d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right) -} \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(-i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right) + \right. \\
& \frac{1}{20 \left(a + a \operatorname{Cos}[c + d x]\right)} 2 i B \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left. \left( \left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right. \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(3 i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] - 3 d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right) -} \right. \right. \\
& \left. \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right. \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(-i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right) + \frac{1}{a + a \operatorname{Cos}[c + d x]} \right. \\
& \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sqrt{\operatorname{Cos}[c + d x]} \left( \frac{2 \left(5 A - 5 B + 10 A \operatorname{Cos}[c] - 16 B \operatorname{Cos}[c]\right) \operatorname{Csc}[c]}{5 d} + \frac{4 (A - B) \operatorname{Cos}[d x] \operatorname{Sin}[c]}{3 d} + \frac{2 B \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{5 d} + \right. \\
& \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] \left(A \operatorname{Sin}\left[\frac{d x}{2}\right] - B \operatorname{Sin}\left[\frac{d x}{2}\right]\right)}{d} + \frac{4 (A - B) \operatorname{Cos}[c] \operatorname{Sin}[d x]}{3 d} + \frac{2 B \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x]}{5 d} \right) - \\
& \left(5 A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \left. \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \frac{\sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{\left(3 d \left(a + a \operatorname{Cos}[c + d x]\right) \sqrt{1 + \operatorname{Cot}[c]^2}\right)} + \right. \\
& \left(5 B \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \left. \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right)
\end{aligned}$$

$$\sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \Bigg/ \left( 3d (a + a \cos[c + dx]) \sqrt{1 + \cot[c]^2} \right)$$

- **Problem 146: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{3/2} (A + B \cos[c + dx])}{a + a \cos[c + dx]} dx$$

Optimal (type 4, 123 leaves, 5 steps):

$$\frac{3(A - B) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} - \frac{(3A - 5B) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3ad} - \frac{(3A - 5B) \sqrt{\cos[c + dx]} \sin[c + dx]}{3ad} + \frac{(A - B) \cos[c + dx]^{3/2} \sin[c + dx]}{d(a + a \cos[c + dx])}$$

Result (type 5, 1129 leaves):

$$\frac{1}{4(a + a \cos[c + dx])} 3iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) \Bigg/ (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ \left. \left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) \Bigg/ (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) - \frac{1}{4(a + a \cos[c + dx])} 3iB \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\ \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) \Bigg/ (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ \left. \left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) \Bigg/ (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \\ \frac{1}{a + a \cos[c + dx]} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \left( -\frac{2(A - B)(1 + 2\cos[c]) \text{Csc}[c]}{d} + \frac{4B \cos[dx] \sin[c]}{3d} - \right. \\ \left. \frac{2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right])}{d} + \frac{4B \cos[c] \sin[dx]}{3d} \right) +$$

$$\left( A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\ \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\ \left( d (a + a \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \left( 5 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right. \\ \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\ \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right)$$

■ **Problem 147: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c + dx]} (A + B \cos[c + dx])}{a + a \cos[c + dx]} dx$$

Optimal (type 4, 85 leaves, 4 steps):

$$-\frac{(A - 3B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{(A - B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{(A - B) \sqrt{\cos[c + dx]} \sin[c + dx]}{d(a + a \cos[c + dx])}$$

Result (type 5, 1098 leaves):

$$-\frac{1}{4(a + a \cos[c + dx])} i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\ \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) + \frac{1}{4(a + a \cos[c + dx])} 3 i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\ \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right.$$

$$\begin{aligned}
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c] \right) + \\
& \frac{\cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \sqrt{\cos [c + d x]} \left( -\frac{2 (-A+B+2 B \cos [c]) \operatorname{Csc}[c]}{d} + \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (A \sin \left[ \frac{d x}{2} \right] - B \sin \left[ \frac{d x}{2} \right])}{d} \right)}{a + a \cos [c + d x]} - \\
& \left( A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]] \right]^2 \right) \operatorname{Sec} \left[ \frac{c}{2} \right] \\
& \quad \operatorname{Sec} [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \\
& \quad \left. \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right) / \left( d (a + a \cos [c + d x]) \sqrt{1 + \cot [c]^2} \right) + \\
& \left( B \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]] \right]^2 \right) \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} [d x - \operatorname{ArcTan}[\cot [c]]] \\
& \quad \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \\
& \quad \left. \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right) / \left( d (a + a \cos [c + d x]) \sqrt{1 + \cot [c]^2} \right)
\end{aligned}$$

- **Problem 148: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos [c + d x]}{\sqrt{\cos [c + d x]} (a + a \cos [c + d x])} dx$$

Optimal (type 4, 83 leaves, 4 steps):

$$\frac{(A - B) \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right]}{a d} + \frac{(A + B) \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right]}{a d} - \frac{(A - B) \sqrt{\cos [c + d x]} \sin [c + d x]}{d (a + a \cos [c + d x])}$$

Result (type 5, 1094 leaves):

$$\frac{1}{4 (a + a \cos [c + d x])} i A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \\
\left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right.$$



$$\begin{aligned}
& \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( 3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c] \right) - \\
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \\
& \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( -id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c] \right) - \frac{1}{4(a + a \cos[c + dx])} i B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \\
& \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \left( \left( 2 e^{2idx} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( 3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c] \right) - \right. \\
& \left. \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( -id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c] \right) \right) + \\
& \frac{\operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \sqrt{\cos[c + dx]} \left( -\frac{2(A-B) \operatorname{Csc}[c]}{d} - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c+dx}{2} \right] (A \operatorname{Sin} \left[ \frac{dx}{2} \right] - B \operatorname{Sin} \left[ \frac{dx}{2} \right])}{d} \right)}{a + a \cos[c + dx]} - \\
& \left( A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \right. \\
& \left. \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( d(a + a \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left( B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( d(a + a \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right)
\end{aligned}$$

■ **Problem 149: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx]}{\cos[c + dx]^{3/2} (a + a \cos[c + dx])} dx$$

Optimal (type 4, 119 leaves, 5 steps):

$$-\frac{(3A-B) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{ad} - \frac{(A-B) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{ad} + \frac{(3A-B) \operatorname{Sin}[c+dx]}{ad \sqrt{\operatorname{Cos}[c+dx]}} - \frac{(A-B) \operatorname{Sin}[c+dx]}{d \sqrt{\operatorname{Cos}[c+dx]} (a+a \operatorname{Cos}[c+dx])}$$

Result (type 5, 1130 leaves):

$$\begin{aligned} & -\frac{1}{4(a+a \operatorname{Cos}[c+dx])} 3iA \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ & \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1+e^{2idx}) \operatorname{Cos}[c] - 3d(-1+e^{2idx}) \operatorname{Sin}[c]) - \right. \\ & \quad \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \right. \\ & \quad \left. \left. \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1+e^{2idx}) \operatorname{Cos}[c] + d(-1+e^{2idx}) \operatorname{Sin}[c]) \right) + \frac{1}{4(a+a \operatorname{Cos}[c+dx])} iB \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\ & \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1+e^{2idx}) \operatorname{Cos}[c] - 3d(-1+e^{2idx}) \operatorname{Sin}[c]) - \right. \\ & \quad \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \right. \\ & \quad \left. \left. \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1+e^{2idx}) \operatorname{Cos}[c] + d(-1+e^{2idx}) \operatorname{Sin}[c]) \right) + \\ & \frac{1}{a+a \operatorname{Cos}[c+dx]} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\operatorname{Cos}[c+dx]} \left( \frac{(2A+A \operatorname{Cos}[c] - B \operatorname{Cos}[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{d} + \right. \\ & \quad \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \operatorname{Sin}\left[\frac{dx}{2}\right] - B \operatorname{Sin}\left[\frac{dx}{2}\right])}{d} + \frac{4A \operatorname{Sec}[c] \operatorname{Sec}[c+dx] \operatorname{Sin}[dx]}{d} \right) + \\ & \left( A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\ & \quad \left. \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\ & \quad \left. \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / (d(a+a \operatorname{Cos}[c+dx]) \sqrt{1 + \operatorname{Cot}[c]^2}) - \end{aligned}$$

$$\left( B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\ \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\ \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( d (a + a \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right)$$

■ **Problem 150: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx]}{\cos[c + dx]^{5/2} (a + a \cos[c + dx])} dx$$

Optimal (type 4, 153 leaves, 6 steps):

$$\frac{3(A - B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{(5A - 3B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3ad} + \\ \frac{(5A - 3B) \sin[c + dx]}{3ad \cos[c + dx]^{3/2}} - \frac{3(A - B) \sin[c + dx]}{ad \sqrt{\cos[c + dx]}} - \frac{(A - B) \sin[c + dx]}{d \cos[c + dx]^{3/2} (a + a \cos[c + dx])}$$

Result (type 5, 1167 leaves):

$$\frac{1}{4(a + a \cos[c + dx])} - 3iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx}\cos[2c] + ie^{2idx}\sin[2c]} \right) / (3id(1 + e^{2idx})\cos[c] - 3d(-1 + e^{2idx})\sin[c]) - \right. \\ \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx}\cos[2c] + ie^{2idx}\sin[2c]} \right) / (-id(1 + e^{2idx})\cos[c] + d(-1 + e^{2idx})\sin[c]) \right) - \frac{1}{4(a + a \cos[c + dx])} - 3iB \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\ \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx}\cos[2c] + ie^{2idx}\sin[2c]} \right) / (3id(1 + e^{2idx})\cos[c] - 3d(-1 + e^{2idx})\sin[c]) - \right. \\ \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx}\cos[2c] + ie^{2idx}\sin[2c]} \right) / (-id(1 + e^{2idx})\cos[c] + d(-1 + e^{2idx})\sin[c]) \right) + \frac{1}{a + a \cos[c + dx]} \right)$$

$$\begin{aligned} & \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \left( -\frac{(A-B)(2+\cos[c])\csc\left[\frac{c}{2}\right]\sec\left[\frac{c}{2}\right]\sec[c]}{d} - \frac{2\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2} + \frac{dx}{2}\right](A\sin\left[\frac{dx}{2}\right] - B\sin\left[\frac{dx}{2}\right])}{d} \right. \\ & \left. + \frac{4A\sec[c]\sec[c+dx]^2\sin[dx]}{3d} + \frac{4\sec[c]\sec[c+dx](A\sin[c] - 3A\sin[dx] + 3B\sin[dx])}{3d} \right) - \\ & \left( 5A\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \csc\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \sec[dx - \operatorname{ArcTan}[\cot[c]]] \right. \\ & \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\ & \left( 3d(a + a\cos[c+dx])\sqrt{1 + \cot[c]^2} \right) + \left( B\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \csc\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \right. \\ & \left. \sec\left[\frac{c}{2}\right] \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\ & \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( d(a + a\cos[c+dx])\sqrt{1 + \cot[c]^2} \right) \end{aligned}$$

- **Problem 151: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{7/2} (A + B\cos[c+dx])}{(a + a\cos[c+dx])^2} dx$$

Optimal (type 4, 203 leaves, 7 steps):

$$\begin{aligned} & -\frac{7(5A-8B)\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5a^2d} + \frac{5(2A-3B)\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3a^2d} + \frac{5(2A-3B)\sqrt{\cos[c+dx]}\sin[c+dx]}{3a^2d} - \\ & \frac{7(5A-8B)\cos[c+dx]^{3/2}\sin[c+dx]}{15a^2d} + \frac{(2A-3B)\cos[c+dx]^{5/2}\sin[c+dx]}{a^2d(1+\cos[c+dx])} + \frac{(A-B)\cos[c+dx]^{7/2}\sin[c+dx]}{3d(a+a\cos[c+dx])^2} \end{aligned}$$

Result (type 5, 1262 leaves):

$$\begin{aligned} & -\frac{1}{2(a+a\cos[c+dx])^2} 7iA\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] \\ & \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i\sin[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\cos[c] + 2i(-1+e^{2idx})\sin[c])} \right. \right. \\ & \left. \left. \sqrt{1+e^{2idx}\cos[2c] + ie^{2idx}\sin[2c]} \right) / (3id(1+e^{2idx})\cos[c] - 3d(-1+e^{2idx})\sin[c]) - \right. \end{aligned}$$

$$\begin{aligned}
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) + \\
& \frac{1}{5 (a + a \operatorname{Cos}[c + d x])^2} 28 i B \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \\
& \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left( 3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \right. \\
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \\
& \left( 20 A \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \operatorname{Cos}[c + d x])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \\
& \left( 10 B \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left( d (a + a \operatorname{Cos}[c + d x])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a + a \operatorname{Cos}[c + d x])^2} \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \\
& \sqrt{\operatorname{Cos}[c + d x]} \\
& \left( \frac{4 (15 A - 20 B + 20 A \operatorname{Cos}[c] - 36 B \operatorname{Cos}[c]) \operatorname{Csc}[c]}{5 d} + \frac{8 (A - 2 B) \operatorname{Cos}[d x] \operatorname{Sin}[c]}{3 d} + \frac{4 B \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{5 d} + \right. \\
& \quad \left. \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (3 A \operatorname{Sin} \left[ \frac{d x}{2} \right] - 4 B \operatorname{Sin} \left[ \frac{d x}{2} \right])}{d} - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (A \operatorname{Sin} \left[ \frac{d x}{2} \right] - B \operatorname{Sin} \left[ \frac{d x}{2} \right])}{3 d} + \right.
\end{aligned}$$

$$\left. \frac{8(A-2B)\cos[c]\sin[dx]}{3d} + \frac{4B\cos[2c]\sin[2dx]}{5d} - \frac{2(A-B)\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right)$$

- **Problem 152: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{5/2} (A+B\cos[c+dx])}{(a+a\cos[c+dx])^2} dx$$

Optimal (type 4, 166 leaves, 6 steps):

$$\frac{(4A-7B)\text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} - \frac{5(A-2B)\text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3a^2 d} - \frac{5(A-2B)\sqrt{\cos[c+dx]}\sin[c+dx]}{3a^2 d} + \frac{(4A-7B)\cos[c+dx]^{3/2}\sin[c+dx]}{3a^2 d(1+\cos[c+dx])} + \frac{(A-B)\cos[c+dx]^{5/2}\sin[c+dx]}{3d(a+a\cos[c+dx])^2}$$

Result (type 5, 1218 leaves):

$$\frac{1}{(a+a\cos[c+dx])^2} 2iA\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c]+i\sin[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\cos[c]+2i(-1+e^{2idx})\sin[c])}\right. \right. \right. \\ \left. \left. \sqrt{1+e^{2idx}\cos[2c]+ie^{2idx}\sin[2c]} \right) / (3id(1+e^{2idx})\cos[c]-3d(-1+e^{2idx})\sin[c]) - \left( 2\text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c]+i\sin[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\cos[c]+2i(-1+e^{2idx})\sin[c])}\right. \right. \right. \\ \left. \left. \sqrt{1+e^{2idx}\cos[2c]+ie^{2idx}\sin[2c]} \right) / (-id(1+e^{2idx})\cos[c]+d(-1+e^{2idx})\sin[c]) \right) - \frac{1}{2(a+a\cos[c+dx])^2} 7iB\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\ \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c]+i\sin[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\cos[c]+2i(-1+e^{2idx})\sin[c])}\right. \right. \right. \\ \left. \left. \sqrt{1+e^{2idx}\cos[2c]+ie^{2idx}\sin[2c]} \right) / (3id(1+e^{2idx})\cos[c]-3d(-1+e^{2idx})\sin[c]) - \left( 2\text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c]+i\sin[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\cos[c]+2i(-1+e^{2idx})\sin[c])}\right. \right. \right. \\ \left. \left. \sqrt{1+e^{2idx}\cos[2c]+ie^{2idx}\sin[2c]} \right) / (-id(1+e^{2idx})\cos[c]+d(-1+e^{2idx})\sin[c]) \right) + \left( 10A\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]\right]^2 \right) \sec\left[\frac{c}{2}\right] \sec[dx - \text{ArcTan}[\text{Cot}[c]]] \right. \\ \left. \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right)$$

$$\begin{aligned}
& \left. \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \left( 3 d (a + a \cos[c + d x])^2 \sqrt{1 + \text{Cot}[c]^2} \right) - \\
& \left( 20 B \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \right. \\
& \left. \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left( 3 d (a + a \cos[c + d x])^2 \sqrt{1 + \text{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + d x])^2} \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \sqrt{\cos[c + d x]} \\
& \left( -\frac{4(2A - 3B + 2A \cos[c] - 4B \cos[c]) \text{Csc}[c]}{d} + \frac{8B \cos[d x] \sin[c]}{3d} - \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (2A \sin\left[\frac{d x}{2}\right] - 3B \sin\left[\frac{d x}{2}\right])}{d} \right. \\
& \left. \frac{2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^3 (A \sin\left[\frac{d x}{2}\right] - B \sin\left[\frac{d x}{2}\right])}{3d} + \frac{8B \cos[c] \sin[d x]}{3d} + \frac{2(A - B) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Tan}\left[\frac{c}{2}\right]}{3d} \right)
\end{aligned}$$

■ **Problem 153: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^{3/2} (A + B \cos[c + d x])}{(a + a \cos[c + d x])^2} dx$$

Optimal (type 4, 136 leaves, 5 steps):

$$\begin{aligned}
& -\frac{(A - 4B) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{a^2 d} + \frac{(2A - 5B) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 a^2 d} + \\
& \frac{(2A - 5B) \sqrt{\cos[c + d x]} \sin[c + d x]}{3 a^2 d (1 + \cos[c + d x])} + \frac{(A - B) \cos[c + d x]^{3/2} \sin[c + d x]}{3 d (a + a \cos[c + d x])^2}
\end{aligned}$$

Result (type 5, 1184 leaves):

$$\begin{aligned}
& -\frac{1}{2(a + a \cos[c + d x])^2} i A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \\
& \left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2(1 + e^{2 i d x}) \cos[c] + 2i(-1 + e^{2 i d x}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2 i d x} \cos[2c] + i e^{2 i d x} \sin[2c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right. \\
& \left. \left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2(1 + e^{2 i d x}) \cos[c] + 2i(-1 + e^{2 i d x}) \sin[c])} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]}}{(-i d (1 + e^{2idx}) \cos[c] + d (-1 + e^{2idx}) \sin[c])} + \frac{1}{(a + a \cos[c + dx])^2} 2 i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \right. \\
& \left. \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c])} \right. \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]}}{(3 i d (1 + e^{2idx}) \cos[c] - 3 d (-1 + e^{2idx}) \sin[c])} - \right. \right. \\
& \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]}}{(-i d (1 + e^{2idx}) \cos[c] + d (-1 + e^{2idx}) \sin[c])} \right) - \right. \\
& \left. \left( 4 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \right. \\
& \left. \frac{\sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \left. \frac{\sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]}}{(3 d (a + a \cos[c + dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2})} + \right. \right. \\
& \left. \left( 10 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \right. \\
& \left. \left. \frac{\sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]}} \right) \right. \\
& \left. \left( 3 d (a + a \cos[c + dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2} + \frac{1}{(a + a \cos[c + dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \right. \right. \\
& \left. \left. - \frac{4 (-A + 2 B + 2 B \cos[c]) \operatorname{Csc}[c]}{d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - 2 B \sin\left[\frac{dx}{2}\right])}{d} - \right. \right. \\
& \left. \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right])}{3 d} - \frac{2 (A - B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right) \right)
\end{aligned}$$

- **Problem 154: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c + dx]} (A + B \cos[c + dx])}{(a + a \cos[c + dx])^2} dx$$

Optimal (type 4, 121 leaves, 5 steps):



$$-\frac{B \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} + \frac{(A+2B) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3a^2 d} + \frac{B \sqrt{\cos[c+dx]} \sin[c+dx]}{a^2 d (1+\cos[c+dx])} + \frac{(A-B) \sqrt{\cos[c+dx]} \sin[c+dx]}{3d (a+a \cos[c+dx])^2}$$

Result (type 5, 815 leaves):

$$-\frac{1}{2(a+a \cos[c+dx])^2} i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1+e^{2 i dx}) \cos[c] + 2i(-1+e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1+e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1+e^{2 i dx}) \cos[c] - 3 d (-1+e^{2 i dx}) \sin[c]) - \right.$$

$$\left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1+e^{2 i dx}) \cos[c] + 2i(-1+e^{2 i dx}) \sin[c])} \right.$$

$$\left. \left. \sqrt{1+e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1+e^{2 i dx}) \cos[c] + d (-1+e^{2 i dx}) \sin[c]) \right) -$$

$$\left( 2 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right.$$

$$\left. \frac{\sqrt{1-\sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a+a \cos[c+dx])^2 \sqrt{1+\operatorname{Cot}[c]^2} \right) -$$

$$\left( 4 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right.$$

$$\left. \frac{\sqrt{1-\sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) /$$

$$\left( 3 d (a+a \cos[c+dx])^2 \sqrt{1+\operatorname{Cot}[c]^2} \right) + \frac{1}{(a+a \cos[c+dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]}$$

$$\left( \frac{4 B \operatorname{Csc}[c]}{d} + \frac{4 B \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right])}{3 d} + \frac{2 (A-B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right)$$

■ **Problem 155: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A+B \cos[c+dx]}{\sqrt{\cos[c+dx]} (a+a \cos[c+dx])^2} dx$$

Optimal (type 4, 121 leaves, 5 steps) :

$$\frac{A \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} + \frac{(2A+B) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 a^2 d} - \frac{A \sqrt{\cos[c+dx]} \sin[c+dx]}{a^2 d (1+\cos[c+dx])} - \frac{(A-B) \sqrt{\cos[c+dx]} \sin[c+dx]}{3 d (a+a \cos[c+dx])^2}$$

Result (type 5, 815 leaves) :

$$\begin{aligned} & \frac{1}{2(a+a \cos[c+dx])^2} i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ & \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1+e^{2 i dx}) \cos[c] + 2 i (-1+e^{2 i dx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1+e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1+e^{2 i dx}) \cos[c] - 3 d (-1+e^{2 i dx}) \sin[c]) - \right. \\ & \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1+e^{2 i dx}) \cos[c] + 2 i (-1+e^{2 i dx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1+e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1+e^{2 i dx}) \cos[c] + d (-1+e^{2 i dx}) \sin[c]) \right) - \\ & \left( 4 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\ & \quad \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\ & \quad \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a+a \cos[c+dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\ & \left( 2 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\ & \quad \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\ & \left( 3 d (a+a \cos[c+dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a+a \cos[c+dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \\ & \left( -\frac{4 A \operatorname{Csc}[c]}{d} - \frac{4 A \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right])}{3 d} - \frac{2 (A-B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right) \end{aligned}$$

■ **Problem 156: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx]}{\cos[c + dx]^{3/2} (a + a \cos[c + dx])^2} dx$$

Optimal (type 4, 168 leaves, 6 steps):

$$-\frac{(4A - B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{a^2 d} - \frac{(5A - 2B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3a^2 d} +$$

$$\frac{(4A - B) \sin[c + dx]}{a^2 d \sqrt{\cos[c + dx]}} - \frac{(5A - 2B) \sin[c + dx]}{3a^2 d \sqrt{\cos[c + dx]} (1 + \cos[c + dx])} - \frac{(A - B) \sin[c + dx]}{3d \sqrt{\cos[c + dx]} (a + a \cos[c + dx])^2}$$

Result (type 5, 1217 leaves):

$$-\frac{1}{(a + a \cos[c + dx])^2} 2iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right.$$

$$\left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \frac{1}{2(a + a \cos[c + dx])^2} iB \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4$$

$$\operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right.$$

$$\left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) +$$

$$\left( 10A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right.$$

$$\left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right.$$

$$\left. \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3d(a + a \cos[c + dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) -$$

$$\left( 4 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\ \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\ \left( 3 d (a + a \cos[c + dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \\ \left( \frac{2 (2 A + 2 A \cos[c] - B \cos[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right])}{3 d} \right) + \\ \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (2 A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right])}{d} + \frac{8 A \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \sin[dx]}{d} + \frac{2 (A - B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right)$$

■ **Problem 157: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx]}{\cos[c + dx]^{5/2} (a + a \cos[c + dx])^2} dx$$

Optimal (type 4, 201 leaves, 7 steps):

$$\frac{(7A - 4B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{a^2 d} + \frac{5(2A - B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3a^2 d} + \frac{5(2A - B) \sin[c + dx]}{3a^2 d \cos[c + dx]^{3/2}} - \\ \frac{(7A - 4B) \sin[c + dx]}{a^2 d \sqrt{\cos[c + dx]}} - \frac{(7A - 4B) \sin[c + dx]}{3a^2 d \cos[c + dx]^{3/2} (1 + \cos[c + dx])} - \frac{(A - B) \sin[c + dx]}{3d \cos[c + dx]^{3/2} (a + a \cos[c + dx])^2}$$

Result (type 5, 1258 leaves):

$$\frac{1}{2(a + a \cos[c + dx])^2} 7i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ \left( \left( 2 e^{2i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1 + e^{2i dx}) \cos[c] + 2i(-1 + e^{2i dx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2i dx} \cos[2c] + i e^{2i dx} \sin[2c]} \right) / (3i d (1 + e^{2i dx}) \cos[c] - 3d(-1 + e^{2i dx}) \sin[c]) - \right. \\ \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1 + e^{2i dx}) \cos[c] + 2i(-1 + e^{2i dx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2i dx} \cos[2c] + i e^{2i dx} \sin[2c]} \right) / (-i d (1 + e^{2i dx}) \cos[c] + d(-1 + e^{2i dx}) \sin[c]) \right) - \frac{1}{(a + a \cos[c + dx])^2} 2i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\ \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1 + e^{2i dx}) \cos[c] + 2i(-1 + e^{2i dx}) \sin[c])} \right. \right.$$

$$\begin{aligned}
& \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( 3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c] \right) - \\
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \\
& \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( -id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c] \right) - \\
& \left( 20A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \right. \\
& \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( 3d(a + a \cos[c + dx])^2 \sqrt{1 + \cot[c]^2} \right) + \\
& \left( 10B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \right. \\
& \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left( 3d(a + a \cos[c + dx])^2 \sqrt{1 + \cot[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \sqrt{\cos[c + dx]} \\
& \left( -\frac{2(4A - 2B + 3A \cos[c] - 2B \cos[c]) \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[c]}{d} - \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] (3A \sin \left[ \frac{dx}{2} \right] - 2B \sin \left[ \frac{dx}{2} \right])}{d} - \right. \\
& \left. \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 (A \sin \left[ \frac{dx}{2} \right] - B \sin \left[ \frac{dx}{2} \right])}{3d} + \frac{8A \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \sin[dx]}{3d} + \right. \\
& \left. \frac{8 \operatorname{Sec}[c] \operatorname{Sec}[c + dx] (A \sin[c] - 6A \sin[dx] + 3B \sin[dx])}{3d} - \frac{2(A - B) \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Tan} \left[ \frac{c}{2} \right]}{3d} \right)
\end{aligned}$$

- **Problem 158: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{9/2} (A + B \cos[c + dx])}{(a + a \cos[c + dx])^3} dx$$

Optimal (type 4, 254 leaves, 8 steps):

$$\begin{aligned}
& - \frac{7 (17 A - 33 B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] + (11 A - 21 B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{10 a^3 d} + \frac{(11 A - 21 B) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx] - 7 (17 A - 33 B) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{2 a^3 d} + \\
& \frac{(A - B) \operatorname{Cos}[c + dx]^{9/2} \operatorname{Sin}[c + dx]}{5 d (a + a \operatorname{Cos}[c + dx])^3} + \frac{(7 A - 12 B) \operatorname{Cos}[c + dx]^{7/2} \operatorname{Sin}[c + dx]}{15 a d (a + a \operatorname{Cos}[c + dx])^2} + \frac{3 (11 A - 21 B) \operatorname{Cos}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{10 d (a^3 + a^3 \operatorname{Cos}[c + dx])}
\end{aligned}$$

Result (type 5, 1346 leaves):

$$\begin{aligned}
& - \frac{1}{10 (a + a \operatorname{Cos}[c + dx])^3} 119 i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \right. \\
& \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) + \\
& \frac{1}{10 (a + a \operatorname{Cos}[c + dx])^3} 231 i B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \right. \\
& \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) - \\
& \left( 22 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \left. \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \left. \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / (d (a + a \operatorname{Cos}[c + dx])^3 \sqrt{1 + \operatorname{Cot}[c]^2}) +
\end{aligned}$$

$$\left( 42 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\ \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\ \left( d (a + a \cos[c + dx])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]} \\ \left( \frac{4 (59 A - 99 B + 60 A \cos[c] - 132 B \cos[c]) \operatorname{Csc}[c]}{5 d} + \frac{16 (A - 3 B) \cos[dx] \sin[c]}{3 d} + \frac{8 B \cos[2 dx] \sin[2 c]}{5 d} + \right. \\ \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (59 A \sin\left[\frac{dx}{2}\right] - 99 B \sin\left[\frac{dx}{2}\right])}{5 d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (19 A \sin\left[\frac{dx}{2}\right] - 24 B \sin\left[\frac{dx}{2}\right])}{15 d} + \\ \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right])}{5 d} + \frac{16 (A - 3 B) \cos[c] \sin[dx]}{3 d} + \\ \left. \frac{8 B \cos[2 c] \sin[2 dx]}{5 d} - \frac{4 (19 A - 24 B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} + \frac{2 (A - B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right)$$

- **Problem 159: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{7/2} (A + B \cos[c + dx])}{(a + a \cos[c + dx])^3} dx$$

Optimal (type 4, 219 leaves, 7 steps):

$$\frac{7 (7 A - 17 B) \operatorname{EllipticE}\left[\frac{1}{2} (c + dx), 2\right]}{10 a^3 d} - \frac{(13 A - 33 B) \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), 2\right]}{6 a^3 d} - \frac{(13 A - 33 B) \sqrt{\cos[c + dx]} \sin[c + dx]}{6 a^3 d} + \\ \frac{(A - B) \cos[c + dx]^{7/2} \sin[c + dx]}{5 d (a + a \cos[c + dx])^3} + \frac{(A - 2 B) \cos[c + dx]^{5/2} \sin[c + dx]}{3 a d (a + a \cos[c + dx])^2} + \frac{7 (7 A - 17 B) \cos[c + dx]^{3/2} \sin[c + dx]}{30 d (a^3 + a^3 \cos[c + dx])}$$

Result (type 5, 1306 leaves):

$$\frac{1}{10 (a + a \cos[c + dx])^3} 49 i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2 i dx} \cos[2 c] + i e^{2 i dx} \sin[2 c]} \right) \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) -$$

$$\begin{aligned}
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2(1+e^{2ix}) \cos[c] + 2i(-1+e^{2ix}) \sin[c])} \right. \\
& \quad \left. \sqrt{1+e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]} \right) / \left( -i d (1+e^{2ix}) \cos[c] + d (-1+e^{2ix}) \sin[c] \right) - \\
& \frac{1}{10 (a+a \cos[c+dx])^3} 119 i B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \\
& \left( \left( 2 e^{2ix} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2(1+e^{2ix}) \cos[c] + 2i(-1+e^{2ix}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1+e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]} \right) / \left( 3 i d (1+e^{2ix}) \cos[c] - 3 d (-1+e^{2ix}) \sin[c] \right) - \right. \\
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2(1+e^{2ix}) \cos[c] + 2i(-1+e^{2ix}) \sin[c])} \right. \\
& \quad \left. \sqrt{1+e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]} \right) / \left( -i d (1+e^{2ix}) \cos[c] + d (-1+e^{2ix}) \sin[c] \right) \Big) + \\
& \left( 26 A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} [dx - \operatorname{ArcTan}[\cot[c]]] \right. \\
& \quad \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( 3 d (a+a \cos[c+dx])^3 \sqrt{1 + \cot[c]^2} \right) - \\
& \left( 22 B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} [dx - \operatorname{ArcTan}[\cot[c]]] \right. \\
& \quad \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left( d (a+a \cos[c+dx])^3 \sqrt{1 + \cot[c]^2} \right) + \frac{1}{(a+a \cos[c+dx])^3} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \sqrt{\cos[c+dx]} \\
& \left( -\frac{4 (29 A - 59 B + 20 A \cos[c] - 60 B \cos[c]) \operatorname{Csc}[c]}{5 d} + \frac{16 B \cos[dx] \sin[c]}{3 d} - \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] (29 A \sin \left[ \frac{dx}{2} \right] - 59 B \sin \left[ \frac{dx}{2} \right])}{5 d} \right. \\
& \quad \left. + \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 (14 A \sin \left[ \frac{dx}{2} \right] - 19 B \sin \left[ \frac{dx}{2} \right])}{15 d} - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 (A \sin \left[ \frac{dx}{2} \right] - B \sin \left[ \frac{dx}{2} \right])}{5 d} \right) +
\end{aligned}$$



$$\left. \frac{16 B \cos[c] \sin[dx]}{3 d} + \frac{4 (14 A - 19 B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15 d} - \frac{2 (A - B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5 d} \right)$$

- **Problem 160: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{5/2} (A + B \cos[c + dx])}{(a + a \cos[c + dx])^3} dx$$

Optimal (type 4, 188 leaves, 6 steps):

$$-\frac{(9 A - 49 B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10 a^3 d} + \frac{(3 A - 13 B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6 a^3 d} +$$

$$\frac{(A - B) \cos[c + dx]^{5/2} \sin[c + dx]}{5 d (a + a \cos[c + dx])^3} + \frac{(3 A - 8 B) \cos[c + dx]^{3/2} \sin[c + dx]}{15 a d (a + a \cos[c + dx])^2} + \frac{(3 A - 13 B) \sqrt{\cos[c + dx]} \sin[c + dx]}{6 d (a^3 + a^3 \cos[c + dx])}$$

Result (type 5, 1273 leaves):

$$-\frac{1}{10 (a + a \cos[c + dx])^3} 9 i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2 c] + i e^{2 i dx} \sin[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right.$$

$$\left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2 c] + i e^{2 i dx} \sin[2 c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) +$$

$$\frac{1}{10 (a + a \cos[c + dx])^3} 49 i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2 c] + i e^{2 i dx} \sin[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right.$$

$$\left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2 c] + i e^{2 i dx} \sin[2 c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) -$$

$$\left( 2 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)$$

$$\begin{aligned}
& \left. \begin{aligned}
& \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \left. \right) / \left( d (a + a \cos[c + d x])^3 \sqrt{1 + \text{Cot}[c]^2} \right) + \\
& \left( 26 B \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \right. \\
& \left. \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left( 3 d (a + a \cos[c + d x])^3 \sqrt{1 + \text{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + d x])^3} \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \sqrt{\cos[c + d x]} \\
& \left( -\frac{4(-9A + 29B + 20B \cos[c]) \text{Csc}[c]}{5d} + \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (9A \sin\left[\frac{d x}{2}\right] - 29B \sin\left[\frac{d x}{2}\right])}{5d} - \right. \\
& \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^3 (9A \sin\left[\frac{d x}{2}\right] - 14B \sin\left[\frac{d x}{2}\right])}{15d} + \frac{2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^5 (A \sin\left[\frac{d x}{2}\right] - B \sin\left[\frac{d x}{2}\right])}{5d} - \\
& \left. \frac{4(9A - 14B) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Tan}\left[\frac{c}{2}\right]}{15d} + \frac{2(A - B) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \text{Tan}\left[\frac{c}{2}\right]}{5d} \right)
\end{aligned}
\end{aligned}$$

■ **Problem 161: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^{3/2} (A + B \cos[c + d x])}{(a + a \cos[c + d x])^3} dx$$

Optimal (type 4, 180 leaves, 6 steps):

$$\begin{aligned}
& -\frac{(A + 9B) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{10 a^3 d} + \frac{(A + 3B) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{6 a^3 d} + \\
& \frac{(A - B) \cos[c + d x]^{3/2} \sin[c + d x]}{5 d (a + a \cos[c + d x])^3} + \frac{(A - 6B) \sqrt{\cos[c + d x]} \sin[c + d x]}{15 a d (a + a \cos[c + d x])^2} + \frac{(A + 9B) \sqrt{\cos[c + d x]} \sin[c + d x]}{10 d (a^3 + a^3 \cos[c + d x])}
\end{aligned}$$

Result (type 5, 1265 leaves):

$$-\frac{1}{10 (a + a \cos[c + d x])^3} i A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right]$$

$$\begin{aligned}
& \left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \\
& \left( 2 \text{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \Big) - \\
& \frac{1}{10 (a + a \cos [c + d x])^3} 9 i B \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \csc \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} \right] \\
& \left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \\
& \left( 2 \text{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \Big) - \\
& \left( 2 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \csc \left[ \frac{c}{2} \right] \text{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \right) \sec \left[ \frac{c}{2} \right] \sec [d x - \text{ArcTan}[\text{Cot}[c]]] \\
& \quad \sqrt{1 - \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin [c] \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \quad \left. \sqrt{1 + \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \left( 3 d (a + a \cos [c + d x])^3 \sqrt{1 + \text{Cot}[c]^2} \right) - \\
& \left( 2 B \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \csc \left[ \frac{c}{2} \right] \text{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \right) \sec \left[ \frac{c}{2} \right] \sec [d x - \text{ArcTan}[\text{Cot}[c]]] \\
& \quad \sqrt{1 - \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin [c] \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \Big) / \\
& \left( d (a + a \cos [c + d x])^3 \sqrt{1 + \text{Cot}[c]^2} \right) + \frac{1}{(a + a \cos [c + d x])^3} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \\
& \quad \sqrt{\cos [c + d x]}
\end{aligned}$$

$$\left( \frac{4 (A + 9 B) \operatorname{Csc}[c]}{5 d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (4 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 9 B \operatorname{Sin}\left[\frac{dx}{2}\right])}{15 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \operatorname{Sin}\left[\frac{dx}{2}\right] - B \operatorname{Sin}\left[\frac{dx}{2}\right])}{5 d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \operatorname{Sin}\left[\frac{dx}{2}\right] + 9 B \operatorname{Sin}\left[\frac{dx}{2}\right])}{5 d} + \frac{4 (4 A - 9 B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} - \frac{2 (A - B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right)$$

■ **Problem 162: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Cos}[c + dx]} (A + B \operatorname{Cos}[c + dx])}{(a + a \operatorname{Cos}[c + dx])^3} dx$$

Optimal (type 4, 178 leaves, 6 steps):

$$\frac{(A - B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10 a^3 d} + \frac{(A + B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6 a^3 d} + \frac{(A - B) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{5 d (a + a \operatorname{Cos}[c + dx])^3} + \frac{(A + 4 B) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{15 a d (a + a \operatorname{Cos}[c + dx])^2} - \frac{(A - B) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{10 d (a^3 + a^3 \operatorname{Cos}[c + dx])}$$

Result (type 5, 1264 leaves):

$$\frac{1}{10 (a + a \operatorname{Cos}[c + dx])^3} i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) - \frac{1}{10 (a + a \operatorname{Cos}[c + dx])^3} i B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) -$$

$$\left( 2 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\ \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\ \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \cos[c + dx])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) -$$

$$\left( 2 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\ \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) /$$

$$\left( 3 d (a + a \cos[c + dx])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]}$$

$$\left( -\frac{4(A - B) \operatorname{Csc}[c]}{5 d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right])}{5 d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right])}{5 d} + \right. \\ \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] + 4 B \sin\left[\frac{dx}{2}\right])}{15 d} + \frac{4(A + 4 B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} + \frac{2(A - B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right)$$

- **Problem 163: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx]}{\sqrt{\cos[c + dx]} (a + a \cos[c + dx])^3} dx$$

Optimal (type 4, 182 leaves, 6 steps):

$$\frac{(9A + B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10 a^3 d} + \frac{(3A + B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6 a^3 d} - \\ \frac{(A - B) \sqrt{\cos[c + dx]} \sin[c + dx]}{5 d (a + a \cos[c + dx])^3} - \frac{(6A - B) \sqrt{\cos[c + dx]} \sin[c + dx]}{15 a d (a + a \cos[c + dx])^2} - \frac{(9A + B) \sqrt{\cos[c + dx]} \sin[c + dx]}{10 d (a^3 + a^3 \cos[c + dx])}$$

Result (type 5, 1265 leaves):

$$\frac{1}{10 (a + a \cos[c + dx])^3} {}_9F_1 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{(3id(1 + e^{2ix}) \cos[c] - 3d(-1 + e^{2ix}) \sin[c]) -} \right. \\
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2(1 + e^{2ix}) \cos[c] + 2i(-1 + e^{2ix}) \sin[c])} \right. \\
& \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{(-id(1 + e^{2ix}) \cos[c] + d(-1 + e^{2ix}) \sin[c])} \right) + \frac{1}{10(a + a \cos[c + dx])^3} i B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \\
& \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \left( \left( 2 e^{2ix} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2(1 + e^{2ix}) \cos[c] + 2i(-1 + e^{2ix}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{(3id(1 + e^{2ix}) \cos[c] - 3d(-1 + e^{2ix}) \sin[c]) -} \right. \right. \\
& \left. \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2(1 + e^{2ix}) \cos[c] + 2i(-1 + e^{2ix}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{(-id(1 + e^{2ix}) \cos[c] + d(-1 + e^{2ix}) \sin[c])} \right) - \right. \\
& \left. \left( 2 A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \right) \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \left. \frac{\sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \frac{\sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( d(a + a \cos[c + dx])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left( 2 B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \right) \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \left. \frac{\sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left( 3d(a + a \cos[c + dx])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^3} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \sqrt{\cos[c + dx]} \\
& \left( -\frac{4(9A + B) \operatorname{Csc}[c]}{5d} - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 (A \sin \left[ \frac{dx}{2} \right] - B \sin \left[ \frac{dx}{2} \right])}{5d} - \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 (6A \sin \left[ \frac{dx}{2} \right] - B \sin \left[ \frac{dx}{2} \right])}{15d} - \right. \\
& \left. \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] (9A \sin \left[ \frac{dx}{2} \right] + B \sin \left[ \frac{dx}{2} \right])}{5d} - \frac{4(6A - B) \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Tan} \left[ \frac{c}{2} \right]}{15d} - \frac{2(A - B) \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Tan} \left[ \frac{c}{2} \right]}{5d} \right)
\end{aligned}$$

■ **Problem 164:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \cos[c + dx]}{\cos[c + dx]^{3/2} (a + a \cos[c + dx])^3} dx$$

Optimal (type 4, 221 leaves, 7 steps):

$$\begin{aligned} & - \frac{(49A - 9B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10a^3 d} - \frac{(13A - 3B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6a^3 d} + \frac{(49A - 9B) \sin[c + dx]}{10a^3 d \sqrt{\cos[c + dx]}} - \\ & \frac{(A - B) \sin[c + dx]}{5d \sqrt{\cos[c + dx]} (a + a \cos[c + dx])^3} - \frac{(8A - 3B) \sin[c + dx]}{15ad \sqrt{\cos[c + dx]} (a + a \cos[c + dx])^2} - \frac{(13A - 3B) \sin[c + dx]}{6d \sqrt{\cos[c + dx]} (a^3 + a^3 \cos[c + dx])} \end{aligned}$$

Result (type 5, 1305 leaves):

$$\begin{aligned} & - \frac{1}{10(a + a \cos[c + dx])^3} 49iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ & \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ & \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \\ & \frac{1}{10(a + a \cos[c + dx])^3} 9iB \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ & \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ & \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \\ & \left( 26A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\ & \quad \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) \end{aligned}$$

$$\left. \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \left( 3 d (a + a \cos[c + d x])^3 \sqrt{1 + \text{Cot}[c]^2} \right) -$$

$$\left( 2 B \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \right.$$

$$\left. \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) /$$

$$\left( d (a + a \cos[c + d x])^3 \sqrt{1 + \text{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + d x])^3} \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\sqrt{\cos[c + d x]}$$

$$\left( \frac{2 (20 A + 29 A \cos[c] - 9 B \cos[c]) \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c]}{5 d} + \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (29 A \sin\left[\frac{d x}{2}\right] - 9 B \sin\left[\frac{d x}{2}\right])}{5 d} + \right.$$

$$\left. \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^3 (11 A \sin\left[\frac{d x}{2}\right] - 6 B \sin\left[\frac{d x}{2}\right])}{15 d} + \frac{2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^5 (A \sin\left[\frac{d x}{2}\right] - B \sin\left[\frac{d x}{2}\right])}{5 d} + \right.$$

$$\left. \frac{16 A \text{Sec}[c] \text{Sec}[c + d x] \sin[d x]}{d} + \frac{4 (11 A - 6 B) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Tan}\left[\frac{c}{2}\right]}{15 d} + \frac{2 (A - B) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \text{Tan}\left[\frac{c}{2}\right]}{5 d} \right)$$

■ **Problem 165: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + d x]}{\cos[c + d x]^{5/2} (a + a \cos[c + d x])^3} dx$$

Optimal (type 4, 254 leaves, 8 steps):

$$\frac{7 (17 A - 7 B) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{10 a^3 d} + \frac{(33 A - 13 B) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{6 a^3 d} + \frac{(33 A - 13 B) \sin[c + d x]}{6 a^3 d \cos[c + d x]^{3/2}} - \frac{7 (17 A - 7 B) \sin[c + d x]}{10 a^3 d \sqrt{\cos[c + d x]}}$$

$$\frac{(A - B) \sin[c + d x]}{5 d \cos[c + d x]^{3/2} (a + a \cos[c + d x])^3} - \frac{(2 A - B) \sin[c + d x]}{3 a d \cos[c + d x]^{3/2} (a + a \cos[c + d x])^2} - \frac{7 (17 A - 7 B) \sin[c + d x]}{30 d \cos[c + d x]^{3/2} (a^3 + a^3 \cos[c + d x])}$$

Result (type 5, 1346 leaves):

$$\frac{1}{10 (a + a \cos[c + d x])^3} 119 i A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) \right) / \left( 3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c] \right) -$$



$$\begin{aligned}
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c] \right) - \\
& \frac{1}{10 (a + a \cos [c + d x])^3} 49 i B \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \\
& \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / \left( 3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c] \right) - \right. \\
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c] \right) - \\
& \left( 22 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} [d x - \operatorname{ArcTan}[\cot [c]]] \right. \\
& \quad \left. \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right. \\
& \quad \left. \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right) / \left( d (a + a \cos [c + d x])^3 \sqrt{1 + \cot [c]^2} \right) + \\
& \left( 26 B \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} [d x - \operatorname{ArcTan}[\cot [c]]] \right. \\
& \quad \left. \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right) / \\
& \left( 3 d (a + a \cos [c + d x])^3 \sqrt{1 + \cot [c]^2} \right) + \frac{1}{(a + a \cos [c + d x])^3} \\
& \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \left( -\frac{2 (60 A - 20 B + 59 A \cos [c] - 29 B \cos [c]) \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} [c]}{5 d} - \right. \\
& \quad \left. \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (59 A \sin \left[ \frac{d x}{2} \right] - 29 B \sin \left[ \frac{d x}{2} \right])}{5 d} - \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (16 A \sin \left[ \frac{d x}{2} \right] - 11 B \sin \left[ \frac{d x}{2} \right])}{15 d} \right)
\end{aligned}$$

$$\frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \operatorname{Sin}\left[\frac{dx}{2}\right] - B \operatorname{Sin}\left[\frac{dx}{2}\right])}{5 d} + \frac{16 A \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \operatorname{Sin}[dx]}{3 d} + \left. \frac{16 \operatorname{Sec}[c] \operatorname{Sec}[c + dx] (A \operatorname{Sin}[c] - 9 A \operatorname{Sin}[dx] + 3 B \operatorname{Sin}[dx])}{3 d} - \frac{4 (16 A - 11 B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} - \frac{2 (A - B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right)$$

■ **Problem 166: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Cos}[c + dx]^{5/2} \sqrt{a + a \operatorname{Cos}[c + dx]} (A + B \operatorname{Cos}[c + dx]) dx$$

Optimal (type 3, 221 leaves, 6 steps):

$$\frac{5 \sqrt{a} (8 A + 7 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{64 d} + \frac{5 a (8 A + 7 B) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{64 d \sqrt{a + a \operatorname{Cos}[c + dx]}} + \frac{5 a (8 A + 7 B) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{96 d \sqrt{a + a \operatorname{Cos}[c + dx]}} + \frac{a (8 A + 7 B) \operatorname{Cos}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{24 d \sqrt{a + a \operatorname{Cos}[c + dx]}} + \frac{a B \operatorname{Cos}[c + dx]^{7/2} \operatorname{Sin}[c + dx]}{4 d \sqrt{a + a \operatorname{Cos}[c + dx]}}$$

Result (type 3, 330 leaves):

$$\frac{1}{768 d} \sqrt{a (1 + \operatorname{Cos}[c + dx])} \operatorname{Sec}\left[\frac{1}{2} (c + dx)\right] \left( \left( 15 i (8 A + 7 B) e^{\frac{idx}{2}} \left( \operatorname{ArcTanh}\left[\left(\operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right]} - \operatorname{Log}\left[ 2 \left( e^{idx} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right] \right) \sqrt{e^{-idx} \left( 2 (1 + e^{2idx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) / \left( \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) + 4 \sqrt{\operatorname{Cos}[c + dx]} (152 A + 133 B + 2 (40 A + 53 B) \operatorname{Cos}[c + dx] + 4 (8 A + 7 B) \operatorname{Cos}[2 (c + dx)] + 12 B \operatorname{Cos}[3 (c + dx)]) \operatorname{Sin}\left[\frac{1}{2} (c + dx)\right] \right)$$

■ **Problem 167: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + dx]^{3/2} \sqrt{a + a \operatorname{Cos}[c + dx]} (A + B \operatorname{Cos}[c + dx]) dx$$

Optimal (type 3, 176 leaves, 5 steps):

$$\frac{\sqrt{a} (6 A + 5 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{8 d} + \frac{a (6 A + 5 B) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{8 d \sqrt{a + a \operatorname{Cos}[c + dx]}} + \frac{a (6 A + 5 B) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{12 d \sqrt{a + a \operatorname{Cos}[c + dx]}} + \frac{a B \operatorname{Cos}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{3 d \sqrt{a + a \operatorname{Cos}[c + dx]}}$$

Result (type 3, 620 leaves):

$$\begin{aligned}
& \frac{1}{48 d \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}} \sqrt{\cos [c + d x]} \sqrt{a (1 + \cos [c + d x])} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \\
& \left( -3 i (6 A + 5 B) \cos \left[\frac{d x}{2}\right] \operatorname{Log}\left[2 \left( e^{i d x} \cos \left[\frac{c}{2}\right] + i e^{i d x} \sin \left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right)\right] \right) + \\
& 3 i (6 A + 5 B) \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right] + i \sin \left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right] \left(\cos \left[\frac{d x}{2}\right] + i \sin \left[\frac{d x}{2}\right]\right) + \\
& 18 A \operatorname{Log}\left[2 \left( e^{i d x} \cos \left[\frac{c}{2}\right] + i e^{i d x} \sin \left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right)\right] \sin \left[\frac{d x}{2}\right] + \\
& 15 B \operatorname{Log}\left[2 \left( e^{i d x} \cos \left[\frac{c}{2}\right] + i e^{i d x} \sin \left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right)\right] \sin \left[\frac{d x}{2}\right] + \\
& 24 \sqrt{2} A \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \sin \left[\frac{1}{2} (c + d x)\right] + \\
& 28 \sqrt{2} B \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \sin \left[\frac{1}{2} (c + d x)\right] + 12 \sqrt{2} A \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \sin \left[\frac{3}{2} (c + d x)\right] + \\
& 6 \sqrt{2} B \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \sin \left[\frac{3}{2} (c + d x)\right] + 4 \sqrt{2} B \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \sin \left[\frac{5}{2} (c + d x)\right] \Big]
\end{aligned}$$

- **Problem 168: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]} (A + B \cos [c + d x]) dx$$

Optimal (type 3, 131 leaves, 4 steps):

$$\frac{\sqrt{a} (4 A + 3 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}}\right]}{4 d} + \frac{a (4 A + 3 B) \sqrt{\cos [c + d x]} \sin [c + d x]}{4 d \sqrt{a + a \cos [c + d x]}} + \frac{a B \cos [c + d x]^{3/2} \sin [c + d x]}{2 d \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 536 leaves):

$$\frac{1}{8 d \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}} \sqrt{\cos [c + d x]} \sqrt{a (1 + \cos [c + d x])} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]$$

$$\left(-i (4 A + 3 B) \cos\left[\frac{d x}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right]\right) +$$

$$i (4 A + 3 B) \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right] \left(\cos\left[\frac{d x}{2}\right] + i \sin\left[\frac{d x}{2}\right]\right) +$$

$$4 A \operatorname{Log}\left[2\left(e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] \sin\left[\frac{d x}{2}\right] +$$

$$3 B \operatorname{Log}\left[2\left(e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] \sin\left[\frac{d x}{2}\right] +$$

$$8 \sqrt{2} A \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin\left[\frac{1}{2} (c + d x)\right] +$$

$$4 \sqrt{2} B \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin\left[\frac{1}{2} (c + d x)\right] + 2 \sqrt{2} B \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin\left[\frac{3}{2} (c + d x)\right]$$

- **Problem 169: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \cos [c + d x]} (A + B \cos [c + d x])}{\sqrt{\cos [c + d x]}} dx$$

Optimal (type 3, 78 leaves, 3 steps):

$$\frac{\sqrt{a} (2 A + B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}}\right]}{d} + \frac{a B \sqrt{\cos [c + d x]} \sin [c + d x]}{d \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 447 leaves):

$$\frac{1}{2 d \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}} \sqrt{\cos [c + d x]} \sqrt{a (1 + \cos [c + d x])} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]$$

$$\left(-i (2 A + B) \cos\left[\frac{d x}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right]\right) +$$

$$i (2 A + B) \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right] \left(\cos\left[\frac{d x}{2}\right] + i \sin\left[\frac{d x}{2}\right]\right) +$$

$$2 A \operatorname{Log}\left[2\left(e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] \sin\left[\frac{d x}{2}\right] +$$

$$B \operatorname{Log}\left[2\left(e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] \sin\left[\frac{d x}{2}\right] +$$

$$2 \sqrt{2} B \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin\left[\frac{1}{2} (c + d x)\right]$$

- **Problem 170: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \cos[c + dx]} (A + B \cos[c + dx])}{\cos[c + dx]^{3/2}} dx$$

Optimal (type 3, 76 leaves, 3 steps):

$$\frac{2 \sqrt{a} B \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} + \frac{2 a A \sin[c + dx]}{d \sqrt{\cos[c + dx]} \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 294 leaves):

$$\frac{1}{\sqrt{2} d \sqrt{\cos[c + dx]} \sqrt{\cos[c + dx]} (\cos[dx] + i \sin[dx])} \sqrt{a (1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{1}{2} (c + dx)\right] \\ \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) \left(i B \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2i dx}) \cos[c] + i (-1 + e^{2i dx}) \sin[c]}\right]\right) \cos[c + dx] - \\ i B \cos[c + dx] \operatorname{Log}\left[2 \left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2i dx}) \cos[c] + i (-1 + e^{2i dx}) \sin[c]}\right)\right] + \\ 2 \sqrt{2} A \left(\cos\left[\frac{dx}{2}\right] - i \sin\left[\frac{dx}{2}\right]\right) \sqrt{\cos[c + dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2} (c + dx)\right]$$

- **Problem 174: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos[c + dx]^{3/2} (a + a \cos[c + dx])^{3/2} (A + B \cos[c + dx]) dx$$

Optimal (type 3, 227 leaves, 6 steps):

$$\frac{a^{3/2} (88 A + 75 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{64 d} + \frac{a^2 (88 A + 75 B) \sqrt{\cos[c + dx]} \sin[c + dx]}{64 d \sqrt{a + a \cos[c + dx]}} + \\ \frac{a^2 (88 A + 75 B) \cos[c + dx]^{3/2} \sin[c + dx]}{96 d \sqrt{a + a \cos[c + dx]}} + \frac{a^2 (8 A + 9 B) \cos[c + dx]^{5/2} \sin[c + dx]}{24 d \sqrt{a + a \cos[c + dx]}} + \frac{a B \cos[c + dx]^{5/2} \sqrt{a + a \cos[c + dx]} \sin[c + dx]}{4 d}$$

Result (type 3, 356 leaves):

$$\begin{aligned}
& - \frac{1}{768 d \sqrt{2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c]}} \\
& (a (1 + \operatorname{Cos}[c + d x]))^{3/2} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^3 \left(-3 i (88 A + 75 B) e^{\frac{i d x}{2}} \left(\operatorname{ArcTanh}\left[\left(\operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \operatorname{Cos}[c] + i (-1 + e^{2 i d x}) \operatorname{Sin}[c]}\right]\right) - \right. \\
& \quad \left. \operatorname{Log}\left[2 \left(e^{i d x} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{i d x} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \operatorname{Cos}[c] + i (-1 + e^{2 i d x}) \operatorname{Sin}[c]}\right]\right)\right] \right) \\
& \sqrt{e^{-i d x} \left((1 + e^{2 i d x}) \operatorname{Cos}[c] + i (-1 + e^{2 i d x}) \operatorname{Sin}[c]\right)} - 4 \sqrt{\operatorname{Cos}[c + d x]} (296 A + 285 B + 2 (88 A + 93 B) \operatorname{Cos}[c + d x] + \\
& \quad 4 (8 A + 15 B) \operatorname{Cos}[2 (c + d x)] + 12 B \operatorname{Cos}[3 (c + d x)]) \sqrt{\operatorname{Cos}[c + d x]} (\operatorname{Cos}[d x] + i \operatorname{Sin}[d x]) \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \Big)
\end{aligned}$$

- **Problem 175: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Cos}[c + d x])^{3/2} (A + B \operatorname{Cos}[c + d x]) dx$$

Optimal (type 3, 180 leaves, 5 steps):

$$\begin{aligned}
& \frac{a^{3/2} (14 A + 11 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c + d x]}{\sqrt{a + a \operatorname{Cos}[c + d x]}}\right]}{8 d} + \frac{a^2 (14 A + 11 B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{8 d \sqrt{a + a \operatorname{Cos}[c + d x]}} + \\
& \frac{a^2 (6 A + 7 B) \operatorname{Cos}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{12 d \sqrt{a + a \operatorname{Cos}[c + d x]}} + \frac{a B \operatorname{Cos}[c + d x]^{3/2} \sqrt{a + a \operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{3 d}
\end{aligned}$$

Result (type 3, 621 leaves):

$$\begin{aligned}
& \frac{1}{48 d \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}} a \sqrt{\cos [c + d x]} \sqrt{a (1 + \cos [c + d x])} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \\
& \left( -3 i (14 A + 11 B) \cos\left[\frac{d x}{2}\right] \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right)\right] + \right. \\
& \quad \left. 3 i (14 A + 11 B) \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right] \left(\cos\left[\frac{d x}{2}\right] + i \sin\left[\frac{d x}{2}\right]\right) + \right. \\
& \quad \left. 42 A \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right)\right] \sin\left[\frac{d x}{2}\right] + \right. \\
& \quad \left. 33 B \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right)\right] \sin\left[\frac{d x}{2}\right] + \right. \\
& \quad \left. 72 \sqrt{2} A \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \sin\left[\frac{1}{2} (c + d x)\right] + \right. \\
& \quad \left. 52 \sqrt{2} B \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \sin\left[\frac{1}{2} (c + d x)\right] + 12 \sqrt{2} A \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \sin\left[\frac{3}{2} (c + d x)\right] + \right. \\
& \quad \left. 18 \sqrt{2} B \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \sin\left[\frac{3}{2} (c + d x)\right] + 4 \sqrt{2} B \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \sin\left[\frac{5}{2} (c + d x)\right] \right)
\end{aligned}$$

- **Problem 176: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^{3/2} (A + B \cos [c + d x])}{\sqrt{\cos [c + d x]}} dx$$

Optimal (type 3, 133 leaves, 4 steps):

$$\frac{a^{3/2} (12 A + 7 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}}\right]}{4 d} + \frac{a^2 (4 A + 5 B) \sqrt{\cos [c + d x]} \sin [c + d x]}{4 d \sqrt{a + a \cos [c + d x]}} + \frac{a B \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{2 d}$$

Result (type 3, 537 leaves):

$$\begin{aligned}
& \frac{1}{8 d \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}} a \sqrt{\cos [c + d x]} \sqrt{a (1 + \cos [c + d x])} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \\
& \left( -i (12 A + 7 B) \cos \left[\frac{d x}{2}\right] \operatorname{Log}\left[2 \left(e^{i d x} \cos \left[\frac{c}{2}\right] + i e^{i d x} \sin \left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] + \right. \\
& \quad i (12 A + 7 B) \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right] + i \sin \left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right] \left(\cos \left[\frac{d x}{2}\right] + i \sin \left[\frac{d x}{2}\right]\right) + \\
& \quad 12 A \operatorname{Log}\left[2 \left(e^{i d x} \cos \left[\frac{c}{2}\right] + i e^{i d x} \sin \left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] \sin \left[\frac{d x}{2}\right] + \\
& \quad 7 B \operatorname{Log}\left[2 \left(e^{i d x} \cos \left[\frac{c}{2}\right] + i e^{i d x} \sin \left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] \sin \left[\frac{d x}{2}\right] + \\
& \quad 8 \sqrt{2} A \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \sin \left[\frac{1}{2} (c + d x)\right] + \\
& \quad \left. 12 \sqrt{2} B \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \sin \left[\frac{1}{2} (c + d x)\right] + 2 \sqrt{2} B \sqrt{\cos [c + d x]} (\cos [d x] + i \sin [d x]) \sin \left[\frac{3}{2} (c + d x)\right] \right)
\end{aligned}$$

- **Problem 177: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^{3/2} (A + B \cos [c + d x])}{\cos [c + d x]^{3/2}} dx$$

Optimal (type 3, 126 leaves, 4 steps):

$$\frac{a^{3/2} (2 A + 3 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}}\right]}{d} - \frac{a^2 (2 A - B) \sqrt{\cos [c + d x]} \sin [c + d x]}{d \sqrt{a + a \cos [c + d x]}} + \frac{2 a A \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{d \sqrt{\cos [c + d x]}}$$

Result (type 3, 869 leaves):



$$\begin{aligned}
& \frac{1}{4} (2A + 3B) (a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \\
& \left( \frac{1}{2} i \operatorname{Sin}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right)\right] \right) \left( \cos\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) - \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \Big) + \\
& \frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right)\right] \right) \left( \cos\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) + \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \Big) \Big) + \\
& \sqrt{\cos[c + dx]} (a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left( \frac{B \cos\left[\frac{dx}{2}\right] \operatorname{Sin}\left[\frac{c}{2}\right]}{2d} + \frac{B \cos\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{2d} + \right. \\
& \quad \left. \frac{A \operatorname{Sec}[c + dx] \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]}{d} \right)
\end{aligned}$$

■ **Problem 178: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx])^{3/2} (A + B \cos[c + dx])}{\cos[c + dx]^{5/2}} dx$$

Optimal (type 3, 125 leaves, 4 steps):

$$\frac{2 a^{3/2} B \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} + \frac{2 a^2 (4 A + 3 B) \operatorname{Sin}[c + dx]}{3 d \sqrt{\cos[c + dx]} \sqrt{a + a \cos[c + dx]}} + \frac{2 a A \sqrt{a + a \cos[c + dx]} \operatorname{Sin}[c + dx]}{3 d \cos[c + dx]^{3/2}}$$

Result (type 3, 702 leaves):

$$\begin{aligned}
& \frac{1}{12 \sqrt{2} d \cos [c+d x]^{3/2} \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])}} \\
& (a(1+\cos [c+d x]))^{3/2} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 \left(\frac{3}{2} i B e^{-\frac{3}{2} i d x} \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right]\right. \\
& \left. \cos \left[\frac{c}{2}\right]^2 \left((1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]\right)^2 - \frac{3}{2} i B e^{-\frac{3}{2} i d x} \cos \left[\frac{c}{2}\right]^2\right. \\
& \left. \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right]\right] \left((1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]\right)^2 + \frac{3}{2} i B \\
& e^{-\frac{3}{2} i d x} \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right] \sin \left[\frac{c}{2}\right]^2 \left((1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]\right)^2 - \\
& \frac{3}{2} i B e^{-\frac{3}{2} i d x} \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right] \sin \left[\frac{c}{2}\right]^2 \\
& \left.\left((1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]\right)^2 + 4 A \sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]} \sin \left[\frac{1}{2}(c+d x)\right] +\right. \\
& \left. 20 A \cos [c+d x] \sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]} \sin \left[\frac{1}{2}(c+d x)\right] +\right. \\
& \left. 12 B \cos [c+d x] \sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]} \sin \left[\frac{1}{2}(c+d x)\right]\right)
\end{aligned}$$

■ **Problem 182: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos [c+d x]^{3/2} (a+a \cos [c+d x])^{5/2} (A+B \cos [c+d x]) dx$$

Optimal (type 3, 274 leaves, 7 steps):

$$\begin{aligned}
& \frac{a^{5/2} (326 A+283 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{128 d} + \frac{a^3 (326 A+283 B) \sqrt{\cos [c+d x]} \sin [c+d x]}{128 d \sqrt{a+a \cos [c+d x]}} + \\
& \frac{a^3 (326 A+283 B) \cos [c+d x]^{3/2} \sin [c+d x]}{192 d \sqrt{a+a \cos [c+d x]}} + \frac{a^3 (170 A+157 B) \cos [c+d x]^{5/2} \sin [c+d x]}{240 d \sqrt{a+a \cos [c+d x]}} + \\
& \frac{a^2 (10 A+13 B) \cos [c+d x]^{5/2} \sqrt{a+a \cos [c+d x]} \sin [c+d x]}{40 d} + \frac{a B \cos [c+d x]^{5/2} (a+a \cos [c+d x])^{3/2} \sin [c+d x]}{5 d}
\end{aligned}$$

Result (type 3, 377 leaves):

$$\begin{aligned}
& - \frac{1}{15360 d \sqrt{2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c]}} (a (1 + \cos[c + d x]))^{5/2} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^5 \\
& \left( -15 i (326 A + 283 B) e^{\frac{i d x}{2}} \left( \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]}\right] - \right. \right. \\
& \quad \left. \left. \operatorname{Log}\left[2 \left(e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]}\right)\right] \right) \right) \\
& \sqrt{e^{-i d x} \left( (1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c] \right) - 4 \sqrt{\cos[c + d x]}} \\
& (5810 A + 5521 B + (3620 A + 3874 B) \cos[c + d x] + 4 (230 A + 331 B) \cos[2 (c + d x)] + 120 A \cos[3 (c + d x)] + \\
& \quad 348 B \cos[3 (c + d x)] + 48 B \cos[4 (c + d x)]) \sqrt{\cos[c + d x] (\cos[d x] + i \sin[d x])} \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \Big)
\end{aligned}$$

■ **Problem 183: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\cos[c + d x]} (a + a \cos[c + d x])^{5/2} (A + B \cos[c + d x]) dx$$

Optimal (type 3, 227 leaves, 6 steps):

$$\begin{aligned}
& \frac{a^{5/2} (200 A + 163 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c + d x]}{\sqrt{a + a \cos[c + d x]}}\right]}{64 d} + \frac{a^3 (200 A + 163 B) \sqrt{\cos[c + d x]} \sin[c + d x]}{64 d \sqrt{a + a \cos[c + d x]}} + \frac{a^3 (104 A + 95 B) \cos[c + d x]^{3/2} \sin[c + d x]}{96 d \sqrt{a + a \cos[c + d x]}} + \\
& \frac{a^2 (8 A + 11 B) \cos[c + d x]^{3/2} \sqrt{a + a \cos[c + d x]} \sin[c + d x]}{24 d} + \frac{a B \cos[c + d x]^{3/2} (a + a \cos[c + d x])^{3/2} \sin[c + d x]}{4 d}
\end{aligned}$$

Result (type 3, 355 leaves):

$$\begin{aligned}
& - \frac{1}{1536 d \sqrt{2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c]}} (a (1 + \cos[c + d x]))^{5/2} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^5 \\
& \left( -3 i (200 A + 163 B) e^{\frac{i d x}{2}} \left( \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]}\right] - \right. \right. \\
& \quad \left. \left. \operatorname{Log}\left[2 \left(e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]}\right)\right] \right) \right) \\
& \sqrt{e^{-i d x} \left( (1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c] \right) - 4 \sqrt{\cos[c + d x]}} (632 A + 581 B + (272 A + 362 B) \cos[c + d x] + \\
& \quad 4 (8 A + 23 B) \cos[2 (c + d x)] + 12 B \cos[3 (c + d x)]) \sqrt{\cos[c + d x] (\cos[d x] + i \sin[d x])} \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \Big)
\end{aligned}$$

■ **Problem 184: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^{5/2} (A + B \cos[c + d x])}{\sqrt{\cos[c + d x]}} dx$$

Optimal (type 3, 180 leaves, 5 steps):

$$\frac{a^{5/2} (38 A + 25 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{8 d} + \frac{a^3 (54 A + 49 B) \sqrt{\cos[c+dx]} \sin[c+dx]}{24 d \sqrt{a+a \cos[c+dx]}} +$$

$$\frac{a^2 (2 A + 3 B) \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{4 d} + \frac{a B \sqrt{\cos[c+dx]} (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{3 d}$$

Result (type 3, 623 leaves):

$$\frac{1}{48 d \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]}} a^2 \sqrt{\cos[c+dx]} \sqrt{a(1+\cos[c+dx])} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]$$

$$\left(-3 i (38 A + 25 B) \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2 \left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]}\right)\right] +\right.$$

$$\left.3 i (38 A + 25 B) \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) +\right.$$

$$\left.114 A \operatorname{Log}\left[2 \left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] +\right.$$

$$\left.75 B \operatorname{Log}\left[2 \left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] +\right.$$

$$\left.120 \sqrt{2} A \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] +\right.$$

$$\left.124 \sqrt{2} B \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + 12 \sqrt{2} A \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] +\right.$$

$$\left.30 \sqrt{2} B \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] + 4 \sqrt{2} B \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{5}{2}(c+dx)\right]\right)$$

■ **Problem 185: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \cos[c+dx])^{5/2} (A+B \cos[c+dx])}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 3, 178 leaves, 5 steps):

$$\frac{a^{5/2} (20 A + 19 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{4 d} - \frac{a^3 (4 A - 9 B) \sqrt{\cos[c+dx]} \sin[c+dx]}{4 d \sqrt{a+a \cos[c+dx]}} -$$

$$\frac{a^2 (4 A - B) \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{2 d} + \frac{2 a A (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{d \sqrt{\cos[c+dx]}}$$

Result (type 3, 926 leaves):

$$\begin{aligned}
& \frac{1}{32} (20 A + 19 B) (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \\
& \left( \frac{1}{2} i \operatorname{Sin} \left[ \frac{c}{2} \right] \left( - \left( 2 i e^{\frac{i d x}{2}} \operatorname{Log} \left[ 2 \left( e^{i d x} \cos \left[ \frac{c}{2} \right] + i e^{i d x} \operatorname{Sin} \left[ \frac{c}{2} \right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \operatorname{Sin} [c]} \right) \right] \left( \cos \left[ \frac{c}{2} \right] - i \operatorname{Sin} \left[ \frac{c}{2} \right] \right) \right. \right. \\
& \quad \left. \left. \sqrt{e^{-i d x} \left( (1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \operatorname{Sin} [c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin} [c]} \right) - \right. \\
& \quad \left( 2 i e^{\frac{i d x}{2}} \operatorname{ArcTanh} \left[ \left( \cos \left[ \frac{c}{2} \right] + i \operatorname{Sin} \left[ \frac{c}{2} \right] \right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \operatorname{Sin} [c]} \right] \left( \cos \left[ \frac{c}{2} \right] + i \operatorname{Sin} \left[ \frac{c}{2} \right] \right) \right. \\
& \quad \left. \left. \sqrt{e^{-i d x} \left( (1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \operatorname{Sin} [c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin} [c]} \right) \right) + \\
& \frac{1}{2} \cos \left[ \frac{c}{2} \right] \left( - \left( 2 i e^{\frac{i d x}{2}} \operatorname{Log} \left[ 2 \left( e^{i d x} \cos \left[ \frac{c}{2} \right] + i e^{i d x} \operatorname{Sin} \left[ \frac{c}{2} \right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \operatorname{Sin} [c]} \right) \right] \left( \cos \left[ \frac{c}{2} \right] - i \operatorname{Sin} \left[ \frac{c}{2} \right] \right) \right. \right. \\
& \quad \left. \left. \sqrt{e^{-i d x} \left( (1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \operatorname{Sin} [c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin} [c]} \right) + \right. \\
& \quad \left( 2 i e^{\frac{i d x}{2}} \operatorname{ArcTanh} \left[ \left( \cos \left[ \frac{c}{2} \right] + i \operatorname{Sin} \left[ \frac{c}{2} \right] \right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \operatorname{Sin} [c]} \right] \left( \cos \left[ \frac{c}{2} \right] + i \operatorname{Sin} \left[ \frac{c}{2} \right] \right) \right. \\
& \quad \left. \left. \sqrt{e^{-i d x} \left( (1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \operatorname{Sin} [c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin} [c]} \right) \right) \right) + \\
& \sqrt{\cos [c + d x]} (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \left( \frac{(2 A + 5 B) \cos \left[ \frac{d x}{2} \right] \operatorname{Sin} \left[ \frac{c}{2} \right]}{8 d} + \frac{B \cos \left[ \frac{3 d x}{2} \right] \operatorname{Sin} \left[ \frac{3 c}{2} \right]}{16 d} + \right. \\
& \quad \left. \frac{(2 A + 5 B) \cos \left[ \frac{c}{2} \right] \operatorname{Sin} \left[ \frac{d x}{2} \right]}{8 d} + \right. \\
& \quad \left. \frac{B \cos \left[ \frac{3 c}{2} \right] \operatorname{Sin} \left[ \frac{3 d x}{2} \right]}{16 d} + \frac{A \operatorname{Sec} [c + d x] \operatorname{Sin} \left[ \frac{c}{2} + \frac{d x}{2} \right]}{2 d} \right)
\end{aligned}$$

■ **Problem 186: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x])}{\cos [c + d x]^{5/2}} dx$$

Optimal (type 3, 173 leaves, 5 steps):

$$\begin{aligned}
& \frac{a^{5/2} (2 A + 5 B) \operatorname{ArcSin} \left[ \frac{\sqrt{a} \operatorname{Sin} [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right]}{d} - \frac{a^3 (14 A + 3 B) \sqrt{\cos [c + d x]} \operatorname{Sin} [c + d x]}{3 d \sqrt{a + a \cos [c + d x]}} + \\
& \frac{2 a^2 (2 A + B) \sqrt{a + a \cos [c + d x]} \operatorname{Sin} [c + d x]}{d \sqrt{\cos [c + d x]}} + \frac{2 a A (a + a \cos [c + d x])^{3/2} \operatorname{Sin} [c + d x]}{3 d \cos [c + d x]^{3/2}}
\end{aligned}$$

Result (type 3, 920 leaves):

$$\begin{aligned}
& \frac{1}{8} (2A + 5B) (a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \\
& \left( \frac{1}{2} i \operatorname{Sin}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]}\right)\right] \left( \cos\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) - \right. \\
& \quad \left. \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]}\right] \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right) \right) + \\
& \frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]}\right)\right] \left( \cos\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) + \right. \\
& \quad \left. \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]}\right] \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right) \right) \right) + \\
& \sqrt{\cos[c + dx]} (a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \frac{B \cos\left[\frac{dx}{2}\right] \operatorname{Sin}\left[\frac{c}{2}\right]}{4d} + \frac{B \cos\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{4d} + \right. \\
& \quad \frac{A \operatorname{Sec}[c + dx]^2 \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]}{6d} + \\
& \quad \left. \frac{\operatorname{Sec}[c + dx] \left( 8A \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right] + 3B \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right] \right)}{6d} \right)
\end{aligned}$$

■ **Problem 187: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx])^{5/2} (A + B \cos[c + dx])}{\cos[c + dx]^{7/2}} dx$$

Optimal (type 3, 172 leaves, 5 steps):

$$\begin{aligned}
& \frac{2 a^{5/2} B \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} + \frac{2 a^3 (32 A + 35 B) \operatorname{Sin}[c + dx]}{15 d \sqrt{\cos[c + dx]} \sqrt{a + a \cos[c + dx]}} + \\
& \frac{2 a^2 (8 A + 5 B) \sqrt{a + a \cos[c + dx]} \operatorname{Sin}[c + dx]}{15 d \cos[c + dx]^{3/2}} + \frac{2 a A (a + a \cos[c + dx])^{3/2} \operatorname{Sin}[c + dx]}{5 d \cos[c + dx]^{5/2}}
\end{aligned}$$

Result (type 3, 800 leaves):

$$\begin{aligned}
& \frac{1}{120 \sqrt{2} d \cos [c+d x]^{5/2} \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])}} (a(1+\cos [c+d x]))^{5/2} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 \\
& \left(\frac{15}{4} B e^{-\frac{5}{2} i d x} \cos \left[\frac{c}{2}\right]^2 \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right)\right]\right) \\
& \left(i\left(1+e^{2 i d x}\right) \cos [c]-\left(-1+e^{2 i d x}\right) \sin [c]\right)^3+\frac{15}{4} B e^{-\frac{5}{2} i d x} \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right)\right] \sin \left[\frac{c}{2}\right]^2\left(i\left(1+e^{2 i d x}\right) \cos [c]-\left(-1+e^{2 i d x}\right) \sin [c]\right)^3+ \\
& \frac{15}{4} i B e^{-\frac{5}{2} i d x} \operatorname{ArcTan h}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right] \cos \left[\frac{c}{2}\right]^2 \\
& \left(\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]\right)^3+\frac{15}{4} i B e^{-\frac{5}{2} i d x} \operatorname{ArcTan h}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right] \\
& \sin \left[\frac{c}{2}\right]^2\left(\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]\right)^3+12 A \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]} \sin \left[\frac{1}{2}(c+d x)\right]+ \\
& 56 A \cos [c+d x] \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]} \sin \left[\frac{1}{2}(c+d x)\right]+ \\
& 20 B \cos [c+d x] \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]} \sin \left[\frac{1}{2}(c+d x)\right]+ \\
& 172 \sqrt{2} A \cos [c+d x]^2 \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])} \sin \left[\frac{1}{2}(c+d x)\right]+ \\
& 160 \sqrt{2} B \cos [c+d x]^2 \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])} \sin \left[\frac{1}{2}(c+d x)\right]
\end{aligned}$$

■ **Problem 191: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c+d x]^{3/2} (A+B \cos [c+d x])}{\sqrt{a+a \cos [c+d x]}} d x$$

Optimal (type 3, 190 leaves, 7 steps):

$$\begin{aligned}
& -\frac{(4 A-7 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{4 \sqrt{a} d}+\frac{\sqrt{2}(A-B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right]}{\sqrt{a} d}+ \\
& \frac{(4 A-B) \sqrt{\cos [c+d x]} \sin [c+d x]}{4 d \sqrt{a+a \cos [c+d x]}}+\frac{B \cos [c+d x]^{3/2} \sin [c+d x]}{2 d \sqrt{a+a \cos [c+d x]}}
\end{aligned}$$

Result (type 3, 348 leaves):

$$\frac{1}{8\sqrt{a}(1+\cos[c+dx])} \cos\left[\frac{1}{2}(c+dx)\right] \left( 1 / \left( d\sqrt{1+e^{2i(c+dx)}} \right) \sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})} \left( -4Adx + 7Bdx + i(4A-7B) \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] - 8i\sqrt{2}(A-B) \right. \right. \\ \left. \left. \operatorname{Log}\left[1+e^{i(c+dx)}\right] - 4iA \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] + 7iB \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] + 8i\sqrt{2}A \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] - \right. \right. \\ \left. \left. 8i\sqrt{2}B \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] \right) + \frac{4\sqrt{\cos[c+dx]}(4A-B+2B\cos[c+dx])\sin\left[\frac{1}{2}(c+dx)\right]}{d} \right)$$

- **Problem 192: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]}(A+B\cos[c+dx])}{\sqrt{a+a\cos[c+dx]}} dx$$

Optimal (type 3, 141 leaves, 6 steps):

$$\frac{(2A-B) \operatorname{ArcSin}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{\sqrt{a}d} - \frac{\sqrt{2}(A-B) \operatorname{ArcTan}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{2}\sqrt{\cos[c+dx]}\sqrt{a+a\cos[c+dx]}}\right]}{\sqrt{a}d} + \frac{B\sqrt{\cos[c+dx]}\sin[c+dx]}{d\sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 333 leaves):

$$\frac{1}{2\sqrt{a}(1+\cos[c+dx])} \cos\left[\frac{1}{2}(c+dx)\right] \left( 1 / \left( d\sqrt{1+e^{2i(c+dx)}} \right) \sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})} \right. \\ \left( 2Adx - Bdx - i(2A-B) \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] + 2i\sqrt{2}(A-B) \operatorname{Log}\left[1+e^{i(c+dx)}\right] + 2iA \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] - iB \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] - \right. \\ \left. \left. 2i\sqrt{2}A \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] + 2i\sqrt{2}B \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] \right) + \frac{4B\sqrt{\cos[c+dx]}\sin\left[\frac{1}{2}(c+dx)\right]}{d} \right)$$

- **Problem 193: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A+B\cos[c+dx]}{\sqrt{\cos[c+dx]}\sqrt{a+a\cos[c+dx]}} dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$\frac{2B \operatorname{ArcSin}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{\sqrt{a}d} + \frac{\sqrt{2}(A-B) \operatorname{ArcTan}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{2}\sqrt{\cos[c+dx]}\sqrt{a+a\cos[c+dx]}}\right]}{\sqrt{a}d}$$

Result (type 3, 255 leaves):



$$\left( (1 + e^{i(c+dx)}) \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \right. \\ \left. \left( B dx - i B \operatorname{ArcSinh}[e^{i(c+dx)}] - i \sqrt{2} (A - B) \operatorname{Log}[1 + e^{i(c+dx)}] + i B \operatorname{Log}[1 + \sqrt{1 + e^{2i(c+dx)}}] + i \sqrt{2} A \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] - \right. \right. \\ \left. \left. i \sqrt{2} B \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) \right) / \left( \sqrt{2} d \sqrt{1 + e^{2i(c+dx)}} \sqrt{a(1 + \operatorname{Cos}[c + dx])} \right)$$

■ **Problem 194: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \operatorname{Cos}[c + dx]}{\operatorname{Cos}[c + dx]^{3/2} \sqrt{a + a \operatorname{Cos}[c + dx]}} dx$$

Optimal (type 3, 99 leaves, 4 steps):

$$- \frac{\sqrt{2} (A - B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{\sqrt{a} d} + \frac{2 A \operatorname{Sin}[c + dx]}{d \sqrt{\operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Cos}[c + dx]}}$$

Result (type 3, 177 leaves):

$$\frac{1}{d \sqrt{a(1 + \operatorname{Cos}[c + dx])}} \\ 2 \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] \left( 1 / \left( \sqrt{1 + e^{2i(c+dx)}} \right) i (A - B) e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( \operatorname{Log}[1 + e^{i(c+dx)}] - \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) \right) + \\ \frac{2 A \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{\operatorname{Cos}[c + dx]}}$$

■ **Problem 195: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \operatorname{Cos}[c + dx]}{\operatorname{Cos}[c + dx]^{5/2} \sqrt{a + a \operatorname{Cos}[c + dx]}} dx$$

Optimal (type 3, 142 leaves, 5 steps):

$$\frac{\sqrt{2} (A - B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{\sqrt{a} d} + \frac{2 A \operatorname{Sin}[c + dx]}{3 d \operatorname{Cos}[c + dx]^{3/2} \sqrt{a + a \operatorname{Cos}[c + dx]}} - \frac{2 (A - 3 B) \operatorname{Sin}[c + dx]}{3 d \sqrt{\operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Cos}[c + dx]}}$$

Result (type 3, 206 leaves):

$$\frac{1}{3 d \sqrt{a (1 + \operatorname{Cos}[c + d x])}} 2 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]$$

$$\left(-1 / \left(\sqrt{1 + e^{2 i (c+d x)}}\right) 3 i (A - B) e^{\frac{1}{2} i (c+d x)} \sqrt{e^{-i (c+d x)} (1 + e^{2 i (c+d x)})} \left(\operatorname{Log}\left[1 + e^{i (c+d x)}\right] - \operatorname{Log}\left[1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}\right]\right)\right) +$$

$$\left(\frac{2 A \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{\operatorname{Cos}[c + d x]^{3/2}} - \frac{2 (A - 3 B) \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{\operatorname{Cos}[c + d x]}}\right)$$

■ **Problem 196: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \operatorname{Cos}[c + d x]}{\operatorname{Cos}[c + d x]^{7/2} \sqrt{a + a \operatorname{Cos}[c + d x]}} dx$$

Optimal (type 3, 187 leaves, 6 steps):

$$-\frac{\sqrt{2} (A - B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c+d x]}{\sqrt{2} \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+a \operatorname{Cos}[c+d x]}}\right]}{\sqrt{a} d} + \frac{2 A \operatorname{Sin}[c + d x]}{5 d \operatorname{Cos}[c + d x]^{5/2} \sqrt{a + a \operatorname{Cos}[c + d x]}} -$$

$$\frac{2 (A - 5 B) \operatorname{Sin}[c + d x]}{15 d \operatorname{Cos}[c + d x]^{3/2} \sqrt{a + a \operatorname{Cos}[c + d x]}} + \frac{2 (13 A - 5 B) \operatorname{Sin}[c + d x]}{15 d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + a \operatorname{Cos}[c + d x]}}$$

Result (type 3, 235 leaves):

$$\left(2 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right.$$

$$\left.\left(1 / \left(\sqrt{1 + e^{2 i (c+d x)}}\right) 15 i (A - B) e^{\frac{1}{2} i (c+d x)} \sqrt{e^{-i (c+d x)} (1 + e^{2 i (c+d x)})} \left(\operatorname{Log}\left[1 + e^{i (c+d x)}\right] - \operatorname{Log}\left[1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}\right]\right)\right) +$$

$$\left.\frac{6 A \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{\operatorname{Cos}[c + d x]^{5/2}} - \frac{2 (A - 5 B) \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{\operatorname{Cos}[c + d x]^{3/2}} + \frac{2 (13 A - 5 B) \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{\operatorname{Cos}[c + d x]}}\right) / \left(15 d \sqrt{a (1 + \operatorname{Cos}[c + d x])}\right)$$

■ **Problem 197: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[c + d x]^{3/2} (A + B \operatorname{Cos}[c + d x])}{(a + a \operatorname{Cos}[c + d x])^{3/2}} dx$$

Optimal (type 3, 197 leaves, 7 steps):

$$\frac{(2A - 3B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{3/2} d} - \frac{(5A - 9B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{2\sqrt{2} a^{3/2} d} +$$

$$\frac{(A - B) \cos[c+dx]^{3/2} \sin[c+dx]}{2d(a+a \cos[c+dx])^{3/2}} - \frac{(A - 3B) \sqrt{\cos[c+dx]} \sin[c+dx]}{2ad \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 362 leaves):

$$\frac{1}{2(a(1+\cos[c+dx]))^{3/2}} \cos\left[\frac{1}{2}(c+dx)\right]^3 \left( 1 / \left( d \sqrt{1+e^{2i(c+dx)}} \right) \sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})} \right.$$

$$\left( 4Adx - 6Bdx - 2i(2A-3B) \operatorname{ArcSinh}[e^{i(c+dx)}] + i\sqrt{2}(5A-9B) \operatorname{Log}[1+e^{i(c+dx)}] + 4iA \operatorname{Log}[1+\sqrt{1+e^{2i(c+dx)}}] - \right.$$

$$\left. 6iB \operatorname{Log}[1+\sqrt{1+e^{2i(c+dx)}}] - 5i\sqrt{2}A \operatorname{Log}[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}] + 9i\sqrt{2}B \operatorname{Log}[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}] \right) +$$

$$\left. \frac{2\sqrt{\cos[c+dx]}(-A+3B+2B\cos[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{d} \right)$$

■ **Problem 198: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]}(A+B\cos[c+dx])}{(a+a\cos[c+dx])^{3/2}} dx$$

Optimal (type 3, 145 leaves, 6 steps):

$$\frac{2B \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{3/2} d} + \frac{(A - 5B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{2\sqrt{2} a^{3/2} d} + \frac{(A - B) \sqrt{\cos[c+dx]} \sin[c+dx]}{2d(a+a \cos[c+dx])^{3/2}}$$

Result (type 3, 313 leaves):

$$\frac{1}{2(a(1+\cos[c+dx]))^{3/2}} \cos\left[\frac{1}{2}(c+dx)\right]^3$$

$$\left( 1 / \left( d \sqrt{1+e^{2i(c+dx)}} \right) \sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})} \left( 4Bdx - 4iB \operatorname{ArcSinh}[e^{i(c+dx)}] - i\sqrt{2}(A-5B) \operatorname{Log}[1+e^{i(c+dx)}] + \right.$$

$$\left. 4iB \operatorname{Log}[1+\sqrt{1+e^{2i(c+dx)}}] + i\sqrt{2}A \operatorname{Log}[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}] - 5i\sqrt{2}B \operatorname{Log}[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}] \right) +$$

$$\left. \frac{2(A-B) \sqrt{\cos[c+dx]} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{d} \right)$$

■ **Problem 199: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos [c + d x]}{\sqrt{\cos [c + d x]} (a + a \cos [c + d x])^{3/2}} dx$$

Optimal (type 3, 107 leaves, 4 steps):

$$\frac{(3 A + B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{(A - B) \sqrt{\cos [c + d x]} \sin [c + d x]}{2 d (a + a \cos [c + d x])^{3/2}}$$

Result (type 3, 195 leaves):

$$\frac{1}{(a (1 + \cos [c + d x]))^{3/2}} \cos \left[ \frac{1}{2} (c + d x) \right]^3$$

$$\left( -1 / \left( d \sqrt{1 + e^{2 i (c+d x)}} \right) i (3 A + B) e^{\frac{1}{2} i (c+d x)} \sqrt{e^{-i (c+d x)} (1 + e^{2 i (c+d x)})} \left( \operatorname{Log} [1 + e^{i (c+d x)}] - \operatorname{Log} [1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}] \right) - \frac{(A - B) \sqrt{\cos [c + d x]} \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right] \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right]}{d} \right)$$

■ **Problem 200: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos [c + d x]}{\cos [c + d x]^{3/2} (a + a \cos [c + d x])^{3/2}} dx$$

Optimal (type 3, 156 leaves, 5 steps):

$$- \frac{(7 A - 3 B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{(A - B) \sin [c + d x]}{2 d \sqrt{\cos [c + d x]} (a + a \cos [c + d x])^{3/2}} + \frac{(5 A - B) \sin [c + d x]}{2 a d \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 209 leaves):

$$\frac{1}{(a (1 + \cos [c + d x]))^{3/2}} \cos \left[ \frac{1}{2} (c + d x) \right]^3$$

$$\left( 1 / \left( d \sqrt{1 + e^{2 i (c+d x)}} \right) i (7 A - 3 B) e^{\frac{1}{2} i (c+d x)} \sqrt{e^{-i (c+d x)} (1 + e^{2 i (c+d x)})} \left( \operatorname{Log} [1 + e^{i (c+d x)}] - \operatorname{Log} [1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}] \right) + \frac{(4 A + (5 A - B) \cos [c + d x]) \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right] \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right]}{d \sqrt{\cos [c + d x]}} \right)$$

■ **Problem 201: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos [c + d x]}{\cos [c + d x]^{5/2} (a + a \cos [c + d x])^{3/2}} dx$$

Optimal (type 3, 203 leaves, 6 steps) :

$$\frac{(11 A - 7 B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{(A - B) \sin[c+dx]}{2 d \cos[c+dx]^{3/2} (a + a \cos[c+dx])^{3/2}} +$$

$$\frac{(7 A - 3 B) \sin[c+dx]}{6 a d \cos[c+dx]^{3/2} \sqrt{a+a \cos[c+dx]}} - \frac{(19 A - 15 B) \sin[c+dx]}{6 a d \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 230 leaves) :

$$\frac{1}{(a (1 + \cos[c+dx]))^{3/2}} \cos\left[\frac{1}{2} (c+dx)\right]^3$$

$$\left( -1 / \left( d \sqrt{1 + e^{2i(c+dx)}} \right) i (11 A - 7 B) e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( \operatorname{Log}[1 + e^{i(c+dx)}] - \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) \right) -$$

$$\frac{(11 A - 15 B + 24 (A - B) \cos[c+dx] + (19 A - 15 B) \cos[2(c+dx)]) \operatorname{Sec}\left[\frac{1}{2} (c+dx)\right] \operatorname{Tan}\left[\frac{1}{2} (c+dx)\right]}{6 d \cos[c+dx]^{3/2}}$$

■ **Problem 202: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^{5/2} (A + B \cos[c+dx])}{(a + a \cos[c+dx])^{5/2}} dx$$

Optimal (type 3, 246 leaves, 8 steps) :

$$\frac{(2 A - 5 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{5/2} d} - \frac{(43 A - 115 B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} +$$

$$\frac{(A - B) \cos[c+dx]^{5/2} \sin[c+dx]}{4 d (a + a \cos[c+dx])^{5/2}} + \frac{(7 A - 15 B) \cos[c+dx]^{3/2} \sin[c+dx]}{16 a d (a + a \cos[c+dx])^{3/2}} - \frac{(11 A - 35 B) \sqrt{\cos[c+dx]} \sin[c+dx]}{16 a^2 d \sqrt{a + a \cos[c+dx]}}$$

Result (type 3, 376 leaves) :

$$\frac{1}{8 d (a (1 + \operatorname{Cos}[c + d x]))^{5/2}} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^5 \left( \frac{1}{\sqrt{1 + e^{2i(c+dx)}}} \sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1 + e^{2i(c+dx)})} \left( 32 A d x - 80 B d x - 16 i (2 A - 5 B) \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] + \right. \right. \\ \left. \left. i \sqrt{2} (43 A - 115 B) \operatorname{Log}\left[1 + e^{i(c+dx)}\right] + 32 i A \operatorname{Log}\left[1 + \sqrt{1 + e^{2i(c+dx)}}\right] - 80 i B \operatorname{Log}\left[1 + \sqrt{1 + e^{2i(c+dx)}}\right] - \right. \right. \\ \left. \left. 43 i \sqrt{2} A \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] + 115 i \sqrt{2} B \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] \right) + \\ \left. \sqrt{\operatorname{Cos}[c + d x]} (-11 A + 43 B + (-15 A + 55 B) \operatorname{Cos}[c + d x] + 8 B \operatorname{Cos}[2(c + d x)]) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^3 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] \right)$$

■ **Problem 203: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[c + d x]^{3/2} (A + B \operatorname{Cos}[c + d x])}{(a + a \operatorname{Cos}[c + d x])^{5/2}} dx$$

Optimal (type 3, 194 leaves, 7 steps):

$$\frac{2 B \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{a^{5/2} d} + \frac{(3 A - 43 B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} + \\ \frac{(A - B) \operatorname{Cos}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{4 d (a + a \operatorname{Cos}[c + d x])^{5/2}} + \frac{(3 A - 11 B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{16 a d (a + a \operatorname{Cos}[c + d x])^{3/2}}$$

Result (type 3, 329 leaves):

$$\frac{1}{8 d (a (1 + \operatorname{Cos}[c + d x]))^{5/2}} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^5 \left( \frac{1}{\sqrt{1 + e^{2i(c+dx)}}} \sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1 + e^{2i(c+dx)})} \left( 32 B d x - 32 i B \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] - i \sqrt{2} (3 A - 43 B) \operatorname{Log}\left[1 + e^{i(c+dx)}\right] + \right. \right. \\ \left. \left. 32 i B \operatorname{Log}\left[1 + \sqrt{1 + e^{2i(c+dx)}}\right] + 3 i \sqrt{2} A \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] - 43 i \sqrt{2} B \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] \right) + \\ \left. \sqrt{\operatorname{Cos}[c + d x]} (3 A - 11 B + (7 A - 15 B) \operatorname{Cos}[c + d x]) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^3 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] \right)$$

■ **Problem 204: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{\operatorname{Cos}[c + d x]} (A + B \operatorname{Cos}[c + d x])}{(a + a \operatorname{Cos}[c + d x])^{5/2}} dx$$

Optimal (type 3, 154 leaves, 5 steps) :

$$\frac{(5A + 3B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} + \frac{(A-B) \sqrt{\cos[c+dx]} \sin[c+dx]}{4d (a+a \cos[c+dx])^{5/2}} + \frac{(A+7B) \sqrt{\cos[c+dx]} \sin[c+dx]}{16ad (a+a \cos[c+dx])^{3/2}}$$

Result (type 3, 215 leaves) :

$$\left( \cos\left[\frac{1}{2}(c+dx)\right]^5 \left( -1 / \left( \sqrt{1+e^{2i(c+dx)}} \right) i (5A+3B) e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})} \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) + \frac{1}{2} \sqrt{\cos[c+dx]} (5A+3B + (A+7B) \cos[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) / (4d (a(1+\cos[c+dx]))^{5/2})$$

■ **Problem 205: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B \cos[c+dx]}{\sqrt{\cos[c+dx]} (a+a \cos[c+dx])^{5/2}} dx$$

Optimal (type 3, 156 leaves, 5 steps) :

$$\frac{(19A+5B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A-B) \sqrt{\cos[c+dx]} \sin[c+dx]}{4d (a+a \cos[c+dx])^{5/2}} - \frac{(9A-B) \sqrt{\cos[c+dx]} \sin[c+dx]}{16ad (a+a \cos[c+dx])^{3/2}}$$

Result (type 3, 217 leaves) :

$$\left( \cos\left[\frac{1}{2}(c+dx)\right]^5 \left( -1 / \left( \sqrt{1+e^{2i(c+dx)}} \right) i (19A+5B) e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})} \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) - \frac{1}{2} \sqrt{\cos[c+dx]} (13A-5B + (9A-B) \cos[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) / (4d (a(1+\cos[c+dx]))^{5/2})$$

■ **Problem 206: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B \cos[c+dx]}{\cos[c+dx]^{3/2} (a+a \cos[c+dx])^{5/2}} dx$$

Optimal (type 3, 203 leaves, 6 steps) :

$$\frac{(75A-19B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A-B) \sin[c+dx]}{4d \sqrt{\cos[c+dx]} (a+a \cos[c+dx])^{5/2}} - \frac{(13A-5B) \sin[c+dx]}{16ad \sqrt{\cos[c+dx]} (a+a \cos[c+dx])^{3/2}} + \frac{(49A-9B) \sin[c+dx]}{16a^2 d \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 234 leaves) :

$$\begin{aligned} & \left( \cos \left[ \frac{1}{2} (c + d x) \right] \right)^5 \\ & \left( 1 / \left( \sqrt{1 + e^{2i(c+dx)}} \right) i (75 A - 19 B) e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( \text{Log} [1 + e^{i(c+dx)}] - \text{Log} [1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) \right) + \\ & 1 / \left( 4 \sqrt{\cos [c + d x]} \right) (113 A - 9 B + 2 (85 A - 13 B) \cos [c + d x] + (49 A - 9 B) \cos [2 (c + d x)]) \\ & \left. \sec \left[ \frac{1}{2} (c + d x) \right]^3 \tan \left[ \frac{1}{2} (c + d x) \right] \right) \Big/ (4 d (a (1 + \cos [c + d x]))^{5/2}) \end{aligned}$$

■ **Problem 207: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos [c + d x]}{\cos [c + d x]^{5/2} (a + a \cos [c + d x])^{5/2}} dx$$

Optimal (type 3, 250 leaves, 7 steps) :

$$\begin{aligned} & \frac{(163 A - 75 B) \text{ArcTan} \left[ \frac{\sqrt{a} \sin [c+dx]}{\sqrt{2} \sqrt{\cos [c+dx]} \sqrt{a+a \cos [c+dx]}} \right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A - B) \sin [c + d x]}{4 d \cos [c + d x]^{3/2} (a + a \cos [c + d x])^{5/2}} - \\ & \frac{(17 A - 9 B) \sin [c + d x]}{16 a d \cos [c + d x]^{3/2} (a + a \cos [c + d x])^{3/2}} + \frac{(95 A - 39 B) \sin [c + d x]}{48 a^2 d \cos [c + d x]^{3/2} \sqrt{a + a \cos [c + d x]}} - \frac{(299 A - 147 B) \sin [c + d x]}{48 a^2 d \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]}} \end{aligned}$$

Result (type 3, 256 leaves) :

$$\begin{aligned} & - \left( \cos \left[ \frac{1}{2} (c + d x) \right] \right)^5 \\ & \left( 1 / \left( \sqrt{1 + e^{2i(c+dx)}} \right) 3 i (163 A - 75 B) e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( \text{Log} [1 + e^{i(c+dx)}] - \text{Log} [1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) \right) + \\ & 1 / \left( 8 \cos [c + d x]^{3/2} \right) (878 A - 510 B + (1537 A - 825 B) \cos [c + d x] + 2 (503 A - 255 B) \cos [2 (c + d x)] + \\ & 299 A \cos [3 (c + d x)] - 147 B \cos [3 (c + d x)]) \sec \left[ \frac{1}{2} (c + d x) \right]^3 \tan \left[ \frac{1}{2} (c + d x) \right] \Big/ (12 d (a (1 + \cos [c + d x]))^{5/2}) \end{aligned}$$

■ **Problem 208: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^{7/2} (A + B \cos [c + d x])}{(a + a \cos [c + d x])^{7/2}} dx$$

Optimal (type 3, 293 leaves, 9 steps) :



$$\frac{(2A - 7B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{7/2} d} - \frac{(177A - 637B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{64 \sqrt{2} a^{7/2} d} + \frac{(A - B) \cos[c+dx]^{7/2} \sin[c+dx]}{6d (a+a \cos[c+dx])^{7/2}} +$$

$$\frac{(3A - 7B) \cos[c+dx]^{5/2} \sin[c+dx]}{16ad (a+a \cos[c+dx])^{5/2}} + \frac{(79A - 259B) \cos[c+dx]^{3/2} \sin[c+dx]}{192a^2 d (a+a \cos[c+dx])^{3/2}} - \frac{7(7A - 27B) \sqrt{\cos[c+dx]} \sin[c+dx]}{64a^3 d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 639 leaves):

$$\frac{1}{8 \sqrt{2} d \sqrt{1 + e^{2i(c+dx)}} (a(1 + \cos[c+dx]))^{7/2}}$$

$$e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left(128A dx - 448B dx - 64i(2A - 7B) \operatorname{ArcSinh}[e^{i(c+dx)}] +\right.$$

$$i \sqrt{2} (177A - 637B) \operatorname{Log}[1 + e^{i(c+dx)}] + 128iA \operatorname{Log}[1 + \sqrt{1 + e^{2i(c+dx)}}] - 448iB \operatorname{Log}[1 + \sqrt{1 + e^{2i(c+dx)}}] -$$

$$\left.177i \sqrt{2} A \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] + 637i \sqrt{2} B \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}]\right) + \frac{1}{(a(1 + \cos[c+dx]))^{7/2}}$$

$$\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\cos[c+dx]} \left(\frac{16B \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{d} + \frac{16B \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (11A \sin\left[\frac{dx}{2}\right] - 15B \sin\left[\frac{dx}{2}\right])}{4d} +\right.$$

$$\frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{3d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (-247A \sin\left[\frac{dx}{2}\right] + 523B \sin\left[\frac{dx}{2}\right])}{24d} -$$

$$\left.\frac{(247A - 523B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{24d} + \frac{(11A - 15B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{4d} - \frac{(A - B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \tan\left[\frac{c}{2}\right]}{3d}\right)$$

■ **Problem 209: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^{5/2} (A + B \cos[c+dx])}{(a + a \cos[c+dx])^{7/2}} dx$$

Optimal (type 3, 241 leaves, 8 steps):

$$\frac{2B \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{7/2} d} + \frac{(5A - 177B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{64 \sqrt{2} a^{7/2} d} +$$

$$\frac{(A - B) \cos[c+dx]^{5/2} \sin[c+dx]}{6d (a+a \cos[c+dx])^{7/2}} + \frac{(5A - 17B) \cos[c+dx]^{3/2} \sin[c+dx]}{48ad (a+a \cos[c+dx])^{5/2}} + \frac{(5A - 49B) \sqrt{\cos[c+dx]} \sin[c+dx]}{64a^2 d (a+a \cos[c+dx])^{3/2}}$$

Result (type 3, 350 leaves):

$$\frac{1}{48 d (a (1 + \cos [c + d x]))^{7/2}} \cos \left[ \frac{1}{2} (c + d x) \right]^7$$

$$\left( \frac{1}{\sqrt{1 + e^{2i(c+dx)}}} 3 \sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( 128 B d x - 128 i B \operatorname{ArcSinh} [e^{i(c+dx)}] - i \sqrt{2} (5 A - 177 B) \operatorname{Log} [1 + e^{i(c+dx)}] + \right. \right.$$

$$\left. 128 i B \operatorname{Log} [1 + \sqrt{1 + e^{2i(c+dx)}}] + 5 i \sqrt{2} A \operatorname{Log} [1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] - 177 i \sqrt{2} B \operatorname{Log} [1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) +$$

$$\left. \frac{1}{4} \sqrt{\cos [c + d x]} (97 A - 541 B + 4 (25 A - 181 B) \cos [c + d x] + (67 A - 247 B) \cos [2 (c + d x)]) \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right]^5 \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] \right)$$

■ **Problem 210: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c + d x]^{3/2} (A + B \cos [c + d x])}{(a + a \cos [c + d x])^{7/2}} dx$$

Optimal (type 3, 201 leaves, 6 steps):

$$\frac{(7 A + 5 B) \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{2} \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]}} \right]}{64 \sqrt{2} a^{7/2} d} + \frac{(A - B) \cos [c + d x]^{3/2} \sin [c + d x]}{6 d (a + a \cos [c + d x])^{7/2}} +$$

$$\frac{(A - 13 B) \sqrt{\cos [c + d x]} \sin [c + d x]}{48 a d (a + a \cos [c + d x])^{5/2}} + \frac{(17 A + 67 B) \sqrt{\cos [c + d x]} \sin [c + d x]}{192 a^2 d (a + a \cos [c + d x])^{3/2}}$$

Result (type 3, 234 leaves):

$$\left( \cos \left[ \frac{1}{2} (c + d x) \right] \right)^7$$

$$\left( -1 / \left( \sqrt{1 + e^{2i(c+dx)}} \right) 3 i (7 A + 5 B) e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( \operatorname{Log} [1 + e^{i(c+dx)}] - \operatorname{Log} [1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) \right) +$$

$$\frac{1}{8} \sqrt{\cos [c + d x]} (59 A + 97 B + 20 (7 A + 5 B) \cos [c + d x] + (17 A + 67 B) \cos [2 (c + d x)])$$

$$\operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right]^5 \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] \right) / (24 d (a (1 + \cos [c + d x]))^{7/2})$$

■ **Problem 211: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{\cos [c + d x]} (A + B \cos [c + d x])}{(a + a \cos [c + d x])^{7/2}} dx$$

Optimal (type 3, 201 leaves, 6 steps):

$$\frac{(13A + 7B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right]}{64 \sqrt{2} a^{7/2} d} + \frac{(A - B) \sqrt{\cos[c+dx]} \sin[c+dx]}{6d (a + a \cos[c+dx])^{7/2}} +$$

$$\frac{(A + 3B) \sqrt{\cos[c+dx]} \sin[c+dx]}{16ad (a + a \cos[c+dx])^{5/2}} - \frac{(5A - 17B) \sqrt{\cos[c+dx]} \sin[c+dx]}{192a^2 d (a + a \cos[c+dx])^{3/2}}$$

Result (type 3, 232 leaves):

$$\left( \cos\left[\frac{1}{2}(c+dx)\right]^7 \right.$$

$$\left. \left( -1 / \left( \sqrt{1 + e^{2i(c+dx)}} \right) 3i (13A + 7B) e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( \log[1 + e^{i(c+dx)}] - \log[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) \right) \right.$$

$$\left. \left. \frac{1}{8} \sqrt{\cos[c+dx]} (73A + 59B + 4(A + 35B) \cos[c+dx] + (-5A + 17B) \cos[2(c+dx)]) \right) \right.$$

$$\left. \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \right) / (24d (a(1 + \cos[c+dx]))^{7/2})$$

■ **Problem 212: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos[c+dx]}{\sqrt{\cos[c+dx]} (a + a \cos[c+dx])^{7/2}} dx$$

Optimal (type 3, 203 leaves, 6 steps):

$$\frac{(63A + 13B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right]}{64 \sqrt{2} a^{7/2} d} - \frac{(A - B) \sqrt{\cos[c+dx]} \sin[c+dx]}{6d (a + a \cos[c+dx])^{7/2}} -$$

$$\frac{(5A - B) \sqrt{\cos[c+dx]} \sin[c+dx]}{16ad (a + a \cos[c+dx])^{5/2}} - \frac{(103A + 5B) \sqrt{\cos[c+dx]} \sin[c+dx]}{192a^2 d (a + a \cos[c+dx])^{3/2}}$$

Result (type 3, 233 leaves):

$$- \left( \cos\left[\frac{1}{2}(c+dx)\right]^7 \right.$$

$$\left. \left( 1 / \left( \sqrt{1 + e^{2i(c+dx)}} \right) 3i (63A + 13B) e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( \log[1 + e^{i(c+dx)}] - \log[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) \right) \right.$$

$$\left. \left. \frac{1}{8} \sqrt{\cos[c+dx]} (493A - 73B + (532A - 4B) \cos[c+dx] + (103A + 5B) \cos[2(c+dx)]) \right) \right.$$

$$\left. \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \right) / (24d (a(1 + \cos[c+dx]))^{7/2})$$

■ **Problem 213: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos [c + d x]}{\cos [c + d x]^{3/2} (a + a \cos [c + d x])^{7/2}} dx$$

Optimal (type 3, 250 leaves, 7 steps):

$$\frac{3 (121 A - 21 B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right]}{64 \sqrt{2} a^{7/2} d} - \frac{(A - B) \sin [c + d x]}{6 d \sqrt{\cos [c + d x]} (a + a \cos [c + d x])^{7/2}} - \frac{(19 A - 7 B) \sin [c + d x]}{48 a d \sqrt{\cos [c + d x]} (a + a \cos [c + d x])^{5/2}} - \frac{(199 A - 43 B) \sin [c + d x]}{192 a^2 d \sqrt{\cos [c + d x]} (a + a \cos [c + d x])^{3/2}} + \frac{(691 A - 103 B) \sin [c + d x]}{192 a^3 d \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 257 leaves):

$$\left( \cos \left[ \frac{1}{2} (c + d x) \right] \right)^7 \left( \frac{1}{\left( \sqrt{1 + e^{2i(c+d x)}} \right)} 9i (121 A - 21 B) e^{\frac{1}{2} i (c+d x)} \sqrt{e^{-i(c+d x)} (1 + e^{2i(c+d x)})} \left( \log [1 + e^{i(c+d x)}] - \log [1 - e^{i(c+d x)} + \sqrt{2} \sqrt{1 + e^{2i(c+d x)}}] \right) \right) + \frac{1}{\left( 16 \sqrt{\cos [c + d x]} \right)} (5284 A - 532 B + 9 (941 A - 121 B) \cos [c + d x] + 4 (937 A - 133 B) \cos [2 (c + d x)] + 691 A \cos [3 (c + d x)] - 103 B \cos [3 (c + d x)]) \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right]^5 \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right] \Bigg) / (24 d (a (1 + \cos [c + d x]))^{7/2})$$

■ **Problem 214: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos [c + d x]}{\cos [c + d x]^{5/2} (a + a \cos [c + d x])^{7/2}} dx$$

Optimal (type 3, 297 leaves, 8 steps):

$$\frac{(1015 A - 363 B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right]}{64 \sqrt{2} a^{7/2} d} - \frac{(A - B) \sin [c + d x]}{6 d \cos [c + d x]^{3/2} (a + a \cos [c + d x])^{7/2}} - \frac{(23 A - 11 B) \sin [c + d x]}{48 a d \cos [c + d x]^{3/2} (a + a \cos [c + d x])^{5/2}} - \frac{(109 A - 41 B) \sin [c + d x]}{64 a^2 d \cos [c + d x]^{3/2} (a + a \cos [c + d x])^{3/2}} + \frac{(579 A - 199 B) \sin [c + d x]}{192 a^3 d \cos [c + d x]^{3/2} \sqrt{a + a \cos [c + d x]}} - \frac{(1887 A - 691 B) \sin [c + d x]}{192 a^3 d \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 524 leaves):

$$\begin{aligned}
& - \left( i (1015 A - 363 B) e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i (c+dx)} (1 + e^{2 i (c+dx)})} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^7 \left( \log [1 + e^{i (c+dx)}] - \log [1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}] \right) \right) / \\
& \left( 8 d \sqrt{1 + e^{2 i (c+dx)}} (a (1 + \cos [c + dx]))^{7/2} \right) + \frac{1}{(a (1 + \cos [c + dx]))^{7/2}} \\
& \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^7 \sqrt{\cos [c + dx]} \left( \frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 (-A \sin \left[ \frac{dx}{2} \right] + B \sin \left[ \frac{dx}{2} \right])}{3 d} + \right. \\
& \frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 (-13 A \sin \left[ \frac{dx}{2} \right] + 9 B \sin \left[ \frac{dx}{2} \right])}{4 d} + \frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 (-607 A \sin \left[ \frac{dx}{2} \right] + 307 B \sin \left[ \frac{dx}{2} \right])}{24 d} + \\
& \frac{32 A \sec [c + dx]^2 \sin \left[ \frac{c}{2} + \frac{dx}{2} \right]}{3 d} - \frac{32 \sec [c + dx] (10 A \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] - 3 B \sin \left[ \frac{c}{2} + \frac{dx}{2} \right])}{3 d} - \\
& \left. \frac{(607 A - 307 B) \sec \left[ \frac{c}{2} + \frac{dx}{2} \right] \tan \left[ \frac{c}{2} \right]}{24 d} - \frac{(13 A - 9 B) \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \tan \left[ \frac{c}{2} \right]}{4 d} - \frac{(A - B) \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \tan \left[ \frac{c}{2} \right]}{3 d} \right)
\end{aligned}$$

■ **Problem 218: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + dx]) (A + B \cos [c + dx]) \sec [c + dx] dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$(A b + a B) x + \frac{a A \operatorname{ArcTanh}[\sin [c + dx]]}{d} + \frac{b B \sin [c + dx]}{d}$$

Result (type 3, 104 leaves):

$$A b x + a B x - \frac{a A \log [\cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right]]}{d} + \frac{a A \log [\cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right]]}{d} + \frac{b B \cos [dx] \sin [c]}{d} + \frac{b B \cos [c] \sin [dx]}{d}$$

■ **Problem 219: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + dx]) (A + B \cos [c + dx]) \sec [c + dx]^2 dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$b B x + \frac{(A b + a B) \operatorname{ArcTanh}[\sin [c + dx]]}{d} + \frac{a A \tan [c + dx]}{d}$$

Result (type 3, 159 leaves):

$$\begin{aligned}
& b B x - \frac{A b \log [\cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right]]}{d} - \frac{a B \log [\cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right]]}{d} + \\
& \frac{A b \log [\cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right]]}{d} + \frac{a B \log [\cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right]]}{d} + \frac{a A \tan [c + dx]}{d}
\end{aligned}$$

■ **Problem 220: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x]) (A + B \cos [c + d x]) \sec [c + d x]^3 dx$$

Optimal (type 3, 61 leaves, 6 steps):

$$\frac{(a A + 2 b B) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{(A b + a B) \tan [c + d x]}{d} + \frac{a A \sec [c + d x] \tan [c + d x]}{2 d}$$

Result (type 3, 164 leaves):

$$\frac{1}{4 d} \left( -2 (a A + 2 b B) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \right. \\ \left. 2 a A \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] + 4 b B \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \right. \\ \left. \frac{a A}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \frac{a A}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + 4 (A b + a B) \tan [c + d x] \right)$$

■ **Problem 222: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x]) (A + B \cos [c + d x]) \sec [c + d x]^5 dx$$

Optimal (type 3, 114 leaves, 7 steps):

$$\frac{(3 a A + 4 b B) \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} + \frac{(A b + a B) \tan [c + d x]}{d} + \\ \frac{(3 a A + 4 b B) \sec [c + d x] \tan [c + d x]}{8 d} + \frac{a A \sec [c + d x]^3 \tan [c + d x]}{4 d} + \frac{(A b + a B) \tan [c + d x]^3}{3 d}$$

Result (type 3, 403 leaves):

$$- \frac{3 a A \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{8 d} - \frac{b B \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{2 d} + \frac{3 a A \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{8 d} + \\ \frac{b B \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{2 d} + \frac{a A}{16 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^4} + \frac{3 a A}{16 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \\ \frac{b B}{4 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \frac{a A}{16 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^4} - \frac{3 a A}{16 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \\ \frac{b B}{4 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{2 a b \tan [c + d x]}{3 d} + \frac{2 a B \tan [c + d x]}{3 d} + \frac{A b \sec [c + d x]^2 \tan [c + d x]}{3 d} + \frac{a B \sec [c + d x]^2 \tan [c + d x]}{3 d}$$

■ **Problem 228: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^2 (A + B \cos [c + d x]) \sec [c + d x]^3 dx$$

Optimal (type 3, 80 leaves, 4 steps):

$$b^2 B x + \frac{(a^2 A + 2 A b^2 + 4 a b B) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{a (2 A b + a B) \tan [c + d x]}{d} + \frac{a^2 A \sec [c + d x] \tan [c + d x]}{2 d}$$

Result (type 3, 225 leaves):

$$\frac{1}{4 d} \left( 4 b^2 B c + 4 b^2 B d x - 2 (a^2 A + 2 A b^2 + 4 a b B) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] + 2 a^2 A \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \right. \\ \left. 4 A b^2 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] + 8 a b B \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \right. \\ \left. \frac{a^2 A}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \frac{a^2 A}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + 4 a (2 A b + a B) \tan [c + d x] \right)$$

■ **Problem 230: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^2 (A + B \cos [c + d x]) \sec [c + d x]^5 dx$$

Optimal (type 3, 156 leaves, 7 steps):

$$\frac{(3 a^2 A + 4 A b^2 + 8 a b B) \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} + \frac{(4 a A b + 2 a^2 B + 3 b^2 B) \tan [c + d x]}{3 d} + \\ \frac{(3 a^2 A + 4 A b^2 + 8 a b B) \sec [c + d x] \tan [c + d x]}{8 d} + \frac{a (2 A b + a B) \sec [c + d x]^2 \tan [c + d x]}{3 d} + \frac{a^2 A \sec [c + d x]^3 \tan [c + d x]}{4 d}$$

Result (type 3, 457 leaves):

$$\frac{1}{48 d} \left( -6 (3 a^2 A + 4 A b^2 + 8 a b B) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] + 6 (3 a^2 A + 4 A b^2 + 8 a b B) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \right. \\ \left. \frac{3 a^2 A}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^4} + \frac{12 A b^2 + 8 a b (A + 3 B) + a^2 (9 A + 4 B)}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{8 a (2 A b + a B) \sin \left[ \frac{1}{2} (c + d x) \right]}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^3} + \right. \\ \left. \frac{16 (4 a A b + 2 a^2 B + 3 b^2 B) \sin \left[ \frac{1}{2} (c + d x) \right]}{\cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right]} - \frac{3 a^2 A}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^4} + \right. \\ \left. \frac{8 a (2 A b + a B) \sin \left[ \frac{1}{2} (c + d x) \right]}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^3} - \frac{12 A b^2 + 8 a b (A + 3 B) + a^2 (9 A + 4 B)}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{16 (4 a A b + 2 a^2 B + 3 b^2 B) \sin \left[ \frac{1}{2} (c + d x) \right]}{\cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right]} \right)$$

■ **Problem 236: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^3 (A + B \cos [c + d x]) \sec [c + d x]^3 dx$$

Optimal (type 3, 124 leaves, 5 steps):

$$b^2 (A b + 3 a B) x + \frac{a (a^2 A + 6 A b^2 + 6 a b B) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} - \frac{b^2 (a A - 2 b B) \sin [c + d x]}{2 d} + \frac{a^2 (2 A b + a B) \tan [c + d x]}{d} + \frac{a A (a + b \cos [c + d x])^2 \sec [c + d x] \tan [c + d x]}{2 d}$$

Result (type 3, 277 leaves):

$$\frac{1}{4 d} \left( 4 b^2 (A b + 3 a B) (c + d x) - 2 a (a^2 A + 6 A b^2 + 6 a b B) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] + 2 a (a^2 A + 6 A b^2 + 6 a b B) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \frac{a^3 A}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{4 a^2 (3 A b + a B) \sin \left[ \frac{1}{2} (c + d x) \right]}{\cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right]} - \frac{a^3 A}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{4 a^2 (3 A b + a B) \sin \left[ \frac{1}{2} (c + d x) \right]}{\cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right]} + 4 b^3 B \sin [c + d x] \right)$$

■ **Problem 237: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^3 (A + B \cos [c + d x]) \sec [c + d x]^4 dx$$

Optimal (type 3, 145 leaves, 5 steps):

$$b^3 B x + \frac{(3 a^2 A b + 2 A b^3 + a^3 B + 6 a b^2 B) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{a (2 a^2 A + 8 A b^2 + 9 a b B) \tan [c + d x]}{3 d} + \frac{a^2 (5 A b + 3 a B) \sec [c + d x] \tan [c + d x]}{6 d} + \frac{a A (a + b \cos [c + d x])^2 \sec [c + d x]^2 \tan [c + d x]}{3 d}$$

Result (type 3, 392 leaves):



$$\frac{1}{12d} \left( 12b^3B(c+dx) - 6(3a^2Ab + 2Ab^3 + a^3B + 6ab^2B) \operatorname{Log} \left[ \cos \left[ \frac{1}{2}(c+dx) \right] - \sin \left[ \frac{1}{2}(c+dx) \right] \right] + \right. \\ \left. 6(3a^2Ab + 2Ab^3 + a^3B + 6ab^2B) \operatorname{Log} \left[ \cos \left[ \frac{1}{2}(c+dx) \right] + \sin \left[ \frac{1}{2}(c+dx) \right] \right] + \frac{a^2(9Ab + a(A+3B))}{\left( \cos \left[ \frac{1}{2}(c+dx) \right] - \sin \left[ \frac{1}{2}(c+dx) \right] \right)^2} + \right. \\ \left. \frac{2a^3A \sin \left[ \frac{1}{2}(c+dx) \right]}{\left( \cos \left[ \frac{1}{2}(c+dx) \right] - \sin \left[ \frac{1}{2}(c+dx) \right] \right)^3} + \frac{4a(2a^2A + 9Ab^2 + 9abB) \sin \left[ \frac{1}{2}(c+dx) \right]}{\cos \left[ \frac{1}{2}(c+dx) \right] - \sin \left[ \frac{1}{2}(c+dx) \right]} + \right. \\ \left. \frac{2a^3A \sin \left[ \frac{1}{2}(c+dx) \right]}{\left( \cos \left[ \frac{1}{2}(c+dx) \right] + \sin \left[ \frac{1}{2}(c+dx) \right] \right)^3} - \frac{a^2(9Ab + a(A+3B))}{\left( \cos \left[ \frac{1}{2}(c+dx) \right] + \sin \left[ \frac{1}{2}(c+dx) \right] \right)^2} + \frac{4a(2a^2A + 9Ab^2 + 9abB) \sin \left[ \frac{1}{2}(c+dx) \right]}{\cos \left[ \frac{1}{2}(c+dx) \right] + \sin \left[ \frac{1}{2}(c+dx) \right]} \right)$$

■ **Problem 238: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + dx])^3 (A + B \cos[c + dx]) \sec[c + dx]^5 dx$$

Optimal (type 3, 188 leaves, 7 steps):

$$\frac{(3a^3A + 12aAb^2 + 12a^2bB + 8b^3B) \operatorname{ArcTanh}[\sin[c + dx]]}{8d} + \\ \frac{(6a^2Ab + 3Ab^3 + 2a^3B + 9ab^2B) \tan[c + dx]}{3d} + \frac{a(3a^2A + 10Ab^2 + 12abB) \sec[c + dx] \tan[c + dx]}{8d} + \\ \frac{a^2(3Ab + 2aB) \sec[c + dx]^2 \tan[c + dx]}{6d} + \frac{aA(a + b \cos[c + dx])^2 \sec[c + dx]^3 \tan[c + dx]}{4d}$$

Result (type 3, 639 leaves):

$$\begin{aligned}
& \frac{(-3 a^3 A - 12 a A b^2 - 12 a^2 b B - 8 b^3 B) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8 d} + \\
& \frac{(3 a^3 A + 12 a A b^2 + 12 a^2 b B + 8 b^3 B) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8 d} + \frac{a^3 A}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \\
& \frac{9 a^3 A + 12 a^2 A b + 36 a A b^2 + 4 a^3 B + 36 a^2 b B}{48 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{a^3 A}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{-9 a^3 A - 12 a^2 A b - 36 a A b^2 - 4 a^3 B - 36 a^2 b B}{48 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \\
& \frac{3 a^2 A b \sin\left[\frac{1}{2}(c+dx)\right] + a^3 B \sin\left[\frac{1}{2}(c+dx)\right]}{6 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{3 a^2 A b \sin\left[\frac{1}{2}(c+dx)\right] + a^3 B \sin\left[\frac{1}{2}(c+dx)\right]}{6 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} + \\
& \frac{6 a^2 A b \sin\left[\frac{1}{2}(c+dx)\right] + 3 A b^3 \sin\left[\frac{1}{2}(c+dx)\right] + 2 a^3 B \sin\left[\frac{1}{2}(c+dx)\right] + 9 a b^2 B \sin\left[\frac{1}{2}(c+dx)\right]}{3 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)} + \\
& \frac{6 a^2 A b \sin\left[\frac{1}{2}(c+dx)\right] + 3 A b^3 \sin\left[\frac{1}{2}(c+dx)\right] + 2 a^3 B \sin\left[\frac{1}{2}(c+dx)\right] + 9 a b^2 B \sin\left[\frac{1}{2}(c+dx)\right]}{3 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}
\end{aligned}$$

■ **Problem 246: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + dx])^4 (A + B \cos[c + dx]) \sec[c + dx]^4 dx$$

Optimal (type 3, 198 leaves, 6 steps):

$$\begin{aligned}
& b^3 (A b + 4 a B) x + \frac{a (4 a^2 A b + 8 A b^3 + a^3 B + 12 a b^2 B) \operatorname{ArcTanh}[\sin[c + dx]]}{2 d} - \\
& \frac{b^2 (8 a A b + 3 a^2 B - 6 b^2 B) \sin[c + dx]}{6 d} + \frac{a^2 (2 a^2 A + 9 A b^2 + 9 a b B) \tan[c + dx]}{3 d} + \\
& \frac{a (2 A b + a B) (a + b \cos[c + dx])^2 \sec[c + dx] \tan[c + dx]}{2 d} + \frac{a A (a + b \cos[c + dx])^3 \sec[c + dx]^2 \tan[c + dx]}{3 d}
\end{aligned}$$

Result (type 3, 831 leaves):

$$\begin{aligned}
& \frac{b^3 (A b + 4 a B) (c + d x) \operatorname{Cos}[c + d x]^4 (b + a \operatorname{Sec}[c + d x])^4}{d (a + b \operatorname{Cos}[c + d x])^4} + \frac{1}{2 d (a + b \operatorname{Cos}[c + d x])^4} \\
& (-4 a^3 A b - 8 a A b^3 - a^4 B - 12 a^2 b^2 B) \operatorname{Cos}[c + d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (b + a \operatorname{Sec}[c + d x])^4 + \\
& \frac{1}{2 d (a + b \operatorname{Cos}[c + d x])^4} (4 a^3 A b + 8 a A b^3 + a^4 B + 12 a^2 b^2 B) \operatorname{Cos}[c + d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (b + a \operatorname{Sec}[c + d x])^4 + \\
& \frac{(a^4 A + 12 a^3 A b + 3 a^4 B) \operatorname{Cos}[c + d x]^4 (b + a \operatorname{Sec}[c + d x])^4}{12 d (a + b \operatorname{Cos}[c + d x])^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{a^4 A \operatorname{Cos}[c + d x]^4 (b + a \operatorname{Sec}[c + d x])^4 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{6 d (a + b \operatorname{Cos}[c + d x])^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^3} + \\
& \frac{a^4 A \operatorname{Cos}[c + d x]^4 (b + a \operatorname{Sec}[c + d x])^4 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{6 d (a + b \operatorname{Cos}[c + d x])^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^3} + \frac{(-a^4 A - 12 a^3 A b - 3 a^4 B) \operatorname{Cos}[c + d x]^4 (b + a \operatorname{Sec}[c + d x])^4}{12 d (a + b \operatorname{Cos}[c + d x])^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \\
& \left(2 \operatorname{Cos}[c + d x]^4 (b + a \operatorname{Sec}[c + d x])^4 \left(a^4 A \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 9 a^2 A b^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 6 a^3 b B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)\right) / \\
& \left(3 d (a + b \operatorname{Cos}[c + d x])^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)\right) + \\
& \left(2 \operatorname{Cos}[c + d x]^4 (b + a \operatorname{Sec}[c + d x])^4 \left(a^4 A \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 9 a^2 A b^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 6 a^3 b B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)\right) / \\
& \left(3 d (a + b \operatorname{Cos}[c + d x])^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)\right) + \frac{b^4 B \operatorname{Cos}[c + d x]^4 (b + a \operatorname{Sec}[c + d x])^4 \operatorname{Sin}[c + d x]}{d (a + b \operatorname{Cos}[c + d x])^4}
\end{aligned}$$

■ **Problem 256: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x]^3}{a + b \operatorname{Cos}[c + d x]} dx$$

Optimal (type 3, 143 leaves, 6 steps):

$$-\frac{2 b^2 (A b - a B) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a+b}}\right]}{a^3 \sqrt{a-b} \sqrt{a+b} d} + \frac{(a^2 A + 2 A b^2 - 2 a b B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 a^3 d} - \frac{(A b - a B) \operatorname{Tan}[c + d x]}{a^2 d} + \frac{A \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 a d}$$

Result (type 3, 300 leaves):

$$\frac{1}{4 a^3 d} \left( \frac{8 b^2 (A b - a B) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}} - 2 (a^2 A + 2 A b^2 - 2 a b B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) +$$

$$2 (a^2 A + 2 A b^2 - 2 a b B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \frac{a^2 A}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} +$$

$$\left. \frac{4 a (-A b + a B) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]} - \frac{a^2 A}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{4 a (-A b + a B) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]} \right)$$

■ **Problem 257: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \operatorname{Cos}[c + dx]) \operatorname{Sec}[c + dx]^4}{a + b \operatorname{Cos}[c + dx]} dx$$

Optimal (type 3, 187 leaves, 7 steps):

$$\frac{2 b^3 (A b - a B) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^4 \sqrt{a-b} \sqrt{a+b} d} - \frac{(a^2 + 2 b^2) (A b - a B) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2 a^4 d} +$$

$$\frac{(2 a^2 A + 3 A b^2 - 3 a b B) \operatorname{Tan}[c + dx]}{3 a^3 d} - \frac{(A b - a B) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2 a^2 d} + \frac{A \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{3 a d}$$

Result (type 3, 422 leaves):

$$\frac{1}{12 a^4 d} \left( \frac{24 b^3 (-A b + a B) \operatorname{ArcTan}\left[\frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}} - 6 (a^2 + 2 b^2) (-A b + a B) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + \right.$$

$$6 (a^2 + 2 b^2) (-A b + a B) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] + \frac{a^2 (-3 A b + a (A + 3 B))}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} +$$

$$\frac{2 a^3 A \sin\left[\frac{1}{2}(c+dx)\right]}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{4 a (2 a^2 A + 3 A b^2 - 3 a b B) \sin\left[\frac{1}{2}(c+dx)\right]}{\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]} +$$

$$\left. \frac{2 a^3 A \sin\left[\frac{1}{2}(c+dx)\right]}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{a^2 (-3 A b + a (A + 3 B))}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{4 a (2 a^2 A + 3 A b^2 - 3 a b B) \sin\left[\frac{1}{2}(c+dx)\right]}{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]} \right)$$

■ **Problem 273: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^4 (A + B \cos[c+dx])}{(a + b \cos[c+dx])^4} dx$$

Optimal (type 3, 409 leaves, 7 steps):

$$\frac{(A b - 4 a B) x}{b^5} - \frac{1}{(a - b)^{7/2} b^5 (a + b)^{7/2} d}$$

$$a \left( 2 a^6 A b - 7 a^4 A b^3 + 8 a^2 A b^5 - 8 A b^7 - 8 a^7 B + 28 a^5 b^2 B - 35 a^3 b^4 B + 20 a b^6 B \right) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right] -$$

$$\frac{(3 a^3 A b - 8 a A b^3 - 12 a^4 B + 23 a^2 b^2 B - 6 b^4 B) \sin[c+dx]}{6 b^4 (a^2 - b^2)^2 d} + \frac{a (A b - a B) \cos[c+dx]^3 \sin[c+dx]}{3 b (a^2 - b^2) d (a + b \cos[c+dx])^3} +$$

$$\frac{a (a^2 A b - 6 A b^3 - 4 a^3 B + 9 a b^2 B) \cos[c+dx]^2 \sin[c+dx]}{6 b^2 (a^2 - b^2)^2 d (a + b \cos[c+dx])^2} - \frac{a^2 (a^4 A b - 2 a^2 A b^3 + 6 A b^5 - 4 a^5 B + 11 a^3 b^2 B - 12 a b^4 B) \sin[c+dx]}{2 b^4 (a^2 - b^2)^3 d (a + b \cos[c+dx])}$$

Result (type 3, 1278 leaves):

$$\begin{aligned}
& - \frac{1}{b^5 (a^2 - b^2)^3 \sqrt{-a^2 + b^2} d} a (-2 a^6 A b + 7 a^4 A b^3 - 8 a^2 A b^5 + 8 A b^7 + 8 a^7 B - 28 a^5 b^2 B + 35 a^3 b^4 B - 20 a b^6 B) \operatorname{ArcTanh} \left[ \frac{(a - b) \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right]}{\sqrt{-a^2 + b^2}} \right] + \\
& \frac{1}{24 b^5 (-a^2 + b^2)^3 d (a + b \operatorname{Cos}[c + d x])^3} \\
& \left( -24 a^9 A b (c + d x) + 36 a^7 A b^3 (c + d x) + 36 a^5 A b^5 (c + d x) - 84 a^3 A b^7 (c + d x) + 36 a A b^9 (c + d x) + 96 a^{10} B (c + d x) - 144 a^8 b^2 B (c + d x) - \right. \\
& 144 a^6 b^4 B (c + d x) + 336 a^4 b^6 B (c + d x) - 144 a^2 b^8 B (c + d x) - 72 a^8 A b^2 (c + d x) \operatorname{Cos}[c + d x] + 198 a^6 A b^4 (c + d x) \operatorname{Cos}[c + d x] - \\
& 162 a^4 A b^6 (c + d x) \operatorname{Cos}[c + d x] + 18 a^2 A b^8 (c + d x) \operatorname{Cos}[c + d x] + 18 A b^{10} (c + d x) \operatorname{Cos}[c + d x] + 288 a^9 b B (c + d x) \operatorname{Cos}[c + d x] - \\
& 792 a^7 b^3 B (c + d x) \operatorname{Cos}[c + d x] + 648 a^5 b^5 B (c + d x) \operatorname{Cos}[c + d x] - 72 a^3 b^7 B (c + d x) \operatorname{Cos}[c + d x] - 72 a b^9 B (c + d x) \operatorname{Cos}[c + d x] - \\
& 36 a^7 A b^3 (c + d x) \operatorname{Cos}[2 (c + d x)] + 108 a^5 A b^5 (c + d x) \operatorname{Cos}[2 (c + d x)] - 108 a^3 A b^7 (c + d x) \operatorname{Cos}[2 (c + d x)] + \\
& 36 a A b^9 (c + d x) \operatorname{Cos}[2 (c + d x)] + 144 a^8 b^2 B (c + d x) \operatorname{Cos}[2 (c + d x)] - 432 a^6 b^4 B (c + d x) \operatorname{Cos}[2 (c + d x)] + \\
& 432 a^4 b^6 B (c + d x) \operatorname{Cos}[2 (c + d x)] - 144 a^2 b^8 B (c + d x) \operatorname{Cos}[2 (c + d x)] - 6 a^6 A b^4 (c + d x) \operatorname{Cos}[3 (c + d x)] + \\
& 18 a^4 A b^6 (c + d x) \operatorname{Cos}[3 (c + d x)] - 18 a^2 A b^8 (c + d x) \operatorname{Cos}[3 (c + d x)] + 6 A b^{10} (c + d x) \operatorname{Cos}[3 (c + d x)] + 24 a^7 b^3 B (c + d x) \operatorname{Cos}[3 (c + d x)] - \\
& 72 a^5 b^5 B (c + d x) \operatorname{Cos}[3 (c + d x)] + 72 a^3 b^7 B (c + d x) \operatorname{Cos}[3 (c + d x)] - 24 a b^9 B (c + d x) \operatorname{Cos}[3 (c + d x)] + 24 a^8 A b^2 \operatorname{Sin}[c + d x] - \\
& 57 a^6 A b^4 \operatorname{Sin}[c + d x] + 72 a^4 A b^6 \operatorname{Sin}[c + d x] + 36 a^2 A b^8 \operatorname{Sin}[c + d x] - 96 a^9 b B \operatorname{Sin}[c + d x] + 228 a^7 b^3 B \operatorname{Sin}[c + d x] - \\
& 135 a^5 b^5 B \operatorname{Sin}[c + d x] - 90 a^3 b^7 B \operatorname{Sin}[c + d x] + 18 a b^9 B \operatorname{Sin}[c + d x] + 30 a^7 A b^3 \operatorname{Sin}[2 (c + d x)] - 90 a^5 A b^5 \operatorname{Sin}[2 (c + d x)] + \\
& 120 a^3 A b^7 \operatorname{Sin}[2 (c + d x)] - 120 a^8 b^2 B \operatorname{Sin}[2 (c + d x)] + 336 a^6 b^4 B \operatorname{Sin}[2 (c + d x)] - 300 a^4 b^6 B \operatorname{Sin}[2 (c + d x)] + \\
& 18 a^2 b^8 B \operatorname{Sin}[2 (c + d x)] + 6 b^{10} B \operatorname{Sin}[2 (c + d x)] + 11 a^6 A b^4 \operatorname{Sin}[3 (c + d x)] - 32 a^4 A b^6 \operatorname{Sin}[3 (c + d x)] + \\
& 36 a^2 A b^8 \operatorname{Sin}[3 (c + d x)] - 44 a^7 b^3 B \operatorname{Sin}[3 (c + d x)] + 125 a^5 b^5 B \operatorname{Sin}[3 (c + d x)] - 114 a^3 b^7 B \operatorname{Sin}[3 (c + d x)] + \\
& 18 a b^9 B \operatorname{Sin}[3 (c + d x)] - 3 a^6 b^4 B \operatorname{Sin}[4 (c + d x)] + 9 a^4 b^6 B \operatorname{Sin}[4 (c + d x)] - 9 a^2 b^8 B \operatorname{Sin}[4 (c + d x)] + 3 b^{10} B \operatorname{Sin}[4 (c + d x)] \left. \right)
\end{aligned}$$

■ **Problem 274: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^3 (A + B \operatorname{Cos}[c + d x])}{(a + b \operatorname{Cos}[c + d x])^4} dx$$

Optimal (type 3, 301 leaves, 6 steps):

$$\begin{aligned}
& \frac{B x}{b^4} - \frac{(3 a^2 A b^5 + 2 A b^7 + 2 a^7 B - 7 a^5 b^2 B + 8 a^3 b^4 B - 8 a b^6 B) \operatorname{ArcTan} \left[ \frac{\sqrt{a-b} \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right]}{\sqrt{a+b}} \right]}{(a - b)^{7/2} b^4 (a + b)^{7/2} d} + \frac{a (A b - a B) \operatorname{Cos}[c + d x]^2 \operatorname{Sin}[c + d x]}{3 b (a^2 - b^2) d (a + b \operatorname{Cos}[c + d x])^3} + \\
& \frac{a^2 (5 A b^3 + 3 a^3 B - 8 a b^2 B) \operatorname{Sin}[c + d x]}{6 b^3 (a^2 - b^2)^2 d (a + b \operatorname{Cos}[c + d x])^2} - \frac{a (a^2 A b^3 - 16 A b^5 + 9 a^5 B - 28 a^3 b^2 B + 34 a b^4 B) \operatorname{Sin}[c + d x]}{6 b^3 (a^2 - b^2)^3 d (a + b \operatorname{Cos}[c + d x])}
\end{aligned}$$

Result (type 3, 717 leaves):

$$\frac{1}{24 b^4 d} \left( \frac{24 (3 a^2 A b^5 + 2 A b^7 + 2 a^7 B - 7 a^5 b^2 B + 8 a^3 b^4 B - 8 a b^6 B) \operatorname{ArcTanh} \left[ \frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]}{\sqrt{-a^2+b^2}} \right]}{(-a^2+b^2)^{7/2}} + \right.$$

$$\left. \frac{1}{(a^2-b^2)^3 (a+b \operatorname{Cos}[c+dx])^3} (24 a^9 B c - 36 a^7 b^2 B c - 36 a^5 b^4 B c + 84 a^3 b^6 B c - 36 a b^8 B c + 24 a^9 B dx - 36 a^7 b^2 B dx - 36 a^5 b^4 B dx + \right.$$

$$84 a^3 b^6 B dx - 36 a b^8 B dx + 18 b (a^2 - b^2)^3 (4 a^2 + b^2) B (c+dx) \operatorname{Cos}[c+dx] + 36 a b^2 (a^2 - b^2)^3 B (c+dx) \operatorname{Cos}[2(c+dx)] +$$

$$6 a^6 b^3 B c \operatorname{Cos}[3(c+dx)] - 18 a^4 b^5 B c \operatorname{Cos}[3(c+dx)] + 18 a^2 b^7 B c \operatorname{Cos}[3(c+dx)] - 6 b^9 B c \operatorname{Cos}[3(c+dx)] +$$

$$6 a^6 b^3 B dx \operatorname{Cos}[3(c+dx)] - 18 a^4 b^5 B dx \operatorname{Cos}[3(c+dx)] + 18 a^2 b^7 B dx \operatorname{Cos}[3(c+dx)] - 6 b^9 B dx \operatorname{Cos}[3(c+dx)] +$$

$$18 a^5 A b^4 \operatorname{Sin}[c+dx] + 39 a^3 A b^6 \operatorname{Sin}[c+dx] + 18 a A b^8 \operatorname{Sin}[c+dx] - 24 a^8 b B \operatorname{Sin}[c+dx] + 57 a^6 b^3 B \operatorname{Sin}[c+dx] -$$

$$72 a^4 b^5 B \operatorname{Sin}[c+dx] - 36 a^2 b^7 B \operatorname{Sin}[c+dx] + 6 a^4 A b^5 \operatorname{Sin}[2(c+dx)] + 54 a^2 A b^7 \operatorname{Sin}[2(c+dx)] - 30 a^7 b^2 B \operatorname{Sin}[2(c+dx)] +$$

$$90 a^5 b^4 B \operatorname{Sin}[2(c+dx)] - 120 a^3 b^6 B \operatorname{Sin}[2(c+dx)] + 2 a^5 A b^4 \operatorname{Sin}[3(c+dx)] - 5 a^3 A b^6 \operatorname{Sin}[3(c+dx)] +$$

$$\left. \left. 18 a A b^8 \operatorname{Sin}[3(c+dx)] - 11 a^6 b^3 B \operatorname{Sin}[3(c+dx)] + 32 a^4 b^5 B \operatorname{Sin}[3(c+dx)] - 36 a^2 b^7 B \operatorname{Sin}[3(c+dx)] \right) \right)$$

■ **Problem 283: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx] (a B + b B \operatorname{Cos}[c+dx])}{a + b \operatorname{Cos}[c+dx]} dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\frac{B \operatorname{Sin}[c+dx]}{d}$$

Result (type 3, 23 leaves):

$$B \left( \frac{\operatorname{Cos}[dx] \operatorname{Sin}[c]}{d} + \frac{\operatorname{Cos}[c] \operatorname{Sin}[dx]}{d} \right)$$

■ **Problem 285: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a B + b B \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a + b \operatorname{Cos}[c+dx]} dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$\frac{B \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{d}$$

Result (type 3, 70 leaves):

$$B \left( -\frac{\operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{\operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} \right)$$

- **Problem 301: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \cos[c + dx]} (A + B \cos[c + dx]) \operatorname{Sec}[c + dx]^2 dx$$

Optimal (type 4, 213 leaves, 9 steps):

$$\begin{aligned} & -\frac{A \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \frac{(aA + 2bB) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{d \sqrt{a + b \cos[c + dx]}} + \\ & \frac{(Ab + 2aB) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{d \sqrt{a + b \cos[c + dx]}} + \frac{A \sqrt{a + b \cos[c + dx]} \operatorname{Tan}[c + dx]}{d} \end{aligned}$$

Result (type 4, 484 leaves):



$$\begin{aligned}
& \frac{1}{4d} \left( \frac{8bB \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b \cos[c+dx]}} + \right. \\
& \frac{2(Ab+4aB) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b \cos[c+dx]}} + \left( 2iAb \sqrt{\frac{b-b \cos[c+dx]}{a+b}} \sqrt{-\frac{b+b \cos[c+dx]}{a-b}} \cos[2(c+dx)] \right. \\
& \left. \left( 2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin[c+dx] \right) / \\
& \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]}^2 \sqrt{-\frac{a^2-b^2-2a(a+b \cos[c+dx])+(a+b \cos[c+dx])^2}{b^2}} \right. \\
& \left. \left. \left. (2a^2-b^2-4a(a+b \cos[c+dx])+2(a+b \cos[c+dx])^2) \right) \right) + \frac{A \sqrt{a+b \cos[c+dx]} \tan[c+dx]}{d}
\end{aligned}$$

- **Problem 302: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+b \cos[c+dx]} (A+B \cos[c+dx]) \sec[c+dx]^3 dx$$

Optimal (type 4, 292 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(A b + 4 a B) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{4 a d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} + \\
& \frac{(3 A b + 4 a B) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right] + (4 a^2 A - A b^2 + 4 a b B) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{4 d \sqrt{a + b \cos [c + d x]}} + \\
& \frac{(A b + 4 a B) \sqrt{a + b \cos [c + d x]} \operatorname{Tan}[c + d x]}{4 a d} + \frac{A \sqrt{a + b \cos [c + d x]} \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 d}
\end{aligned}$$

Result (type 4, 552 leaves):

$$\begin{aligned}
& \frac{1}{16 a d} \left( \frac{8 a A b \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{\sqrt{a + b \cos [c + d x]}} + \frac{2(8 a^2 A - 3 A b^2 + 4 a b B) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{\sqrt{a + b \cos [c + d x]}} - \right. \\
& \left. \left( 2 i (-A b^2 - 4 a b B) \sqrt{\frac{b - b \cos [c + d x]}{a + b}} \sqrt{\frac{b + b \cos [c + d x]}{a - b}} \operatorname{Cos}[2(c + d x)] \right. \right. \\
& \left. \left. \left( 2 a (a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos [c + d x]}\right], \frac{a + b}{a - b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos [c + d x]}\right], \frac{a + b}{a - b}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a + b}{a - b}\right] - b \operatorname{EllipticPi}\left[\frac{a + b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos [c + d x]}\right], \frac{a + b}{a - b}\right]\right) \right) \operatorname{Sin}[c + d x] \right) / \left( a \sqrt{-\frac{1}{a + b}} \sqrt{1 - \cos [c + d x]^2} \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2 - b^2 - 2 a (a + b \cos [c + d x]) + (a + b \cos [c + d x])^2}{b^2}} (2 a^2 - b^2 - 4 a (a + b \cos [c + d x]) + 2 (a + b \cos [c + d x])^2) \right) \right) \right) + \\
& \frac{\sqrt{a + b \cos [c + d x]} \left( \frac{\operatorname{Sec}[c + d x] (A b \operatorname{Sin}[c + d x] + 4 a B \operatorname{Sin}[c + d x])}{4 a} + \frac{1}{2} A \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x] \right)}{d}
\end{aligned}$$

■ **Problem 303: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a + b \cos [c + d x]} (A + B \cos [c + d x]) \operatorname{Sec}[c + d x]^4 dx$$

Optimal (type 4, 378 leaves, 11 steps):

$$\begin{aligned}
 & - \frac{(16 a^2 A - 3 A b^2 + 6 a b B) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{24 a^2 d \sqrt{\frac{a + b \cos[c + d x]}{a + b}}} + \frac{(16 a^2 A - A b^2 + 18 a b B) \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{24 a d \sqrt{a + b \cos[c + d x]}} + \\
 & \frac{(4 a^2 A b + A b^3 + 8 a^3 B - 2 a b^2 B) \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{8 a^2 d \sqrt{a + b \cos[c + d x]}} + \frac{(16 a^2 A - 3 A b^2 + 6 a b B) \sqrt{a + b \cos[c + d x]} \operatorname{Tan}[c + d x]}{24 a^2 d} + \\
 & \frac{(A b + 6 a B) \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{12 a d} + \frac{A \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d}
 \end{aligned}$$

Result (type 4, 635 leaves):

$$\begin{aligned}
& \frac{1}{96 a^2 d} \left( \frac{2 (4 a A b^2 + 24 a^2 b B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2 (8 a^2 A b + 9 A b^3 + 48 a^3 B - 18 a b^2 B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\
& \left( 2 i (-16 a^2 A b + 3 A b^3 - 6 a b^2 B) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{\sec [c+d x]^2 (A b \sin [c+d x]+6 a B \sin [c+d x])}{12 a} + \right. \\
& \left. \frac{\sec [c+d x] (16 a^2 A \sin [c+d x]-3 A b^2 \sin [c+d x]+6 a b B \sin [c+d x])}{24 a^2} + \frac{1}{3} A \sec [c+d x]^2 \tan [c+d x] \right)
\end{aligned}$$

■ **Problem 307: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos [c+d x])^{3/2} (A+B \cos [c+d x]) \sec [c+d x] dx$$

Optimal (type 4, 236 leaves, 9 steps):

$$\frac{2(3Ab + 4aB)\sqrt{a+b\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{3d\sqrt{\frac{a+b\cos[c+dx]}{a+b}}} + \frac{2(3aAb - a^2B + b^2B)\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{3d\sqrt{a+b\cos[c+dx]}}$$

$$\frac{2a^2A\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{d\sqrt{a+b\cos[c+dx]}} + \frac{2bB\sqrt{a+b\cos[c+dx]}\sin[c+dx]}{3d}$$

Result (type 4, 406 leaves):

$$\frac{1}{6d}$$

$$\left( \frac{4(6aAb + 3a^2B + b^2B)\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \frac{2(6a^2A + 3Ab^2 + 4abB)\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} \right) +$$

$$\frac{1}{ab\sqrt{-\frac{1}{a+b}}} 2i(3Ab + 4aB) \sqrt{-\frac{b(-1 + \cos[c+dx])}{a+b}} \sqrt{\frac{b(1 + \cos[c+dx])}{-a+b}} \operatorname{Csc}[c+dx]$$

$$\left( -2a(a-b)\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b\left( -2a\operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b\operatorname{EllipticPi}\left[\frac{a+b}{a}, i\operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] \right) \right) + 4bB\sqrt{a+b\cos[c+dx]}\sin[c+dx]$$

■ **Problem 308: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b\cos[c+dx])^{3/2} (A+B\cos[c+dx]) \operatorname{Sec}[c+dx]^2 dx$$

Optimal (type 4, 232 leaves, 9 steps):

$$\begin{aligned}
& - \frac{(aA - 2bB) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] + (a^2A + 2Ab^2 + 2abB) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \frac{(a^2A + 2Ab^2 + 2abB) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{d \sqrt{a + b \cos[c + dx]}} + \\
& \frac{a(3Ab + 2aB) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{d \sqrt{a + b \cos[c + dx]}} + \frac{aA \sqrt{a + b \cos[c + dx]} \operatorname{Tan}[c + dx]}{d}
\end{aligned}$$

Result (type 4, 398 leaves):

$$\begin{aligned}
& \frac{1}{4d} \left( \frac{8b(Ab + 2aB) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} + \right. \\
& \frac{2(5aAb + 4a^2B + 2b^2B) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} + \frac{1}{ab \sqrt{-\frac{1}{a+b}}} 2i(-aA + 2bB) \sqrt{-\frac{b(-1 + \cos[c + dx])}{a+b}} \\
& \sqrt{\frac{b(1 + \cos[c + dx])}{-a+b}} \operatorname{Csc}[c + dx] \left( -2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right] + \right. \\
& b \left( -2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right] + \right. \\
& \left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right]\right) \right) + 4aA \sqrt{a + b \cos[c + dx]} \operatorname{Tan}[c + dx] \left. \right)
\end{aligned}$$

■ **Problem 309: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + b \cos[c + dx])^{3/2} (A + B \cos[c + dx]) \operatorname{Sec}[c + dx]^3 dx$$

Optimal (type 4, 295 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(5Ab + 4aB) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \\
& \frac{(7aAb + 4a^2B + 8b^2B) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] + (4a^2A + 3Ab^2 + 12abB) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4d \sqrt{a + b \cos[c + dx]}} + \\
& \frac{(5Ab + 4aB) \sqrt{a + b \cos[c + dx]} \tan[c + dx]}{4d} + \frac{aA \sqrt{a + b \cos[c + dx]} \operatorname{Sec}[c + dx] \tan[c + dx]}{2d}
\end{aligned}$$

Result (type 4, 556 leaves):

$$\begin{aligned}
& \frac{1}{16d} \left( \frac{2(4aAb + 16b^2B) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} + \frac{2(8a^2A + Ab^2 + 20abB) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} \right) - \\
& \left( 2i(-5Ab^2 - 4abB) \sqrt{\frac{b - b \cos[c + dx]}{a + b}} \sqrt{\frac{b + b \cos[c + dx]}{a - b}} \cos[2(c + dx)] \right. \\
& \left. \left( 2a(a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin[c + dx] \Big/ \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1 - \cos[c + dx]^2} \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2 - b^2 - 2a(a + b \cos[c + dx]) + (a + b \cos[c + dx])^2}{b^2}} (2a^2 - b^2 - 4a(a + b \cos[c + dx]) + 2(a + b \cos[c + dx])^2) \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a + b \cos[c + dx]} \left( \frac{1}{4} \operatorname{Sec}[c + dx] (5Ab \sin[c + dx] + 4aB \sin[c + dx]) + \frac{1}{2} aA \operatorname{Sec}[c + dx] \tan[c + dx] \right)
\end{aligned}$$

■ **Problem 310: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + b \cos[c + dx])^{3/2} (A + B \cos[c + dx]) \operatorname{Sec}[c + dx]^4 dx$$

Optimal (type 4, 375 leaves, 11 steps):

$$\begin{aligned}
 & - \frac{(16 a^2 A + 3 A b^2 + 30 a b B) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{24 a d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}} + \\
 & \frac{(16 a^2 A + 17 A b^2 + 42 a b B) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{24 d \sqrt{a + b \cos[c + d x]}} + \\
 & \frac{(12 a^2 A b - A b^3 + 8 a^3 B + 6 a b^2 B) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{8 a d \sqrt{a + b \cos[c + d x]}} + \frac{(16 a^2 A + 3 A b^2 + 30 a b B) \sqrt{a + b \cos[c + d x]} \operatorname{Tan}[c + d x]}{24 a d} + \\
 & \frac{(7 A b + 6 a B) \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{12 d} + \frac{a A \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d}
 \end{aligned}$$

Result (type 4, 634 leaves):



$$\begin{aligned}
& \frac{1}{96 a d} \left( \frac{2 (28 a A b^2 + 24 a^2 b B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2 (56 a^2 A b - 9 A b^3 + 48 a^3 B + 6 a b^2 B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\
& \left( 2 i (-16 a^2 A b - 3 A b^3 - 30 a b^2 B) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{1}{12} \sec [c+d x]^2 (7 A b \sin [c+d x]+6 a B \sin [c+d x]) + \right. \\
& \left. \frac{\sec [c+d x] (16 a^2 A \sin [c+d x]+3 A b^2 \sin [c+d x]+30 a b B \sin [c+d x])}{24 a} + \frac{1}{3} a A \sec [c+d x]^2 \tan [c+d x] \right)
\end{aligned}$$

■ **Problem 314: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos [c+d x])^{5/2} (A+B \cos [c+d x]) \sec [c+d x] dx$$

Optimal (type 4, 292 leaves, 10 steps):

$$\begin{aligned}
& \frac{2 (35 a A b + 23 a^2 B + 9 b^2 B) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{15 d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}} + \\
& \frac{2 (10 a^2 A b + 5 A b^3 - 8 a^3 B + 8 a b^2 B) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{15 d \sqrt{a + b \cos[c + d x]}} + \frac{2 a^3 A \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{2 b (5 A b + 8 a B) \sqrt{a + b \cos[c + d x]} \sin[c + d x]}{15 d} + \frac{2 b B (a + b \cos[c + d x])^{3/2} \sin[c + d x]}{5 d}
\end{aligned}$$

Result (type 4, 453 leaves):

$$\begin{aligned}
& \frac{1}{30 d} \left( \frac{4 (45 a^2 A b + 5 A b^3 + 15 a^3 B + 17 a b^2 B) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + d x]}} + \right. \\
& \frac{2 (30 a^3 A + 35 a A b^2 + 23 a^2 b B + 9 b^3 B) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + d x]}} + \\
& \frac{1}{a b \sqrt{-\frac{1}{a+b}}} 2 i (35 a A b + 23 a^2 B + 9 b^2 B) \sqrt{-\frac{b(-1 + \cos[c + d x])}{a+b}} \sqrt{-\frac{b(1 + \cos[c + d x])}{a-b}} \\
& \left. \operatorname{Csc}[c + d x] \left( -2 a (a - b) \operatorname{EllipticE}\left[\operatorname{i} \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + d x]}\right], \frac{a+b}{a-b}\right] + b \left( -2 a \operatorname{EllipticF}\left[\right. \right. \right. \right. \\
& \left. \left. \left. \operatorname{i} \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + d x]}\right], \frac{a+b}{a-b}\right] + b \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{i} \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + d x]}\right], \frac{a+b}{a-b}\right]\right)\right) \right) + \\
& \left. 4 b \sqrt{a + b \cos[c + d x]} (5 A b + 11 a B + 3 b B \cos[c + d x]) \sin[c + d x] \right)
\end{aligned}$$

■ **Problem 315: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + b \cos [c + d x])^{5/2} (A + B \cos [c + d x]) \sec [c + d x]^2 dx$$

Optimal (type 4, 296 leaves, 10 steps):

$$\begin{aligned} & - \frac{(3 a^2 A - 6 A b^2 - 14 a b B) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{3 d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} + \\ & \frac{(3 a^3 A + 12 a A b^2 + 4 a^2 b B + 2 b^3 B) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right] - a^2 (5 A b + 2 a B) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{3 d \sqrt{a + b \cos [c + d x]} + d \sqrt{a + b \cos [c + d x]}} - \\ & \frac{b (3 a A - 2 b B) \sqrt{a + b \cos [c + d x]} \sin [c + d x]}{3 d} + \frac{a A (a + b \cos [c + d x])^{3/2} \tan [c + d x]}{d} \end{aligned}$$

Result (type 4, 560 leaves):

$$\begin{aligned}
& \frac{1}{12d} \left( \frac{2(36aAb^2 + 36a^2bB + 4b^3B) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \right. \\
& \frac{2(27a^2Ab + 6Ab^3 + 12a^3B + 14ab^2B) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} - \\
& \left. \left( 2i(-3a^2Ab + 6Ab^3 + 14ab^2B) \sqrt{\frac{b-b\cos[c+dx]}{a+b}} \sqrt{\frac{b+b\cos[c+dx]}{a-b}} \cos[2(c+dx)] \right. \right. \\
& \left. \left( 2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin[c+dx] \Big/ \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]^2} \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2a(a+b\cos[c+dx])+(a+b\cos[c+dx])^2}{b^2}} (2a^2-b^2-4a(a+b\cos[c+dx])+2(a+b\cos[c+dx])^2) \right) \right) \right) + \\
& \frac{\sqrt{a+b\cos[c+dx]} \left( \frac{2}{3}b^2B \sin[c+dx] + a^2A \tan[c+dx] \right)}{d}
\end{aligned}$$

■ **Problem 316: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b\cos[c+dx])^{5/2} (A+B\cos[c+dx]) \sec[c+dx]^3 dx$$

Optimal (type 4, 315 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(9 a A b + 4 a^2 B - 8 b^2 B) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{4 d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}} + \\
& \frac{(11 a^2 A b + 8 A b^3 + 4 a^3 B + 16 a b^2 B) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{4 d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{a (4 a^2 A + 15 A b^2 + 20 a b B) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{4 d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{a (7 A b + 4 a B) \sqrt{a + b \cos[c + d x]} \tan[c + d x]}{4 d} + \frac{a A (a + b \cos[c + d x])^{3/2} \sec[c + d x] \tan[c + d x]}{2 d}
\end{aligned}$$

Result (type 4, 589 leaves):

$$\begin{aligned}
& \frac{1}{16d} \left( \frac{2(4a^2Ab + 16Ab^3 + 48ab^2B) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \right. \\
& \frac{2(8a^3A + 21aAb^2 + 36a^2bB + 8b^3B) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} - \\
& \left( 2i(-9aAb^2 - 4a^2bB + 8b^3B) \sqrt{\frac{b-b\cos[c+dx]}{a+b}} \sqrt{-\frac{b+b\cos[c+dx]}{a-b}} \cos[2(c+dx)] \right. \\
& \left. \left( 2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin[c+dx] \Big/ \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]^2} \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2a(a+b\cos[c+dx])+(a+b\cos[c+dx])^2}{b^2}} (2a^2-b^2-4a(a+b\cos[c+dx])+2(a+b\cos[c+dx])^2) \right) \right) \right) + \\
& \left. \frac{1}{d} \sqrt{a+b\cos[c+dx]} \left( \frac{1}{4} \sec[c+dx] (9aAb \sin[c+dx] + 4a^2B \sin[c+dx]) + \frac{1}{2} a^2A \sec[c+dx] \tan[c+dx] \right) \right)
\end{aligned}$$

■ **Problem 317: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b\cos[c+dx])^{5/2} (A+B\cos[c+dx]) \sec[c+dx]^4 dx$$

Optimal (type 4, 376 leaves, 11 steps):

$$\begin{aligned}
& - \frac{(16 a^2 A + 33 A b^2 + 54 a b B) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{24 d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} + \\
& \frac{(16 a^3 A + 59 a A b^2 + 66 a^2 b B + 48 b^3 B) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{24 d \sqrt{a + b \cos [c + d x]}} + \\
& \frac{(20 a^2 A b + 5 A b^3 + 8 a^3 B + 30 a b^2 B) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{8 d \sqrt{a + b \cos [c + d x]}} + \\
& \frac{(16 a^2 A + 33 A b^2 + 54 a b B) \sqrt{a + b \cos [c + d x]} \tan [c + d x]}{24 d} + \\
& \frac{a(3 A b + 2 a B) \sqrt{a + b \cos [c + d x]} \sec [c + d x] \tan [c + d x]}{4 d} + \frac{a A (a + b \cos [c + d x])^{3/2} \sec [c + d x]^2 \tan [c + d x]}{3 d}
\end{aligned}$$

Result (type 4, 639 leaves):

$$\begin{aligned}
& \frac{1}{96 d} \left( \frac{2 (52 a A b^2 + 24 a^2 b B + 96 b^3 B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2 (104 a^2 A b - 3 A b^3 + 48 a^3 B + 126 a b^2 B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\
& \left. \left( 2 i (-16 a^2 A b - 33 A b^3 - 54 a b^2 B) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \right. \\
& \left. \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{1}{12} \sec [c+d x]^2 (13 a A b \sin [c+d x] + 6 a^2 B \sin [c+d x]) + \right. \\
& \left. \frac{1}{24} \sec [c+d x] (16 a^2 A \sin [c+d x] + 33 A b^2 \sin [c+d x] + 54 a b B \sin [c+d x]) + \frac{1}{3} a^2 A \sec [c+d x]^2 \tan [c+d x] \right)
\end{aligned}$$

■ **Problem 318: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos [c+d x])^{5/2} (A+B \cos [c+d x]) \sec [c+d x]^5 dx$$

Optimal (type 4, 465 leaves, 12 steps):



$$\begin{aligned}
& - \frac{(284 a^2 A b + 15 A b^3 + 128 a^3 B + 264 a b^2 B) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{192 a d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}} + \\
& \frac{(356 a^2 A b + 133 A b^3 + 128 a^3 B + 472 a b^2 B) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{192 d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{(48 a^4 A + 120 a^2 A b^2 - 5 A b^4 + 160 a^3 b B + 40 a b^3 B) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{64 a d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{(284 a^2 A b + 15 A b^3 + 128 a^3 B + 264 a b^2 B) \sqrt{a + b \cos[c + d x]} \tan[c + d x]}{192 a d} + \\
& \frac{(36 a^2 A + 59 A b^2 + 104 a b B) \sqrt{a + b \cos[c + d x]} \sec[c + d x] \tan[c + d x]}{96 d} + \\
& \frac{a (11 A b + 8 a B) \sqrt{a + b \cos[c + d x]} \sec[c + d x]^2 \tan[c + d x]}{24 d} + \frac{a A (a + b \cos[c + d x])^{3/2} \sec[c + d x]^3 \tan[c + d x]}{4 d}
\end{aligned}$$

Result (type 4, 729 leaves):

$$\begin{aligned}
& \frac{1}{768 a d} \left( \frac{2 (144 a^3 A b + 236 a A b^3 + 416 a^2 b^2 B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{1}{\sqrt{a+b \cos [c+d x]}} \right. \\
& 2 (288 a^4 A + 436 a^2 A b^2 - 45 A b^4 + 832 a^3 b B - 24 a b^3 B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] - \\
& \left. \left( 2 i (-284 a^2 A b^2 - 15 A b^4 - 128 a^3 b B - 264 a b^3 B) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \right. \\
& \left. \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{1}{24} \sec [c+d x]^3 (17 a A b \sin [c+d x] + 8 a^2 B \sin [c+d x]) + \right. \\
& \frac{1}{96} \sec [c+d x]^2 (36 a^2 A \sin [c+d x] + 59 A b^2 \sin [c+d x] + 104 a b B \sin [c+d x]) + \\
& 1 / (192 a) \sec [c+d x] (284 a^2 A b \sin [c+d x] + 15 A b^3 \sin [c+d x] + 128 a^3 B \sin [c+d x] + 264 a b^2 B \sin [c+d x]) + \\
& \left. \frac{1}{4} a^2 A \sec [c+d x]^3 \tan [c+d x] \right)
\end{aligned}$$

■ **Problem 324: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B \cos [c+d x]) \sec [c+d x]^2}{\sqrt{a+b \cos [c+d x]}} dx$$

Optimal (type 4, 216 leaves, 9 steps):

$$\begin{aligned}
& - \frac{A \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{a d \sqrt{\frac{a+b \cos [c+d x]}{a+b}}} + \frac{A \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{d \sqrt{a+b \cos [c+d x]}} \\
& \frac{(A b-2 a B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{a d \sqrt{a+b \cos [c+d x]}} + \frac{A \sqrt{a+b \cos [c+d x]} \operatorname{Tan}[c+d x]}{a d}
\end{aligned}$$

Result (type 4, 320 leaves):

$$\begin{aligned}
& \frac{1}{4 a d} \left( \frac{2(-3 A b+4 a B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \right. \\
& \left. 1 / \left( a b \sqrt{-\frac{1}{a+b}} \right) 2 i A \sqrt{-\frac{b(-1+\cos [c+d x])}{a+b}} \sqrt{\frac{b(1+\cos [c+d x])}{-a+b}} \operatorname{Csc}[c+d x] \right. \\
& \left. \left( -2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( -2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] + b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] \right) \right) + 4 A \sqrt{a+b \cos [c+d x]} \operatorname{Tan}[c+d x] \right)
\end{aligned}$$

■ **Problem 325: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B \cos [c+d x]) \operatorname{Sec}[c+d x]^3}{\sqrt{a+b \cos [c+d x]}} dx$$

Optimal (type 4, 299 leaves, 10 steps):

$$\frac{(3Ab - 4aB) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4a^2 d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} -$$

$$\frac{(Ab - 4aB) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] + (4a^2 A + 3Ab^2 - 4abB) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4ad \sqrt{a + b \cos[c + dx]} + 4a^2 d \sqrt{a + b \cos[c + dx]}} -$$

$$\frac{(3Ab - 4aB) \sqrt{a + b \cos[c + dx]} \tan[c + dx]}{4a^2 d} + \frac{A \sqrt{a + b \cos[c + dx]} \sec[c + dx] \tan[c + dx]}{2ad}$$

Result (type 4, 556 leaves):

$$\frac{1}{16a^2 d} \left( \frac{8aAb \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} + \frac{2(8a^2 A + 9Ab^2 - 12abB) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} - \right.$$

$$\left. \left( 2i(3Ab^2 - 4abB) \sqrt{\frac{b - b \cos[c + dx]}{a + b}} \sqrt{\frac{b + b \cos[c + dx]}{a - b}} \cos[2(c + dx)] \right) \right.$$

$$\left. \left( 2a(a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right], \right. \right. \right.$$

$$\left. \left. \frac{a+b}{a-b} \right) - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right] \right) \sin[c + dx] \Big/ \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1 - \cos[c + dx]^2} \right.$$

$$\left. \left. \sqrt{-\frac{a^2 - b^2 - 2a(a + b \cos[c + dx]) + (a + b \cos[c + dx])^2}{b^2}} (2a^2 - b^2 - 4a(a + b \cos[c + dx]) + 2(a + b \cos[c + dx])^2) \right) \right) +$$

$$\frac{\sqrt{a + b \cos[c + dx]} \left( \frac{\sec[c + dx] (-3Ab \sin[c + dx] + 4aB \sin[c + dx])}{4a^2} + \frac{A \sec[c + dx] \tan[c + dx]}{2a} \right)}{d}$$

■ **Problem 330: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]}{(a + b \cos[c + dx])^{3/2}} dx$$

Optimal (type 4, 190 leaves, 7 steps) :

$$\begin{aligned}
 & - \frac{2 (A b - a B) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{a (a^2 - b^2) d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} + \\
 & \frac{2 A \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{a d \sqrt{a + b \cos [c + d x]}} + \frac{2 b (A b - a B) \sin [c + d x]}{a (a^2 - b^2) d \sqrt{a + b \cos [c + d x]}}
 \end{aligned}$$

Result (type 4, 614 leaves) :

$$\begin{aligned}
 & - \frac{2 \cos [c + d x] (B + A \sec [c + d x]) (-A b^2 \sin [c + d x] + a b B \sin [c + d x])}{a (a^2 - b^2) d \sqrt{a + b \cos [c + d x]} (A + B \cos [c + d x])} - \\
 & \frac{1}{2 a (-a + b) (a + b) d (A + B \cos [c + d x])} \cos [c + d x] (B + A \sec [c + d x]) \\
 & \left( \frac{2 (-2 a A b + 2 a^2 B) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{\sqrt{a + b \cos [c + d x]}} + \frac{2 (2 a^2 A - 3 A b^2 + a b B) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{\sqrt{a + b \cos [c + d x]}} - \right. \\
 & \left. \left( 2 i (-A b^2 + a b B) \sqrt{\frac{b - b \cos [c + d x]}{a + b}} \sqrt{-\frac{b + b \cos [c + d x]}{a - b}} \cos [2 (c + d x)] \right. \right. \\
 & \left. \left( 2 a (a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos [c + d x]}\right], \frac{a + b}{a - b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos [c + d x]}\right], \right. \right. \right. \right. \\
 & \left. \left. \left. \frac{a + b}{a - b}\right] - b \operatorname{EllipticPi}\left[\frac{a + b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos [c + d x]}\right], \frac{a + b}{a - b}\right]\right) \right) \sin [c + d x] \left. \right) / \left( a \sqrt{-\frac{1}{a + b}} \right. \\
 & \left. \left. \left. \sqrt{1 - \cos [c + d x]^2} \sqrt{-\frac{a^2 - b^2 - 2 a (a + b \cos [c + d x]) + (a + b \cos [c + d x])^2}{b^2}} (2 a^2 - b^2 - 4 a (a + b \cos [c + d x]) + 2 (a + b \cos [c + d x])^2) \right) \right) \right)
 \end{aligned}$$

- **Problem 331: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]^2}{(a + b \cos[c + dx])^{3/2}} dx$$

Optimal (type 4, 303 leaves, 10 steps):

$$\begin{aligned} & - \frac{(a^2 A - 3 A b^2 + 2 a b B) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] + A \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{a^2 (a^2 - b^2) d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \frac{A d \sqrt{a + b \cos[c + dx]}}{a d \sqrt{a + b \cos[c + dx]}} - \\ & \frac{(3 A b - 2 a B) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{a^2 d \sqrt{a + b \cos[c + dx]}} + \frac{b (a^2 A - 3 A b^2 + 2 a b B) \sin[c + dx]}{a^2 (a^2 - b^2) d \sqrt{a + b \cos[c + dx]}} + \frac{A \tan[c + dx]}{a d \sqrt{a + b \cos[c + dx]}} \end{aligned}$$

Result (type 4, 608 leaves):

$$\begin{aligned}
& \frac{1}{4 a^2 (a-b)(a+b) d} \left( \frac{2 (4 a A b^2 - 4 a^2 b B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2 (-7 a^2 A b + 9 A b^3 + 4 a^3 B - 6 a b^2 B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\
& \left( 2 i (-a^2 A b + 3 A b^3 - 2 a b^2 B) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \left. \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) + \\
& \frac{\sqrt{a+b \cos [c+d x]} \left( \frac{2(-A b^3 \sin [c+d x]+a b^2 B \sin [c+d x])}{a^2(a^2-b^2)(a+b \cos [c+d x])} + \frac{A \tan [c+d x]}{a^2} \right)}{d}
\end{aligned}$$

■ **Problem 332: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B \cos [c+d x]) \sec [c+d x]^3}{(a+b \cos [c+d x])^{3/2}} dx$$

Optimal (type 4, 398 leaves, 11 steps):

$$\begin{aligned}
& \frac{(7 a^2 A b - 15 A b^3 - 4 a^3 B + 12 a b^2 B) \sqrt{a + b \operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{4 a^3 (a^2 - b^2) d \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{a+b}}} - \\
& \frac{(5 A b - 4 a B) \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{4 a^2 d \sqrt{a + b \operatorname{Cos}[c + d x]}} + \frac{(4 a^2 A + 15 A b^2 - 12 a b B) \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{4 a^3 d \sqrt{a + b \operatorname{Cos}[c + d x]}} - \\
& \frac{b (7 a^2 A b - 15 A b^3 - 4 a^3 B + 12 a b^2 B) \operatorname{Sin}[c + d x]}{4 a^3 (a^2 - b^2) d \sqrt{a + b \operatorname{Cos}[c + d x]}} - \frac{(5 A b - 4 a B) \operatorname{Tan}[c + d x]}{4 a^2 d \sqrt{a + b \operatorname{Cos}[c + d x]}} + \frac{A \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 a d \sqrt{a + b \operatorname{Cos}[c + d x]}}
\end{aligned}$$

Result (type 4, 678 leaves):



$$\begin{aligned}
& - \frac{1}{16 a^3 (-a+b)(a+b) d} \left( \frac{2 (4 a^3 A b - 20 a A b^3 + 16 a^2 b^2 B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2 (8 a^4 A + 29 a^2 A b^2 - 45 A b^4 - 28 a^3 b B + 36 a b^3 B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\
& \left( 2 i (7 a^2 A b^2 - 15 A b^4 - 4 a^3 b B + 12 a b^3 B) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \\
& \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} \right. \\
& \left. \left. \left( 2 a^2 - b^2 - 4 a(a+b \cos [c+d x]) + 2(a+b \cos [c+d x])^2 \right) \right) \right) + \frac{1}{d} \\
& \sqrt{a+b \cos [c+d x]} \left( \frac{\sec [c+d x] (-7 A b \sin [c+d x] + 4 a B \sin [c+d x])}{4 a^3} - \frac{2 (-A b^4 \sin [c+d x] + a b^3 B \sin [c+d x])}{a^3 (a^2 - b^2) (a+b \cos [c+d x])} + \right. \\
& \left. \frac{A \sec [c+d x] \tan [c+d x]}{2 a^2} \right)
\end{aligned}$$

- **Problem 338: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A+B \cos [c+d x]) \sec [c+d x]}{(a+b \cos [c+d x])^{5/2}} dx$$

Optimal (type 4, 349 leaves, 10 steps):

$$\begin{aligned}
& - \frac{2 (7 a^2 A b - 3 A b^3 - 4 a^3 B) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{3 a^2 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} + \frac{2 (A b - a B) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{3 a (a^2 - b^2) d \sqrt{a + b \cos [c + d x]}} + \\
& \frac{2 A \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{a^2 d \sqrt{a + b \cos [c + d x]}} + \frac{2 b (A b - a B) \sin [c + d x]}{3 a (a^2 - b^2) d (a + b \cos [c + d x])^{3/2}} + \frac{2 b (7 a^2 A b - 3 A b^3 - 4 a^3 B) \sin [c + d x]}{3 a^2 (a^2 - b^2)^2 d \sqrt{a + b \cos [c + d x]}}
\end{aligned}$$

Result (type 4, 743 leaves):

$$\begin{aligned}
& \frac{1}{6 a^2 (a-b)^2 (a+b)^2 d (A+B \cos [c+d x])} \\
& \cos [c+d x] (B+A \sec [c+d x]) \left( \frac{2 \left(-12 a^3 A b+4 a A b^3+6 a^4 B+2 a^2 b^2 B\right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2\left(6 a^4 A-19 a^2 A b^2+9 A b^4+4 a^3 b B\right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\
& \left. \left(2 i\left(-7 a^2 A b^2+3 A b^4+4 a^3 b B\right) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)]\left(2 a(a-b) \right. \right. \right. \\
& \left. \left. \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]+b\left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]-\right. \right. \right. \\
& \left. \left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right)\right) \sin [c+d x]\right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2}\right) \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}}\left(2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2\right)\right)\right)\right) + \\
& \frac{1}{d(A+B \cos [c+d x])} \cos [c+d x] \sqrt{a+b \cos [c+d x]} (B+A \sec [c+d x]) \\
& \left(-\frac{2(-A b^2 \sin [c+d x]+a b B \sin [c+d x])}{3 a\left(a^2-b^2\right)(a+b \cos [c+d x])^2}-\right. \\
& \left.\frac{2\left(-7 a^2 A b^2 \sin [c+d x]+3 A b^4 \sin [c+d x]+4 a^3 b B \sin [c+d x]\right)}{3 a^2\left(a^2-b^2\right)^2(a+b \cos [c+d x])}\right)
\end{aligned}$$

■ **Problem 339: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B \cos [c+d x]) \sec [c+d x]^2}{(a+b \cos [c+d x])^{5/2}} dx$$

Optimal (type 4, 437 leaves, 11 steps):

$$\begin{aligned}
 & - \frac{(3 a^4 A - 26 a^2 A b^2 + 15 A b^4 + 14 a^3 b B - 6 a b^3 B) \sqrt{a + b \operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{3 a^3 (a^2 - b^2)^2 d \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{a+b}}} + \\
 & \frac{(3 a^2 A - 5 A b^2 + 2 a b B) \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right] - (5 A b - 2 a B) \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{3 a^2 (a^2 - b^2) d \sqrt{a + b \operatorname{Cos}[c + d x]} - a^3 d \sqrt{a + b \operatorname{Cos}[c + d x]}} + \\
 & \frac{b (3 a^2 A - 5 A b^2 + 2 a b B) \operatorname{Sin}[c + d x]}{3 a^2 (a^2 - b^2) d (a + b \operatorname{Cos}[c + d x])^{3/2}} + \frac{b (3 a^4 A - 26 a^2 A b^2 + 15 A b^4 + 14 a^3 b B - 6 a b^3 B) \operatorname{Sin}[c + d x]}{3 a^3 (a^2 - b^2)^2 d \sqrt{a + b \operatorname{Cos}[c + d x]}} + \frac{A \operatorname{Tan}[c + d x]}{a d (a + b \operatorname{Cos}[c + d x])^{3/2}}
 \end{aligned}$$

Result (type 4, 750 leaves):

$$\begin{aligned}
& \frac{1}{12 a^3 (-a+b)^2 (a+b)^2 d} \left( \frac{2 (36 a^3 A b^2 - 20 a A b^4 - 24 a^4 b B + 8 a^2 b^3 B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{1}{\sqrt{a+b \cos [c+d x]}} \right. \\
& 2 (-33 a^4 A b + 86 a^2 A b^3 - 45 A b^5 + 12 a^5 B - 38 a^3 b^2 B + 18 a b^4 B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] - \\
& \left( 2 i (-3 a^4 A b + 26 a^2 A b^3 - 15 A b^5 - 14 a^3 b^2 B + 6 a b^4 B) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{2(-A b^3 \sin [c+d x]+a b^2 B \sin [c+d x])}{3 a^2(a^2-b^2)(a+b \cos [c+d x])^2} + \right. \\
& \left. \frac{2(-10 a^2 A b^3 \sin [c+d x]+6 A b^5 \sin [c+d x]+7 a^3 b^2 B \sin [c+d x]-3 a b^4 B \sin [c+d x])}{3 a^3(a^2-b^2)^2(a+b \cos [c+d x])} + \frac{A \tan [c+d x]}{a^3} \right)
\end{aligned}$$

■ **Problem 340: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B \cos [c+d x]) \sec [c+d x]^3}{(a+b \cos [c+d x])^{5/2}} dx$$

Optimal (type 4, 532 leaves, 12 steps):

$$\begin{aligned}
& \left( (33 a^4 A b - 170 a^2 A b^3 + 105 A b^5 - 12 a^5 B + 104 a^3 b^2 B - 60 a b^4 B) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right] \right) / \\
& \left( 12 a^4 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \right) - \frac{(27 a^2 A b - 35 A b^3 - 12 a^3 B + 20 a b^2 B) \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{12 a^3 (a^2 - b^2) d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{(4 a^2 A + 35 A b^2 - 20 a b B) \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{4 a^4 d \sqrt{a + b \cos[c + d x]}} - \frac{b (27 a^2 A b - 35 A b^3 - 12 a^3 B + 20 a b^2 B) \sin[c + d x]}{12 a^3 (a^2 - b^2) d (a + b \cos[c + d x])^{3/2}} - \\
& \frac{b (33 a^4 A b - 170 a^2 A b^3 + 105 A b^5 - 12 a^5 B + 104 a^3 b^2 B - 60 a b^4 B) \sin[c + d x]}{12 a^4 (a^2 - b^2)^2 d \sqrt{a + b \cos[c + d x]}} - \frac{(7 A b - 4 a B) \tan[c + d x]}{4 a^2 d (a + b \cos[c + d x])^{3/2}} + \frac{A \sec[c + d x] \tan[c + d x]}{2 a d (a + b \cos[c + d x])^{3/2}}
\end{aligned}$$

Result (type 4, 820 leaves):

$$\begin{aligned}
& \frac{1}{48 a^4 (a-b)^2 (a+b)^2 d} \left( \frac{1}{\sqrt{a+b \cos [c+d x]}} \right. \\
& 2 \left( 12 a^5 A b - 216 a^3 A b^3 + 140 a A b^5 + 144 a^4 b^2 B - 80 a^2 b^4 B \right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] + \frac{1}{\sqrt{a+b \cos [c+d x]}} \\
& 2 \left( 24 a^6 A + 195 a^4 A b^2 - 566 a^2 A b^4 + 315 A b^6 - 132 a^5 b B + 344 a^3 b^3 B - 180 a b^5 B \right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] - \\
& \left( 2 i \left( 33 a^4 A b^2 - 170 a^2 A b^4 + 105 A b^6 - 12 a^5 b B + 104 a^3 b^3 B - 60 a b^5 B \right) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \right. \\
& \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} \left( 2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2 \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{\operatorname{Sec}[c+d x](-11 A b \sin [c+d x]+4 a B \sin [c+d x])}{4 a^4} - \frac{2(-A b^4 \sin [c+d x]+a b^3 B \sin [c+d x])}{3 a^3(a^2-b^2)(a+b \cos [c+d x])^2} - \right. \\
& \left. \frac{2(-13 a^2 A b^4 \sin [c+d x]+9 A b^6 \sin [c+d x]+10 a^3 b^3 B \sin [c+d x]-6 a b^5 B \sin [c+d x])}{3 a^4(a^2-b^2)^2(a+b \cos [c+d x])} + \frac{A \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{2 a^3} \right)
\end{aligned}$$

■ **Problem 344: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a B + b B \cos [c+d x]) \operatorname{Sec}[c+d x]}{(a+b \cos [c+d x])^{5/2}} dx$$

Optimal (type 4, 179 leaves, 8 steps):

$$-\frac{2 b B \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{a(a^2-b^2) d \sqrt{\frac{a+b \cos [c+d x]}{a+b}}} + \frac{2 B \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{a d \sqrt{a+b \cos [c+d x]}} + \frac{2 b^2 B \sin [c+d x]}{a(a^2-b^2) d \sqrt{a+b \cos [c+d x]}}$$

Result (type 4, 522 leaves) :

$$\begin{aligned}
 & \left( \frac{2 b^2 \sin [c+d x]}{a\left(a^2-b^2\right) d \sqrt{a+b \cos [c+d x]}} - \frac{1}{2 a(-a+b)(a+b) d} \right. \\
 & \left. - \frac{4 a b \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{2\left(2 a^2-3 b^2\right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
 & \left. \left( 2 i b^2 \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \left( 2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + \right. \right. \right. \\
 & \left. \left. b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] - \right. \right. \right. \\
 & \left. \left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \right. \\
 & \left. \left. \left. \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} \left( 2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2 \right) \right) \right) \right) \right) \right)
 \end{aligned}$$

■ **Problem 369: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+B \cos [c+d x]}{\cos [c+d x]^{3 / 2}(a+b \cos [c+d x])} d x$$

Optimal (type 4, 86 leaves, 5 steps) :

$$-\frac{2 A \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{a d} - \frac{2(A b-a B) \operatorname{EllipticPi}\left[\frac{2 b}{a+b}, \frac{1}{2}(c+d x), 2\right]}{a(a+b) d} + \frac{2 A \sin [c+d x]}{a d \sqrt{\cos [c+d x]}}$$

Result (type 4, 210 leaves) :



$$\frac{1}{2ad} \left( \frac{2(-3Ab + 2aB) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a+b} - \frac{2aA \left( 2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] - \frac{2a \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a+b} \right)}{b} + \right. \\ \left. \frac{4A \operatorname{Sin}[c+dx]}{\sqrt{\operatorname{Cos}[c+dx]}} - 1 \Big/ \left( ab \sqrt{\operatorname{Sin}[c+dx]^2} \right) 2A \left( -2ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] + \right. \right. \\ \left. \left. 2a(a+b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] + (2a^2 - b^2) \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] \right) \operatorname{Sin}[c+dx] \right)$$

■ **Problem 393: Result more than twice size of optimal antiderivative.**

$$\int \frac{aB + bB \operatorname{Cos}[c+dx]}{\operatorname{Cos}[c+dx]^{3/2} (a+b \operatorname{Cos}[c+dx])^2} dx$$

Optimal (type 4, 80 leaves, 6 steps):

$$-\frac{2B \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{ad} - \frac{2bB \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a(a+b)d} + \frac{2B \operatorname{Sin}[c+dx]}{ad \sqrt{\operatorname{Cos}[c+dx]}}$$

Result (type 4, 200 leaves):

$$-\frac{1}{2ad} B \left( \frac{6b \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a+b} + \frac{2a \left( 2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] - \frac{2a \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a+b} \right)}{b} - \right. \\ \left. \frac{4 \operatorname{Sin}[c+dx]}{\sqrt{\operatorname{Cos}[c+dx]}} + 1 \Big/ \left( ab \sqrt{\operatorname{Sin}[c+dx]^2} \right) 2 \left( -2ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] + \right. \right. \\ \left. \left. 2a(a+b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] + (2a^2 - b^2) \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] \right) \operatorname{Sin}[c+dx] \right)$$

■ **Problem 395: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+b \operatorname{Cos}[c+dx]} (A+B \operatorname{Cos}[c+dx]) dx$$

Optimal (type 4, 560 leaves, 8 steps):

$$\begin{aligned}
& -\frac{1}{24ab^2d} (a-b)\sqrt{a+b} (6aAb - 3a^2B + 16b^2B) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{24b^2d} \sqrt{a+b} (a+2b) (6Ab - 3aB + 8bB) \operatorname{Cot}[c+dx] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{8b^3d} \\
& \sqrt{a+b} (2a^2Ab - 8Ab^3 - a^3B - 4ab^2B) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(6aAb - 3a^2B + 16b^2B) \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{24b^2d\sqrt{\cos[c+dx]}} + \\
& \frac{(2Ab - aB) \sqrt{\cos[c+dx]} \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{4bd} + \frac{B\sqrt{\cos[c+dx]} (a+b)\cos[c+dx]^{3/2} \sin[c+dx]}{3bd}
\end{aligned}$$

Result (type 4, 1224 leaves):

$$\begin{aligned}
& -\frac{1}{48bd} \left( \left( 4a(-18aAb + a^2B - 16b^2B) \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx] \right) - 4a(-24Ab^2 - 28abB) \\
& \left( \left( \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right/ \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right/ \left(b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) \right) + \\
& 2(-6aAb + 3a^2B - 16b^2B) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b}2a \left( \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right/ \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) - \right. \\
& \left. \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.
\end{aligned}$$

$$\left. \left. \left( \left( \text{Csc}[c + dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \right. \right.$$

$$\left. \left. \left. \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right) \right) +$$

$$\frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{(6Ab+aB) \sin[c+dx]}{12b} + \frac{1}{6} B \sin[2(c+dx)] \right)}{d}$$

■ **Problem 396: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} (A+B \cos[c+dx]) dx$$

Optimal (type 4, 473 leaves, 7 steps):

$$-\frac{1}{4abd}$$

$$(a-b) \sqrt{a+b} (4Ab+aB) \cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{4bd} \sqrt{a+b} (4Ab+(a+2b)B) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{4b^2d} \sqrt{a+b} (4aAb-a^2B+4b^2B) \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}}$$

$$\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(4Ab+aB) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{4bd \sqrt{\cos[c+dx]}} + \frac{B \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2d}$$

Result (type 4, 1175 leaves):

$$\frac{B \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2d} + \frac{1}{8d}$$

$$\begin{aligned}
& \left( - \left( 4 a (4 A b + 3 a B) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a \right. \\
& \quad (8 a A + 4 b B) \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \quad 2 (4 A b + a B) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\ \left. \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right)$$

■ **Problem 397: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} (A+B \operatorname{Cos}[c+dx])}{\sqrt{\operatorname{Cos}[c+dx]}} dx$$

Optimal (type 4, 385 leaves, 6 steps):

$$\begin{aligned}
& -\frac{1}{ad} (a-b) \sqrt{a+b} B \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{d} \\
& \sqrt{a+b} (2A+B) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{bd} \\
& \sqrt{a+b} (2Ab+aB) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{B \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}}
\end{aligned}$$

Result (type 4, 3054 leaves):

$$\begin{aligned}
& \left( (1+\cos[c+dx])^{3/2} \left( \frac{A \sqrt{a+b \cos[c+dx]}}{\sqrt{\cos[c+dx]}} + B \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \left( 2(a+b) B \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - 4(Ab+a(-A+B)) \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \right. \\
& \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - 8Ab \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - \right. \\
& \left. 4aB \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + bB \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \right. \\
& \left. \left. \sec\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] + 2aB \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] - bB \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) / \\
& \left( 4d \sqrt{a+b \cos[c+dx]} \left( \frac{1}{8(a+b \cos[c+dx])^{3/2}} b(1+\cos[c+dx])^{3/2} \sec\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] \right. \right. \\
& \left. \left. \left( 2(a+b) B \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 4 (A b + a (-A + B)) \sqrt{\frac{a + b \cos[c + d x]}{(a + b) (1 + \cos[c + d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] - \\
& 8 A b \sqrt{\frac{a + b \cos[c + d x]}{(a + b) (1 + \cos[c + d x])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] - 4 a B \sqrt{\frac{a + b \cos[c + d x]}{(a + b) (1 + \cos[c + d x])}} \\
& \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] + b B \sqrt{\frac{\cos[c + d x]}{1 + \cos[c + d x]}} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \operatorname{Sin}\left[\frac{3}{2} (c + d x)\right] + \\
& 2 a B \sqrt{\frac{\cos[c + d x]}{1 + \cos[c + d x]}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - b B \sqrt{\frac{\cos[c + d x]}{1 + \cos[c + d x]}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \left. \right) - \frac{1}{8 \sqrt{a + b \cos[c + d x]}} \\
& 3 \sqrt{1 + \cos[c + d x]} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Sin}[c + d x] \left( 2 (a + b) B \sqrt{\frac{a + b \cos[c + d x]}{(a + b) (1 + \cos[c + d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] - \right. \\
& 4 (A b + a (-A + B)) \sqrt{\frac{a + b \cos[c + d x]}{(a + b) (1 + \cos[c + d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] - \\
& 8 A b \sqrt{\frac{a + b \cos[c + d x]}{(a + b) (1 + \cos[c + d x])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] - 4 a B \sqrt{\frac{a + b \cos[c + d x]}{(a + b) (1 + \cos[c + d x])}} \\
& \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] + b B \sqrt{\frac{\cos[c + d x]}{1 + \cos[c + d x]}} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \operatorname{Sin}\left[\frac{3}{2} (c + d x)\right] + \\
& 2 a B \sqrt{\frac{\cos[c + d x]}{1 + \cos[c + d x]}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - b B \sqrt{\frac{\cos[c + d x]}{1 + \cos[c + d x]}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \left. \right) + \frac{1}{4 \sqrt{a + b \cos[c + d x]}} (1 + \cos[c + d x])^{3/2} \\
& \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \left( 2 (a + b) B \sqrt{\frac{a + b \cos[c + d x]}{(a + b) (1 + \cos[c + d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] - \right. \\
& 4 (A b + a (-A + B)) \sqrt{\frac{a + b \cos[c + d x]}{(a + b) (1 + \cos[c + d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] - \\
& 8 A b \sqrt{\frac{a + b \cos[c + d x]}{(a + b) (1 + \cos[c + d x])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] -
\end{aligned}$$



$$\begin{aligned}
& 4 a B \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]+b B \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \\
& \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right]+2 a B \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]-b B \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+ \\
& \frac{1}{4 \sqrt{a+b \cos [c+d x]}}(1+\cos [c+d x])^{3 / 2} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\left(\frac{3}{2} b B \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \cos \left[\frac{3}{2}(c+d x)\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]+ \right. \\
& a B \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2-\frac{1}{2} b B \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2+\frac{1}{\sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}}(a+b) B \\
& \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]\left(-\frac{b \operatorname{Sin}[c+d x]}{(a+b)(1+\cos [c+d x])}+\frac{(a+b \cos [c+d x]) \operatorname{Sin}[c+d x]}{(a+b)(1+\cos [c+d x])^2}\right)-\frac{1}{\sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} \right. \\
& \left. 2(A b+a(-A+B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]\left(-\frac{b \operatorname{Sin}[c+d x]}{(a+b)(1+\cos [c+d x])}+\frac{(a+b \cos [c+d x]) \operatorname{Sin}[c+d x]}{(a+b)(1+\cos [c+d x])^2}\right)- \right. \\
& \left. \frac{1}{\sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} 4 A b \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \right. \\
& \left. \left(-\frac{b \operatorname{Sin}[c+d x]}{(a+b)(1+\cos [c+d x])}+\frac{(a+b \cos [c+d x]) \operatorname{Sin}[c+d x]}{(a+b)(1+\cos [c+d x])^2}\right)-\frac{1}{\sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} \right. \\
& \left. 2 a B \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]\left(-\frac{b \operatorname{Sin}[c+d x]}{(a+b)(1+\cos [c+d x])}+\frac{(a+b \cos [c+d x]) \operatorname{Sin}[c+d x]}{(a+b)(1+\cos [c+d x])^2}\right)+ \right. \\
& \left. b B \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]\left(\frac{\cos [c+d x] \operatorname{Sin}[c+d x]}{(1+\cos [c+d x])^2}-\frac{\operatorname{Sin}[c+d x]}{1+\cos [c+d x]}\right) \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right]+a B\left(\frac{\cos [c+d x] \operatorname{Sin}[c+d x]}{(1+\cos [c+d x])^2}-\frac{\operatorname{Sin}[c+d x]}{1+\cos [c+d x]}\right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \right. \\
& \left. \frac{2 \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}}{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}} \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{b B \left( \frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} - \frac{\sin[c+dx]}{1+\cos[c+dx]} \right) \tan\left[\frac{1}{2}(c+dx)\right]}{2 \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}}} + \frac{1}{2} b B \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sec\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] \tan\left[\frac{1}{2}(c+dx)\right] - \\
& \frac{2(Ab + a(-A+B)) \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sec\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{1 - \frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} + \frac{4Ab \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sec\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{1 - \frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} + \\
& \frac{2aB \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sec\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{1 - \frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} + \\
& \left. \left. \left. \frac{(a+b) B \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sec\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}{\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) \right) \right) \right)
\end{aligned}$$

- **Problem 398: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \cos[c+dx]} (A+B \cos[c+dx])}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 351 leaves, 5 steps):

$$\begin{aligned}
& \frac{1}{ad} 2A(a-b) \sqrt{a+b} \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{ad} \\
& 2\sqrt{a+b} (Ab - a(A-B)) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\
& \frac{1}{d} 2\sqrt{a+b} B \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}
\end{aligned}$$

Result (type 4, 1161 leaves):

$$\begin{aligned}
& - \left( 4 a^2 B \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \\
& \frac{1}{d} 4 a (-a A + b B) \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left. \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \frac{2 A \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{d \sqrt{\operatorname{Cos}[c+dx]}} - \frac{1}{d} 2 A b \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\ \left. \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\cos[c+dx]}} \right)$$

- **Problem 399: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \cos[c+dx]} (A+B \cos[c+dx])}{\cos[c+dx]^{5/2}} dx$$

Optimal (type 4, 284 leaves, 4 steps):

$$\frac{1}{3 a^2 d} 2 (a-b) \sqrt{a+b} (A b+3 a B) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{1}{3 a d} 2 (a-b) \sqrt{a+b} (A-3 B) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{2 A \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 d \operatorname{Cos}[c+d x]^{3 / 2}}$$

Result (type 4, 1229 leaves):

$$\frac{1}{3 a d}$$

$$\left(-\left(\left(4 a\left(a^2 A-A b^2\right) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x]\right.\right.\right.$$

$$\left.\left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right],-\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]}\right)-\right.$$

$$\left.\left.4 a(-a A b-3 a^2 B)\left(\left(\sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+\right.\right.\right.$$

$$\left.\left.\left.d x\right] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right],-\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]}\right)-\right.$$

$$\left.\left.\left(\sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x]\right)\right)$$

$$\begin{aligned}
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4 \right/ \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) \right\} + \\
& 2(-A b^2-3 a b B) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \sec [c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \sec [c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \sec [c+d x]}{a+b}} \right. + \\
& \left. \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right. \\
& \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4 \right) \right/ \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\
& \left. \left. \left. \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4 \right) \right/ \right. \\
& \left. \left. \left. \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) \right) \right) +
\end{aligned}$$

$$\frac{\sqrt{\cos[c+dx]} \sqrt{a+b\cos[c+dx]} \left( \frac{2\sec[c+dx](Ab\sin[c+dx]+3aB\sin[c+dx])}{3a} + \frac{2}{3}A\sec[c+dx]\tan[c+dx] \right)}{d}$$

■ **Problem 400: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b\cos[c+dx]} (A+B\cos[c+dx])}{\cos[c+dx]^{7/2}} dx$$

Optimal (type 4, 350 leaves, 5 steps):

$$\frac{1}{15a^3d} 2(a-b)\sqrt{a+b} (9a^2A - 2Ab^2 + 5abB) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{15a^2d}$$

$$2(a-b)\sqrt{a+b} (9aA + 2Ab - 5aB) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}}$$

$$\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{2A\sqrt{a+b\cos[c+dx]}\sin[c+dx]}{5d\cos[c+dx]^{5/2}} + \frac{2(Ab+5aB)\sqrt{a+b\cos[c+dx]}\sin[c+dx]}{15ad\cos[c+dx]^{3/2}}$$

Result (type 4, 1315 leaves):

$$-\frac{1}{15a^2d} \left( \left( 4a(2a^2Ab - 2Ab^3 - 5a^3B + 5ab^2B) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ \left. \left. \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} - 4a(9a^3A - 2aAb^2 + 5a^2bB) \right. \\ \left. \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right) \right)$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right/ \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right/ \left(b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) \right) + \\
& 2(9a^2Ab - 2Ab^3 + 5ab^2B) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b}2a \left( \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right/ \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) - \right. \\
& \left. \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \left. \left( \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \\
& \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2 \sec[c+dx]^2 (Ab \sin[c+dx] + 5 a B \sin[c+dx])}{15 a} + \right. \\
& \frac{2 \sec[c+dx] (9 a^2 A \sin[c+dx] - 2 A b^2 \sin[c+dx] + 5 a b B \sin[c+dx])}{15 a^2} + \\
& \frac{2}{5} \\
& A \\
& \sec[c+dx]^2 \\
& \left. \tan[c+dx] \right)
\end{aligned}$$

- **Problem 401: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \cos[c+dx]} (A+B \cos[c+dx])}{\cos[c+dx]^{9/2}} dx$$

Optimal (type 4, 433 leaves, 6 steps):

$$\begin{aligned}
& \frac{1}{105 a^4 d} 2 (a-b) \sqrt{a+b} (19 a^2 A b + 8 A b^3 + 63 a^3 B - 14 a b^2 B) \cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{105 a^3 d} 2 (a-b) \sqrt{a+b} (8 A b^2 + a^2 (25 A - 63 B) + 2 a b (3 A - 7 B)) \\
& \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{2 A \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{7 d \cos[c+dx]^{7/2}} + \frac{2 (A b + 7 a B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{35 a d \cos[c+dx]^{5/2}} + \frac{2 (25 a^2 A - 4 A b^2 + 7 a b B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{105 a^2 d \cos[c+dx]^{3/2}}
\end{aligned}$$

Result (type 4, 1408 leaves):

$$\begin{aligned}
& \frac{1}{105 a^3 d} \left( - \left( 4 a (25 a^4 A - 17 a^2 A b^2 - 8 A b^4 - 14 a^3 b B + 14 a b^3 B) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a (-19 a^3 A b - 8 a A b^3 - 63 a^4 B + 14 a^2 b^2 B) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2 (-19 a^2 A b^2 - 8 A b^4 - 63 a^3 b B + 14 a b^3 B) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} - 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \quad \left. \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) + \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2 \operatorname{Sec}[c+dx]^3 (A b \operatorname{Sin}[c+dx] + 7 a B \operatorname{Sin}[c+dx])}{35 a} + \right. \\
& \quad \frac{2 \operatorname{Sec}[c+dx]^2 (25 a^2 A \operatorname{Sin}[c+dx] - 4 A b^2 \operatorname{Sin}[c+dx] + 7 a b B \operatorname{Sin}[c+dx])}{105 a^2} + \\
& \quad \frac{2 \operatorname{Sec}[c+dx] (19 a^2 A b \operatorname{Sin}[c+dx] + 8 A b^3 \operatorname{Sin}[c+dx] + 63 a^3 B \operatorname{Sin}[c+dx] - 14 a b^2 B \operatorname{Sin}[c+dx])}{105 a^3} + \\
& \quad \left. \frac{2}{7} A \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx] \right)
\end{aligned}$$

■ **Problem 402: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Cos}[c+dx]^{3/2} (a+b \operatorname{Cos}[c+dx])^{3/2} (A+B \operatorname{Cos}[c+dx]) dx$$

Optimal (type 4, 670 leaves, 9 steps):

$$\begin{aligned}
& -\frac{1}{192 a b^2 d} (a-b) \sqrt{a+b} (24 a^2 A b+128 A b^3-9 a^3 B+156 a b^2 B) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \\
& \frac{1}{192 b^2 d} \sqrt{a+b} (9 a^3 B-6 a^2 b(4 A+B)-8 b^3(16 A+9 B)-4 a b^2(28 A+39 B)) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{64 b^3 d} \\
& \sqrt{a+b} (8 a^3 A b-96 a A b^3-3 a^4 B-24 a^2 b^2 B-48 b^4 B) \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{(24 a^2 A b+128 A b^3-9 a^3 B+156 a b^2 B) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{192 b^2 d \sqrt{\operatorname{Cos}[c+d x]}} + \\
& \frac{(8 a A b-3 a^2 B+12 b^2 B) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{32 b d} + \\
& \frac{(8 A b-3 a B) \sqrt{\operatorname{Cos}[c+d x]} (a+b \operatorname{Cos}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{24 b d} + \frac{B \sqrt{\operatorname{Cos}[c+d x]} (a+b \operatorname{Cos}[c+d x])^{5/2} \operatorname{Sin}[c+d x]}{4 b d}
\end{aligned}$$

Result (type 4, 1284 leaves):

$$\begin{aligned}
& -\frac{1}{384 b d} \left( \left( 4 a (-136 a^2 A b-128 A b^3+3 a^3 B-228 a b^2 B) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (-416 a A b^2-228 a^2 b B-144 b^3 B) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right/ \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right/ \left(b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) \right) + \\
& 2(-24a^2Ab - 128Ab^3 + 9a^3B - 156ab^2B) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b}2a \left( \left( a \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right/ \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left( \text{Csc}[c + d x] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \text{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \text{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \right. \\
& \left. \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \text{Sin}[c+d x]}{b \sqrt{\cos [c+d x]}} \right) + \\
& \frac{1}{d} \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left( \frac{(56 a A b + 3 a^2 B + 42 b^2 B) \text{Sin}[c+d x]}{96 b} + \right. \\
& \frac{1}{48} \\
& (8 A b + 9 a B) \\
& \text{Sin}[2(c+d x)] + \frac{1}{16} \\
& b \\
& B \\
& \left. \text{Sin}[3(c+d x)] \right)
\end{aligned}$$

- **Problem 403: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos [c+d x]} (a+b \cos [c+d x])^{3/2} (A+B \cos [c+d x]) dx$$

Optimal (type 4, 566 leaves, 8 steps):

$$\begin{aligned}
& -\frac{1}{24abd} (a-b) \sqrt{a+b} (30aAb + 3a^2B + 16b^2B) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{24bd} \sqrt{a+b} (30aAb + 12Ab^2 + 3a^2B + 14abB + 16b^2B) \\
& \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{8b^2d} \\
& \sqrt{a+b} (6a^2Ab + 8Ab^3 - a^3B + 12ab^2B) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{(30aAb + 3a^2B + 16b^2B) \sqrt{a+b}\cos[c+dx] \operatorname{Sin}[c+dx]}{24bd\sqrt{\cos[c+dx]}} + \\
& \frac{(6Ab + 7aB) \sqrt{\cos[c+dx]} \sqrt{a+b}\cos[c+dx] \operatorname{Sin}[c+dx]}{12d} + \frac{bB\cos[c+dx]^{3/2} \sqrt{a+b}\cos[c+dx] \operatorname{Sin}[c+dx]}{3d}
\end{aligned}$$

Result (type 4, 1227 leaves):

$$\begin{aligned}
& \frac{1}{48d} \left( - \left( 4a (42aAb + 17a^2B + 16b^2B) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a (48a^2A + 24Ab^2 + 52abB) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) \right) + \\
& 2\left(30 a A b+3 a^2 B+16 b^2 B\right) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Sec}[c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Sec}[c+d x]}{a+b}} \right. + \\
& \left. \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \right. \\
& \left. \left. \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right) \right) \right)
\end{aligned}$$



$$\left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left(b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}\right) \right) +$$

$$\left. \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left(\frac{1}{12}(6Ab+7aB) \sin[c+dx] + \frac{1}{6}bB \sin[2(c+dx)]\right)}{d}$$

■ **Problem 404: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{3/2} (A+B \cos[c+dx])}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 472 leaves, 7 steps):

$$-\frac{1}{4ad}$$

$$(a-b) \sqrt{a+b} (4Ab+5aB) \cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{4d} \sqrt{a+b} (8aA+4Ab+5aB+2bB) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{4bd} \sqrt{a+b} (12aAb+3a^2B+4b^2B) \cot[c+dx]$$

$$\text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{(4Ab+5aB) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{4d \sqrt{\cos[c+dx]}} + \frac{bB \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2d}$$

Result (type 4, 1198 leaves):

$$\frac{bB \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2d} +$$

$$\begin{aligned}
& \frac{1}{8d} \left( - \left( 4a(8a^2A + 4Ab^2 + 7abB) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - 4a(16aAb + 8a^2B + 4b^2B) \\
& \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) + \\
& \left. 2(4Ab^2 + 5abB) \frac{\left( i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\operatorname{EllipticE}\left[i\operatorname{ArcSin}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx] \right. \right. \\
& \left. \left. b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx] \sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}} \right) \right)}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx] \sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}}} +
\end{aligned}$$

$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\ \left. \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right)$$

- **Problem 405: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{3/2} (A+B \operatorname{Cos}[c+dx])}{\operatorname{Cos}[c+dx]^{3/2}} dx$$

Optimal (type 4, 449 leaves, 7 steps):

$$\frac{1}{ad} (a-b) \sqrt{a+b} (2aA-bB) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{d}$$

$$\sqrt{a+b} (2a(A-B) - b(4A+B)) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{d} \sqrt{a+b} (2Ab+3aB) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2aA\sqrt{a+b}\cos[c+dx] \sin[c+dx]}{d\sqrt{\cos[c+dx]}} - \frac{(2aA-bB)\sqrt{a+b}\cos[c+dx] \sin[c+dx]}{d\sqrt{\cos[c+dx]}}$$

Result (type 4, 1196 leaves):

$$\frac{2aA\sqrt{a+b}\cos[c+dx] \sin[c+dx]}{d\sqrt{\cos[c+dx]}} +$$

$$\frac{1}{2d} \left( \left( 4a(-2aAb - 2a^2B - b^2B) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx] \right) + 4a(2a^2A - 2Ab^2 - 4abB)$$

$$\left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx] \right) -$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& 2(2aAb - b^2B) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) /
\end{aligned}$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}}$$

■ **Problem 406: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{3/2} (A+B \cos[c+dx])}{\cos[c+dx]^{5/2}} dx$$

Optimal (type 4, 419 leaves, 6 steps):

$$\frac{1}{3ad} 2(a-b) \sqrt{a+b} (4Ab+3aB) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{3ad} 2\sqrt{a+b} (3Ab^2+a^2(A-3B)-a(4Ab-6bB)) \cot[c+dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{d}$$

$$2b\sqrt{a+b} B \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2aA \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{3d \cos[c+dx]^{3/2}}$$

Result (type 4, 1236 leaves):

$$\frac{1}{3d} \left( \left( 4a(a^2A-Ab^2+3aB) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right]}, -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right/$$

$$\left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a(-4aAb-3a^2B+3b^2B)$$

$$\begin{aligned}
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2(-4Ab^2 - 3abB) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\ \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\ \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} + \frac{1}{d} \\ \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2}{3} \operatorname{Sec}[c+dx] (4Ab \sin[c+dx] + 3aB \sin[c+dx]) + \frac{2}{3} aA \operatorname{Sec}[c+dx] \right. \\ \left. \tan[c+dx] \right)$$

■ **Problem 407: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{3/2} (A+B \cos[c+dx])}{\cos[c+dx]^{7/2}} dx$$

Optimal (type 4, 353 leaves, 5 steps):

$$\frac{1}{15a^2d} 2(a-b) \sqrt{a+b} (9a^2A + 3Ab^2 + 20abB) \cot[c+dx] \\ \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{15ad} \\ 2(a-b) \sqrt{a+b} (9aA - 3Ab - 5aB + 15bB) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \\ \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{2aA \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{5d \cos[c+dx]^{5/2}} + \frac{2(6Ab + 5aB) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{15d \cos[c+dx]^{3/2}}$$

Result (type 4, 1314 leaves):



$$\begin{aligned}
& -\frac{1}{15ad} \left( - \left( 4a(-3a^2Ab + 3Ab^3 - 5a^3B + 5ab^2B) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - 4a(9a^3A + 3aAb^2 + 20a^2bB) \\
& \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) + \\
& 2(9a^2Ab + 3Ab^3 + 20a^2b^2B) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\operatorname{Sec}[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\operatorname{Sec}[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \Bigg) + \\
& \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2}{15} \operatorname{Sec}[c+dx]^2 (6Ab \operatorname{Sin}[c+dx] + 5aB \operatorname{Sin}[c+dx]) + \right. \\
& \quad \left. \frac{2 \operatorname{Sec}[c+dx] (9a^2 A \operatorname{Sin}[c+dx] + 3Ab^2 \operatorname{Sin}[c+dx] + 20abB \operatorname{Sin}[c+dx])}{15a} + \right. \\
& \quad \left. \frac{2}{5} a A \operatorname{Sec}[c+dx]^2 \right. \\
& \quad \left. \operatorname{Tan}[c+dx] \right)
\end{aligned}$$

■ **Problem 408:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \cos[c + dx])^{3/2} (A + B \cos[c + dx])}{\cos[c + dx]^{9/2}} dx$$

Optimal (type 4, 433 leaves, 6 steps):

$$\frac{1}{105 a^3 d} 2 (a - b) \sqrt{a + b} (82 a^2 A b - 6 A b^3 + 63 a^3 B + 21 a b^2 B) \cot[c + dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a (1 + \sec[c + dx])}{a - b}} - \frac{1}{105 a^2 d}$$

$$2 (a - b) \sqrt{a + b} (6 A b^2 - a^2 (25 A - 63 B) + 3 a b (19 A - 7 B)) \cot[c + dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right]$$

$$\sqrt{\frac{a (1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a (1 + \sec[c + dx])}{a - b}} + \frac{2 a A \sqrt{a + b \cos[c + dx]} \sin[c + dx]}{7 d \cos[c + dx]^{7/2}} +$$

$$\frac{2 (8 A b + 7 a B) \sqrt{a + b \cos[c + dx]} \sin[c + dx]}{35 d \cos[c + dx]^{5/2}} + \frac{2 (25 a^2 A + 3 A b^2 + 42 a b B) \sqrt{a + b \cos[c + dx]} \sin[c + dx]}{105 a d \cos[c + dx]^{3/2}}$$

Result (type 4, 1407 leaves):

$$\frac{1}{105 a^2 d} \left( - \left( 4 a (25 a^4 A - 31 a^2 A b^2 + 6 A b^4 + 21 a^3 b B - 21 a b^3 B) \sqrt{\frac{(a + b) \cot\left[\frac{1}{2} (c + dx)\right]^2}{-a + b}} \sqrt{\frac{(a + b) \cos[c + dx] \csc\left[\frac{1}{2} (c + dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a + b \cos[c + dx]) \csc\left[\frac{1}{2} (c + dx)\right]^2}{a}} \csc[c + dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a + b \cos[c + dx]) \csc\left[\frac{1}{2} (c + dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a + b}\right] \sin\left[\frac{1}{2} (c + dx)\right]^4 \right) /$$

$$\left( (a + b) \sqrt{\cos[c + dx]} \sqrt{a + b \cos[c + dx]} \right) - 4 a (-82 a^3 A b + 6 a A b^3 - 63 a^4 B - 21 a^2 b^2 B)$$

$$\left( \left( \sqrt{\frac{(a + b) \cot\left[\frac{1}{2} (c + dx)\right]^2}{-a + b}} \sqrt{\frac{(a + b) \cos[c + dx] \csc\left[\frac{1}{2} (c + dx)\right]^2}{a}} \sqrt{\frac{(a + b \cos[c + dx]) \csc\left[\frac{1}{2} (c + dx)\right]^2}{a}} \csc[c + dx] \right. \right.$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \csc [c+d x] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) \right) + \\
& 2 \left(-82 a^2 A b^2 + 6 A b^4 - 63 a^3 b B - 21 a b^3 B\right) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \sec [c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \sec [c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \sec [c+d x]}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \csc [c+d x] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right.
\end{aligned}$$

$$\left. \left( \text{Csc}[c + dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{(a+b \text{Cos}[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \text{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right/$$

$$\left. \left. \left( b \sqrt{\text{Cos}[c+dx]} \sqrt{a+b \text{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \text{Cos}[c+dx]} \text{Sin}[c+dx]}{b \sqrt{\text{Cos}[c+dx]}} \right) \right)$$

$$\frac{1}{d} \sqrt{\text{Cos}[c+dx]} \sqrt{a+b \text{Cos}[c+dx]} \left( \frac{2}{35} \text{Sec}[c+dx]^3 (8Ab \text{Sin}[c+dx] + 7aB \text{Sin}[c+dx]) + \right.$$

$$\frac{2 \text{Sec}[c+dx]^2 (25a^2 A \text{Sin}[c+dx] + 3Ab^2 \text{Sin}[c+dx] + 42abB \text{Sin}[c+dx])}{105a} +$$

$$\frac{2 \text{Sec}[c+dx] (82a^2 Ab \text{Sin}[c+dx] - 6Ab^3 \text{Sin}[c+dx] + 63a^3 B \text{Sin}[c+dx] + 21ab^2 B \text{Sin}[c+dx])}{105a^2} +$$

$$\left. \left. \frac{2}{7} aA \text{Sec}[c+dx]^3 \text{Tan}[c+dx] \right) \right)$$

■ **Problem 409: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \text{Cos}[c+dx])^{3/2} (A+B \text{Cos}[c+dx])}{\text{Cos}[c+dx]^{11/2}} dx$$

Optimal (type 4, 522 leaves, 7 steps):

$$\frac{1}{315 a^4 d} 2 (a-b) \sqrt{a+b} (147 a^4 A + 33 a^2 A b^2 + 8 A b^4 + 246 a^3 b B - 18 a b^3 B) \text{Cot}[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\text{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} + \frac{1}{315 a^3 d}$$

$$2(a-b) \sqrt{a+b} (8Ab^3 - a^3(147A - 75B) + 3a^2b(13A - 57B) + 6ab^2(A - 3B)) \text{Cot}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\text{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} + \frac{2aA \sqrt{a+b \text{Cos}[c+dx]} \text{Sin}[c+dx]}{9d \text{Cos}[c+dx]^{9/2}} + \frac{2(10Ab+9aB) \sqrt{a+b \text{Cos}[c+dx]} \text{Sin}[c+dx]}{63d \text{Cos}[c+dx]^{7/2}} +$$

$$\frac{2(49a^2A + 3Ab^2 + 72abB) \sqrt{a+b \text{Cos}[c+dx]} \text{Sin}[c+dx]}{315ad \text{Cos}[c+dx]^{5/2}} + \frac{2(88a^2Ab - 4Ab^3 + 75a^3B + 9ab^2B) \sqrt{a+b \text{Cos}[c+dx]} \text{Sin}[c+dx]}{315a^2d \text{Cos}[c+dx]^{3/2}}$$

Result (type 4, 1515 leaves):

$$\begin{aligned}
& -\frac{1}{315 a^3 d} \\
& \left( -4 a \left( -39 a^4 A b + 31 a^2 A b^3 + 8 A b^5 - 75 a^5 B + 93 a^3 b^2 B - 18 a b^4 B \right) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \\
& \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a \left( 147 a^5 A + 33 a^3 A b^2 + 8 a A b^4 + 246 a^4 b B - 18 a^2 b^3 B \right) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \\
& \left. \left. \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \right)
\end{aligned}$$

$$\left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + 2 (147 a^4 A b + 33 a^2 A b^3 + 8 A b^5 + 246 a^3 b^2 B - 18 a b^4 B)$$

$$\left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) +$$

$$\frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right.$$

$$\left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) -$$

$$\left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right.$$

$$\left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) +$$

$$\frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2}{63} \operatorname{Sec}[c+dx]^4 (10 A b \sin[c+dx] + 9 a B \sin[c+dx]) + \right.$$

$$\begin{aligned}
& \frac{2 \operatorname{Sec}[c + d x]^3 (49 a^2 A \operatorname{Sin}[c + d x] + 3 A b^2 \operatorname{Sin}[c + d x] + 72 a b B \operatorname{Sin}[c + d x])}{315 a} + \\
& \frac{1}{315 a^2} \\
& 2 \operatorname{Sec}[c + d x]^2 (88 a^2 A b \operatorname{Sin}[c + d x] - 4 A b^3 \operatorname{Sin}[c + d x] + 75 a^3 B \operatorname{Sin}[c + d x] + 9 a b^2 B \operatorname{Sin}[c + d x]) + \\
& \frac{1}{315 a^3} \\
& 2 \operatorname{Sec}[c + d x] (147 a^4 A \operatorname{Sin}[c + d x] + 33 a^2 A b^2 \operatorname{Sin}[c + d x] + 8 A b^4 \operatorname{Sin}[c + d x] + 246 a^3 b B \operatorname{Sin}[c + d x] - 18 a b^3 B \operatorname{Sin}[c + d x]) + \\
& \left. \frac{2}{9} a A \operatorname{Sec}[c + d x]^4 \operatorname{Tan}[c + d x] \right)
\end{aligned}$$

- **Problem 410: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Cos}[c + d x]^{3/2} (a + b \operatorname{Cos}[c + d x])^{5/2} (A + B \operatorname{Cos}[c + d x]) dx$$

Optimal (type 4, 779 leaves, 10 steps):



$$\begin{aligned}
& - \frac{1}{1920 a b^2 d} (a-b) \sqrt{a+b} (150 a^3 A b + 2840 a A b^3 - 45 a^4 B + 1692 a^2 b^2 B + 1024 b^4 B) \\
& \quad \text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\
& \frac{1}{1920 b^2 d} \sqrt{a+b} (45 a^4 B - 30 a^3 b (5A+B) - 16 b^4 (45A+64B) - 8 a b^3 (355A+193B) - 4 a^2 b^2 (295A+423B)) \text{Cot}[c+dx] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{1}{128 b^3 d} \sqrt{a+b} (10 a^4 A b - 240 a^2 A b^3 - 96 A b^5 - 3 a^5 B - 40 a^3 b^2 B - 240 a b^4 B) \text{Cot}[c+dx] \\
& \quad \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{(150 a^3 A b + 2840 a A b^3 - 45 a^4 B + 1692 a^2 b^2 B + 1024 b^4 B) \sqrt{a+b} \cos[c+dx] \sin[c+dx]}{1920 b^2 d \sqrt{\cos[c+dx]}} + \\
& \frac{(50 a^2 A b + 120 A b^3 - 15 a^3 B + 172 a b^2 B) \sqrt{\cos[c+dx]} \sqrt{a+b} \cos[c+dx] \sin[c+dx]}{320 b d} + \\
& \frac{(50 a A b - 15 a^2 B + 64 b^2 B) \sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{240 b d} + \\
& \frac{(10 A b - 3 a B) \sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{5/2} \sin[c+dx]}{40 b d} + \frac{B \sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{7/2} \sin[c+dx]}{5 b d}
\end{aligned}$$

Result (type 4, 1353 leaves):

$$\begin{aligned}
& - \frac{1}{3840 b d} \\
& \left( - \left( 4 a (-1330 a^3 A b - 3560 a A b^3 + 15 a^4 B - 3236 a^2 b^2 B - 1024 b^4 B) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \quad \left. \left. \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right]}, -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a \left( -6440 a^2 A b^2 - 1440 A b^4 - 2292 a^3 b B - 4624 a b^3 B \right) \\
& \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + 2 \left( -150 a^3 A b - 2840 a A b^3 + 45 a^4 B - 1692 a^2 b^2 B - 1024 b^4 B \right) \\
& \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.
\end{aligned}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right] / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) -$$

$$\left(a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}\right.$$

$$\left.\operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right] /$$

$$\left.\left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}}\right) +$$

$$\frac{1}{d} \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left(\frac{(590 a^2 A b+420 A b^3+15 a^3 B+898 a b^2 B) \sin [c+d x]}{960 b} +$$

$$\frac{1}{480}$$

$$(170 a A b+93 a^2 B+88 b^2 B) \sin [2(c+d x)] +$$

$$\frac{1}{160} b(10 A b+21 a B) \sin [3(c+d x)] + \frac{1}{40} b^2 B \sin [4(c+d x)]\right)$$

■ **Problem 411: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\cos [c+d x]} (a+b \cos [c+d x])^{5/2} (A+B \cos [c+d x]) dx$$

Optimal (type 4, 664 leaves, 9 steps):

$$\begin{aligned}
& -\frac{1}{192abd} (a-b) \sqrt{a+b} (264a^2Ab + 128Ab^3 + 15a^3B + 284ab^2B) \operatorname{Cot}[c+dx] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{1}{192bd} \sqrt{a+b} (15a^3B + 8b^3(16A+9B) + 2a^2b(132A+59B) + 4ab^2(52A+71B)) \operatorname{Cot}[c+dx] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{64b^2d} \\
& \sqrt{a+b} (40a^3Ab + 160aAb^3 - 5a^4B + 120a^2b^2B + 48b^4B) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(264a^2Ab + 128Ab^3 + 15a^3B + 284ab^2B) \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{192bd\sqrt{\cos[c+dx]}} + \\
& \frac{(24aAb + 5a^2B + 12b^2B) \sqrt{\cos[c+dx]} \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{32d} + \\
& \frac{(8Ab + 11aB) \sqrt{\cos[c+dx]} (a+b\cos[c+dx])^{3/2} \sin[c+dx]}{24d} + \frac{bB\cos[c+dx]^{3/2} (a+b\cos[c+dx])^{3/2} \sin[c+dx]}{4d}
\end{aligned}$$

Result (type 4, 1287 leaves):

$$\begin{aligned}
& \frac{1}{384d} \left( - \left( 4a (472a^2Ab + 128Ab^3 + 133a^3B + 356ab^2B) \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b\cos[c+dx]} \right) - 4a (384a^3A + 608aAb^2 + 644a^2bB + 144b^3B) \\
& \left( \left( \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) + \\
& 2(264a^2Ab + 128Ab^3 + 15a^3B + 284ab^2B) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b}2a \left( \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right. \\
& \left. \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right) \right)
\end{aligned}$$

$$\left. \left( \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right/$$

$$\left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) + \frac{\sqrt{a+b\cos[c+dx]}\sin[c+dx]}{b\sqrt{\cos[c+dx]}} + \frac{1}{d}$$

$$\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \left( \frac{1}{96} (104aAb + 59a^2B + 42b^2B) \sin[c+dx] + \frac{1}{48} b(8Ab + 17aB) \right.$$

$$\left. \sin[2(c+dx)] + \frac{1}{16} b^2B \sin[3(c+dx)] \right)$$

■ **Problem 412: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b\cos[c+dx])^{5/2} (A+B\cos[c+dx])}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 564 leaves, 8 steps):

$$-\frac{1}{24ad} (a-b)\sqrt{a+b} (54aAb + 33a^2B + 16b^2B) \cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{24d} \sqrt{a+b} (4b^2(3A+4B) + a^2(48A+33B) + a(54Ab+26bB))$$

$$\cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{8bd}$$

$$\sqrt{a+b} (30a^2Ab + 8Ab^3 + 5a^3B + 20ab^2B) \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(54aAb + 33a^2B + 16b^2B) \sqrt{a+b\cos[c+dx]}\sin[c+dx]}{24d\sqrt{\cos[c+dx]}} +$$

$$\frac{b(2Ab+3aB)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\sin[c+dx]}{4d} + \frac{bB\sqrt{\cos[c+dx]}(a+b\cos[c+dx])^{3/2}\sin[c+dx]}{3d}$$

Result (type 4, 1251 leaves):

$$\begin{aligned}
& \frac{1}{48d} \left( - \left( 4a(48a^3A + 66aAb^2 + 59a^2bB + 16b^3B) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right]}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - 4a(144a^2Ab + 24Ab^3 + 48a^3B + 76ab^2B) \\
& \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right]}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right]}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \Bigg) + \\
& 2(54aAb^2 + 33a^2bB + 16b^3B) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\operatorname{Sec}[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\operatorname{Sec}[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \Bigg) + \\
& \quad \frac{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{1}{12} b (6 A b + 13 a B) \operatorname{Sin}[c+dx] + \frac{1}{6} b^2 B \operatorname{Sin}[2(c+dx)] \right)}{d}
\end{aligned}$$

■ **Problem 413: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{5/2} (A+B \operatorname{Cos}[c+dx])}{\operatorname{Cos}[c+dx]^{3/2}} dx$$

Optimal (type 4, 547 leaves, 8 steps):



$$\begin{aligned}
& \frac{1}{4ad} (a-b) \sqrt{a+b} (8a^2A - 4Ab^2 - 9abB) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{4d} \sqrt{a+b} (8a^2(A-B) - 2b^2(2A+B) - 3ab(8A+3B)) \\
& \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\
& \frac{1}{4d} \sqrt{a+b} (20aAb + 15a^2B + 4b^2B) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{(8a^2A - 4Ab^2 - 9abB) \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{4d\sqrt{\cos[c+dx]}} - \\
& \frac{b(4aA - bB) \sqrt{\cos[c+dx]} \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{2d} + \frac{2aA(a+b)\cos[c+dx]^{3/2} \sin[c+dx]}{d\sqrt{\cos[c+dx]}}
\end{aligned}$$

Result (type 4, 1241 leaves):

$$\begin{aligned}
& \frac{1}{8d} \left( \left( 4a(-16a^2Ab - 4Ab^3 - 8a^3B - 11ab^2B) \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx] \right) + 4a(8a^3A - 24aAb^2 - 24a^2bB - 4b^3B) \\
& \left( \left( \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) - \\
& 2(8a^2Ab - 4Ab^3 - 9ab^2B) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b}2a \left( \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right. \right. \\
& \left. \left. \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right) \right) \right) -
\end{aligned}$$

$$\left. \left. \left. \left. \text{EllipticPi} \left[ -\frac{a}{b}, \text{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}} \right], -\frac{2a}{-a+b} \right] \sin \left[ \frac{1}{2}(c+dx) \right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) \right) \right)$$

$$\left. \left. \left. \left. \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right) \right) + \frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{1}{2} b^2 B \sin[c+dx] + 2 a^2 A \tan[c+dx] \right)}{d}$$

■ **Problem 414: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{5/2} (A+B \cos[c+dx])}{\cos[c+dx]^{5/2}} dx$$

Optimal (type 4, 536 leaves, 8 steps):

$$\frac{1}{3 a d} (a-b) \sqrt{a+b} \left( 14 a A b + 6 a^2 B - 3 b^2 B \right) \cot[c+dx] \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}} \right], -\frac{a+b}{a-b} \right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{3 d} \sqrt{a+b} \left( 2 a b (7 A - 9 B) - 2 a^2 (A - 3 B) - 3 b^2 (6 A + B) \right)$$

$$\cot[c+dx] \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{d}$$

$$b \sqrt{a+b} (2 A b + 5 a B) \cot[c+dx] \text{EllipticPi} \left[ \frac{a+b}{b}, \text{ArcSin} \left[ \frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2 a (2 A b + a B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}}$$

$$\frac{(14 a A b + 6 a^2 B - 3 b^2 B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{3 d \sqrt{\cos[c+dx]}} + \frac{2 a A (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{3 d \cos[c+dx]^{3/2}}$$

Result (type 4, 1269 leaves):

$$\begin{aligned}
& \frac{1}{6d} \left( - \left( 4a \left( 2a^3 A + 4aAb^2 + 12a^2bB + 3b^3B \right) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4a \left( -14a^2Ab + 6Ab^3 - 6a^3B + 18ab^2B \right) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2 \left( -14aAb^2 - 6a^2bB + 3b^3B \right) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \Bigg) + \\
& \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2}{3} \operatorname{Sec}[c+dx] (7 a A b \operatorname{Sin}[c+dx] + 3 a^2 B \operatorname{Sin}[c+dx]) + \right. \\
& \quad \left. \frac{2}{3} a^2 A \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx] \right)
\end{aligned}$$

- **Problem 415: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{5/2} (A+B \operatorname{Cos}[c+dx])}{\operatorname{Cos}[c+dx]^{7/2}} dx$$

Optimal (type 4, 493 leaves, 7 steps):

$$\frac{1}{15 a d} 2 (a-b) \sqrt{a+b} (9 a^2 A + 23 A b^2 + 35 a b B) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{15 a d} 2 \sqrt{a+b} (15 A b^3 - a b^2 (23 A - 45 B) + a^2 b (17 A - 35 B) - a^3 (9 A - 5 B))$$

$$\operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{d}$$

$$2 b^2 \sqrt{a+b} B \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{2 a (8 A b + 5 a B) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{15 d \operatorname{Cos}[c+d x]^{3/2}} + \frac{2 a A (a+b) \operatorname{Cos}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{5 d \operatorname{Cos}[c+d x]^{5/2}}$$

Result (type 4, 1319 leaves):

$$\frac{1}{15 d} \left( \left( 4 a (-8 a^2 A b + 8 A b^3 - 5 a^3 B - 10 a b^2 B) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right/$$

$$\left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b} \operatorname{Cos}[c+d x] \right) + 4 a (9 a^3 A + 23 a A b^2 + 35 a^2 b B - 15 b^3 B)$$

$$\left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right/ \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b} \operatorname{Cos}[c+d x] \right) -$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& 2(9a^2Ab + 23Ab^3 + 35ab^2B) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left( \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) +$$

$$\frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2}{15} \sec[c+dx]^2 (11 a A b \sin[c+dx] + 5 a^2 B \sin[c+dx]) + \right.$$

$$\left. \frac{2}{15} \sec[c+dx] \right.$$

$$\left. (9 a^2 A \sin[c+dx] + 23 A b^2 \sin[c+dx] + 35 a b B \sin[c+dx]) + \frac{2}{5} \right.$$

$$\left. a^2 A \sec[c+dx]^2 \tan[c+dx] \right)$$

■ **Problem 416: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{5/2} (A+B \cos[c+dx])}{\cos[c+dx]^{9/2}} dx$$

Optimal (type 4, 434 leaves, 6 steps):

$$\frac{1}{105 a^2 d} 2 (a-b) \sqrt{a+b} (145 a^2 A b + 15 A b^3 + 63 a^3 B + 161 a b^2 B) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{105 a d}$$

$$2 (a-b) \sqrt{a+b} (a^2 (25 A - 63 B) + 15 b^2 (A - 7 B) - 8 a b (15 A - 7 B)) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{2 a (10 A b + 7 a B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{35 d \cos[c+dx]^{5/2}} +$$

$$\frac{2 (25 a^2 A + 45 A b^2 + 77 a b B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{105 d \cos[c+dx]^{3/2}} + \frac{2 a A (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{7 d \cos[c+dx]^{7/2}}$$

Result (type 4, 1409 leaves):

$$\frac{1}{105 a d} \left( - \left( 4 a (25 a^4 A - 10 a^2 A b^2 - 15 A b^4 + 56 a^3 b B - 56 a b^3 B) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right) \right)$$



$$\left( \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a \left( -145a^3Ab - 15aAb^3 - 63a^4B - 161a^2b^2B \right)$$

$$\left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.$$

$$\left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right.$$

$$\left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + 2 \left( -145a^2Ab^2 - 15Ab^4 - 63a^3bB - 161ab^3B \right)$$

$$\left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) +$$

$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \Bigg) + \\
& \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2}{35} \operatorname{Sec}[c+dx]^3 (15 a A b \operatorname{Sin}[c+dx] + 7 a^2 B \operatorname{Sin}[c+dx]) + \right. \\
& \quad \frac{2}{105} \operatorname{Sec}[c+dx]^2 \\
& \quad \left. (25 a^2 A \operatorname{Sin}[c+dx] + 45 A b^2 \operatorname{Sin}[c+dx] + 77 a b B \operatorname{Sin}[c+dx]) + \right. \\
& \quad \left. 1 / (105 a)^2 \operatorname{Sec}[c+dx] (145 a^2 A b \operatorname{Sin}[c+dx] + 15 A b^3 \operatorname{Sin}[c+dx] + 63 a^3 B \operatorname{Sin}[c+dx] + 161 a b^2 B \operatorname{Sin}[c+dx]) + \right. \\
& \quad \left. \frac{2}{7} a^2 A \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx] \right)
\end{aligned}$$

■ **Problem 417: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{5/2} (A+B \operatorname{Cos}[c+dx])}{\operatorname{Cos}[c+dx]^{11/2}} dx$$

Optimal (type 4, 522 leaves, 7 steps) :

$$\frac{1}{315 a^3 d} 2 (a-b) \sqrt{a+b} (147 a^4 A + 279 a^2 A b^2 - 10 A b^4 + 435 a^3 b B + 45 a b^3 B)$$

$$\text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{315 a^2 d} 2 (a-b) \sqrt{a+b} (10 A b^3 - 6 a^2 b (19 A - 60 B) + 3 a^3 (49 A - 25 B) + 15 a b^2 (11 A - 3 B)) \text{Cot}[c+dx]$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2 a (4 A b + 3 a B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{21 d \cos[c+dx]^{7/2}} + \frac{2 (49 a^2 A + 75 A b^2 + 135 a b B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{315 d \cos[c+dx]^{5/2}} +$$

$$\frac{2 (163 a^2 A b + 5 A b^3 + 75 a^3 B + 135 a b^2 B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{315 a d \cos[c+dx]^{3/2}} + \frac{2 a A (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{9 d \cos[c+dx]^{9/2}}$$

Result (type 4, 1517 leaves) :

$$-\frac{1}{315 a^2 d}$$

$$\left( - \left( 4 a (-114 a^4 A b + 124 a^2 A b^3 - 10 A b^5 - 75 a^5 B + 30 a^3 b^2 B + 45 a b^4 B) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.$$

$$\left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4 a (147 a^5 A + 279 a^3 A b^2 - 10 a A b^4 + 435 a^4 b B + 45 a^2 b^3 B)$$

$$\left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right.$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\text{Sin}\left[\frac{1}{2}(c+dx)\right]^4\right) / \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) - \\
& \left( \sqrt{\frac{(a+b)\text{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \text{Csc}[c+dx]\text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\text{Sin}\left[\frac{1}{2}(c+dx)\right]^4\right) / \right. \\
& \left. \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) + 2(147a^4Ab + 279a^2Ab^3 - 10Ab^5 + 435a^3b^2B + 45ab^4B) \\
& \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\text{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\text{Sec}[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\text{Sec}[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\text{Sec}[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b}2a \left( a\sqrt{\frac{(a+b)\text{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \text{Csc}[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\text{Sin}\left[\frac{1}{2}(c+dx)\right]^4\right) / \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
\left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
\left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \Bigg) + \\
\frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2}{63} \operatorname{Sec}[c+dx]^4 (19 a A b \operatorname{Sin}[c+dx] + 9 a^2 B \operatorname{Sin}[c+dx]) + \right. \\
\frac{2}{315} \operatorname{Sec}[c+dx]^3 (49 a^2 A \operatorname{Sin}[c+dx] + 75 A b^2 \operatorname{Sin}[c+dx] + 135 a b B \operatorname{Sin}[c+dx]) + \\
\frac{1}{315 a} \\
2 \operatorname{Sec}[c+dx]^2 (163 a^2 A b \operatorname{Sin}[c+dx] + 5 A b^3 \operatorname{Sin}[c+dx] + 75 a^3 B \operatorname{Sin}[c+dx] + 135 a b^2 B \operatorname{Sin}[c+dx]) + \\
\frac{1}{315 a^2} \\
2 \operatorname{Sec}[c+dx] (147 a^4 A \operatorname{Sin}[c+dx] + 279 a^2 A b^2 \operatorname{Sin}[c+dx] - 10 A b^4 \operatorname{Sin}[c+dx] + 435 a^3 b B \operatorname{Sin}[c+dx] + 45 a b^3 B \operatorname{Sin}[c+dx]) + \\
\left. \frac{2}{9} a^2 A \operatorname{Sec}[c+dx]^4 \operatorname{Tan}[c+dx] \right)$$

- **Problem 418: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{5/2} (A+B \operatorname{Cos}[c+dx])}{\operatorname{Cos}[c+dx]^{13/2}} dx$$

Optimal (type 4, 622 leaves, 8 steps):

$$\frac{1}{3465 a^4 d} 2 (a-b) \sqrt{a+b} (3705 a^4 A b + 255 a^2 A b^3 + 40 A b^5 + 1617 a^5 B + 3069 a^3 b^2 B - 110 a b^4 B)$$

$$\text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{3465 a^3 d} 2 (a-b) \sqrt{a+b} (40 A b^4 + 3 a^4 (225 A - 539 B) - 6 a^3 b (505 A - 209 B) + 15 a^2 b^2 (19 A - 121 B) + 10 a b^3 (3 A - 11 B))$$

$$\text{Cot}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2 a (14 A b + 11 a B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{99 d \cos[c+dx]^{9/2}} + \frac{2 (81 a^2 A + 113 A b^2 + 209 a b B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{693 d \cos[c+dx]^{7/2}} +$$

$$\frac{2 (1145 a^2 A b + 15 A b^3 + 539 a^3 B + 825 a b^2 B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{3465 a d \cos[c+dx]^{5/2}} +$$

$$\frac{2 (675 a^4 A + 1025 a^2 A b^2 - 20 A b^4 + 1793 a^3 B b + 55 a b^3 B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{3465 a^2 d \cos[c+dx]^{3/2}} + \frac{2 a A (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{11 d \cos[c+dx]^{11/2}}$$

Result (type 4, 1640 leaves):

$$\frac{1}{3465 a^3 d} \left( - \left( 4 a (675 a^6 A - 390 a^4 A b^2 - 245 a^2 A b^4 - 40 A b^6 + 1254 a^5 b B - 1364 a^3 b^3 B + 110 a b^5 B) \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right.$$

$$\left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}}{a \sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.$$

$$\left. 4 a (-3705 a^5 A b - 255 a^3 A b^3 - 40 a A b^5 - 1617 a^6 B - 3069 a^4 b^2 B + 110 a^2 b^4 B) \right.$$

$$\left. \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \right.$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right] / \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) -$$

$$\left(\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}}\sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\csc[c+dx]\right.$$

$$\left.\text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right] / \left(b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) +$$

$$2(-3705a^4Ab^2 - 255a^2Ab^4 - 40Ab^6 - 1617a^5bB - 3069a^3b^3B + 110ab^5B)$$

$$\left(\frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}}\right) +$$

$$\frac{1}{b}2a\left(\left(a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}}\sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\csc[c+dx]\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right] / \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) -$$

$$\left(a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}}\sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\right)$$

$$\left. \text{Csc}[c + dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right)$$

$$\begin{aligned} & \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2}{99} \sec[c+dx]^5 (23 a A b \sin[c+dx] + 11 a^2 B \sin[c+dx]) + \right. \\ & \frac{2}{693} \sec[c+dx]^4 (81 a^2 A \sin[c+dx] + 113 A b^2 \sin[c+dx] + 209 a b B \sin[c+dx]) + \frac{1}{3465 a} \\ & 2 \sec[c+dx]^3 (1145 a^2 A b \sin[c+dx] + 15 A b^3 \sin[c+dx] + 539 a^3 B \sin[c+dx] + 825 a b^2 B \sin[c+dx]) + \frac{1}{3465 a^2} \\ & 2 \sec[c+dx]^2 (675 a^4 A \sin[c+dx] + 1025 a^2 A b^2 \sin[c+dx] - 20 A b^4 \sin[c+dx] + 1793 a^3 b B \sin[c+dx] + 55 a b^3 B \sin[c+dx]) + \\ & \frac{1}{3465 a^3} 2 \sec[c+dx] (3705 a^4 A b \sin[c+dx] + 255 a^2 A b^3 \sin[c+dx] + 40 A b^5 \sin[c+dx] + \\ & \left. 1617 a^5 B \sin[c+dx] + 3069 a^3 b^2 B \sin[c+dx] - 110 a b^4 B \sin[c+dx]) + \frac{2}{11} a^2 A \sec[c+dx]^5 \tan[c+dx] \right) \end{aligned}$$

- **Problem 419: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \cos[c + dx])^{5/2} \left( \frac{3bB}{2a} + B \cos[c + dx] \right)}{\cos[c + dx]^{5/2}} dx$$

Optimal (type 4, 418 leaves, 6 steps):



$$\frac{1}{ad} 2(a-b)\sqrt{a+b}(a^2+3b^2)B\cot[c+dx]\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}}\right], -\frac{a+b}{a-b}\right]\sqrt{\frac{a(1-\sec[c+dx])}{a+b}}\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{ad}(a-3b)\sqrt{a+b}(2a^2-ab+3b^2)B\cot[c+dx]\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}}\right], -\frac{a+b}{a-b}\right]\sqrt{\frac{a(1-\sec[c+dx])}{a+b}}$$

$$\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{d}b\sqrt{a+b}\left(5a+\frac{3b^2}{a}\right)B\cot[c+dx]\operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}}\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{bB(a+b\cos[c+dx])^{3/2}\sin[c+dx]}{d\cos[c+dx]^{3/2}}$$

Result (type 4, 1236 leaves):

$$-\frac{1}{2ad}$$

$$B\left(-\left(4a(-5a^3b-3ab^3)\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}}\sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\csc\left[\frac{1}{2}(c+dx)\right]\right.\right.$$

$$\left.\left.\csc[c+dx]\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right)/\right.$$

$$\left.\left(\frac{(a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}}{a}\right) - 4a(2a^4+a^2b^2-3b^4)\right)$$

$$\left(\left(\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}}\sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\csc[c+dx]\right.\right.\right.$$

$$\left.\left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right)/\left(\frac{(a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}}{a}\right) -$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \Bigg) + \\
& 2(2a^3b + 6ab^3) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \csc[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \Bigg) + \frac{1}{d}$$

$$\frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \sec[c+dx] (2a^2 B \sin[c+dx] + 7b^2 B \sin[c+dx]) + \right.}{a} \\ \left. \frac{b}{B} \sec[c+dx] \tan[c+dx] \right)$$

- **Problem 420: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{3/2} (A+B \cos[c+dx])}{\sqrt{a+b \cos[c+dx]}} dx$$

Optimal (type 4, 479 leaves, 7 steps):

$$-\frac{1}{4ab^2d} (a-b) \sqrt{a+b} (4Ab-3aB) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{4b^2d}$$

$$\sqrt{a+b} (4Ab-3aB+2bB) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{4b^3d} \sqrt{a+b} (4aAb-3a^2B-4b^2B) \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}}$$

$$\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(4Ab-3aB) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{4b^2d \sqrt{\cos[c+dx]}} + \frac{B \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2bd}$$

Result (type 4, 1175 leaves):

$$\frac{B \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2bd} + \frac{1}{8bd}$$

$$\begin{aligned}
& \left( - \left( 4 a (4 A b - a B) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad 16 a b B \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \quad 2 (4 A b - 3 a B) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) +
\end{aligned}$$

$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\ \left. \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right)$$

- **Problem 421: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Cos}[c+dx]} (A+B \operatorname{Cos}[c+dx])}{\sqrt{a+b \operatorname{Cos}[c+dx]}} dx$$

Optimal (type 4, 427 leaves, 7 steps):

$$\begin{aligned}
& -\frac{1}{abd} (a-b) \sqrt{a+b} B \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{\sqrt{a+b} B \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{bd} - \frac{1}{b^2 d} \\
& \sqrt{a+b} (2Ab - aB) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{aB \sin[c+dx]}{bd \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}} + \frac{B \sqrt{\cos[c+dx]} \sin[c+dx]}{d \sqrt{a+b \cos[c+dx]}}
\end{aligned}$$

Result (type 4, 4017 leaves):

$$\begin{aligned}
& \left( (1 + \cos[c+dx])^{3/2} \left( \frac{A \sqrt{\cos[c+dx]}}{\sqrt{a+b \cos[c+dx]}} + \frac{B \cos[c+dx]^{3/2}}{\sqrt{a+b \cos[c+dx]}} \right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \left. \left( 2i(a-b)B \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + \right. \right. \\
& \left. \left. 4i(Ab - aB) \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] - \right. \right. \\
& \left. \left. 8iAb \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + \right. \right. \\
& \left. \left. 4iaB \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + b \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \right. \right. \\
& \left. \left. \sec\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] + 2a \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] - b \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( 4 b \sqrt{\frac{a-b}{a+b}} d \sqrt{a+b \cos[c+dx]} \right) \left( \frac{1}{8 \sqrt{\frac{a-b}{a+b}} (a+b \cos[c+dx])^{3/2}} (1 + \cos[c+dx])^{3/2} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] \right. \\
& \left( 2 i (a-b) B \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + \right. \\
& 4 i (Ab - aB) \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] - 8 i Ab \\
& \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + 4 i a B \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \\
& \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + b \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] + \\
& \left. 2 a \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - b \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) - \\
& \frac{1}{8 b \sqrt{\frac{a-b}{a+b}} \sqrt{a+b \cos[c+dx]}} 3 \sqrt{1 + \cos[c+dx]} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] \\
& \left( 2 i (a-b) B \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + \right. \\
& 4 i (Ab - aB) \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] - 8 i Ab \\
& \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + 4 i a B \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \\
& \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + b \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] +
\end{aligned}$$

$$\begin{aligned}
& \left. 2 a \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - b \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \right) + \\
& \frac{1}{4 b \sqrt{\frac{a-b}{a+b}} \sqrt{a+b \cos [c+d x]}} (1+\cos [c+d x])^{3 / 2} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \\
& \left( 2 i (a-b) B \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] + \right. \\
& 4 i (A b-a B) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] - 8 i A b \\
& \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] + 4 i a B \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \\
& \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] + b \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right] + \\
& \left. 2 a \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - b \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \right) + \\
& \frac{1}{4 b \sqrt{\frac{a-b}{a+b}} \sqrt{a+b \cos [c+d x]}} (1+\cos [c+d x])^{3 / 2} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \left( \frac{3}{2} b \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Cos}\left[\frac{3}{2}(c+d x)\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] + \right. \\
& a \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 - \frac{1}{2} b \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 + \frac{1}{\sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} \\
& \left. i (a-b) B \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] \left( -\frac{b \operatorname{Sin}[c+d x]}{(a+b)(1+\cos [c+d x])} + \frac{(a+b \cos [c+d x]) \operatorname{Sin}[c+d x]}{(a+b)(1+\cos [c+d x])^2} \right) + \right.
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{\sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} 2 i(A b-a B) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] \\
& \left(-\frac{b \sin [c+d x]}{(a+b)(1+\cos [c+d x])}+\frac{(a+b \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2}\right)-\frac{1}{\sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} 4 i A b \\
& \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right]\left(-\frac{b \sin [c+d x]}{(a+b)(1+\cos [c+d x])}+\frac{(a+b \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2}\right)+ \\
& \frac{1}{\sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} 2 i a B \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right]\left(-\frac{b \sin [c+d x]}{(a+b)(1+\cos [c+d x])}+\right. \\
& \left.\frac{(a+b \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2}\right)+\frac{b \sqrt{\frac{a-b}{a+b}} B \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]\left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2}-\frac{\sin [c+d x]}{1+\cos [c+d x]}\right) \sin \left[\frac{3}{2}(c+d x)\right]}{2 \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}}+ \\
& \frac{a \sqrt{\frac{a-b}{a+b}} B\left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2}-\frac{\sin [c+d x]}{1+\cos [c+d x]}\right) \tan \left[\frac{1}{2}(c+d x)\right]-b \sqrt{\frac{a-b}{a+b}} B\left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2}-\frac{\sin [c+d x]}{1+\cos [c+d x]}\right) \tan \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}}-\frac{1}{2} b \sqrt{\frac{a-b}{a+b}} \\
& B \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \sin \left[\frac{3}{2}(c+d x)\right] \tan \left[\frac{1}{2}(c+d x)\right]-\frac{2 \sqrt{\frac{a-b}{a+b}}(A b-a B) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{\sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{1+\frac{(a-b) \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}} \\
& \frac{(a-b) \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}}{\sqrt{1+\frac{(a-b) \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}+
\end{aligned}$$

$$\frac{4 A b \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{\sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{1+\frac{(a-b) \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}$$

$$\left. \frac{2 a \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{\sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{1+\frac{(a-b) \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}\right)$$

■ **Problem 423: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A+B \cos [c+d x]}{\cos [c+d x]^{3 / 2} \sqrt{a+b \cos [c+d x]}} d x$$

Optimal (type 4, 230 leaves, 3 steps):

$$\frac{1}{a^2 d} 2 A (a-b) \sqrt{a+b} \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} -$$

$$\frac{1}{a d} 2 \sqrt{a+b} (A-B) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}$$

Result (type 4, 1164 leaves):

$$\frac{2 A \sqrt{a+b \cos [c+d x]} \operatorname{Sin}[c+d x]}{a d \sqrt{\cos [c+d x]}}$$

$$\frac{1}{a d} \left( - \left( \left( 4 a (A b - a B) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right.$$

$$\left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \right)$$

$$\begin{aligned}
& 4 a^2 A \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2 A b \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\ \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\ \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \Bigg)$$

■ **Problem 424: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Cos}[c + dx]}{\operatorname{Cos}[c + dx]^{5/2} \sqrt{a + b \operatorname{Cos}[c + dx]}} dx$$

Optimal (type 4, 290 leaves, 4 steps):

$$-\frac{1}{3a^3d} \\ 2(a-b)\sqrt{a+b} (2Ab - 3aB) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \\ \frac{1}{3a^2d} 2\sqrt{a+b} (2Ab + a(A-3B)) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\ \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{2A\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{3ad \operatorname{Cos}[c+dx]^{3/2}}$$

Result (type 4, 1238 leaves):

$$\frac{1}{3a^2d}$$

$$\begin{aligned}
& \left( - \left( 4 a (a^2 A + 2 A b^2 - 3 a b B) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left. 4 a (2 a A b - 3 a^2 B) \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+ \right. \right. \right. \\
& \quad \left. \left. \left. dx\right] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left. \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \quad \left. 2 (2 A b^2 - 3 a b B) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \right.
\end{aligned}$$

$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\ \left. \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right) + \\ \frac{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2 \operatorname{Sec}[c+dx] (-2 A b \operatorname{Sin}[c+dx] + 3 a B \operatorname{Sin}[c+dx])}{3 a^2} + \frac{2 A \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{3 a} \right)}{d}$$

■ **Problem 425: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Cos}[c + dx]}{\operatorname{Cos}[c + dx]^{7/2} \sqrt{a + b \operatorname{Cos}[c + dx]}} dx$$

Optimal (type 4, 363 leaves, 5 steps):

$$\frac{1}{15 a^4 d} 2 (a-b) \sqrt{a+b} (9 a^2 A + 8 A b^2 - 10 a b B) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{15 a^3 d}$$

$$2 \sqrt{a+b} (8 A b^2 + a^2 (9 A - 5 B) - 2 a b (A + 5 B)) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{2 A \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{5 a d \operatorname{Cos}[c+d x]^{5/2}} - \frac{2 (4 A b - 5 a B) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{15 a^2 d \operatorname{Cos}[c+d x]^{3/2}}$$

Result (type 4, 1319 leaves):

$$-\frac{1}{15 a^3 d} \left( - \left( 4 a (7 a^2 A b + 8 A b^3 - 5 a^3 B - 10 a b^2 B) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\ \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right) / \\ \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (9 a^3 A + 8 a A b^2 - 10 a^2 b B) \\ \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right. \\ \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) -$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& 2(9a^2Ab + 8Ab^3 - 10ab^2B) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \right. \\
& \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \left. \csc[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$



$$\left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) +$$

$$\frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2 \sec^2[c+dx] (-4Ab \sin[c+dx] + 5aB \sin[c+dx])}{15a^2} + \right.$$

$$\frac{2 \sec[c+dx] (9a^2A \sin[c+dx] + 8Ab^2 \sin[c+dx] - 10abB \sin[c+dx])}{15a^3} +$$

$$\left. \frac{2A \sec[c+dx]^2 \tan[c+dx]}{5a} \right)$$

■ **Problem 426: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{3/2} (A+B \cos[c+dx])}{(a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 500 leaves, 7 steps):

$$\frac{1}{ab^2 \sqrt{a+b} d} (2aAb - 3a^2B + b^2B) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{b^2 \sqrt{a+b} d}$$

$$(2Ab - (3a+b)B) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{b^3 d}$$

$$\sqrt{a+b} (2Ab - 3aB) \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2a(Ab - aB) \sqrt{\cos[c+dx]} \sin[c+dx]}{b(a^2 - b^2) d \sqrt{a+b \cos[c+dx]}} - \frac{(2aAb - 3a^2B + b^2B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b^2(a^2 - b^2) d \sqrt{\cos[c+dx]}}$$

Result (type 4, 1234 leaves):

$$\frac{2 \sqrt{\cos[c+dx]} (-aAb \sin[c+dx] + a^2B \sin[c+dx])}{b(-a^2 + b^2) d \sqrt{a+b \cos[c+dx]}} + \frac{1}{2(a-b)b(a+b)d}$$

$$\begin{aligned}
& \left( - \left( 4 a (a^2 B - b^2 B) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left. 4 a (-2 A b^2 + 2 a b B) \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+ \right. \right. \right. \\
& \quad \left. \left. \left. dx \right] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left. \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \quad \left. 2 (-2 a A b + 3 a^2 B - b^2 B) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx] \right. \right. \\
& \quad \left. \left. b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) \right) +
\end{aligned}$$

$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\ \left. \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right)$$

- **Problem 427: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Cos}[c+dx]} (A+B \operatorname{Cos}[c+dx])}{(a+b \operatorname{Cos}[c+dx])^{3/2}} dx$$

Optimal (type 4, 416 leaves, 6 steps):

$$\begin{aligned}
& \frac{2 (A b - a B) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}}}{a b \sqrt{a+b} d} + \\
& \frac{2 (A b - a B) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}}}{a b \sqrt{a+b} d} - \frac{1}{b^2 d} \\
& \frac{2 \sqrt{a+b} B \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}}}{2 a (A b - a B) \operatorname{Sin}[c + d x]} + \\
& \frac{b (a^2 - b^2) d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a+b \operatorname{Cos}[c + d x]}}{b (a^2 - b^2) d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a+b \operatorname{Cos}[c + d x]}}
\end{aligned}$$

Result (type 4, 1012 leaves):

$$\begin{aligned}
& \frac{2 \sqrt{\operatorname{Cos}[c + d x]} (-A b \operatorname{Sin}[c + d x] + a B \operatorname{Sin}[c + d x])}{(a^2 - b^2) d \sqrt{a+b \operatorname{Cos}[c + d x]}} - \frac{1}{(-a+b)(a+b)d} \\
& \left( -4 a (a A - b B) \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x]} \right. \right. \right. \\
& \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x]} \right. \\
& \left. \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 2 (A b - a B) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Sec}[c+d x]}{a+b}}}} + \right. \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \\
& \left. \left. \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \right. \\
& \left. \left. \left( b \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{b \sqrt{\operatorname{Cos}[c+d x]}} \right) \right)
\end{aligned}$$

■ **Problem 428: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Cos}[c+d x]}{\sqrt{\operatorname{Cos}[c+d x]} (a+b \operatorname{Cos}[c+d x])^{3/2}} dx$$

Optimal (type 4, 284 leaves, 4 steps):

$$\frac{2 (A b - a B) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}}}{a^2 \sqrt{a+b} d} +$$

$$\frac{2 (A + B) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}}}{a \sqrt{a+b} d} - \frac{2 (A b - a B) \operatorname{Sin}[c + d x]}{(a^2 - b^2) d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + b \operatorname{Cos}[c + d x]}}$$

Result (type 4, 1223 leaves):

$$-\frac{2 \sqrt{\operatorname{Cos}[c + d x]} (-A b^2 \operatorname{Sin}[c + d x] + a b B \operatorname{Sin}[c + d x])}{a (a^2 - b^2) d \sqrt{a + b \operatorname{Cos}[c + d x]}} + \frac{1}{a (a - b) (a + b) d}$$

$$\left( - \left( 4 a (a^2 A - A b^2) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a \right.$$

$$\left. (-a A b + a^2 B) \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - \right.$$

$$\left. \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right)$$

$$\begin{aligned}
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right/ \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) \right\} + \\
& 2(-A b^2 + a b B) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \sec [c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \sec [c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \sec [c+d x]}{a+b}} \right. + \\
& \left. \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \csc [c+d x] \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right/ \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\
& \left. \left. \left. \csc [c+d x] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right/ \right. \right. \\
& \left. \left. \left. \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) \right) \right)
\end{aligned}$$

■ **Problem 429: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx]}{\cos[c + dx]^{3/2} (a + b \cos[c + dx])^{3/2}} dx$$

Optimal (type 4, 305 leaves, 4 steps):

$$\frac{1}{a^3 \sqrt{a+b} d} 2 (a^2 A - 2 A b^2 + a b B) \cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c + dx]}{\sqrt{a+b} \sqrt{\cos[c + dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a+b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a-b}} -$$

$$\frac{1}{a^2 \sqrt{a+b} d} 2 (2 A b + a(A - B)) \cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c + dx]}{\sqrt{a+b} \sqrt{\cos[c + dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a+b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a-b}} +$$

$$\frac{2 b (A b - a B) \sin[c + dx]}{a (a^2 - b^2) d \sqrt{\cos[c + dx]} \sqrt{a + b \cos[c + dx]}}$$

Result (type 4, 1281 leaves):

$$\frac{1}{a^2 (-a+b) (a+b) d} \left( - \left( 4 a (2 a^2 A b - 2 A b^3 - a^3 B + a b^2 B) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a \sqrt{2}}\right]}, -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.$$

$$\left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4 a (a^3 A - 2 a A b^2 + a^2 b B)$$

$$\left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a \sqrt{2}}\right]}, -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) -$$



$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \Bigg) + \\
& 2(a^2 A b - 2 A b^3 + a b^2 B) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) \Bigg) +
\end{aligned}$$

$$\left. \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) + \frac{\sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left( \frac{2(-A b^3 \sin [c+d x]+a b^2 B \sin [c+d x])}{a^2(a^2-b^2)(a+b \cos [c+d x])} + \frac{2 A \tan [c+d x]}{a^2} \right)}{d}$$

■ **Problem 430: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A+B \cos [c+d x]}{\cos [c+d x]^{5/2} (a+b \cos [c+d x])^{3/2}} dx$$

Optimal (type 4, 393 leaves, 5 steps):

$$-\frac{1}{3 a^4 \sqrt{a+b} d} 2 \left( 5 a^2 A b - 8 A b^3 - 3 a^3 B + 6 a b^2 B \right) \cot [c+d x]$$

$$\text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \frac{1}{3 a^3 \sqrt{a+b} d}$$

$$2(a+2b)(4Ab+a(A-3B)) \cot [c+d x] \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} +$$

$$\frac{2b(Ab-aB) \sin [c+d x]}{a(a^2-b^2)d \cos [c+d x]^{3/2} \sqrt{a+b \cos [c+d x]}} + \frac{2(a^2A-4Ab^2+3abB) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{3a^2(a^2-b^2)d \cos [c+d x]^{3/2}}$$

Result (type 4, 1357 leaves):

$$\frac{1}{3 a^3 (a-b)(a+b) d} \left( - \left( 4 a \left( a^4 A + 7 a^2 A b^2 - 8 A b^4 - 6 a^3 b B + 6 a b^3 B \right) \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos [c+d x] \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \csc [c+d x] \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right) \right) /$$

$$\left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - 4 a \left( 5 a^3 A b - 8 a A b^3 - 3 a^4 B + 6 a^2 b^2 B \right)$$

$$\begin{aligned}
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \left. \right) + \\
& 2 \left( 5 a^2 A b^2 - 8 A b^4 - 3 a^3 b B + 6 a b^3 B \right) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
\left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
\left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \left. \right) + \\
\frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2 \operatorname{Sec}[c+dx] (-5Ab \operatorname{Sin}[c+dx] + 3aB \operatorname{Sin}[c+dx])}{3a^3} - \right. \\
\left. \frac{2(-Ab^4 \operatorname{Sin}[c+dx] + ab^3B \operatorname{Sin}[c+dx])}{a^3(a^2-b^2)(a+b \operatorname{Cos}[c+dx])} + \right. \\
\left. \frac{2A \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{3a^2} \right)$$

■ **Problem 431: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^{5/2} (A+B \operatorname{Cos}[c+dx])}{(a+b \operatorname{Cos}[c+dx])^{5/2}} dx$$

Optimal (type 4, 674 leaves, 8 steps):

$$\frac{1}{3 a (a-b) b^3 (a+b)^{3/2} d} (6 a^3 A b - 14 a A b^3 - 15 a^4 B + 26 a^2 b^2 B - 3 b^4 B) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} -$$

$$\frac{1}{3 (a-b) b^3 (a+b)^{3/2} d} (6 a^2 A b + 2 a A b^2 - 12 A b^3 - 15 a^3 B - 5 a^2 b B + 21 a b^2 B + 3 b^3 B) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{b^4 d}$$

$$\sqrt{a+b} (2 A b - 5 a B) \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{2 a (A b - a B) \operatorname{Cos}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{3 b (a^2 - b^2) d (a+b \operatorname{Cos}[c+d x])^{3/2}} + \frac{2 a (2 a^2 A b - 6 A b^3 - 5 a^3 B + 9 a b^2 B) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 b^2 (a^2 - b^2)^2 d \sqrt{a+b \operatorname{Cos}[c+d x]}} -$$

$$\frac{(6 a^3 A b - 14 a A b^3 - 15 a^4 B + 26 a^2 b^2 B - 3 b^4 B) \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 b^3 (a^2 - b^2)^2 d \sqrt{\operatorname{Cos}[c+d x]}}$$

Result (type 4, 1396 leaves):

$$\frac{1}{d} \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]}$$

$$\left( -\frac{2(-a^2 A b \operatorname{Sin}[c+d x] + a^3 B \operatorname{Sin}[c+d x])}{3 b^2 (-a^2 + b^2) (a+b \operatorname{Cos}[c+d x])^2} - \frac{2(-3 a^3 A b \operatorname{Sin}[c+d x] + 7 a A b^3 \operatorname{Sin}[c+d x] + 6 a^4 B \operatorname{Sin}[c+d x] - 10 a^2 b^2 B \operatorname{Sin}[c+d x])}{3 b^2 (-a^2 + b^2)^2 (a+b \operatorname{Cos}[c+d x])} \right) +$$

$$\frac{1}{6 (a-b)^2 b^2 (a+b)^2 d} \left( -\left( 4 a (-2 a^3 A b + 2 a A b^3 + 5 a^4 B - 8 a^2 b^2 B + 3 b^4 B) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) /$$

$$\left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (2 a^2 A b^2 + 6 A b^4 + 4 a^3 b B - 12 a b^3 B)$$

$$\left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) -$$

$$\left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right.$$

$$\left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + 2 \left( -6 a^3 A b + 14 a A b^3 + 15 a^4 B - 26 a^2 b^2 B + 3 b^4 B \right)$$

$$\left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) +$$

$$\frac{1}{b} 2 a \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right)$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right/ \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+d x] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right/ \right. \\
& \left. \left. \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) \right)
\end{aligned}$$

- **Problem 432: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^{3/2} (A+B \cos [c+d x])}{(a+b \cos [c+d x])^{5/2}} dx$$

Optimal (type 4, 545 leaves, 7 steps):

$$\frac{1}{3 a (a-b) b^2 (a+b)^{3/2} d}$$

$$2 (4 A b^3 + 3 a^3 B - 7 a b^2 B) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{1}{3 a (a-b) b^2 (a+b)^{3/2} d} 2 (a A b^2 - 3 A b^3 - 3 a^3 B - a^2 b B + 6 a b^2 B) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{b^3 d}$$

$$2 \sqrt{a+b} B \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{2 a (A b - a B) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 b (a^2 - b^2) d (a+b \operatorname{Cos}[c+d x])^{3/2}} - \frac{2 a (4 A b^3 + 3 a^3 B - 7 a b^2 B) \operatorname{Sin}[c+d x]}{3 b^2 (a^2 - b^2)^2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]}}$$

Result (type 4, 1342 leaves):

$$\frac{1}{d} \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \left( \frac{2 (-a A b \operatorname{Sin}[c+d x] + a^2 B \operatorname{Sin}[c+d x])}{3 b (-a^2 + b^2) (a+b \operatorname{Cos}[c+d x])^2} + \frac{2 (4 A b^3 \operatorname{Sin}[c+d x] + 3 a^3 B \operatorname{Sin}[c+d x] - 7 a b^2 B \operatorname{Sin}[c+d x])}{3 b (-a^2 + b^2)^2 (a+b \operatorname{Cos}[c+d x])} \right) -$$

$$\frac{1}{3 (a-b)^2 b (a+b)^2 d} \left( - \left( 4 a (-a^2 A b + A b^3 + a^3 B - a b^2 B) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \right.$$

$$\left. \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (4 a A b^2 - a^2 b B - 3 b^3 B) \right.$$

$$\left. \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right.$$



$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \csc [c+d x] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) \right) + \\
& 2\left(4 A b^3+3 a^3 B-7 a b^2 B\right) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \sec [c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \sec [c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \sec [c+d x]}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \csc [c+d x] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \right) \right)
\end{aligned}$$

$$\left( \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}}$$

■ **Problem 433: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]} (A+B \cos[c+dx])}{(a+b \cos[c+dx])^{5/2}} dx$$

Optimal (type 4, 391 leaves, 5 steps):

$$-\frac{1}{3a^2(a-b)(a+b)^{3/2}d} 2(3a^2A+Ab^2-4abB) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{3a(a-b)(a+b)^{3/2}d}$$

$$2(3aA-Ab+aB-3bB) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{2(Ab-aB) \sqrt{\cos[c+dx]} \sin[c+dx]}{3(a^2-b^2)d(a+b \cos[c+dx])^{3/2}} + \frac{2(3a^2A+Ab^2-4abB) \sin[c+dx]}{3(a^2-b^2)^2d \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}}$$

Result (type 4, 1335 leaves):

$$\frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2(-Ab \sin[c+dx]+aB \sin[c+dx])}{3(a^2-b^2)(a+b \cos[c+dx])^2} - \frac{2(3a^2Ab \sin[c+dx]+Ab^3 \sin[c+dx]-4ab^2B \sin[c+dx])}{3a(a^2-b^2)^2(a+b \cos[c+dx])} \right) +$$

$$\frac{1}{3a(a-b)^2(a+b)^2d} \left( - \left( 4a(-a^2Ab+Ab^3+a^3B-ab^2B) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right) \right)$$

$$\begin{aligned}
& \left. \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right)} \right/ \\
& \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - 4 a \left( 3 a^3 A+a A b^2-4 a^2 b B \right) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right. \\
& \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right) \right) \right/ \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \\
& \left. \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right) \right) \right/ \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) \right) + \\
& 2 \left( 3 a^2 A b+A b^3-4 a b^2 B \right) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Sec}[c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Sec}[c+d x]}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right.
\end{aligned}$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) -$$

$$\left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right.$$

$$\left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/$$

$$\left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right)$$

■ **Problem 434: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx]}{\sqrt{\cos[c + dx]} (a + b \cos[c + dx])^{5/2}} dx$$

Optimal (type 4, 429 leaves, 5 steps):

$$\frac{1}{3a^3(a-b)(a+b)^{3/2}d} 2(6a^2Ab - 2Ab^3 - 3a^3B - ab^2B) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{3a^2 \sqrt{a+b} (a^2-b^2) d}$$

$$2(2Ab^2 - 3a^2(A+B) + ab(3A+B)) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}}$$

$$\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{2b(Ab-ab) \sqrt{\cos[c+dx]} \sin[c+dx]}{3a(a^2-b^2)d(a+b \cos[c+dx])^{3/2}} - \frac{2(6a^2Ab - 2Ab^3 - 3a^3B - ab^2B) \sin[c+dx]}{3a(a^2-b^2)^2 d \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}}$$

Result (type 4, 1384 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \\
& \left( -\frac{2(-Ab^2 \sin[c+dx] + abB \sin[c+dx])}{3a(a^2-b^2)(a+b \cos[c+dx])^2} - \frac{2(-6a^2Ab^2 \sin[c+dx] + 2Ab^4 \sin[c+dx] + 3a^3bB \sin[c+dx] + ab^3B \sin[c+dx])}{3a^2(a^2-b^2)^2(a+b \cos[c+dx])} \right) + \\
& \frac{1}{3a^2(a-b)^2(a+b)^2d} \left( -\left( 4a(3a^4A - 5a^2Ab^2 + 2Ab^4 - a^3bB + ab^3B) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a(-6a^3Ab + 2aAb^3 + 3a^4B + a^2b^2B) \\
& \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 2 \left( -6 a^2 A b^2 + 2 A b^4 + 3 a^3 b B + a b^3 B \right) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right)
\end{aligned}$$

■ **Problem 435: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Cos}[c+dx]}{\operatorname{Cos}[c+dx]^{3/2} (a+b \operatorname{Cos}[c+dx])^{5/2}} dx$$

Optimal (type 4, 456 leaves, 5 steps):

$$\frac{1}{3 a^4 (a-b) (a+b)^{3/2} d} \left( 3 a^4 A - 15 a^2 A b^2 + 8 A b^4 + 6 a^3 b B - 2 a b^3 B \right) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{3 a^3 \sqrt{a+b} (a^2-b^2) d} \left( 8 A b^3 - 3 a^3 (A-B) + 2 a b^2 (3 A-B) - 3 a^2 b (3 A+B) \right)$$

$$\operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{2 b (A b - a B) \operatorname{Sin}[c+d x]}{3 a (a^2-b^2) d \sqrt{\operatorname{Cos}[c+d x]} (a+b \operatorname{Cos}[c+d x])^{3/2}} + \frac{2 b (8 a^2 A b - 4 A b^3 - 5 a^3 B + a b^2 B) \operatorname{Sin}[c+d x]}{3 a^2 (a^2-b^2)^2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]}}$$

Result (type 4, 1431 leaves):

$$-\frac{1}{3 a^3 (a-b)^2 (a+b)^2 d}$$

$$\left( -4 a \left( 9 a^4 A b - 17 a^2 A b^3 + 8 A b^5 - 3 a^5 B + 5 a^3 b^2 B - 2 a b^4 B \right) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right.$$

$$\left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) /$$

$$\left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a \left( 3 a^5 A - 15 a^3 A b^2 + 8 a A b^4 + 6 a^4 b B - 2 a^2 b^3 B \right)$$

$$\left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) -$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + 2 \left( 3 a^4 A b - 15 a^2 A b^3 + 8 A b^5 + 6 a^3 b^2 B - 2 a b^4 B \right) \\
& \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right) -
\end{aligned}$$





$$\begin{aligned}
& - \frac{1}{3 a^5 (a-b) (a+b)^{3/2} d} 2 \left( 8 a^4 A b - 28 a^2 A b^3 + 16 A b^5 - 3 a^5 B + 15 a^3 b^2 B - 8 a b^4 B \right) \\
& \quad \text{Cot}[c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \\
& \frac{1}{3 a^4 \sqrt{a+b} (a^2-b^2) d} 2 \left( 16 A b^4 - a^4 (A-3 B) + 4 a b^3 (3 A-2 B) - 9 a^3 b (A-B) - 2 a^2 b^2 (8 A+3 B) \right) \text{Cot}[c+d x] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \\
& \quad \frac{2 b (A b-a B) \sin [c+d x]}{3 a (a^2-b^2) d \cos [c+d x]^{3/2} (a+b \cos [c+d x])^{3/2}} + \frac{2 b (10 a^2 A b-6 A b^3-7 a^3 B+3 a b^2 B) \sin [c+d x]}{3 a^2 (a^2-b^2)^2 d \cos [c+d x]^{3/2} \sqrt{a+b \cos [c+d x]}} + \\
& \quad \frac{2 (a^4 A-13 a^2 A b^2+8 A b^4+8 a^3 b B-4 a b^3 B) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{3 a^3 (a^2-b^2)^2 d \cos [c+d x]^{3/2}}
\end{aligned}$$

Result (type 4, 1499 leaves):

$$\begin{aligned}
& \frac{1}{3 a^4 (a-b)^2 (a+b)^2 d} \\
& \left( - \left( 4 a (a^6 A+15 a^4 A b^2-32 a^2 A b^4+16 A b^6-9 a^5 b B+17 a^3 b^3 B-8 a b^5 B) \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\
& \quad \left. \left. \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \csc [c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}}{a \sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right) \right) / \\
& \quad \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - 4 a (8 a^5 A b-28 a^3 A b^3+16 a A b^5-3 a^6 B+15 a^4 b^2 B-8 a^2 b^4 B) \\
& \quad \left( \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \csc [c+d x] \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \\
& \left. (b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}) \right) + 2 (8 a^4 A b^2 - 28 a^2 A b^4 + 16 A b^6 - 3 a^5 b B + 15 a^3 b^3 B - 8 a b^5 B) \\
& \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \text{Csc}[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
\left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
\left. \left. \frac{\sqrt{a+b \cos[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right) + \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \\
\left( \frac{2 \operatorname{Sec}[c+dx] (-8 A b \operatorname{Sin}[c+dx] + 3 a B \operatorname{Sin}[c+dx])}{3 a^4} - \frac{2 (-A b^4 \operatorname{Sin}[c+dx] + a b^3 B \operatorname{Sin}[c+dx])}{3 a^3 (a^2 - b^2) (a+b \cos[c+dx])^2} - \right. \\
\left. \frac{2 (-12 a^2 A b^4 \operatorname{Sin}[c+dx] + 8 A b^6 \operatorname{Sin}[c+dx] + 9 a^3 b^3 B \operatorname{Sin}[c+dx] - 5 a b^5 B \operatorname{Sin}[c+dx])}{3 a^4 (a^2 - b^2)^2 (a+b \cos[c+dx])} + \right. \\
\left. \frac{2 A \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{3 a^3} \right)$$

■ **Problem 437: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^{3/2} (a B + b B \cos[c+dx])}{(a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 419 leaves, 9 steps):

$$\begin{aligned}
& -\frac{1}{abd} (a-b) \sqrt{a+b} B \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \\
& \frac{\sqrt{a+b} B \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}}}{bd} + \frac{1}{b^2 d} \\
& a \sqrt{a+b} B \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \\
& \frac{a B \operatorname{Sin}[c+dx]}{bd \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]}} + \frac{B \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{d \sqrt{a+b \operatorname{Cos}[c+dx]}}
\end{aligned}$$

Result (type 4, 480 leaves) :

$$\begin{aligned}
& \frac{1}{2 b \sqrt{\frac{a-b}{a+b}} d \sqrt{\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{a+b \operatorname{Cos}[c+dx]}} \\
& B \sqrt{\operatorname{Cos}[c+dx]} \left( 2 i (a-b) \sqrt{\frac{a+b \operatorname{Cos}[c+dx]}{(a+b)(1+\operatorname{Cos}[c+dx])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] - \right. \\
& 4 i a \sqrt{\frac{a+b \operatorname{Cos}[c+dx]}{(a+b)(1+\operatorname{Cos}[c+dx])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + \\
& \left. 4 i a \sqrt{\frac{a+b \operatorname{Cos}[c+dx]}{(a+b)(1+\operatorname{Cos}[c+dx])}} \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + b \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + 2 a \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - b \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right)
\end{aligned}$$

- **Problem 440: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a B + b B \operatorname{Cos}[c+dx]}{\operatorname{Cos}[c+dx]^{3/2} (a+b \operatorname{Cos}[c+dx])^{3/2}} dx$$

Optimal (type 4, 226 leaves, 4 steps) :

$$\frac{1}{a^2 d} 2 (a-b) \sqrt{a+b} B \cot [c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} -$$

$$\frac{2 \sqrt{a+b} B \cot [c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}}{a d}$$

Result (type 4, 896 leaves):

$$B \left( \frac{1}{d} 4 a \left( \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \csc [c+d x] \right. \right. \right.$$

$$\left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \right.$$

$$\left. \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \csc [c+d x] \right. \right.$$

$$\left. \left. \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) \right) +$$

$$\frac{2 \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{a d \sqrt{\cos [c+d x]}} - \frac{1}{a \sqrt{\frac{a-b}{a+b}} d \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{a+b \cos [c+d x]}} \sqrt{\cos [c+d x]}$$

$$\left( 2 i (a-b) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] - \right.$$

$$\begin{aligned}
& 4 i a \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right]+ \\
& 4 i a \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right]+b \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right]+2 a \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[\frac{1}{2}(c+d x)\right]-b \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[\frac{1}{2}(c+d x)\right]\right)
\end{aligned}$$

■ **Problem 441: Unable to integrate problem.**

$$\int \frac{1+\cos [c+d x]}{\cos [c+d x]^{3 / 2} \sqrt{2+3 \cos [c+d x]}} d x$$

Optimal (type 4, 72 leaves, 1 step):

$$\frac{\operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{2+3 \cos [c+d x]}}{\sqrt{5} \sqrt{\cos [c+d x]}}\right], 5\right] \sqrt{-1-\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]}}{d}$$

Result (type 8, 35 leaves):

$$\int \frac{1+\cos [c+d x]}{\cos [c+d x]^{3 / 2} \sqrt{2+3 \cos [c+d x]}} d x$$

■ **Problem 442: Attempted integration timed out after 120 seconds.**

$$\int \frac{1+\cos [c+d x]}{\cos [c+d x]^{3 / 2} \sqrt{-2+3 \cos [c+d x]}} d x$$

Optimal (type 4, 70 leaves, 1 step):

$$\frac{\sqrt{5} \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-2+3 \cos [c+d x]}}{\sqrt{\cos [c+d x]}}\right], \frac{1}{5}\right] \sqrt{-1+\operatorname{Sec}[c+d x]} \sqrt{1+\operatorname{Sec}[c+d x]}}{d}$$

Result (type 1, 1 leaves):

???

■ **Problem 443: Attempted integration timed out after 120 seconds.**

$$\int \frac{1+\cos [c+d x]}{\sqrt{2-3 \cos [c+d x]} \cos [c+d x]^{3 / 2}} d x$$

Optimal (type 4, 93 leaves, 2 steps):

$$\frac{1}{d} \sqrt{5} \sqrt{-\cos[c+dx]} \sqrt{\cos[c+dx]} \csc[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{2-3\cos[c+dx]}}{\sqrt{-\cos[c+dx]}}\right], \frac{1}{5}\right] \sqrt{-1+\sec[c+dx]} \sqrt{1+\sec[c+dx]}$$

Result (type 1, 1 leaves):

???

■ **Problem 444: Unable to integrate problem.**

$$\int \frac{1 + \cos[c+dx]}{\sqrt{-2-3\cos[c+dx]} \cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 95 leaves, 2 steps):

$$\frac{1}{d} \sqrt{-\cos[c+dx]} \sqrt{\cos[c+dx]} \csc[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-2-3\cos[c+dx]}}{\sqrt{5} \sqrt{-\cos[c+dx]}}\right], 5\right] \sqrt{-1-\sec[c+dx]} \sqrt{1-\sec[c+dx]}$$

Result (type 8, 35 leaves):

$$\int \frac{1 + \cos[c+dx]}{\sqrt{-2-3\cos[c+dx]} \cos[c+dx]^{3/2}} dx$$

■ **Problem 445: Unable to integrate problem.**

$$\int \frac{1 + \cos[c+dx]}{\cos[c+dx]^{3/2} \sqrt{3+2\cos[c+dx]}} dx$$

Optimal (type 4, 72 leaves, 1 step):

$$\frac{2 \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{3+2\cos[c+dx]}}{\sqrt{5} \sqrt{\cos[c+dx]}}\right], -5\right] \sqrt{1-\sec[c+dx]} \sqrt{1+\sec[c+dx]}}{3d}$$

Result (type 8, 35 leaves):

$$\int \frac{1 + \cos[c+dx]}{\cos[c+dx]^{3/2} \sqrt{3+2\cos[c+dx]}} dx$$

■ **Problem 446: Unable to integrate problem.**

$$\int \frac{1 + \cos[c+dx]}{\sqrt{3-2\cos[c+dx]} \cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 74 leaves, 1 step):

$$\frac{2\sqrt{5} \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{3-2\cos[c+dx]}}{\sqrt{\cos[c+dx]}}\right], -\frac{1}{5}\right] \sqrt{1-\sec[c+dx]} \sqrt{1+\sec[c+dx]}}{3d}$$

Result (type 8, 35 leaves):



$$\int \frac{1 + \cos [c + d x]}{\sqrt{3 - 2 \cos [c + d x]} \cos [c + d x]^{3/2}} dx$$

- **Problem 447: Attempted integration timed out after 120 seconds.**

$$\int \frac{1 + \cos [c + d x]}{\cos [c + d x]^{3/2} \sqrt{-3 + 2 \cos [c + d x]}} dx$$

Optimal (type 4, 98 leaves, 2 steps):

$$-\frac{1}{3d} 2 \sqrt{5} \sqrt{-\cos [c + d x]} \sqrt{\cos [c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-3 + 2 \cos [c + d x]}}{\sqrt{-\cos [c + d x]}}\right], -\frac{1}{5}\right] \sqrt{1 - \operatorname{Sec}[c + d x]} \sqrt{1 + \operatorname{Sec}[c + d x]}$$

Result (type 1, 1 leaves):

???

- **Problem 448: Unable to integrate problem.**

$$\int \frac{1 + \cos [c + d x]}{\sqrt{-3 - 2 \cos [c + d x]} \cos [c + d x]^{3/2}} dx$$

Optimal (type 4, 96 leaves, 2 steps):

$$-\frac{1}{3d} 2 \sqrt{-\cos [c + d x]} \sqrt{\cos [c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-3 - 2 \cos [c + d x]}}{\sqrt{5} \sqrt{-\cos [c + d x]}}\right], -5\right] \sqrt{1 - \operatorname{Sec}[c + d x]} \sqrt{1 + \operatorname{Sec}[c + d x]}$$

Result (type 8, 35 leaves):

$$\int \frac{1 + \cos [c + d x]}{\sqrt{-3 - 2 \cos [c + d x]} \cos [c + d x]^{3/2}} dx$$

- **Problem 451: Result more than twice size of optimal antiderivative.**

$$\int (c \cos [e + f x])^m (a + b \cos [e + f x])^3 (A + B \cos [e + f x]) dx$$

Optimal (type 5, 406 leaves, 6 steps):

$$\begin{aligned}
& \frac{b \left( b^2 B (3+m) + 3 a A b (4+m) + 2 a^2 B (5+m) \right) (c \operatorname{Cos}[e+f x])^{1+m} \operatorname{Sin}[e+f x]}{c f (2+m) (4+m)} + \\
& \frac{b^2 (A b (4+m) + a B (6+m)) \operatorname{Cos}[e+f x] (c \operatorname{Cos}[e+f x])^{1+m} \operatorname{Sin}[e+f x]}{c f (3+m) (4+m)} + \frac{b B (c \operatorname{Cos}[e+f x])^{1+m} (a+b \operatorname{Cos}[e+f x])^2 \operatorname{Sin}[e+f x]}{c f (4+m)} - \\
& \left( \left( a^2 (2+m) (b B (1+m) + a A (4+m)) + b (1+m) (b^2 B (3+m) + 3 a A b (4+m) + 2 a^2 B (5+m)) \right) (c \operatorname{Cos}[e+f x])^{1+m} \right. \\
& \quad \left. \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Cos}[e+f x]^2 \right] \operatorname{Sin}[e+f x] \right) / \left( c f (1+m) (2+m) (4+m) \sqrt{\operatorname{Sin}[e+f x]^2} \right) - \\
& \left( \left( A b^3 (2+m) + 3 a b^2 B (2+m) + 3 a^2 A b (3+m) + a^3 B (3+m) \right) (c \operatorname{Cos}[e+f x])^{2+m} \right. \\
& \quad \left. \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \operatorname{Cos}[e+f x]^2 \right] \operatorname{Sin}[e+f x] \right) / \left( c^2 f (2+m) (3+m) \sqrt{\operatorname{Sin}[e+f x]^2} \right)
\end{aligned}$$

Result (type 5, 967 leaves):

$$\begin{aligned}
& \left( (c \cos[e + f x])^m (a + b \cos[e + f x])^3 (A + B \cos[e + f x]) \sec[e + f x]^4 (\sec[e + f x]^2)^{\frac{1-m}{2}} \right. \\
& \left( a^3 A \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 3 + \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] \tan[e + f x] + 3 a A b^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 3 + \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] \tan[e + f x] + \right. \\
& 3 a^2 b B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 3 + \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] \tan[e + f x] + b^3 B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 3 + \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] \tan[e + f x] + \\
& 3 a^2 A b \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{7}{2} + \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] \tan[e + f x] + \\
& A b^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{7}{2} + \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] \tan[e + f x] + a^3 B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{7}{2} + \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] \tan[e + f x] + \\
& 3 a b^2 B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{7}{2} + \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] \tan[e + f x] + \frac{2}{3} a^3 A \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, 3 + \frac{m}{2}, \frac{5}{2}, -\tan[e + f x]^2\right] \\
& \tan[e + f x]^3 + a A b^2 \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, 3 + \frac{m}{2}, \frac{5}{2}, -\tan[e + f x]^2\right] \tan[e + f x]^3 + \\
& a^2 b B \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, 3 + \frac{m}{2}, \frac{5}{2}, -\tan[e + f x]^2\right] \tan[e + f x]^3 + 2 a^2 A b \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{7}{2} + \frac{m}{2}, \frac{5}{2}, -\tan[e + f x]^2\right] \\
& \tan[e + f x]^3 + \frac{1}{3} A b^3 \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{7}{2} + \frac{m}{2}, \frac{5}{2}, -\tan[e + f x]^2\right] \tan[e + f x]^3 + \frac{2}{3} a^3 B \\
& \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{7}{2} + \frac{m}{2}, \frac{5}{2}, -\tan[e + f x]^2\right] \tan[e + f x]^3 + a b^2 B \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{7}{2} + \frac{m}{2}, \frac{5}{2}, -\tan[e + f x]^2\right] \tan[e + f x]^3 + \\
& \frac{1}{5} a^3 A \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, 3 + \frac{m}{2}, \frac{7}{2}, -\tan[e + f x]^2\right] \tan[e + f x]^5 + \frac{3}{5} a^2 A b \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, \frac{7}{2} + \frac{m}{2}, \frac{7}{2}, -\tan[e + f x]^2\right] \\
& \tan[e + f x]^5 + \frac{1}{5} a^3 B \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, \frac{7}{2} + \frac{m}{2}, \frac{7}{2}, -\tan[e + f x]^2\right] \tan[e + f x]^5 \left. \right) / \\
& \left( f \left( a^3 \left( B + A \sqrt{\sec[e + f x]^2} \right) + 3 a b^2 \left( B + A \sqrt{\sec[e + f x]^2} \right) + 3 a^2 b \left( A + B \sqrt{\sec[e + f x]^2} \right) + b^3 \left( A + B \sqrt{\sec[e + f x]^2} \right) \right) + \right. \\
& \left( A b^3 + 2 a^3 \left( B + A \sqrt{\sec[e + f x]^2} \right) + 3 a b^2 \left( B + A \sqrt{\sec[e + f x]^2} \right) + 3 a^2 b \left( 2 A + B \sqrt{\sec[e + f x]^2} \right) \right) \tan[e + f x]^2 + \\
& \left. a^2 \left( 3 A b + a B + a A \sqrt{\sec[e + f x]^2} \right) \tan[e + f x]^4 \right)
\end{aligned}$$

■ **Problem 454: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c \cos[e + f x])^m (A + B \cos[e + f x])}{a + b \cos[e + f x]} dx$$

Optimal (type 6, 286 leaves, 7 steps):

$$\frac{1}{b(a^2 - b^2)f} a(Ab - aB)c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1-m}{2}, 1, \frac{3}{2}, \sin[e+fx]^2, -\frac{b^2 \sin[e+fx]^2}{a^2 - b^2}\right] (c \cos[e+fx])^{-1+m} (\cos[e+fx]^2)^{\frac{1-m}{2}} \sin[e+fx] -$$

$$\frac{1}{(a^2 - b^2)f} (Ab - aB) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, \sin[e+fx]^2, -\frac{b^2 \sin[e+fx]^2}{a^2 - b^2}\right] (c \cos[e+fx])^m (\cos[e+fx]^2)^{-m/2} \sin[e+fx] -$$

$$\frac{B(c \cos[e+fx])^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[e+fx]^2\right] \sin[e+fx]}{bcf(1+m)\sqrt{\sin[e+fx]^2}}$$

Result (type 6, 10482 leaves):

$$\left( \cos[e+fx]^{-1+m} (c \cos[e+fx])^m \left( \frac{A \cos[e+fx]^m}{a + b \cos[e+fx]} + \frac{B \cos[e+fx]^{1+m}}{a + b \cos[e+fx]} \right) \sin[e+fx] \left( \frac{B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1 + \frac{m}{2}, \frac{3}{2}, -\tan[e+fx]^2\right]}{b} + \right. \right.$$

$$\frac{A \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3}{2}, -\tan[e+fx]^2\right]}{b} - \frac{a B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3}{2}, -\tan[e+fx]^2\right]}{b^2} +$$

$$\left. \left( 3 a^2 A (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\tan[e+fx]^2, \frac{a^2 \tan[e+fx]^2}{-a^2 + b^2}\right] (1 + \tan[e+fx]^2)^{\frac{1-m}{2}-\frac{m}{2}} \right) / \right.$$

$$\left. \left( b \left( -3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\tan[e+fx]^2, -\frac{a^2 \tan[e+fx]^2}{a^2 - b^2}\right] + \right. \right.$$

$$\left. \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\tan[e+fx]^2, -\frac{a^2 \tan[e+fx]^2}{a^2 - b^2}\right] + \right. \right.$$

$$\left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[e+fx]^2, -\frac{a^2 \tan[e+fx]^2}{a^2 - b^2}\right] \right) \tan[e+fx]^2 (-b^2 + a^2 (1 + \tan[e+fx]^2)) \right) -$$

$$\left( 3 a^3 (a^2 - b^2) B \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\tan[e+fx]^2, \frac{a^2 \tan[e+fx]^2}{-a^2 + b^2}\right] (1 + \tan[e+fx]^2)^{\frac{1-m}{2}-\frac{m}{2}} \right) /$$

$$\left( b^2 \left( -3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\tan[e+fx]^2, -\frac{a^2 \tan[e+fx]^2}{a^2 - b^2}\right] + \right. \right.$$

$$\left. \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\tan[e+fx]^2, -\frac{a^2 \tan[e+fx]^2}{a^2 - b^2}\right] + \right. \right.$$

$$\left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[e+fx]^2, -\frac{a^2 \tan[e+fx]^2}{a^2 - b^2}\right] \right) \tan[e+fx]^2 (-b^2 + a^2 (1 + \tan[e+fx]^2)) \right) -$$

$$\left( 3 a A (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[e+fx]^2, -\frac{a^2 \tan[e+fx]^2}{a^2 - b^2}\right] (1 + \tan[e+fx]^2)^{-m/2} \right) /$$

$$\left( \left( -3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[e+fx]^2, -\frac{a^2 \tan[e+fx]^2}{a^2 - b^2}\right] + \right. \right.$$

$$\begin{aligned}
& \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \\
& \quad \left. (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] \operatorname{Tan}[e+f x]^2 \right) (-b^2+a^2(1+\operatorname{Tan}[e+f x]^2)) \Big) + \\
& \left( 3 a^2 (a^2-b^2) \operatorname{B AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, \frac{a^2 \operatorname{Tan}[e+f x]^2}{-a^2+b^2} \right] (1+\operatorname{Tan}[e+f x]^2)^{-m/2} \right) / \\
& \left( b \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] \operatorname{Tan}[e+f x]^2 \right) (-b^2+a^2(1+\operatorname{Tan}[e+f x]^2)) \right) \Big) \Big) / \\
& \left( f \left( \operatorname{Sec}[e+f x]^2 \left( \frac{\operatorname{B Hypergeometric2F1} \left[ \frac{1}{2}, 1+\frac{m}{2}, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2 \right]}{b} + \frac{\operatorname{A Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1+m}{2}, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2 \right]}{b} - \right. \right. \right. \\
& \quad \left. \left. \frac{a \operatorname{B Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1+m}{2}, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2 \right]}{b^2} + \right. \right. \\
& \quad \left. \left( 3 a^2 \operatorname{A} (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, \frac{a^2 \operatorname{Tan}[e+f x]^2}{-a^2+b^2} \right] (1+\operatorname{Tan}[e+f x]^2)^{\frac{1-m}{2}} \right) / \right. \\
& \quad \left( b \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] \operatorname{Tan}[e+f x]^2 \right) (-b^2+a^2(1+\operatorname{Tan}[e+f x]^2)) \right) \Big) - \\
& \left( 3 a^3 (a^2-b^2) \operatorname{B AppellF1} \left[ \frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, \frac{a^2 \operatorname{Tan}[e+f x]^2}{-a^2+b^2} \right] (1+\operatorname{Tan}[e+f x]^2)^{\frac{1-m}{2}} \right) / \\
& \left( b^2 \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] \operatorname{Tan}[e+f x]^2 \right) (-b^2+a^2(1+\operatorname{Tan}[e+f x]^2)) \right) \Big) -
\end{aligned}$$



$$\begin{aligned}
& \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \\
& \quad \left. (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] \operatorname{Tan}[e+f x]^2 \right) (-b^2+a^2(1+\operatorname{Tan}[e+f x]^2))^2 \Big) - \\
& \left( 6 a^4 (a^2-b^2) \operatorname{B AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, \frac{a^2 \operatorname{Tan}[e+f x]^2}{-a^2+b^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] (1+\operatorname{Tan}[e+f x]^2)^{-m/2} \right) / \\
& \left( b \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[e+f x]^2 \right) (-b^2+a^2(1+\operatorname{Tan}[e+f x]^2))^2 \Big) + \\
& \left( 6 a^2 A (a^2-b^2) \left( \frac{1}{2} - \frac{m}{2} \right) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, \frac{a^2 \operatorname{Tan}[e+f x]^2}{-a^2+b^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right. \\
& \quad \left. (1+\operatorname{Tan}[e+f x]^2)^{-\frac{1}{2}-\frac{m}{2}} \right) / \left( b \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[e+f x]^2 \right) (-b^2+a^2(1+\operatorname{Tan}[e+f x]^2))^2 \Big) - \\
& \left( 6 a^3 (a^2-b^2) \operatorname{B} \left( \frac{1}{2} - \frac{m}{2} \right) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, \frac{a^2 \operatorname{Tan}[e+f x]^2}{-a^2+b^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right. \\
& \quad \left. (1+\operatorname{Tan}[e+f x]^2)^{-\frac{1}{2}-\frac{m}{2}} \right) / \left( b^2 \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[e+f x]^2 \right) (-b^2+a^2(1+\operatorname{Tan}[e+f x]^2))^2 \Big) + \\
& \left( 3 a^2 A (a^2-b^2) \left( -\frac{1}{3} (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1+\frac{1}{2} (-1+m), 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \frac{a^2 \operatorname{Tan}[e+f x]^2}{-a^2+b^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + \right. \right. \\
& \quad \left. \left. 1 / (3(-a^2+b^2)) 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \frac{a^2 \operatorname{Tan}[e+f x]^2}{-a^2+b^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right. \\
& \quad \left. (1+\operatorname{Tan}[e+f x]^2)^{\frac{1}{2}-\frac{m}{2}} \right) / \left( b \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \\
& \quad \left. (a^2-b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] \operatorname{Tan}[e+f x]^2 \right) (-b^2+a^2 (1+\operatorname{Tan}[e+f x]^2)) \Big) - \\
& \left( 3 a^3 (a^2-b^2) B \left( -\frac{1}{3} (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1+\frac{1}{2} (-1+m), 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \frac{a^2 \operatorname{Tan}[e+f x]^2}{-a^2+b^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + \right. \right. \\
& \quad \left. \left. 1 / (3 (-a^2+b^2)) 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \frac{a^2 \operatorname{Tan}[e+f x]^2}{-a^2+b^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) \\
& (1+\operatorname{Tan}[e+f x]^2)^{\frac{1}{2}-\frac{m}{2}} \Big) / \left( b^2 \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] \operatorname{Tan}[e+f x]^2 \right) \right) (-b^2+a^2 (1+\operatorname{Tan}[e+f x]^2)) \Big) + \\
& \left( 3 a A (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] (1+\operatorname{Tan}[e+f x]^2)^{-1-\frac{m}{2}} \right) / \\
& \left( \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] \operatorname{Tan}[e+f x]^2 \right) \right) (-b^2+a^2 (1+\operatorname{Tan}[e+f x]^2)) \Big) - \\
& \left( 3 a^2 (a^2-b^2) B m \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, \frac{a^2 \operatorname{Tan}[e+f x]^2}{-a^2+b^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] (1+\operatorname{Tan}[e+f x]^2)^{-1-\frac{m}{2}} \right) / \\
& \left( b \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] \operatorname{Tan}[e+f x]^2 \right) \right) (-b^2+a^2 (1+\operatorname{Tan}[e+f x]^2)) \Big) - \\
& \left( 3 a A (a^2-b^2) \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+\frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{a^2 \operatorname{Tan}[e+f x]^2}{a^2-b^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \frac{2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, -\frac{a^2 \tan[e+f x]^2}{a^2-b^2}\right] \operatorname{Sec}[e+f x]^2 \tan[e+f x]}{3\left(a^2-b^2\right)}\left(1+\tan[e+f x]^2\right)^{-m / 2} \Bigg/ \left(\left(-3\left(a^2-b^2\right)\right.\right. \\
& \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[e+f x]^2, -\frac{a^2 \tan[e+f x]^2}{a^2-b^2}\right]+\left(2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, -\frac{a^2 \tan[e+f x]^2}{a^2-b^2}\right]+ \right. \\
& \left.\left.\left(a^2-b^2\right) m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[e+f x]^2, -\frac{a^2 \tan[e+f x]^2}{a^2-b^2}\right]\right) \tan[e+f x]^2\left(-b^2+a^2\left(1+\tan[e+f x]^2\right)\right)\right)+ \\
& \left(3 a^2\left(a^2-b^2\right) B\left[-\frac{1}{3} m \operatorname{AppellF1}\left[\frac{3}{2}, 1+\frac{m}{2}, 1, \frac{5}{2}, -\tan[e+f x]^2, \frac{a^2 \tan[e+f x]^2}{-a^2+b^2}\right] \operatorname{Sec}[e+f x]^2 \tan[e+f x]+ \right. \\
& \left. \frac{2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, \frac{a^2 \tan[e+f x]^2}{-a^2+b^2}\right] \operatorname{Sec}[e+f x]^2 \tan[e+f x]}{3\left(-a^2+b^2\right)}\right)\left(1+\tan[e+f x]^2\right)^{-m / 2} \Bigg/ \left(b\left(-3\left(a^2-b^2\right)\right.\right. \\
& \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[e+f x]^2, -\frac{a^2 \tan[e+f x]^2}{a^2-b^2}\right]+\left(2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, -\frac{a^2 \tan[e+f x]^2}{a^2-b^2}\right]+ \right. \\
& \left.\left.\left(a^2-b^2\right) m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[e+f x]^2, -\frac{a^2 \tan[e+f x]^2}{a^2-b^2}\right]\right) \tan[e+f x]^2\left(-b^2+a^2\left(1+\tan[e+f x]^2\right)\right)\right)+ \\
& \frac{1}{b} A \operatorname{Csc}[e+f x] \operatorname{Sec}[e+f x]\left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3}{2}, -\tan[e+f x]^2\right]+\left(1+\tan[e+f x]^2\right)^{\frac{1}{2}(-1-m)}\right)-\frac{1}{b^2} \\
& a B \operatorname{Csc}[e+f x] \operatorname{Sec}[e+f x]\left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3}{2}, -\tan[e+f x]^2\right]+\left(1+\tan[e+f x]^2\right)^{\frac{1}{2}(-1-m)}\right)+ \\
& \frac{1}{b} B \operatorname{Csc}[e+f x] \operatorname{Sec}[e+f x]\left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+\frac{m}{2}, \frac{3}{2}, -\tan[e+f x]^2\right]+\left(1+\tan[e+f x]^2\right)^{-1-\frac{m}{2}}\right)- \\
& \left(3 a^2 A\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\tan[e+f x]^2, \frac{a^2 \tan[e+f x]^2}{-a^2+b^2}\right]\right. \\
& \left.\left(1+\tan[e+f x]^2\right)^{\frac{1}{2}-\frac{m}{2}}\left(2\left(2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\tan[e+f x]^2, -\frac{a^2 \tan[e+f x]^2}{a^2-b^2}\right]+ \right.\right. \\
& \left.\left.\left(a^2-b^2\right)(-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[e+f x]^2, -\frac{a^2 \tan[e+f x]^2}{a^2-b^2}\right]\right) \operatorname{Sec}[e+f x]^2 \tan[e+f x]- \right. \\
& \left. 3\left(a^2-b^2\right)\left(-\frac{1}{3}(-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 1+\frac{1}{2}(-1+m), 1, \frac{5}{2}, -\tan[e+f x]^2, -\frac{a^2 \tan[e+f x]^2}{a^2-b^2}\right] \operatorname{Sec}[e+f x]^2 \tan[e+f x]- \right. \\
& \left. \frac{1}{3\left(a^2-b^2\right)} 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\tan[e+f x]^2, -\frac{a^2 \tan[e+f x]^2}{a^2-b^2}\right] \operatorname{Sec}[e+f x]^2 \tan[e+f x]\right)+
\end{aligned}$$

$$\begin{aligned}
& \tan[e + f x]^2 \left( 2 a^2 \left( -\frac{3}{5} (-1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1}{2} (-1+m), 2, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \right. \right. \\
& \quad \left. \left. \frac{1}{5 (a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2} (-1+m), 3, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) + \right. \\
& \quad (a^2 - b^2) (-1+m) \left( -\frac{1}{5 (a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1+m}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \right. \\
& \quad \left. \frac{3}{5} (1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1+m}{2}, 1, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) \Bigg) \Bigg) / \\
& \left( b \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), \right. \right. \right. \right. \\
& \quad \left. \left. \left. 2, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] + (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \tan[e + f x]^2 \right)^2 (-b^2 + a^2 (1 + \tan[e + f x]^2)) \Bigg) + \\
& \left( 3 a^3 (a^2 - b^2) \operatorname{B AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{a^2 \tan[e + f x]^2}{-a^2 + b^2} \right] (1 + \tan[e + f x]^2)^{\frac{1}{2} - \frac{m}{2}} \right. \\
& \quad \left. \left( 2 \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \right. \right. \\
& \quad \left. \left. 3 (a^2 - b^2) \left( -\frac{1}{3} (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{1}{2} (-1+m), 1, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{3 (a^2 - b^2)} 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) + \right. \right. \\
& \quad \left. \left. \tan[e + f x]^2 \left( 2 a^2 \left( -\frac{3}{5} (-1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1}{2} (-1+m), 2, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{5 (a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2} (-1+m), 3, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (-1+m) \left( -\frac{1}{5 (a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1+m}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \right. \right. \\
& \quad \left. \left. \frac{3}{5} (1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1+m}{2}, 1, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) \right) \Bigg) \Bigg) \Bigg) /
\end{aligned}$$

$$\begin{aligned}
& \left( b^2 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{a^2 \operatorname{Tan}[e+fx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), \right. \right. \right. \\
& \quad \left. \left. \left. 2, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{a^2 \operatorname{Tan}[e+fx]^2}{a^2 - b^2} \right] + (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{a^2 \operatorname{Tan}[e+fx]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e+fx]^2 \left( -b^2 + a^2 (1 + \operatorname{Tan}[e+fx]^2) \right) \right) + \\
& \left( 3 a A (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{a^2 \operatorname{Tan}[e+fx]^2}{a^2 - b^2} \right] (1 + \operatorname{Tan}[e+fx]^2)^{-m/2} \right. \\
& \quad \left( 2 \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{a^2 \operatorname{Tan}[e+fx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{a^2 \operatorname{Tan}[e+fx]^2}{a^2 - b^2} \right] \right) \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - \right. \\
& \quad 3 (a^2 - b^2) \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{a^2 \operatorname{Tan}[e+fx]^2}{a^2 - b^2} \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - \right. \\
& \quad \left. \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{a^2 \operatorname{Tan}[e+fx]^2}{a^2 - b^2} \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx]}{3 (a^2 - b^2)} \right) \right) + \\
& \quad \operatorname{Tan}[e+fx]^2 \left( 2 a^2 \left( -\frac{3}{5} m \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{m}{2}, 2, \frac{7}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{a^2 \operatorname{Tan}[e+fx]^2}{a^2 - b^2} \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - \right. \right. \\
& \quad \left. \left. \frac{1}{5 (a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{m}{2}, 3, \frac{7}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{a^2 \operatorname{Tan}[e+fx]^2}{a^2 - b^2} \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) \right) + \\
& \quad (a^2 - b^2) m \left( -\frac{1}{5 (a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{2+m}{2}, 2, \frac{7}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{a^2 \operatorname{Tan}[e+fx]^2}{a^2 - b^2} \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - \right. \\
& \quad \left. \left. \frac{3}{5} (2+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{2+m}{2}, 1, \frac{7}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{a^2 \operatorname{Tan}[e+fx]^2}{a^2 - b^2} \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) \right) \right) \Bigg) / \\
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{a^2 \operatorname{Tan}[e+fx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \operatorname{Tan}[e+fx]^2}{a^2 - b^2} \right] + (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, -\frac{a^2 \operatorname{Tan}[e+fx]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e+fx]^2 \right)^2
\end{aligned}$$

$$\begin{aligned}
& (-b^2 + a^2 (1 + \tan[e + f x]^2)) - \left( 3 a^2 (a^2 - b^2) \text{B AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{a^2 \tan[e + f x]^2}{-a^2 + b^2} \right] \right. \\
& (1 + \tan[e + f x]^2)^{-m/2} \left( 2 \left( 2 a^2 \text{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) m \text{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \text{Sec}[e + f x]^2 \tan[e + f x] - \right. \\
& 3 (a^2 - b^2) \left( -\frac{1}{3} m \text{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \text{Sec}[e + f x]^2 \tan[e + f x] - \right. \\
& \left. \left. \frac{2 a^2 \text{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \text{Sec}[e + f x]^2 \tan[e + f x]}{3 (a^2 - b^2)} \right) \right) + \\
& \tan[e + f x]^2 \left( 2 a^2 \left( -\frac{3}{5} m \text{AppellF1} \left[ \frac{5}{2}, 1 + \frac{m}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \text{Sec}[e + f x]^2 \tan[e + f x] - \right. \right. \\
& \left. \left. \frac{1}{5 (a^2 - b^2)} 12 a^2 \text{AppellF1} \left[ \frac{5}{2}, \frac{m}{2}, 3, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \text{Sec}[e + f x]^2 \tan[e + f x] \right) + \right. \\
& \left. (a^2 - b^2) m \left( -\frac{1}{5 (a^2 - b^2)} 6 a^2 \text{AppellF1} \left[ \frac{5}{2}, \frac{2+m}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \text{Sec}[e + f x]^2 \tan[e + f x] - \right. \right. \\
& \left. \left. \frac{3}{5} (2+m) \text{AppellF1} \left[ \frac{5}{2}, 1 + \frac{2+m}{2}, 1, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \text{Sec}[e + f x]^2 \tan[e + f x] \right) \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( b \left( -3 (a^2 - b^2) \text{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \left( 2 a^2 \text{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, \right. \right. \right. \\
& \left. \left. -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] + (a^2 - b^2) m \text{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, \right. \right. \\
& \left. \left. -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \tan[e + f x]^2 \right)^2 (-b^2 + a^2 (1 + \tan[e + f x]^2)) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 459: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + d x]) (A + B \cos[c + d x]) \sec[c + d x]^{7/2} dx$$

Optimal (type 4, 172 leaves, 9 steps):

$$\begin{aligned}
& - \frac{2 a (3 A + 5 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} + \frac{2 a (A + B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{3 d} \\
& + \frac{2 a (3 A + 5 B) \sqrt{\sec [c + d x]} \sin [c + d x]}{5 d} + \frac{2 a (A + B) \sec [c + d x]^{3 / 2} \sin [c + d x]}{3 d} + \frac{2 a A \sec [c + d x]^{5 / 2} \sin [c + d x]}{5 d}
\end{aligned}$$

Result (type 5, 381 leaves):

$$\begin{aligned}
& \frac{1}{30 d} a (1 + \cos [c + d x]) \sec \left[ \frac{1}{2}(c + d x) \right]^2 \\
& \left( -9 \sqrt{2} A e^{-i(2 c + d x)} \sqrt{\frac{e^{i(c + d x)}}{1 + e^{2 i(c + d x)}}} \operatorname{Csc}[c] \left( 1 + e^{2 i(c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right] \right) - \right. \\
& 15 \sqrt{2} B e^{-i(2 c + d x)} \sqrt{\frac{e^{i(c + d x)}}{1 + e^{2 i(c + d x)}}} \operatorname{Csc}[c] \left( 1 + e^{2 i(c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right] \right) + \\
& 10 A \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]} + 10 B \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]} + \\
& \left. 2 \sqrt{\sec [c + d x]} (3(3 A + 5 B) \cos [d x] \operatorname{Csc}[c] + (5(A + B) + 3 A \sec [c + d x]) \tan [c + d x]) \right)
\end{aligned}$$

■ **Problem 460: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c + d x]) (A + B \cos [c + d x]) \sec [c + d x]^{5 / 2} dx$$

Optimal (type 4, 135 leaves, 8 steps):

$$\begin{aligned}
& - \frac{2 a (A + B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{d} + \\
& \frac{2 a (A + 3 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{3 d} + \frac{2 a (A + B) \sqrt{\sec [c + d x]} \sin [c + d x]}{d} + \frac{2 a A \sec [c + d x]^{3 / 2} \sin [c + d x]}{3 d}
\end{aligned}$$

Result (type 5, 227 leaves):

$$\begin{aligned}
& \frac{1}{3 d (1 + e^{2 i(c + d x)})} a e^{-i(c + d x)} (1 + \cos [c + d x]) \left( (A + 3 B) e^{i(c + d x)} (1 + e^{2 i(c + d x)}) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] - \right. \\
& \left. i \left( -3 A - 3 B - A e^{i(c + d x)} - 3 A e^{2 i(c + d x)} - 3 B e^{2 i(c + d x)} + A e^{3 i(c + d x)} + 3(A + B) (1 + e^{2 i(c + d x)})^{3 / 2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right] \right) \right) \\
& \sec \left[ \frac{1}{2}(c + d x) \right]^2 \sqrt{\sec [c + d x]}
\end{aligned}$$

■ **Problem 461: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c + d x]) (A + B \cos [c + d x]) \sec [c + d x]^{3/2} dx$$

Optimal (type 4, 106 leaves, 7 steps):

$$-\frac{2 a (A - B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{d} + \frac{2 a (A + B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{d} + \frac{2 a A \sqrt{\sec [c + d x]} \sin [c + d x]}{d}$$

Result (type 5, 124 leaves):

$$\frac{1}{d} 2 a \left( (A + B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] - \frac{1}{2} i e^{-i(c+d x)} \left( -2 A + B + B e^{2 i(c+d x)} + 2 (A - B) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] \right) \right) \sqrt{\sec [c + d x]}$$

■ **Problem 462: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c + d x]) (A + B \cos [c + d x]) \sqrt{\sec [c + d x]} dx$$

Optimal (type 4, 110 leaves, 7 steps):

$$\frac{2 a (A + B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{d} + \frac{2 a (3 A + B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{3 d} + \frac{2 a B \sin [c + d x]}{3 d \sqrt{\sec [c + d x]}}$$

Result (type 5, 163 leaves):

$$\frac{1}{3 d} a e^{-i(2 c+d x)} \sqrt{\sec [c + d x]} \left( 2 (3 A + B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + 6 i (A + B) e^{-i(c+d x)} \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + 2 \cos [c + d x] (-3 i (A + B) + B \sin [c + d x]) \right) (\cos [2 c + d x] + i \sin [2 c + d x])$$

■ **Problem 463: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \cos [c + d x]) (A + B \cos [c + d x])}{\sqrt{\sec [c + d x]}} dx$$

Optimal (type 4, 141 leaves, 8 steps):

$$\frac{2 a (5 A + 3 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} +$$

$$\frac{2 a (A + B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{3 d} + \frac{2 a B \sin [c + d x]}{5 d \sec [c + d x]^{3/2}} + \frac{2 a (A + B) \sin [c + d x]}{3 d \sqrt{\sec [c + d x]}}$$

Result (type 5, 149 leaves):

$$\frac{1}{30 d} a \sqrt{\sec [c + d x]}$$

$$\left( 20 (A + B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + 12 i (5 A + 3 B) e^{-i(c + d x)} \sqrt{1 + e^{2 i(c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right] + \right.$$

$$\left. 2 \cos [c + d x] (-6 i (5 A + 3 B) + 10 (A + B) \sin [c + d x] + 3 B \sin [2(c + d x)]) \right)$$

■ **Problem 464: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \cos [c + d x]) (A + B \cos [c + d x])}{\sec [c + d x]^{3/2}} dx$$

Optimal (type 4, 172 leaves, 9 steps):

$$\frac{6 a (A + B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} +$$

$$\frac{2 a (7 A + 5 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{21 d} + \frac{2 a B \sin [c + d x]}{7 d \sec [c + d x]^{5/2}} + \frac{2 a (A + B) \sin [c + d x]}{5 d \sec [c + d x]^{3/2}} + \frac{2 a (7 A + 5 B) \sin [c + d x]}{21 d \sqrt{\sec [c + d x]}}$$

Result (type 5, 162 leaves):

$$\frac{1}{420 d} a \sqrt{\sec [c + d x]}$$

$$\left( 40 (7 A + 5 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + 504 i (A + B) e^{-i(c + d x)} \sqrt{1 + e^{2 i(c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right] + \right.$$

$$\left. 2 \cos [c + d x] (-252 i (A + B) + 5 (28 A + 23 B) \sin [c + d x] + 42 (A + B) \sin [2(c + d x)] + 15 B \sin [3(c + d x)]) \right)$$

■ **Problem 465: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c + d x])^2 (A + B \cos [c + d x]) \sec [c + d x]^{7/2} dx$$

Optimal (type 4, 199 leaves, 9 steps):

$$\begin{aligned}
& - \frac{4 a^2 (4 A+5 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} + \frac{4 a^2 (A+2 B) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 d} + \\
& \frac{4 a^2 (4 A+5 B) \sqrt{\sec [c+d x]} \sin [c+d x]}{5 d} + \frac{2 a^2 (7 A+5 B) \sec [c+d x]^{3 / 2} \sin [c+d x]}{15 d} + \frac{2 A \sec [c+d x]^{3 / 2}\left(a^2+a^2 \sec [c+d x]\right) \sin [c+d x]}{5 d}
\end{aligned}$$

Result (type 5, 386 leaves):

$$\begin{aligned}
& \frac{1}{30 d} a^2 (1+\cos [c+d x])^2 \sec \left[\frac{1}{2}(c+d x)\right]^4 \\
& \left( -12 \sqrt{2} A e^{-i(2 c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}} \operatorname{Csc}[c] \left(1+e^{2 i(c+d x)}+(-1+e^{2 i c}) \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]\right) - \right. \\
& \left. 15 \sqrt{2} B e^{-i(2 c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}} \operatorname{Csc}[c] \left(1+e^{2 i(c+d x)}+(-1+e^{2 i c}) \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]\right) + \right. \\
& \left. 10 A \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]} + 20 B \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]} + \right. \\
& \left. \sqrt{\sec [c+d x]} (6(4 A+5 B) \cos [d x] \operatorname{Csc}[c] + (5(2 A+B) + 3 A \sec [c+d x]) \tan [c+d x]) \right)
\end{aligned}$$

■ **Problem 466: Result unnecessarily involves higher level functions.**

$$\int (a+a \cos [c+d x])^2 (A+B \cos [c+d x]) \sec [c+d x]^{5 / 2} d x$$

Optimal (type 4, 160 leaves, 8 steps):

$$\begin{aligned}
& - \frac{4 a^2 A \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{d} + \frac{4 a^2 (2 A+3 B) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 d} + \\
& \frac{2 a^2 (5 A+3 B) \sqrt{\sec [c+d x]} \sin [c+d x]}{3 d} + \frac{2 A \sqrt{\sec [c+d x]} (a^2+a^2 \sec [c+d x]) \sin [c+d x]}{3 d}
\end{aligned}$$

Result (type 5, 188 leaves):

$$\begin{aligned}
& - \frac{1}{3 d} i a^2 \sec [c+d x]^{3 / 2} \\
& \left( -6 A - 6 A \cos [2(c+d x)] + 6 A e^{-2 i(c+d x)} (1+e^{2 i(c+d x)})^{3 / 2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + 2(2 A+3 B) e^{-i(c+d x)} \right. \\
& \left. (1+e^{2 i(c+d x)})^{3 / 2} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] + 2 i A \sin [c+d x] + 6 i A \sin [2(c+d x)] + 3 i B \sin [2(c+d x)] \right)
\end{aligned}$$



■ **Problem 467: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c + d x])^2 (A + B \cos [c + d x]) \sec [c + d x]^{3/2} dx$$

Optimal (type 4, 160 leaves, 8 steps):

$$\frac{4 a^2 B \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{d} + \frac{4 a^2 (3 A + 2 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{3 d} + \frac{2 a^2 (3 A - B) \sqrt{\sec [c + d x]} \sin [c + d x]}{3 d} + \frac{2 B (a^2 + a^2 \sec [c + d x]) \sin [c + d x]}{3 d \sqrt{\sec [c + d x]}}$$

Result (type 5, 152 leaves):

$$\left( 2 a^2 \left( 12 i B \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] - 4 i (3 A + 2 B) e^{i (c + d x)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c + d x)}\right] + \sqrt{1 + e^{2 i (c + d x)}} (-6 i B + B \sin [c + d x] + 3 A \tan [c + d x]) \right) \right) / \left( 3 d \sqrt{1 + e^{2 i (c + d x)}} \sqrt{\sec [c + d x]} \right)$$

■ **Problem 468: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c + d x])^2 (A + B \cos [c + d x]) \sqrt{\sec [c + d x]} dx$$

Optimal (type 4, 166 leaves, 8 steps):

$$\frac{4 a^2 (5 A + 4 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} + \frac{4 a^2 (2 A + B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{3 d} + \frac{2 a^2 (5 A + 7 B) \sin [c + d x]}{15 d \sqrt{\sec [c + d x]}} + \frac{2 B (a^2 + a^2 \sec [c + d x]) \sin [c + d x]}{5 d \sec [c + d x]^{3/2}}$$

Result (type 5, 155 leaves):

$$\frac{1}{30 d} a^2 \sqrt{\sec [c + d x]} \left( 40 (2 A + B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + 24 i (5 A + 4 B) e^{-i (c + d x)} \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] + 2 \cos [c + d x] (-12 i (5 A + 4 B) + 10 (A + 2 B) \sin [c + d x] + 3 B \sin [2 (c + d x)]) \right)$$

■ **Problem 469: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \cos [c + d x])^2 (A + B \cos [c + d x])}{\sqrt{\sec [c + d x]}} dx$$

Optimal (type 4, 201 leaves, 9 steps):

$$\frac{4 a^2 (4 A + 3 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} + \frac{4 a^2 (7 A + 6 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{21 d} +$$

$$\frac{2 a^2 (7 A + 9 B) \sin [c + d x]}{35 d \sec [c + d x]^{3/2}} + \frac{4 a^2 (7 A + 6 B) \sin [c + d x]}{21 d \sqrt{\sec [c + d x]}} + \frac{2 B \left(a^2 + a^2 \sec [c + d x]\right) \sin [c + d x]}{7 d \sec [c + d x]^{5/2}}$$

Result (type 5, 208 leaves):

$$\frac{1}{420 d} a^2 e^{-i(2 c + d x)} \sqrt{\sec [c + d x]} \left( 80 (7 A + 6 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + \right.$$

$$336 i (4 A + 3 B) e^{-i(c + d x)} \sqrt{1 + e^{2 i(c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right] + 2 \cos [c + d x]$$

$$\left. (-168 i (4 A + 3 B) + 5 (56 A + 51 B) \sin [c + d x] + 42 (A + 2 B) \sin [2(c + d x)] + 15 B \sin [3(c + d x)]) \right) (\cos [2 c + d x] + i \sin [2 c + d x])$$

■ **Problem 470: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^3 (A + B \cos [c + d x]) \sec [c + d x]^{9/2} dx$$

Optimal (type 4, 244 leaves, 10 steps):

$$-\frac{4 a^3 (7 A + 9 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} +$$

$$\frac{4 a^3 (13 A + 21 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{21 d} +$$

$$\frac{4 a^3 (7 A + 9 B) \sqrt{\sec [c + d x]} \sin [c + d x]}{5 d} + \frac{4 a^3 (41 A + 42 B) \sec [c + d x]^{3/2} \sin [c + d x]}{105 d} +$$

$$\frac{2 a A \sec [c + d x]^{3/2} (a + a \sec [c + d x])^2 \sin [c + d x]}{7 d} + \frac{2 (11 A + 7 B) \sec [c + d x]^{3/2} (a^3 + a^3 \sec [c + d x]) \sin [c + d x]}{35 d}$$

Result (type 5, 605 leaves):

$$\begin{aligned}
& -\frac{1}{10\sqrt{2}d} 7A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \\
& \left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 - \frac{1}{10\sqrt{2}d} 9B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \\
& (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 + \\
& \frac{13A\sqrt{\cos[c+dx]}(a+a\cos[c+dx])^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]}}{42d} + \\
& \frac{B\sqrt{\cos[c+dx]}(a+a\cos[c+dx])^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]}}{2d} + (a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]} \\
& \left(\frac{(7A+9B)\cos[dx]\operatorname{Csc}[c]}{10d} + \frac{A\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^3\sin[dx]}{28d} + \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^2(5A\sin[c]+21A\sin[dx]+7B\sin[dx])}{140d} + \right. \\
& \left. \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx](63A\sin[c]+21B\sin[c]+130A\sin[dx]+105B\sin[dx])}{420d} + \frac{(26A+21B)\tan[c]}{84d}\right)
\end{aligned}$$

- **Problem 471: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+a\cos[c+dx])^3 (A+B\cos[c+dx]) \operatorname{Sec}[c+dx]^{7/2} dx$$

Optimal (type 4, 211 leaves, 9 steps):

$$\begin{aligned}
& -\frac{4a^3(9A+5B)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{5d} + \\
& \frac{4a^3(3A+5B)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{3d} + \frac{4a^3(21A+20B)\sqrt{\operatorname{Sec}[c+dx]}\sin[c+dx]}{15d} + \\
& \frac{2aA\sqrt{\operatorname{Sec}[c+dx]}(a+a\operatorname{Sec}[c+dx])^2\sin[c+dx]}{5d} + \frac{2(9A+5B)\sqrt{\operatorname{Sec}[c+dx]}(a^3+a^3\operatorname{Sec}[c+dx])\sin[c+dx]}{15d}
\end{aligned}$$

Result (type 5, 582 leaves):

$$\begin{aligned}
& -\frac{1}{10\sqrt{2}d} 9A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \\
& \left( 1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 - \frac{1}{2\sqrt{2}d} \\
& B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \left( 1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 + \frac{A\sqrt{\cos[c+dx]}(a+a\cos[c+dx])^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]}}{2d} + \\
& \frac{5B\sqrt{\cos[c+dx]}(a+a\cos[c+dx])^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]}}{6d} + \\
& (a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]} \left( -\frac{(-36A-25B+5B\cos[2c])\cos[dx]\operatorname{Csc}[c]}{40d} + \frac{B\cos[c]\sin[dx]}{4d} + \right. \\
& \left. \frac{A\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^2\sin[dx]}{20d} + \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx](3A\sin[c]+15A\sin[dx]+5B\sin[dx])}{60d} + \frac{(3A+B)\tan[c]}{12d} \right)
\end{aligned}$$

■ **Problem 472: Result unnecessarily involves higher level functions.**

$$\int (a+a\cos[c+dx])^3 (A+B\cos[c+dx]) \operatorname{Sec}[c+dx]^{5/2} dx$$

Optimal (type 4, 199 leaves, 9 steps):

$$\begin{aligned}
& -\frac{4a^3(A-B)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{d} + \frac{20a^3(A+B)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{3d} \\
& \frac{4a^3(4A+B)\sqrt{\operatorname{Sec}[c+dx]}\sin[c+dx]}{3d} + \frac{2aB(a+a\operatorname{Sec}[c+dx])^2\sin[c+dx]}{3d\sqrt{\operatorname{Sec}[c+dx]}} + \frac{2(A-B)\sqrt{\operatorname{Sec}[c+dx]}(a^3+a^3\operatorname{Sec}[c+dx])\sin[c+dx]}{3d}
\end{aligned}$$

Result (type 5, 192 leaves):

$$\begin{aligned}
& \frac{1}{6d} a^3 \operatorname{Sec}[c+dx]^{3/2} \left( 12iA - 12iB + 12iA\cos[2(c+dx)] - 12iB\cos[2(c+dx)] + \right. \\
& 40(A+B)\cos[c+dx]^{3/2} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] - 12i(A-B)e^{-2i(c+dx)}(1+e^{2i(c+dx)})^{3/2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + \\
& \left. 4A\sin[c+dx] + B\sin[c+dx] + 18A\sin[2(c+dx)] + 6B\sin[2(c+dx)] + B\sin[3(c+dx)] \right)
\end{aligned}$$

■ **Problem 473: Result unnecessarily involves higher level functions.**

$$\int (a+a\cos[c+dx])^3 (A+B\cos[c+dx]) \operatorname{Sec}[c+dx]^{3/2} dx$$

Optimal (type 4, 211 leaves, 9 steps):

$$\frac{4 a^3 (5 A + 9 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} + \frac{4 a^3 (5 A + 3 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{3 d} + \frac{4 a^3 (5 A - 6 B) \sqrt{\sec [c + d x]} \sin [c + d x]}{15 d} + \frac{2 a B (a + a \sec [c + d x])^2 \sin [c + d x]}{5 d \sec [c + d x]^{3/2}} + \frac{2 (5 A + 9 B) (a^3 + a^3 \sec [c + d x]) \sin [c + d x]}{15 d \sqrt{\sec [c + d x]}}$$

Result (type 5, 220 leaves):

$$\frac{1}{30 d} a^3 e^{-i(2 c + d x)} \sqrt{\sec [c + d x]} \left( -120 i A \cos [c + d x] - 216 i B \cos [c + d x] + 40 (5 A + 3 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + 24 i (5 A + 9 B) e^{-i(c + d x)} \sqrt{1 + e^{2 i(c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right] + 60 A \sin [c + d x] + 3 B \sin [c + d x] + 10 A \sin [2(c + d x)] + 30 B \sin [2(c + d x)] + 3 B \sin [3(c + d x)] \right) (\cos [2 c + d x] + i \sin [2 c + d x])$$

■ **Problem 474: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c + d x])^3 (A + B \cos [c + d x]) \sqrt{\sec [c + d x]} dx$$

Optimal (type 4, 211 leaves, 9 steps):

$$\frac{4 a^3 (9 A + 7 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} + \frac{4 a^3 (21 A + 13 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{21 d} + \frac{4 a^3 (42 A + 41 B) \sin [c + d x]}{105 d \sqrt{\sec [c + d x]}} + \frac{2 a B (a + a \sec [c + d x])^2 \sin [c + d x]}{7 d \sec [c + d x]^{5/2}} + \frac{2 (7 A + 11 B) (a^3 + a^3 \sec [c + d x]) \sin [c + d x]}{35 d \sec [c + d x]^{3/2}}$$

Result (type 5, 208 leaves):

$$\frac{1}{420 d} a^3 e^{-i(2 c + d x)} \sqrt{\sec [c + d x]} \left( 80 (21 A + 13 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + 336 i (9 A + 7 B) e^{-i(c + d x)} \sqrt{1 + e^{2 i(c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right] + 2 \cos [c + d x] (-168 i (9 A + 7 B) + 5 (84 A + 107 B) \sin [c + d x] + 42 (A + 3 B) \sin [2(c + d x)] + 15 B \sin [3(c + d x)]) \right) (\cos [2 c + d x] + i \sin [2 c + d x])$$

■ **Problem 475: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \cos [c + d x])^3 (A + B \cos [c + d x])}{\sqrt{\sec [c + d x]}} dx$$

Optimal (type 4, 244 leaves, 10 steps):

$$\frac{4 a^3 (21 A + 17 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{15 d} +$$

$$\frac{4 a^3 (13 A + 11 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{21 d} + \frac{4 a^3 (24 A + 23 B) \sin [c + d x]}{105 d \sec [c + d x]^{3/2}} +$$

$$\frac{4 a^3 (13 A + 11 B) \sin [c + d x]}{21 d \sqrt{\sec [c + d x]}} + \frac{2 a B (a + a \sec [c + d x])^2 \sin [c + d x]}{9 d \sec [c + d x]^{7/2}} + \frac{2 (9 A + 13 B) (a^3 + a^3 \sec [c + d x]) \sin [c + d x]}{63 d \sec [c + d x]^{5/2}}$$

Result (type 5, 197 leaves):

$$\frac{1}{2520 d} a^3 \sqrt{\sec [c + d x]} \left( 480 (13 A + 11 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + \right.$$

$$672 i (21 A + 17 B) e^{-i(c + d x)} \sqrt{1 + e^{2 i(c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right] + 2 \cos [c + d x] (-7056 i A - 5712 i B +$$

$$30 (107 A + 97 B) \sin [c + d x] + 14 (54 A + 73 B) \sin [2(c + d x)] + 90 A \sin [3(c + d x)] + 270 B \sin [3(c + d x)] + 35 B \sin [4(c + d x)] \left. \right)$$

■ **Problem 476: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + d x]) \sec [c + d x]^{5/2}}{a + a \cos [c + d x]} dx$$

Optimal (type 4, 193 leaves, 9 steps):

$$\frac{3 (A - B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{a d} + \frac{(5 A - 3 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{3 a d} -$$

$$\frac{3 (A - B) \sqrt{\sec [c + d x]} \sin [c + d x]}{a d} + \frac{(5 A - 3 B) \sec [c + d x]^{3/2} \sin [c + d x]}{3 a d} - \frac{(A - B) \sec [c + d x]^{5/2} \sin [c + d x]}{d (a + a \sec [c + d x])}$$

Result (type 5, 631 leaves):

$$\frac{1}{\sqrt{2} d (a + a \cos [c + dx])} - 3 A e^{-i (2c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2i (c+dx)}}} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Csc} \left[ \frac{c}{2} \right]$$

$$\left( 1 + e^{2i (c+dx)} + (-1 + e^{2i c}) \sqrt{1 + e^{2i (c+dx)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i (c+dx)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] - \frac{1}{\sqrt{2} d (a + a \cos [c + dx])}$$

$$3 B e^{-i (2c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2i (c+dx)}}} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Csc} \left[ \frac{c}{2} \right] \left( 1 + e^{2i (c+dx)} + (-1 + e^{2i c}) \sqrt{1 + e^{2i (c+dx)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i (c+dx)} \right] \right)$$

$$\operatorname{Sec} \left[ \frac{c}{2} \right] + \frac{5 A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \sqrt{\cos [c + dx]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + dx), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + dx]} \sin [c]}{3 d (a + a \cos [c + dx])} -$$

$$\frac{B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \sqrt{\cos [c + dx]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + dx), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + dx]} \sin [c]}{d (a + a \cos [c + dx])} + \frac{1}{a + a \cos [c + dx]}$$

$$\cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \sqrt{\sec [c + dx]} \left( -\frac{3 (A - B) \cos [dx] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{d} + \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] (A \sin \left[ \frac{dx}{2} \right] - B \sin \left[ \frac{dx}{2} \right])}{d} \right) +$$

$$\frac{4 A \operatorname{Sec} [c] \operatorname{Sec} [c + dx] \sin [dx]}{3 d} + \frac{2 (2 A + 5 A \cos [c] - 3 B \cos [c]) \operatorname{Sec} [c] \operatorname{Tan} \left[ \frac{c}{2} \right]}{3 d}$$

- **Problem 477: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + dx]) \operatorname{Sec} [c + dx]^{3/2}}{a + a \cos [c + dx]} dx$$

Optimal (type 4, 159 leaves, 8 steps):

$$-\frac{(3 A - B) \sqrt{\cos [c + dx]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + dx), 2 \right] \sqrt{\sec [c + dx]}}{a d} -$$

$$\frac{(A - B) \sqrt{\cos [c + dx]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + dx), 2 \right] \sqrt{\sec [c + dx]}}{a d} + \frac{(3 A - B) \sqrt{\sec [c + dx]} \sin [c + dx]}{a d} - \frac{(A - B) \operatorname{Sec} [c + dx]^{3/2} \sin [c + dx]}{d (a + a \sec [c + dx])}$$

Result (type 5, 595 leaves):

$$\begin{aligned}
& - \frac{1}{\sqrt{2} d (a + a \cos [c + dx])} 3 A e^{-i (2c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2i (c+dx)}}} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Csc} \left[ \frac{c}{2} \right] \\
& \left( 1 + e^{2i (c+dx)} + (-1 + e^{2i c}) \sqrt{1 + e^{2i (c+dx)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i (c+dx)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] + \frac{1}{\sqrt{2} d (a + a \cos [c + dx])} \\
& B e^{-i (2c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2i (c+dx)}}} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Csc} \left[ \frac{c}{2} \right] \left( 1 + e^{2i (c+dx)} + (-1 + e^{2i c}) \sqrt{1 + e^{2i (c+dx)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i (c+dx)} \right] \right) \\
& \operatorname{Sec} \left[ \frac{c}{2} \right] - \frac{A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \sqrt{\cos [c + dx]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + dx), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + dx]} \sin [c]}{d (a + a \cos [c + dx])} + \\
& \frac{B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \sqrt{\cos [c + dx]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + dx), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + dx]} \sin [c]}{d (a + a \cos [c + dx])} + \frac{1}{a + a \cos [c + dx]} \\
& \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \sqrt{\sec [c + dx]} \left( \frac{(3A - B) \cos [dx] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{d} - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] (A \sin \left[ \frac{dx}{2} \right] - B \sin \left[ \frac{dx}{2} \right])}{d} - \frac{2(A - B) \tan \left[ \frac{c}{2} \right]}{d} \right)
\end{aligned}$$

■ **Problem 478: Result unnecessarily involves higher level functions.**

$$\int \frac{(A + B \cos [c + dx]) \sqrt{\sec [c + dx]}}{a + a \cos [c + dx]} dx$$

Optimal (type 4, 123 leaves, 7 steps):

$$\frac{(A - B) \sqrt{\cos [c + dx]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + dx), 2 \right] \sqrt{\sec [c + dx]}}{a d} + \frac{(A + B) \sqrt{\cos [c + dx]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + dx), 2 \right] \sqrt{\sec [c + dx]}}{a d} - \frac{(A - B) \sqrt{\sec [c + dx]} \sin [c + dx]}{d (a + a \sec [c + dx])}$$

Result (type 5, 185 leaves):

$$\frac{1}{2 a (1 + \cos [c + dx])} \cos \left[ \frac{1}{2} (c + dx) \right]^2 \left( \frac{4 (A + B) \sqrt{\cos [c + dx]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + dx), 2 \right] \sqrt{\sec [c + dx]}}{d} - 1 / (d (1 + e^{i (c+dx)})) \right) 4 i \\
(A - B) e^{-i (c+dx)} \left( 1 + e^{2i (c+dx)} - (1 + e^{i (c+dx)}) \sqrt{1 + e^{2i (c+dx)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i (c+dx)} \right] \right) \sqrt{\sec [c + dx]}$$

■ **Problem 479: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos [c + dx]}{(a + a \cos [c + dx]) \sqrt{\sec [c + dx]}} dx$$



Optimal (type 4, 125 leaves, 7 steps) :

$$-\frac{(A-3B)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{ad} + \frac{(A-B)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{ad} + \frac{(A-B)\sqrt{\sec[c+dx]}\sin[c+dx]}{d(a+a\sec[c+dx])}$$

Result (type 5, 402 leaves) :

$$\frac{1}{2ad(1+\cos[c+dx])}\cos\left[\frac{1}{2}(c+dx)\right]^2 - 2\sqrt{2}Ae^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\operatorname{Csc}[c]\left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) + 6\sqrt{2}Be^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\operatorname{Csc}[c]\left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) + \frac{2\left((A-2B)\cos\left[\frac{1}{2}(c-dx)\right]-B\cos\left[\frac{1}{2}(3c+dx)\right]\right)\operatorname{Csc}\left[\frac{c}{2}\right]\operatorname{Sec}\left[\frac{c}{2}\right]\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\sec[c+dx]}} + \left(4A\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}-4B\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}\right)$$

■ **Problem 480: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A+B\cos[c+dx]}{(a+a\cos[c+dx])\sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 163 leaves, 8 steps) :

$$\frac{3(A-B)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{ad} - \frac{(3A-5B)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{3ad} - \frac{(3A-5B)\sin[c+dx]}{3ad\sqrt{\sec[c+dx]}} + \frac{(A-B)\sin[c+dx]}{d\sqrt{\sec[c+dx]}(a+a\sec[c+dx])}$$

Result (type 5, 654 leaves) :

$$\frac{1}{\sqrt{2} d (a + a \cos [c + d x])} 3 A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Csc} \left[ \frac{c}{2} \right]$$

$$\left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] - \frac{1}{\sqrt{2} d (a + a \cos [c + d x])}$$

$$3 B e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Csc} \left[ \frac{c}{2} \right] \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right)$$

$$\operatorname{Sec} \left[ \frac{c}{2} \right] - \frac{A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{d (a + a \cos [c + d x])} +$$

$$\frac{5 B \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])} + \frac{1}{a + a \cos [c + d x]}$$

$$\cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \sqrt{\sec [c + d x]} \left( -\frac{(A - B) (2 + \cos [2 c]) \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{d} + \frac{2 B \cos [2 d x] \sin [2 c]}{3 d} + \right.$$

$$\left. \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (A \sin \left[ \frac{d x}{2} \right] - B \sin \left[ \frac{d x}{2} \right])}{d} + \frac{4 (A - B) \cos [c] \sin [d x]}{d} + \frac{2 B \cos [2 c] \sin [2 d x]}{3 d} + \frac{2 (A - B) \tan \left[ \frac{c}{2} \right]}{d} \right)$$

- **Problem 481: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos [c + d x]}{(a + a \cos [c + d x]) \operatorname{Sec} [c + d x]^{5/2}} dx$$

Optimal (type 4, 196 leaves, 9 steps):

$$\frac{3 (5 A - 7 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]} + 5 (A - B) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{5 a d} + \frac{5 (A - B) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{3 a d}$$

$$\frac{(5 A - 7 B) \sin [c + d x]}{5 a d \operatorname{Sec} [c + d x]^{3/2}} + \frac{5 (A - B) \sin [c + d x]}{3 a d \sqrt{\sec [c + d x]}} + \frac{(A - B) \sin [c + d x]}{d \operatorname{Sec} [c + d x]^{3/2} (a + a \sec [c + d x])}$$

Result (type 5, 498 leaves):

$$\frac{1}{60 a d (1 + \operatorname{Cos}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^2$$

$$\left( -180 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Csc}[c] \left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) + \right.$$

$$252 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Csc}[c] \left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) +$$

$$200 A \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} - 200 B \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} +$$

$$\sqrt{\operatorname{Sec}[c + d x]} \left( 3 (40 A - 51 B + (20 A - 33 B) \operatorname{Cos}[2c]) \operatorname{Cos}[d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] + \right.$$

$$40 (A - B) \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c] + 12 B \operatorname{Cos}[3 d x] \operatorname{Sin}[3 c] - 120 (A - B) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sin}\left[\frac{d x}{2}\right] -$$

$$\left. \left. 12 (20 A - 33 B) \operatorname{Cos}[c] \operatorname{Sin}[d x] + 40 (A - B) \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x] + 12 B \operatorname{Cos}[3 c] \operatorname{Sin}[3 d x] - 120 (A - B) \operatorname{Tan}\left[\frac{c}{2}\right] \right) \right)$$

■ **Problem 482: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x]^{3/2}}{(a + a \operatorname{Cos}[c + d x])^2} dx$$

Optimal (type 4, 208 leaves, 9 steps):

$$-\frac{(4A - B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{a^2 d} - \frac{(5A - 2B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 a^2 d} +$$

$$\frac{(4A - B) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{a^2 d} - \frac{(5A - 2B) \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{3 a^2 d (1 + \operatorname{Sec}[c + d x])} - \frac{(A - B) \operatorname{Sec}[c + d x]^{5/2} \operatorname{Sin}[c + d x]}{3 d (a + a \operatorname{Sec}[c + d x])^2}$$

Result (type 5, 689 leaves):

$$\begin{aligned}
& - \frac{1}{d (a + a \cos [c + dx])^2} 4 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \\
& \left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] + \frac{1}{d (a + a \cos [c + dx])^2} \\
& \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)} \right] \right) \\
& \operatorname{Sec} \left[ \frac{c}{2} \right] - \frac{10 A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \sqrt{\cos [c + dx]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + dx), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + dx]} \sin [c]}{3 d (a + a \cos [c + dx])^2} + \\
& \frac{4 B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \sqrt{\cos [c + dx]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + dx), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + dx]} \sin [c]}{3 d (a + a \cos [c + dx])^2} + \frac{1}{(a + a \cos [c + dx])^2} \\
& \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \sqrt{\sec [c + dx]} \left( \frac{2 (4 A - B) \cos [dx] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{d} - \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] (5 A \sin \left[ \frac{dx}{2} \right] - 2 B \sin \left[ \frac{dx}{2} \right])}{3 d} - \right. \\
& \left. \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 (A \sin \left[ \frac{dx}{2} \right] - B \sin \left[ \frac{dx}{2} \right])}{3 d} - \frac{4 (5 A - 2 B) \tan \left[ \frac{c}{2} \right]}{3 d} - \frac{2 (A - B) \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \tan \left[ \frac{c}{2} \right]}{3 d} \right)
\end{aligned}$$

■ **Problem 483: Result unnecessarily involves higher level functions.**

$$\int \frac{(A + B \cos [c + dx]) \sqrt{\sec [c + dx]}}{(a + a \cos [c + dx])^2} dx$$

Optimal (type 4, 161 leaves, 8 steps):

$$\begin{aligned}
& \frac{A \sqrt{\cos [c + dx]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + dx), 2 \right] \sqrt{\sec [c + dx]}}{a^2 d} + \\
& \frac{(2 A + B) \sqrt{\cos [c + dx]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + dx), 2 \right] \sqrt{\sec [c + dx]}}{3 a^2 d} - \frac{A \sqrt{\sec [c + dx]} \sin [c + dx]}{a^2 d (1 + \sec [c + dx])} - \frac{(A - B) \sec [c + dx]^{3/2} \sin [c + dx]}{3 d (a + a \sec [c + dx])^2}
\end{aligned}$$

Result (type 5, 263 leaves):

$$\frac{1}{6 a^2 d (1 + \operatorname{Cos}[c + d x])^2} e^{-i(2c+dx)} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{\operatorname{Sec}[c+dx]} \left( 3 i A e^{-2i(c+dx)} (1 + e^{i(c+dx)})^3 \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + \right. \\ \left. 8 (2A+B) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \left( \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + i \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) - \right. \\ \left. 2 i \operatorname{Cos}[c+dx] (7A-B + (5A+B) \operatorname{Cos}[c+dx] - i(A-B) \operatorname{Sin}[c+dx]) \right) \left( \operatorname{Cos}\left[\frac{1}{2}(3c+dx)\right] + i \operatorname{Sin}\left[\frac{1}{2}(3c+dx)\right] \right)$$

■ **Problem 484: Result unnecessarily involves higher level functions.**

$$\int \frac{A + B \operatorname{Cos}[c + d x]}{(a + a \operatorname{Cos}[c + d x])^2 \sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 168 leaves, 8 steps):

$$-\frac{B \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{a^2 d} + \\ \frac{(A + 2 B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 a^2 d} + \frac{(A + 2 B) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{3 a^2 d (1 + \operatorname{Sec}[c + d x])} - \frac{(A - B) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{3 d (a + a \operatorname{Sec}[c + d x])^2}$$

Result (type 5, 264 leaves):

$$\frac{1}{6 a^2 d (1 + \operatorname{Cos}[c + d x])^2} e^{-i(2c+dx)} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{\operatorname{Sec}[c+dx]} \\ \left( 8 (A + 2 B) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \left( \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + i \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) - \right. \\ \left. i \left( 3 B e^{-2i(c+dx)} (1 + e^{i(c+dx)})^3 \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + \right. \right. \\ \left. \left. 2 \operatorname{Cos}[c+dx] (-A - 5B + (A - 7B) \operatorname{Cos}[c+dx] + i(A - B) \operatorname{Sin}[c+dx]) \right) \right) \left( \operatorname{Cos}\left[\frac{1}{2}(3c+dx)\right] + i \operatorname{Sin}\left[\frac{1}{2}(3c+dx)\right] \right)$$

■ **Problem 485: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Cos}[c + d x]}{(a + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 4, 176 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(A - 4B) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{a^2 d} + \\
& \frac{(2A - 5B) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{3a^2 d} + \frac{(2A - 5B) \sqrt{\sec[c + dx]} \sin[c + dx]}{3a^2 d (1 + \sec[c + dx])} + \frac{(A - B) \sqrt{\sec[c + dx]} \sin[c + dx]}{3d (a + a \sec[c + dx])^2}
\end{aligned}$$

Result (type 5, 708 leaves):

$$\begin{aligned}
& - \frac{1}{d (a + a \cos[c + dx])^2} \sqrt{2} A e^{-i(2c + dx)} \sqrt{\frac{e^{i(c + dx)}}{1 + e^{2i(c + dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \\
& \left(1 + e^{2i(c + dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c + dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c + dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] + \frac{1}{d (a + a \cos[c + dx])^2} \\
& 4 \sqrt{2} B e^{-i(2c + dx)} \sqrt{\frac{e^{i(c + dx)}}{1 + e^{2i(c + dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c + dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c + dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c + dx)}\right]\right) \\
& \operatorname{Sec}\left[\frac{c}{2}\right] + \frac{4A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\sec[c + dx]} \sin[c]}{3d (a + a \cos[c + dx])^2} - \\
& \frac{10B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\sec[c + dx]} \sin[c]}{3d (a + a \cos[c + dx])^2} + \frac{1}{(a + a \cos[c + dx])^2} \\
& \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\sec[c + dx]} \left( - \frac{2(-A + 3B + B \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (4A \sin\left[\frac{dx}{2}\right] - 7B \sin\left[\frac{dx}{2}\right])}{3d} + \right. \\
& \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right])}{3d} + \frac{8B \cos[c] \sin[dx]}{d} - \frac{4(4A - 7B) \tan\left[\frac{c}{2}\right]}{3d} + \frac{2(A - B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right)
\end{aligned}$$

■ **Problem 486: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx]}{(a + a \cos[c + dx])^2 \sec[c + dx]^{5/2}} dx$$

Optimal (type 4, 206 leaves, 9 steps):

$$\begin{aligned}
& \frac{(4A - 7B) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{a^2 d} - \frac{5(A - 2B) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{3a^2 d} - \\
& \frac{5(A - 2B) \sin[c + dx]}{3a^2 d \sqrt{\sec[c + dx]}} + \frac{(4A - 7B) \sin[c + dx]}{3a^2 d \sqrt{\sec[c + dx]} (1 + \sec[c + dx])} + \frac{(A - B) \sin[c + dx]}{3d \sqrt{\sec[c + dx]} (a + a \sec[c + dx])^2}
\end{aligned}$$

Result (type 5, 753 leaves):

$$\begin{aligned}
& \frac{1}{d (a + a \cos [c + d x])^2} 4 \sqrt{2} A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \\
& \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] - \frac{1}{d (a + a \cos [c + d x])^2} \\
& 7 \sqrt{2} B e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \\
& \operatorname{Sec} \left[ \frac{c}{2} \right] - \frac{10 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^2} + \\
& \frac{20 B \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^2} + \frac{1}{(a + a \cos [c + d x])^2} \\
& \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\sec [c + d x]} \left( -\frac{2 (3 A - 5 B + A \cos [2 c] - 2 B \cos [2 c]) \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{d} + \frac{4 B \cos [2 d x] \sin [2 c]}{3 d} \right) + \\
& \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (7 A \sin \left[ \frac{d x}{2} \right] - 10 B \sin \left[ \frac{d x}{2} \right])}{3 d} - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (A \sin \left[ \frac{d x}{2} \right] - B \sin \left[ \frac{d x}{2} \right])}{3 d} + \\
& \left. \frac{8 (A - 2 B) \cos [c] \sin [d x]}{d} + \frac{4 B \cos [2 c] \sin [2 d x]}{3 d} + \frac{4 (7 A - 10 B) \tan \left[ \frac{c}{2} \right]}{3 d} - \frac{2 (A - B) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \tan \left[ \frac{c}{2} \right]}{3 d} \right)
\end{aligned}$$

- **Problem 487: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + d x]) \operatorname{Sec} [c + d x]^{3/2}}{(a + a \cos [c + d x])^3} dx$$

Optimal (type 4, 261 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(49 A - 9 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{10 a^3 d} - \\
& \frac{(13 A - 3 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{6 a^3 d} + \frac{(49 A - 9 B) \sqrt{\sec [c + d x]} \sin [c + d x]}{10 a^3 d} - \\
& \frac{(A - B) \operatorname{Sec} [c + d x]^{7/2} \sin [c + d x]}{5 d (a + a \operatorname{Sec} [c + d x])^3} - \frac{(8 A - 3 B) \operatorname{Sec} [c + d x]^{5/2} \sin [c + d x]}{15 a d (a + a \operatorname{Sec} [c + d x])^2} - \frac{(13 A - 3 B) \operatorname{Sec} [c + d x]^{3/2} \sin [c + d x]}{6 d (a^3 + a^3 \operatorname{Sec} [c + d x])}
\end{aligned}$$

Result (type 5, 778 leaves):

$$\begin{aligned}
& - \frac{1}{5 d (a + a \cos [c + d x])^3} 49 \sqrt{2} A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \\
& \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] + \frac{1}{5 d (a + a \cos [c + d x])^3} \\
& 9 \sqrt{2} B e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \\
& \operatorname{Sec} \left[ \frac{c}{2} \right] - \frac{26 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^3} + \\
& \frac{2 B \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{d (a + a \cos [c + d x])^3} + \frac{1}{(a + a \cos [c + d x])^3} \\
& \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\sec [c + d x]} \left( \frac{2 (49 A - 9 B) \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{5 d} - \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (8 A \sin \left[ \frac{d x}{2} \right] - 3 B \sin \left[ \frac{d x}{2} \right])}{15 d} \right. \\
& \left. \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (13 A \sin \left[ \frac{d x}{2} \right] - 3 B \sin \left[ \frac{d x}{2} \right])}{3 d} - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 (A \sin \left[ \frac{d x}{2} \right] - B \sin \left[ \frac{d x}{2} \right])}{5 d} \right. \\
& \left. \frac{4 (13 A - 3 B) \tan \left[ \frac{c}{2} \right]}{3 d} - \frac{4 (8 A - 3 B) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \tan \left[ \frac{c}{2} \right]}{15 d} - \frac{2 (A - B) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \tan \left[ \frac{c}{2} \right]}{5 d} \right)
\end{aligned}$$

■ **Problem 488: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + d x]) \sqrt{\sec [c + d x]}}{(a + a \cos [c + d x])^3} dx$$

Optimal (type 4, 222 leaves, 9 steps):

$$\begin{aligned}
& \frac{(9 A + B) \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{10 a^3 d} + \frac{(3 A + B) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{6 a^3 d} \\
& \frac{(A - B) \sec [c + d x]^{5/2} \sin [c + d x]}{5 d (a + a \sec [c + d x])^3} - \frac{(6 A - B) \sec [c + d x]^{3/2} \sin [c + d x]}{15 a d (a + a \sec [c + d x])^2} - \frac{(9 A + B) \sqrt{\sec [c + d x]} \sin [c + d x]}{10 d (a^3 + a^3 \sec [c + d x])}
\end{aligned}$$

Result (type 5, 773 leaves):



$$\begin{aligned}
& \frac{1}{5 d (a + a \cos [c + d x])^3} 9 \sqrt{2} A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \\
& \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] + \frac{1}{5 d (a + a \cos [c + d x])^3} \\
& \sqrt{2} B e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \\
& \operatorname{Sec} \left[ \frac{c}{2} \right] + \frac{2 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{d (a + a \cos [c + d x])^3} + \\
& \frac{2 B \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^3} + \\
& \frac{1}{(a + a \cos [c + d x])^3} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\sec [c + d x]} \\
& \left( -\frac{2 (9 A + B) \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{5 d} + \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 (A \sin \left[ \frac{d x}{2} \right] - B \sin \left[ \frac{d x}{2} \right])}{5 d} + \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (3 A \sin \left[ \frac{d x}{2} \right] + B \sin \left[ \frac{d x}{2} \right])}{3 d} \right) \\
& \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (3 A \sin \left[ \frac{d x}{2} \right] + 2 B \sin \left[ \frac{d x}{2} \right])}{15 d} + \frac{4 (3 A + B) \tan \left[ \frac{c}{2} \right]}{3 d} + \frac{4 (3 A + 2 B) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \tan \left[ \frac{c}{2} \right]}{15 d} + \frac{2 (A - B) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \tan \left[ \frac{c}{2} \right]}{5 d} \right)
\end{aligned}$$

- **Problem 489: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos [c + d x]}{(a + a \cos [c + d x])^3 \sqrt{\sec [c + d x]}} dx$$

Optimal (type 4, 216 leaves, 9 steps):

$$\begin{aligned}
& \frac{(A - B) \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{10 a^3 d} + \frac{(A + B) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{6 a^3 d} - \\
& \frac{(A - B) \operatorname{Sec} [c + d x]^{3/2} \sin [c + d x]}{5 d (a + a \operatorname{Sec} [c + d x])^3} - \frac{(4 A + B) \sqrt{\sec [c + d x]} \sin [c + d x]}{15 a d (a + a \operatorname{Sec} [c + d x])^2} + \frac{(A + B) \sqrt{\sec [c + d x]} \sin [c + d x]}{6 d (a^3 + a^3 \operatorname{Sec} [c + d x])}
\end{aligned}$$

Result (type 5, 772 leaves):

$$\begin{aligned}
& \frac{1}{5 d (a + a \operatorname{Cos}[c + d x])^3} \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \\
& \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] - \frac{1}{5 d (a + a \operatorname{Cos}[c + d x])^3} \\
& \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \\
& \operatorname{Sec}\left[\frac{c}{2}\right] + \frac{2 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c]}{3 d (a + a \operatorname{Cos}[c + d x])^3} + \\
& \frac{2 B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c]}{3 d (a + a \operatorname{Cos}[c + d x])^3} + \\
& \frac{1}{(a + a \operatorname{Cos}[c + d x])^3} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c + d x]} \\
& \left(-\frac{2(A - B) \operatorname{Cos}[d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5 d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (2 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 7 B \operatorname{Sin}\left[\frac{dx}{2}\right])}{15 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \operatorname{Sin}\left[\frac{dx}{2}\right] - B \operatorname{Sin}\left[\frac{dx}{2}\right])}{5 d} + \right. \\
& \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \operatorname{Sin}\left[\frac{dx}{2}\right] + B \operatorname{Sin}\left[\frac{dx}{2}\right])}{3 d} + \frac{4(A + B) \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} + \frac{4(2A - 7B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} - \frac{2(A - B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d}\right)
\end{aligned}$$

- **Problem 490: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Cos}[c + d x]}{(a + a \operatorname{Cos}[c + d x])^3 \operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 4, 222 leaves, 9 steps):

$$\begin{aligned}
& -\frac{(A + 9 B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{10 a^3 d} + \frac{(A + 3 B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{6 a^3 d} - \\
& \frac{(A - B) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{5 d (a + a \operatorname{Sec}[c + d x])^3} + \frac{(2 A + 3 B) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{15 a d (a + a \operatorname{Sec}[c + d x])^2} + \frac{(A + 3 B) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{6 d (a^3 + a^3 \operatorname{Sec}[c + d x])}
\end{aligned}$$

Result (type 5, 773 leaves):

$$\begin{aligned}
& - \frac{1}{5d (a + a \cos[c + dx])^3} \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \\
& \left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] - \frac{1}{5d (a + a \cos[c + dx])^3} \\
& 9\sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \\
& \operatorname{Sec}\left[\frac{c}{2}\right] + \frac{2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\sec[c + dx]} \sin[c]}{3d (a + a \cos[c + dx])^3} + \\
& \frac{2B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\sec[c + dx]} \sin[c]}{d (a + a \cos[c + dx])^3} + \\
& \frac{1}{(a + a \cos[c + dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\sec[c + dx]} \\
& \left( \frac{2(A + 9B) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (7A \sin\left[\frac{dx}{2}\right] - 12B \sin\left[\frac{dx}{2}\right])}{15d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - 9B \sin\left[\frac{dx}{2}\right])}{3d} \right. \\
& \left. + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right])}{5d} + \frac{4(A - 9B) \tan\left[\frac{c}{2}\right]}{3d} - \frac{4(7A - 12B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} + \frac{2(A - B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right)
\end{aligned}$$

- **Problem 491: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx]}{(a + a \cos[c + dx])^3 \operatorname{Sec}[c + dx]^{5/2}} dx$$

Optimal (type 4, 228 leaves, 9 steps):

$$\begin{aligned}
& - \frac{(9A - 49B) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{10a^3d} + \frac{(3A - 13B) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{6a^3d} + \\
& \frac{(A - B) \sqrt{\sec[c + dx]} \sin[c + dx]}{5d (a + a \sec[c + dx])^3} + \frac{(3A - 8B) \sqrt{\sec[c + dx]} \sin[c + dx]}{15ad (a + a \sec[c + dx])^2} + \frac{(3A - 13B) \sqrt{\sec[c + dx]} \sin[c + dx]}{6d (a^3 + a^3 \sec[c + dx])}
\end{aligned}$$

Result (type 5, 797 leaves):

$$\begin{aligned}
& - \frac{1}{5 d (a + a \cos [c + d x])^3} 9 \sqrt{2} A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \\
& \quad \operatorname{Csc} \left[ \frac{c}{2} \right] \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] + \\
& \frac{1}{5 d (a + a \cos [c + d x])^3} 49 \sqrt{2} B e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \\
& \quad \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] + \\
& \frac{2 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{d (a + a \cos [c + d x])^3} - \\
& \frac{26 B \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^3} + \frac{1}{(a + a \cos [c + d x])^3} \\
& \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\sec [c + d x]} \left( - \frac{2 (-9 A + 39 B + 10 B \cos [2 c]) \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{5 d} - \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (9 A \sin \left[ \frac{d x}{2} \right] - 23 B \sin \left[ \frac{d x}{2} \right])}{3 d} + \right. \\
& \quad \left. \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (12 A \sin \left[ \frac{d x}{2} \right] - 17 B \sin \left[ \frac{d x}{2} \right])}{15 d} - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 (A \sin \left[ \frac{d x}{2} \right] - B \sin \left[ \frac{d x}{2} \right])}{5 d} + \right. \\
& \quad \left. \frac{16 B \cos [c] \sin [d x]}{d} - \frac{4 (9 A - 23 B) \tan \left[ \frac{c}{2} \right]}{3 d} + \frac{4 (12 A - 17 B) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \tan \left[ \frac{c}{2} \right]}{15 d} - \frac{2 (A - B) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \tan \left[ \frac{c}{2} \right]}{5 d} \right)
\end{aligned}$$

■ **Problem 492: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos [c + d x]}{(a + a \cos [c + d x])^3 \operatorname{Sec} [c + d x]^{7/2}} dx$$

Optimal (type 4, 259 leaves, 10 steps):

$$\begin{aligned}
& \frac{7 (7 A - 17 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{10 a^3 d} - \\
& \frac{(13 A - 33 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{6 a^3 d} - \frac{(13 A - 33 B) \sin [c + d x]}{6 a^3 d \sqrt{\sec [c + d x]}} + \\
& \frac{(A - B) \sin [c + d x]}{5 d \sqrt{\sec [c + d x]} (a + a \sec [c + d x])^3} + \frac{(A - 2 B) \sin [c + d x]}{3 a d \sqrt{\sec [c + d x]} (a + a \sec [c + d x])^2} + \frac{7 (7 A - 17 B) \sin [c + d x]}{30 d \sqrt{\sec [c + d x]} (a^3 + a^3 \sec [c + d x])}
\end{aligned}$$

Result (type 5, 842 leaves):

$$\begin{aligned}
& \frac{1}{5 d (a + a \operatorname{Cos}[c + d x])^3} 49 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \\
& \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] - \\
& \frac{1}{5 d (a + a \operatorname{Cos}[c + d x])^3} 119 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \\
& \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] - \\
& \frac{26 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c]}{3 d (a + a \operatorname{Cos}[c + d x])^3} + \\
& \frac{22 B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c]}{d (a + a \operatorname{Cos}[c + d x])^3} + \\
& \frac{1}{(a + a \operatorname{Cos}[c + d x])^3} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c + d x]} \left(-\frac{2(39 A - 89 B + 10 A \operatorname{Cos}[2c] - 30 B \operatorname{Cos}[2c]) \operatorname{Cos}[d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5 d} + \right. \\
& \frac{8 B \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{3 d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (23 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 43 B \operatorname{Sin}\left[\frac{dx}{2}\right])}{3 d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (17 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 22 B \operatorname{Sin}\left[\frac{dx}{2}\right])}{15 d} + \\
& \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \operatorname{Sin}\left[\frac{dx}{2}\right] - B \operatorname{Sin}\left[\frac{dx}{2}\right])}{5 d} + \frac{16 (A - 3 B) \operatorname{Cos}[c] \operatorname{Sin}[d x]}{d} + \frac{8 B \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x]}{3 d} + \\
& \left. \frac{4 (23 A - 43 B) \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} - \frac{4 (17 A - 22 B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} + \frac{2 (A - B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right)
\end{aligned}$$

■ **Problem 497: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \operatorname{Cos}[c + d x]} (A + B \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x]^{3/2} dx$$

Optimal (type 3, 96 leaves, 4 steps):

$$\frac{2 \sqrt{a} B \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \operatorname{Cos}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]} + \frac{2 a A \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{d \sqrt{a+a \operatorname{Cos}[c+dx]}}}{d}$$

Result (type 3, 299 leaves):

$$\frac{1}{d \sqrt{(1 + e^{2i dx}) \cos[c] + i(-1 + e^{2i dx}) \sin[c]}} \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{\operatorname{Sec}[c + dx]}$$

$$\left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) \left(i B \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2i dx}) \cos[c] + i(-1 + e^{2i dx}) \sin[c]}\right] \cos[c + dx] -\right.$$

$$\left. i B \cos[c + dx] \operatorname{Log}\left[2 \left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2i dx}) \cos[c] + i(-1 + e^{2i dx}) \sin[c]}\right)\right] +\right.$$

$$\left. 2\sqrt{2} A \left(\cos\left[\frac{dx}{2}\right] - i \sin\left[\frac{dx}{2}\right]\right) \sqrt{\cos[c + dx] (\cos[dx] + i \sin[dx])} \sin\left[\frac{1}{2}(c + dx)\right]\right)$$

- **Problem 498: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \cos[c + dx]} (A + B \cos[c + dx]) \sqrt{\operatorname{Sec}[c + dx]} dx}{d}$$

Optimal (type 3, 98 leaves, 4 steps):

$$\frac{\sqrt{a} (2A + B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right] \sqrt{\cos[c + dx]} \sqrt{\operatorname{Sec}[c + dx]}}{d} + \frac{a B \sin[c + dx]}{d \sqrt{a + a \cos[c + dx]} \sqrt{\operatorname{Sec}[c + dx]}}$$

Result (type 3, 442 leaves):

$$\frac{1}{2\sqrt{2} d \sqrt{\operatorname{Sec}[c + dx]} \sqrt{\cos[c + dx]} (\cos[dx] + i \sin[dx])} \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]$$

$$\left(-i(2A + B) \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2 \left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2i dx}) \cos[c] + i(-1 + e^{2i dx}) \sin[c]}\right)\right] +\right.$$

$$\left. i(2A + B) \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2i dx}) \cos[c] + i(-1 + e^{2i dx}) \sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) +\right.$$

$$\left. 2A \operatorname{Log}\left[2 \left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2i dx}) \cos[c] + i(-1 + e^{2i dx}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] +\right.$$

$$\left. B \operatorname{Log}\left[2 \left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2i dx}) \cos[c] + i(-1 + e^{2i dx}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] +\right.$$

$$\left. 2\sqrt{2} B \sqrt{\cos[c + dx] (\cos[dx] + i \sin[dx])} \sin\left[\frac{1}{2}(c + dx)\right]\right)$$

- **Problem 499: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \cos[c + dx]} (A + B \cos[c + dx])}{\sqrt{\operatorname{Sec}[c + dx]}} dx$$

Optimal (type 3, 151 leaves, 5 steps):

$$\frac{\sqrt{a} (4A + 3B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{4d} +$$

$$\frac{aB \sin[c+dx]}{2d \sqrt{a+a\cos[c+dx]} \sec[c+dx]^{3/2}} + \frac{a(4A+3B) \sin[c+dx]}{4d \sqrt{a+a\cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 531 leaves):

$$\frac{1}{8\sqrt{2}d\sqrt{\sec[c+dx]}\sqrt{\cos[c+dx]}(\cos[dx]+i\sin[dx])}\sqrt{a(1+\cos[c+dx])}\sec\left[\frac{1}{2}(c+dx)\right]$$

$$\left(-i(4A+3B)\cos\left[\frac{dx}{2}\right]\operatorname{Log}\left[2\left(e^{i dx}\cos\left[\frac{c}{2}\right]+ie^{i dx}\sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2i dx})\cos[c]+i(-1+e^{2i dx})\sin[c]}\right)\right]\right)+$$

$$i(4A+3B)\operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right]+i\sin\left[\frac{c}{2}\right]\right)\sqrt{(1+e^{2i dx})\cos[c]+i(-1+e^{2i dx})\sin[c]}\right]\left(\cos\left[\frac{dx}{2}\right]+i\sin\left[\frac{dx}{2}\right]\right)+$$

$$4A\operatorname{Log}\left[2\left(e^{i dx}\cos\left[\frac{c}{2}\right]+ie^{i dx}\sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2i dx})\cos[c]+i(-1+e^{2i dx})\sin[c]}\right)\right]\sin\left[\frac{dx}{2}\right]+$$

$$3B\operatorname{Log}\left[2\left(e^{i dx}\cos\left[\frac{c}{2}\right]+ie^{i dx}\sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2i dx})\cos[c]+i(-1+e^{2i dx})\sin[c]}\right)\right]\sin\left[\frac{dx}{2}\right]+$$

$$8\sqrt{2}A\sqrt{\cos[c+dx]}(\cos[dx]+i\sin[dx])\sin\left[\frac{1}{2}(c+dx)\right]+$$

$$4\sqrt{2}B\sqrt{\cos[c+dx]}(\cos[dx]+i\sin[dx])\sin\left[\frac{1}{2}(c+dx)\right]+2\sqrt{2}B\sqrt{\cos[c+dx]}(\cos[dx]+i\sin[dx])\sin\left[\frac{3}{2}(c+dx)\right]$$

■ **Problem 500: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+a\cos[c+dx]}(A+B\cos[c+dx])}{\sec[c+dx]^{3/2}} dx$$

Optimal (type 3, 196 leaves, 6 steps):

$$\frac{\sqrt{a} (6A + 5B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{8d} +$$

$$\frac{aB \sin[c+dx]}{3d \sqrt{a+a\cos[c+dx]} \sec[c+dx]^{5/2}} + \frac{a(6A+5B) \sin[c+dx]}{12d \sqrt{a+a\cos[c+dx]} \sec[c+dx]^{3/2}} + \frac{a(6A+5B) \sin[c+dx]}{8d \sqrt{a+a\cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 615 leaves):

$$\begin{aligned}
& \frac{1}{48 \sqrt{2} d \sqrt{\sec[c+dx]} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx])} \sqrt{a(1+\cos[c+dx])} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \\
& \left( -3 i (6 A + 5 B) \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2 i dx}) \cos[c] + i(-1+e^{2 i dx}) \sin[c]} \right) \right] + \right. \\
& \quad 3 i (6 A + 5 B) \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i dx}) \cos[c] + i(-1+e^{2 i dx}) \sin[c]} \right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) + \\
& \quad 18 A \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2 i dx}) \cos[c] + i(-1+e^{2 i dx}) \sin[c]} \right) \right] \sin\left[\frac{dx}{2}\right] + \\
& \quad 15 B \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2 i dx}) \cos[c] + i(-1+e^{2 i dx}) \sin[c]} \right) \right] \sin\left[\frac{dx}{2}\right] + \\
& \quad 24 \sqrt{2} A \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + \\
& \quad 28 \sqrt{2} B \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + 12 \sqrt{2} A \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] + \\
& \quad \left. 6 \sqrt{2} B \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] + 4 \sqrt{2} B \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{5}{2}(c+dx)\right] \right)
\end{aligned}$$

- **Problem 505: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c+dx])^{3/2} (A + B \cos[c+dx]) \operatorname{Sec}[c+dx]^{5/2} dx$$

Optimal (type 3, 145 leaves, 5 steps):

$$\begin{aligned}
& \frac{2 a^{3/2} B \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} + \\
& \frac{2 a^2 (4 A + 3 B) \sqrt{\sec[c+dx]} \sin[c+dx]}{3 d \sqrt{a+a \cos[c+dx]}} + \frac{2 a A \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{3 d}
\end{aligned}$$

Result (type 3, 774 leaves):



$$\begin{aligned}
& \frac{1}{12 d \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}} (a (1 + \cos [c + d x]))^{3/2} \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^3 \sqrt{\operatorname{Sec}[c + d x]} \\
& \left(3 B e^{-\frac{1}{2} i d x} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right] \cos\left[\frac{c}{2}\right]^2 (i (1 + e^{2 i d x}) \cos [c] - (-1 + e^{2 i d x}) \sin [c]) + \right. \\
& \left. 3 B e^{-\frac{1}{2} i d x} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right] \right. \\
& \left. \sin\left[\frac{c}{2}\right]^2 (i (1 + e^{2 i d x}) \cos [c] - (-1 + e^{2 i d x}) \sin [c]) - 3 i B e^{-\frac{1}{2} i d x} \cos\left[\frac{c}{2}\right]^2 \right. \\
& \left. \operatorname{Log}\left[2\left(e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] \left((1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]\right) - \right. \\
& \left. 3 i B e^{-\frac{1}{2} i d x} \operatorname{Log}\left[2\left(e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] \right] \sin\left[\frac{c}{2}\right]^2 \\
& \left. \left((1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]\right) + 20 A \cos\left[\frac{c}{2}\right] \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \sin\left[\frac{d x}{2}\right] + \right. \\
& \left. 12 B \cos\left[\frac{c}{2}\right] \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \sin\left[\frac{d x}{2}\right] + 20 \sqrt{2} A \cos\left[\frac{d x}{2}\right] \sin\left[\frac{c}{2}\right] \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} + \right. \\
& \left. 12 \sqrt{2} B \cos\left[\frac{d x}{2}\right] \sin\left[\frac{c}{2}\right] \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} + 4 \sqrt{2} A \operatorname{Sec}[c + d x] \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin\left[\frac{1}{2}(c + d x)\right] \right)
\end{aligned}$$

■ **Problem 506: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{3/2} (A + B \cos [c + d x]) \operatorname{Sec}[c + d x]^{3/2} dx$$

Optimal (type 3, 146 leaves, 5 steps):

$$\begin{aligned}
& \frac{a^{3/2} (2 A + 3 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}}\right] \sqrt{\cos [c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}{d} - \\
& \frac{a^2 (2 A - B) \sin [c + d x]}{d \sqrt{a + a \cos [c + d x]} \sqrt{\operatorname{Sec}[c + d x]}} + \frac{2 a A \sqrt{a + a \cos [c + d x]} \sqrt{\operatorname{Sec}[c + d x]} \sin [c + d x]}{d}
\end{aligned}$$

Result (type 3, 885 leaves):

$$\begin{aligned}
& \frac{1}{4 d \sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}} a \sqrt{a(1+\cos [c+d x])} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \sqrt{\operatorname{Sec}[c+d x]} \\
& \left(-i(2 A+3 B) \cos \left[c+\frac{d x}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right]-\right. \\
& \quad 2 i A \cos \left[c+\frac{3 d x}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right]- \\
& \quad \left.3 i B \cos \left[c+\frac{3 d x}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right]+ \right. \\
& \quad 2 i(2 A+3 B) \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right] \cos [c+d x] \left(\cos \left[\frac{d x}{2}\right]+i \sin \left[\frac{d x}{2}\right]\right)- \\
& \quad 2 A \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right] \sin \left[c+\frac{d x}{2}\right]- \\
& \quad \left.3 B \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right] \sin \left[c+\frac{d x}{2}\right]+ \right. \\
& \quad 8 \sqrt{2} A \sqrt{\cos [c+d x]}\left(\cos [d x]+i \sin [d x]\right) \sin \left[\frac{1}{2}(c+d x)\right]- \\
& \quad 2 \sqrt{2} B \sqrt{\cos [c+d x]}\left(\cos [d x]+i \sin [d x]\right) \sin \left[\frac{1}{2}(c+d x)\right]+2 \sqrt{2} B \sqrt{\cos [c+d x]}\left(\cos [d x]+i \sin [d x]\right) \sin \left[\frac{3}{2}(c+d x)\right]+ \\
& \quad 2 A \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right] \sin \left[c+\frac{3 d x}{2}\right]+ \\
& \quad \left.3 B \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right] \sin \left[c+\frac{3 d x}{2}\right]\right)
\end{aligned}$$

■ **Problem 507: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a+a \cos [c+d x])^{3 / 2}(A+B \cos [c+d x]) \sqrt{\operatorname{Sec}[c+d x]} d x$$

Optimal (type 3, 153 leaves, 5 steps):

$$\frac{a^{3 / 2}(12 A+7 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{4 d}+\frac{a^2(4 A+5 B) \sin [c+d x]}{4 d \sqrt{a+a \cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}+\frac{a B \sqrt{a+a \cos [c+d x]} \sin [c+d x]}{2 d \sqrt{\operatorname{Sec}[c+d x]}}$$

Result (type 3, 532 leaves):

$$\begin{aligned}
& \frac{1}{8 \sqrt{2} d \sqrt{\sec[c+dx]} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx])} a \sqrt{a(1+\cos[c+dx])} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \\
& \left( -i(12A+7B) \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2\left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]}\right)\right] \right) + \\
& i(12A+7B) \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) + \\
& 12A \operatorname{Log}\left[2\left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] + \\
& 7B \operatorname{Log}\left[2\left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] + \\
& 8\sqrt{2} A \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + \\
& 12\sqrt{2} B \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + 2\sqrt{2} B \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right]
\end{aligned}$$

■ **Problem 508: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \cos[c+dx])^{3/2} (A+B \cos[c+dx])}{\sqrt{\sec[c+dx]}} dx$$

Optimal (type 3, 200 leaves, 6 steps):

$$\begin{aligned}
& \frac{a^{3/2} (14A+11B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{8d} + \\
& \frac{a^2 (6A+7B) \sin[c+dx]}{12d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2}} + \frac{aB \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{3d \sec[c+dx]^{3/2}} + \frac{a^2 (14A+11B) \sin[c+dx]}{8d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}
\end{aligned}$$

Result (type 3, 616 leaves):

$$\begin{aligned}
& \frac{1}{48 \sqrt{2} d \sqrt{\sec[c+dx]} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx])} a \sqrt{a(1+\cos[c+dx])} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \\
& \left( -3 i (14 A + 11 B) \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right) \right] \right) + \\
& 3 i (14 A + 11 B) \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) + \\
& 42 A \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right) \right] \sin\left[\frac{dx}{2}\right] + \\
& 33 B \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right) \right] \sin\left[\frac{dx}{2}\right] + \\
& 72 \sqrt{2} A \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + \\
& 52 \sqrt{2} B \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + 12 \sqrt{2} A \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] + \\
& 18 \sqrt{2} B \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] + 4 \sqrt{2} B \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{5}{2}(c+dx)\right] \Big)
\end{aligned}$$

■ **Problem 509: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \cos[c+dx])^{3/2} (A + B \cos[c+dx])}{\sec[c+dx]^{3/2}} dx$$

Optimal (type 3, 247 leaves, 7 steps):

$$\begin{aligned}
& \frac{a^{3/2} (88 A + 75 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{64 d} + \frac{a^2 (8 A + 9 B) \sin[c+dx]}{24 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{5/2}} + \\
& \frac{a B \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{4 d \sec[c+dx]^{5/2}} + \frac{a^2 (88 A + 75 B) \sin[c+dx]}{96 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2}} + \frac{a^2 (88 A + 75 B) \sin[c+dx]}{64 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}
\end{aligned}$$

Result (type 3, 371 leaves):

$$\begin{aligned}
& - \frac{1}{768 d \sqrt{2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c]}} (a (1 + \cos [c + d x]))^{3/2} \sec \left[ \frac{1}{2} (c + d x) \right]^3 \sqrt{\sec [c + d x]} \\
& \left( - \frac{1}{\sqrt{2}} 3 i (88 A + 75 B) e^{-\frac{1}{2} i d x} \left( \operatorname{ArcTanh} \left[ \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right] - \right. \right. \\
& \quad \left. \left. \operatorname{Log} \left[ 2 \left( e^{i d x} \cos \left[ \frac{c}{2} \right] + i e^{i d x} \sin \left[ \frac{c}{2} \right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) \right] \right) \left( (1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c] \right) + \right. \\
& \quad \left. (296 A + 285 B + 2 (88 A + 93 B) \cos [c + d x] + 4 (8 A + 15 B) \cos [2 (c + d x)] + 12 B \cos [3 (c + d x)]) \right) \\
& \quad \left. \sqrt{2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c]} \left( \sin \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{3}{2} (c + d x) \right] \right) \right)
\end{aligned}$$

- **Problem 514: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x]) \sec [c + d x]^{7/2} dx$$

Optimal (type 3, 192 leaves, 6 steps):

$$\begin{aligned}
& \frac{2 a^{5/2} B \operatorname{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{d} + \frac{2 a^3 (32 A + 35 B) \sqrt{\sec [c + d x]} \sin [c + d x]}{15 d \sqrt{a + a \cos [c + d x]}} + \\
& \frac{2 a^2 (8 A + 5 B) \sqrt{a + a \cos [c + d x]} \sec [c + d x]^{3/2} \sin [c + d x]}{15 d} + \frac{2 a A (a + a \cos [c + d x])^{3/2} \sec [c + d x]^{5/2} \sin [c + d x]}{5 d}
\end{aligned}$$

Result (type 3, 946 leaves):

$$\begin{aligned}
& \frac{1}{4} B \sqrt{\cos[c+dx]} (a(1+\cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c+dx]} \\
& \left( \frac{1}{2} i \sin\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]} \right) - \right. \\
& \quad \left. \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]} \right) \right) + \\
& \quad \frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]} \right) + \right. \\
& \quad \left. \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]} \right) \right) \right) + \\
& (a(1+\cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c+dx]} \left( \frac{(43A+40B) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{30d} + \frac{(43A+40B) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{30d} + \right. \\
& \quad \frac{A \sec[c+dx]^2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{10d} + \\
& \quad \left. \frac{\sec[c+dx] \left( 14A \sin\left[\frac{c}{2} + \frac{dx}{2}\right] + 5B \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)}{30d} \right)
\end{aligned}$$

■ **Problem 515: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a+a \cos[c+dx])^{5/2} (A+B \cos[c+dx]) \sec[c+dx]^{5/2} dx$$

Optimal (type 3, 193 leaves, 6 steps):

$$\begin{aligned}
& \frac{a^{5/2} (2A+5B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} - \frac{a^3 (14A+3B) \sin[c+dx]}{3d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}} + \\
& \frac{2a^2 (2A+B) \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]} \sin[c+dx]}{d} + \frac{2aA (a+a \cos[c+dx])^{3/2} \sec[c+dx]^{3/2} \sin[c+dx]}{3d}
\end{aligned}$$

Result (type 3, 946 leaves):

$$\begin{aligned}
& \frac{1}{8} (2A + 5B) \sqrt{\cos[c + dx]} (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c + dx]} \\
& \left( \frac{1}{2} i \sin\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]} \right) - \right. \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]} \right) \right) + \\
& \frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]} \right) + \right. \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]} \right) \right) \right) + \\
& (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c + dx]} \left( \frac{(32A + 9B) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{24d} + \frac{B \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{8d} + \right. \\
& \quad \left. \frac{(32A + 9B) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{24d} + \right. \\
& \quad \left. \frac{B \cos\left[\frac{3c}{2}\right] \sin\left[\frac{3dx}{2}\right]}{8d} + \frac{A \sec[c + dx] \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{6d} \right)
\end{aligned}$$

■ **Problem 516: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{5/2} (A + B \cos[c + dx]) \sec[c + dx]^{3/2} dx$$

Optimal (type 3, 198 leaves, 6 steps):

$$\begin{aligned}
& \frac{a^{5/2} (20A + 19B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right] \sqrt{\cos[c + dx]} \sqrt{\sec[c + dx]}}{4d} - \frac{a^3 (4A - 9B) \sin[c + dx]}{4d \sqrt{a + a \cos[c + dx]} \sqrt{\sec[c + dx]}} - \\
& \frac{a^2 (4A - B) \sqrt{a + a \cos[c + dx]} \sin[c + dx]}{2d \sqrt{\sec[c + dx]}} + \frac{2aA (a + a \cos[c + dx])^{3/2} \sqrt{\sec[c + dx]} \sin[c + dx]}{d}
\end{aligned}$$

Result (type 3, 973 leaves):

$$\begin{aligned}
& \frac{1}{32} (20A + 19B) \sqrt{\cos[c + dx]} (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c + dx]} \\
& \left( \frac{1}{2} i \sin\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]} \right) - \right. \\
& \quad \left. \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]} \right) \right) + \\
& \frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]} \right) + \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]} \right) \right) \Bigg) + \\
& (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c + dx]} \left( \frac{3(4A - 3B) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{32d} + \frac{(2A + 5B) \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{16d} + \right. \\
& \quad \frac{B \cos\left[\frac{5dx}{2}\right] \sin\left[\frac{5c}{2}\right]}{32d} + \frac{3(4A - 3B) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{32d} + \\
& \quad \left. \frac{(2A + 5B) \cos\left[\frac{3c}{2}\right] \sin\left[\frac{3dx}{2}\right]}{16d} + \frac{B \cos\left[\frac{5c}{2}\right] \sin\left[\frac{5dx}{2}\right]}{32d} \right)
\end{aligned}$$

■ **Problem 517: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{5/2} (A + B \cos[c + dx]) \sqrt{\sec[c + dx]} dx$$

Optimal (type 3, 200 leaves, 6 steps):

$$\begin{aligned}
& \frac{a^{5/2} (38A + 25B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right] \sqrt{\cos[c + dx]} \sqrt{\sec[c + dx]}}{8d} + \\
& \frac{a^3 (54A + 49B) \sin[c + dx]}{24d \sqrt{a + a \cos[c + dx]} \sqrt{\sec[c + dx]}} + \frac{a^2 (2A + 3B) \sqrt{a + a \cos[c + dx]} \sin[c + dx]}{4d \sqrt{\sec[c + dx]}} + \frac{aB (a + a \cos[c + dx])^{3/2} \sin[c + dx]}{3d \sqrt{\sec[c + dx]}}
\end{aligned}$$

Result (type 3, 618 leaves):



$$\begin{aligned}
& \frac{1}{48 \sqrt{2} d \sqrt{\sec[c+dx]} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx])} a^2 \sqrt{a(1+\cos[c+dx])} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \\
& \left( -3 i (38 A + 25 B) \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right) \right] \right) + \\
& 3 i (38 A + 25 B) \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) + \\
& 114 A \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right) \right] \sin\left[\frac{dx}{2}\right] + \\
& 75 B \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right) \right] \sin\left[\frac{dx}{2}\right] + \\
& 120 \sqrt{2} A \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + \\
& 124 \sqrt{2} B \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + 12 \sqrt{2} A \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] + \\
& 30 \sqrt{2} B \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] + 4 \sqrt{2} B \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{5}{2}(c+dx)\right] \Big)
\end{aligned}$$

- **Problem 518: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c+dx])^{5/2} (A + B \cos[c+dx])}{\sqrt{\sec[c+dx]}} dx$$

Optimal (type 3, 247 leaves, 7 steps):

$$\begin{aligned}
& \frac{a^{5/2} (200 A + 163 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{64 d} + \frac{a^3 (104 A + 95 B) \sin[c+dx]}{96 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2}} + \\
& \frac{a^2 (8 A + 11 B) \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{24 d \sec[c+dx]^{3/2}} + \frac{a B (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{4 d \sec[c+dx]^{3/2}} + \frac{a^3 (200 A + 163 B) \sin[c+dx]}{64 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}
\end{aligned}$$

Result (type 3, 1081 leaves):

$$\begin{aligned}
& \frac{1}{512} (200 A + 163 B) \sqrt{\cos[c + dx]} (a (1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c + dx]} \\
& \left( \frac{1}{2} i \sin\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]}\right)\right] \right) \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]} \right) - \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]}\right] \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]} \right) \Bigg) + \\
& \frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]}\right)\right] \right) \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]} \right) + \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]}\right] \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]} \right) \Bigg) \Bigg) + \\
& (a (1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c + dx]} \left( - \frac{(376 A + 265 B) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{1536 d} + \frac{(64 A + 55 B) \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{192 d} + \right. \\
& \quad \frac{(40 A + 47 B) \cos\left[\frac{5dx}{2}\right] \sin\left[\frac{5c}{2}\right]}{512 d} + \frac{(2 A + 5 B) \cos\left[\frac{7dx}{2}\right] \sin\left[\frac{7c}{2}\right]}{192 d} + \\
& \quad \frac{B \cos\left[\frac{9dx}{2}\right] \sin\left[\frac{9c}{2}\right]}{256 d} - \frac{(376 A + 265 B) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{1536 d} + \\
& \quad \frac{(64 A + 55 B) \cos\left[\frac{3c}{2}\right] \sin\left[\frac{3dx}{2}\right]}{192 d} + \frac{(40 A + 47 B) \cos\left[\frac{5c}{2}\right] \sin\left[\frac{5dx}{2}\right]}{512 d} + \\
& \quad \left. \frac{(2 A + 5 B) \cos\left[\frac{7c}{2}\right] \sin\left[\frac{7dx}{2}\right]}{192 d} + \frac{B \cos\left[\frac{9c}{2}\right] \sin\left[\frac{9dx}{2}\right]}{256 d} \right)
\end{aligned}$$

■ **Problem 519: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx])^{5/2} (A + B \cos[c + dx])}{\sec[c + dx]^{3/2}} dx$$

Optimal (type 3, 294 leaves, 8 steps):

$$\frac{a^{5/2} (326 A + 283 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{128 d} +$$

$$\frac{a^3 (170 A + 157 B) \sin[c+dx]}{240 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{5/2}} + \frac{a^2 (10 A + 13 B) \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{40 d \sec[c+dx]^{5/2}} +$$

$$\frac{a B (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{5 d \sec[c+dx]^{5/2}} + \frac{a^3 (326 A + 283 B) \sin[c+dx]}{192 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2}} + \frac{a^3 (326 A + 283 B) \sin[c+dx]}{128 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 1135 leaves):

$$\frac{1}{1024} (326 A + 283 B) \sqrt{\cos[c+dx]} (a (1 + \cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c+dx]}$$

$$\left(\frac{1}{2} i \sin\left[\frac{c}{2}\right] \left(-\left(2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right]\right) \left(\cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right]\right)\right.\right.\right.$$

$$\left.\left.\frac{\sqrt{e^{-idx} \left((1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]\right)}}{\left(d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]}\right)} - \right.\right.$$

$$\left.\left(2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right] \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right)\right.\right.$$

$$\left.\left.\frac{\sqrt{e^{-idx} \left((1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]\right)}}{\left(d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]}\right)}\right)\right) +$$

$$\frac{1}{2} \cos\left[\frac{c}{2}\right] \left(-\left(2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right]\right) \left(\cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right]\right)\right.\right.$$

$$\left.\left.\frac{\sqrt{e^{-idx} \left((1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]\right)}}{\left(d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]}\right)} + \right.\right.$$

$$\left.\left(2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right] \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right)\right.\right.$$

$$\left.\left.\frac{\sqrt{e^{-idx} \left((1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]\right)}}{\left(d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]}\right)}\right)\right) +$$

$$(a (1 + \cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c+dx]} \left(-\frac{(2650 A + 2309 B) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{15360 d} + \right.$$

$$\frac{(550 A + 509 B) \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{1920 d} +$$

$$\frac{(94 A + 95 B) \cos\left[\frac{5dx}{2}\right] \sin\left[\frac{5c}{2}\right]}{1024 d} +$$

$$\frac{(25 A + 32 B) \cos\left[\frac{7dx}{2}\right] \sin\left[\frac{7c}{2}\right]}{960 d} +$$

$$\frac{(2 A + 5 B) \cos\left[\frac{9dx}{2}\right] \sin\left[\frac{9c}{2}\right]}{512 d} +$$

$$\frac{B \operatorname{Cos}\left[\frac{11dx}{2}\right] \operatorname{Sin}\left[\frac{11c}{2}\right]}{640d} -$$

$$\frac{(2650A + 2309B) \operatorname{Cos}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{15360d} +$$

$$\frac{(550A + 509B) \operatorname{Cos}\left[\frac{3c}{2}\right] \operatorname{Sin}\left[\frac{3dx}{2}\right]}{1920d} +$$

$$\frac{(94A + 95B) \operatorname{Cos}\left[\frac{5c}{2}\right] \operatorname{Sin}\left[\frac{5dx}{2}\right]}{1024d} + \frac{(25A + 32B) \operatorname{Cos}\left[\frac{7c}{2}\right] \operatorname{Sin}\left[\frac{7dx}{2}\right]}{960d} +$$

$$\left( \frac{(2A + 5B) \operatorname{Cos}\left[\frac{9c}{2}\right] \operatorname{Sin}\left[\frac{9dx}{2}\right]}{512d} + \frac{B \operatorname{Cos}\left[\frac{11c}{2}\right] \operatorname{Sin}\left[\frac{11dx}{2}\right]}{640d} \right)$$

■ **Problem 520: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \operatorname{Cos}[c + dx]) \operatorname{Sec}[c + dx]^{11/2}}{\sqrt{a + a \operatorname{Cos}[c + dx]}} dx$$

Optimal (type 3, 295 leaves, 9 steps):

$$-\frac{\sqrt{2} (A - B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Cos}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{\sqrt{a} d} +$$

$$\frac{2 (257A - 129B) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{315d \sqrt{a+a \operatorname{Cos}[c+dx]}} - \frac{2 (29A - 93B) \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{315d \sqrt{a+a \operatorname{Cos}[c+dx]}} +$$

$$\frac{2 (19A - 3B) \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{105d \sqrt{a+a \operatorname{Cos}[c+dx]}} - \frac{2 (A - 9B) \operatorname{Sec}[c+dx]^{7/2} \operatorname{Sin}[c+dx]}{63d \sqrt{a+a \operatorname{Cos}[c+dx]}} + \frac{2A \operatorname{Sec}[c+dx]^{9/2} \operatorname{Sin}[c+dx]}{9d \sqrt{a+a \operatorname{Cos}[c+dx]}}$$

Result (type 3, 264 leaves):

$$\frac{1}{\sqrt{a} (1 + \operatorname{Cos}[c + dx])}$$

$$\operatorname{Cos}\left[\frac{1}{2} (c + dx)\right] \left( 1 / d 2 i (A - B) e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2 i (c+dx)}}} \sqrt{1 + e^{2 i (c+dx)}} \left( \operatorname{Log}[1 + e^{i (c+dx)}] - \operatorname{Log}[1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}] \right) \right) +$$

$$1 / (630d) (1279A - 423B + (-214A + 918B) \operatorname{Cos}[c + dx] + 8 (157A - 69B) \operatorname{Cos}[2 (c + dx)] - 58A \operatorname{Cos}[3 (c + dx)] +$$

$$186B \operatorname{Cos}[3 (c + dx)] + 257A \operatorname{Cos}[4 (c + dx)] - 129B \operatorname{Cos}[4 (c + dx)]) \operatorname{Sec}[c + dx]^{9/2} \operatorname{Sin}\left[\frac{1}{2} (c + dx)\right]$$

■ **Problem 521: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]^{9/2}}{\sqrt{a + a \cos[c + dx]}} dx$$

Optimal (type 3, 250 leaves, 8 steps) :

$$\frac{\sqrt{2} (A - B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a} d} - \frac{2 (43 A - 91 B) \sqrt{\sec[c+dx]} \sin[c+dx]}{105 d \sqrt{a + a \cos[c + dx]}} +$$

$$\frac{2 (31 A - 7 B) \sec[c + dx]^{3/2} \sin[c + dx]}{105 d \sqrt{a + a \cos[c + dx]}} - \frac{2 (A - 7 B) \sec[c + dx]^{5/2} \sin[c + dx]}{35 d \sqrt{a + a \cos[c + dx]}} + \frac{2 A \sec[c + dx]^{7/2} \sin[c + dx]}{7 d \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 242 leaves) :

$$\frac{1}{\sqrt{a} (1 + \cos[c + dx])}$$

$$\cos\left[\frac{1}{2} (c + dx)\right] \left( -1 / d 2 i (A - B) e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2 i (c+dx)}}} \sqrt{1 + e^{2 i (c+dx)}} \left( \operatorname{Log}[1 + e^{i (c+dx)}] - \operatorname{Log}[1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2 i (c+dx)}}] \right) - \right.$$

$$\left. \frac{1}{(105 d) (-122 A + 14 B + 3 (47 A - 119 B) \cos[c + dx] + (-62 A + 14 B) \cos[2 (c + dx)] + 43 A \cos[3 (c + dx)] - 91 B \cos[3 (c + dx)])} \right)$$

$$\sec[c + dx]^{7/2} \sin\left[\frac{1}{2} (c + dx)\right]$$

■ **Problem 522: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]^{7/2}}{\sqrt{a + a \cos[c + dx]}} dx$$

Optimal (type 3, 207 leaves, 7 steps) :

$$\frac{\sqrt{2} (A - B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a} d} +$$

$$\frac{2 (13 A - 5 B) \sqrt{\sec[c + dx]} \sin[c + dx]}{15 d \sqrt{a + a \cos[c + dx]}} - \frac{2 (A - 5 B) \sec[c + dx]^{3/2} \sin[c + dx]}{15 d \sqrt{a + a \cos[c + dx]}} + \frac{2 A \sec[c + dx]^{5/2} \sin[c + dx]}{5 d \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 215 leaves) :

$$\left( 2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \left( 15i(A-B) e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left( \operatorname{Log}[1+e^{i(c+dx)}] - \operatorname{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) + \right. \right. \\ \left. \left. (19A-5B-2(A-5B)\operatorname{Cos}[c+dx] + (13A-5B)\operatorname{Cos}[2(c+dx)]) \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right) / (15d\sqrt{a(1+\operatorname{Cos}[c+dx])})$$

■ **Problem 523: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B\operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]^{5/2}}{\sqrt{a+a\operatorname{Cos}[c+dx]}} dx$$

Optimal (type 3, 162 leaves, 6 steps):

$$\frac{\sqrt{2}(A-B) \operatorname{ArcTan}\left[\frac{\sqrt{a}\operatorname{Sin}[c+dx]}{\sqrt{2}\sqrt{\operatorname{Cos}[c+dx]}\sqrt{a+a\operatorname{Cos}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{\sqrt{a}d} - \\ \frac{2(A-3B)\sqrt{\operatorname{Sec}[c+dx]}\operatorname{Sin}[c+dx]}{3d\sqrt{a+a\operatorname{Cos}[c+dx]}} + \frac{2A\operatorname{Sec}[c+dx]^{3/2}\operatorname{Sin}[c+dx]}{3d\sqrt{a+a\operatorname{Cos}[c+dx]}}$$

Result (type 3, 198 leaves):

$$- \frac{1}{3d\sqrt{a(1+\operatorname{Cos}[c+dx])}} - \\ 2i\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \left( 3(A-B) e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left( \operatorname{Log}[1+e^{i(c+dx)}] - \operatorname{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) - \right. \\ \left. 2i(-A+(A-3B)\operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 524: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B\operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]^{3/2}}{\sqrt{a+a\operatorname{Cos}[c+dx]}} dx$$

Optimal (type 3, 119 leaves, 5 steps):

$$- \frac{\sqrt{2}(A-B) \operatorname{ArcTan}\left[\frac{\sqrt{a}\operatorname{Sin}[c+dx]}{\sqrt{2}\sqrt{\operatorname{Cos}[c+dx]}\sqrt{a+a\operatorname{Cos}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{\sqrt{a}d} + \frac{2A\sqrt{\operatorname{Sec}[c+dx]}\operatorname{Sin}[c+dx]}{d\sqrt{a+a\operatorname{Cos}[c+dx]}}$$

Result (type 3, 179 leaves):

$$\frac{1}{d \sqrt{a(1 + \cos[c + dx])}}$$

$$2 \cos\left[\frac{1}{2}(c + dx)\right] \left( i(A - B) e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \sqrt{1 + e^{2i(c+dx)}} \left( \log[1 + e^{i(c+dx)}] - \log[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) + \right.$$

$$\left. 2A \sqrt{\sec[c + dx]} \sin\left[\frac{1}{2}(c + dx)\right] \right)$$

- **Problem 525: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \cos[c + dx]) \sqrt{\sec[c + dx]}}{\sqrt{a + a \cos[c + dx]}} dx$$

Optimal (type 3, 140 leaves, 6 steps):

$$\frac{2B \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} + \sqrt{2} (A - B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a} d}$$

Result (type 3, 267 leaves):

$$\frac{1}{d \sqrt{a(1 + \cos[c + dx])}}$$

$$\sqrt{2} e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \sqrt{1 + e^{2i(c+dx)}} \cos\left[\frac{1}{2}(c + dx)\right] \left( B dx - iB \operatorname{ArcSinh}[e^{i(c+dx)}] - i\sqrt{2} (A - B) \log[1 + e^{i(c+dx)}] + \right.$$

$$\left. iB \log\left[1 + \sqrt{1 + e^{2i(c+dx)}}\right] + i\sqrt{2} A \log\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] - i\sqrt{2} B \log\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] \right)$$

- **Problem 526: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx]}{\sqrt{a + a \cos[c + dx]} \sqrt{\sec[c + dx]}} dx$$

Optimal (type 3, 181 leaves, 7 steps):

$$\frac{(2A - B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a} d} -$$

$$\frac{\sqrt{2} (A - B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a} d} + \frac{B \sin[c + dx]}{d \sqrt{a + a \cos[c + dx]} \sqrt{\sec[c + dx]}}$$

Result (type 3, 542 leaves) :

$$\frac{1}{4 d \sqrt{a (1 + \cos [c + d x])}} e^{-2 i (c+d x)} \left( 1 + e^{i (c+d x)} \right) \left( i B - i B e^{i (c+d x)} + i B e^{2 i (c+d x)} - i B e^{3 i (c+d x)} + 2 A d e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} x - B d e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} x - \right. \\ \left. i (2 A - B) e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{ArcSinh}\left[e^{i (c+d x)}\right] + 2 i \sqrt{2} (A - B) e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 + e^{i (c+d x)}\right] + \right. \\ \left. 2 i A e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c+d x)}}\right] - i B e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c+d x)}}\right] - 2 i \sqrt{2} A e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \right. \\ \left. \operatorname{Log}\left[1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}\right] + 2 i \sqrt{2} B e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}\right] \right) \sqrt{\sec [c + d x]}$$

■ **Problem 527: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos [c + d x]}{\sqrt{a + a \cos [c + d x]} \sec [c + d x]^{3/2}} dx$$

Optimal (type 3, 230 leaves, 8 steps) :

$$\frac{(4 A - 7 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]} + \sqrt{2} (A - B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{4 \sqrt{a} d} + \frac{B \sin [c+d x]}{2 d \sqrt{a+a \cos [c+d x]} \sec [c+d x]^{3/2}} + \frac{(4 A - B) \sin [c+d x]}{4 d \sqrt{a+a \cos [c+d x]} \sqrt{\sec [c+d x]}}$$

Result (type 3, 638 leaves) :

$$\frac{1}{16 d \sqrt{a (1 + \cos [c + d x])}} e^{-3 i (c+d x)} \left( 1 + e^{i (c+d x)} \right) \left( i B + 4 i A e^{i (c+d x)} - 2 i B e^{i (c+d x)} - 4 i A e^{2 i (c+d x)} + 3 i B e^{2 i (c+d x)} + 4 i A e^{3 i (c+d x)} - 3 i B e^{3 i (c+d x)} - 4 i A e^{4 i (c+d x)} + 2 i B e^{4 i (c+d x)} - \right. \\ \left. i B e^{5 i (c+d x)} - 4 A d e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} x + 7 B d e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} x + i (4 A - 7 B) e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{ArcSinh}\left[e^{i (c+d x)}\right] - \right. \\ \left. 8 i \sqrt{2} (A - B) e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 + e^{i (c+d x)}\right] - 4 i A e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c+d x)}}\right] + \right. \\ \left. 7 i B e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c+d x)}}\right] + 8 i \sqrt{2} A e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}\right] - \right. \\ \left. 8 i \sqrt{2} B e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}\right] \right) \sqrt{\sec [c + d x]}$$

■ **Problem 528: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a A + (A b + a B) \cos [c + d x] + b B \cos [c + d x]^2) \sqrt{\sec [c + d x]}}{\sqrt{a + a \cos [c + d x]}} dx$$

Optimal (type 3, 192 leaves, 7 steps) :



$$\frac{(2Ab + 2aB - bB) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a} d} +$$

$$\frac{\sqrt{2} (a-b) (A-B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a} d} + \frac{bB \sin[c+dx]}{d \sqrt{a+a\cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 807 leaves):

$$\frac{1}{4d\sqrt{a(1+\cos[c+dx])}}$$

$$e^{-2i(c+dx)} (1+e^{i(c+dx)}) \left( i b B - i b B e^{i(c+dx)} + i b B e^{2i(c+dx)} - i b B e^{3i(c+dx)} + 2Abd e^{i(c+dx)} \sqrt{1+e^{2i(c+dx)}} x + 2aBd e^{i(c+dx)} \sqrt{1+e^{2i(c+dx)}} x - \right.$$

$$b B d e^{i(c+dx)} \sqrt{1+e^{2i(c+dx)}} x - i (2Ab + 2aB - bB) e^{i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \operatorname{ArcSinh}[e^{i(c+dx)}] -$$

$$2i\sqrt{2} (a-b) (A-B) e^{i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \operatorname{Log}[1+e^{i(c+dx)}] + 2iAb e^{i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \operatorname{Log}[1+\sqrt{1+e^{2i(c+dx)}}] +$$

$$2iaB e^{i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \operatorname{Log}[1+\sqrt{1+e^{2i(c+dx)}}] - i b B e^{i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \operatorname{Log}[1+\sqrt{1+e^{2i(c+dx)}}] +$$

$$2i\sqrt{2} aA e^{i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}\right] - 2i\sqrt{2} Ab e^{i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}\right] +$$

$$2i\sqrt{2} bB e^{i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}\right] \left. \right) \sqrt{\sec[c+dx]}$$

■ **Problem 529: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B\cos[c+dx]) \sec[c+dx]^{9/2}}{(a+a\cos[c+dx])^{3/2}} dx$$

Optimal (type 3, 317 leaves, 9 steps):

$$\frac{(19A - 15B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{2\sqrt{2} a^{3/2} d} -$$

$$\frac{(1201A - 1029B) \sqrt{\sec[c+dx]} \sin[c+dx]}{210ad\sqrt{a+a\cos[c+dx]}} + \frac{(397A - 273B) \sec[c+dx]^{3/2} \sin[c+dx]}{210ad\sqrt{a+a\cos[c+dx]}} -$$

$$\frac{(67A - 63B) \sec[c+dx]^{5/2} \sin[c+dx]}{70ad\sqrt{a+a\cos[c+dx]}} - \frac{(A-B) \sec[c+dx]^{7/2} \sin[c+dx]}{2d(a+a\cos[c+dx])^{3/2}} + \frac{(11A - 7B) \sec[c+dx]^{7/2} \sin[c+dx]}{14ad\sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 279 leaves):

$$\frac{1}{(a(1 + \cos[c + dx]))^{3/2}}$$

$$\cos\left[\frac{1}{2}(c + dx)\right]^3 \left( -1 / di (19A - 15B) e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \sqrt{1 + e^{2i(c+dx)}} \left( \log[1 + e^{i(c+dx)}] - \log[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) - \right.$$

$$\left. 1 / (840d) (2339A - 2751B + 24(213A - 217B) \cos[c + dx] + 60(67A - 63B) \cos[2(c + dx)] + 1608A \cos[3(c + dx)] - \right.$$

$$\left. 1512B \cos[3(c + dx)] + 1201A \cos[4(c + dx)] - 1029B \cos[4(c + dx)]) \sec\left[\frac{1}{2}(c + dx)\right] \sec[c + dx]^{7/2} \tan\left[\frac{1}{2}(c + dx)\right] \right)$$

■ **Problem 530: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]^{7/2}}{(a + a \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 270 leaves, 8 steps):

$$-\frac{(15A - 11B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{2\sqrt{2} a^{3/2} d} + \frac{(147A - 95B) \sqrt{\sec[c+dx]} \sin[c+dx]}{30ad \sqrt{a+a \cos[c+dx]}}$$

$$\frac{(39A - 35B) \sec[c+dx]^{3/2} \sin[c+dx]}{30ad \sqrt{a+a \cos[c+dx]}} - \frac{(A - B) \sec[c+dx]^{5/2} \sin[c+dx]}{2d(a+a \cos[c+dx])^{3/2}} + \frac{(9A - 5B) \sec[c+dx]^{5/2} \sin[c+dx]}{10ad \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 253 leaves):

$$\frac{1}{60d(a(1 + \cos[c + dx]))^{3/2}}$$

$$\cos\left[\frac{1}{2}(c + dx)\right]^3 \left( 60i(15A - 11B) e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \sqrt{1 + e^{2i(c+dx)}} \left( \log[1 + e^{i(c+dx)}] - \log[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) + \right.$$

$$\left. (264A - 120B + (393A - 205B) \cos[c + dx] + 24(9A - 5B) \cos[2(c + dx)] + 147A \cos[3(c + dx)] - 95B \cos[3(c + dx)]) \right.$$

$$\left. \sec\left[\frac{1}{2}(c + dx)\right] \sec[c + dx]^{5/2} \tan\left[\frac{1}{2}(c + dx)\right] \right)$$

■ **Problem 531: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]^{5/2}}{(a + a \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 223 leaves, 7 steps):

$$\frac{(11A - 7B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{2\sqrt{2} a^{3/2} d}$$

$$\frac{(19A - 15B) \sqrt{\sec[c+dx]} \sin[c+dx]}{6ad \sqrt{a+a\cos[c+dx]}} - \frac{(A - B) \sec[c+dx]^{3/2} \sin[c+dx]}{2d(a+a\cos[c+dx])^{3/2}} + \frac{(7A - 3B) \sec[c+dx]^{3/2} \sin[c+dx]}{6ad \sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 232 leaves):

$$\frac{1}{(a(1+\cos[c+dx]))^{3/2}} \cos\left[\frac{1}{2}(c+dx)\right]^3 \left( -1/di(11A - 7B) e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left( \operatorname{Log}[1+e^{i(c+dx)}] - \operatorname{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) - \right. \\ \left. 1/(6d)(11A - 15B + 24(A - B)\cos[c+dx] + (19A - 15B)\cos[2(c+dx)]) \sec\left[\frac{1}{2}(c+dx)\right] \sec[c+dx]^{3/2} \tan\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 532: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B\cos[c+dx]) \sec[c+dx]^{3/2}}{(a+a\cos[c+dx])^{3/2}} dx$$

Optimal (type 3, 176 leaves, 6 steps):

$$\frac{(7A - 3B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{2\sqrt{2} a^{3/2} d}$$

$$\frac{(A - B) \sqrt{\sec[c+dx]} \sin[c+dx]}{2d(a+a\cos[c+dx])^{3/2}} + \frac{(5A - B) \sqrt{\sec[c+dx]} \sin[c+dx]}{2ad \sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 208 leaves):

$$\frac{1}{d(a(1+\cos[c+dx]))^{3/2}} \cos\left[\frac{1}{2}(c+dx)\right]^3 \left( i(7A - 3B) e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left( \operatorname{Log}[1+e^{i(c+dx)}] - \operatorname{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) + \right. \\ \left. (4A + (5A - B)\cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{\sec[c+dx]} \tan\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 533: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \cos [c + d x]) \sqrt{\sec [c + d x]}}{(a + a \cos [c + d x])^{3/2}} dx$$

Optimal (type 3, 127 leaves, 5 steps):

$$\frac{(3 A + B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{2 \sqrt{2} a^{3/2} d} - \frac{(A - B) \sin [c+d x]}{2 d (a + a \cos [c+d x])^{3/2} \sqrt{\sec [c+d x]}}$$

Result (type 3, 213 leaves):

$$-\frac{1}{d (a (1 + \cos [c + d x]))^{3/2}} + i \cos \left[ \frac{1}{2} (c + d x) \right]^3 \left( (3 A + B) e^{-\frac{1}{2} i (c+d x)} \sqrt{\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}}} \sqrt{1 + e^{2 i (c+d x)}} \left( \operatorname{Log} [1 + e^{i (c+d x)}] - \operatorname{Log} [1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}] \right) \right) + \frac{1}{2} i (A - B) \sec \left[ \frac{1}{2} (c + d x) \right]^2 \sqrt{\sec [c + d x]} \left( \sin \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{3}{2} (c + d x) \right] \right)$$

■ **Problem 534: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos [c + d x]}{(a + a \cos [c + d x])^{3/2} \sqrt{\sec [c + d x]}} dx$$

Optimal (type 3, 185 leaves, 7 steps):

$$\frac{2 B \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{a^{3/2} d} + \frac{(A - 5 B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{2 \sqrt{2} a^{3/2} d} + \frac{(A - B) \sin [c+d x]}{2 d (a + a \cos [c+d x])^{3/2} \sqrt{\sec [c+d x]}}$$

Result (type 3, 326 leaves):

$$\frac{1}{2 d (a (1 + \cos [c + d x]))^{3/2}}$$

$$\cos \left[ \frac{1}{2} (c + d x) \right]^3 \left( \sqrt{2} e^{-\frac{1}{2} i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \sqrt{1 + e^{2 i (c + d x)}} \left( 4 B d x - 4 i B \operatorname{ArcSinh} \left[ e^{i (c + d x)} \right] - i \sqrt{2} (A - 5 B) \operatorname{Log} \left[ 1 + e^{i (c + d x)} \right] + \right. \right.$$

$$\left. 4 i B \operatorname{Log} \left[ 1 + \sqrt{1 + e^{2 i (c + d x)}} \right] + i \sqrt{2} A \operatorname{Log} \left[ 1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}} \right] - 5 i \sqrt{2} B \operatorname{Log} \left[ 1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}} \right] \right) +$$

$$\left. (A - B) \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \sqrt{\operatorname{Sec} [c + d x]} \left( -\sin \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{3}{2} (c + d x) \right] \right) \right)$$

- **Problem 535: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos [c + d x]}{(a + a \cos [c + d x])^{3/2} \operatorname{Sec} [c + d x]^{3/2}} dx$$

Optimal (type 3, 237 leaves, 8 steps):

$$\frac{(2 A - 3 B) \operatorname{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\operatorname{Sec} [c + d x]} - (5 A - 9 B) \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{2} \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\operatorname{Sec} [c + d x]}}{a^{3/2} d} + \frac{(A - B) \sin [c + d x]}{2 d (a + a \cos [c + d x])^{3/2} \operatorname{Sec} [c + d x]^{3/2}} - \frac{(A - 3 B) \sin [c + d x]}{2 a d \sqrt{a + a \cos [c + d x]} \sqrt{\operatorname{Sec} [c + d x]}}$$

Result (type 3, 944 leaves):

$$\begin{aligned}
& \frac{1}{d (a (1 + \cos [c + dx]))^{3/2}} i A e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2i (c+dx)}}} \sqrt{1 + e^{2i (c+dx)}} \\
& \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \left( \log [1 + e^{i (c+dx)}] - \log [1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2i (c+dx)}}] \right) - \frac{1}{d (a (1 + \cos [c + dx]))^{3/2}} \\
& 3 i B e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2i (c+dx)}}} \sqrt{1 + e^{2i (c+dx)}} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \left( \log [1 + e^{i (c+dx)}] - \log [1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2i (c+dx)}}] \right) + \\
& \frac{1}{d (a (1 + \cos [c + dx]))^{3/2}} 2 \sqrt{2} A e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2i (c+dx)}}} \sqrt{1 + e^{2i (c+dx)}} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \\
& \left( dx - i \operatorname{ArcSinh} [e^{i (c+dx)}] + i \sqrt{2} \log [1 + e^{i (c+dx)}] + i \log [1 + \sqrt{1 + e^{2i (c+dx)}}] - i \sqrt{2} \log [1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2i (c+dx)}}] \right) - \\
& \frac{1}{d (a (1 + \cos [c + dx]))^{3/2}} 3 \sqrt{2} B e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2i (c+dx)}}} \sqrt{1 + e^{2i (c+dx)}} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \\
& \left( dx - i \operatorname{ArcSinh} [e^{i (c+dx)}] + i \sqrt{2} \log [1 + e^{i (c+dx)}] + i \log [1 + \sqrt{1 + e^{2i (c+dx)}}] - i \sqrt{2} \log [1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2i (c+dx)}}] \right) + \\
& \frac{1}{(a (1 + \cos [c + dx]))^{3/2}} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sqrt{\sec [c + dx]} \left( -\frac{2 A \cos \left[ \frac{dx}{2} \right] \sin \left[ \frac{c}{2} \right]}{d} + \frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right] (A \sin \left[ \frac{c}{2} \right] - B \sin \left[ \frac{c}{2} \right])}{d} + \right. \\
& \left. \frac{2 B \cos \left[ \frac{3dx}{2} \right] \sin \left[ \frac{3c}{2} \right]}{d} - \frac{2 A \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right]}{d} + \frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 (A \sin \left[ \frac{dx}{2} \right] - B \sin \left[ \frac{dx}{2} \right])}{d} + \frac{2 B \cos \left[ \frac{3c}{2} \right] \sin \left[ \frac{3dx}{2} \right]}{d} \right)
\end{aligned}$$

■ **Problem 536: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \cos [c + dx]) \sec [c + dx]^{7/2}}{(a + a \cos [c + dx])^{5/2}} dx$$

Optimal (type 3, 317 leaves, 9 steps):

$$\begin{aligned}
& \frac{(283 A - 163 B) \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin [c+dx]}{\sqrt{2} \sqrt{\cos [c+dx]} \sqrt{a+a \cos [c+dx]}} \right] \sqrt{\cos [c+dx]} \sqrt{\sec [c+dx]}}{16 \sqrt{2} a^{5/2} d} + \\
& \frac{(2671 A - 1495 B) \sqrt{\sec [c+dx]} \sin [c+dx]}{240 a^2 d \sqrt{a+a \cos [c+dx]}} - \frac{(787 A - 475 B) \sec [c+dx]^{3/2} \sin [c+dx]}{240 a^2 d \sqrt{a+a \cos [c+dx]}} - \\
& \frac{(A - B) \sec [c+dx]^{5/2} \sin [c+dx]}{4 d (a + a \cos [c + dx])^{5/2}} - \frac{(21 A - 13 B) \sec [c + dx]^{5/2} \sin [c + dx]}{16 a d (a + a \cos [c + dx])^{3/2}} + \frac{(157 A - 85 B) \sec [c + dx]^{5/2} \sin [c + dx]}{80 a^2 d \sqrt{a + a \cos [c + dx]}}
\end{aligned}$$

Result (type 3, 278 leaves) :

$$\frac{1}{960 d (a (1 + \cos [c + d x]))^{5/2}}$$

$$\cos \left[ \frac{1}{2} (c + d x) \right]^5 \left( 240 i (283 A - 163 B) e^{-\frac{1}{2} i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \sqrt{1 + e^{2 i (c + d x)}} \left( \log [1 + e^{i (c + d x)}] - \log [1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}] \right) + \right.$$

$$\left. (15053 A - 7685 B + 10 (2605 A - 1381 B) \cos [c + d x] + 108 (157 A - 85 B) \cos [2 (c + d x)] + 9110 A \cos [3 (c + d x)] - \right.$$

$$\left. 5030 B \cos [3 (c + d x)] + 2671 A \cos [4 (c + d x)] - 1495 B \cos [4 (c + d x)] \right) \sec \left[ \frac{1}{2} (c + d x) \right]^3 \sec [c + d x]^{5/2} \tan \left[ \frac{1}{2} (c + d x) \right]$$

■ **Problem 537: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \cos [c + d x]) \sec [c + d x]^{5/2}}{(a + a \cos [c + d x])^{5/2}} dx$$

Optimal (type 3, 270 leaves, 8 steps) :

$$\frac{(163 A - 75 B) \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{2} \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]} - (299 A - 147 B) \sqrt{\sec [c + d x]} \sin [c + d x]}{16 \sqrt{2} a^{5/2} d} - \frac{(A - B) \sec [c + d x]^{3/2} \sin [c + d x]}{4 d (a + a \cos [c + d x])^{5/2}} - \frac{(17 A - 9 B) \sec [c + d x]^{3/2} \sin [c + d x]}{16 a d (a + a \cos [c + d x])^{3/2}} + \frac{(95 A - 39 B) \sec [c + d x]^{3/2} \sin [c + d x]}{48 a^2 d \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 261 leaves) :

$$\frac{1}{4 (a (1 + \cos [c + d x]))^{5/2}}$$

$$\cos \left[ \frac{1}{2} (c + d x) \right]^5 \left( -1 / d i (163 A - 75 B) e^{-\frac{1}{2} i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \sqrt{1 + e^{2 i (c + d x)}} \left( \log [1 + e^{i (c + d x)}] - \log [1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}] \right) - \right.$$

$$\left. 1 / (24 d) (878 A - 510 B + (1537 A - 825 B) \cos [c + d x] + 2 (503 A - 255 B) \cos [2 (c + d x)] + 299 A \cos [3 (c + d x)] - 147 B \cos [3 (c + d x)] \right)$$

$$\left. \sec \left[ \frac{1}{2} (c + d x) \right]^3 \sec [c + d x]^{3/2} \tan \left[ \frac{1}{2} (c + d x) \right] \right)$$

■ **Problem 538: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \cos [c + d x]) \sec [c + d x]^{3/2}}{(a + a \cos [c + d x])^{5/2}} dx$$

Optimal (type 3, 223 leaves, 7 steps) :

$$\frac{(75 A - 19 B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{16 \sqrt{2} a^{5/2} d}$$

$$\frac{(A - B) \sqrt{\sec[c+dx]} \sin[c+dx]}{4 d (a + a \cos[c+dx])^{5/2}} - \frac{(13 A - 5 B) \sqrt{\sec[c+dx]} \sin[c+dx]}{16 a d (a + a \cos[c+dx])^{3/2}} + \frac{(49 A - 9 B) \sqrt{\sec[c+dx]} \sin[c+dx]}{16 a^2 d \sqrt{a + a \cos[c+dx]}}$$

Result (type 3, 236 leaves):

$$\left( \cos\left[\frac{1}{2}(c+dx)\right]^5 \left( i (75 A - 19 B) e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) + \frac{1}{4} (113 A - 9 B + 2 (85 A - 13 B) \cos[c+dx] + (49 A - 9 B) \cos[2(c+dx)]) \right. \\ \left. \sec\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\sec[c+dx]} \tan\left[\frac{1}{2}(c+dx)\right] \right) / (4 d (a (1 + \cos[c+dx]))^{5/2})$$

■ **Problem 539: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \cos[c+dx]) \sqrt{\sec[c+dx]}}{(a + a \cos[c+dx])^{5/2}} dx$$

Optimal (type 3, 176 leaves, 6 steps):

$$\frac{(19 A + 5 B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{16 \sqrt{2} a^{5/2} d} - \frac{(A - B) \sin[c+dx]}{4 d (a + a \cos[c+dx])^{5/2} \sqrt{\sec[c+dx]}} - \frac{(9 A - B) \sin[c+dx]}{16 a d (a + a \cos[c+dx])^{3/2} \sqrt{\sec[c+dx]}}$$

Result (type 3, 233 leaves):

$$- \left( i \cos\left[\frac{1}{2}(c+dx)\right]^5 \left( (19 A + 5 B) e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) + \frac{1}{4} i \right. \\ \left. (13 A - 5 B + (9 A - B) \cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^4 \sqrt{\sec[c+dx]} \left( \sin\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{3}{2}(c+dx)\right] \right) \right) / (4 d (a (1 + \cos[c+dx]))^{5/2})$$



■ **Problem 540: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos [c + d x]}{(a + a \cos [c + d x])^{5/2} \sqrt{\sec [c + d x]}} dx$$

Optimal (type 3, 174 leaves, 6 steps) :

$$\frac{(5 A + 3 B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{16 \sqrt{2} a^{5/2} d} + \frac{(A - B) \sin [c+d x]}{4 d (a + a \cos [c+d x])^{5/2} \sqrt{\sec [c+d x]}} + \frac{(A + 7 B) \sin [c+d x]}{16 a d (a + a \cos [c+d x])^{3/2} \sqrt{\sec [c+d x]}}$$

Result (type 3, 230 leaves) :

$$\left( \cos \left[ \frac{1}{2} (c + d x) \right] \right)^5 \left( -i (5 A + 3 B) e^{-\frac{1}{2} i (c+d x)} \sqrt{\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}}} \sqrt{1 + e^{2 i (c+d x)}} \left( \log [1 + e^{i (c+d x)}] - \log [1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}] \right) + \frac{1}{4} (5 A + 3 B + (A + 7 B) \cos [c + d x]) \sec \left[ \frac{1}{2} (c + d x) \right]^4 \sqrt{\sec [c + d x]} \left( -\sin \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{3}{2} (c + d x) \right] \right) \right) \Bigg/ (4 d (a (1 + \cos [c + d x]))^{5/2})$$

■ **Problem 541: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos [c + d x]}{(a + a \cos [c + d x])^{5/2} \sec [c + d x]^{3/2}} dx$$

Optimal (type 3, 234 leaves, 8 steps) :

$$\frac{2 B \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{a^{5/2} d} + \frac{(3 A - 43 B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{16 \sqrt{2} a^{5/2} d} + \frac{(A - B) \sin [c+d x]}{4 d (a + a \cos [c+d x])^{5/2} \sec [c+d x]^{3/2}} + \frac{(3 A - 11 B) \sin [c+d x]}{16 a d (a + a \cos [c+d x])^{3/2} \sqrt{\sec [c+d x]}}$$

Result (type 3, 347 leaves) :

$$\frac{1}{8 d (a (1 + \operatorname{Cos}[c + d x]))^{5/2}}$$

$$\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^5 \left( \sqrt{2} e^{-\frac{1}{2} i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \sqrt{1 + e^{2 i (c + d x)}} \left( 32 B d x - 32 i B \operatorname{ArcSinh}\left[e^{i (c + d x)}\right] - i \sqrt{2} (3 A - 43 B) \operatorname{Log}\left[1 + e^{i (c + d x)}\right] + \right. \right.$$

$$\left. \left. 32 i B \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c + d x)}}\right] + 3 i \sqrt{2} A \operatorname{Log}\left[1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}\right] - 43 i \sqrt{2} B \operatorname{Log}\left[1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}\right] \right) + \right.$$

$$\left. \frac{1}{2} (3 A - 11 B + (7 A - 15 B) \operatorname{Cos}[c + d x]) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \sqrt{\operatorname{Sec}[c + d x]} \left( -\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{3}{2}(c + d x)\right] \right) \right)$$

- **Problem 542: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Cos}[c + d x]}{(a + a \operatorname{Cos}[c + d x])^{5/2} \operatorname{Sec}[c + d x]^{5/2}} dx$$

Optimal (type 3, 286 leaves, 9 steps):

$$\frac{(2 A - 5 B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c + d x]}{\sqrt{a + a \operatorname{Cos}[c + d x]}}\right] \sqrt{\operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]} - (43 A - 115 B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c + d x]}{\sqrt{2} \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + a \operatorname{Cos}[c + d x]}}\right] \sqrt{\operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}{a^{5/2} d} + \frac{16 \sqrt{2} a^{5/2} d}{(A - B) \operatorname{Sin}[c + d x]} + \frac{(7 A - 15 B) \operatorname{Sin}[c + d x]}{4 d (a + a \operatorname{Cos}[c + d x])^{5/2} \operatorname{Sec}[c + d x]^{5/2}} - \frac{(11 A - 35 B) \operatorname{Sin}[c + d x]}{16 a d (a + a \operatorname{Cos}[c + d x])^{3/2} \operatorname{Sec}[c + d x]^{3/2}} - \frac{(11 A - 35 B) \operatorname{Sin}[c + d x]}{16 a^2 d \sqrt{a + a \operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 3, 1037 leaves):

$$\begin{aligned}
& \left( 11 i A e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \text{Log}[1+e^{i(c+dx)}] - \text{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\
& (4 d (a (1 + \text{Cos}[c + dx]))^{5/2}) - \\
& \left( 35 i B e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \text{Log}[1+e^{i(c+dx)}] - \text{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\
& (4 d (a (1 + \text{Cos}[c + dx]))^{5/2}) + \frac{1}{d (a (1 + \text{Cos}[c + dx]))^{5/2}} 4 \sqrt{2} A e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \\
& \left( dx - i \text{ArcSinh}[e^{i(c+dx)}] + i \sqrt{2} \text{Log}[1+e^{i(c+dx)}] + i \text{Log}[1+\sqrt{1+e^{2i(c+dx)}}] - i \sqrt{2} \text{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) - \\
& \frac{1}{d (a (1 + \text{Cos}[c + dx]))^{5/2}} 10 \sqrt{2} B e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \\
& \left( dx - i \text{ArcSinh}[e^{i(c+dx)}] + i \sqrt{2} \text{Log}[1+e^{i(c+dx)}] + i \text{Log}[1+\sqrt{1+e^{2i(c+dx)}}] - i \sqrt{2} \text{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) + \\
& \frac{1}{(a (1 + \text{Cos}[c + dx]))^{5/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\text{Sec}[c + dx]} \\
& \left( \frac{15 (-A + B) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{2 d} + \frac{4 B \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{d} - \frac{15 (A - B) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{2 d} + \frac{\text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (19 A \sin\left[\frac{dx}{2}\right] - 27 B \sin\left[\frac{dx}{2}\right])}{4 d} + \right. \\
& \left. \frac{\text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{2 d} + \frac{4 B \cos\left[\frac{3c}{2}\right] \sin\left[\frac{3dx}{2}\right]}{d} + \frac{(19 A - 27 B) \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \text{Tan}\left[\frac{c}{2}\right]}{4 d} - \frac{(A - B) \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \text{Tan}\left[\frac{c}{2}\right]}{2 d} \right)
\end{aligned}$$

■ **Problem 543: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \text{Cos}[c + dx]) \text{Sec}[c + dx]^{5/2}}{(a + a \text{Cos}[c + dx])^{7/2}} dx$$

Optimal (type 3, 317 leaves, 9 steps):

$$\frac{(1015 A - 363 B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{64 \sqrt{2} a^{7/2} d}$$

$$\frac{(1887 A - 691 B) \sqrt{\sec[c+dx]} \sin[c+dx]}{192 a^3 d \sqrt{a+a \cos[c+dx]}} - \frac{(A - B) \sec[c+dx]^{3/2} \sin[c+dx]}{6 d (a+a \cos[c+dx])^{7/2}} - \frac{(23 A - 11 B) \sec[c+dx]^{3/2} \sin[c+dx]}{48 a d (a+a \cos[c+dx])^{5/2}}$$

$$\frac{(109 A - 41 B) \sec[c+dx]^{3/2} \sin[c+dx]}{64 a^2 d (a+a \cos[c+dx])^{3/2}} + \frac{(579 A - 199 B) \sec[c+dx]^{3/2} \sin[c+dx]}{192 a^3 d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 532 leaves):

$$\begin{aligned} & - \left( i (1015 A - 363 B) e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left( \operatorname{Log}\left[1+e^{i(c+dx)}\right] - \operatorname{Log}\left[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}\right] \right) \right) / \\ & (8 d (a (1 + \cos[c+dx]))^{7/2}) + \frac{1}{(a (1 + \cos[c+dx]))^{7/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\sec[c+dx]} \\ & \left( - \frac{(1887 A - 691 B) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{12 d} - \frac{(1887 A - 691 B) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{12 d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (451 A \sin\left[\frac{dx}{2}\right] - 199 B \sin\left[\frac{dx}{2}\right])}{24 d} + \right. \\ & \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (31 A \sin\left[\frac{dx}{2}\right] - 19 B \sin\left[\frac{dx}{2}\right])}{12 d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right])}{3 d} + \frac{32 A \sec[c+dx] \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{3 d} + \\ & \left. \frac{(451 A - 199 B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{24 d} + \frac{(31 A - 19 B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{12 d} + \frac{(A - B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \tan\left[\frac{c}{2}\right]}{3 d} \right) \end{aligned}$$

■ **Problem 544: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \cos[c+dx]) \sec[c+dx]^{3/2}}{(a + a \cos[c+dx])^{7/2}} dx$$

Optimal (type 3, 270 leaves, 8 steps):

$$\begin{aligned} & - \frac{3 (121 A - 21 B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{64 \sqrt{2} a^{7/2} d} - \frac{(A - B) \sqrt{\sec[c+dx]} \sin[c+dx]}{6 d (a+a \cos[c+dx])^{7/2}} - \\ & \frac{(19 A - 7 B) \sqrt{\sec[c+dx]} \sin[c+dx]}{48 a d (a+a \cos[c+dx])^{5/2}} - \frac{(199 A - 43 B) \sqrt{\sec[c+dx]} \sin[c+dx]}{192 a^2 d (a+a \cos[c+dx])^{3/2}} + \frac{(691 A - 103 B) \sqrt{\sec[c+dx]} \sin[c+dx]}{192 a^3 d \sqrt{a+a \cos[c+dx]}} \end{aligned}$$

Result (type 3, 259 leaves):

$$\frac{1}{24 d (a (1 + \cos [c + d x]))^{7/2}}$$

$$\cos \left[ \frac{1}{2} (c + d x) \right]^7 \left( 9 i (121 A - 21 B) e^{-\frac{1}{2} i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \sqrt{1 + e^{2 i (c + d x)}} \left( \log [1 + e^{i (c + d x)}] - \log [1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}] \right) + \right.$$

$$\frac{1}{16} (5284 A - 532 B + 9 (941 A - 121 B) \cos [c + d x] + 4 (937 A - 133 B) \cos [2 (c + d x)] + 691 A \cos [3 (c + d x)] - 103 B \cos [3 (c + d x)])$$

$$\left. \sec \left[ \frac{1}{2} (c + d x) \right]^5 \sqrt{\sec [c + d x]} \tan \left[ \frac{1}{2} (c + d x) \right] \right)$$

■ **Problem 545: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + d x]) \sqrt{\sec [c + d x]}}{(a + a \cos [c + d x])^{7/2}} dx$$

Optimal (type 3, 223 leaves, 7 steps):

$$\frac{(63 A + 13 B) \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{2} \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{64 \sqrt{2} a^{7/2} d} -$$

$$\frac{(A - B) \sin [c + d x]}{6 d (a + a \cos [c + d x])^{7/2} \sqrt{\sec [c + d x]}} - \frac{(5 A - B) \sin [c + d x]}{16 a d (a + a \cos [c + d x])^{5/2} \sqrt{\sec [c + d x]}} - \frac{(103 A + 5 B) \sin [c + d x]}{192 a^2 d (a + a \cos [c + d x])^{3/2} \sqrt{\sec [c + d x]}}$$

Result (type 3, 505 leaves):

$$- \left( i (63 A + 13 B) e^{-\frac{1}{2} i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \sqrt{1 + e^{2 i (c + d x)}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^7 \left( \log [1 + e^{i (c + d x)}] - \log [1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}] \right) \right) /$$

$$(8 d (a (1 + \cos [c + d x]))^{7/2}) + \frac{1}{(a (1 + \cos [c + d x]))^{7/2}}$$

$$\cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^7 \sqrt{\sec [c + d x]} \left( - \frac{(103 A + 5 B) \cos \left[ \frac{d x}{2} \right] \sin \left[ \frac{c}{2} \right]}{12 d} - \frac{(103 A + 5 B) \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right]}{12 d} + \frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 (A \sin \left[ \frac{d x}{2} \right] - B \sin \left[ \frac{d x}{2} \right])}{3 d} + \right.$$

$$\frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 (7 A \sin \left[ \frac{d x}{2} \right] + 5 B \sin \left[ \frac{d x}{2} \right])}{12 d} + \frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 (43 A \sin \left[ \frac{d x}{2} \right] + 17 B \sin \left[ \frac{d x}{2} \right])}{24 d} +$$

$$\left. \frac{(43 A + 17 B) \sec \left[ \frac{c}{2} + \frac{d x}{2} \right] \tan \left[ \frac{c}{2} \right]}{24 d} + \frac{(7 A + 5 B) \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \tan \left[ \frac{c}{2} \right]}{12 d} + \frac{(A - B) \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \tan \left[ \frac{c}{2} \right]}{3 d} \right)$$

- **Problem 546: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos [c + d x]}{(a + a \cos [c + d x])^{7/2} \sqrt{\sec [c + d x]}} dx$$

Optimal (type 3, 221 leaves, 7 steps) :

$$\frac{(13 A + 7 B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{64 \sqrt{2} a^{7/2} d} + \frac{(A - B) \sin [c+d x]}{6 d (a + a \cos [c+d x])^{7/2} \sqrt{\sec [c+d x]}} + \frac{(A + 3 B) \sin [c+d x]}{16 a d (a + a \cos [c+d x])^{5/2} \sqrt{\sec [c+d x]}} - \frac{(5 A - 17 B) \sin [c+d x]}{192 a^2 d (a + a \cos [c+d x])^{3/2} \sqrt{\sec [c+d x]}}$$

Result (type 3, 505 leaves) :

$$-\left( i (13 A + 7 B) e^{-\frac{1}{2} i (c+d x)} \sqrt{\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}}} \sqrt{1 + e^{2 i (c+d x)}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^7 \left( \operatorname{Log} [1 + e^{i (c+d x)}] - \operatorname{Log} [1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}] \right) \right) / \left( (8 d (a (1 + \cos [c+d x]))^{7/2}) + \frac{1}{(a (1 + \cos [c+d x]))^{7/2}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^7 \sqrt{\sec [c+d x]} \left( -\frac{(5 A - 17 B) \cos \left[ \frac{d x}{2} \right] \sin \left[ \frac{c}{2} \right]}{12 d} - \frac{(5 A - 17 B) \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right]}{12 d} + \frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 (5 A \sin \left[ \frac{d x}{2} \right] - 17 B \sin \left[ \frac{d x}{2} \right])}{12 d} + \frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 (-A \sin \left[ \frac{d x}{2} \right] + B \sin \left[ \frac{d x}{2} \right])}{3 d} + \frac{\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 (17 A \sin \left[ \frac{d x}{2} \right] + 19 B \sin \left[ \frac{d x}{2} \right])}{24 d} + \frac{(17 A + 19 B) \sec \left[ \frac{c}{2} + \frac{d x}{2} \right] \tan \left[ \frac{c}{2} \right]}{24 d} + \frac{(5 A - 17 B) \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \tan \left[ \frac{c}{2} \right]}{12 d} - \frac{(A - B) \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \tan \left[ \frac{c}{2} \right]}{3 d} \right)$$

- **Problem 547: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos [c + d x]}{(a + a \cos [c + d x])^{7/2} \sec [c + d x]^{3/2}} dx$$

Optimal (type 3, 221 leaves, 7 steps) :

$$\frac{(7 A + 5 B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{64 \sqrt{2} a^{7/2} d} + \frac{(A - B) \sin [c+d x]}{6 d (a + a \cos [c+d x])^{7/2} \sec [c+d x]^{3/2}} + \frac{(A - 13 B) \sin [c+d x]}{48 a d (a + a \cos [c+d x])^{5/2} \sqrt{\sec [c+d x]}} + \frac{(17 A + 67 B) \sin [c+d x]}{192 a^2 d (a + a \cos [c+d x])^{3/2} \sqrt{\sec [c+d x]}}$$

Result (type 3, 505 leaves) :

$$\begin{aligned}
& - \left( i (7A + 5B) e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\
& (8d (a(1+\cos[c+dx]))^{7/2}) + \frac{1}{(a(1+\cos[c+dx]))^{7/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\sec[c+dx]} \\
& \left( \frac{(17A + 67B) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{12d} + \frac{(17A + 67B) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{12d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (19A \sin\left[\frac{dx}{2}\right] - 151B \sin\left[\frac{c}{2}\right])}{24d} + \right. \\
& \left. \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{c}{2}\right])}{3d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (-17A \sin\left[\frac{dx}{2}\right] + 29B \sin\left[\frac{c}{2}\right])}{12d} + \right. \\
& \left. \frac{(19A - 151B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{24d} - \frac{(17A - 29B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{12d} + \frac{(A - B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \tan\left[\frac{c}{2}\right]}{3d} \right)
\end{aligned}$$

■ **Problem 548: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx]}{(a + a \cos[c + dx])^{7/2} \sec[c + dx]^{5/2}} dx$$

Optimal (type 3, 281 leaves, 9 steps):

$$\begin{aligned}
& \frac{2B \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{a^{7/2} d} + \frac{(5A - 177B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{64 \sqrt{2} a^{7/2} d} + \\
& \frac{(A - B) \sin[c+dx]}{6d (a + a \cos[c+dx])^{7/2} \sec[c+dx]^{5/2}} + \frac{(5A - 17B) \sin[c+dx]}{48ad (a + a \cos[c+dx])^{5/2} \sec[c+dx]^{3/2}} + \frac{(5A - 49B) \sin[c+dx]}{64a^2 d (a + a \cos[c+dx])^{3/2} \sqrt{\sec[c+dx]}}
\end{aligned}$$

Result (type 3, 621 leaves):

$$\left( e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left( 128 B dx - 128 i B \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] - i \sqrt{2} (5A - 177B) \operatorname{Log}\left[1+e^{i(c+dx)}\right] + \right. \right. \\ \left. \left. 128 i B \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] + 5 i \sqrt{2} A \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] - 177 i \sqrt{2} B \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] \right) \right) / \\ \left( 8 \sqrt{2} d (a(1+\cos[c+dx]))^{7/2} + \frac{1}{(a(1+\cos[c+dx]))^{7/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\sec[c+dx]} \right. \\ \left. \left( \frac{(67A - 247B) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{12d} + \frac{(67A - 247B) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{12d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (29A \sin\left[\frac{dx}{2}\right] - 41B \sin\left[\frac{dx}{2}\right])}{12d} + \right. \right. \\ \left. \left. \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{3d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (-151A \sin\left[\frac{dx}{2}\right] + 379B \sin\left[\frac{dx}{2}\right])}{24d} - \right. \right. \\ \left. \left. \frac{(151A - 379B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{24d} + \frac{(29A - 41B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{12d} - \frac{(A - B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \tan\left[\frac{c}{2}\right]}{3d} \right) \right)$$

- **Problem 549: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx]}{(a + a \cos[c + dx])^{7/2} \sec[c + dx]^{7/2}} dx$$

Optimal (type 3, 333 leaves, 10 steps):

$$\frac{(2A - 7B) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{a^{7/2} d} - \\ \frac{(177A - 637B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{64 \sqrt{2} a^{7/2} d} + \frac{(A - B) \sin[c+dx]}{6d (a + a \cos[c+dx])^{7/2} \sec[c+dx]^{7/2}} + \\ \frac{(3A - 7B) \sin[c+dx]}{16ad (a + a \cos[c+dx])^{5/2} \sec[c+dx]^{5/2}} + \frac{(79A - 259B) \sin[c+dx]}{192a^2 d (a + a \cos[c+dx])^{3/2} \sec[c+dx]^{3/2}} - \frac{7(7A - 27B) \sin[c+dx]}{64a^3 d \sqrt{a + a \cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 1125 leaves):



$$\begin{aligned}
& \left( 49 i A e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\
& \left( 8 d (a (1 + \cos[c + dx]))^{7/2} \right) - \\
& \left( 189 i B e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\
& \left( 8 d (a (1 + \cos[c + dx]))^{7/2} \right) + \frac{1}{d (a (1 + \cos[c + dx]))^{7/2}} 8 \sqrt{2} A e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \\
& \left( dx - i \operatorname{ArcSinh}[e^{i(c+dx)}] + i \sqrt{2} \log[1+e^{i(c+dx)}] + i \log[1+\sqrt{1+e^{2i(c+dx)}}] - i \sqrt{2} \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) - \\
& \frac{1}{d (a (1 + \cos[c + dx]))^{7/2}} 28 \sqrt{2} B e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \\
& \left( dx - i \operatorname{ArcSinh}[e^{i(c+dx)}] + i \sqrt{2} \log[1+e^{i(c+dx)}] + i \log[1+\sqrt{1+e^{2i(c+dx)}}] - i \sqrt{2} \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) + \\
& \frac{1}{(a (1 + \cos[c + dx]))^{7/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sqrt{\sec[c + dx]} \left( \frac{(-247 A + 427 B) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{12 d} + \frac{8 B \cos\left[\frac{3 dx}{2}\right] \sin\left[\frac{3 c}{2}\right]}{d} - \right. \\
& \frac{(247 A - 427 B) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{12 d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (379 A \sin\left[\frac{dx}{2}\right] - 703 B \sin\left[\frac{dx}{2}\right])}{24 d} + \\
& \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right])}{3 d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (-41 A \sin\left[\frac{dx}{2}\right] + 53 B \sin\left[\frac{dx}{2}\right])}{12 d} + \\
& \left. \frac{8 B \cos\left[\frac{3 c}{2}\right] \sin\left[\frac{3 dx}{2}\right]}{d} + \frac{(379 A - 703 B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{24 d} - \frac{(41 A - 53 B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{12 d} + \frac{(A - B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \tan\left[\frac{c}{2}\right]}{3 d} \right)
\end{aligned}$$

■ **Problem 571: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx]}{(a + b \cos[c + dx]) \sec[c + dx]^{3/2}} dx$$

Optimal (type 4, 197 leaves, 10 steps):

$$\frac{2 (A b - a B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{b^2 d} -$$

$$\frac{2 (3 a A b - 3 a^2 B - b^2 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{3 b^3 d} +$$

$$\frac{2 a^2 (A b - a B) \sqrt{\cos [c + d x]} \operatorname{EllipticPi}\left[\frac{2 b}{a + b}, \frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{b^3 (a + b) d} + \frac{2 B \sin [c + d x]}{3 b d \sqrt{\sec [c + d x]}}$$

Result (type 4, 548 leaves):

$$-\frac{1}{6 b d} \left( \left( 4 B \cos [c + d x]^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec [c + d x]}\right], -1\right] (b + a \sec [c + d x]) \sqrt{1 - \sec [c + d x]^2} \sin [c + d x] \right) / \right.$$

$$\left. \left( (a + b \cos [c + d x]) (1 - \cos [c + d x]^2) \right) + \right.$$

$$\left. \left( 2 (-3 A b + a B) \cos [c + d x]^2 \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec [c + d x]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec [c + d x]}\right], -1\right] \right) \right) \right.$$

$$\left. \left( (b + a \sec [c + d x]) \sqrt{1 - \sec [c + d x]^2} \sin [c + d x] \right) / \left( a (a + b \cos [c + d x]) (1 - \cos [c + d x]^2) \right) + \right.$$

$$\left. \left( (-3 A b + 3 a B) \cos [2 (c + d x)] (b + a \sec [c + d x]) \left( -4 a b + 4 a b \sec [c + d x]^2 - 4 a b \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec [c + d x]}\right], -1\right] \right) \right) \right.$$

$$\left. \left( \sqrt{\sec [c + d x]} \sqrt{1 - \sec [c + d x]^2} + 2 (2 a - b) b \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec [c + d x]}\right], -1\right] \sqrt{\sec [c + d x]} \sqrt{1 - \sec [c + d x]^2} + \right) \right.$$

$$\left. \left( 4 a^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec [c + d x]}\right], -1\right] \sqrt{\sec [c + d x]} \sqrt{1 - \sec [c + d x]^2} - \right) \right.$$

$$\left. \left( 2 b^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec [c + d x]}\right], -1\right] \sqrt{\sec [c + d x]} \sqrt{1 - \sec [c + d x]^2} \right) \sin [c + d x] \right) /$$

$$\left( a b^2 (a + b \cos [c + d x]) (1 - \cos [c + d x]^2) \sqrt{\sec [c + d x]} (2 - \sec [c + d x]^2) \right) + \frac{B \sqrt{\sec [c + d x]} \sin [2 (c + d x)]}{3 b d}$$

■ **Problem 573: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + d x]) \sec [c + d x]^{3/2}}{(a + b \cos [c + d x])^2} dx$$

Optimal (type 4, 316 leaves, 11 steps):

$$\begin{aligned}
& - \frac{(2a^2A - 3Ab^2 + abB) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a^2(a^2-b^2)d} + \\
& \frac{(Ab - aB) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a(a^2-b^2)d} - \\
& \frac{(5a^2Ab - 3Ab^3 - 3a^3B + ab^2B) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a^2(a-b)(a+b)^2d} + \\
& \frac{(2a^2A - 3Ab^2 + abB) \sqrt{\sec[c+dx]} \sin[c+dx]}{a^2(a^2-b^2)d} + \frac{b(Ab - aB) \sec[c+dx]^{3/2} \sin[c+dx]}{a(a^2-b^2)d(b+a\sec[c+dx])}
\end{aligned}$$

Result (type 4, 687 leaves):

$$\begin{aligned}
& - \frac{1}{4a^2(a-b)(a+b)d} \\
& \left( - \left( 2(4a^3A - 8aAb^2 + 4a^2bB) \cos[c+dx]^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] (b+a\sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \right. \right. \\
& \quad \left. \left. \sin[c+dx] \right) / \left( b(a+b\cos[c+dx])(1-\cos[c+dx]^2) \right) + \left( 2(10a^2Ab - 9Ab^3 - 4a^3B + 3ab^2B) \right. \right. \\
& \quad \left. \left. \cos[c+dx]^2 \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right) \right. \right. \\
& \quad \left. \left. (b+a\sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / \left( a(a+b\cos[c+dx])(1-\cos[c+dx]^2) \right) + \right. \\
& \quad \left( (2a^2Ab - 3Ab^3 + ab^2B) \cos[2(c+dx)] (b+a\sec[c+dx]) \left( -4ab + 4ab\sec[c+dx]^2 - 4ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right. \right. \right. \\
& \quad \left. \left. \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + 2(2a-b)b \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + \right. \right. \\
& \quad \left. \left. 4a^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} - \right. \right. \\
& \quad \left. \left. 2b^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \right) \sin[c+dx] \right) / \\
& \quad \left( ab^2(a+b\cos[c+dx])(1-\cos[c+dx]^2) \sqrt{\sec[c+dx]} (2-\sec[c+dx]^2) \right) + \\
& \frac{\sqrt{\sec[c+dx]} \left( \frac{(2a^2A - 3Ab^2 + abB) \sin[c+dx]}{a^2(a^2-b^2)} + \frac{Ab^2 \sin[c+dx] - abB \sin[c+dx]}{a(a^2-b^2)(a+b\cos[c+dx])} \right)}{d}
\end{aligned}$$

■ **Problem 574: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c+dx]) \sqrt{\sec[c+dx]}}{(a + b \cos[c+dx])^2} dx$$

Optimal (type 4, 260 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(Ab - aB) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a(a^2 - b^2)d} - \frac{(Ab - aB) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{b(a^2 - b^2)d} + \\
& \frac{(3a^2Ab - Ab^3 - a^3B - ab^2B) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a(a-b)b(a+b)^2d} + \frac{b(Ab - aB) \sqrt{\sec[c+dx]} \sin[c+dx]}{a(a^2 - b^2)d(b + a \sec[c+dx])}
\end{aligned}$$

Result (type 4, 645 leaves):

$$\begin{aligned}
& \frac{1}{4a(-a+b)(a+b)d} \\
& \left( - \left( 2(4aAb - 4a^2B) \cos[c+dx]^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] (b + a \sec[c+dx]) \sqrt{1 - \sec[c+dx]^2} \sin[c+dx] \right) / \right. \\
& \quad \left. (b(a+b \cos[c+dx]) (1 - \cos[c+dx]^2)) + \right. \\
& \quad \left( 2(-4a^2A + 3Ab^2 + abB) \cos[c+dx]^2 \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right) \right. \\
& \quad \left. (b + a \sec[c+dx]) \sqrt{1 - \sec[c+dx]^2} \sin[c+dx] \right) / (a(a+b \cos[c+dx]) (1 - \cos[c+dx]^2)) + \\
& \quad \left( (Ab^2 - abB) \cos[2(c+dx)] (b + a \sec[c+dx]) \left( -4ab + 4ab \sec[c+dx]^2 - 4ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right. \right. \\
& \quad \left. \left. \sqrt{\sec[c+dx]} \sqrt{1 - \sec[c+dx]^2} + 2(2a-b)b \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1 - \sec[c+dx]^2} + \right. \right. \\
& \quad \left. \left. 4a^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1 - \sec[c+dx]^2} - \right. \right. \\
& \quad \left. \left. 2b^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1 - \sec[c+dx]^2} \right) \sin[c+dx] \right) / \\
& \quad \left. (ab^2(a+b \cos[c+dx]) (1 - \cos[c+dx]^2) \sqrt{\sec[c+dx]} (2 - \sec[c+dx]^2)) \right) + \\
& \frac{\sqrt{\sec[c+dx]} \left( -\frac{(-Ab+aB) \sin[c+dx]}{a(a^2-b^2)} + \frac{-Ab \sin[c+dx] + aB \sin[c+dx]}{(a^2-b^2)(a+b \cos[c+dx])} \right)}{d}
\end{aligned}$$

■ **Problem 575: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c+dx]}{(a+b \cos[c+dx])^2 \sqrt{\sec[c+dx]}} dx$$

Optimal (type 4, 258 leaves, 10 steps):

$$\begin{aligned}
& \frac{(Ab - aB) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{b(a^2 - b^2)d} + \frac{(aAb + a^2B - 2b^2B) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{b^2(a^2 - b^2)d} - \\
& \frac{(a^2Ab + Ab^3 + a^3B - 3ab^2B) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{(a-b)b^2(a+b)^2d} - \frac{(Ab - aB) \sqrt{\sec[c+dx]} \sin[c+dx]}{(a^2 - b^2)d(b + a \sec[c+dx])}
\end{aligned}$$

Result (type 4, 632 leaves):

$$\frac{1}{4(a-b)(a+b)d} \left( - \left( 2(4aA - 4bB) \cos[c+dx]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] (b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / \right. \\ \left. (b(a+b \cos[c+dx]) (1-\cos[c+dx]^2)) + \right. \\ \left. \left( 2(-Ab+aB) \cos[c+dx]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \right) \right. \right. \\ \left. \left. (b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / (a(a+b \cos[c+dx]) (1-\cos[c+dx]^2)) + \right. \\ \left. \left( (Ab-aB) \cos[2(c+dx)] (b+a \sec[c+dx]) \left( -4ab+4ab \sec[c+dx]^2 - 4ab \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \right. \right. \right. \\ \left. \left. \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + 2(2a-b)b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + \right. \right. \\ \left. \left. 4a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} - \right. \right. \\ \left. \left. 2b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \right) \sin[c+dx] \right) / \\ \left. \left( ab^2(a+b \cos[c+dx]) (1-\cos[c+dx]^2) \sqrt{\sec[c+dx]} (2-\sec[c+dx]^2) \right) \right) + \\ \frac{\sqrt{\sec[c+dx]} \left( \frac{(Ab-aB) \sin[c+dx]}{b(-a^2+b^2)} + \frac{-aAb \sin[c+dx] + a^2B \sin[c+dx]}{b(-a^2+b^2)(a+b \cos[c+dx])} \right)}{d}$$

■ **Problem 576: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+B \cos[c+dx]}{(a+b \cos[c+dx])^2 \sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 284 leaves, 10 steps):

$$- \frac{(aAb - 3a^2B + 2b^2B) \sqrt{\cos[c+dx]} \operatorname{EllipticE} \left[ \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{b^2(a^2-b^2)d} + \\ \frac{(a^2Ab - 2Ab^3 - 3a^3B + 4ab^2B) \sqrt{\cos[c+dx]} \operatorname{EllipticF} \left[ \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{b^3(a^2-b^2)d} - \\ \frac{a(a^2Ab - 3Ab^3 - 3a^3B + 5ab^2B) \sqrt{\cos[c+dx]} \operatorname{EllipticPi} \left[ \frac{2b}{a+b}, \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{(a-b)b^3(a+b)^2d} + \frac{a(Ab-aB) \sqrt{\sec[c+dx]} \sin[c+dx]}{b(a^2-b^2)d(b+a \sec[c+dx])}$$

Result (type 4, 661 leaves):

$$\frac{1}{4 b (-a + b) (a + b) d} \left( - \left( 2 (4 A b^2 - 4 a b B) \cos [c + d x]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c + d x]} \right], -1 \right] (b + a \sec [c + d x]) \sqrt{1 - \sec [c + d x]^2} \sin [c + d x] \right) / \right. \\ \left. (b (a + b \cos [c + d x]) (1 - \cos [c + d x]^2)) + \right. \\ \left. \left( 2 (-a A b - a^2 B + 2 b^2 B) \cos [c + d x]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c + d x]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c + d x]} \right], -1 \right] \right) \right. \right. \\ \left. \left. (b + a \sec [c + d x]) \sqrt{1 - \sec [c + d x]^2} \sin [c + d x] \right) / \left( a (a + b \cos [c + d x]) (1 - \cos [c + d x]^2) \right) + \right. \\ \left. \left( (a A b - 3 a^2 B + 2 b^2 B) \cos [2 (c + d x)] (b + a \sec [c + d x]) \left( -4 a b + 4 a b \sec [c + d x]^2 - 4 a b \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c + d x]} \right], -1 \right] \right. \right. \right. \\ \left. \left. \sqrt{\sec [c + d x]} \sqrt{1 - \sec [c + d x]^2} + 2 (2 a - b) b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c + d x]} \right], -1 \right] \sqrt{\sec [c + d x]} \sqrt{1 - \sec [c + d x]^2} + \right. \right. \\ \left. \left. 4 a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c + d x]} \right], -1 \right] \sqrt{\sec [c + d x]} \sqrt{1 - \sec [c + d x]^2} - \right. \right. \\ \left. \left. 2 b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c + d x]} \right], -1 \right] \sqrt{\sec [c + d x]} \sqrt{1 - \sec [c + d x]^2} \right) \sin [c + d x] \right) / \\ \left. \left( a b^2 (a + b \cos [c + d x]) (1 - \cos [c + d x]^2) \sqrt{\sec [c + d x]} (2 - \sec [c + d x]^2) \right) \right) + \\ \frac{\sqrt{\sec [c + d x]} \left( -\frac{a (-A b + a B) \sin [c + d x]}{b^2 (a^2 - b^2)} + \frac{a^2 A b \sin [c + d x] - a^3 B \sin [c + d x]}{b^2 (-a^2 + b^2) (a + b \cos [c + d x])} \right)}{d}$$

■ **Problem 590: Attempted integration timed out after 120 seconds.**

$$\int \sqrt{a + b \cos [c + d x]} (A + B \cos [c + d x]) \sec [c + d x]^{9/2} dx$$

Optimal (type 4, 473 leaves, 7 steps):

$$\frac{1}{105 a^4 d \sqrt{\sec [c + d x]}} 2 (a - b) \sqrt{a + b} (19 a^2 A b + 8 A b^3 + 63 a^3 B - 14 a b^2 B) \sqrt{\cos [c + d x]} \operatorname{Csc}[c + d x] \\ \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}} \right], -\frac{a + b}{a - b} \right] \sqrt{\frac{a (1 - \sec [c + d x])}{a + b}} \sqrt{\frac{a (1 + \sec [c + d x])}{a - b}} + \frac{1}{105 a^3 d \sqrt{\sec [c + d x]}} \\ 2 (a - b) \sqrt{a + b} (8 A b^2 + a^2 (25 A - 63 B) + 2 a b (3 A - 7 B)) \sqrt{\cos [c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}} \right], -\frac{a + b}{a - b} \right] \\ \sqrt{\frac{a (1 - \sec [c + d x])}{a + b}} \sqrt{\frac{a (1 + \sec [c + d x])}{a - b}} + \frac{2 (25 a^2 A - 4 A b^2 + 7 a b B) \sqrt{a + b \cos [c + d x]} \sec [c + d x]^{3/2} \sin [c + d x]}{105 a^2 d} + \\ \frac{2 (A b + 7 a B) \sqrt{a + b \cos [c + d x]} \sec [c + d x]^{5/2} \sin [c + d x]}{35 a d} + \frac{2 A \sqrt{a + b \cos [c + d x]} \sec [c + d x]^{7/2} \sin [c + d x]}{7 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 591: Unable to integrate problem.**

$$\int \sqrt{a + b \cos[c + dx]} (A + B \cos[c + dx]) \sec[c + dx]^{7/2} dx$$

Optimal (type 4, 390 leaves, 6 steps):

$$\frac{1}{15 a^3 d \sqrt{\sec[c + dx]}} 2 (a - b) \sqrt{a + b} (9 a^2 A - 2 A b^2 + 5 a b B) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right]$$

$$\sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \frac{1}{15 a^2 d \sqrt{\sec[c + dx]}} 2 (a - b) \sqrt{a + b} (9 a A + 2 A b - 5 a B) \sqrt{\cos[c + dx]}$$

$$\operatorname{Csc}[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} +$$

$$\frac{2 (A b + 5 a B) \sqrt{a + b \cos[c + dx]} \sec[c + dx]^{3/2} \sin[c + dx]}{15 a d} + \frac{2 A \sqrt{a + b \cos[c + dx]} \sec[c + dx]^{5/2} \sin[c + dx]}{5 d}$$

Result (type 8, 37 leaves):

$$\int \sqrt{a + b \cos[c + dx]} (A + B \cos[c + dx]) \sec[c + dx]^{7/2} dx$$

■ **Problem 592: Attempted integration timed out after 120 seconds.**

$$\int \sqrt{a + b \cos[c + dx]} (A + B \cos[c + dx]) \sec[c + dx]^{5/2} dx$$

Optimal (type 4, 324 leaves, 5 steps):

$$\frac{1}{3 a^2 d \sqrt{\sec[c + dx]}} 2 (a - b) \sqrt{a + b} (A b + 3 a B) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \frac{1}{3 a d \sqrt{\sec[c + dx]}}$$

$$2 (a - b) \sqrt{a + b} (A - 3 B) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right]$$

$$\sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \frac{2 A \sqrt{a + b \cos[c + dx]} \sec[c + dx]^{3/2} \sin[c + dx]}{3 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 595: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \cos [c+d x]} (A+B \cos [c+d x])}{\sqrt{\sec [c+d x]}} dx$$

Optimal (type 4, 533 leaves, 8 steps):

$$\begin{aligned} & -\frac{1}{4 a b d \sqrt{\sec [c+d x]}} (a-b) \sqrt{a+b} (4 A b+a B) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \\ & \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \\ & \frac{1}{4 b d \sqrt{\sec [c+d x]}} \sqrt{a+b} (4 A b+(a+2 b) B) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \\ & \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \frac{1}{4 b^2 d \sqrt{\sec [c+d x]}} \sqrt{a+b} (4 a A b-a^2 B+4 b^2 B) \sqrt{\cos [c+d x]} \\ & \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \\ & \frac{B \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{2 d \sqrt{\sec [c+d x]}} + \frac{(4 A b+a B) \sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]} \sin [c+d x]}{4 b d} \end{aligned}$$

Result (type 4, 1133 leaves):

$$\begin{aligned} & \frac{B \sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]} \sin [2(c+d x)]}{4 d} + \frac{1}{4 b d \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right)^{3/2}} \sqrt{\frac{a+b+a \tan \left[\frac{1}{2}(c+d x)\right]^2-b \tan \left[\frac{1}{2}(c+d x)\right]^2}{1+\tan \left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{\frac{1}{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}} \\ & \left(4 a A b \tan \left[\frac{1}{2}(c+d x)\right]+4 A b^2 \tan \left[\frac{1}{2}(c+d x)\right]+a^2 B \tan \left[\frac{1}{2}(c+d x)\right]+a b B \tan \left[\frac{1}{2}(c+d x)\right]-8 A b^2 \tan \left[\frac{1}{2}(c+d x)\right]^3 - \right. \\ & \left.2 a b B \tan \left[\frac{1}{2}(c+d x)\right]^3-4 a A b \tan \left[\frac{1}{2}(c+d x)\right]^5+4 A b^2 \tan \left[\frac{1}{2}(c+d x)\right]^5-a^2 B \tan \left[\frac{1}{2}(c+d x)\right]^5+a b B \tan \left[\frac{1}{2}(c+d x)\right]^5 - \right) \end{aligned}$$



$$\begin{aligned}
& 8 a A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 2 a^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 8 b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 8 a A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 2 a^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 8 b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& (a+b)(4 A b+a B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 2 b(4 a A-a B+2 b B) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}
\end{aligned}$$

■ **Problem 596: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \operatorname{Cos}[c+d x]} (A+B \operatorname{Cos}[c+d x])}{\operatorname{Sec}[c+d x]^{3/2}} dx$$

Optimal (type 4, 620 leaves, 9 steps):

$$\begin{aligned}
& - \frac{1}{24 a b^2 d \sqrt{\sec[c+d x]}} (a-b) \sqrt{a+b} (6 a A b-3 a^2 B+16 b^2 B) \sqrt{\cos[c+d x]} \operatorname{Csc}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+d x]}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} + \frac{1}{24 b^2 d \sqrt{\sec[c+d x]}} \\
& \sqrt{a+b} (a+2 b) (6 A b-3 a B+8 b B) \sqrt{\cos[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+d x]}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} + \frac{1}{8 b^3 d \sqrt{\sec[c+d x]}} \\
& \sqrt{a+b} (2 a^2 A b-8 A b^3-a^3 B-4 a b^2 B) \sqrt{\cos[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+d x]}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} + \frac{(2 A b-a B) \sqrt{a+b} \cos[c+d x] \sin[c+d x]}{4 b d \sqrt{\sec[c+d x]}} + \\
& \frac{B(a+b \cos[c+d x])^{3/2} \sin[c+d x]}{3 b d \sqrt{\sec[c+d x]}} + \frac{(6 a A b-3 a^2 B+16 b^2 B) \sqrt{a+b} \cos[c+d x] \sqrt{\sec[c+d x]} \sin[c+d x]}{24 b^2 d}
\end{aligned}$$

Result (type 4, 3687 leaves):

$$\begin{aligned}
& \frac{\sqrt{a+b} \cos[c+d x] \sqrt{\sec[c+d x]} \left(\frac{1}{12} B \sin[c+d x] + \frac{(6 A b+a B) \sin[2(c+d x)]}{24 b} + \frac{1}{12} B \sin[3(c+d x)]\right)}{d} + \\
& \left( \frac{A b}{2 \sqrt{a+b} \cos[c+d x] \sqrt{\sec[c+d x]}} + \frac{7 a B}{12 \sqrt{a+b} \cos[c+d x] \sqrt{\sec[c+d x]}} + \frac{3 a A \sqrt{\sec[c+d x]}}{8 \sqrt{a+b} \cos[c+d x]} - \frac{a^2 B \sqrt{\sec[c+d x]}}{48 b \sqrt{a+b} \cos[c+d x]} + \right. \\
& \left. \frac{b B \sqrt{\sec[c+d x]}}{3 \sqrt{a+b} \cos[c+d x]} + \frac{a A \cos[2(c+d x)] \sqrt{\sec[c+d x]}}{8 \sqrt{a+b} \cos[c+d x]} - \frac{a^2 B \cos[2(c+d x)] \sqrt{\sec[c+d x]}}{16 b \sqrt{a+b} \cos[c+d x]} + \frac{b B \cos[2(c+d x)] \sqrt{\sec[c+d x]}}{3 \sqrt{a+b} \cos[c+d x]} \right)
\end{aligned}$$

$$\left( \frac{(6 a A b - 3 a^2 B + 16 b^2 B) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}}{24 b^2 \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}} + \right.$$

$$\left. \left( (a+b) (-6 a A b + 3 a^2 B - 16 b^2 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] + \right. \right.$$

$$2 b (-6 a A b + 12 A b^2 - a^2 B + 14 a b B) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] + 6 (-2 a^2 A b + 8 A b^3 + a^3 B + 4 a b^2 B)$$

$$\left. \left. \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^4}\right) \right) /$$

$$\left. \left. \left( 24 b^2 (a+b) \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)^2 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) \right) \right) /$$

$$d \left( \frac{(6 a A b - 3 a^2 B + 16 b^2 B) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}}{48 b^2 \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}} - \right.$$

$$\left. \left( (a+b) (-6 a A b + 3 a^2 B - 16 b^2 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] + 2 b (-6 a A b + 12 A b^2 - a^2 B + 14 a b B) \operatorname{EllipticF}\left[\right. \right. \right.$$

$$\left. \left. \left. \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] + 6 (-2 a^2 A b + 8 A b^3 + a^3 B + 4 a b^2 B) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \right) \right) \right)$$

$$\begin{aligned}
& \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}}\right] / \left(24b^2(a+b)\right. \\
& \left. \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^4}\right) - \\
& \left( \left( (a+b) (-6aAb + 3a^2B - 16b^2B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + 2b (-6aAb + 12Ab^2 - a^2B + 14abB) \operatorname{EllipticF}\left[\right. \right. \\
& \quad \left. \left. \operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + 6(-2a^2Ab + 8Ab^3 + a^3B + 4ab^2B) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \\
& \left( a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - b \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \\
& \left. \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^4}\right) / \\
& \left( 48b^2(a+b)^2 \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \left(\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right)^{3/2} \right) - \\
& \left( \left( (a+b) (-6aAb + 3a^2B - 16b^2B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + 2b (-6aAb + 12Ab^2 - a^2B + 14abB) \operatorname{EllipticF}\left[\right. \right. \\
& \quad \left. \left. \operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + 6(-2a^2Ab + 8Ab^3 + a^3B + 4ab^2B) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \\
& \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^4}\right) / \\
& \left( 24b^2(a+b) \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( (a+b) (-6 a A b + 3 a^2 B - 16 b^2 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + 2b (-6 a A b + 12 A b^2 - a^2 B + 14 a b B) \operatorname{EllipticF}\left[\right. \right. \\
& \quad \left. \left. \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + 6 (-2 a^2 A b + 8 A b^3 + a^3 B + 4 a b^2 B) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \Bigg) / \\
& \quad \left( 48 b^2 (a+b) \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) - \\
& \quad \left( (6 a A b - 3 a^2 B + 16 b^2 B) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \quad \left. \left( \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \right) / \left( 48 b^2 \left( \frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3/2} \right) + \\
& \quad \left( (6 a A b - 3 a^2 B + 16 b^2 B) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left( \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} - \right. \right. \\
& \quad \left. \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \right) / \\
& \quad \left( 48 b^2 \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \\
& \quad \left( (a+b) (-6 a A b + 3 a^2 B - 16 b^2 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + 2b (-6 a A b + 12 A b^2 - a^2 B + 14 a b B) \operatorname{EllipticF}\left[\right. \right. \\
& \quad \left. \left. \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + 6 (-2 a^2 A b + 8 A b^3 + a^3 B + 4 a b^2 B) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^4} \left( \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - b \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} - \right. \\
& \left. \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left( a + b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)}{\left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \Bigg/ \\
& \left( 48 b^2 (a+b) \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right. \\
& \left. \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \left( \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^4} \right. \\
& \left. \frac{b(-6aAb + 12A^2b^2 - a^2B + 14abB) \sec\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{1 - \frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} - \frac{3(-2a^2Ab + 8Ab^3 + a^3B + 4ab^2B) \sec\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{1 - \frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} \right. \\
& \left. \frac{(a+b)(-6aAb + 3a^2B - 16b^2B) \sec\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}{2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) \Bigg/ \\
& \left( 24 b^2 (a+b) \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 597: Attempted integration timed out after 120 seconds.**

$$\int (a + b \cos[c + dx])^{3/2} (A + B \cos[c + dx]) \sec[c + dx]^{11/2} dx$$

Optimal (type 4, 562 leaves, 8 steps):

$$\frac{1}{315 a^4 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (147 a^4 A + 33 a^2 A b^2 + 8 A b^4 + 246 a^3 b B - 18 a b^3 B) \sqrt{\cos[c+dx]}$$

$$\operatorname{Csc}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{315 a^3 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (8 A b^3 - a^3 (147 A - 75 B) + 3 a^2 b (13 A - 57 B) + 6 a b^2 (A - 3 B)) \sqrt{\cos[c+dx]}$$

$$\operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2 (88 a^2 A b - 4 A b^3 + 75 a^3 B + 9 a b^2 B) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{315 a^2 d} +$$

$$\frac{2 (49 a^2 A + 3 A b^2 + 72 a b B) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{5/2} \sin[c+dx]}{315 a d} +$$

$$\frac{2 (10 A b + 9 a B) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{7/2} \sin[c+dx]}{63 d} + \frac{2 a A \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{9/2} \sin[c+dx]}{9 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 598: Attempted integration timed out after 120 seconds.**

$$\int (a+b \cos[c+dx])^{3/2} (A+B \cos[c+dx]) \sec[c+dx]^{9/2} dx$$

Optimal (type 4, 473 leaves, 7 steps):

$$\frac{1}{105 a^3 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (82 a^2 A b - 6 A b^3 + 63 a^3 B + 21 a b^2 B) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{105 a^2 d \sqrt{\sec[c+dx]}}$$

$$2 (a-b) \sqrt{a+b} (6 A b^2 - a^2 (25 A - 63 B) + 3 a b (19 A - 7 B)) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{2 (25 a^2 A + 3 A b^2 + 42 a b B) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{105 a d} +$$

$$\frac{2 (8 A b + 7 a B) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{5/2} \sin[c+dx]}{35 d} + \frac{2 a A \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{7/2} \sin[c+dx]}{7 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 599: Attempted integration timed out after 120 seconds.**

$$\int (a+b \cos[c+dx])^{3/2} (A+B \cos[c+dx]) \sec[c+dx]^{7/2} dx$$

Optimal (type 4, 393 leaves, 6 steps):

$$\frac{1}{15 a^2 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (9 a^2 A + 3 A b^2 + 20 a b B) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{15 a d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (9 a A - 3 A b - 5 a B + 15 b B)$$

$$\sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2 (6 A b + 5 a B) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{15 d} + \frac{2 a A \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{5/2} \sin[c+dx]}{5 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 602: Result more than twice size of optimal antiderivative.**

$$\int (a+b \cos[c+dx])^{3/2} (A+B \cos[c+dx]) \sqrt{\sec[c+dx]} dx$$



Optimal (type 4, 532 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{1}{4 a d \sqrt{\operatorname{Sec}[c+d x]}} (a-b) \sqrt{a+b} (4 A b+5 a B) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \\
 & \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{4 d \sqrt{\operatorname{Sec}[c+d x]}} \\
 & \sqrt{a+b} (8 a A+4 A b+5 a B+2 b B) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
 & \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{4 b d \sqrt{\operatorname{Sec}[c+d x]}} \sqrt{a+b} (12 a A b+3 a^2 B+4 b^2 B) \sqrt{\operatorname{Cos}[c+d x]} \\
 & \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \\
 & \frac{b B \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{2 d \sqrt{\operatorname{Sec}[c+d x]}} + \frac{(4 A b+5 a B) \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d}
 \end{aligned}$$

Result (type 4, 1146 leaves):

$$\begin{aligned}
 & \frac{b B \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[2(c+d x)]}{4 d} + \frac{1}{4 d \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)^{3/2}} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \\
 & \left(4 a A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+4 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+5 a^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+5 a b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]-8 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3-\right. \\
 & \left.10 a b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3-4 a A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+4 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5-5 a^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+5 a b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5-\right. \\
 & \left.24 a A b \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}-\right. \\
 & \left.6 a^2 B \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}-\right.
 \end{aligned}$$

$$\begin{aligned}
& 8 b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]\right], \frac{-a+b}{a+b} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 24 a A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]\right], \frac{-a+b}{a+b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 6 a^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]\right], \frac{-a+b}{a+b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 8 b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]\right], \frac{-a+b}{a+b} \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b)(4 A b+5 a B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]\right], \frac{-a+b}{a+b} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2\left(4 a^2(A-B)-2 b^2 B+a b(-8 A+B)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]\right], \frac{-a+b}{a+b} \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}
\end{aligned}$$

■ **Problem 603: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{3/2} (A+B \operatorname{Cos}[c+dx])}{\sqrt{\operatorname{Sec}[c+dx]}} dx$$

Optimal (type 4, 626 leaves, 9 steps):

$$\begin{aligned}
& - \frac{1}{24 a b d \sqrt{\operatorname{Sec}[c+d x]}} (a-b) \sqrt{a+b} \left(30 a A b+3 a^2 B+16 b^2 B\right) \sqrt{\operatorname{Cos}[c+d x]} \\
& \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}}+ \\
& \frac{1}{24 b d \sqrt{\operatorname{Sec}[c+d x]}} \sqrt{a+b} \left(30 a A b+12 A b^2+3 a^2 B+14 a b B+16 b^2 B\right) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}}-\frac{1}{8 b^2 d \sqrt{\operatorname{Sec}[c+d x]}} \\
& \sqrt{a+b} \left(6 a^2 A b+8 A b^3-a^3 B+12 a b^2 B\right) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}}+\frac{b B \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 d \operatorname{Sec}[c+d x]^{3 / 2}}+ \\
& \frac{(6 A b+7 a B) \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{12 d \sqrt{\operatorname{Sec}[c+d x]}}+\frac{\left(30 a A b+3 a^2 B+16 b^2 B\right) \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{24 b d}
\end{aligned}$$

Result (type 4, 1505 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \left(\frac{1}{12} b B \operatorname{Sin}[c+d x]+\frac{1}{24}(6 A b+7 a B) \operatorname{Sin}[2(c+d x)]+\frac{1}{12} b B \operatorname{Sin}[3(c+d x)]\right)+ \\
& \frac{1}{24 b d \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \\
& \left(30 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+30 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+3 a^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+3 a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+16 a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+ \right. \\
& \left. 16 b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]-60 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3-6 a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3-32 b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3-30 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+ \right. \\
& \left. 30 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5-3 a^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+3 a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5-16 a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+16 b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5-\right.
\end{aligned}$$

$$\begin{aligned}
& 36 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 48 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 6 a^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 72 a b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 36 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 48 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 6 a^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 72 a b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + (a+b) (30 a A b+3 a^2 B+16 b^2 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 b (12 A b^2+a^2 (24 A-7 B)+a (-6 A b+26 b B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right]
\end{aligned}$$

$$\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}$$

■ **Problem 604: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{3/2} (A+B \cos[c+dx])}{\sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 730 leaves, 10 steps):

$$\begin{aligned} & - \frac{1}{192 a b^2 d \sqrt{\sec[c+dx]}} (a-b) \sqrt{a+b} (24 a^2 A b + 128 A b^3 - 9 a^3 B + 156 a b^2 B) \sqrt{\cos[c+dx]} \\ & \quad \text{Csc}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\ & \frac{1}{192 b^2 d \sqrt{\sec[c+dx]}} \sqrt{a+b} (9 a^3 B - 6 a^2 b (4 A + B) - 8 b^3 (16 A + 9 B) - 4 a b^2 (28 A + 39 B)) \sqrt{\cos[c+dx]} \text{Csc}[c+dx] \\ & \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\ & \frac{1}{64 b^3 d \sqrt{\sec[c+dx]}} \sqrt{a+b} (8 a^3 A b - 96 a A b^3 - 3 a^4 B - 24 a^2 b^2 B - 48 b^4 B) \sqrt{\cos[c+dx]} \text{Csc}[c+dx] \\ & \quad \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\ & \frac{(8 a A b - 3 a^2 B + 12 b^2 B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{32 b d \sqrt{\sec[c+dx]}} + \frac{(8 A b - 3 a B) (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{24 b d \sqrt{\sec[c+dx]}} + \\ & \frac{B (a+b \cos[c+dx])^{5/2} \sin[c+dx]}{4 b d \sqrt{\sec[c+dx]}} + \frac{(24 a^2 A b + 128 A b^3 - 9 a^3 B + 156 a b^2 B) \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]} \sin[c+dx]}{192 b^2 d} \end{aligned}$$

Result (type 4, 1907 leaves):

$$\begin{aligned} & \frac{1}{d} \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]} \\ & \left( \frac{1}{96} (8 A b + 9 a B) \sin[c+dx] + \frac{(56 a A b + 3 a^2 B + 48 b^2 B) \sin[2(c+dx)]}{192 b} + \frac{1}{96} (8 A b + 9 a B) \sin[3(c+dx)] + \frac{1}{32} b B \sin[4(c+dx)] \right) + \end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left( -24a^3Ab\tan\left[\frac{1}{2}(c+dx)\right] - 24a^2Ab^2\tan\left[\frac{1}{2}(c+dx)\right] - \right. \right. \\
& 128aAb^3\tan\left[\frac{1}{2}(c+dx)\right] - 128Ab^4\tan\left[\frac{1}{2}(c+dx)\right] + 9a^4B\tan\left[\frac{1}{2}(c+dx)\right] + 9a^3bB\tan\left[\frac{1}{2}(c+dx)\right] - 156a^2b^2B\tan\left[\frac{1}{2}(c+dx)\right] - \\
& 156ab^3B\tan\left[\frac{1}{2}(c+dx)\right] + 48a^2Ab^2\tan\left[\frac{1}{2}(c+dx)\right]^3 + 256Ab^4\tan\left[\frac{1}{2}(c+dx)\right]^3 - 18a^3bB\tan\left[\frac{1}{2}(c+dx)\right]^3 + \\
& 312ab^3B\tan\left[\frac{1}{2}(c+dx)\right]^3 + 24a^3Ab\tan\left[\frac{1}{2}(c+dx)\right]^5 - 24a^2Ab^2\tan\left[\frac{1}{2}(c+dx)\right]^5 + 128aAb^3\tan\left[\frac{1}{2}(c+dx)\right]^5 - \\
& 128Ab^4\tan\left[\frac{1}{2}(c+dx)\right]^5 - 9a^4B\tan\left[\frac{1}{2}(c+dx)\right]^5 + 9a^3bB\tan\left[\frac{1}{2}(c+dx)\right]^5 + 156a^2b^2B\tan\left[\frac{1}{2}(c+dx)\right]^5 - 156ab^3B\tan\left[\frac{1}{2}(c+dx)\right]^5 - \\
& 48a^3Ab\text{EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 576aAb^3\text{EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 18a^4B\text{EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 144a^2b^2B\text{EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 288b^4B\text{EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 48a^3Ab\text{EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 576aAb^3\text{EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 18a^4B\text{EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right]
\end{aligned}$$

$$\begin{aligned}
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 144 a^2 b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 288 b^4 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (-24 a^2 A b - 128 A b^3 + 9 a^3 B - 156 a b^2 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 b (2 a^2 b (28 A - 57 B) - 4 a b^2 (52 A - 9 B) + 3 a^3 B - 72 b^3 B) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(192 b^2 d \sqrt{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(b \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - a \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right)
\end{aligned}$$

■ **Problem 605: Attempted integration timed out after 120 seconds.**

$$\int (a+b \cos[c+dx])^{5/2} (A+B \cos[c+dx]) \sec[c+dx]^{13/2} dx$$

Optimal (type 4, 662 leaves, 9 steps):

$$\begin{aligned}
& \frac{1}{3465 a^4 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (3705 a^4 A b + 255 a^2 A b^3 + 40 A b^5 + 1617 a^5 B + 3069 a^3 b^2 B - 110 a b^4 B) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{3465 a^3 d \sqrt{\sec[c+dx]}} \\
& 2 (a-b) \sqrt{a+b} (40 A b^4 + 3 a^4 (225 A - 539 B) - 6 a^3 b (505 A - 209 B) + 15 a^2 b^2 (19 A - 121 B) + 10 a b^3 (3 A - 11 B)) \sqrt{\cos[c+dx]} \\
& \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{2 (675 a^4 A + 1025 a^2 A b^2 - 20 A b^4 + 1793 a^3 b B + 55 a b^3 B) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{3465 a^2 d} + \\
& \frac{2 (1145 a^2 A b + 15 A b^3 + 539 a^3 B + 825 a b^2 B) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{5/2} \sin[c+dx]}{3465 a d} + \\
& \frac{2 (81 a^2 A + 113 A b^2 + 209 a b B) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{7/2} \sin[c+dx]}{693 d} + \\
& \frac{2 a (14 A b + 11 a B) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{9/2} \sin[c+dx]}{99 d} + \frac{2 a A (a+b \cos[c+dx])^{3/2} \sec[c+dx]^{11/2} \sin[c+dx]}{11 d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 606: Attempted integration timed out after 120 seconds.**

$$\int (a+b \cos[c+dx])^{5/2} (A+B \cos[c+dx]) \sec[c+dx]^{11/2} dx$$

Optimal (type 4, 562 leaves, 8 steps):



$$\frac{1}{315 a^3 d \sqrt{\sec[c + d x]}} 2 (a - b) \sqrt{a + b} (147 a^4 A + 279 a^2 A b^2 - 10 A b^4 + 435 a^3 b B + 45 a b^3 B) \sqrt{\cos[c + d x]}$$

$$\operatorname{Csc}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + d x])}{a + b}} \sqrt{\frac{a(1 + \sec[c + d x])}{a - b}} -$$

$$\frac{1}{315 a^2 d \sqrt{\sec[c + d x]}} 2 (a - b) \sqrt{a + b} (10 A b^3 - 6 a^2 b (19 A - 60 B) + 3 a^3 (49 A - 25 B) + 15 a b^2 (11 A - 3 B)) \sqrt{\cos[c + d x]}$$

$$\operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + d x])}{a + b}} \sqrt{\frac{a(1 + \sec[c + d x])}{a - b}} +$$

$$\frac{2 (163 a^2 A b + 5 A b^3 + 75 a^3 B + 135 a b^2 B) \sqrt{a + b \cos[c + d x]} \sec[c + d x]^{3/2} \sin[c + d x]}{315 a d} +$$

$$\frac{2 (49 a^2 A + 75 A b^2 + 135 a b B) \sqrt{a + b \cos[c + d x]} \sec[c + d x]^{5/2} \sin[c + d x]}{315 d} +$$

$$\frac{2 a (4 A b + 3 a B) \sqrt{a + b \cos[c + d x]} \sec[c + d x]^{7/2} \sin[c + d x]}{21 d} + \frac{2 a A (a + b \cos[c + d x])^{3/2} \sec[c + d x]^{9/2} \sin[c + d x]}{9 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 607: Attempted integration timed out after 120 seconds.**

$$\int (a + b \cos[c + d x])^{5/2} (A + B \cos[c + d x]) \sec[c + d x]^{9/2} dx$$

Optimal (type 4, 474 leaves, 7 steps):

$$\frac{1}{105 a^2 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (145 a^2 A b + 15 A b^3 + 63 a^3 B + 161 a b^2 B) \sqrt{\cos[c+dx]}$$

$$\operatorname{Csc}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{105 a d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (a^2 (25 A - 63 B) + 15 b^2 (A - 7 B) - 8 a b (15 A - 7 B)) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2 (25 a^2 A + 45 A b^2 + 77 a b B) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{105 d} +$$

$$\frac{2 a (10 A b + 7 a B) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{5/2} \sin[c+dx]}{35 d} + \frac{2 a A (a+b \cos[c+dx])^{3/2} \sec[c+dx]^{7/2} \sin[c+dx]}{7 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 610: Result more than twice size of optimal antiderivative.**

$$\int (a+b \cos[c+dx])^{5/2} (A+B \cos[c+dx]) \sec[c+dx]^{3/2} dx$$

Optimal (type 4, 607 leaves, 9 steps):

$$\frac{1}{4 a d \sqrt{\operatorname{Sec}[c+d x]}} (a-b) \sqrt{a+b} \left(8 a^2 A-4 A b^2-9 a b B\right) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} -$$

$$\frac{1}{4 d \sqrt{\operatorname{Sec}[c+d x]}} \sqrt{a+b} \left(8 a^2(A-B)-2 b^2(2 A+B)-3 a b(8 A+3 B)\right) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{4 d \sqrt{\operatorname{Sec}[c+d x]}}$$

$$\sqrt{a+b} \left(20 a A b+15 a^2 B+4 b^2 B\right) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{b(4 a A-b B) \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{2 d \sqrt{\operatorname{Sec}[c+d x]}}$$

$$\frac{\left(8 a^2 A-4 A b^2-9 a b B\right) \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d} + \frac{2 a A(a+b \operatorname{Cos}[c+d x])^{3 / 2} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{d}$$

Result (type 4, 1290 leaves):

$$\frac{\sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \left(2 a^2 A \operatorname{Sin}[c+d x]+\frac{1}{4} b^2 B \operatorname{Sin}[2(c+d x)]\right)}{d} +$$

$$\frac{1}{4 d \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)^{3 / 2}} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}$$

$$\left(-8 a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]-8 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+4 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+4 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+9 a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+9 a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+16 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3-8 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3-18 a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3+8 a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5-8 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5-4 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+4 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5-9 a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+9 a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5-\right.$$

$$\begin{aligned}
& 40 a A b^2 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 30 a^2 b B \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 8 b^3 B \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 40 a A b^2 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 30 a^2 b B \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 8 b^3 B \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - (a+b) (8 a^2 A - 4 A b^2 - 9 a b B) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 2 (12 a^2 b (A - B) - 2 b^3 B + a b^2 (-12 A + B) + 4 a^3 (A + B)) \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}
\end{aligned}$$

■ **Problem 611: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + dx])^{5/2} (A + B \cos[c + dx]) \sqrt{\sec[c + dx]} dx$$

Optimal (type 4, 624 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{1}{24 a d \sqrt{\sec [c+d x]}} (a-b) \sqrt{a+b} \left(54 a A b+33 a^2 B+16 b^2 B\right) \sqrt{\cos [c+d x]} \\
 & \quad \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}+ \\
 & \frac{1}{24 d \sqrt{\sec [c+d x]}} \sqrt{a+b} \left(4 b^2(3 A+4 B)+a^2(48 A+33 B)+a(54 A b+26 b B)\right) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \\
 & \quad \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}-\frac{1}{8 b d \sqrt{\sec [c+d x]}} \\
 & \sqrt{a+b} \left(30 a^2 A b+8 A b^3+5 a^3 B+20 a b^2 B\right) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \\
 & \quad \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}+\frac{b(2 A b+3 a B) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{4 d \sqrt{\sec [c+d x]}}+ \\
 & \frac{b B(a+b \cos [c+d x])^{3 / 2} \sin [c+d x]}{3 d \sqrt{\sec [c+d x]}}+\frac{\left(54 a A b+33 a^2 B+16 b^2 B\right) \sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]} \sin [c+d x]}{24 d}
 \end{aligned}$$

Result (type 4, 1521 leaves):

$$\begin{aligned}
 & \frac{1}{d} \sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]} \left(\frac{1}{12} b^2 B \sin [c+d x]+\frac{1}{24} b(6 A b+13 a B) \sin [2(c+d x)]+\frac{1}{12} b^2 B \sin [3(c+d x)]\right)- \\
 & \frac{1}{24 d\left(1+\tan \left[\frac{1}{2}(c+d x)\right]\right)^{3 / 2}} \sqrt{\frac{a+b a \tan \left[\frac{1}{2}(c+d x)\right]^2-b \tan \left[\frac{1}{2}(c+d x)\right]^2}{1+\tan \left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \\
 & \left(-54 a^2 A b \tan \left[\frac{1}{2}(c+d x)\right]-54 a A b^2 \tan \left[\frac{1}{2}(c+d x)\right]-33 a^3 B \tan \left[\frac{1}{2}(c+d x)\right]-33 a^2 b B \tan \left[\frac{1}{2}(c+d x)\right]-16 a b^2 B \tan \left[\frac{1}{2}(c+d x)\right]-\right. \\
 & \quad 16 b^3 B \tan \left[\frac{1}{2}(c+d x)\right]+108 a A b^2 \tan \left[\frac{1}{2}(c+d x)\right]^3+66 a^2 b B \tan \left[\frac{1}{2}(c+d x)\right]^3+32 b^3 B \tan \left[\frac{1}{2}(c+d x)\right]^3+54 a^2 A b \tan \left[\frac{1}{2}(c+d x)\right]^5- \\
 & \quad \left.54 a A b^2 \tan \left[\frac{1}{2}(c+d x)\right]^5+33 a^3 B \tan \left[\frac{1}{2}(c+d x)\right]^5-33 a^2 b B \tan \left[\frac{1}{2}(c+d x)\right]^5+16 a b^2 B \tan \left[\frac{1}{2}(c+d x)\right]^5-16 b^3 B \tan \left[\frac{1}{2}(c+d x)\right]^5+\right.
 \end{aligned}$$

$$\begin{aligned}
& 180 a^2 A b \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 48 A b^3 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 30 a^3 B \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 120 a b^2 B \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 180 a^2 A b \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 48 A b^3 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 30 a^3 B \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 120 a b^2 B \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - (a+b) (54 a A b + 33 a^2 B + 16 b^2 B) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 (-12 A b^3 + 2 a b^2 (3 A - 19 B) + 24 a^3 (A - B) + a^2 (-72 A b + 13 b B)) \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right]
\end{aligned}$$

$$\sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c + dx)\right]^2\right) \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}}$$

■ **Problem 612: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \cos[c + dx])^{5/2} (A + B \cos[c + dx])}{\sqrt{\sec[c + dx]}} dx$$

Optimal (type 4, 724 leaves, 10 steps):

$$\begin{aligned} & - \frac{1}{192 a b d \sqrt{\sec[c + dx]}} (a - b) \sqrt{a + b} (264 a^2 A b + 128 A b^3 + 15 a^3 B + 284 a b^2 B) \sqrt{\cos[c + dx]} \\ & \quad \text{Csc}[c + dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \\ & \frac{1}{192 b d \sqrt{\sec[c + dx]}} \sqrt{a + b} (15 a^3 B + 8 b^3 (16 A + 9 B) + 2 a^2 b (132 A + 59 B) + 4 a b^2 (52 A + 71 B)) \sqrt{\cos[c + dx]} \\ & \quad \text{Csc}[c + dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \\ & \frac{1}{64 b^2 d \sqrt{\sec[c + dx]}} \sqrt{a + b} (40 a^3 A b + 160 a A b^3 - 5 a^4 B + 120 a^2 b^2 B + 48 b^4 B) \sqrt{\cos[c + dx]} \text{Csc}[c + dx] \\ & \quad \text{EllipticPi}\left[\frac{a + b}{b}, \text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \\ & \frac{b B (a + b \cos[c + dx])^{3/2} \sin[c + dx]}{4 d \sec[c + dx]^{3/2}} + \frac{(24 a A b + 5 a^2 B + 12 b^2 B) \sqrt{a + b \cos[c + dx]} \sin[c + dx]}{32 d \sqrt{\sec[c + dx]}} + \\ & \frac{(8 A b + 11 a B) (a + b \cos[c + dx])^{3/2} \sin[c + dx]}{24 d \sqrt{\sec[c + dx]}} + \frac{(264 a^2 A b + 128 A b^3 + 15 a^3 B + 284 a b^2 B) \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} \sin[c + dx]}{192 b d} \end{aligned}$$

Result (type 4, 1877 leaves):

$$\begin{aligned} & \frac{1}{d} \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} \left( \frac{1}{96} b (8 A b + 17 a B) \sin[c + dx] + \right. \\ & \left. \frac{1}{192} (104 a A b + 59 a^2 B + 48 b^2 B) \sin[2(c + dx)] + \frac{1}{96} b (8 A b + 17 a B) \sin[3(c + dx)] + \frac{1}{32} b^2 B \sin[4(c + dx)] \right) + \end{aligned}$$

$$\begin{aligned}
& \frac{1}{192 b d \left(1 + \tan\left[\frac{1}{2}(c + d x)\right]\right)^2} \sqrt{\frac{1}{1 + \tan\left[\frac{1}{2}(c + d x)\right]^2}} \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + d x)\right]^2}} \\
& \left( 264 a^3 A b \tan\left[\frac{1}{2}(c + d x)\right] + 264 a^2 A b^2 \tan\left[\frac{1}{2}(c + d x)\right] + 128 a A b^3 \tan\left[\frac{1}{2}(c + d x)\right] + 128 A b^4 \tan\left[\frac{1}{2}(c + d x)\right] + 15 a^4 B \tan\left[\frac{1}{2}(c + d x)\right] + \right. \\
& 15 a^3 b B \tan\left[\frac{1}{2}(c + d x)\right] + 284 a^2 b^2 B \tan\left[\frac{1}{2}(c + d x)\right] + 284 a b^3 B \tan\left[\frac{1}{2}(c + d x)\right] - 528 a^2 A b^2 \tan\left[\frac{1}{2}(c + d x)\right]^3 - \\
& 256 A b^4 \tan\left[\frac{1}{2}(c + d x)\right]^3 - 30 a^3 b B \tan\left[\frac{1}{2}(c + d x)\right]^3 - 568 a b^3 B \tan\left[\frac{1}{2}(c + d x)\right]^3 - 264 a^3 A b \tan\left[\frac{1}{2}(c + d x)\right]^5 + \\
& 264 a^2 A b^2 \tan\left[\frac{1}{2}(c + d x)\right]^5 - 128 a A b^3 \tan\left[\frac{1}{2}(c + d x)\right]^5 + 128 A b^4 \tan\left[\frac{1}{2}(c + d x)\right]^5 - 15 a^4 B \tan\left[\frac{1}{2}(c + d x)\right]^5 + \\
& \left. 15 a^3 b B \tan\left[\frac{1}{2}(c + d x)\right]^5 - 284 a^2 b^2 B \tan\left[\frac{1}{2}(c + d x)\right]^5 + 284 a b^3 B \tan\left[\frac{1}{2}(c + d x)\right]^5 - \right. \\
& 240 a^3 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{-a + b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + d x)\right]^2} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + d x)\right]^2 - b \tan\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} - \\
& 960 a A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{-a + b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + d x)\right]^2} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + d x)\right]^2 - b \tan\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} + \\
& 30 a^4 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{-a + b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + d x)\right]^2} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + d x)\right]^2 - b \tan\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} - \\
& 720 a^2 b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{-a + b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + d x)\right]^2} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + d x)\right]^2 - b \tan\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} - \\
& 288 b^4 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{-a + b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + d x)\right]^2} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + d x)\right]^2 - b \tan\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} - \\
& 240 a^3 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{-a + b}{a + b}\right] \tan\left[\frac{1}{2}(c + d x)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c + d x)\right]^2}
\end{aligned}$$



$$\begin{aligned}
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}-960 a A b^3 \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+30 a^4 B \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}- \\
& 720 a^2 b^2 B \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}-288 b^4 B \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+ \\
& (a+b)\left(264 a^2 A b+128 A b^3+15 a^3 B+284 a b^2 B\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}- \\
& 2 b\left(a^3(192 A-59 B)+4 a b^2(76 A-9 B)+72 b^3 B+a^2(-104 A b+322 b B)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}
\end{aligned}$$

■ **Problem 613: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+d x])^{5/2}(A+B \operatorname{Cos}[c+d x])}{\operatorname{Sec}[c+d x]^{3/2}} d x$$

Optimal (type 4, 839 leaves, 11 steps):

$$\begin{aligned}
& - \frac{1}{1920 a b^2 d \sqrt{\text{Sec}[c+d x]}} (a-b) \sqrt{a+b} \left(150 a^3 A b + 2840 a A b^3 - 45 a^4 B + 1692 a^2 b^2 B + 1024 b^4 B\right) \sqrt{\text{Cos}[c+d x]} \\
& \quad \text{Csc}[c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\text{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+d x])}{a-b}} - \\
& \frac{1}{1920 b^2 d \sqrt{\text{Sec}[c+d x]}} \sqrt{a+b} \left(45 a^4 B - 30 a^3 b(5 A+B) - 16 b^4(45 A+64 B) - 8 a b^3(355 A+193 B) - 4 a^2 b^2(295 A+423 B)\right) \\
& \quad \sqrt{\text{Cos}[c+d x]} \text{Csc}[c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\text{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+d x])}{a-b}} + \\
& \frac{1}{128 b^3 d \sqrt{\text{Sec}[c+d x]}} \sqrt{a+b} \left(10 a^4 A b - 240 a^2 A b^3 - 96 A b^5 - 3 a^5 B - 40 a^3 b^2 B - 240 a b^4 B\right) \sqrt{\text{Cos}[c+d x]} \text{Csc}[c+d x] \\
& \quad \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \text{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\text{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+d x])}{a-b}} + \\
& \frac{(50 a^2 A b + 120 A b^3 - 15 a^3 B + 172 a b^2 B) \sqrt{a+b \text{Cos}[c+d x]} \text{Sin}[c+d x]}{320 b d \sqrt{\text{Sec}[c+d x]}} + \frac{(50 a A b - 15 a^2 B + 64 b^2 B) (a+b \text{Cos}[c+d x])^{3/2} \text{Sin}[c+d x]}{240 b d \sqrt{\text{Sec}[c+d x]}} + \\
& \frac{(10 A b - 3 a B) (a+b \text{Cos}[c+d x])^{5/2} \text{Sin}[c+d x]}{40 b d \sqrt{\text{Sec}[c+d x]}} + \frac{B (a+b \text{Cos}[c+d x])^{7/2} \text{Sin}[c+d x]}{5 b d \sqrt{\text{Sec}[c+d x]}} + \frac{1}{1920 b^2 d} \\
& \frac{(150 a^3 A b + 2840 a A b^3 - 45 a^4 B + 1692 a^2 b^2 B + 1024 b^4 B) \sqrt{a+b \text{Cos}[c+d x]} \sqrt{\text{Sec}[c+d x]} \text{Sin}[c+d x]}{1920 b^2 d}
\end{aligned}$$

Result (type 4, 4552 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{a+b \text{Cos}[c+d x]} \sqrt{\text{Sec}[c+d x]} \left( \frac{1}{960} (170 a A b + 93 a^2 B + 88 b^2 B) \text{Sin}[c+d x] + \frac{(590 a^2 A b + 480 A b^3 + 15 a^3 B + 1024 a b^2 B) \text{Sin}[2(c+d x)]}{1920 b} + \right. \\
& \left. \frac{1}{960} (170 a A b + 93 a^2 B + 100 b^2 B) \text{Sin}[3(c+d x)] + \frac{1}{320} b (10 A b + 21 a B) \text{Sin}[4(c+d x)] + \frac{1}{80} b^2 B \text{Sin}[5(c+d x)] \right) + \\
& \left( \left( \frac{161 a^2 A b}{96 \sqrt{a+b \text{Cos}[c+d x]} \sqrt{\text{Sec}[c+d x]}} + \frac{3 A b^3}{8 \sqrt{a+b \text{Cos}[c+d x]} \sqrt{\text{Sec}[c+d x]}} + \frac{191 a^3 B}{320 \sqrt{a+b \text{Cos}[c+d x]} \sqrt{\text{Sec}[c+d x]}} + \right. \right. \\
& \frac{289 a b^2 B}{240 \sqrt{a+b \text{Cos}[c+d x]} \sqrt{\text{Sec}[c+d x]}} + \frac{133 a^3 A \sqrt{\text{Sec}[c+d x]}}{384 \sqrt{a+b \text{Cos}[c+d x]}} + \frac{89 a A b^2 \sqrt{\text{Sec}[c+d x]}}{96 \sqrt{a+b \text{Cos}[c+d x]}} - \frac{a^4 B \sqrt{\text{Sec}[c+d x]}}{256 b \sqrt{a+b \text{Cos}[c+d x]}} + \\
& \left. \frac{809 a^2 b B \sqrt{\text{Sec}[c+d x]}}{960 \sqrt{a+b \text{Cos}[c+d x]}} + \frac{4 b^3 B \sqrt{\text{Sec}[c+d x]}}{15 \sqrt{a+b \text{Cos}[c+d x]}} + \frac{5 a^3 A \text{Cos}[2(c+d x)] \sqrt{\text{Sec}[c+d x]}}{128 \sqrt{a+b \text{Cos}[c+d x]}} + \frac{71 a A b^2 \text{Cos}[2(c+d x)] \sqrt{\text{Sec}[c+d x]}}{96 \sqrt{a+b \text{Cos}[c+d x]}} - \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{3 a^4 B \cos[2(c+dx)] \sqrt{\sec[c+dx]}}{256 b \sqrt{a+b \cos[c+dx]}} + \frac{141 a^2 b B \cos[2(c+dx)] \sqrt{\sec[c+dx]}}{320 \sqrt{a+b \cos[c+dx]}} + \frac{4 b^3 B \cos[2(c+dx)] \sqrt{\sec[c+dx]}}{15 \sqrt{a+b \cos[c+dx]}} \right) \\
& \left( \left( (150 a^3 A b + 2840 a A b^3 - 45 a^4 B + 1692 a^2 b^2 B + 1024 b^4 B) \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) \right) / \right. \\
& \left( 1920 b^2 \sqrt{\frac{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \\
& \left( (a+b) (-150 a^3 A b - 2840 a A b^3 + 45 a^4 B - 1692 a^2 b^2 B - 1024 b^4 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& 2 b (720 A b^4 - 8 a b^3 (45 A - 289 B) + 4 a^2 b^2 (805 A - 193 B) - 15 a^4 B + a^3 (-590 A b + 1146 b B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \right. \\
& \left. \frac{-a+b}{a+b}\right] + 30 (-10 a^4 A b + 240 a^2 A b^3 + 96 A b^5 + 3 a^5 B + 40 a^3 b^2 B + 240 a b^4 B) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \\
& \left. \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^4}} \right) / \\
& \left( 1920 b^2 (a+b) \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) / \\
& \left( \left( (150 a^3 A b + 2840 a A b^3 - 45 a^4 B + 1692 a^2 b^2 B + 1024 b^4 B) \sec\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) \right) / \right. \\
& \left( 3840 b^2 \sqrt{\frac{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( (a+b) (-150 a^3 A b - 2840 a A b^3 + 45 a^4 B - 1692 a^2 b^2 B - 1024 b^4 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& 2b(720 A b^4 - 8 a b^3(45 A - 289 B) + 4 a^2 b^2(805 A - 193 B) - 15 a^4 B + a^3(-590 A b + 1146 b B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \right. \\
& \left. \frac{-a+b}{a+b}\right] + 30(-10 a^4 A b + 240 a^2 A b^3 + 96 A b^5 + 3 a^5 B + 40 a^3 b^2 B + 240 a b^4 B) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) / \left( 1920 b^2 (a+b) \right. \\
& \left. \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \right) - \\
& \left( (a+b) (-150 a^3 A b - 2840 a A b^3 + 45 a^4 B - 1692 a^2 b^2 B - 1024 b^4 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& 2b(720 A b^4 - 8 a b^3(45 A - 289 B) + 4 a^2 b^2(805 A - 193 B) - 15 a^4 B + a^3(-590 A b + 1146 b B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \right. \\
& \left. \frac{-a+b}{a+b}\right] + 30(-10 a^4 A b + 240 a^2 A b^3 + 96 A b^5 + 3 a^5 B + 40 a^3 b^2 B + 240 a b^4 B) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \left( a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \right) / \\
& \left( 3840 b^2 (a+b)^2 \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \left(\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right)^{3/2} \right) - \\
& \left( (a+b) (-150 a^3 A b - 2840 a A b^3 + 45 a^4 B - 1692 a^2 b^2 B - 1024 b^4 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 b (720 A b^4 - 8 a b^3 (45 A - 289 B) + 4 a^2 b^2 (805 A - 193 B) - 15 a^4 B + a^3 (-590 A b + 1146 b B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \right. \\
& \left. \frac{-a+b}{a+b}\right] + 30 (-10 a^4 A b + 240 a^2 A b^3 + 96 A b^5 + 3 a^5 B + 40 a^3 b^2 B + 240 a b^4 B) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^4} \Big/ \\
& \left(1920 b^2(a+b) \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)^2 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}\right) - \\
& \left(\left((a+b)\left(-150 a^3 A b-2840 a A b^3+45 a^4 B-1692 a^2 b^2 B-1024 b^4 B\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]+ \right. \right. \\
& \left. \left. 2 b(720 A b^4-8 a b^3(45 A-289 B)+4 a^2 b^2(805 A-193 B)-15 a^4 B+a^3(-590 A b+1146 b B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \right. \right. \right. \\
& \left. \left. \frac{-a+b}{a+b}\right]+30(-10 a^4 A b+240 a^2 A b^3+96 A b^5+3 a^5 B+40 a^3 b^2 B+240 a b^4 B) \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]\right) \\
& \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^4} \Big/ \\
& \left(3840 b^2(a+b)\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)^2 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}\right) - \\
& \left((150 a^3 A b+2840 a A b^3-45 a^4 B+1692 a^2 b^2 B+1024 b^4 B) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right. \\
& \left.\left.\left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}+\frac{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right)}{\left(1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right)^2}\right)\right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \left( 3840 b^2 \left( \frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]}{1 - \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{3/2} \right) + \left( (150 a^3 A b + 2840 a A b^3 - 45 a^4 B + 1692 a^2 b^2 B + 1024 b^4 B) \tan\left[\frac{1}{2}(c+dx)\right] \right. \\
& \left. \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - b \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} - \right. \\
& \left. \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left( a + b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)}{\left( 1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^2} \right) \Bigg) / \\
& \left( 3840 b^2 \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \\
& \left( (a+b) (-150 a^3 A b - 2840 a A b^3 + 45 a^4 B - 1692 a^2 b^2 B - 1024 b^4 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& \left. 2 b (720 A b^4 - 8 a b^3 (45 A - 289 B) + 4 a^2 b^2 (805 A - 193 B) - 15 a^4 B + a^3 (-590 A b + 1146 b B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \right. \right. \\
& \left. \left. \frac{-a+b}{a+b}\right] + 30 (-10 a^4 A b + 240 a^2 A b^3 + 96 A b^5 + 3 a^5 B + 40 a^3 b^2 B + 240 a b^4 B) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^4} \left( \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - b \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} - \right. \\
& \left. \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left( a + b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)}{\left( 1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^2} \right) \Bigg) / \left( 3840 b^2 (a+b) \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) \\
& \left( -1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \\
& \left( \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^4} \left( b (720 A b^4 - 8 a b^3 (45 A - 289 B) + 4 a^2 b^2 (805 A - 193 B) - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left( \frac{15 a^4 B + a^3 (-590 A b + 1146 b B) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1 - \frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) - \right. \\
& \left. \frac{15 (-10 a^4 A b + 240 a^2 A b^3 + 96 A b^5 + 3 a^5 B + 40 a^3 b^2 B + 240 a b^4 B) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} (1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2)} \sqrt{1 - \frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) + \left( (a+b) (-150 a^3 A b - 2840 a A b^3 + \right. \\
& \left. 45 a^4 B - 1692 a^2 b^2 B - 1024 b^4 B) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) / \left( 2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) \right) \Bigg) / \\
& \left( \frac{1920 b^2 (a+b) \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} (-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2) \sqrt{\frac{a+b + a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg)
\end{aligned}$$

■ **Problem 614: Unable to integrate problem.**

$$\int \frac{(A + B \cos[c + dx]) \operatorname{Sec}[c + dx]^{7/2}}{\sqrt{a + b \cos[c + dx]}} dx$$

Optimal (type 4, 403 leaves, 6 steps):

$$\begin{aligned}
& \frac{1}{15 a^4 d \sqrt{\operatorname{Sec}[c + dx]}} 2 (a - b) \sqrt{a + b} (9 a^2 A + 8 A b^2 - 10 a b B) \sqrt{\cos[c + dx]} \\
& \operatorname{Csc}[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + dx])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + dx])}{a - b}} - \\
& \frac{1}{15 a^3 d \sqrt{\operatorname{Sec}[c + dx]}} 2 \sqrt{a + b} (8 A b^2 + a^2 (9 A - 5 B) - 2 a b (A + 5 B)) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + dx])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + dx])}{a - b}} - \\
& \frac{2 (4 A b - 5 a B) \sqrt{a + b \cos[c + dx]} \operatorname{Sec}[c + dx]^{3/2} \sin[c + dx]}{15 a^2 d} + \frac{2 A \sqrt{a + b \cos[c + dx]} \operatorname{Sec}[c + dx]^{5/2} \sin[c + dx]}{5 a d}
\end{aligned}$$

Result (type 8, 37 leaves) :

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]^{7/2}}{\sqrt{a + b \cos[c + dx]}} dx$$

■ **Problem 615: Unable to integrate problem.**

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]^{5/2}}{\sqrt{a + b \cos[c + dx]}} dx$$

Optimal (type 4, 330 leaves, 5 steps) :

$$\begin{aligned} & -\frac{1}{3a^3 d \sqrt{\sec[c + dx]}} 2(a - b) \sqrt{a + b} (2Ab - 3aB) \sqrt{\cos[c + dx]} \csc[c + dx] \\ & \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \frac{1}{3a^2 d \sqrt{\sec[c + dx]}} \\ & 2\sqrt{a + b} (2Ab + a(A - 3B)) \sqrt{\cos[c + dx]} \csc[c + dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \\ & \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \frac{2A\sqrt{a + b \cos[c + dx]} \sec[c + dx]^{3/2} \sin[c + dx]}{3ad} \end{aligned}$$

Result (type 8, 37 leaves) :

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]^{5/2}}{\sqrt{a + b \cos[c + dx]}} dx$$

■ **Problem 616: Unable to integrate problem.**

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]^{3/2}}{\sqrt{a + b \cos[c + dx]}} dx$$

Optimal (type 4, 270 leaves, 4 steps) :

$$\begin{aligned} & \frac{1}{a^2 d \sqrt{\sec[c + dx]}} 2A(a - b) \sqrt{a + b} \sqrt{\cos[c + dx]} \csc[c + dx] \\ & \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \frac{1}{ad \sqrt{\sec[c + dx]}} \\ & 2\sqrt{a + b} (A - B) \sqrt{\cos[c + dx]} \csc[c + dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} \end{aligned}$$



Result (type 8, 37 leaves) :

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]^{3/2}}{\sqrt{a + b \cos[c + dx]}} dx$$

■ **Problem 618: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx]}{\sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]}} dx$$

Optimal (type 4, 487 leaves, 8 steps) :

$$-\frac{1}{abd\sqrt{\sec[c + dx]}}(a - b)\sqrt{a + b}B\sqrt{\cos[c + dx]}\csc[c + dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \frac{1}{bd\sqrt{\sec[c + dx]}}$$

$$\sqrt{a + b}B\sqrt{\cos[c + dx]}\csc[c + dx]\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} -$$

$$\frac{1}{b^2 d \sqrt{\sec[c + dx]}} \sqrt{a + b} (2Ab - aB) \sqrt{\cos[c + dx]}\csc[c + dx]\text{EllipticPi}\left[\frac{a + b}{b}, \text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right]$$

$$\sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \frac{B \sin[c + dx]}{d \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]}} + \frac{aB \sqrt{\sec[c + dx]} \sin[c + dx]}{bd \sqrt{a + b \cos[c + dx]}}$$

Result (type 4, 1091 leaves) :

$$\sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}}$$

$$\left( a \sqrt{\frac{a - b}{a + b}} B \tan\left[\frac{1}{2}(c + dx)\right] + b \sqrt{\frac{a - b}{a + b}} B \tan\left[\frac{1}{2}(c + dx)\right] - 2b \sqrt{\frac{a - b}{a + b}} B \tan\left[\frac{1}{2}(c + dx)\right]^3 - a \sqrt{\frac{a - b}{a + b}} B \tan\left[\frac{1}{2}(c + dx)\right]^5 + \right.$$

$$\left. b \sqrt{\frac{a - b}{a + b}} B \tan\left[\frac{1}{2}(c + dx)\right]^5 - 4i a B \text{EllipticPi}\left[\frac{a + b}{a - b}, i \text{ArcSinh}\left[\sqrt{\frac{a - b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]\right], -\frac{a + b}{a - b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \right.$$

$$\left. \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} + 2i a B \text{EllipticPi}\left[\frac{a + b}{a - b}, i \text{ArcSinh}\left[\sqrt{\frac{a - b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]\right], -\frac{a + b}{a - b}\right] \right)$$

$$\begin{aligned}
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 4i \operatorname{AbEllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2i \operatorname{aB EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& i(a-b) \operatorname{B EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2i \operatorname{(Ab-aB) EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \\
& \left. \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) / \\
& \left( b \sqrt{\frac{a-b}{a+b}} d \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left( b \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - a \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \right) \right)
\end{aligned}$$

■ **Problem 619: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c+dx]}{\sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 539 leaves, 8 steps):

$$\begin{aligned}
& - \frac{1}{4 a b^2 d \sqrt{\operatorname{Sec}[c+d x]}} (a-b) \sqrt{a+b} (4 A b-3 a B) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{4 b^2 d \sqrt{\operatorname{Sec}[c+d x]}} \\
& \sqrt{a+b} (4 A b-3 a B+2 b B) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{4 b^3 d \sqrt{\operatorname{Sec}[c+d x]}} \sqrt{a+b} (4 a A b-3 a^2 B-4 b^2 B) \sqrt{\operatorname{Cos}[c+d x]} \\
& \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \\
& \frac{B \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{2 b d \sqrt{\operatorname{Sec}[c+d x]}} + \frac{(4 A b-3 a B) \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 b^2 d}
\end{aligned}$$

Result (type 4, 1169 leaves):

$$\begin{aligned}
& \frac{B \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[2(c+d x)]}{4 b d} + \left( \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right. \\
& \left( -4 a A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 4 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 3 a^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 3 a b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 8 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - \right. \\
& \left. 6 a b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 4 a A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 4 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 3 a^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 3 a b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \right. \\
& \left. 8 a A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& \left. 6 a^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \right. \\
& \left. 8 b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \right.
\end{aligned}$$

$$\begin{aligned}
& 8 a A b \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 6 a^2 B \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 8 b^2 B \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (-4 A b + 3 a B) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2 (a-2 b) b B \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(4 b^2 d \sqrt{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(b \left(-1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) - a \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right)
\end{aligned}$$

■ **Problem 620: Attempted integration timed out after 120 seconds.**

$$\int \frac{(A + B \text{Cos}[c + dx]) \text{Sec}[c + dx]^{5/2}}{(a + b \text{Cos}[c + dx])^{3/2}} dx$$

Optimal (type 4, 433 leaves, 6 steps):

$$\begin{aligned}
& - \left( 2 (5 a^2 A b - 8 A b^3 - 3 a^3 B + 6 a b^2 B) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \right. \\
& \quad \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + d x])}{a + b}} \sqrt{\frac{a(1 + \sec[c + d x])}{a - b}} \right) / (3 a^4 \sqrt{a + b} d \sqrt{\sec[c + d x]}) + \\
& \left( 2 (a + 2 b) (4 A b + a (A - 3 B)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1 - \sec[c + d x])}{a + b}} \sqrt{\frac{a(1 + \sec[c + d x])}{a - b}} \right) / (3 a^3 \sqrt{a + b} d \sqrt{\sec[c + d x]}) + \\
& \frac{2 b (A b - a B) \sec[c + d x]^{3/2} \sin[c + d x]}{a (a^2 - b^2) d \sqrt{a + b \cos[c + d x]}} + \frac{2 (a^2 A - 4 A b^2 + 3 a b B) \sqrt{a + b \cos[c + d x]} \sec[c + d x]^{3/2} \sin[c + d x]}{3 a^2 (a^2 - b^2) d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 621: Unable to integrate problem.**

$$\int \frac{(A + B \cos[c + d x]) \sec[c + d x]^{3/2}}{(a + b \cos[c + d x])^{3/2}} dx$$

Optimal (type 4, 345 leaves, 5 steps):

$$\begin{aligned}
& \left( 2 (a^2 A - 2 A b^2 + a b B) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \right. \\
& \quad \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + d x])}{a + b}} \sqrt{\frac{a(1 + \sec[c + d x])}{a - b}} \right) / (a^3 \sqrt{a + b} d \sqrt{\sec[c + d x]}) - \\
& \left( 2 (2 A b + a (A - B)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + d x])}{a + b}} \right. \\
& \quad \left. \sqrt{\frac{a(1 + \sec[c + d x])}{a - b}} \right) / (a^2 \sqrt{a + b} d \sqrt{\sec[c + d x]}) + \frac{2 b (A b - a B) \sqrt{\sec[c + d x]} \sin[c + d x]}{a (a^2 - b^2) d \sqrt{a + b \cos[c + d x]}}
\end{aligned}$$

Result (type 8, 37 leaves) :

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]^{3/2}}{(a + b \cos[c + dx])^{3/2}} dx$$

■ **Problem 622: Unable to integrate problem.**

$$\int \frac{(A + B \cos[c + dx]) \sqrt{\sec[c + dx]}}{(a + b \cos[c + dx])^{3/2}} dx$$

Optimal (type 4, 324 leaves, 5 steps) :

$$\left( 2 (Ab - aB) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} \right) /$$

$$\left( a^2 \sqrt{a + b} d \sqrt{\sec[c + dx]} \right) + \frac{1}{a \sqrt{a + b} d \sqrt{\sec[c + dx]}}$$

$$2 (A + B) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} -$$

$$\frac{2 (Ab - aB) \sqrt{\sec[c + dx]} \operatorname{Sin}[c + dx]}{(a^2 - b^2) d \sqrt{a + b \cos[c + dx]}}$$

Result (type 8, 37 leaves) :

$$\int \frac{(A + B \cos[c + dx]) \sqrt{\sec[c + dx]}}{(a + b \cos[c + dx])^{3/2}} dx$$

■ **Problem 623: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx]}{(a + b \cos[c + dx])^{3/2} \sqrt{\sec[c + dx]}} dx$$

Optimal (type 4, 476 leaves, 7 steps) :

$$\begin{aligned}
& - \left( 2 (A b - a B) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} \right) / \\
& \quad \left( a b \sqrt{a+b} d \sqrt{\operatorname{Sec}[c+d x]} \right) + \\
& \left( 2 (A b - a B) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} \right) / \\
& \quad \left( a b \sqrt{a+b} d \sqrt{\operatorname{Sec}[c+d x]} \right) - \frac{1}{b^2 d \sqrt{\operatorname{Sec}[c+d x]}} \\
& 2 \sqrt{a+b} B \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \quad \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{2 a (A b - a B) \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{b \left(a^2 - b^2\right) d \sqrt{a+b \cos [c+d x]}}
\end{aligned}$$

Result (type 4, 1403 leaves):

$$\begin{aligned}
& \frac{\sqrt{a+b \cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \left( \frac{2 (A b - a B) \operatorname{Sin}[c+d x]}{b \left(-a^2 + b^2\right)} - \frac{2 (a A b \operatorname{Sin}[c+d x] - a^2 B \operatorname{Sin}[c+d x])}{b \left(-a^2 + b^2\right) (a+b \cos [c+d x])} \right)}{d} + \\
& \left( 2 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \left( a A b \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + A b^2 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \right. \right. \\
& \quad a^2 \sqrt{\frac{a-b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a b \sqrt{\frac{a-b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 2 A b^2 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 2 a b \sqrt{\frac{a-b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - \\
& \quad a A b \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + A b^2 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + a^2 \sqrt{\frac{a-b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - a b \sqrt{\frac{a-b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \\
& \quad \left. \left. 2 i a^2 B \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \right. \right. \\
& \quad \left. \left. \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 2 i b^2 B \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2i a^2 \text{B EllipticPi}\left[\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2i b^2 \text{B EllipticPi}\left[\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& i(a-b)(-Ab+aB) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + i(a-b)(-Ab+(2a+b)B) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \\
& \left. \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) / \\
& \left( b \sqrt{\frac{a-b}{a+b}} (a^2 - b^2) d \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(b \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - a \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right) \right)
\end{aligned}$$

■ **Problem 624: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c+dx]}{(a+b \cos[c+dx])^{3/2} \sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 560 leaves, 8 steps):



$$\begin{aligned}
& \left( (2 a A b - 3 a^2 B + b^2 B) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / (a b^2 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]}) - \\
& \left( (2 A b - (3 a + b) B) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / (b^2 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]}) - \frac{1}{b^3 d \sqrt{\operatorname{Sec}[c + d x]}} \\
& \sqrt{a + b} (2 A b - 3 a B) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticPi}\left[\frac{a + b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b}}\right], -\frac{a + b}{a - b}\right] \\
& \quad \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} + \frac{2 a (A b - a B) \sin[c + d x]}{b (a^2 - b^2) d \sqrt{a + b \cos[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}} - \\
& \quad \frac{(2 a A b - 3 a^2 B + b^2 B) \sqrt{a + b \cos[c + d x]} \sqrt{\operatorname{Sec}[c + d x]} \sin[c + d x]}{b^2 (a^2 - b^2) d}
\end{aligned}$$

Result (type 4, 1567 leaves):

$$\begin{aligned}
& \frac{\sqrt{a + b \cos[c + d x]} \sqrt{\operatorname{Sec}[c + d x]} \left( -\frac{2 a (-A b + a B) \sin[c + d x]}{b^2 (a^2 - b^2)} + \frac{2 (a^2 A b \sin[c + d x] - a^3 B \sin[c + d x])}{b^2 (-a^2 + b^2) (a + b \cos[c + d x])} \right)}{d} - \\
& \left( \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + d x)\right]^2}} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + d x)\right]^2 - b \tan\left[\frac{1}{2}(c + d x)\right]^2}{1 + \tan\left[\frac{1}{2}(c + d x)\right]^2}} \right. \\
& \quad \left( 2 a^2 A b \tan\left[\frac{1}{2}(c + d x)\right] + 2 a A b^2 \tan\left[\frac{1}{2}(c + d x)\right] - 3 a^3 B \tan\left[\frac{1}{2}(c + d x)\right] - 3 a^2 b B \tan\left[\frac{1}{2}(c + d x)\right] + a b^2 B \tan\left[\frac{1}{2}(c + d x)\right] + \right. \\
& \quad \left. b^3 B \tan\left[\frac{1}{2}(c + d x)\right] - 4 a A b^2 \tan\left[\frac{1}{2}(c + d x)\right]^3 + 6 a^2 b B \tan\left[\frac{1}{2}(c + d x)\right]^3 - 2 b^3 B \tan\left[\frac{1}{2}(c + d x)\right]^3 - 2 a^2 A b \tan\left[\frac{1}{2}(c + d x)\right]^5 + \right. \\
& \quad \left. 2 a A b^2 \tan\left[\frac{1}{2}(c + d x)\right]^5 + 3 a^3 B \tan\left[\frac{1}{2}(c + d x)\right]^5 - 3 a^2 b B \tan\left[\frac{1}{2}(c + d x)\right]^5 - a b^2 B \tan\left[\frac{1}{2}(c + d x)\right]^5 + b^3 B \tan\left[\frac{1}{2}(c + d x)\right]^5 + \right.
\end{aligned}$$

$$\begin{aligned}
& 4 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 4 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 6 a^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 6 a b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 4 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 4 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 6 a^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 6 a b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& (a+b)(-2 a A b+3 a^2 B-b^2 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)^2 \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 2 b(a+b)(-A b+a B) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]
\end{aligned}$$

$$\left. \left( \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) /$$

$$\left( b^2(-a^2+b^2)d \sqrt{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \left( b \left( -1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) - a \left( 1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right)$$

■ **Problem 625: Attempted integration timed out after 120 seconds.**

$$\int \frac{(A+B\cos[c+dx]) \operatorname{Sec}[c+dx]^{5/2}}{(a+b\cos[c+dx])^{5/2}} dx$$

Optimal (type 4, 607 leaves, 7 steps):

$$- \left( 2 \left( 8a^4Ab - 28a^2Ab^3 + 16Ab^5 - 3a^5B + 15a^3b^2B - 8ab^4B \right) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \right.$$

$$\left. \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} \right) / \left( 3a^5(a-b)(a+b)^{3/2}d\sqrt{\operatorname{Sec}[c+dx]} \right) -$$

$$\left( 2 \left( 16Ab^4 - a^4(A-3B) + 4ab^3(3A-2B) - 9a^3b(A-B) - 2a^2b^2(8A+3B) \right) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \right.$$

$$\left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} \right) /$$

$$\left( 3a^4\sqrt{a+b}(a^2-b^2)d\sqrt{\operatorname{Sec}[c+dx]} \right) + \frac{2b(Ab-aB)\operatorname{Sec}[c+dx]^{3/2}\operatorname{Sin}[c+dx]}{3a(a^2-b^2)d(a+b\cos[c+dx])^{3/2}} +$$

$$\frac{2b(10a^2Ab-6Ab^3-7a^3B+3ab^2B)\operatorname{Sec}[c+dx]^{3/2}\operatorname{Sin}[c+dx]}{3a^2(a^2-b^2)^2d\sqrt{a+b\cos[c+dx]}} +$$

$$\frac{2(a^4A-13a^2Ab^2+8Ab^4+8a^3bB-4ab^3B)\sqrt{a+b\cos[c+dx]}\operatorname{Sec}[c+dx]^{3/2}\operatorname{Sin}[c+dx]}{3a^3(a^2-b^2)^2d}$$

Result (type 1, 1 leaves):

???

■ **Problem 626: Attempted integration timed out after 120 seconds.**

$$\int \frac{(A + B \cos[c + dx]) \sec[c + dx]^{3/2}}{(a + b \cos[c + dx])^{5/2}} dx$$

Optimal (type 4, 496 leaves, 6 steps):

$$\left( 2 \left( 3 a^4 A - 15 a^2 A b^2 + 8 A b^4 + 6 a^3 b B - 2 a b^3 B \right) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \right. \\ \left. \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} \right) / \left( 3 a^4 (a - b) (a + b)^{3/2} d \sqrt{\sec[c + dx]} \right) + \\ \left( 2 \left( 8 A b^3 - 3 a^3 (A - B) + 2 a b^2 (3 A - B) - 3 a^2 b (3 A + B) \right) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \right. \\ \left. \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} \right) / \left( 3 a^3 \sqrt{a + b} (a^2 - b^2) d \sqrt{\sec[c + dx]} \right) + \\ \frac{2 b (A b - a B) \sqrt{\sec[c + dx]} \operatorname{Sin}[c + dx]}{3 a (a^2 - b^2) d (a + b \cos[c + dx])^{3/2}} + \frac{2 b (8 a^2 A b - 4 A b^3 - 5 a^3 B + a b^2 B) \sqrt{\sec[c + dx]} \operatorname{Sin}[c + dx]}{3 a^2 (a^2 - b^2)^2 d \sqrt{a + b \cos[c + dx]}}$$

Result (type 1, 1 leaves):

???

■ **Problem 627: Unable to integrate problem.**

$$\int \frac{(A + B \cos[c + dx]) \sqrt{\sec[c + dx]}}{(a + b \cos[c + dx])^{5/2}} dx$$

Optimal (type 4, 469 leaves, 6 steps):

$$\begin{aligned}
& \left( 2 (6 a^2 A b - 2 A b^3 - 3 a^3 B - a b^2 B) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} \right) / \left( 3 a^3 (a-b) (a+b)^{3/2} d \sqrt{\sec [c+d x]} \right) - \\
& \left( 2 (2 A b^2 - 3 a^2 (A+B) + a b (3 A+B)) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} \right) / \left( 3 a^2 \sqrt{a+b} (a^2-b^2) d \sqrt{\sec [c+d x]} \right) + \\
& \frac{2 b (A b - a B) \sin [c+d x]}{3 a (a^2-b^2) d (a+b \cos [c+d x])^{3/2} \sqrt{\sec [c+d x]}} - \frac{2 (6 a^2 A b - 2 A b^3 - 3 a^3 B - a b^2 B) \sqrt{\sec [c+d x]} \sin [c+d x]}{3 a (a^2-b^2)^2 d \sqrt{a+b \cos [c+d x]}}
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \frac{(A+B \cos [c+d x]) \sqrt{\sec [c+d x]}}{(a+b \cos [c+d x])^{5/2}} dx$$

■ **Problem 628: Unable to integrate problem.**

$$\int \frac{A+B \cos [c+d x]}{(a+b \cos [c+d x])^{5/2} \sqrt{\sec [c+d x]}} dx$$

Optimal (type 4, 431 leaves, 6 steps):

$$\begin{aligned}
& - \left( 2 (3 a^2 A + A b^2 - 4 a b B) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a^2 (a - b) (a + b)^{3/2} d \sqrt{\operatorname{Sec}[c + d x]} \right) + \\
& \left( 2 (a (3 A + B) - b (A + 3 B)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a (a - b) (a + b)^{3/2} d \sqrt{\operatorname{Sec}[c + d x]} \right) - \\
& \frac{2 (A b - a B) \sin[c + d x]}{3 (a^2 - b^2) d (a + b \cos[c + d x])^{3/2} \sqrt{\operatorname{Sec}[c + d x]}} + \frac{2 (3 a^2 A + A b^2 - 4 a b B) \sqrt{\operatorname{Sec}[c + d x]} \sin[c + d x]}{3 (a^2 - b^2)^2 d \sqrt{a + b \cos[c + d x]}}
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \frac{A + B \cos[c + d x]}{(a + b \cos[c + d x])^{5/2} \sqrt{\operatorname{Sec}[c + d x]}} dx$$

- **Problem 629: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + d x]}{(a + b \cos[c + d x])^{5/2} \operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 4, 602 leaves, 8 steps):

$$\begin{aligned}
& \left( 2 (4 A b^3 + 3 a^3 B - 7 a b^2 B) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} \right) / \left( 3 a (a-b) b^2 (a+b)^{3/2} d \sqrt{\sec[c+dx]} \right) - \\
& \left( 2 (3 A b^3 + 3 a^3 B + a^2 b B - a b^2 (A+6 B)) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} \right) / \left( 3 a (a-b) b^2 (a+b)^{3/2} d \sqrt{\sec[c+dx]} \right) - \\
& \frac{1}{b^3 d \sqrt{\sec[c+dx]}} 2 \sqrt{a+b} B \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \quad \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{2 a (A b - a B) \sin[c+dx]}{3 b (a^2 - b^2) d (a+b \cos[c+dx])^{3/2} \sqrt{\sec[c+dx]}} - \frac{2 a (4 A b^3 + 3 a^3 B - 7 a b^2 B) \sqrt{\sec[c+dx]} \sin[c+dx]}{3 b^2 (a^2 - b^2)^2 d \sqrt{a+b \cos[c+dx]}}
\end{aligned}$$

Result (type 4, 1994 leaves) :

$$\begin{aligned}
& \frac{1}{d} \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]} \left( \frac{2 (4 A b^3 + 3 a^3 B - 7 a b^2 B) \sin[c+dx]}{3 b^2 (-a^2 + b^2)^2} - \right. \\
& \quad \left. \frac{2 (-a^2 A b \sin[c+dx] + a^3 B \sin[c+dx])}{3 b^2 (-a^2 + b^2) (a+b \cos[c+dx])^2} - \frac{2 (-a^3 A b \sin[c+dx] + 5 a A b^3 \sin[c+dx] + 4 a^4 B \sin[c+dx] - 8 a^2 b^2 B \sin[c+dx])}{3 b^2 (-a^2 + b^2)^2 (a+b \cos[c+dx])} \right) - \\
& \left( 2 \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left( 4 a A b^3 \sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right] + 4 A b^4 \sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right] \right) + \right. \\
& \quad \left. 3 a^4 \sqrt{\frac{a-b}{a+b}} B \tan\left[\frac{1}{2}(c+dx)\right] + 3 a^3 b \sqrt{\frac{a-b}{a+b}} B \tan\left[\frac{1}{2}(c+dx)\right] - 7 a^2 b^2 \sqrt{\frac{a-b}{a+b}} B \tan\left[\frac{1}{2}(c+dx)\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& 7 a b^3 \sqrt{\frac{a-b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right] - 8 A b^4 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 - 6 a^3 b \sqrt{\frac{a-b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right]^3 + \\
& 14 a b^3 \sqrt{\frac{a-b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right]^3 - 4 a A b^3 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 4 A b^4 \sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - \\
& 3 a^4 \sqrt{\frac{a-b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 3 a^3 b \sqrt{\frac{a-b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 7 a^2 b^2 \sqrt{\frac{a-b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right]^5 - \\
& 7 a b^3 \sqrt{\frac{a-b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 6 i a^4 \operatorname{B EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 12 i a^2 b^2 \operatorname{B EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 6 i b^4 \operatorname{B EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 6 i a^4 \operatorname{B EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 12 i a^2 b^2 \operatorname{B EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 6 i b^4 \operatorname{B EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} +
\end{aligned}$$



$$i (a - b) (4 A b^3 + 3 a^3 B - 7 a b^2 B) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{a - b}{a + b}} \tan\left[\frac{1}{2} (c + d x)\right]\right], -\frac{a + b}{a - b}\right]$$

$$\sqrt{1 - \tan\left[\frac{1}{2} (c + d x)\right]^2} \left(1 + \tan\left[\frac{1}{2} (c + d x)\right]^2\right) \sqrt{\frac{a + b + a \tan\left[\frac{1}{2} (c + d x)\right]^2 - b \tan\left[\frac{1}{2} (c + d x)\right]^2}{a + b}} -$$

$$i (a - b) (3 b^3 (A - B) + 6 a^3 B + 4 a^2 b B - a b^2 (A + 9 B)) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{a - b}{a + b}} \tan\left[\frac{1}{2} (c + d x)\right]\right], -\frac{a + b}{a - b}\right]$$

$$\sqrt{1 - \tan\left[\frac{1}{2} (c + d x)\right]^2} \left(1 + \tan\left[\frac{1}{2} (c + d x)\right]^2\right) \sqrt{\frac{a + b + a \tan\left[\frac{1}{2} (c + d x)\right]^2 - b \tan\left[\frac{1}{2} (c + d x)\right]^2}{a + b}} \Bigg) /$$

$$\left(3 b^2 \sqrt{\frac{a - b}{a + b}} (a^2 - b^2)^2 d \left(-1 + \tan\left[\frac{1}{2} (c + d x)\right]^2\right) \sqrt{\frac{1 + \tan\left[\frac{1}{2} (c + d x)\right]^2}{1 - \tan\left[\frac{1}{2} (c + d x)\right]^2}} \left(b \left(-1 + \tan\left[\frac{1}{2} (c + d x)\right]^2\right) - a \left(1 + \tan\left[\frac{1}{2} (c + d x)\right]^2\right)\right)\right)$$

■ **Problem 630: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + d x]}{(a + b \cos[c + d x])^{5/2} \sec[c + d x]^{5/2}} dx$$

Optimal (type 4, 733 leaves, 9 steps):

$$\begin{aligned}
& \left( (6 a^3 A b - 14 a A b^3 - 15 a^4 B + 26 a^2 b^2 B - 3 b^4 B) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} \right) / (3 a(a-b) b^3 (a+b)^{3/2} d \sqrt{\sec[c+dx]}) + \\
& \left( (3 b^3 (4 A - B) + 15 a^3 B - a b^2 (2 A + 21 B) - a^2 (6 A b - 5 b B)) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} \right) / (3(a-b) b^3 (a+b)^{3/2} d \sqrt{\sec[c+dx]}) - \frac{1}{b^4 d \sqrt{\sec[c+dx]}} \\
& \sqrt{a+b} (2 A b - 5 a B) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \quad \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \quad \frac{2 a (A b - a B) \sin[c+dx]}{3 b (a^2 - b^2) d (a+b \cos[c+dx])^{3/2} \sec[c+dx]^{3/2}} + \frac{2 a (2 a^2 A b - 6 A b^3 - 5 a^3 B + 9 a b^2 B) \sin[c+dx]}{3 b^2 (a^2 - b^2)^2 d \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]}} - \\
& \quad \frac{(6 a^3 A b - 14 a A b^3 - 15 a^4 B + 26 a^2 b^2 B - 3 b^4 B) \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]} \sin[c+dx]}{3 b^3 (a^2 - b^2)^2 d}
\end{aligned}$$

Result (type 4, 2342 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]} \left( -\frac{2 a (-3 a^2 A b + 7 A b^3 + 6 a^3 B - 10 a b^2 B) \sin[c+dx]}{3 b^3 (a^2 - b^2)^2} + \right. \\
& \quad \left. \frac{2 (-a^3 A b \sin[c+dx] + a^4 B \sin[c+dx])}{3 b^3 (-a^2 + b^2) (a+b \cos[c+dx])^2} + \frac{2 (-4 a^4 A b \sin[c+dx] + 8 a^2 A b^3 \sin[c+dx] + 7 a^5 B \sin[c+dx] - 11 a^3 b^2 B \sin[c+dx])}{3 b^3 (-a^2 + b^2)^2 (a+b \cos[c+dx])} \right) + \\
& \left( \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( 6 a^4 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 6 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 14 a^2 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 14 a A b^4 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 15 a^5 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \right. \\
& 15 a^4 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 26 a^3 b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 26 a^2 b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 3 a b^4 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 3 b^5 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \\
& 12 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 28 a A b^4 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 30 a^4 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - 52 a^2 b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 6 b^5 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - \\
& 6 a^4 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 6 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 14 a^2 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 14 a A b^4 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 15 a^5 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \\
& 15 a^4 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 26 a^3 b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 26 a^2 b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 3 a b^4 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 3 b^5 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + \\
& 12 a^4 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 24 a^2 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 12 A b^5 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 30 a^5 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 60 a^3 b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 30 a b^4 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 12 a^4 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \left. \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 24 a^2 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 12Ab^5 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 30a^5 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 60a^3 b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 30a^4 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& (a+b) (-6a^3 Ab + 14aAb^3 + 15a^4 B - 26a^2 b^2 B + 3b^4 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 2b(a+b) (3Ab^3 + 3ab^2(A-2B) + 5a^3 B - a^2 b(2A+3B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \left. \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) \Bigg/ \\
& \left( 3b^3 (a^2 - b^2)^2 d \sqrt{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \left( b \left( -1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) - a \left( 1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right)
\end{aligned}$$

■ **Problem 631: Unable to integrate problem.**

$$\int \frac{(aB + bB \cos[c+dx]) \sec[c+dx]^{3/2}}{(a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 266 leaves, 5 steps) :

$$\frac{1}{a^2 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} B \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{a d \sqrt{\sec[c+dx]}}$$

$$2 \sqrt{a+b} B \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}$$

Result (type 8, 40 leaves) :

$$\int \frac{(a B + b B \cos[c+dx]) \sec[c+dx]^{3/2}}{(a+b \cos[c+dx])^{3/2}} dx$$

■ **Problem 633: Unable to integrate problem.**

$$\int \frac{a B + b B \cos[c+dx]}{(a+b \cos[c+dx])^{3/2} \sqrt{\sec[c+dx]}} dx$$

Optimal (type 4, 137 leaves, 3 steps) :

$$-\frac{1}{b d \sqrt{\sec[c+dx]}}$$

$$2 \sqrt{a+b} B \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}$$

Result (type 8, 40 leaves) :

$$\int \frac{a B + b B \cos[c+dx]}{(a+b \cos[c+dx])^{3/2} \sqrt{\sec[c+dx]}} dx$$

■ **Problem 634: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a B + b B \cos[c+dx]}{(a+b \cos[c+dx])^{3/2} \sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 479 leaves, 10 steps) :

$$\begin{aligned}
& - \frac{1}{a b d \sqrt{\sec[c+d x]}} (a-b) \sqrt{a+b} B \sqrt{\cos[c+d x]} \operatorname{Csc}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+d x]}}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} + \frac{1}{b d \sqrt{\sec[c+d x]}} \\
& \sqrt{a+b} B \sqrt{\cos[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+d x]}}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} + \\
& \frac{1}{b^2 d \sqrt{\sec[c+d x]}} a \sqrt{a+b} B \sqrt{\cos[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+d x]}}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} + \frac{B \sin[c+d x]}{d \sqrt{a+b \cos[c+d x]} \sqrt{\sec[c+d x]}} + \frac{a B \sqrt{\sec[c+d x]} \sin[c+d x]}{b d \sqrt{a+b \cos[c+d x]}}
\end{aligned}$$

Result (type 4, 760 leaves):

$$\begin{aligned}
& \frac{1}{b \sqrt{\frac{a-b}{a+b}} d \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^{3/2} \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}}} B \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left( a \sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} + b \sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} + \right. \\
& \left. a \sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} - b \sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} + \right. \\
& \left. 2 i a \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \right. \\
& \left. 2 i a \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \right. \\
& \left. i (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \right. \\
& \left. 2 i a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right)
\end{aligned}$$

■ **Problem 637: Result more than twice size of optimal antiderivative.**

$$\int (a+b \cos[ex+fx])^3 (A+B \cos[ex+fx]) (c \sec[ex+fx])^m dx$$

Optimal (type 5, 455 leaves, 9 steps):

$$\begin{aligned}
& - \left( c^5 (a^3 A (8 - 6m + m^2) + 3 a A b^2 (4 - 5m + m^2) + 3 a^2 b B (4 - 5m + m^2) + b^3 B (3 - 4m + m^2)) \right. \\
& \quad \left. \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5-m}{2}, \frac{7-m}{2}, \text{Cos}[e+f x]^2 \right] (c \text{Sec}[e+f x])^{-5+m} \text{Sin}[e+f x] \right) / \left( f (1-m) (3-m) (5-m) \sqrt{\text{Sin}[e+f x]^2} \right) - \\
& \left( c^4 (A b^3 (2-m) + 3 a b^2 B (2-m) + 3 a^2 A b (3-m) + a^3 B (3-m)) \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{4-m}{2}, \frac{6-m}{2}, \text{Cos}[e+f x]^2 \right] \right. \\
& \quad \left. (c \text{Sec}[e+f x])^{-4+m} \text{Sin}[e+f x] \right) / \left( f (2-m) (4-m) \sqrt{\text{Sin}[e+f x]^2} \right) - \\
& \frac{a c^4 (3 a b B (1-m) + a^2 A (2-m) - 2 A b^2 m) (c \text{Sec}[e+f x])^{-4+m} \text{Tan}[e+f x]}{f (1-m) (3-m)} - \\
& \frac{a^2 c^4 (a B (1-m) - A b (1+m)) \text{Sec}[e+f x] (c \text{Sec}[e+f x])^{-4+m} \text{Tan}[e+f x]}{f (1-m) (2-m)} - \\
& \frac{a A c^4 (c \text{Sec}[e+f x])^{-4+m} (b + a \text{Sec}[e+f x])^2 \text{Tan}[e+f x]}{f (1-m)}
\end{aligned}$$

Result (type 5, 966 leaves):



$$\begin{aligned}
& \left( 8 \cos[e + f x]^4 (c \sec[e + f x])^m (\sec[e + f x]^2)^{\frac{1-m}{2}} (b + a \sec[e + f x])^3 (B + A \sec[e + f x]) \right. \\
& \left( a^3 A \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 3 - \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] \tan[e + f x] + 3 a A b^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 3 - \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] \tan[e + f x] + \right. \\
& 3 a^2 b B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 3 - \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] \tan[e + f x] + b^3 B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 3 - \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] \tan[e + f x] + \\
& 3 a^2 A b \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{7}{2} - \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] \tan[e + f x] + \\
& A b^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{7}{2} - \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] \tan[e + f x] + a^3 B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{7}{2} - \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] \tan[e + f x] + \\
& 3 a b^2 B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{7}{2} - \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] \tan[e + f x] + \frac{2}{3} a^3 A \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, 3 - \frac{m}{2}, \frac{5}{2}, -\tan[e + f x]^2\right] \\
& \tan[e + f x]^3 + a A b^2 \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, 3 - \frac{m}{2}, \frac{5}{2}, -\tan[e + f x]^2\right] \tan[e + f x]^3 + \\
& a^2 b B \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, 3 - \frac{m}{2}, \frac{5}{2}, -\tan[e + f x]^2\right] \tan[e + f x]^3 + 2 a^2 A b \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{7}{2} - \frac{m}{2}, \frac{5}{2}, -\tan[e + f x]^2\right] \\
& \tan[e + f x]^3 + \frac{1}{3} A b^3 \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{7}{2} - \frac{m}{2}, \frac{5}{2}, -\tan[e + f x]^2\right] \tan[e + f x]^3 + \frac{2}{3} a^3 B \\
& \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{7}{2} - \frac{m}{2}, \frac{5}{2}, -\tan[e + f x]^2\right] \tan[e + f x]^3 + a b^2 B \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{7}{2} - \frac{m}{2}, \frac{5}{2}, -\tan[e + f x]^2\right] \tan[e + f x]^3 + \\
& \frac{1}{5} a^3 A \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, 3 - \frac{m}{2}, \frac{7}{2}, -\tan[e + f x]^2\right] \tan[e + f x]^5 + \frac{3}{5} a^2 A b \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, \frac{7}{2} - \frac{m}{2}, \frac{7}{2}, -\tan[e + f x]^2\right] \\
& \tan[e + f x]^5 + \frac{1}{5} a^3 B \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, \frac{7}{2} - \frac{m}{2}, \frac{7}{2}, -\tan[e + f x]^2\right] \tan[e + f x]^5 \left. \right) / \\
& \left( f \left( 24 a^2 A b + 4 A b^3 + 8 a^3 B + 12 a b^2 B + 8 a^3 A \sqrt{\sec[e + f x]^2} + 12 a A b^2 \sqrt{\sec[e + f x]^2} + 12 a^2 b B \sqrt{\sec[e + f x]^2} + \right. \right. \\
& 3 b^3 B \sqrt{\sec[e + f x]^2} + b^3 B \cos[4(e + f x)] \sqrt{\sec[e + f x]^2} + \\
& \left. \left. 4 b \cos[2(e + f x)] \left( A b \left( b + 3 a \sqrt{\sec[e + f x]^2} \right) + B \left( 3 a b + 3 a^2 \sqrt{\sec[e + f x]^2} + b^2 \sqrt{\sec[e + f x]^2} \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 639: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b \cos[e + f x]) (A + B \cos[e + f x]) (c \sec[e + f x])^m dx$$

Optimal (type 5, 217 leaves, 7 steps):

$$\begin{aligned}
& - \left( c^3 (b B (1-m) + a A (2-m)) \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3-m}{2}, \frac{5-m}{2}, \cos[e+fx]^2 \right] (c \operatorname{Sec}[e+fx])^{-3+m} \sin[e+fx] \right) / \\
& \left( f (1-m) (3-m) \sqrt{\sin[e+fx]^2} \right) - \\
& \frac{(A b + a B) c^2 \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{2-m}{2}, \frac{4-m}{2}, \cos[e+fx]^2 \right] (c \operatorname{Sec}[e+fx])^{-2+m} \sin[e+fx]}{f (2-m) \sqrt{\sin[e+fx]^2}} - \frac{a A c^2 (c \operatorname{Sec}[e+fx])^{-2+m} \tan[e+fx]}{f (1-m)}
\end{aligned}$$

Result (type 6, 16794 leaves):

$$\begin{aligned}
& \left( 6 \operatorname{Sec}[e+fx]^{-m} (c \operatorname{Sec}[e+fx])^m \right. \\
& (b B \operatorname{Sec}[e+fx]^{-2+m} + a A \operatorname{Sec}[e+fx]^m + \cos[e+fx] (A b \operatorname{Sec}[e+fx]^m + a B \operatorname{Sec}[e+fx]^m)) \tan \left[ \frac{1}{2} (e+fx) \right] \left( \frac{1}{1 - \tan \left[ \frac{1}{2} (e+fx) \right]^2} \right)^m \\
& \left. \left( 1 + \tan \left[ \frac{1}{2} (e+fx) \right]^2 \right)^{-3+m} \left( \left( a A \operatorname{AppellF1} \left[ \frac{1}{2}, m, 1-m, \frac{3}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \left( 1 + \tan \left[ \frac{1}{2} (e+fx) \right]^2 \right)^2 \right) / \right. \right. \\
& \left. \left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, m, 1-m, \frac{3}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + 2 \left( (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, m, 2-m, \frac{5}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (e+fx) \right]^2 \right) - \right. \\
& \left. \left( A b \operatorname{AppellF1} \left[ \frac{1}{2}, m, 1-m, \frac{3}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \left( 1 + \tan \left[ \frac{1}{2} (e+fx) \right]^2 \right)^2 \right) / \right. \\
& \left. \left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, m, 1-m, \frac{3}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + 2 \left( (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, m, 2-m, \frac{5}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (e+fx) \right]^2 \right) - \right. \\
& \left. \left( a B \operatorname{AppellF1} \left[ \frac{1}{2}, m, 1-m, \frac{3}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \left( 1 + \tan \left[ \frac{1}{2} (e+fx) \right]^2 \right)^2 \right) / \right. \\
& \left. \left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, m, 1-m, \frac{3}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + 2 \left( (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, m, 2-m, \frac{5}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (e+fx) \right]^2 \right) + \right. \\
& \left. \left( b B \operatorname{AppellF1} \left[ \frac{1}{2}, m, 1-m, \frac{3}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \left( 1 + \tan \left[ \frac{1}{2} (e+fx) \right]^2 \right)^2 \right) / \right. \\
& \left. \left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, m, 1-m, \frac{3}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + 2 \left( (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, m, 2-m, \frac{5}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \right. \right.
\end{aligned}$$





























$$\int \frac{(A + B \cos[e + f x]) (c \sec[e + f x])^m}{a + b \cos[e + f x]} dx$$

Optimal (type 6, 299 leaves, 10 steps):

$$\begin{aligned} & - \frac{1}{(a^2 - b^2) c f} (A b - a B) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, \sin[e + f x]^2, -\frac{b^2 \sin[e + f x]^2}{a^2 - b^2} \right] \cos[e + f x] (\cos[e + f x]^2)^{m/2} (c \sec[e + f x])^{1+m} \sin[e + f x] + \\ & \frac{1}{b (a^2 - b^2) c f} a (A b - a B) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1+m}{2}, 1, \frac{3}{2}, \sin[e + f x]^2, -\frac{b^2 \sin[e + f x]^2}{a^2 - b^2} \right] (\cos[e + f x]^2)^{\frac{1+m}{2}} (c \sec[e + f x])^{1+m} \sin[e + f x] - \\ & \frac{B c \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1-m}{2}, \frac{3-m}{2}, \cos[e + f x]^2 \right] (c \sec[e + f x])^{-1+m} \sin[e + f x]}{b f (1-m) \sqrt{\sin[e + f x]^2}} \end{aligned}$$

Result (type 6, 10630 leaves):

$$\begin{aligned} & \left( \sec[e + f x]^{1-m} (c \sec[e + f x])^m \left( \frac{B \sec[e + f x]^{-1+m}}{a + b \cos[e + f x]} + \frac{A \sec[e + f x]^m}{a + b \cos[e + f x]} \right) \sin[e + f x] \left( \frac{A \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{2} - \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2 \right]}{b} - \right. \right. \\ & \frac{a B \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{2} - \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2 \right]}{b^2} + \frac{B \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, 1 - \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2 \right]}{b} + \\ & \left. \left( 3 a^2 A (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1-m), 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{a^2 \tan[e + f x]^2}{-a^2 + b^2} \right] (1 + \tan[e + f x]^2)^{\frac{1+m}{2}} \right) / \right. \\ & \left. \left( b \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1-m), 1, \frac{3}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \right. \\ & \left. \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1-m), 2, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] - \right. \right. \right. \\ & \left. \left. \left. (a^2 - b^2) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1-m}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \tan[e + f x]^2 \right) (-b^2 + a^2 (1 + \tan[e + f x]^2)) \right) \left. \right) - \\ & \left( 3 a^3 (a^2 - b^2) B \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1-m), 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{a^2 \tan[e + f x]^2}{-a^2 + b^2} \right] (1 + \tan[e + f x]^2)^{\frac{1+m}{2}} \right) / \\ & \left( b^2 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1-m), 1, \frac{3}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\ & \left. \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1-m), 2, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] - \right. \right. \right. \\ & \left. \left. \left. (a^2 - b^2) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1-m}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \tan[e + f x]^2 \right) (-b^2 + a^2 (1 + \tan[e + f x]^2)) \right) \left. \right) + \\ & \left( 3 a A (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] (1 + \tan[e + f x]^2)^{m/2} \right) / \end{aligned}$$



$$\begin{aligned}
& \left( \left( 3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - \right. \\
& \quad \left. \left. 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \left( -b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2) \right) \right) - \\
& \left( 3 a^2 (a^2 - b^2) \operatorname{B AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{a^2 \operatorname{Tan}[e + f x]^2}{-a^2 + b^2} \right] (1 + \operatorname{Tan}[e + f x]^2)^{m/2} \right) / \\
& \left( b \left( 3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - \right. \\
& \quad \left. \left. 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \left( -b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2) \right) \right) \right) / \\
& \left( f \left( \operatorname{Sec}[e + f x]^2 \left( \frac{\operatorname{A Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{2} - \frac{m}{2}, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2 \right]}{b} - \frac{a \operatorname{B Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{2} - \frac{m}{2}, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2 \right]}{b^2} \right) + \right. \right. \\
& \quad \left. \frac{\operatorname{B Hypergeometric2F1} \left[ \frac{1}{2}, 1 - \frac{m}{2}, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2 \right]}{b} + \right. \\
& \quad \left( 3 a^2 \operatorname{A} (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{a^2 \operatorname{Tan}[e + f x]^2}{-a^2 + b^2} \right] (1 + \operatorname{Tan}[e + f x]^2)^{\frac{1+m}{2}} \right) / \\
& \quad \left( b \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 - m), 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - \right. \\
& \quad \left. \left. (a^2 - b^2) (1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1 - m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \left( -b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2) \right) \right) - \\
& \quad \left( 3 a^3 (a^2 - b^2) \operatorname{B AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{a^2 \operatorname{Tan}[e + f x]^2}{-a^2 + b^2} \right] (1 + \operatorname{Tan}[e + f x]^2)^{\frac{1+m}{2}} \right) / \\
& \quad \left( b^2 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 - m), 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (a^2 - b^2) (1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1-m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Tan}[e + f x]^2 \left( -b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2) \right) \Bigg) + \\
& \left( 3 a A (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] (1 + \operatorname{Tan}[e + f x]^2)^{m/2} \right) / \\
& \left( \left( 3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \right) \\
& \left( -b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2) \right) \Bigg) - \left( 3 a^2 (a^2 - b^2) B \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{a^2 \operatorname{Tan}[e + f x]^2}{-a^2 + b^2} \right] \right. \\
& \left. (1 + \operatorname{Tan}[e + f x]^2)^{m/2} \right) / \left( b \left( 3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - \right. \\
& \quad \left. \left. 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \right) \left( -b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2) \right) \Bigg) + \\
& \operatorname{Tan}[e + f x] \left[ - \left( 6 a^4 A (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{a^2 \operatorname{Tan}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right. \right. \\
& \quad \left. \left. (1 + \operatorname{Tan}[e + f x]^2)^{\frac{1+m}{2}} \right) / \left( b \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 - m), 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - (a^2 - b^2) (1 + m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1-m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \right) \left( -b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2) \right)^2 \Bigg) + \\
& \left( 6 a^5 (a^2 - b^2) B \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{a^2 \operatorname{Tan}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] (1 + \operatorname{Tan}[e + f x]^2)^{\frac{1+m}{2}} \right) / \\
& \left( b^2 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 - m), 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - \right. \\
& \quad \left. \left. (a^2 - b^2) (1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1-m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \right) \left( -b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2) \right)^2 \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& \left( 6 a^3 A (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] (1 + \operatorname{Tan}[e + f x]^2)^{m/2} \right) / \\
& \left( \left( 3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( (a^2 - b^2)^m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - \right. \right. \\
& \quad \left. \left. 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2))^2 + \\
& \left( 6 a^4 (a^2 - b^2) B \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{a^2 \operatorname{Tan}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] (1 + \operatorname{Tan}[e + f x]^2)^{m/2} \right) / \\
& \left( b \left( 3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( (a^2 - b^2)^m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - \right. \right. \\
& \quad \left. \left. 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2))^2 + \\
& \left( 3 a^2 A (a^2 - b^2) \left( -\frac{1}{3} (-1 - m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{1}{2} (-1 - m), 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{a^2 \operatorname{Tan}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] + \right. \right. \\
& \quad \left. \left. 1 / (3 (-a^2 + b^2)) 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 - m), 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{a^2 \operatorname{Tan}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) \right. \\
& \quad \left. (1 + \operatorname{Tan}[e + f x]^2)^{\frac{1+m}{2}} \right) / \left( b \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 - m), 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1 - m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2)) \right) - \\
& \left( 3 a^3 (a^2 - b^2) B \left( -\frac{1}{3} (-1 - m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{1}{2} (-1 - m), 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{a^2 \operatorname{Tan}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] + \right. \right. \\
& \quad \left. \left. 1 / (3 (-a^2 + b^2)) 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 - m), 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{a^2 \operatorname{Tan}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) \right. \\
& \quad \left. (1 + \operatorname{Tan}[e + f x]^2)^{\frac{1+m}{2}} \right) / \left( b^2 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 - m), 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. (a^2 - b^2) (1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1-m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Tan}[e + f x]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2)) \Bigg) + \\
& \left( 3 a^2 A (a^2 - b^2) (1 + m) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{a^2 \operatorname{Tan}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right. \\
& \left. (1 + \operatorname{Tan}[e + f x]^2)^{-1 + \frac{1+m}{2}} \right) / \left( b \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 - m), 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) (1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1-m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Tan}[e + f x]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2)) \right) \right) - \\
& \left( 3 a^3 (a^2 - b^2) B (1 + m) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{a^2 \operatorname{Tan}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right. \\
& \left. (1 + \operatorname{Tan}[e + f x]^2)^{-1 + \frac{1+m}{2}} \right) / \left( b^2 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 - m), 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) (1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1-m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Tan}[e + f x]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2)) \right) \right) + \\
& \left( 3 a A (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] (1 + \operatorname{Tan}[e + f x]^2)^{-1 + \frac{m}{2}} \right) / \\
& \left( \left( 3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - \right. \right. \\
& \left. \left. 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Tan}[e + f x]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2)) \right) - \\
& \left( 3 a^2 (a^2 - b^2) B m \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{a^2 \operatorname{Tan}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] (1 + \operatorname{Tan}[e + f x]^2)^{-1 + \frac{m}{2}} \right) / \\
& \left( b \left( 3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - \right. \right. \\
& \left. \left. 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Tan}[e + f x]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2)) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left( 3 a A (a^2 - b^2) \left( \frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \right. \right. \\
& \left. \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x]}{3 (a^2 - b^2)} \right) (1 + \tan[e + f x]^2)^{m/2} \right) / \\
& \left( \left( 3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, \right. \right. \right. \\
& \left. \left. -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] - 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \tan[e + f x]^2 \right) \\
& \left. (-b^2 + a^2 (1 + \tan[e + f x]^2)) \right) - \left( 3 a^2 (a^2 - b^2) B \left[ \frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{a^2 \tan[e + f x]^2}{-a^2 + b^2} \right] \right. \right. \\
& \left. \left. \operatorname{Sec}[e + f x]^2 \tan[e + f x] + \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{a^2 \tan[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x]}{3 (-a^2 + b^2)} \right) \right) \\
& \left. (1 + \tan[e + f x]^2)^{m/2} \right) / \left( b \left( 3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] - \right. \right. \\
& \left. \left. 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \tan[e + f x]^2 \right) (-b^2 + a^2 (1 + \tan[e + f x]^2)) \right) + \\
& \frac{1}{b} B \operatorname{Csc}[e + f x] \operatorname{Sec}[e + f x] \left( -\operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, 1 - \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2 \right] + (1 + \tan[e + f x]^2)^{-1 + \frac{m}{2}} \right) + \frac{1}{b} \\
& A \operatorname{Csc}[e + f x] \operatorname{Sec}[e + f x] \left( -\operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{2} - \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2 \right] + (1 + \tan[e + f x]^2)^{-\frac{1}{2} + \frac{m}{2}} \right) - \frac{1}{b^2} \\
& a B \operatorname{Csc}[e + f x] \operatorname{Sec}[e + f x] \left( -\operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{2} - \frac{m}{2}, \frac{3}{2}, -\tan[e + f x]^2 \right] + (1 + \tan[e + f x]^2)^{-\frac{1}{2} + \frac{m}{2}} \right) - \\
& \left( 3 a^2 A (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{a^2 \tan[e + f x]^2}{-a^2 + b^2} \right] \right. \\
& \left. (1 + \tan[e + f x]^2)^{\frac{1+m}{2}} \left( 2 \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 - m), 2, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] - \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) (1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1 - m}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 3 (a^2 - b^2) \left( -\frac{1}{3} (-1 - m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{1}{2} (-1 - m), 1, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \right. \\
& \quad \left. \frac{1}{3 (a^2 - b^2)} 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 - m), 2, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) + \\
& \tan[e + f x]^2 \left( 2 a^2 \left( -\frac{3}{5} (-1 - m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1}{2} (-1 - m), 2, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \right. \right. \\
& \quad \left. \left. \frac{1}{5 (a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2} (-1 - m), 3, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) - \right. \\
& \quad (a^2 - b^2) (1 + m) \left( -\frac{3}{5} (1 - m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1 - m}{2}, 1, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \right. \\
& \quad \left. \frac{1}{5 (a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1 - m}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) \right) \Bigg) \Bigg) / \\
& \left( b \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 - m), \right. \right. \right. \\
& \quad \left. \left. \left. 2, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] - (a^2 - b^2) (1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1 - m}{2}, 1, \frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \tan[e + f x]^2 \left( -b^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) \right) + \\
& \left( 3 a^3 (a^2 - b^2) \operatorname{B AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{a^2 \tan[e + f x]^2}{-a^2 + b^2} \right] (1 + \tan[e + f x]^2)^{\frac{1+m}{2}} \right. \\
& \quad \left. \left( 2 \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 - m), 2, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] - \right. \right. \right. \\
& \quad \left. \left. \left. (a^2 - b^2) (1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1 - m}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \right. \right. \\
& \quad \left. \left. 3 (a^2 - b^2) \left( -\frac{1}{3} (-1 - m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{1}{2} (-1 - m), 1, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{3 (a^2 - b^2)} 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 - m), 2, \frac{5}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) \right) + \right. \\
& \quad \left. \tan[e + f x]^2 \left( 2 a^2 \left( -\frac{3}{5} (-1 - m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1}{2} (-1 - m), 2, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{5 (a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2} (-1 - m), 3, \frac{7}{2}, -\tan[e + f x]^2, -\frac{a^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (a^2 - b^2) (1 + m) \left( -\frac{3}{5} (1 - m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1 - m}{2}, 1, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] - \right. \\
& \left. \frac{1}{5 (a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1 - m}{2}, 2, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( b^2 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 - m), 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 - m), \right. \right. \right. \right. \\
& \left. \left. \left. 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - (a^2 - b^2) (1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1 - m}{2}, 1, \frac{5}{2}, \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \right)^2 (-b^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2)) \Bigg) - \\
& \left( 3 a A (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] (1 + \operatorname{Tan}[e + f x]^2)^{m/2} \right. \\
& \left. \left( 2 \left( (a^2 - b^2)^m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] - \right. \right. \right. \\
& \left. \left. \left. 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] + \right. \right. \\
& \left. \left. 3 (a^2 - b^2) \left( \frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] - \right. \right. \right. \\
& \left. \left. \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{3 (a^2 - b^2)} \right) + \operatorname{Tan}[e + f x]^2 \right. \right. \\
& \left. \left( (a^2 - b^2)^m \left( -\frac{1}{5 (a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, 1 - \frac{m}{2}, 2, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] - \right. \right. \right. \\
& \left. \left. \left. \frac{6}{5} \left( 1 - \frac{m}{2} \right) \operatorname{AppellF1} \left[ \frac{5}{2}, 2 - \frac{m}{2}, 1, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) - \right. \right. \\
& \left. \left. 2 a^2 \left( \frac{3}{5} m \operatorname{AppellF1} \left[ \frac{5}{2}, 1 - \frac{m}{2}, 2, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] - \frac{1}{5 (a^2 - b^2)} \right. \right. \right. \\
& \left. \left. \left. 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, -\frac{m}{2}, 3, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{a^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) \right) \right) \Bigg) \Bigg) \Bigg) /
\end{aligned}$$

$$\begin{aligned} & \left( \left( 3 (a^2 - b^2) \text{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\text{Tan}[e + f x]^2, -\frac{a^2 \text{Tan}[e + f x]^2}{a^2 - b^2} \right] + \left( (a^2 - b^2) m \text{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, \right. \right. \right. \right. \\ & \quad \left. \left. \left. -\text{Tan}[e + f x]^2, -\frac{a^2 \text{Tan}[e + f x]^2}{a^2 - b^2} \right] - 2 a^2 \text{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{a^2 \text{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \text{Tan}[e + f x]^2 \right)^2 \\ & \left. \left( -b^2 + a^2 (1 + \text{Tan}[e + f x]^2) \right) \right) + \left( 3 a^2 (a^2 - b^2) \text{BAppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\text{Tan}[e + f x]^2, \frac{a^2 \text{Tan}[e + f x]^2}{-a^2 + b^2} \right] \right. \\ & \left. (1 + \text{Tan}[e + f x]^2)^{m/2} \left( 2 \left( (a^2 - b^2) m \text{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{a^2 \text{Tan}[e + f x]^2}{a^2 - b^2} \right] - \right. \right. \right. \\ & \quad \left. \left. \left. 2 a^2 \text{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{a^2 \text{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \text{Sec}[e + f x]^2 \text{Tan}[e + f x] + \right. \right. \\ & \quad \left. \left. 3 (a^2 - b^2) \left( \frac{1}{3} m \text{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{a^2 \text{Tan}[e + f x]^2}{a^2 - b^2} \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] - \right. \right. \right. \\ & \quad \left. \left. \left. \frac{2 a^2 \text{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{a^2 \text{Tan}[e + f x]^2}{a^2 - b^2} \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x]}{3 (a^2 - b^2)} \right) \right) + \text{Tan}[e + f x]^2 \right. \\ & \quad \left. \left( (a^2 - b^2) m \left( -\frac{1}{5 (a^2 - b^2)} 6 a^2 \text{AppellF1} \left[ \frac{5}{2}, 1 - \frac{m}{2}, 2, \frac{7}{2}, -\text{Tan}[e + f x]^2, -\frac{a^2 \text{Tan}[e + f x]^2}{a^2 - b^2} \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] - \right. \right. \right. \\ & \quad \left. \frac{6}{5} \left( 1 - \frac{m}{2} \right) \text{AppellF1} \left[ \frac{5}{2}, 2 - \frac{m}{2}, 1, \frac{7}{2}, -\text{Tan}[e + f x]^2, -\frac{a^2 \text{Tan}[e + f x]^2}{a^2 - b^2} \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] \right) - \\ & \quad \left. 2 a^2 \left( \frac{3}{5} m \text{AppellF1} \left[ \frac{5}{2}, 1 - \frac{m}{2}, 2, \frac{7}{2}, -\text{Tan}[e + f x]^2, -\frac{a^2 \text{Tan}[e + f x]^2}{a^2 - b^2} \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] - \frac{1}{5 (a^2 - b^2)} \right. \right. \\ & \quad \left. \left. \left. \left. \left. 12 a^2 \text{AppellF1} \left[ \frac{5}{2}, -\frac{m}{2}, 3, \frac{7}{2}, -\text{Tan}[e + f x]^2, -\frac{a^2 \text{Tan}[e + f x]^2}{a^2 - b^2} \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] \right) \right) \right) \right) \Bigg/ \\ & \left( b \left( 3 (a^2 - b^2) \text{AppellF1} \left[ \frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, -\text{Tan}[e + f x]^2, -\frac{a^2 \text{Tan}[e + f x]^2}{a^2 - b^2} \right] + \left( (a^2 - b^2) m \text{AppellF1} \left[ \frac{3}{2}, 1 - \frac{m}{2}, 1, \right. \right. \right. \right. \\ & \quad \left. \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{a^2 \text{Tan}[e + f x]^2}{a^2 - b^2} \right] - 2 a^2 \text{AppellF1} \left[ \frac{3}{2}, -\frac{m}{2}, 2, \frac{5}{2}, \right. \\ & \quad \left. \left. \left. -\text{Tan}[e + f x]^2, -\frac{a^2 \text{Tan}[e + f x]^2}{a^2 - b^2} \right] \text{Tan}[e + f x]^2 \left( -b^2 + a^2 (1 + \text{Tan}[e + f x]^2) \right) \right) \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) \end{aligned}$$



**Problem 641: Attempted integration timed out after 120 seconds.**

$$\int (a + b \cos[e + f x])^{3/2} (A + B \cos[e + f x]) (c \sec[e + f x])^m dx$$

Optimal (type 9, 209 leaves, 2 steps):

$$\frac{2 b B \cos[e + f x] \sqrt{a + b \cos[e + f x]} (c \sec[e + f x])^m \sin[e + f x]}{f (5 - 2 m)} + \frac{1}{c (5 - 2 m)}$$

$$2 (c \cos[e + f x])^m (c \sec[e + f x])^m \text{Unintegrable}\left[1 / \left(\sqrt{a + b \cos[e + f x]}\right) (c \cos[e + f x])^{-m} \left(\frac{1}{2} a c \left(2 b B (1 - m) + 2 a A \left(\frac{5}{2} - m\right)\right) + \frac{1}{2} c \left(b^2 B (3 - 2 m) + a (2 A b + a B) (5 - 2 m)\right) \cos[e + f x] + \frac{1}{2} b c (A b (5 - 2 m) + 2 a B (3 - m)) \cos[e + f x]^2\right], x\right]$$

Result (type 1, 1 leaves):

???

## Test results for the 393 problems in "4.2.4.1 (a+b cos)^m (A+B cos+C cos^2).m"

- **Problem 5: Result more than twice size of optimal antiderivative.**

$$\int (A + C \cos[c + d x]^2) \sec[c + d x] dx$$

Optimal (type 3, 24 leaves, 2 steps):

$$\frac{A \text{ArcTanh}[\sin[c + d x]]}{d} + \frac{C \sin[c + d x]}{d}$$

Result (type 3, 92 leaves):

$$-\frac{A \text{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{A \text{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{C \cos[dx] \sin[c]}{d} + \frac{C \cos[c] \sin[dx]}{d}$$

- **Problem 8: Result more than twice size of optimal antiderivative.**

$$\int (A + C \cos[c + d x]^2) \sec[c + d x]^7 dx$$

Optimal (type 3, 98 leaves, 4 steps):

$$\frac{(5 A + 6 C) \text{ArcTanh}[\sin[c + d x]]}{16 d} + \frac{(5 A + 6 C) \sec[c + d x] \tan[c + d x]}{16 d} + \frac{(5 A + 6 C) \sec[c + d x]^3 \tan[c + d x]}{24 d} + \frac{A \sec[c + d x]^5 \tan[c + d x]}{6 d}$$

Result (type 3, 445 leaves):

$$\begin{aligned}
& - \frac{5 A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{16 d} - \frac{3 C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8 d} + \\
& \frac{5 A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{16 d} + \frac{3 C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8 d} + \frac{A}{48 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^6} + \\
& \frac{A}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{C}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{5 A}{32 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \\
& \frac{3 C}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{A}{48 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6} - \frac{A}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \\
& \frac{C}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{5 A}{32 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{3 C}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}
\end{aligned}$$

- **Problem 34: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (b \cos[c+dx])^m (A+C \cos[c+dx])^2 dx$$

Optimal (type 5, 117 leaves, 2 steps):

$$\frac{c (b \cos[c+dx])^{1+m} \sin[c+dx]}{b d (2+m)} - \frac{(C(1+m) + A(2+m)) (b \cos[c+dx])^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]}{b d (1+m) (2+m) \sqrt{\sin[c+dx]^2}}$$

Result (type 5, 294 leaves):

$$\begin{aligned}
& \frac{1}{4 d} (b \cos[c+dx])^m \\
& \left( \frac{1}{(2+m) i} 2^{-m} C e^{-2 i (c+dx)} (1+e^{2 i (c+dx)})^{-m} (e^{-i (c+dx)} (1+e^{2 i (c+dx)}))^m \cos[c+dx]^{-m} \operatorname{Hypergeometric2F1}\left[-1-\frac{m}{2}, -m, -\frac{m}{2}, -e^{2 i (c+dx)}\right] + \right. \\
& \left. \frac{1}{(-2+m) i} 2^{-m} C e^{2 i (c+dx)} (1+e^{2 i (c+dx)})^{-m} (e^{-i (c+dx)} (1+e^{2 i (c+dx)}))^m \cos[c+dx]^{-m} \operatorname{Hypergeometric2F1}\left[1-\frac{m}{2}, -m, 2-\frac{m}{2}, -e^{2 i (c+dx)}\right] - \right. \\
& \left. \frac{2(2A+C) \cot[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[c+dx]^2\right] \sqrt{\sin[c+dx]^2}}{1+m} \right)
\end{aligned}$$

- **Problem 66: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A+C \cos[c+dx])^2 \sec[c+dx]}{\sqrt{b \cos[c+dx]}} dx$$

Optimal (type 4, 71 leaves, 4 steps):

$$-\frac{2(A-C)\sqrt{b\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{bd\sqrt{\cos[c+dx]}} + \frac{2A\sin[c+dx]}{d\sqrt{b\cos[c+dx]}}$$

Result (type 5, 200 leaves):

$$-\frac{1}{3d\sqrt{b\cos[c+dx]}}\operatorname{Csc}[c]\left(-6A\cos[dx]+3C\cos[dx]+3C\cos[2c+dx]+3(A-C)\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c]+i\sin[c])^2\right](\cos[dx]-i\sin[dx])\sqrt{1+\cos[2(c+dx)]+i\sin[2(c+dx)]}\right) + (A-C)\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c]+i\sin[c])^2\right](\cos[dx]+i\sin[dx])\sqrt{1+\cos[2(c+dx)]+i\sin[2(c+dx)]}$$

■ **Problem 67: Result unnecessarily involves higher level functions.**

$$\int \frac{(A+C\cos[c+dx])^2 \operatorname{Sec}[c+dx]^2}{\sqrt{b\cos[c+dx]}} dx$$

Optimal (type 4, 73 leaves, 4 steps):

$$\frac{2(A+3C)\sqrt{b\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d\sqrt{b\cos[c+dx]}} + \frac{2Ab\sin[c+dx]}{3d(b\cos[c+dx])^{3/2}}$$

Result (type 5, 141 leaves):

$$-\left(4b(A+C\cos[c+dx])^2\left((A+3C)\cos[c+dx]^2\sqrt{\cos[dx-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2}\operatorname{Csc}[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right]\operatorname{Sec}[dx-\operatorname{ArcTan}[\operatorname{Cot}[c]]]-A\sqrt{\operatorname{Csc}[c]^2}\sin[c+dx]\right)\right)/\left(3d(b\cos[c+dx])^{3/2}(2A+C+C\cos[2(c+dx)])\sqrt{\operatorname{Csc}[c]^2}\right)$$

■ **Problem 68: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A+C\cos[c+dx])^2 \operatorname{Sec}[c+dx]^3}{\sqrt{b\cos[c+dx]}} dx$$

Optimal (type 4, 112 leaves, 5 steps):

$$-\frac{2(3A+5C)\sqrt{b\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5bd\sqrt{\cos[c+dx]}} + \frac{2Ab^2\sin[c+dx]}{5d(b\cos[c+dx])^{5/2}} + \frac{2(3A+5C)\sin[c+dx]}{5d\sqrt{b\cos[c+dx]}}$$

Result (type 5, 522 leaves):

$$\begin{aligned}
& b \left( -\frac{1}{10 (b \cos [c+d x])^{3/2} (2 A+C+C \cos [2 c+2 d x])} i (3 A+5 C) \cos [c+d x]^{7/2} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (C+A \operatorname{Sec}[c+d x]^2) \right. \\
& \quad \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c]+i \sin [c])^2\right] \sqrt{e^{-i d x} (2 (1+e^{2 i d x}) \cos [c]+2 i (-1+e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1+e^{2 i d x} \cos [2 c]+i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1+e^{2 i d x}) \cos [c]-3 d (-1+e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c]+i \sin [c])^2\right] \sqrt{e^{-i d x} (2 (1+e^{2 i d x}) \cos [c]+2 i (-1+e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \left. \sqrt{1+e^{2 i d x} \cos [2 c]+i e^{2 i d x} \sin [2 c]} \right) / (-i d (1+e^{2 i d x}) \cos [c]+d (-1+e^{2 i d x}) \sin [c]) \right) + \\
& \quad \left( \cos [c+d x]^4 (C+A \operatorname{Sec}[c+d x]^2) \left( \frac{4 (3 A+5 C) \operatorname{Csc}[c] \operatorname{Sec}[c]}{5 d} + \frac{4 A \operatorname{Sec}[c] \operatorname{Sec}[c+d x]^3 \sin [d x]}{5 d} + \right. \right. \\
& \quad \left. \left. \frac{4 \operatorname{Sec}[c] \operatorname{Sec}[c+d x] (3 A \sin [d x]+5 C \sin [d x])}{5 d} + \frac{4 A \operatorname{Sec}[c+d x]^2 \tan [c]}{5 d} \right) \right) / \left( (b \cos [c+d x])^{3/2} (2 A+C+C \cos [2 c+2 d x]) \right) \Big)
\end{aligned}$$

■ **Problem 76: Result unnecessarily involves higher level functions.**

$$\int \frac{(A+C \cos [c+d x]^2) \operatorname{Sec}[c+d x]}{(b \cos [c+d x])^{3/2}} dx$$

Optimal (type 4, 75 leaves, 4 steps):

$$\frac{2 (A+3 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), 2\right]}{3 b d \sqrt{b \cos [c+d x]}} + \frac{2 A \sin [c+d x]}{3 d (b \cos [c+d x])^{3/2}}$$

Result (type 5, 140 leaves):

$$\begin{aligned}
& - \left( 4 (A+C \cos [c+d x]^2) \right. \\
& \quad \left( (A+3 C) \cos [c+d x]^2 \sqrt{\cos [d x - \operatorname{ArcTan}[\cot [c]]]^2} \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \right. \\
& \quad \left. \left. \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] - A \sqrt{\operatorname{Csc}[c]^2} \sin [c+d x] \right) \right) / \left( 3 d (b \cos [c+d x])^{3/2} (2 A+C+C \cos [2 (c+d x)]) \sqrt{\operatorname{Csc}[c]^2} \right)
\end{aligned}$$

■ **Problem 161: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A+C \cos [c+d x]^2) \operatorname{Sec}[c+d x]}{(b \cos [c+d x])^{1/3}} dx$$

Optimal (type 5, 90 leaves, 3 steps):

$$\frac{3 A \sin [c+d x]}{d (b \cos [c+d x])^{1 / 3}}+\frac{3(2 A-C)(b \cos [c+d x])^{5 / 3} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos [c+d x]^2\right] \sin [c+d x]}{5 b^2 d \sqrt{\sin [c+d x]^2}}$$

Result (type 5, 283 leaves):

$$-\left(\left(3 e^{-i d x} \cos [c+d x]^{1 / 3} \operatorname{Csc}[c](\cos [d x]+i \sin [d x])\right.\right. \\ \left.\left(-8 A \cos [d x]+2 C \cos [d x]+2 C \cos [2 c+d x]+2(2 A-C) \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3},-e^{2 i d x}(\cos [c]+i \sin [c])^2\right]\right.\right. \\ \left.\left.(\cos [d x]-i \sin [d x])(1+\cos [2(c+d x)]+i \sin [2(c+d x)])^{1 / 3}+(2 A-C)\right.\right. \\ \left.\left.\operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3},-e^{2 i d x}(\cos [c]+i \sin [c])^2\right](\cos [d x]+i \sin [d x])(1+\cos [2(c+d x)]+i \sin [2(c+d x)])^{1 / 3}\right)\right) / \\ \left.(4 \times 2^{2 / 3} d(b \cos [c+d x])^{1 / 3}\left(e^{-i d x}\left((1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]\right)\right)^{1 / 3}\right)$$

■ **Problem 163: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A+C \cos [c+d x]^2) \operatorname{Sec}[c+d x]^3}{(b \cos [c+d x])^{1 / 3}} d x$$

Optimal (type 5, 92 leaves, 3 steps):

$$\frac{3 A b^2 \sin [c+d x]}{7 d(b \cos [c+d x])^{7 / 3}}+\frac{3(4 A+7 C) \operatorname{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \cos [c+d x]^2\right] \sin [c+d x]}{7 d(b \cos [c+d x])^{1 / 3} \sqrt{\sin [c+d x]^2}}$$

Result (type 5, 481 leaves):

$$b\left(-\left(i(4 A+7 C) \cos [c+d x]^{10 / 3} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right](C+A \operatorname{Sec}[c+d x]^2)\right.\right. \\ \left.\left(-\left(3 i e^{-i d x} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3},-e^{2 i d x}(\cos [c]+i \sin [c])^2\right](1+e^{2 i d x} \cos [2 c]+i e^{2 i d x} \sin [2 c])^{1 / 3}\right)\right) / \\ \left.\left(2^{2 / 3} d\left(e^{-i d x}\left((1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]\right)\right)^{1 / 3}-\right.\right. \\ \left.\left(3 i e^{i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3},-e^{2 i d x}(\cos [c]+i \sin [c])^2\right](1+e^{2 i d x} \cos [2 c]+i e^{2 i d x} \sin [2 c])^{1 / 3}\right)\right) / \\ \left.\left(2 \times 2^{2 / 3} d\left(e^{-i d x}\left((1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]\right)\right)^{1 / 3}\right)\right) / \left(7(b \cos [c+d x])^{4 / 3}(2 A+C+C \cos [2 c+2 d x])\right)+ \\ \left.\left(\cos [c+d x]^4(C+A \operatorname{Sec}[c+d x]^2)\left(\frac{6(4 A+7 C) \operatorname{Csc}[c] \operatorname{Sec}[c]}{7 d}+\frac{6 A \operatorname{Sec}[c] \operatorname{Sec}[c+d x]^3 \sin [d x]}{7 d}+\right.\right.\right. \\ \left.\left.\frac{6 \operatorname{Sec}[c] \operatorname{Sec}[c+d x](4 A \sin [d x]+7 C \sin [d x])}{7 d}+\frac{6 A \operatorname{Sec}[c+d x]^2 \tan [c]}{7 d}\right)\right) / \left((b \cos [c+d x])^{4 / 3}(2 A+C+C \cos [2 c+2 d x])\right)$$

- **Problem 167: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos[c + dx])^2 \sec[c + dx]}{(b \cos[c + dx])^{2/3}} dx$$

Optimal (type 5, 90 leaves, 3 steps):

$$\frac{3 A \sin[c + dx]}{2 d (b \cos[c + dx])^{2/3}} + \frac{3 (A - 2 C) (b \cos[c + dx])^{4/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos[c + dx]^2\right] \sin[c + dx]}{8 b^2 d \sqrt{\sin[c + dx]^2}}$$

Result (type 5, 277 leaves):

$$-\left( \left( 3 e^{-i dx} \cos[c + dx]^{2/3} \operatorname{Csc}[c] (\cos[dx] + i \sin[dx]) \right. \right. \\ \left. \left( 10 ((-A + C) \cos[dx] + C \cos[2c + dx]) + 5 (A - 2 C) \operatorname{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{2}{3}, \frac{5}{6}, -e^{2i dx} (\cos[c] + i \sin[c])^2\right] \right. \right. \\ \left. (\cos[dx] - i \sin[dx]) (1 + \cos[2(c + dx)] + i \sin[2(c + dx)])^{2/3} + (A - 2 C) \right. \\ \left. \left. \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{5}{6}, \frac{11}{6}, -e^{2i dx} (\cos[c] + i \sin[c])^2\right] (\cos[dx] + i \sin[dx]) (1 + \cos[2(c + dx)] + i \sin[2(c + dx)])^{2/3} \right) \right) / \\ \left. (10 \times 2^{1/3} d (b \cos[c + dx])^{2/3} (e^{-i dx} ((1 + e^{2i dx}) \cos[c] + i (-1 + e^{2i dx}) \sin[c]))^{2/3} \right)$$

- **Problem 169: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos[c + dx])^2 \sec[c + dx]^3}{(b \cos[c + dx])^{2/3}} dx$$

Optimal (type 5, 92 leaves, 3 steps):

$$\frac{3 A b^2 \sin[c + dx]}{8 d (b \cos[c + dx])^{8/3}} + \frac{3 (5 A + 8 C) \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos[c + dx]^2\right] \sin[c + dx]}{16 d (b \cos[c + dx])^{2/3} \sqrt{\sin[c + dx]^2}}$$

Result (type 5, 473 leaves):

$$\begin{aligned}
& b \left( - \left( i (5A + 8C) \cos[c + dx]^{11/3} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (C + A \operatorname{Sec}[c + dx]^2) \right. \right. \\
& \quad \left. \left( - \left( 3 i e^{-idx} \operatorname{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{2}{3}, \frac{5}{6}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] (2 + 2 e^{2idx} \cos[2c] + 2 i e^{2idx} \sin[2c])^{2/3} \right) \right) \right) / \\
& \quad \left( d (e^{-idx} ((1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]))^{2/3} - \right. \\
& \quad \left. \left( 3 i e^{idx} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{5}{6}, \frac{11}{6}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] (2 + 2 e^{2idx} \cos[2c] + 2 i e^{2idx} \sin[2c])^{2/3} \right) \right) / \\
& \quad \left. \left( 5 d (e^{-idx} ((1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]))^{2/3} \right) \right) / \left( (32 (b \cos[c + dx])^{5/3} (2A + C + C \cos[2c + 2dx])) \right) + \\
& \quad \left( \cos[c + dx]^4 (C + A \operatorname{Sec}[c + dx]^2) \left( \frac{3 (5A + 8C) \operatorname{Csc}[c] \operatorname{Sec}[c]}{8d} + \frac{3A \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^3 \sin[dx]}{4d} + \right. \right. \\
& \quad \left. \left. \frac{3 \operatorname{Sec}[c] \operatorname{Sec}[c + dx] (5A \sin[dx] + 8C \sin[dx])}{8d} + \frac{3A \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c]}{4d} \right) \right) / \left( (b \cos[c + dx])^{5/3} (2A + C + C \cos[2c + 2dx]) \right) \Big)
\end{aligned}$$

■ **Problem 182: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a \cos[c + dx])^m (b \cos[c + dx])^n (A + C \cos[c + dx]^2) dx$$

Optimal (type 5, 144 leaves, 3 steps):

$$\frac{C (a \cos[c + dx])^{1+m} (b \cos[c + dx])^n \sin[c + dx]}{a d (2 + m + n)} -$$

$$\left( (C (1 + m + n) + A (2 + m + n)) (a \cos[c + dx])^{1+m} (b \cos[c + dx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (1 + m + n), \frac{1}{2} (3 + m + n), \cos[c + dx]^2\right] \right. \\
\left. \sin[c + dx] \right) / \left( a d (1 + m + n) (2 + m + n) \sqrt{\sin[c + dx]^2} \right)$$

Result (type 5, 459 leaves):

$$\frac{1}{4d} C \cos[c+dx]^{-m-n} (a \cos[c+dx])^m (b \cos[c+dx])^n$$

$$\left( \frac{1}{(2+m+n)i} 2^{-m-n} e^{-2i(c+dx)} (e^{-i(c+dx)} + e^{i(c+dx)})^{m+n} (1 + e^{2i(c+dx)})^{-m-n} \text{Hypergeometric2F1}\left[-m-n, -1 - \frac{m}{2} - \frac{n}{2}, -\frac{m}{2} - \frac{n}{2}, -e^{2i(c+dx)}\right] + \right.$$

$$\left. \frac{1}{(-2+m+n)i} 2^{-m-n} e^{2i(c+dx)} (e^{-i(c+dx)} + e^{i(c+dx)})^{m+n} (1 + e^{2i(c+dx)})^{-m-n} \text{Hypergeometric2F1}\left[-m-n, 1 - \frac{m}{2} - \frac{n}{2}, 2 - \frac{m}{2} - \frac{n}{2}, -e^{2i(c+dx)}\right] \right) -$$

$$\left( A \cos[c+dx] (a \cos[c+dx])^m (b \cos[c+dx])^n \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}(1+m+n), \frac{1}{2}(3+m+n), \cos[c+dx]^2\right] \sin[c+dx] \right) /$$

$$\left( d(1+m+n) \sqrt{\sin[c+dx]^2} \right) -$$

$$\left( C \cos[c+dx] (a \cos[c+dx])^m (b \cos[c+dx])^n \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}(1+m+n), \frac{1}{2}(3+m+n), \cos[c+dx]^2\right] \sin[c+dx] \right) /$$

$$\left( 2d(1+m+n) \sqrt{\sin[c+dx]^2} \right)$$

■ **Problem 183: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^2 (b \cos[c+dx])^n (A + C \cos[c+dx]^2) dx$$

Optimal (type 5, 117 leaves, 3 steps):

$$\frac{C (b \cos[c+dx])^{3+n} \sin[c+dx]}{b^3 d (4+n)} - \frac{(C(3+n) + A(4+n)) (b \cos[c+dx])^{3+n} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+n}{2}, \frac{5+n}{2}, \cos[c+dx]^2\right] \sin[c+dx]}{b^3 d (3+n) (4+n) \sqrt{\sin[c+dx]^2}}$$

Result (type 5, 342 leaves):

$$\frac{1}{8d} (b \cos[c+dx])^n \cot[c+dx] \left( -\frac{C \text{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos[c+dx]^2\right]}{1+n} + \frac{4(A+C) \text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos[c+dx]^2\right]}{1+n} + \right.$$

$$\frac{6C \cos[c+dx]^2 \text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{3+n}{2}, \frac{5+n}{2}, \cos[c+dx]^2\right]}{3+n} - \frac{4A \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos[c+dx]^2\right]}{1+n} -$$

$$\frac{3C \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos[c+dx]^2\right]}{1+n} - \frac{4A \cos[c+dx]^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+n}{2}, \frac{5+n}{2}, \cos[c+dx]^2\right]}{3+n} -$$

$$\frac{4C \cos[c+dx]^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+n}{2}, \frac{5+n}{2}, \cos[c+dx]^2\right]}{3+n} -$$

$$\left. \frac{C \cos[c+dx]^4 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5+n}{2}, \frac{7+n}{2}, \cos[c+dx]^2\right]}{5+n} \right) \sqrt{\sin[c+dx]^2}$$



- **Problem 185: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (b \cos [c + d x])^n (A + C \cos [c + d x]^2) dx$$

Optimal (type 5, 117 leaves, 2 steps):

$$\frac{C (b \cos [c + d x])^{1+n} \sin [c + d x]}{b d (2+n)} - \frac{(C (1+n) + A (2+n)) (b \cos [c + d x])^{1+n} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos [c + d x]^2\right] \sin [c + d x]}{b d (1+n) (2+n) \sqrt{\sin [c + d x]^2}}$$

Result (type 5, 294 leaves):

$$\frac{1}{4 d} (b \cos [c + d x])^n \left( \begin{aligned} & 1 / (2+n) i 2^{-n} C e^{-2 i (c+d x)} (1 + e^{2 i (c+d x)})^{-n} (e^{-i (c+d x)} (1 + e^{2 i (c+d x)}))^n \cos [c + d x]^{-n} \operatorname{Hypergeometric2F1}\left[-1 - \frac{n}{2}, -n, -\frac{n}{2}, -e^{2 i (c+d x)}\right] + \\ & 1 / (-2+n) i 2^{-n} C e^{2 i (c+d x)} (1 + e^{2 i (c+d x)})^{-n} (e^{-i (c+d x)} (1 + e^{2 i (c+d x)}))^n \cos [c + d x]^{-n} \operatorname{Hypergeometric2F1}\left[1 - \frac{n}{2}, -n, 2 - \frac{n}{2}, -e^{2 i (c+d x)}\right] - \\ & \frac{2 (2 A + C) \cot [c + d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos [c + d x]^2\right] \sqrt{\sin [c + d x]^2}}{1+n} \end{aligned} \right)$$

- **Problem 190: Result more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^{5/2} (b \cos [c + d x])^n (A + C \cos [c + d x]^2) dx$$

Optimal (type 5, 142 leaves, 3 steps):

$$\frac{2 C \cos [c + d x]^{7/2} (b \cos [c + d x])^n \sin [c + d x]}{d (9 + 2 n)} - \frac{\left( 2 (C (7 + 2 n) + A (9 + 2 n)) \cos [c + d x]^{7/2} (b \cos [c + d x])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4} (7 + 2 n), \frac{1}{4} (11 + 2 n), \cos [c + d x]^2\right] \sin [c + d x] \right)}{d (7 + 2 n) (9 + 2 n) \sqrt{\sin [c + d x]^2}}$$

Result (type 5, 400 leaves):

$$\frac{1}{8d} \cos[c+dx]^{3/2} (b \cos[c+dx])^n \operatorname{Csc}[c+dx] \left( -\frac{2 C \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1}{4}(3+2n), \frac{1}{4}(7+2n), \cos[c+dx]^2\right]}{3+2n} + \right.$$

$$\frac{8(A+C) \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{4}(3+2n), \frac{1}{4}(7+2n), \cos[c+dx]^2\right]}{3+2n} +$$

$$\frac{6 C \cos[c+dx]^2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{4}(7+2n), \frac{1}{4}(11+2n), \cos[c+dx]^2\right]}{\frac{7}{2}+n} -$$

$$\frac{8 A \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(3+2n), \frac{1}{4}(7+2n), \cos[c+dx]^2\right]}{3+2n} - \frac{6 C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(3+2n), \frac{1}{4}(7+2n), \cos[c+dx]^2\right]}{3+2n} -$$

$$\frac{8 A \cos[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(7+2n), \frac{1}{4}(11+2n), \cos[c+dx]^2\right]}{7+2n} -$$

$$\frac{8 C \cos[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(7+2n), \frac{1}{4}(11+2n), \cos[c+dx]^2\right]}{7+2n} -$$

$$\left. \frac{2 C \cos[c+dx]^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(11+2n), \frac{1}{4}(15+2n), \cos[c+dx]^2\right]}{11+2n} \right) \sqrt{\sin[c+dx]^2}$$

■ **Problem 198: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + a \cos[ex + fx])^m (A + C \cos[ex + fx]^2) dx$$

Optimal (type 5, 170 leaves, 4 steps):

$$-\frac{C(a + a \cos[ex + fx])^m \sin[ex + fx]}{f(2 + 3m + m^2)} + \frac{C(a + a \cos[ex + fx])^{1+m} \sin[ex + fx]}{af(2 + m)} + \frac{1}{f(1 + m)(2 + m)}$$

$$2^{\frac{1}{2}+m} (C(1 + m + m^2) + A(2 + 3m + m^2)) (1 + \cos[ex + fx])^{-\frac{1}{2}-m} (a + a \cos[ex + fx])^m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{1}{2}(1 - \cos[ex + fx])\right] \sin[ex + fx]$$

Result (type 5, 238 leaves):

$$\frac{1}{f(-2 + m)m(2 + m)} i^{4^{-1-m}} e^{-2i(ex+fx)} (1 + e^{i(ex+fx)})^{-2m} \left( e^{-\frac{1}{2}i(ex+fx)} (1 + e^{i(ex+fx)}) \right)^{2m} \cos\left[\frac{1}{2}(ex + fx)\right]^{-2m}$$

$$(a(1 + \cos[ex + fx]))^m (C(-2 + m) \operatorname{Hypergeometric2F1}[-2 - m, -2m, -1 - m, -e^{i(ex+fx)}] + e^{2i(ex+fx)}(2 + m)$$

$$(C e^{2i(ex+fx)} \operatorname{Hypergeometric2F1}[2 - m, -2m, 3 - m, -e^{i(ex+fx)}] + 2(2A + C)(-2 + m) \operatorname{Hypergeometric2F1}[-2m, -m, 1 - m, -e^{i(ex+fx)}]))$$

■ **Problem 200: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + a \cos[c + dx])^{1/3} (A + C \cos[c + dx]^2) dx$$

Optimal (type 5, 135 leaves, 4 steps):

$$-\frac{9C(a+a\cos[c+dx])^{1/3}\sin[c+dx]}{28d} + \frac{3C(a+a\cos[c+dx])^{4/3}\sin[c+dx]}{7ad} +$$

$$\frac{(28A+13C)(a+a\cos[c+dx])^{1/3}\operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1-\cos[c+dx])\right]\sin[c+dx]}{14 \times 2^{1/6} d (1+\cos[c+dx])^{5/6}}$$

Result (type 5, 240 leaves):

$$\frac{1}{112d} 3(a(1+\cos[c+dx]))^{1/3}$$

$$\left(-4(28A+13C)\cot\left[\frac{C}{2}\right] + 4C\cos[dx]\sin[c] + \left((28A+13C)\operatorname{Csc}\left[\frac{C}{4}\right] \left(2\operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{idx}(\cos[c]+i\sin[c])\right]\right) +\right.\right.$$

$$\left.\left.e^{idx}\operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -e^{idx}(\cos[c]+i\sin[c])\right]\right)\operatorname{Sec}\left[\frac{C}{4}\right] (1+e^{idx}\cos[c]+ie^{idx}\sin[c])^{1/3}\right) /$$

$$\left(\left((1+e^{idx})\cos\left[\frac{C}{2}\right] + i(-1+e^{idx})\sin\left[\frac{C}{2}\right]\right) + 8C\cos[2dx]\sin[2c] + 4C\cos[c]\sin[dx] + 8C\cos[2c]\sin[2dx]\right)$$

■ **Problem 202: Unable to integrate problem.**

$$\int \frac{A+C\cos[c+dx]^2}{(a+a\cos[c+dx])^{2/3}} dx$$

Optimal (type 5, 138 leaves, 4 steps):

$$\frac{3(A+C)\sin[c+dx]}{d(a+a\cos[c+dx])^{2/3}} + \frac{3C(a+a\cos[c+dx])^{1/3}\sin[c+dx]}{4ad} -$$

$$\frac{(4A+7C)(a+a\cos[c+dx])^{1/3}\operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1-\cos[c+dx])\right]\sin[c+dx]}{2 \times 2^{1/6} ad (1+\cos[c+dx])^{5/6}}$$

Result (type 8, 29 leaves):

$$\int \frac{A+C\cos[c+dx]^2}{(a+a\cos[c+dx])^{2/3}} dx$$

■ **Problem 208: Result more than twice size of optimal antiderivative.**

$$\int (a+b\cos[e+fx])^m (A+C\cos[e+fx]^2) dx$$

Optimal (type 6, 285 leaves, 8 steps):

$$\frac{C (a + b \cos[e + f x])^{1+m} \sin[e + f x]}{b f (2+m)} -$$

$$\left( \sqrt{2} a (a+b) C \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, -1-m, \frac{3}{2}, \frac{1}{2} (1 - \cos[e + f x]), \frac{b (1 - \cos[e + f x])}{a+b} \right] (a+b \cos[e + f x])^m \left( \frac{a+b \cos[e + f x]}{a+b} \right)^{-m} \sin[e + f x] \right) /$$

$$\left( b^2 f (2+m) \sqrt{1 + \cos[e + f x]} \right) + \left( \sqrt{2} (a^2 C + b^2 (C (1+m) + A (2+m))) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1 - \cos[e + f x]), \frac{b (1 - \cos[e + f x])}{a+b} \right] \right.$$

$$\left. (a+b \cos[e + f x])^m \left( \frac{a+b \cos[e + f x]}{a+b} \right)^{-m} \sin[e + f x] \right) / \left( b^2 f (2+m) \sqrt{1 + \cos[e + f x]} \right)$$

Result (type 6, 10836 leaves):

$$\left( 6 (a+b) \left( A (a+b \cos[e + f x])^m + \frac{1}{2} C (a+b \cos[e + f x])^m + \frac{1}{2} C (a+b \cos[e + f x])^m \cos[2 (e + f x)] \right) \tan \left[ \frac{1}{2} (e + f x) \right] \right.$$

$$\left. \left( a + \frac{b - b \tan \left[ \frac{1}{2} (e + f x) \right]^2}{1 + \tan \left[ \frac{1}{2} (e + f x) \right]^2} \right)^m \left( \left( A \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\tan \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \tan \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \left( 1 + \tan \left[ \frac{1}{2} (e + f x) \right]^2 \right)^2 \right) \right) /$$

$$\left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\tan \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \tan \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] + \right.$$

$$\left. 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\tan \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \tan \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] - \right.$$

$$\left. (a+b) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\tan \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \tan \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \right) \tan \left[ \frac{1}{2} (e + f x) \right]^2 \right) +$$

$$\left( C \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\tan \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \tan \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \left( 1 + \tan \left[ \frac{1}{2} (e + f x) \right]^2 \right)^2 \right) /$$

$$\left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\tan \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \tan \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] + \right.$$

$$\left. 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\tan \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \tan \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] - \right.$$

$$\left. (a+b) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\tan \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a-b) \tan \left[ \frac{1}{2} (e + f x) \right]^2}{a+b} \right] \right) \tan \left[ \frac{1}{2} (e + f x) \right]^2 \right) -$$

$$\begin{aligned}
& \left( 4 \text{C AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \left( 1 + \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) \right) / \\
& \left( 3 (a+b) \text{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \text{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - \right. \\
& \left. \left. (a+b) (2+m) \text{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) + \\
& \left( 4 \text{C AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) / \\
& \left( 3 (a+b) \text{AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \text{AppellF1} \left[ \frac{3}{2}, 3+m, 1-m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - \right. \\
& \left. \left. (a+b) (3+m) \text{AppellF1} \left[ \frac{3}{2}, 4+m, -m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) \right) / \\
& \left( f \left( 1 + \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) \right)^3 \left( \frac{1}{\left( 1 + \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^3} 6 (a+b) m \text{Tan} \left[ \frac{1}{2} (e+fx) \right] \left( -\frac{b \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \text{Tan} \left[ \frac{1}{2} (e+fx) \right]}{1 + \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2} - \right. \right. \right. \\
& \left. \left. \frac{\text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \text{Tan} \left[ \frac{1}{2} (e+fx) \right] (b - b \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2)}{\left( 1 + \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^2} \right) \left( a + \frac{b - b \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{1 + \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2} \right)^{-1+m} \right) \\
& \left( \left( \text{A AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \left( 1 + \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) \right) \right) / \\
& \left( 3 (a+b) \text{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (1+m) \right. \\
& \quad \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \Bigg) + \\
& \left( C \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^2 \right) \Bigg) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) + \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (1+m) \right. \\
& \quad \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \Bigg) - \\
& \left( 4 C \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^2 \right) \Bigg) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) + \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (2+m) \right. \\
& \quad \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \Bigg) + \\
& \left( 4 C \operatorname{AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \Bigg) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) +
\end{aligned}$$

$$\begin{aligned}
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - (a+b) (3+m) \right. \\
& \quad \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 4+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \Bigg) - \\
& \frac{1}{\left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^4} 18 (a+b) \operatorname{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \left( a + \frac{b-b \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{1 + \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2} \right)^m \\
& \left( \left( \operatorname{A} \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^2 \right) / \right. \\
& \quad \left. \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) + \right. \\
& \quad 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - (a+b) (1+m) \right. \\
& \quad \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \Bigg) + \\
& \left( \operatorname{C} \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^2 \right) / \right. \\
& \quad \left. \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) + \right. \\
& \quad 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - (a+b) (1+m) \right. \\
& \quad \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \Bigg) - \\
& \left( 4 \operatorname{C} \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) \right) / \right.
\end{aligned}$$

$$\begin{aligned}
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (2+m) \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) + \\
& \left( 4 C \operatorname{AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (3+m) \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 4+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) + \\
& \frac{1}{\left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^3} 3 (a+b) \operatorname{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \left( a + \frac{b - b \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2} \right)^m \\
& \left( \left( A \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^2 \right) / \right. \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (1+m) \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) +
\end{aligned}$$



$$\begin{aligned}
& \left( C \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^2 \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - (a+b) (1+m) \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) - \\
& \left( 4 C \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^2 \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - (a+b) (2+m) \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) + \\
& \left( 4 C \operatorname{AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - (a+b) (3+m) \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 4+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\left(1 + \tan\left[\frac{1}{2}(e + fx)\right]\right)^3} 6(a + b) \tan\left[\frac{1}{2}(e + fx)\right] \left( a + \frac{b - b \tan\left[\frac{1}{2}(e + fx)\right]^2}{1 + \tan\left[\frac{1}{2}(e + fx)\right]^2} \right)^m \\
& \left( \left( 2 \text{A AppellF1}\left[\frac{1}{2}, 1 + m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] \text{Sec}\left[\frac{1}{2}(e + fx)\right]^2 \tan\left[\frac{1}{2}(e + fx)\right] \right. \right. \\
& \left. \left. \left(1 + \tan\left[\frac{1}{2}(e + fx)\right]^2\right) \right) \right) / \left( 3(a + b) \text{AppellF1}\left[\frac{1}{2}, 1 + m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] \right) + \right. \\
& 2 \left( (a - b) m \text{AppellF1}\left[\frac{3}{2}, 1 + m, 1 - m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] - (a + b)(1 + m) \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{2}, 2 + m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] \right) \tan\left[\frac{1}{2}(e + fx)\right]^2 \right) + \\
& \left( 2 \text{C AppellF1}\left[\frac{1}{2}, 1 + m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] \text{Sec}\left[\frac{1}{2}(e + fx)\right]^2 \tan\left[\frac{1}{2}(e + fx)\right] \right. \\
& \left. \left. \left(1 + \tan\left[\frac{1}{2}(e + fx)\right]^2\right) \right) \right) / \left( 3(a + b) \text{AppellF1}\left[\frac{1}{2}, 1 + m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] \right) + \right. \\
& 2 \left( (a - b) m \text{AppellF1}\left[\frac{3}{2}, 1 + m, 1 - m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] - (a + b)(1 + m) \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{2}, 2 + m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] \right) \tan\left[\frac{1}{2}(e + fx)\right]^2 \right) + \\
& \left( \left( 1 / (3(a + b)) (a - b) m \text{AppellF1}\left[\frac{3}{2}, 1 + m, 1 - m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] \right. \right. \\
& \left. \left. \text{Sec}\left[\frac{1}{2}(e + fx)\right]^2 \tan\left[\frac{1}{2}(e + fx)\right] - \frac{1}{3}(1 + m) \text{AppellF1}\left[\frac{3}{2}, 2 + m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. - \frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b} \right] \text{Sec}\left[\frac{1}{2}(e + fx)\right]^2 \tan\left[\frac{1}{2}(e + fx)\right] \right) \right) \left(1 + \tan\left[\frac{1}{2}(e + fx)\right]^2\right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (1+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) + \\
& \left( C \left( 1 / (3 (a+b)) (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right] - \frac{1}{3} (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right] \right) \right) \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^2 \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (1+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) - \\
& \left( 4 C \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right] \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (2+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) -
\end{aligned}$$



$$\begin{aligned}
& (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \\
& \frac{1}{3}(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \Bigg) + \\
& 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left( (a-b) m \left( -\frac{1}{5(a+b)} 3(a-b)(1-m) \operatorname{AppellF1}\left[\frac{5}{2}, 1+m, 2-m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(1+m) \operatorname{AppellF1}\left[\frac{5}{2}, 2+m, 1-m, \frac{7}{2}, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) - (a+b)(1+m) \right. \\
& \left. \left( \frac{1}{5(a+b)} 3(a-b) m \operatorname{AppellF1}\left[\frac{5}{2}, 2+m, 1-m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(2+m) \operatorname{AppellF1}\left[\frac{5}{2}, 3+m, -m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \Bigg) / \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \quad \left. \left. -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - \right. \right. \\
& \quad \left. \left. (a+b)(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 - \\
& \left( C \operatorname{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \right. \\
& \left. \left( 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - (a+b)(1+m) \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 3(a+b) \left( \frac{1}{3(a+b)} (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right. \\
& \quad \operatorname{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right] - \frac{1}{3} (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, \right. \\
& \quad \left. \left. -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right] \right) + 2 \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \\
& \left( (a-b) m \left( -\frac{1}{5(a+b)} 3(a-b)(1-m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1+m, 2-m, \frac{7}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right. \right. \\
& \quad \operatorname{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right] - \frac{3}{5} (1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 2+m, 1-m, \frac{7}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, \right. \\
& \quad \left. \left. -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right] \right) - (a+b)(1+m) \left( \frac{1}{5(a+b)} 3(a-b) m \operatorname{AppellF1} \left[ \frac{5}{2}, \right. \right. \\
& \quad 2+m, 1-m, \frac{7}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right] - \frac{3}{5} (2+m) \\
& \quad \left. \left. \left. \operatorname{AppellF1} \left[ \frac{5}{2}, 3+m, -m, \frac{7}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right] \right) \right) \right) \right) \right) / \\
& \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] + 2 \left( (a-b) m \operatorname{AppellF1} \left[ \right. \right. \right. \\
& \quad \left. \left. \left. \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - \right. \right. \\
& \quad \left. \left. (a+b)(1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^2 + \\
& \left( 4 C \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) \right) \\
& \left( 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - (a+b)(2+m) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \text{AppellF1}\left[\frac{3}{2}, 3+m, -m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& 3(a+b) \left( \frac{1}{3(a+b)}(a-b)m \text{AppellF1}\left[\frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right. \\
& \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3}(2+m) \text{AppellF1}\left[\frac{3}{2}, 3+m, -m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
& \left. \left. -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \left. \right) + 2 \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \\
& \left( (a-b)m \left( -\frac{1}{5(a+b)} 3(a-b)(1-m) \text{AppellF1}\left[\frac{5}{2}, 2+m, 2-m, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right. \right. \\
& \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(2+m) \text{AppellF1}\left[\frac{5}{2}, 3+m, 1-m, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
& \left. \left. -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \left. \right) - (a+b)(2+m) \left( \frac{1}{5(a+b)} 3(a-b)m \text{AppellF1}\left[\frac{5}{2}, \right. \right. \\
& \left. \left. 3+m, 1-m, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(3+m) \\
& \left. \left. \text{AppellF1}\left[\frac{5}{2}, 4+m, -m, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \Big/ \\
& \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + 2 \left( (a-b)m \text{AppellF1}\left[ \right. \right. \right. \\
& \left. \left. \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - \right. \\
& \left. \left. (a+b)(2+m) \text{AppellF1}\left[\frac{3}{2}, 3+m, -m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right)^2 - \\
& \left( 4 \text{C AppellF1}\left[\frac{1}{2}, 3+m, -m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \left( 2 \left( (a-b)m \text{AppellF1}\left[\frac{3}{2}, 3+m, 1-m, \right. \right. \right.
\end{aligned}$$





Optimal (type 5, 173 leaves, 5 steps):

$$\frac{3 B \operatorname{Cos}[c+d x]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6}(2+3 m), \frac{1}{6}(8+3 m), \operatorname{Cos}[c+d x]^2\right] \operatorname{Sin}[c+d x]}{b d(2+3 m)(b \operatorname{Cos}[c+d x])^{1/3} \sqrt{\operatorname{Sin}[c+d x]^2}}$$

$$\frac{3 C \operatorname{Cos}[c+d x]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6}(5+3 m), \frac{1}{6}(11+3 m), \operatorname{Cos}[c+d x]^2\right] \operatorname{Sin}[c+d x]}{b d(5+3 m)(b \operatorname{Cos}[c+d x])^{1/3} \sqrt{\operatorname{Sin}[c+d x]^2}}$$

Result (type 6, 4959 leaves):

$$\begin{aligned} & \left( 2 \left( \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2 \right)^{\frac{5}{3}+m} \operatorname{Cos}[c+d x]^{4/3} \left( \operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \right)^{-\frac{1}{3}+m} \right. \\ & \left. \left( \frac{1}{2} C \operatorname{Cos}[c+d x]^{\frac{2}{3}+m} + B \operatorname{Cos}[c+d x]^{\frac{5}{3}+m} + \frac{1}{2} C \operatorname{Cos}[c+d x]^{\frac{2}{3}+m} \operatorname{Cos}[2(c+d x)] + \frac{1}{2} i C \operatorname{Cos}[c+d x]^{\frac{2}{3}+m} \operatorname{Sin}[2(c+d x)] + \right. \right. \\ & \left. \left. \operatorname{Sec}[c+d x] \left( -\frac{1}{2} i C \operatorname{Cos}[c+d x]^{\frac{2}{3}+m} \operatorname{Cos}[2(c+d x)] \operatorname{Sin}[c+d x] + B \operatorname{Cos}[c+d x]^{\frac{2}{3}+m} \operatorname{Sin}[c+d x]^2 + \right. \right. \right. \\ & \left. \left. \left. \operatorname{Sin}[c+d x] \left( -\frac{1}{2} i C \operatorname{Cos}[c+d x]^{\frac{2}{3}+m} + \frac{1}{2} C \operatorname{Cos}[c+d x]^{\frac{2}{3}+m} \operatorname{Sin}[2(c+d x)] \right) \right) \right) \right) \\ & \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \left( \left( 9(B+C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \right) / \right. \\ & \left( 9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] + 2 \left( -(5+3 m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \right. \\ & \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] + (1-3 m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right) + \\ & \left( 5(-B+C) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right) / \\ & \left( -15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] + 2 \left( (5+3 m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \right. \\ & \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] + (-1+3 m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{4}{3}-m, \frac{5}{3}+m, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right) \right) / \\ & \left( d(b \operatorname{Cos}[c+d x])^{4/3} \left( \left( \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2 \right)^{\frac{2}{3}+m} \left( \operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \right)^{-\frac{1}{3}+m} \right. \right. \\ & \left. \left. \left( \left( 9(B+C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \right) / \left( 9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \right. \right. \right. \\ & \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] + 2 \left( -(5+3 m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \right) \right) \right) \right) \end{aligned}$$

$$\begin{aligned}
& (1-3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 + \\
& \left(5(-B+C) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2\right) / \\
& \left(-15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& 2 \left( (5+3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \left. \left. (-1+3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{4}{3}-m, \frac{5}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) - \\
& 2 \left(\frac{5}{3}+m\right) \left(\cos\left[\frac{1}{2}(c+dx)\right]^2\right)^{\frac{2}{3}+m} \left(\cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right)^{-\frac{1}{3}+m} \sin\left[\frac{1}{2}(c+dx)\right]^2 \\
& \left( \left(9(B+C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) / \left(9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]^2\right) + 2 \left(- (5+3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \left. \left. (1-3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
& \left(5(-B+C) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2\right) / \\
& \left(-15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& 2 \left( (5+3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \left. \left. (-1+3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{4}{3}-m, \frac{5}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
& 2 \left(-\frac{1}{3}+m\right) \left(\cos\left[\frac{1}{2}(c+dx)\right]^2\right)^{\frac{5}{3}+m} \left(\cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right)^{-\frac{4}{3}+m} \tan\left[\frac{1}{2}(c+dx)\right] \\
& \left(-\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] + \cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right) \\
& \left( \left(9(B+C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) / \right. \\
& \left. \left(9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}-m, \frac{5}{3}+m, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right.
\end{aligned}$$





$$\begin{aligned}
& \frac{3}{5} \left( \frac{1}{3} - m \right) \text{AppellF1} \left[ \frac{5}{2}, \frac{4}{3} - m, \frac{5}{3} + m, \frac{7}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \tan \left[ \frac{1}{2} (c + d x) \right] \Bigg) + \\
& 2 \tan \left[ \frac{1}{2} (c + d x) \right]^2 \left( (5 + 3 m) \left( -\frac{5}{7} \left( \frac{8}{3} + m \right) \text{AppellF1} \left[ \frac{7}{2}, \frac{1}{3} - m, \frac{11}{3} + m, \frac{9}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] \right. \right. \\
& \quad \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \tan \left[ \frac{1}{2} (c + d x) \right] + \frac{5}{7} \left( \frac{1}{3} - m \right) \text{AppellF1} \left[ \frac{7}{2}, \frac{4}{3} - m, \frac{8}{3} + m, \frac{9}{2}, \right. \\
& \quad \left. \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \tan \left[ \frac{1}{2} (c + d x) \right] \Bigg) + (-1 + 3 m) \\
& \left( -\frac{5}{7} \left( \frac{5}{3} + m \right) \text{AppellF1} \left[ \frac{7}{2}, \frac{4}{3} - m, \frac{8}{3} + m, \frac{9}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \tan \left[ \frac{1}{2} (c + d x) \right] + \right. \\
& \quad \left. \frac{5}{7} \left( \frac{4}{3} - m \right) \text{AppellF1} \left[ \frac{7}{2}, \frac{7}{3} - m, \frac{5}{3} + m, \frac{9}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \tan \left[ \frac{1}{2} (c + d x) \right] \right) \Bigg) \Bigg) \Bigg) / \\
& \left( -15 \text{AppellF1} \left[ \frac{3}{2}, \frac{1}{3} - m, \frac{5}{3} + m, \frac{5}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] + 2 \left( (5 + 3 m) \text{AppellF1} \left[ \frac{5}{2}, \frac{1}{3} - m, \right. \right. \right. \\
& \quad \left. \left. \frac{8}{3} + m, \frac{7}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] + \right. \\
& \quad \left. \left. (-1 + 3 m) \text{AppellF1} \left[ \frac{5}{2}, \frac{4}{3} - m, \frac{5}{3} + m, \frac{7}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (c + d x) \right]^2 \right) \Bigg) \Bigg) \Bigg) /
\end{aligned}$$

■ **Problem 224: Result more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^{5/2} (b \cos [c + d x])^n (B \cos [c + d x] + C \cos [c + d x]^2) dx$$

Optimal (type 5, 163 leaves, 5 steps):

$$\begin{aligned}
& - \left( 2 B \cos [c + d x]^{9/2} (b \cos [c + d x])^n \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{4} (9 + 2 n), \frac{1}{4} (13 + 2 n), \cos [c + d x]^2 \right] \sin [c + d x] \right) / \\
& \quad \left( d (9 + 2 n) \sqrt{\sin [c + d x]^2} \right) - \\
& \left( 2 C \cos [c + d x]^{11/2} (b \cos [c + d x])^n \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{4} (11 + 2 n), \frac{1}{4} (15 + 2 n), \cos [c + d x]^2 \right] \sin [c + d x] \right) / \\
& \quad \left( d (11 + 2 n) \sqrt{\sin [c + d x]^2} \right)
\end{aligned}$$

Result (type 5, 450 leaves):

$$\frac{1}{8d} \operatorname{Cos}[c+dx]^{3/2} (b \operatorname{Cos}[c+dx])^n \operatorname{Csc}[c+dx] \left( -\frac{2 C \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1}{4}(3+2n), \frac{1}{4}(7+2n), \operatorname{Cos}[c+dx]^2\right]}{3+2n} + \frac{8 C \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{4}(3+2n), \frac{1}{4}(7+2n), \operatorname{Cos}[c+dx]^2\right]}{3+2n} + \frac{6 B \operatorname{Cos}[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{4}(5+2n), \frac{1}{4}(9+2n), \operatorname{Cos}[c+dx]^2\right]}{\frac{5}{2}+n} + \frac{6 C \operatorname{Cos}[c+dx]^2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{4}(7+2n), \frac{1}{4}(11+2n), \operatorname{Cos}[c+dx]^2\right]}{\frac{7}{2}+n} - \frac{6 C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(3+2n), \frac{1}{4}(7+2n), \operatorname{Cos}[c+dx]^2\right]}{3+2n} - \frac{6 B \operatorname{Cos}[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(5+2n), \frac{1}{4}(9+2n), \operatorname{Cos}[c+dx]^2\right]}{\frac{5}{2}+n} - \frac{8 C \operatorname{Cos}[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(7+2n), \frac{1}{4}(11+2n), \operatorname{Cos}[c+dx]^2\right]}{7+2n} - \frac{4 B \operatorname{Cos}[c+dx]^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(9+2n), \frac{1}{4}(13+2n), \operatorname{Cos}[c+dx]^2\right]}{9+2n} - \frac{2 C \operatorname{Cos}[c+dx]^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(11+2n), \frac{1}{4}(15+2n), \operatorname{Cos}[c+dx]^2\right]}{11+2n} \right) \sqrt{\operatorname{Sin}[c+dx]^2}$$

- **Problem 228: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(b \operatorname{Cos}[c+dx])^n (B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{3/2}} dx$$

Optimal (type 5, 163 leaves, 5 steps):

$$- \left( 2 B \sqrt{\operatorname{Cos}[c+dx]} (b \operatorname{Cos}[c+dx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(1+2n), \frac{1}{4}(5+2n), \operatorname{Cos}[c+dx]^2\right] \operatorname{Sin}[c+dx] \right) / \left( d(1+2n) \sqrt{\operatorname{Sin}[c+dx]^2} \right) - \left( 2 C \operatorname{Cos}[c+dx]^{3/2} (b \operatorname{Cos}[c+dx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(3+2n), \frac{1}{4}(7+2n), \operatorname{Cos}[c+dx]^2\right] \operatorname{Sin}[c+dx] \right) / \left( d(3+2n) \sqrt{\operatorname{Sin}[c+dx]^2} \right)$$

Result (type 6, 4951 leaves):

$$\left( 2 \left( \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \right)^{\frac{3}{2}+n} \operatorname{Cos}[c+dx]^{-n} (b \operatorname{Cos}[c+dx])^n \left( \operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right)^{-\frac{1}{2}+n} \right)$$

$$\begin{aligned}
& \left( \frac{1}{2} C \cos [c+d x]^{\frac{1}{2}+n} + B \cos [c+d x]^{\frac{3}{2}+n} + \frac{1}{2} C \cos [c+d x]^{\frac{1}{2}+n} \cos [2(c+d x)] + \frac{1}{2} i C \cos [c+d x]^{\frac{1}{2}+n} \sin [2(c+d x)] + \right. \\
& \quad \left. \sec [c+d x] \left( -\frac{1}{2} i C \cos [c+d x]^{\frac{1}{2}+n} \cos [2(c+d x)] \sin [c+d x] + B \cos [c+d x]^{\frac{1}{2}+n} \sin [c+d x]^2 + \right. \right. \\
& \quad \left. \left. \sin [c+d x] \left( -\frac{1}{2} i C \cos [c+d x]^{\frac{1}{2}+n} + \frac{1}{2} C \cos [c+d x]^{\frac{1}{2}+n} \sin [2(c+d x)] \right) \right) \right) \\
& \tan \left[ \frac{1}{2}(c+d x) \right] \left( \left( 9(B+C) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) / \right. \\
& \quad \left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \left( -(3+2n) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}-n, \frac{5}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + (1-2n) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) + \\
& \quad \left( 5(-B+C) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) / \\
& \quad \left( -5 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \left( (3+2n) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2}-n, \frac{5}{2}+n, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + (-1+2n) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) \right) / \\
& \left( 3d \left( \frac{1}{3} \left( \cos \left[ \frac{1}{2}(c+d x) \right]^2 \right)^{\frac{1}{2}+n} \left( \cos [c+d x] \sec \left[ \frac{1}{2}(c+d x) \right]^2 \right)^{-\frac{1}{2}+n} \left( \left( 9(B+C) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, \right. \right. \right. \right. \right. \\
& \quad \left. \left. -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) / \left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \right. \\
& \quad \left( -(3+2n) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}-n, \frac{5}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \\
& \quad \left. (1-2n) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) + \\
& \quad \left( 5(-B+C) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) / \left( -5 \right. \\
& \quad \left. \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \left( (3+2n) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2}-n, \frac{5}{2}+n, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + (-1+2n) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) \right) - \\
& \quad \frac{2}{3} \left( \frac{3}{2}+n \right) \left( \cos \left[ \frac{1}{2}(c+d x) \right]^2 \right)^{\frac{1}{2}+n} \left( \cos [c+d x] \sec \left[ \frac{1}{2}(c+d x) \right]^2 \right)^{-\frac{1}{2}+n} \sin \left[ \frac{1}{2}(c+d x) \right]^2
\end{aligned}$$









$$\left( -5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} - n, \frac{3}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] + \left( (3 + 2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2} - n, \frac{5}{2} + n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \right. \\ \left. \left. (-1 + 2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2} - n, \frac{3}{2} + n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \right)^2 \Bigg) \Bigg)$$

■ **Problem 229: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(b \operatorname{Cos}[c + dx])^n (B \operatorname{Cos}[c + dx] + C \operatorname{Cos}[c + dx]^2)}{\operatorname{Cos}[c + dx]^{5/2}} dx$$

Optimal (type 5, 163 leaves, 5 steps):

$$\frac{2 B (b \operatorname{Cos}[c + dx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(-1 + 2n), \frac{1}{4}(3 + 2n), \operatorname{Cos}[c + dx]^2\right] \operatorname{Sin}[c + dx]}{d(1 - 2n) \sqrt{\operatorname{Cos}[c + dx]} \sqrt{\operatorname{Sin}[c + dx]^2}} - \frac{\left(2 C \sqrt{\operatorname{Cos}[c + dx]} (b \operatorname{Cos}[c + dx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(1 + 2n), \frac{1}{4}(5 + 2n), \operatorname{Cos}[c + dx]^2\right] \operatorname{Sin}[c + dx]\right)}{d(1 + 2n) \sqrt{\operatorname{Sin}[c + dx]^2}}$$

Result (type 6, 4842 leaves):

$$\left(6 \sqrt{\operatorname{Cos}[c + dx]} (b \operatorname{Cos}[c + dx])^n \left( B \operatorname{Cos}[c + dx]^{\frac{1}{2}+n} + \operatorname{Sec}[c + dx] \left( \frac{1}{2} C \operatorname{Cos}[c + dx]^{\frac{1}{2}+n} + \frac{1}{2} C \operatorname{Cos}[c + dx]^{\frac{1}{2}+n} \operatorname{Cos}[2(c + dx)] + \frac{1}{2} i C \operatorname{Cos}[c + dx]^{\frac{1}{2}+n} \operatorname{Sin}[2(c + dx)] \right) \right) \right. \\ \left. \operatorname{Sec}[c + dx]^2 \left( -\frac{1}{2} i C \operatorname{Cos}[c + dx]^{\frac{1}{2}+n} \operatorname{Cos}[2(c + dx)] \operatorname{Sin}[c + dx] + B \operatorname{Cos}[c + dx]^{\frac{1}{2}+n} \operatorname{Sin}[c + dx]^2 + \right. \right. \\ \left. \left. \operatorname{Sin}[c + dx] \left( -\frac{1}{2} i C \operatorname{Cos}[c + dx]^{\frac{1}{2}+n} + \frac{1}{2} C \operatorname{Cos}[c + dx]^{\frac{1}{2}+n} \operatorname{Sin}[2(c + dx)] \right) \right) \right) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \\ \left( \left( (B - C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} - n, \frac{1}{2} + n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \left( -1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \right) \right) \right) / \\ \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} - n, \frac{1}{2} + n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] - \left( (1 + 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} - n, \frac{3}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, \right. \right. \right. \\ \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] + (-1 + 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2} - n, \frac{1}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \right) + \\ \left( 2 B \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2} - n, \frac{1}{2} + n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) / \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2} - n, \frac{1}{2} + n, \frac{3}{2}, \right. \right. \\ \left. \left. \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] - \left( (1 + 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2} - n, \frac{3}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \right.$$







$$\begin{aligned}
& 3 \left( -\frac{1}{3} \left( \frac{1}{2} + n \right) \text{AppellF1} \left[ \frac{3}{2}, \frac{3}{2} - n, \frac{3}{2} + n, \frac{5}{2}, \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \text{Tan} \left[ \frac{1}{2} (c + d x) \right] + \right. \\
& \quad \left. \frac{1}{3} \left( \frac{3}{2} - n \right) \text{AppellF1} \left[ \frac{3}{2}, \frac{5}{2} - n, \frac{1}{2} + n, \frac{5}{2}, \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \text{Tan} \left[ \frac{1}{2} (c + d x) \right] \right) - \\
& \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \left( (1 + 2n) \left( -\frac{3}{5} \left( \frac{3}{2} + n \right) \text{AppellF1} \left[ \frac{5}{2}, \frac{3}{2} - n, \frac{5}{2} + n, \frac{7}{2}, \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \right. \right. \\
& \quad \left. \left. \text{Tan} \left[ \frac{1}{2} (c + d x) \right] + \frac{3}{5} \left( \frac{3}{2} - n \right) \text{AppellF1} \left[ \frac{5}{2}, \frac{5}{2} - n, \frac{3}{2} + n, \frac{7}{2}, \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \right. \right. \\
& \quad \left. \left. \text{Tan} \left[ \frac{1}{2} (c + d x) \right] \right) + (-3 + 2n) \left( -\frac{3}{5} \left( \frac{1}{2} + n \right) \text{AppellF1} \left[ \frac{5}{2}, \frac{5}{2} - n, \frac{3}{2} + n, \frac{7}{2}, \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \text{Tan} \left[ \frac{1}{2} (c + d x) \right] + \frac{3}{5} \left( \frac{5}{2} - n \right) \text{AppellF1} \left[ \frac{5}{2}, \frac{7}{2} - n, \frac{1}{2} + n, \frac{7}{2}, \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \text{Tan} \left[ \frac{1}{2} (c + d x) \right] \right) \right) \right) / \left( 3 \text{AppellF1} \left[ \frac{1}{2}, \frac{3}{2} - n, \frac{1}{2} + n, \frac{3}{2}, \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2, \right. \right. \\
& \quad \left. \left. -\text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right] - \left( (1 + 2n) \text{AppellF1} \left[ \frac{3}{2}, \frac{3}{2} - n, \frac{3}{2} + n, \frac{5}{2}, \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (-3 + 2n) \text{AppellF1} \left[ \frac{3}{2}, \frac{5}{2} - n, \frac{1}{2} + n, \frac{5}{2}, \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right] \right) \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right) \right) \right)
\end{aligned}$$

■ **Problem 230: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(b \cos [c + d x])^n (B \cos [c + d x] + C \cos [c + d x]^2)}{\cos [c + d x]^{7/2}} dx$$

Optimal (type 5, 163 leaves, 5 steps):

$$\begin{aligned}
& \frac{2 B (b \cos [c + d x])^n \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{4} (-3 + 2n), \frac{1}{4} (1 + 2n), \cos [c + d x]^2 \right] \sin [c + d x]}{d (3 - 2n) \cos [c + d x]^{3/2} \sqrt{\sin [c + d x]^2}} + \\
& \frac{2 C (b \cos [c + d x])^n \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{4} (-1 + 2n), \frac{1}{4} (3 + 2n), \cos [c + d x]^2 \right] \sin [c + d x]}{d (1 - 2n) \sqrt{\cos [c + d x]} \sqrt{\sin [c + d x]^2}}
\end{aligned}$$

Result (type 6, 4948 leaves):

$$\left( 2 \cos [c + d x]^{-n} (b \cos [c + d x])^n \right. \\
\left. \left( B \cos [c + d x]^{-\frac{1}{2}+n} + \text{Sec} [c + d x]^2 \left( \frac{1}{2} C \cos [c + d x]^{\frac{1}{2}+n} + \frac{1}{2} C \cos [c + d x]^{\frac{1}{2}+n} \cos [2 (c + d x)] + \frac{1}{2} i C \cos [c + d x]^{\frac{1}{2}+n} \sin [2 (c + d x)] \right) \right) + \right.$$

$$\begin{aligned}
& \operatorname{Sec}[c+dx]^3 \left( -\frac{1}{2} i C \operatorname{Cos}[c+dx]^{\frac{1}{2}+n} \operatorname{Cos}[2(c+dx)] \operatorname{Sin}[c+dx] + B \operatorname{Cos}[c+dx]^{\frac{1}{2}+n} \operatorname{Sin}[c+dx]^2 + \right. \\
& \left. \operatorname{Sin}[c+dx] \left( -\frac{1}{2} i C \operatorname{Cos}[c+dx]^{\frac{1}{2}+n} + \frac{1}{2} C \operatorname{Cos}[c+dx]^{\frac{1}{2}+n} \operatorname{Sin}[2(c+dx)] \right) \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left( 1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^{-\frac{5}{2}+n} \\
& \left( \frac{1}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^{-\frac{1}{2}+n} \left( \left( 9(B+C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) / \right. \\
& \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \left( (1-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + (5-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
& \left( 5(-B+C) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) / \\
& \left( -5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \left( (-1+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}-n, \frac{1}{2}+n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + (-5+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \left. \right) / \\
& \left( 3d \left( -\frac{2}{3} \left( -\frac{5}{2}+n \right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \left( 1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^{-\frac{7}{2}+n} \left( \frac{1}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^{-\frac{1}{2}+n} \right. \right. \\
& \left( \left( 9(B+C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) / \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{3}{2}, \right. \right. \right. \\
& \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \left( (1-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \right. \\
& \left. \left. \left. (5-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) + \right. \\
& \left( 5(-B+C) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) / \\
& \left( -5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \left. \left( (-1+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}-n, \frac{1}{2}+n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + (-5+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \right. \right. \right.
\end{aligned}$$









$$\begin{aligned}
 & \left( -\frac{5}{7} \left( -\frac{1}{2} + n \right) \text{AppellF1} \left[ \frac{7}{2}, \frac{7}{2} - n, \frac{1}{2} + n, \frac{9}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \tan \left[ \frac{1}{2} (c + d x) \right] + \right. \\
 & \quad \left. \frac{5}{7} \left( \frac{7}{2} - n \right) \text{AppellF1} \left[ \frac{7}{2}, \frac{9}{2} - n, -\frac{1}{2} + n, \frac{9}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \tan \left[ \frac{1}{2} (c + d x) \right] \right) \Bigg) / \\
 & \left( -5 \text{AppellF1} \left[ \frac{3}{2}, \frac{5}{2} - n, -\frac{1}{2} + n, \frac{5}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] + \left( (-1 + 2n) \text{AppellF1} \left[ \frac{5}{2}, \frac{5}{2} - n, \right. \right. \right. \\
 & \quad \left. \left. \frac{1}{2} + n, \frac{7}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] + \right. \\
 & \quad \left. \left. (-5 + 2n) \text{AppellF1} \left[ \frac{5}{2}, \frac{7}{2} - n, -\frac{1}{2} + n, \frac{7}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (c + d x) \right]^2 \right) \Bigg)
 \end{aligned}$$

▪ **Problem 231: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(b \cos [c + d x])^n (B \cos [c + d x] + C \cos [c + d x]^2)}{\cos [c + d x]^{9/2}} dx$$

Optimal (type 5, 163 leaves, 5 steps):

$$\begin{aligned}
 & \frac{2 B (b \cos [c + d x])^n \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{4} (-5 + 2 n), \frac{1}{4} (-1 + 2 n), \cos [c + d x]^2 \right] \sin [c + d x]}{d (5 - 2 n) \cos [c + d x]^{5/2} \sqrt{\sin [c + d x]^2}} + \\
 & \frac{2 C (b \cos [c + d x])^n \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{4} (-3 + 2 n), \frac{1}{4} (1 + 2 n), \cos [c + d x]^2 \right] \sin [c + d x]}{d (3 - 2 n) \cos [c + d x]^{3/2} \sqrt{\sin [c + d x]^2}}
 \end{aligned}$$

Result (type 6, 4948 leaves):

$$\begin{aligned}
 & \left( 2 \cos [c + d x]^{-n} (b \cos [c + d x])^n \right. \\
 & \quad \left( B \cos [c + d x]^{-\frac{3}{2}+n} + \text{Sec} [c + d x]^3 \left( \frac{1}{2} C \cos [c + d x]^{\frac{1}{2}+n} + \frac{1}{2} C \cos [c + d x]^{\frac{1}{2}+n} \cos [2 (c + d x)] + \frac{1}{2} i C \cos [c + d x]^{\frac{1}{2}+n} \sin [2 (c + d x)] \right) \right. \\
 & \quad \left. \text{Sec} [c + d x]^4 \left( -\frac{1}{2} i C \cos [c + d x]^{\frac{1}{2}+n} \cos [2 (c + d x)] \sin [c + d x] + B \cos [c + d x]^{\frac{1}{2}+n} \sin [c + d x]^2 + \right. \right. \\
 & \quad \left. \left. \sin [c + d x] \left( -\frac{1}{2} i C \cos [c + d x]^{\frac{1}{2}+n} + \frac{1}{2} C \cos [c + d x]^{\frac{1}{2}+n} \sin [2 (c + d x)] \right) \right) \right) \tan \left[ \frac{1}{2} (c + d x) \right] \left( 1 - \tan \left[ \frac{1}{2} (c + d x) \right]^2 \right)^{-\frac{7}{2}+n} \\
 & \quad \left( \frac{1}{1 + \tan \left[ \frac{1}{2} (c + d x) \right]^2} \right)^{-\frac{3}{2}+n} \left( \left( 9 (B + C) \text{AppellF1} \left[ \frac{1}{2}, \frac{7}{2} - n, -\frac{3}{2} + n, \frac{3}{2}, \tan \left[ \frac{1}{2} (c + d x) \right]^2, -\tan \left[ \frac{1}{2} (c + d x) \right]^2 \right] \right) \right) /
 \end{aligned}$$











$$(-7 + 2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2} - n, -\frac{3}{2} + n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right)^2\right)^2\right)^2\right)$$

■ **Problem 232: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[e + fx])^m (B \cos[e + fx] + C \cos[e + fx]^2) dx$$

Optimal (type 5, 173 leaves, 4 steps):

$$-\frac{(C - B(2 + m))(a + a \cos[e + fx])^m \sin[e + fx]}{f(1 + m)(2 + m)} + \frac{C(a + a \cos[e + fx])^{1+m} \sin[e + fx]}{af(2 + m)} + \frac{1}{f(1 + m)(2 + m)} \\ 2^{\frac{1}{2}+m} (Bm(2 + m) + C(1 + m + m^2)) (1 + \cos[e + fx])^{-\frac{1}{2}-m} (a + a \cos[e + fx])^m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{1}{2}(1 - \cos[e + fx])\right] \sin[e + fx]$$

Result (type 5, 356 leaves):

$$\frac{1}{f(-2 + m)(-1 + m)m(1 + m)(2 + m)} i 4^{-1-m} e^{-2i(e+fx)} (1 + e^{i(e+fx)})^{-2m} \left(e^{-\frac{1}{2}i(e+fx)} (1 + e^{i(e+fx)})\right)^{2m} \\ \cos\left[\frac{1}{2}(e + fx)\right]^{-2m} (a(1 + \cos[e + fx]))^m (Cm(2 - m - 2m^2 + m^3) \operatorname{Hypergeometric2F1}[-2 - m, -2m, -1 - m, -e^{i(e+fx)}] + \\ e^{i(e+fx)}(2 + m)(2Bm(2 - 3m + m^2) \operatorname{Hypergeometric2F1}[-1 - m, -2m, -m, -e^{i(e+fx)}] + \\ e^{i(e+fx)}(1 + m)(2Be^{i(e+fx)}(-2 + m)m \operatorname{Hypergeometric2F1}[1 - m, -2m, 2 - m, -e^{i(e+fx)}] + C(-1 + m) \\ (e^{2i(e+fx)}m \operatorname{Hypergeometric2F1}[2 - m, -2m, 3 - m, -e^{i(e+fx)}] + 2(-2 + m) \operatorname{Hypergeometric2F1}[-2m, -m, 1 - m, -e^{i(e+fx)}]))))$$

■ **Problem 233: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[e + fx])^m (B \cos[e + fx] + C \cos[e + fx]^2) dx$$

Optimal (type 6, 295 leaves, 8 steps):

$$\frac{C(a + b \cos[e + fx])^{1+m} \sin[e + fx]}{bf(2 + m)} - \\ \left(\sqrt{2}(a + b)(aC - bB(2 + m)) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -1 - m, \frac{3}{2}, \frac{1}{2}(1 - \cos[e + fx]), \frac{b(1 - \cos[e + fx])}{a + b}\right] (a + b \cos[e + fx])^m \right. \\ \left. \left(\frac{a + b \cos[e + fx]}{a + b}\right)^{-m} \sin[e + fx]\right) / (b^2 f(2 + m) \sqrt{1 + \cos[e + fx]}) + \\ \left(\sqrt{2}(a^2 C + b^2 C(1 + m) - a b B(2 + m)) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2}(1 - \cos[e + fx]), \frac{b(1 - \cos[e + fx])}{a + b}\right] \right. \\ \left. (a + b \cos[e + fx])^m \left(\frac{a + b \cos[e + fx]}{a + b}\right)^{-m} \sin[e + fx]\right) / (b^2 f(2 + m) \sqrt{1 + \cos[e + fx]})$$

Result (type 6, 13480 leaves):

$$\begin{aligned}
& - \left( \left( 6 (a+b) (B \cos[e+fx] (a+b \cos[e+fx])^m + C \cos[e+fx]^2 (a+b \cos[e+fx])^m) \tan\left[\frac{1}{2}(e+fx)\right] \left( a + \frac{b - b \tan\left[\frac{1}{2}(e+fx)\right]^2}{1 + \tan\left[\frac{1}{2}(e+fx)\right]^2} \right)^m \right. \right. \\
& \left. \left( \left( B \operatorname{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \right) \right) / \right. \\
& \left. \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \right. \\
& \left. \left. 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - \right. \right. \right. \\
& \left. \left. \left. (a+b) (1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \right. \\
& \left. \left( C \operatorname{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \right) \right) / \right. \\
& \left. \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \right. \\
& \left. \left. 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - \right. \right. \right. \\
& \left. \left. \left. (a+b) (1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \right. \\
& \left. \left( 2 B \operatorname{AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right) \right) \right) / \right. \\
& \left. \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \right. \\
& \left. \left. 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. (a+b) (2+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) + \\
& \left( 4 \operatorname{C AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& \left. 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - \right. \right. \\
& \left. \left. (a+b) (2+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) - \right. \\
& \left. \left( 4 \operatorname{C AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) / \right. \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& \left. 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - \right. \right. \\
& \left. \left. (a+b) (3+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 4+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) \right) / \\
& \left( f \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right)^3 \left( -\frac{1}{\left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^3} 6 (a+b) m \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right] \left( -\frac{b \operatorname{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]}{1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2} - \right. \right. \right. \\
& \left. \left. \frac{\operatorname{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right] (b - b \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2)}{\left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^2} \right) \left( a + \frac{b - b \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2} \right)^{-1+m} \right. \\
& \left. \left( \left( \operatorname{B AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) \right) / \right)
\end{aligned}$$

$$\begin{aligned}
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + 2 \right. \\
& \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - \right. \\
& \left. (a+b) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \Big) - \\
& \left( C \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^2 \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + 2 \right. \\
& \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - \right. \\
& \left. (a+b) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \Big) - \\
& \left( 2 B \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + 2 \right. \\
& \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - \right. \\
& \left. (a+b) (2+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \Big) + \\
& \left( 4 C \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + 2 \right. \\
& \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (2+m) \right. \\
& \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) - \left( 4 C \operatorname{AppellF1} \left[ \frac{1}{2}, 3+m, \right. \right. \\
& \left. \left. -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) / \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, \right. \right. \\
& \left. \left. -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - \right. \right. \\
& \left. \left. (a+b) (3+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 4+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) + \\
& \frac{1}{\left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^4} 18 (a+b) \operatorname{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \left( a + \frac{b - b \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2} \right)^m \\
& \left( \left( B \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^2 \right) \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& \left. 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - \right. \right. \\
& \left. \left. (a+b) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) - \\
& \left( C \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^2 \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - \right. \\
& \left. \left. (a+b) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) - \\
& \left( 2 B \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - \right. \\
& \left. \left. (a+b) (2+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) + \\
& \left( 4 C \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - \right. \\
& \left. \left. (a+b) (2+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) - \\
& \left( 4 C \operatorname{AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - \right. \\
& \left. \left. (a+b) (3+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 4+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \Bigg) - \\
& \frac{1}{\left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^3} 3 (a+b) \operatorname{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \left( a + \frac{b - b \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2} \right)^m \\
& \left( \left( \operatorname{B} \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^2 \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - \right. \\
& \left. \left. (a+b) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) - \\
& \left( \operatorname{C} \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^2 \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - \right. \\
& \left. \left. (a+b) (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( 2 \text{B AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \left( 1 + \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) / \\
& \left( 3 (a+b) \text{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \text{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - \right. \\
& \left. \left. (a+b) (2+m) \text{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) + \\
& \left( 4 \text{C AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \left( 1 + \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) / \\
& \left( 3 (a+b) \text{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \text{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - \right. \\
& \left. \left. (a+b) (2+m) \text{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) - \\
& \left( 4 \text{C AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) / \\
& \left( 3 (a+b) \text{AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \text{AppellF1} \left[ \frac{3}{2}, 3+m, 1-m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - \right. \\
& \left. \left. (a+b) (3+m) \text{AppellF1} \left[ \frac{3}{2}, 4+m, -m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \text{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) -
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{\left(1 + \tan\left[\frac{1}{2}(e + fx)\right]\right)^3} 6(a + b) \tan\left[\frac{1}{2}(e + fx)\right] \left(a + \frac{b - b \tan\left[\frac{1}{2}(e + fx)\right]^2}{1 + \tan\left[\frac{1}{2}(e + fx)\right]^2}\right)^m \\
& \left( \left( 2 \text{B AppellF1}\left[\frac{1}{2}, 1 + m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] \text{Sec}\left[\frac{1}{2}(e + fx)\right]^2 \tan\left[\frac{1}{2}(e + fx)\right] \right. \right. \\
& \left. \left. \left(1 + \tan\left[\frac{1}{2}(e + fx)\right]^2\right)\right) \right) / \left( 3(a + b) \text{AppellF1}\left[\frac{1}{2}, 1 + m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] \right) + \right. \\
& 2 \left( (a - b) m \text{AppellF1}\left[\frac{3}{2}, 1 + m, 1 - m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] - \right. \\
& \left. (a + b)(1 + m) \text{AppellF1}\left[\frac{3}{2}, 2 + m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] \right) \tan\left[\frac{1}{2}(e + fx)\right]^2 \right) - \\
& \left( 2 \text{C AppellF1}\left[\frac{1}{2}, 1 + m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] \text{Sec}\left[\frac{1}{2}(e + fx)\right]^2 \tan\left[\frac{1}{2}(e + fx)\right] \right. \\
& \left. \left(1 + \tan\left[\frac{1}{2}(e + fx)\right]^2\right)\right) / \left( 3(a + b) \text{AppellF1}\left[\frac{1}{2}, 1 + m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] \right) + \right. \\
& 2 \left( (a - b) m \text{AppellF1}\left[\frac{3}{2}, 1 + m, 1 - m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] - \right. \\
& \left. (a + b)(1 + m) \text{AppellF1}\left[\frac{3}{2}, 2 + m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] \right) \tan\left[\frac{1}{2}(e + fx)\right]^2 \right) + \\
& \left( \text{B} \left( 1 / (3(a + b)) (a - b) m \text{AppellF1}\left[\frac{3}{2}, 1 + m, 1 - m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] \right. \right. \\
& \left. \left. \text{Sec}\left[\frac{1}{2}(e + fx)\right]^2 \tan\left[\frac{1}{2}(e + fx)\right] - \frac{1}{3}(1 + m) \text{AppellF1}\left[\frac{3}{2}, 2 + m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + fx)\right]^2}{a + b}\right] \right) \right. \\
& \left. \left. \text{Sec}\left[\frac{1}{2}(e + fx)\right]^2 \tan\left[\frac{1}{2}(e + fx)\right] \right) \right) \left( 1 + \tan\left[\frac{1}{2}(e + fx)\right]^2 \right)^2 \right) / \left( 3(a + b) \text{AppellF1}\left[\frac{1}{2}, 1 + m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e + fx)\right]^2, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \Bigg] + 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - \right. \\
& \left. (a+b)(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \Bigg) - \\
& \left( C \left( 1 / (3(a+b)) (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3}(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \Bigg) / \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left. \left. -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - \right. \right. \\
& \left. \left. (a+b)(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) - \\
& \left( 2 B \operatorname{AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) / \\
& \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
& \left. 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - \right. \right. \\
& \left. \left. (a+b)(2+m) \operatorname{AppellF1}\left[\frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& \left( 4 C \operatorname{AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) /
\end{aligned}$$



$$\begin{aligned}
& \left( 4 C \left( 1 / (3 (a + b)) (a - b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 3 + m, 1 - m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a - b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a + b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \right. \right. \\
& \quad \left. \left. \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] - \frac{1}{3} (3 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, 4 + m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a - b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a + b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \right. \right. \\
& \quad \left. \left. \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] \right) \right) / \left( 3 (a + b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3 + m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a - b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a + b} \right] + \right. \\
& \quad \left. 2 \left( (a - b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 3 + m, 1 - m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a - b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a + b} \right] - \right. \right. \\
& \quad \left. \left. (a + b) (3 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, 4 + m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a - b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a + b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right) - \\
& \quad \left( B \operatorname{AppellF1} \left[ \frac{1}{2}, 1 + m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a - b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a + b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right)^2 \right. \\
& \quad \left. \left( 2 \left( (a - b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + m, 1 - m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a - b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a + b} \right] - (a + b) (1 + m) \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2 + m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a - b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a + b} \right] \right) \right) \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] + \right. \\
& \quad \left. 3 (a + b) \left( \frac{1}{3 (a + b)} (a - b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + m, 1 - m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a - b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a + b} \right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] - \frac{1}{3} (1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, 2 + m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{(a - b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a + b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] \right) \right) + 2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \\
& \quad \left( (a - b) m \left( -\frac{1}{5 (a + b)} 3 (a - b) (1 - m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + m, 2 - m, \frac{7}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\frac{(a - b) \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2}{a + b} \right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] - \frac{3}{5} (1 + m) \operatorname{AppellF1} \left[ \frac{5}{2}, 2 + m, 1 - m, \frac{7}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \left] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - (a+b)(1+m) \left( \frac{1}{5(a+b)} {}_3F_2(a-b, m) \operatorname{AppellF1}\left[\frac{5}{2}, \right. \right. \right. \\
& \left. \left. \left. 2+m, 1-m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(2+m) \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{5}{2}, 3+m, -m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \Bigg) \Bigg) / \\
& \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] + \right. \\
& \left. 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] - \right. \right. \\
& \left. \left. (a+b)(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 + \\
& \left( C \operatorname{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \right. \\
& \left. \left( 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] - (a+b)(1+m) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \left. 3(a+b) \left( \frac{1}{3(a+b)} (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \right) \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3}(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left. \left. - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) + 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \left. \left( (a-b) m \left( -\frac{1}{5(a+b)} {}_3F_2(a-b, (1-m)) \operatorname{AppellF1}\left[\frac{5}{2}, 1+m, 2-m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
 & \left( \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(1+m) \operatorname{AppellF1}\left[\frac{5}{2}, 2+m, 1-m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
 & \left. \left. - \frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) - (a+b)(1+m) \left( \frac{1}{5(a+b)} {}_3F_2(a-b)m \operatorname{AppellF1}\left[\frac{5}{2}, \right. \right. \\
 & \left. \left. 2+m, 1-m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(2+m) \right. \\
 & \left. \operatorname{AppellF1}\left[\frac{5}{2}, 3+m, -m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \Big) \Big) \Big) \Big) / \\
 & \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
 & \left. 2 \left( (a-b)m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - \right. \right. \\
 & \left. \left. (a+b)(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 + \\
 & \left( 2 {}_2F_1 \operatorname{AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right) \right) \\
 & \left( 2 \left( (a-b)m \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - (a+b)(2+m) \right. \right. \\
 & \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, 3+m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \\
 & 3(a+b) \left( \frac{1}{3(a+b)} (a-b)m \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \\
 & \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3}(2+m) \operatorname{AppellF1}\left[\frac{3}{2}, 3+m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
 & \left. - \frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + 2 \tan\left[\frac{1}{2}(e+fx)\right]^2
 \end{aligned}$$

$$\begin{aligned}
& \left( (a-b) m \left( -\frac{1}{5(a+b)} 3(a-b)(1-m) \operatorname{AppellF1} \left[ \frac{5}{2}, 2+m, 2-m, \frac{7}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \right. \right. \\
& \quad \operatorname{Sec} \left[ \frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right] - \frac{3}{5}(2+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 3+m, 1-m, \frac{7}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, \right. \\
& \quad \left. \left. -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right] \right) - (a+b)(2+m) \left( \frac{1}{5(a+b)} 3(a-b) m \operatorname{AppellF1} \left[ \frac{5}{2}, \right. \right. \\
& \quad \left. \left. 3+m, 1-m, \frac{7}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right] - \frac{3}{5}(3+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{5}{2}, 4+m, -m, \frac{7}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right] \right) \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \right) + \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] - \right. \\
& \quad \left. (a+b)(2+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2 \right)^2 - \\
& \left( 4 C \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2 \right) \right. \\
& \left. \left( 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] - (a+b)(2+m) \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \right) \operatorname{Sec} \left[ \frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right] + \right. \right. \\
& \left. \left. 3(a+b) \left( \frac{1}{3(a+b)} (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \right) \right. \right. \\
& \quad \left. \left. \operatorname{Sec} \left[ \frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right] - \frac{1}{3}(2+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \left] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \left. \left( (a-b) m \left( -\frac{1}{5(a+b)} 3(a-b)(1-m) \operatorname{AppellF1}\left[\frac{5}{2}, 2+m, 2-m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right. \right. \right. \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(2+m) \operatorname{AppellF1}\left[\frac{5}{2}, 3+m, 1-m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
& \quad \left. \left. -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) - (a+b)(2+m) \left( \frac{1}{5(a+b)} 3(a-b) m \operatorname{AppellF1}\left[\frac{5}{2}, \right. \right. \\
& \quad \left. \left. 3+m, 1-m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(3+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{5}{2}, 4+m, -m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \right) \Bigg/ \\
& \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
& \quad \left. 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - \right. \right. \\
& \quad \left. \left. (a+b)(2+m) \operatorname{AppellF1}\left[\frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 + \\
& \left( 4 C \operatorname{AppellF1}\left[\frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \left( 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 3+m, 1-m, \right. \right. \right. \\
& \quad \left. \left. \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - (a+b)(3+m) \operatorname{AppellF1}\left[\frac{3}{2}, 4+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \quad \left. \left. -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + 3(a+b) \left( \frac{1}{3(a+b)} (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 3+m, 1-m, \right. \right. \right. \\
& \quad \left. \left. \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3}(3+m) \operatorname{AppellF1}\left[\frac{3}{2}, 4+m, \right. \right. \right.
\end{aligned}$$



$$\begin{aligned}
& -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\left]\sec\left[\frac{1}{2}(e+fx)\right]^2\tan\left[\frac{1}{2}(e+fx)\right]\right]+2\tan\left[\frac{1}{2}(e+fx)\right]^2 \\
& \left(\left(a-b\right)m\left(-\frac{1}{5(a+b)}3(a-b)(1-m)\operatorname{AppellF1}\left[\frac{5}{2}, 3+m, 2-m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right.\right. \\
& \left.\left.\sec\left[\frac{1}{2}(e+fx)\right]^2\tan\left[\frac{1}{2}(e+fx)\right]-\frac{3}{5}(3+m)\operatorname{AppellF1}\left[\frac{5}{2}, 4+m, 1-m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right.\right. \\
& \left.\left.-\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\sec\left[\frac{1}{2}(e+fx)\right]^2\tan\left[\frac{1}{2}(e+fx)\right]\right)-\left(a+b\right)(3+m)\left(\frac{1}{5(a+b)}3(a-b)m\operatorname{AppellF1}\left[\frac{5}{2}, \right.\right. \\
& \left.\left.4+m, 1-m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\sec\left[\frac{1}{2}(e+fx)\right]^2\tan\left[\frac{1}{2}(e+fx)\right]-\frac{3}{5}(4+m)\right.\right. \\
& \left.\left.\operatorname{AppellF1}\left[\frac{5}{2}, 5+m, -m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\sec\left[\frac{1}{2}(e+fx)\right]^2\tan\left[\frac{1}{2}(e+fx)\right]\right)\right)\right)\right)/ \\
& \left(3(a+b)\operatorname{AppellF1}\left[\frac{1}{2}, 3+m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]+2\left(\left(a-b\right)m\operatorname{AppellF1}\left[\frac{3}{2}, 3+m, 1-m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]-\right.\right. \\
& \left.\left.\left(a+b\right)(3+m)\operatorname{AppellF1}\left[\frac{3}{2}, 4+m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\tan\left[\frac{1}{2}(e+fx)\right]^2\right)\right)\right)
\end{aligned}$$

■ **Problem 238: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a \cos[e+fx])^m (A+B \cos[e+fx]+C \cos[e+fx]^2) dx$$

Optimal (type 5, 187 leaves, 4 steps):

$$\begin{aligned}
& \frac{C(a \cos[e+fx])^{1+m} \sin[e+fx]}{af(2+m)} - \frac{(C(1+m)+A(2+m))(a \cos[e+fx])^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[e+fx]^2\right] \sin[e+fx]}{af(1+m)(2+m)\sqrt{\sin[e+fx]^2}} - \\
& \frac{B(a \cos[e+fx])^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos[e+fx]^2\right] \sin[e+fx]}{a^2 f(2+m)\sqrt{\sin[e+fx]^2}}
\end{aligned}$$

Result (type 5, 441 leaves):

$$\frac{1}{4f} C \cos[e+fx]^{-m} (a \cos[e+fx])^m$$

$$\left( \frac{1}{(2+m)i} 2^{-m} e^{-2i(e+fx)} (e^{-i(e+fx)} + e^{i(e+fx)})^m (1 + e^{2i(e+fx)})^{-m} \text{Hypergeometric2F1}\left[-1 - \frac{m}{2}, -m, -\frac{m}{2}, -e^{2i(e+fx)}\right] + \right.$$

$$\left. \frac{1}{(-2+m)i} 2^{-m} e^{2i(e+fx)} (e^{-i(e+fx)} + e^{i(e+fx)})^m (1 + e^{2i(e+fx)})^{-m} \text{Hypergeometric2F1}\left[1 - \frac{m}{2}, -m, 2 - \frac{m}{2}, -e^{2i(e+fx)}\right] \right) -$$

$$\frac{A \cos[e+fx] (a \cos[e+fx])^m \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[e+fx]^2\right] \sin[e+fx]}{f(1+m) \sqrt{\sin[e+fx]^2}} -$$

$$\frac{C \cos[e+fx] (a \cos[e+fx])^m \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[e+fx]^2\right] \sin[e+fx]}{2f(1+m) \sqrt{\sin[e+fx]^2}} -$$

$$\frac{B \cos[e+fx]^2 (a \cos[e+fx])^m \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos[e+fx]^2\right] \sin[e+fx]}{f(2+m) \sqrt{\sin[e+fx]^2}}$$

- **Problem 267: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c+dx] + C \cos[c+dx]^2) \sec[c+dx]}{\sqrt{b \cos[c+dx]}} dx$$

Optimal (type 4, 110 leaves, 7 steps):

$$- \frac{2(A-C) \sqrt{b \cos[c+dx]} \text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{bd \sqrt{\cos[c+dx]}} + \frac{2B \sqrt{\cos[c+dx]} \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{d \sqrt{b \cos[c+dx]}} + \frac{2A \sin[c+dx]}{d \sqrt{b \cos[c+dx]}}$$

Result (type 5, 803 leaves):

$$\frac{\cos[c+dx]^2 (B + C \cos[c+dx] + A \sec[c+dx]) \left( -\frac{2(-2A+C \cos[2c]) \csc[c] \sec[c]}{d} + \frac{4A \sec[c] \sec[c+dx] \sin[dx]}{d} \right)}{\sqrt{b \cos[c+dx]} (2A + C + 2B \cos[c+dx] + C \cos[2c + 2dx])}$$

$$\left( 4B \cos[c+dx]^{3/2} \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \right.$$

$$\left. (B + C \cos[c+dx] + A \sec[c+dx]) \sec[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right.$$

$$\left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) /$$

$$\left( d \sqrt{b \cos[c + dx]} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) + \left( 2A \cos[c + dx]^{3/2} \csc[c] (B + C \cos[c + dx] + A \sec[c + dx]) \right)$$

$$\left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[dx + \text{ArcTan}[\tan[c]]]^2 \right] \sin[dx + \text{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left( \sqrt{1 - \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -$$

$$\left( \frac{\frac{\sin[dx + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) /$$

$$\left( d \sqrt{b \cos[c + dx]} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \right) - \left( 2C \cos[c + dx]^{3/2} \csc[c] (B + C \cos[c + dx] + A \sec[c + dx]) \right)$$

$$\left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[dx + \text{ArcTan}[\tan[c]]]^2 \right] \sin[dx + \text{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left( \sqrt{1 - \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -$$

$$\left( \frac{\frac{\sin[dx + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) / \left( d \sqrt{b \cos[c + dx]} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \right)$$

■ **Problem 268: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^2}{\sqrt{b \cos[c + dx]}} dx$$

Optimal (type 4, 139 leaves, 8 steps):

$$-\frac{2B\sqrt{b\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{bd\sqrt{\cos[c+dx]}} + \frac{2(A+3C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d\sqrt{b\cos[c+dx]}} + \frac{2Ab\sin[c+dx]}{3d(b\cos[c+dx])^{3/2}} + \frac{2B\sin[c+dx]}{d\sqrt{b\cos[c+dx]}}$$

Result (type 5, 757 leaves):

$$\begin{aligned} & \left( \cos[c + dx]^3 (C + B \sec[c + dx] + A \sec[c + dx]^2) \right. \\ & \left. \left( \frac{4B \csc[c] \sec[c]}{d} + \frac{4A \sec[c] \sec[c + dx]^2 \sin[dx]}{3d} + \frac{4 \sec[c] \sec[c + dx] (A \sin[c] + 3B \sin[dx])}{3d} \right) \right) / \\ & \left( \sqrt{b \cos[c + dx]} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \right) - \\ & \left( 4A \cos[c + dx]^{5/2} \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] (C + B \sec[c + dx] + A \sec[c + dx]^2) \right. \\ & \left. \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\ & \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( 3d \sqrt{b \cos[c + dx]} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\ & \left( 4C \cos[c + dx]^{5/2} \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \right. \\ & \left. (C + B \sec[c + dx] + A \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\ & \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\ & \left( d \sqrt{b \cos[c + dx]} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) + \left( 2B \cos[c + dx]^{5/2} \csc[c] (C + B \sec[c + dx] + A \sec[c + dx]^2) \right) \end{aligned}$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left( \sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \frac{\frac{\text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) / \left( d \sqrt{b \text{Cos}[c + d x]} (2 A + C + 2 B \text{Cos}[c + d x] + C \text{Cos}[2 c + 2 d x]) \right)$$

- **Problem 276: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \text{Cos}[c + d x] + C \text{Cos}[c + d x]^2) \text{Sec}[c + d x]}{(b \text{Cos}[c + d x])^{3/2}} dx$$

Optimal (type 4, 144 leaves, 8 steps):

$$-\frac{2 B \sqrt{b \text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{b^2 d \sqrt{\text{Cos}[c + d x]}} + \\ \frac{2(A + 3 C) \sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 b d \sqrt{b \text{Cos}[c + d x]}} + \frac{2 A \text{Sin}[c + d x]}{3 d (b \text{Cos}[c + d x])^{3/2}} + \frac{2 B \text{Sin}[c + d x]}{b d \sqrt{b \text{Cos}[c + d x]}}$$

Result (type 5, 761 leaves):

$$\frac{1}{b} \left( \text{Cos}[c + d x]^3 (C + B \text{Sec}[c + d x] + A \text{Sec}[c + d x]^2) \right. \\ \left. \left( \frac{4 B \text{Csc}[c] \text{Sec}[c]}{d} + \frac{4 A \text{Sec}[c] \text{Sec}[c + d x]^2 \text{Sin}[d x]}{3 d} + \frac{4 \text{Sec}[c] \text{Sec}[c + d x] (A \text{Sin}[c] + 3 B \text{Sin}[d x])}{3 d} \right) / \right. \\ \left. \left( \sqrt{b \text{Cos}[c + d x]} (2 A + C + 2 B \text{Cos}[c + d x] + C \text{Cos}[2 c + 2 d x]) \right) - \right. \\ \left. \left( 4 A \text{Cos}[c + d x]^{5/2} \text{Csc}[c] \text{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \right) (C + B \text{Sec}[c + d x] + A \text{Sec}[c + d x]^2) \right)$$

$$\begin{aligned}
& \left. \frac{\sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right. \\
& \left. \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right) / \left( 3 d \sqrt{b \cos [c + d x]} (2 A + C + 2 B \cos [c + d x] + C \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} \right) - \\
& \left( 4 C \cos [c + d x]^{5/2} \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] (C + B \sec [c + d x] + A \sec [c + d x]^2) \right. \\
& \left. \sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right. \\
& \left. \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right) / \left( d \sqrt{b \cos [c + d x]} (2 A + C + 2 B \cos [c + d x] + C \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} \right) + \\
& \left( 2 B \cos [c + d x]^{5/2} \operatorname{Csc}[c] (C + B \sec [c + d x] + A \sec [c + d x]^2) \right. \\
& \left. \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right. \\
& \left. \left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) - \right. \\
& \left. \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right) / \left( d \sqrt{b \cos [c + d x]} (2 A + C + 2 B \cos [c + d x] + C \cos [2 c + 2 d x]) \right)
\end{aligned}$$

■ **Problem 283: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x] (A + B \cos [c + d x] + C \cos [c + d x]^2)}{(b \cos [c + d x])^{5/2}} dx$$

Optimal (type 4, 116 leaves, 7 steps):

$$-\frac{2(A-C)\sqrt{b\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{b^3 d \sqrt{\cos[c+dx]}} + \frac{2B\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{b^2 d \sqrt{b\cos[c+dx]}} + \frac{2A\sin[c+dx]}{b^2 d \sqrt{b\cos[c+dx]}}$$

Result (type 5, 807 leaves):

$$\frac{1}{b^2} \left( \frac{\cos[c+dx]^2 (B+C\cos[c+dx] + A\sec[c+dx]) \left( -\frac{2(-2A+C\cos[2c])\csc[c]\sec[c]}{d} + \frac{4A\sec[c]\sec[c+dx]\sin[dx]}{d} \right)}{\sqrt{b\cos[c+dx]} (2A+C+2B\cos[c+dx] + C\cos[2c+2dx])} - \right.$$

$$\left. \left( 4B\cos[c+dx]^{3/2} \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] (B+C\cos[c+dx] + A\sec[c+dx]) \right. \right.$$

$$\left. \frac{\sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( d \sqrt{b\cos[c+dx]} (2A+C+2B\cos[c+dx] + C\cos[2c+2dx]) \sqrt{1 + \cot[c]^2} \right) +$$

$$\left( 2A\cos[c+dx]^{3/2} \csc[c] (B+C\cos[c+dx] + A\sec[c+dx]) \right.$$

$$\left. \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \right.$$

$$\left. \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) / \right)$$

$$\left( d \sqrt{b \cos[c + dx]} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \right) - \left( 2C \cos[c + dx]^{3/2} \csc[c] (B + C \cos[c + dx] + A \sec[c + dx]) \right)$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[dx + \text{ArcTan}[\tan[c]]]^2 \right] \sin[dx + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right.$$

$$\left. \left( \sqrt{1 - \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin[dx + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) / \left( d \sqrt{b \cos[c + dx]} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \right)$$

■ **Problem 296: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{b \cos[c + dx]} (A + B \cos[c + dx] + C \cos[c + dx]^2)}{\cos[c + dx]^{11/2}} dx$$

Optimal (type 3, 193 leaves, 7 steps):

$$\frac{(3A + 4C) \text{ArcTanh}[\sin[c + dx]] \sqrt{b \cos[c + dx]}}{8d \sqrt{\cos[c + dx]}} + \frac{A \sqrt{b \cos[c + dx]} \sin[c + dx]}{4d \cos[c + dx]^{9/2}} +$$

$$\frac{(3A + 4C) \sqrt{b \cos[c + dx]} \sin[c + dx]}{8d \cos[c + dx]^{5/2}} + \frac{B \sqrt{b \cos[c + dx]} \sin[c + dx]}{d \cos[c + dx]^{3/2}} + \frac{B \sqrt{b \cos[c + dx]} \sin[c + dx]^3}{3d \cos[c + dx]^{7/2}}$$

Result (type 3, 609 leaves):



$$\begin{aligned}
& \frac{(-3A - 4C) \sqrt{b \cos[c + dx]} \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d \sqrt{\cos[c + dx]}} + \frac{(3A + 4C) \sqrt{b \cos[c + dx]} \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d \sqrt{\cos[c + dx]}} + \\
& \frac{A \sqrt{b \cos[c + dx]}}{16d \sqrt{\cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{(9A + 4B + 12C) \sqrt{b \cos[c + dx]}}{48d \sqrt{\cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{B \sqrt{b \cos[c + dx]} \sin\left[\frac{1}{2}(c + dx)\right]}{6d \sqrt{\cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{2B \sqrt{b \cos[c + dx]} \sin\left[\frac{1}{2}(c + dx)\right]}{3d \sqrt{\cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} - \\
& \frac{A \sqrt{b \cos[c + dx]}}{16d \sqrt{\cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{B \sqrt{b \cos[c + dx]} \sin\left[\frac{1}{2}(c + dx)\right]}{6d \sqrt{\cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(-9A - 4B - 12C) \sqrt{b \cos[c + dx]}}{48d \sqrt{\cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{2B \sqrt{b \cos[c + dx]} \sin\left[\frac{1}{2}(c + dx)\right]}{3d \sqrt{\cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)}
\end{aligned}$$

■ **Problem 305: Result more than twice size of optimal antiderivative.**

$$\int \frac{(b \cos[c + dx])^{3/2} (A + B \cos[c + dx] + C \cos[c + dx]^2)}{\cos[c + dx]^{13/2}} dx$$

Optimal (type 3, 198 leaves, 7 steps):

$$\begin{aligned}
& \frac{b(3A + 4C) \operatorname{ArcTanh}[\sin[c + dx]] \sqrt{b \cos[c + dx]}}{8d \sqrt{\cos[c + dx]}} + \frac{Ab \sqrt{b \cos[c + dx]} \sin[c + dx]}{4d \cos[c + dx]^{9/2}} + \\
& \frac{b(3A + 4C) \sqrt{b \cos[c + dx]} \sin[c + dx]}{8d \cos[c + dx]^{5/2}} + \frac{bB \sqrt{b \cos[c + dx]} \sin[c + dx]}{d \cos[c + dx]^{3/2}} + \frac{bB \sqrt{b \cos[c + dx]} \sin[c + dx]^3}{3d \cos[c + dx]^{7/2}}
\end{aligned}$$

Result (type 3, 609 leaves):

$$\begin{aligned}
& \frac{(-3A - 4C)(b \cos[c + dx])^{3/2} \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d \cos[c + dx]^{3/2}} + \frac{(3A + 4C)(b \cos[c + dx])^{3/2} \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d \cos[c + dx]^{3/2}} + \\
& \frac{A(b \cos[c + dx])^{3/2}}{16d \cos[c + dx]^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{(9A + 4B + 12C)(b \cos[c + dx])^{3/2}}{48d \cos[c + dx]^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{B(b \cos[c + dx])^{3/2} \sin\left[\frac{1}{2}(c + dx)\right]}{6d \cos[c + dx]^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{2B(b \cos[c + dx])^{3/2} \sin\left[\frac{1}{2}(c + dx)\right]}{3d \cos[c + dx]^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} - \\
& \frac{A(b \cos[c + dx])^{3/2}}{16d \cos[c + dx]^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{B(b \cos[c + dx])^{3/2} \sin\left[\frac{1}{2}(c + dx)\right]}{6d \cos[c + dx]^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(-9A - 4B - 12C)(b \cos[c + dx])^{3/2}}{48d \cos[c + dx]^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{2B(b \cos[c + dx])^{3/2} \sin\left[\frac{1}{2}(c + dx)\right]}{3d \cos[c + dx]^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)}
\end{aligned}$$

■ **Problem 314: Result more than twice size of optimal antiderivative.**

$$\int \frac{(b \cos[c + dx])^{5/2} (A + B \cos[c + dx] + C \cos[c + dx]^2)}{\cos[c + dx]^{15/2}} dx$$

Optimal (type 3, 208 leaves, 7 steps):

$$\begin{aligned}
& \frac{b^2(3A + 4C) \operatorname{ArcTanh}\left[\frac{\sin[c + dx]}{\sqrt{b \cos[c + dx]}}\right] \sqrt{b \cos[c + dx]}}{8d \sqrt{\cos[c + dx]}} + \frac{A b^2 \sqrt{b \cos[c + dx]} \sin[c + dx]}{4d \cos[c + dx]^{9/2}} + \\
& \frac{b^2(3A + 4C) \sqrt{b \cos[c + dx]} \sin[c + dx]}{8d \cos[c + dx]^{5/2}} + \frac{b^2 B \sqrt{b \cos[c + dx]} \sin[c + dx]}{d \cos[c + dx]^{3/2}} + \frac{b^2 B \sqrt{b \cos[c + dx]} \sin[c + dx]^3}{3d \cos[c + dx]^{7/2}}
\end{aligned}$$

Result (type 3, 609 leaves):

$$\begin{aligned}
& \frac{(-3A - 4C)(b \cos[c + dx])^{5/2} \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d \cos[c + dx]^{5/2}} + \frac{(3A + 4C)(b \cos[c + dx])^{5/2} \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d \cos[c + dx]^{5/2}} + \\
& \frac{A(b \cos[c + dx])^{5/2}}{16d \cos[c + dx]^{5/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{(9A + 4B + 12C)(b \cos[c + dx])^{5/2}}{48d \cos[c + dx]^{5/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{B(b \cos[c + dx])^{5/2} \sin\left[\frac{1}{2}(c + dx)\right]}{6d \cos[c + dx]^{5/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{2B(b \cos[c + dx])^{5/2} \sin\left[\frac{1}{2}(c + dx)\right]}{3d \cos[c + dx]^{5/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} - \\
& \frac{A(b \cos[c + dx])^{5/2}}{16d \cos[c + dx]^{5/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{B(b \cos[c + dx])^{5/2} \sin\left[\frac{1}{2}(c + dx)\right]}{6d \cos[c + dx]^{5/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(-9A - 4B - 12C)(b \cos[c + dx])^{5/2}}{48d \cos[c + dx]^{5/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{2B(b \cos[c + dx])^{5/2} \sin\left[\frac{1}{2}(c + dx)\right]}{3d \cos[c + dx]^{5/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)}
\end{aligned}$$

■ **Problem 322: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx] + C \cos[c + dx]^2}{\cos[c + dx]^{9/2} \sqrt{b \cos[c + dx]}} dx$$

Optimal (type 3, 193 leaves, 7 steps):

$$\begin{aligned}
& \frac{(3A + 4C) \operatorname{ArcTanh}\left[\frac{\sin[c + dx]}{\sqrt{\cos[c + dx]}}\right]}{8d \sqrt{b \cos[c + dx]}} + \frac{A \sin[c + dx]}{4d \cos[c + dx]^{7/2} \sqrt{b \cos[c + dx]}} + \\
& \frac{(3A + 4C) \sin[c + dx]}{8d \cos[c + dx]^{3/2} \sqrt{b \cos[c + dx]}} + \frac{B \sin[c + dx]}{d \sqrt{\cos[c + dx]} \sqrt{b \cos[c + dx]}} + \frac{B \sin[c + dx]^3}{3d \cos[c + dx]^{5/2} \sqrt{b \cos[c + dx]}}
\end{aligned}$$

Result (type 3, 609 leaves):

$$\begin{aligned}
& \frac{(-3A - 4C) \sqrt{\cos[c + dx]} \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d \sqrt{b \cos[c + dx]}} + \frac{(3A + 4C) \sqrt{\cos[c + dx]} \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d \sqrt{b \cos[c + dx]}} + \\
& \frac{A \sqrt{\cos[c + dx]}}{16d \sqrt{b \cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{(9A + 4B + 12C) \sqrt{\cos[c + dx]}}{48d \sqrt{b \cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{B \sqrt{\cos[c + dx]} \sin\left[\frac{1}{2}(c + dx)\right]}{6d \sqrt{b \cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{2B \sqrt{\cos[c + dx]} \sin\left[\frac{1}{2}(c + dx)\right]}{3d \sqrt{b \cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} - \\
& \frac{A \sqrt{\cos[c + dx]}}{16d \sqrt{b \cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{B \sqrt{\cos[c + dx]} \sin\left[\frac{1}{2}(c + dx)\right]}{6d \sqrt{b \cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(-9A - 4B - 12C) \sqrt{\cos[c + dx]}}{48d \sqrt{b \cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{2B \sqrt{\cos[c + dx]} \sin\left[\frac{1}{2}(c + dx)\right]}{3d \sqrt{b \cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)}
\end{aligned}$$

■ **Problem 330: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx] + C \cos[c + dx]^2}{\cos[c + dx]^{7/2} (b \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 208 leaves, 7 steps):

$$\begin{aligned}
& \frac{(3A + 4C) \operatorname{ArcTanh}[\sin[c + dx]] \sqrt{\cos[c + dx]}}{8bd \sqrt{b \cos[c + dx]}} + \frac{A \sin[c + dx]}{4bd \cos[c + dx]^{7/2} \sqrt{b \cos[c + dx]}} + \\
& \frac{(3A + 4C) \sin[c + dx]}{8bd \cos[c + dx]^{3/2} \sqrt{b \cos[c + dx]}} + \frac{B \sin[c + dx]}{bd \sqrt{\cos[c + dx]} \sqrt{b \cos[c + dx]}} + \frac{B \sin[c + dx]^3}{3bd \cos[c + dx]^{5/2} \sqrt{b \cos[c + dx]}}
\end{aligned}$$

Result (type 3, 609 leaves):

$$\begin{aligned}
& \frac{(-3A - 4C) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right]}{8d(b \operatorname{Cos}[c + dx])^{3/2}} + \frac{(3A + 4C) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right]}{8d(b \operatorname{Cos}[c + dx])^{3/2}} + \\
& \frac{A \operatorname{Cos}[c + dx]^{3/2}}{16d(b \operatorname{Cos}[c + dx])^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{(9A + 4B + 12C) \operatorname{Cos}[c + dx]^{3/2}}{48d(b \operatorname{Cos}[c + dx])^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{B \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]}{6d(b \operatorname{Cos}[c + dx])^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{2B \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]}{3d(b \operatorname{Cos}[c + dx])^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)} - \\
& \frac{A \operatorname{Cos}[c + dx]^{3/2}}{16d(b \operatorname{Cos}[c + dx])^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{B \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]}{6d(b \operatorname{Cos}[c + dx])^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(-9A - 4B - 12C) \operatorname{Cos}[c + dx]^{3/2}}{48d(b \operatorname{Cos}[c + dx])^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{2B \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]}{3d(b \operatorname{Cos}[c + dx])^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)}
\end{aligned}$$

■ **Problem 338: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Cos}[c + dx] + C \operatorname{Cos}[c + dx]^2}{\operatorname{Cos}[c + dx]^{5/2} (b \operatorname{Cos}[c + dx])^{5/2}} dx$$

Optimal (type 3, 208 leaves, 7 steps):

$$\begin{aligned}
& \frac{(3A + 4C) \operatorname{ArcTanh}\left[\operatorname{Sin}[c + dx]\right] \sqrt{\operatorname{Cos}[c + dx]}}{8b^2 d \sqrt{b \operatorname{Cos}[c + dx]}} + \frac{A \operatorname{Sin}[c + dx]}{4b^2 d \operatorname{Cos}[c + dx]^{7/2} \sqrt{b \operatorname{Cos}[c + dx]}} + \\
& \frac{(3A + 4C) \operatorname{Sin}[c + dx]}{8b^2 d \operatorname{Cos}[c + dx]^{3/2} \sqrt{b \operatorname{Cos}[c + dx]}} + \frac{B \operatorname{Sin}[c + dx]}{b^2 d \sqrt{\operatorname{Cos}[c + dx]} \sqrt{b \operatorname{Cos}[c + dx]}} + \frac{B \operatorname{Sin}[c + dx]^3}{3b^2 d \operatorname{Cos}[c + dx]^{5/2} \sqrt{b \operatorname{Cos}[c + dx]}}
\end{aligned}$$

Result (type 3, 609 leaves):

$$\begin{aligned}
& \frac{(-3A - 4C) \cos[c + dx]^{5/2} \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d (b \cos[c + dx])^{5/2}} + \frac{(3A + 4C) \cos[c + dx]^{5/2} \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d (b \cos[c + dx])^{5/2}} + \\
& \frac{A \cos[c + dx]^{5/2}}{16d (b \cos[c + dx])^{5/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{(9A + 4B + 12C) \cos[c + dx]^{5/2}}{48d (b \cos[c + dx])^{5/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{B \cos[c + dx]^{5/2} \sin\left[\frac{1}{2}(c + dx)\right]}{6d (b \cos[c + dx])^{5/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{2B \cos[c + dx]^{5/2} \sin\left[\frac{1}{2}(c + dx)\right]}{3d (b \cos[c + dx])^{5/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} - \\
& \frac{A \cos[c + dx]^{5/2}}{16d (b \cos[c + dx])^{5/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{B \cos[c + dx]^{5/2} \sin\left[\frac{1}{2}(c + dx)\right]}{6d (b \cos[c + dx])^{5/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(-9A - 4B - 12C) \cos[c + dx]^{5/2}}{48d (b \cos[c + dx])^{5/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{2B \cos[c + dx]^{5/2} \sin\left[\frac{1}{2}(c + dx)\right]}{3d (b \cos[c + dx])^{5/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)}
\end{aligned}$$

■ **Problem 354: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]}{(b \cos[c + dx])^{1/3}} dx$$

Optimal (type 5, 149 leaves, 5 steps):

$$\begin{aligned}
& \frac{3A \sin[c + dx]}{d (b \cos[c + dx])^{1/3}} - \frac{3B (b \cos[c + dx])^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos[c + dx]^2\right] \sin[c + dx]}{2bd \sqrt{\sin[c + dx]^2}} + \\
& \frac{3(2A - C) (b \cos[c + dx])^{5/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos[c + dx]^2\right] \sin[c + dx]}{5b^2 d \sqrt{\sin[c + dx]^2}}
\end{aligned}$$

Result (type 5, 779 leaves):

$$\begin{aligned}
& \frac{\cos[c + dx]^2 (B + C \cos[c + dx] + A \sec[c + dx]) \left(-\frac{3(-4A + C + C \cos[2c]) \csc[c] \sec[c]}{2d} + \frac{6A \sec[c] \sec[c + dx] \sin[dx]}{d}\right)}{(b \cos[c + dx])^{1/3} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx])} - \\
& \left(2B \cos[c + dx]^{4/3} \cos[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right) \\
& \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \cos[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (B + C \cos[c + dx] + A \sec[c + dx]) \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \Big/ \\
& (d (b \cos[c + dx])^{1/3} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) (\cos[c] \cos[dx] - \sin[c] \sin[dx])^{1/3} (\sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2)^{1/3}) +
\end{aligned}$$

$$\left( 4 A \cos [c + d x]^{4/3} \operatorname{Csc}[c] (B + C \cos [c + d x] + A \operatorname{Sec}[c + d x]) \right.$$

$$\left. \left( \operatorname{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{6}, \frac{5}{6} \right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2 \right\} \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right.$$

$$\left. \left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \left( \cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2} \right)^{1/3} \sqrt{1 + \tan [c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{3 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{2 (\cos [c]^2 + \sin [c]^2)}}{\left( \cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2} \right)^{1/3}} \right) / \left. \right)$$

$$(d (b \cos [c + d x])^{1/3} (2 A + C + 2 B \cos [c + d x] + C \cos [2 c + 2 d x])) - \left( 2 C \cos [c + d x]^{4/3} \operatorname{Csc}[c] (B + C \cos [c + d x] + A \operatorname{Sec}[c + d x]) \right.$$

$$\left. \left( \operatorname{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{6}, \frac{5}{6} \right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2 \right\} \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right.$$

$$\left. \left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \left( \cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2} \right)^{1/3} \sqrt{1 + \tan [c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{3 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{2 (\cos [c]^2 + \sin [c]^2)}}{\left( \cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2} \right)^{1/3}} \right) / \left. \right) (d (b \cos [c + d x])^{1/3} (2 A + C + 2 B \cos [c + d x] + C \cos [2 c + 2 d x]))$$

■ **Problem 355: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + d x] + C \cos [c + d x]^2) \operatorname{Sec}[c + d x]^2}{(b \cos [c + d x])^{1/3}} dx$$

Optimal (type 5, 145 leaves, 5 steps):

$$\frac{3 A b \sin [c+d x]}{4 d (b \cos [c+d x])^{4/3}} + \frac{3 B \operatorname{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \cos [c+d x]^2\right] \sin [c+d x]}{d (b \cos [c+d x])^{1/3} \sqrt{\sin [c+d x]^2}} -$$

$$\frac{3 (A+4 C) (b \cos [c+d x])^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos [c+d x]^2\right] \sin [c+d x]}{8 b d \sqrt{\sin [c+d x]^2}}$$

Result (type 5, 699 leaves):



$$\begin{aligned}
& \left( \cos [c+d x]^3 (C+B \operatorname{Sec}[c+d x]+A \operatorname{Sec}[c+d x]^2) \right. \\
& \quad \left. \left( \frac{6 B \operatorname{Csc}[c] \operatorname{Sec}[c]}{d} + \frac{3 A \operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2 \sin [d x]}{2 d} + \frac{3 \operatorname{Sec}[c] \operatorname{Sec}[c+d x] (A \sin [c]+4 B \sin [d x])}{2 d} \right) \right) / \\
& \quad \left( (b \cos [c+d x])^{1 / 3} (2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]) \right) - \left( A \cos [c+d x]^{7 / 3} \cos [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \right) \\
& \quad \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \cos [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (C+B \operatorname{Sec}[c+d x]+A \operatorname{Sec}[c+d x]^2) \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \Big/ \\
& \quad \left( 2 d (b \cos [c+d x])^{1 / 3} (2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]) (\cos [c] \cos [d x]-\sin [c] \sin [d x])^{1 / 3} (\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2)^{1 / 3} \right) - \\
& \quad \left( 2 C \cos [c+d x]^{7 / 3} \cos [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \cos [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \right. \\
& \quad \left. (C+B \operatorname{Sec}[c+d x]+A \operatorname{Sec}[c+d x]^2) \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \right) / \\
& \quad \left( d (b \cos [c+d x])^{1 / 3} (2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]) (\cos [c] \cos [d x]-\sin [c] \sin [d x])^{1 / 3} (\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2)^{1 / 3} \right) + \\
& \quad \left( 4 B \cos [c+d x]^{7 / 3} \operatorname{Csc}[c] (C+B \operatorname{Sec}[c+d x]+A \operatorname{Sec}[c+d x]^2) \right) \\
& \quad \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{6}\right\},\left\{\frac{5}{6}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\
& \quad \left( \sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} (\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2})^{1 / 3} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \\
& \quad \left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}} + \frac{3 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{2 (\cos [c]^2+\sin [c]^2)}}{(\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2})^{1 / 3}} \right) / \left( d (b \cos [c+d x])^{1 / 3} (2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]) \right)
\end{aligned}$$

■ **Problem 361: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+B \cos [c+d x]+C \cos [c+d x]^2) \operatorname{Sec}[c+d x]}{(b \cos [c+d x])^{4 / 3}} d x$$

Optimal (type 5, 147 leaves, 5 steps):

$$\frac{3 A \sin [c+d x]}{4 d (b \cos [c+d x])^{4 / 3}}+\frac{3 B \operatorname{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \cos [c+d x]^2\right] \sin [c+d x]}{b d (b \cos [c+d x])^{1 / 3} \sqrt{\sin [c+d x]^2}}$$

$$\frac{3(A+4 C)(b \cos [c+d x])^{2 / 3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos [c+d x]^2\right] \sin [c+d x]}{8 b^2 d \sqrt{\sin [c+d x]^2}}$$

Result (type 5, 703 leaves):

$$\frac{1}{b} \left( \left( \cos [c+d x]^3 (C+B \sec [c+d x]+A \sec [c+d x]^2) \right. \right.$$

$$\left. \left( \frac{6 B \csc [c] \sec [c]}{d}+\frac{3 A \sec [c] \sec [c+d x]^2 \sin [d x]}{2 d}+\frac{3 \sec [c] \sec [c+d x](A \sin [c]+4 B \sin [d x])}{2 d} \right) \right) /$$

$$\left( (b \cos [c+d x])^{1 / 3}(2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]) \right)-$$

$$\left( A \cos [c+d x]^{7 / 3} \cos [d x-\operatorname{ArcTan}[\cot [c]]] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \cos [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \right.$$

$$\left. (C+B \sec [c+d x]+A \sec [c+d x]^2) \sin [d x-\operatorname{ArcTan}[\cot [c]]] \right) / \left( 2 d (b \cos [c+d x])^{1 / 3} \right.$$

$$\left. (2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]) (\cos [c] \cos [d x]-\sin [c] \sin [d x])^{1 / 3}(\sin [d x-\operatorname{ArcTan}[\cot [c]]]^2)^{1 / 3} \right)-$$

$$\left( 2 C \cos [c+d x]^{7 / 3} \cos [d x-\operatorname{ArcTan}[\cot [c]]] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \cos [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \right.$$

$$\left. (C+B \sec [c+d x]+A \sec [c+d x]^2) \sin [d x-\operatorname{ArcTan}[\cot [c]]] \right) /$$

$$\left( d (b \cos [c+d x])^{1 / 3}(2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]) (\cos [c] \cos [d x]-\sin [c] \sin [d x])^{1 / 3}(\sin [d x-\operatorname{ArcTan}[\cot [c]]]^2)^{1 / 3} \right)+$$

$$\left( 4 B \cos [c+d x]^{7 / 3} \csc [c](C+B \sec [c+d x]+A \sec [c+d x]^2) \right.$$

$$\left. \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{6}\right\},\left\{\frac{5}{6}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c] \right) \right) /$$

$$\left( \sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} (\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2})^{1 / 3} \right)$$

$$\left. \left( \sqrt{1 + \tan[c]^2} \right) - \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{3 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{2 (\cos[c]^2 + \sin[c]^2)}}{\left( \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2} \right)^{1/3}} \right) /$$

$$\left( d (b \cos[c + d x])^{1/3} (2 A + C + 2 B \cos[c + d x] + C \cos[2 c + 2 d x]) \right)$$

■ **Problem 366: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^m (A + B \cos[c + d x] + C \cos[c + d x]^2)}{(b \cos[c + d x])^{1/3}} dx$$

Optimal (type 5, 229 leaves, 5 steps):

$$\frac{3 C \cos[c + d x]^{1+m} \sin[c + d x]}{d (5 + 3 m) (b \cos[c + d x])^{1/3}} -$$

$$\left( 3 (C (2 + 3 m) + A (5 + 3 m)) \cos[c + d x]^{1+m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6} (2 + 3 m), \frac{1}{6} (8 + 3 m), \cos[c + d x]^2\right] \sin[c + d x] \right) /$$

$$\left( d (2 + 3 m) (5 + 3 m) (b \cos[c + d x])^{1/3} \sqrt{\sin[c + d x]^2} \right) -$$

$$\frac{3 B \cos[c + d x]^{2+m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6} (5 + 3 m), \frac{1}{6} (11 + 3 m), \cos[c + d x]^2\right] \sin[c + d x]}{d (5 + 3 m) (b \cos[c + d x])^{1/3} \sqrt{\sin[c + d x]^2}}$$

Result (type 6, 7630 leaves):

$$\left( 2 \cos[c + d x]^{1/3} \left( \frac{1}{2} B \cos[c + d x]^{\frac{2}{3}+m} \cos[2 (c + d x)] - \right. \right.$$

$$\frac{1}{2} i B \cos[c + d x]^{\frac{2}{3}+m} \sin[2 (c + d x)] + \sec[c + d x] \left( \left( A \cos[c + d x]^{\frac{2}{3}+m} + \frac{1}{2} C \cos[c + d x]^{\frac{2}{3}+m} \right) \cos[2 (c + d x)]^2 - \right.$$

$$\frac{1}{2} i B \cos[c + d x]^{\frac{2}{3}+m} \cos[3 (c + d x)] \sin[2 (c + d x)] - \frac{1}{4} i C \cos[c + d x]^{\frac{2}{3}+m} \cos[4 (c + d x)] \sin[2 (c + d x)] +$$

$$\frac{1}{2} B \cos[c + d x]^{\frac{2}{3}+m} \sin[c + d x] \sin[2 (c + d x)] + \left( A \cos[c + d x]^{\frac{2}{3}+m} + \frac{1}{2} C \cos[c + d x]^{\frac{2}{3}+m} \right) \sin[2 (c + d x)]^2 +$$

$$\left. \left. \cos[2 (c + d x)] \left( \frac{1}{4} C \cos[c + d x]^{\frac{2}{3}+m} + \frac{1}{2} B \cos[c + d x]^{\frac{2}{3}+m} \cos[3 (c + d x)] + \frac{1}{4} C \cos[c + d x]^{\frac{2}{3}+m} \cos[4 (c + d x)] + \right. \right.$$

$$\begin{aligned}
& \left. \frac{1}{2} i B \cos [c+d x]^{\frac{2}{3}+m} \sin [c+d x] + \frac{1}{2} i B \cos [c+d x]^{\frac{2}{3}+m} \sin [3(c+d x)] + \frac{1}{4} i C \cos [c+d x]^{\frac{2}{3}+m} \sin [4(c+d x)] \right) + \\
& \left. \sin [2(c+d x)] \left( -\frac{1}{4} i C \cos [c+d x]^{\frac{2}{3}+m} + \frac{1}{2} B \cos [c+d x]^{\frac{2}{3}+m} \sin [3(c+d x)] + \frac{1}{4} C \cos [c+d x]^{\frac{2}{3}+m} \sin [4(c+d x)] \right) \right) \\
& \tan \left[ \frac{1}{2}(c+d x) \right] \left( 1 - \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right)^{-\frac{1}{3}+m} \left( \frac{1}{1 + \tan \left[ \frac{1}{2}(c+d x) \right]^2} \right)^{\frac{8}{3}+m} \\
& \left( \left( 45(A+B+C) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) / \right. \\
& \left( 9 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + 2 \left( -(8+3m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{3}-m, \frac{11}{3}+m, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + (1-3m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{4}{3}-m, \frac{8}{3}+m, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) + \\
& \left( 50(A-C) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) / \\
& \left( 15 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \\
& \left. 2 \left( -(8+3m) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{3}-m, \frac{11}{3}+m, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \right. \\
& \left. \left. (1-3m) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{4}{3}-m, \frac{8}{3}+m, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) - \\
& \left( 21(A-B+C) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \tan \left[ \frac{1}{2}(c+d x) \right]^4 \right) / \\
& \left( -21 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \\
& \left. 2 \left( (8+3m) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{1}{3}-m, \frac{11}{3}+m, \frac{9}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \right. \\
& \left. \left. (-1+3m) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{4}{3}-m, \frac{8}{3}+m, \frac{9}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) \right) / \\
& \left( 5 d (b \cos [c+d x])^{1/3} \left( -\frac{2}{5} \left( -\frac{1}{3}+m \right) \sec \left[ \frac{1}{2}(c+d x) \right]^2 \tan \left[ \frac{1}{2}(c+d x) \right]^2 \left( 1 - \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right)^{-\frac{4}{3}+m} \left( \frac{1}{1 + \tan \left[ \frac{1}{2}(c+d x) \right]^2} \right)^{\frac{8}{3}+m} \right. \right. \\
& \left. \left( \left( 45(A+B+C) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) / \left( 9 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{3}{2}, \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + 2\left(- (8+3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{11}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \left. (1-3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}-m, \frac{8}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right) + \\
& \left(50(A-C) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2\right) / \left(15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \right. \right. \\
& \left. \left. \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + 2\left(- (8+3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{3}-m, \frac{11}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \left. \left. (1-3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{4}{3}-m, \frac{8}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - \\
& \left(21(A-B+C) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^4\right) / \\
& \left(-21 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \left. 2\left((8+3m) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{1}{3}-m, \frac{11}{3}+m, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (-1+3m) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{4}{3}-m, \frac{8}{3}+m, \right. \right. \right. \\
& \left. \left. \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right) + \frac{1}{5} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{-\frac{1}{3}+m} \\
& \left(\frac{1}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}\right)^{\frac{8}{3}+m} \left(\left(45(A+B+C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) / \right. \\
& \left. \left(9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \left. \left. 2\left(- (8+3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{11}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \right. \\
& \left. \left. (1-3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}-m, \frac{8}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right) + \\
& \left(50(A-C) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2\right) / \\
& \left(15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \left. 2\left(- (8+3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{3}-m, \frac{11}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \left. \left. (1-3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{4}{3}-m, \frac{8}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right) -
\end{aligned}$$











$$\begin{aligned}
& 2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\left((8+3 m)\left(-\frac{7}{9}\left(\frac{11}{3}+m\right) \operatorname{AppellF1}\left[\frac{9}{2}, \frac{1}{3}-m, \frac{14}{3}+m, \frac{11}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]\right.\right. \\
& \quad \left.\left.\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+\frac{7}{9}\left(\frac{1}{3}-m\right) \operatorname{AppellF1}\left[\frac{9}{2}, \frac{4}{3}-m, \frac{11}{3}+m, \frac{11}{2},\right.\right.\right. \\
& \quad \left.\left.\left.\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)\right)+(-1+3 m) \\
& \quad \left(-\frac{7}{9}\left(\frac{8}{3}+m\right) \operatorname{AppellF1}\left[\frac{9}{2}, \frac{4}{3}-m, \frac{11}{3}+m, \frac{11}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+ \right. \\
& \quad \left.\frac{7}{9}\left(\frac{4}{3}-m\right) \operatorname{AppellF1}\left[\frac{9}{2}, \frac{7}{3}-m, \frac{8}{3}+m, \frac{11}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)\right) / \\
& \left(-21 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{3}-m, \frac{8}{3}+m, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]+2\left((8+3 m) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{1}{3}-m,\right.\right.\right. \\
& \quad \left.\left.\left.\frac{11}{3}+m, \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]+ \right.\right. \\
& \quad \left.\left.(-1+3 m) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{4}{3}-m, \frac{8}{3}+m, \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]\right)\right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2) \Bigg)
\end{aligned}$$

▪ **Problem 367: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^m(A+B \cos [c+d x]+C \cos [c+d x]^2)}{(b \cos [c+d x])^{2/3}} d x$$

Optimal (type 5, 227 leaves, 5 steps):

$$\frac{3 C \cos [c+d x]^{1+m} \sin [c+d x]}{d(4+3 m)(b \cos [c+d x])^{2/3}} -$$

$$\left(3(C+3 C m+A(4+3 m)) \cos [c+d x]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6}(1+3 m), \frac{1}{6}(7+3 m), \cos [c+d x]^2\right] \sin [c+d x]\right) /$$

$$\left(d(1+3 m)(4+3 m)(b \cos [c+d x])^{2/3} \sqrt{\sin [c+d x]^2}\right) -$$

$$\frac{3 B \cos [c+d x]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6}(4+3 m), \frac{1}{6}(10+3 m), \cos [c+d x]^2\right] \sin [c+d x]}{d(4+3 m)(b \cos [c+d x])^{2/3} \sqrt{\sin [c+d x]^2}}$$

Result (type 6, 7613 leaves):

$$\left(2 \cos [c+d x]^{2/3}\left(\frac{1}{2} B \cos [c+d x]^{1/3+m} \cos [2(c+d x)]-\right.\right.$$

$$\begin{aligned}
& \frac{1}{2} i B \cos [c+d x]^{\frac{1}{3}+m} \sin [2(c+d x)] + \sec [c+d x] \left( \left( A \cos [c+d x]^{\frac{1}{3}+m} + \frac{1}{2} C \cos [c+d x]^{\frac{1}{3}+m} \right) \cos [2(c+d x)]^2 - \right. \\
& \frac{1}{2} i B \cos [c+d x]^{\frac{1}{3}+m} \cos [3(c+d x)] \sin [2(c+d x)] - \frac{1}{4} i C \cos [c+d x]^{\frac{1}{3}+m} \cos [4(c+d x)] \sin [2(c+d x)] + \\
& \frac{1}{2} B \cos [c+d x]^{\frac{1}{3}+m} \sin [c+d x] \sin [2(c+d x)] + \left. \left( A \cos [c+d x]^{\frac{1}{3}+m} + \frac{1}{2} C \cos [c+d x]^{\frac{1}{3}+m} \right) \sin [2(c+d x)]^2 + \right. \\
& \cos [2(c+d x)] \left( \frac{1}{4} C \cos [c+d x]^{\frac{1}{3}+m} + \frac{1}{2} B \cos [c+d x]^{\frac{1}{3}+m} \cos [3(c+d x)] + \frac{1}{4} C \cos [c+d x]^{\frac{1}{3}+m} \cos [4(c+d x)] + \right. \\
& \left. \frac{1}{2} i B \cos [c+d x]^{\frac{1}{3}+m} \sin [c+d x] + \frac{1}{2} i B \cos [c+d x]^{\frac{1}{3}+m} \sin [3(c+d x)] + \frac{1}{4} i C \cos [c+d x]^{\frac{1}{3}+m} \sin [4(c+d x)] \right) + \\
& \left. \sin [2(c+d x)] \left( -\frac{1}{4} i C \cos [c+d x]^{\frac{1}{3}+m} + \frac{1}{2} B \cos [c+d x]^{\frac{1}{3}+m} \sin [3(c+d x)] + \frac{1}{4} C \cos [c+d x]^{\frac{1}{3}+m} \sin [4(c+d x)] \right) \right) \Bigg) \\
& \tan \left[ \frac{1}{2}(c+d x) \right] \left( 1 - \tan \left[ \frac{1}{2}(c+d x) \right] \right)^{-\frac{2}{3}+m} \left( \frac{1}{1 + \tan \left[ \frac{1}{2}(c+d x) \right]^2} \right)^{\frac{7}{3}+m} \\
& \left( \left( 45(A+B+C) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{2}{3}-m, \frac{7}{3}+m, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) / \right. \\
& \left( 9 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{2}{3}-m, \frac{7}{3}+m, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] - 2 \left( (7+3m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2}{3}-m, \frac{10}{3}+m, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + (-2+3m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{5}{3}-m, \frac{7}{3}+m, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) + \\
& \left( 50(A-C) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2}{3}-m, \frac{7}{3}+m, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) / \\
& \left( 15 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2}{3}-m, \frac{7}{3}+m, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] - \right. \\
& 2 \left( (7+3m) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{2}{3}-m, \frac{10}{3}+m, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \\
& \left. (-2+3m) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{5}{3}-m, \frac{7}{3}+m, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) - \\
& \left( 21(A-B+C) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{2}{3}-m, \frac{7}{3}+m, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \tan \left[ \frac{1}{2}(c+d x) \right]^4 \right) / \\
& \left( -21 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{2}{3}-m, \frac{7}{3}+m, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \\
& \left. 2 \left( (7+3m) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{2}{3}-m, \frac{10}{3}+m, \frac{9}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. (-2 + 3m) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{5}{3} - m, \frac{7}{3} + m, \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right) \Bigg/ \\
& \left( 5d (b \operatorname{Cos}[c + dx])^{2/3} \left( -\frac{2}{5} \left( -\frac{2}{3} + m \right) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \left( 1 - \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \right)^{-\frac{5}{3}+m} \left( \frac{1}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2} \right)^{\frac{7}{3}+m} \right. \right. \\
& \left. \left( \left( 45(A + B + C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3} - m, \frac{7}{3} + m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \Bigg/ \left( 9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3} - m, \frac{7}{3} + m, \frac{3}{2}, \right. \right. \right. \\
& \left. \left. \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] - 2 \left( (7 + 3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3} - m, \frac{10}{3} + m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) + \right. \right. \\
& \left. \left. (-2 + 3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{3} - m, \frac{7}{3} + m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \right) + \\
& \left( 50(A - C) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3} - m, \frac{7}{3} + m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \right) \Bigg/ \left( 15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3} - m, \frac{7}{3} + m, \right. \right. \\
& \left. \left. \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] - 2 \left( (7 + 3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2}{3} - m, \frac{10}{3} + m, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) + \right. \\
& \left. (-2 + 3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{3} - m, \frac{7}{3} + m, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \right) - \\
& \left( 21(A - B + C) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2}{3} - m, \frac{7}{3} + m, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^4 \right) \Bigg/ \\
& \left( -21 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2}{3} - m, \frac{7}{3} + m, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) + \\
& 2 \left( (7 + 3m) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{2}{3} - m, \frac{10}{3} + m, \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] + (-2 + 3m) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{5}{3} - m, \frac{7}{3} + m, \right. \right. \\
& \left. \left. \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \Bigg) + \frac{1}{5} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \left( 1 - \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \right)^{-\frac{2}{3}+m} \\
& \left( \frac{1}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2} \right)^{\frac{7}{3}+m} \left( \left( 45(A + B + C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3} - m, \frac{7}{3} + m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \Bigg/ \right. \\
& \left( 9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3} - m, \frac{7}{3} + m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) - \\
& 2 \left( (7 + 3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3} - m, \frac{10}{3} + m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) + \\
& \left. (-2 + 3m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{3} - m, \frac{7}{3} + m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \Bigg) +
\end{aligned}$$











$$\begin{aligned}
& \left( 2 \left( (7+3m) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{2}{3}-m, \frac{10}{3}+m, \frac{9}{2}, \tan \left[ \frac{1}{2} (c+dx) \right]^2, -\tan \left[ \frac{1}{2} (c+dx) \right]^2 \right] + \right. \right. \\
& \quad \left. (-2+3m) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{5}{3}-m, \frac{7}{3}+m, \frac{9}{2}, \tan \left[ \frac{1}{2} (c+dx) \right]^2, -\tan \left[ \frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \tan \left[ \frac{1}{2} (c+dx) \right] - \\
& \quad 21 \left( -\frac{5}{7} \left( \frac{7}{3}+m \right) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{2}{3}-m, \frac{10}{3}+m, \frac{9}{2}, \tan \left[ \frac{1}{2} (c+dx) \right]^2, -\tan \left[ \frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \tan \left[ \frac{1}{2} (c+dx) \right] + \right. \\
& \quad \left. \frac{5}{7} \left( \frac{2}{3}-m \right) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{5}{3}-m, \frac{7}{3}+m, \frac{9}{2}, \tan \left[ \frac{1}{2} (c+dx) \right]^2, -\tan \left[ \frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \tan \left[ \frac{1}{2} (c+dx) \right] \right) + \\
& \quad 2 \tan \left[ \frac{1}{2} (c+dx) \right]^2 \left( (7+3m) \left( -\frac{7}{9} \left( \frac{10}{3}+m \right) \operatorname{AppellF1} \left[ \frac{9}{2}, \frac{2}{3}-m, \frac{13}{3}+m, \frac{11}{2}, \tan \left[ \frac{1}{2} (c+dx) \right]^2, -\tan \left[ \frac{1}{2} (c+dx) \right]^2 \right] \right. \right. \\
& \quad \left. \operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \tan \left[ \frac{1}{2} (c+dx) \right] + \frac{7}{9} \left( \frac{2}{3}-m \right) \operatorname{AppellF1} \left[ \frac{9}{2}, \frac{5}{3}-m, \frac{10}{3}+m, \frac{11}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[ \frac{1}{2} (c+dx) \right]^2, -\tan \left[ \frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \tan \left[ \frac{1}{2} (c+dx) \right] \right) + (-2+3m) \\
& \quad \left( -\frac{7}{9} \left( \frac{7}{3}+m \right) \operatorname{AppellF1} \left[ \frac{9}{2}, \frac{5}{3}-m, \frac{10}{3}+m, \frac{11}{2}, \tan \left[ \frac{1}{2} (c+dx) \right]^2, -\tan \left[ \frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \tan \left[ \frac{1}{2} (c+dx) \right] + \right. \\
& \quad \left. \frac{7}{9} \left( \frac{5}{3}-m \right) \operatorname{AppellF1} \left[ \frac{9}{2}, \frac{8}{3}-m, \frac{7}{3}+m, \frac{11}{2}, \tan \left[ \frac{1}{2} (c+dx) \right]^2, -\tan \left[ \frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \tan \left[ \frac{1}{2} (c+dx) \right] \right) \right) \Bigg) / \\
& \left( -21 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{2}{3}-m, \frac{7}{3}+m, \frac{7}{2}, \tan \left[ \frac{1}{2} (c+dx) \right]^2, -\tan \left[ \frac{1}{2} (c+dx) \right]^2 \right] + 2 \left( (7+3m) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{2}{3}-m, \right. \right. \right. \\
& \quad \left. \left. \frac{10}{3}+m, \frac{9}{2}, \tan \left[ \frac{1}{2} (c+dx) \right]^2, -\tan \left[ \frac{1}{2} (c+dx) \right]^2 \right] + \right. \\
& \quad \left. \left. (-2+3m) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{5}{3}-m, \frac{7}{3}+m, \frac{9}{2}, \tan \left[ \frac{1}{2} (c+dx) \right]^2, -\tan \left[ \frac{1}{2} (c+dx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (c+dx) \right]^2 \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 368: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+dx]^m (A+B \cos [c+dx]+C \cos [c+dx]^2)}{(b \cos [c+dx])^{4/3}} dx$$

Optimal (type 5, 235 leaves, 5 steps):

$$\frac{3 C \cos [c+d x]^m \sin [c+d x]}{b d (2+3 m) (b \cos [c+d x])^{1/3}} -$$

$$\left( 3 (C (1-3 m) - A (2+3 m)) \cos [c+d x]^m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6}(-1+3 m), \frac{1}{6}(5+3 m), \cos [c+d x]^2\right] \sin [c+d x] \right) /$$

$$\left( b d (1-3 m) (2+3 m) (b \cos [c+d x])^{1/3} \sqrt{\sin [c+d x]^2} \right) -$$

$$\frac{3 B \cos [c+d x]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6}(2+3 m), \frac{1}{6}(8+3 m), \cos [c+d x]^2\right] \sin [c+d x]}{b d (2+3 m) (b \cos [c+d x])^{1/3} \sqrt{\sin [c+d x]^2}}$$

Result (type 6, 7623 leaves):

$$\left( 2 \cos [c+d x]^{4/3} \left( \sec [c+d x] \left( \frac{1}{2} B \cos [c+d x]^{\frac{2}{3}+m} \cos [2(c+d x)] - \frac{1}{2} i B \cos [c+d x]^{\frac{2}{3}+m} \sin [2(c+d x)] \right) + \right. \right.$$

$$\sec [c+d x]^2 \left( \left( A \cos [c+d x]^{\frac{2}{3}+m} + \frac{1}{2} C \cos [c+d x]^{\frac{2}{3}+m} \right) \cos [2(c+d x)]^2 - \right.$$

$$\frac{1}{2} i B \cos [c+d x]^{\frac{2}{3}+m} \cos [3(c+d x)] \sin [2(c+d x)] - \frac{1}{4} i C \cos [c+d x]^{\frac{2}{3}+m} \cos [4(c+d x)] \sin [2(c+d x)] +$$

$$\frac{1}{2} B \cos [c+d x]^{\frac{2}{3}+m} \sin [c+d x] \sin [2(c+d x)] + \left. \left( A \cos [c+d x]^{\frac{2}{3}+m} + \frac{1}{2} C \cos [c+d x]^{\frac{2}{3}+m} \right) \sin [2(c+d x)]^2 + \right.$$

$$\cos [2(c+d x)] \left( \frac{1}{4} C \cos [c+d x]^{\frac{2}{3}+m} + \frac{1}{2} B \cos [c+d x]^{\frac{2}{3}+m} \cos [3(c+d x)] + \frac{1}{4} C \cos [c+d x]^{\frac{2}{3}+m} \cos [4(c+d x)] + \right.$$

$$\frac{1}{2} i B \cos [c+d x]^{\frac{2}{3}+m} \sin [c+d x] + \frac{1}{2} i B \cos [c+d x]^{\frac{2}{3}+m} \sin [3(c+d x)] + \frac{1}{4} i C \cos [c+d x]^{\frac{2}{3}+m} \sin [4(c+d x)] \left. \right) +$$

$$\left. \left. \sin [2(c+d x)] \left( -\frac{1}{4} i C \cos [c+d x]^{\frac{2}{3}+m} + \frac{1}{2} B \cos [c+d x]^{\frac{2}{3}+m} \sin [3(c+d x)] + \frac{1}{4} C \cos [c+d x]^{\frac{2}{3}+m} \sin [4(c+d x)] \right) \right) \right)$$

$$\tan \left[ \frac{1}{2} (c+d x) \right] \left( 1 - \tan \left[ \frac{1}{2} (c+d x) \right]^2 \right)^{-\frac{4}{3}+m} \left( \frac{1}{1 + \tan \left[ \frac{1}{2} (c+d x) \right]^2} \right)^{\frac{5}{3}+m}$$

$$\left( \left( 45 (A+B+C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{4}{3}-m, \frac{5}{3}+m, \frac{3}{2}, \tan \left[ \frac{1}{2} (c+d x) \right]^2, -\tan \left[ \frac{1}{2} (c+d x) \right]^2\right] \right) / \right.$$

$$\left( 9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{4}{3}-m, \frac{5}{3}+m, \frac{3}{2}, \tan \left[ \frac{1}{2} (c+d x) \right]^2, -\tan \left[ \frac{1}{2} (c+d x) \right]^2\right] - 2 \left( (5+3 m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}-m, \frac{8}{3}+m, \frac{5}{2}, \tan \left[ \frac{1}{2} (c+d x) \right]^2, \right. \right.$$

$$\left. \left. -\tan \left[ \frac{1}{2} (c+d x) \right]^2\right] + (-4+3 m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \tan \left[ \frac{1}{2} (c+d x) \right]^2, -\tan \left[ \frac{1}{2} (c+d x) \right]^2\right] \right) \tan \left[ \frac{1}{2} (c+d x) \right]^2 \right) +$$

$$\left( 50 (A-C) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \tan \left[ \frac{1}{2} (c+d x) \right]^2, -\tan \left[ \frac{1}{2} (c+d x) \right]^2\right] \tan \left[ \frac{1}{2} (c+d x) \right]^2 \right) /$$











$$\begin{aligned}
 & \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\Big] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\Big) + (-4+3m) \\
 & \left(-\frac{5}{7}\left(\frac{5}{3}+m\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{7}{3}-m, \frac{8}{3}+m, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \\
 & \left. \frac{5}{7}\left(\frac{7}{3}-m\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{10}{3}-m, \frac{5}{3}+m, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right]\right) \\
 & \left(15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}-m, \frac{5}{3}+m, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - 2\left((5+3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{4}{3}-m, \right. \right. \right. \\
 & \left. \left. \left. \frac{8}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) + \right. \\
 & \left. (-4+3m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{3}-m, \frac{5}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2 + \\
 & \left(21(A-B+C) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{4}{3}-m, \frac{5}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^4 \right. \\
 & \left. + 2\left((5+3m) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{4}{3}-m, \frac{8}{3}+m, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
 & \left. \left. (-4+3m) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{7}{3}-m, \frac{5}{3}+m, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - \right. \\
 & 21\left(-\frac{5}{7}\left(\frac{5}{3}+m\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{4}{3}-m, \frac{8}{3}+m, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \\
 & \left. \frac{5}{7}\left(\frac{4}{3}-m\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{7}{3}-m, \frac{5}{3}+m, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right) + \\
 & 2 \tan\left[\frac{1}{2}(c+dx)\right]^2 \left((5+3m) \left(-\frac{7}{9}\left(\frac{8}{3}+m\right) \operatorname{AppellF1}\left[\frac{9}{2}, \frac{4}{3}-m, \frac{11}{3}+m, \frac{11}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \right. \\
 & \left. \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{7}{9}\left(\frac{4}{3}-m\right) \operatorname{AppellF1}\left[\frac{9}{2}, \frac{7}{3}-m, \frac{8}{3}+m, \frac{11}{2}, \right. \right. \right. \\
 & \left. \left. \left. \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right)\right) + (-4+3m) \\
 & \left(-\frac{7}{9}\left(\frac{5}{3}+m\right) \operatorname{AppellF1}\left[\frac{9}{2}, \frac{7}{3}-m, \frac{8}{3}+m, \frac{11}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{7}{9} \right. \\
 & \left. \left(\frac{7}{3}-m\right) \operatorname{AppellF1}\left[\frac{9}{2}, \frac{10}{3}-m, \frac{5}{3}+m, \frac{11}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right)\right) \\
 & \left(-21 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{4}{3}-m, \frac{5}{3}+m, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + 2\left((5+3m) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{4}{3}-m, \right. \right. \right. \\
 & \left. \left. \left. \frac{8}{3}+m, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) + \right.
 \end{aligned}$$



$$\left. \left. \left. (-4 + 3m) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{7}{3} - m, \frac{5}{3} + m, \frac{9}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right], \tan\left[\frac{1}{2}(c + dx)\right]^2\right]^2\right)\right)\right)$$

■ **Problem 369: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a \cos[c + dx])^m (b \cos[c + dx])^n (A + B \cos[c + dx] + C \cos[c + dx]^2) dx$$

Optimal (type 5, 227 leaves, 5 steps):

$$\frac{C (a \cos[c + dx])^{1+m} (b \cos[c + dx])^n \sin[c + dx]}{a d (2 + m + n)} -$$

$$\left( (C (1 + m + n) + A (2 + m + n)) (a \cos[c + dx])^{1+m} (b \cos[c + dx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (1 + m + n), \frac{1}{2} (3 + m + n), \cos^2[c + dx]\right] \right. \\ \left. \sin[c + dx] \right) / \left( a d (1 + m + n) (2 + m + n) \sqrt{\sin^2[c + dx]} \right) -$$

$$\left( B (a \cos[c + dx])^{2+m} (b \cos[c + dx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (2 + m + n), \frac{1}{2} (4 + m + n), \cos^2[c + dx]\right] \sin[c + dx] \right) / \\ \left( a^2 d (2 + m + n) \sqrt{\sin^2[c + dx]} \right)$$

Result (type 5, 545 leaves):

$$\frac{1}{4 d} C \cos[c + dx]^{-m-n} (a \cos[c + dx])^m (b \cos[c + dx])^n$$

$$\left( 1 / (2 + m + n) i 2^{-m-n} e^{-2i(c+dx)} \left( e^{-i(c+dx)} + e^{i(c+dx)} \right)^{m+n} \left( 1 + e^{2i(c+dx)} \right)^{-m-n} \operatorname{Hypergeometric2F1}\left[-m-n, -1 - \frac{m}{2} - \frac{n}{2}, -\frac{m}{2} - \frac{n}{2}, -e^{2i(c+dx)}\right] + \right. \\ \left. 1 / (-2 + m + n) i 2^{-m-n} e^{2i(c+dx)} \left( e^{-i(c+dx)} + e^{i(c+dx)} \right)^{m+n} \left( 1 + e^{2i(c+dx)} \right)^{-m-n} \operatorname{Hypergeometric2F1}\left[-m-n, 1 - \frac{m}{2} - \frac{n}{2}, 2 - \frac{m}{2} - \frac{n}{2}, -e^{2i(c+dx)}\right] \right) -$$

$$\left( A \cos[c + dx] (a \cos[c + dx])^m (b \cos[c + dx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (1 + m + n), \frac{1}{2} (3 + m + n), \cos^2[c + dx]\right] \sin[c + dx] \right) / \\ \left( d (1 + m + n) \sqrt{\sin^2[c + dx]} \right) -$$

$$\left( C \cos[c + dx] (a \cos[c + dx])^m (b \cos[c + dx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (1 + m + n), \frac{1}{2} (3 + m + n), \cos^2[c + dx]\right] \sin[c + dx] \right) / \\ \left( 2 d (1 + m + n) \sqrt{\sin^2[c + dx]} \right) -$$

$$\left( B \cos^2[c + dx] (a \cos[c + dx])^m (b \cos[c + dx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (2 + m + n), \frac{1}{2} (4 + m + n), \cos^2[c + dx]\right] \sin[c + dx] \right) / \\ \left( d (2 + m + n) \sqrt{\sin^2[c + dx]} \right)$$

- **Problem 370: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^2 (b \cos [c+d x])^n (A+B \cos [c+d x]+C \cos [c+d x]^2) d x$$

Optimal (type 5, 187 leaves, 5 steps):

$$\frac{C (b \cos [c+d x])^{3+n} \sin [c+d x]}{b^3 d (4+n)} - \frac{(C (3+n)+A (4+n)) (b \cos [c+d x])^{3+n} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+n}{2}, \frac{5+n}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{b^3 d (3+n)(4+n) \sqrt{\sin [c+d x]^2}}$$

$$\frac{B (b \cos [c+d x])^{4+n} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{4+n}{2}, \frac{6+n}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{b^4 d (4+n) \sqrt{\sin [c+d x]^2}}$$

Result (type 6, 29753 leaves): Display of huge result suppressed!

- **Problem 372: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (b \cos [c+d x])^n (A+B \cos [c+d x]+C \cos [c+d x]^2) d x$$

Optimal (type 5, 187 leaves, 4 steps):

$$\frac{C (b \cos [c+d x])^{1+n} \sin [c+d x]}{b d (2+n)} - \frac{(C (1+n)+A (2+n)) (b \cos [c+d x])^{1+n} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{b d (1+n)(2+n) \sqrt{\sin [c+d x]^2}}$$

$$\frac{B (b \cos [c+d x])^{2+n} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{b^2 d (2+n) \sqrt{\sin [c+d x]^2}}$$

Result (type 5, 441 leaves):

$$\frac{1}{4d} C \cos[c+dx]^{-n} (b \cos[c+dx])^n$$

$$\left( \frac{1}{(2+n)i} 2^{-n} e^{-2i(c+dx)} (e^{-i(c+dx)} + e^{i(c+dx)})^n (1 + e^{2i(c+dx)})^{-n} \text{Hypergeometric2F1}\left[-1 - \frac{n}{2}, -n, -\frac{n}{2}, -e^{2i(c+dx)}\right] + \right.$$

$$\left. \frac{1}{(-2+n)i} 2^{-n} e^{2i(c+dx)} (e^{-i(c+dx)} + e^{i(c+dx)})^n (1 + e^{2i(c+dx)})^{-n} \text{Hypergeometric2F1}\left[1 - \frac{n}{2}, -n, 2 - \frac{n}{2}, -e^{2i(c+dx)}\right] \right) -$$

$$\frac{A \cos[c+dx] (b \cos[c+dx])^n \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos[c+dx]^2\right] \sin[c+dx]}{d(1+n) \sqrt{\sin[c+dx]^2}} -$$

$$\frac{C \cos[c+dx] (b \cos[c+dx])^n \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos[c+dx]^2\right] \sin[c+dx]}{2d(1+n) \sqrt{\sin[c+dx]^2}} -$$

$$\frac{B \cos[c+dx]^2 (b \cos[c+dx])^n \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \cos[c+dx]^2\right] \sin[c+dx]}{d(2+n) \sqrt{\sin[c+dx]^2}}$$

- **Problem 379: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(b \cos[c+dx])^n (A + B \cos[c+dx] + C \cos[c+dx]^2)}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 5, 221 leaves, 5 steps):

$$\frac{2C \sqrt{\cos[c+dx]} (b \cos[c+dx])^n \sin[c+dx]}{d(3+2n)} -$$

$$\left( 2(C + 2Cn + A(3+2n)) \sqrt{\cos[c+dx]} (b \cos[c+dx])^n \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(1+2n), \frac{1}{4}(5+2n), \cos[c+dx]^2\right] \sin[c+dx] \right) /$$

$$\left( d(1+2n)(3+2n) \sqrt{\sin[c+dx]^2} \right) -$$

$$\left( 2B \cos[c+dx]^{3/2} (b \cos[c+dx])^n \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(3+2n), \frac{1}{4}(7+2n), \cos[c+dx]^2\right] \sin[c+dx] \right) / \left( d(3+2n) \sqrt{\sin[c+dx]^2} \right)$$

Result (type 6, 7602 leaves):

$$\left( 2 \cos[c+dx]^{-n} (b \cos[c+dx])^n \right.$$

$$\left( \frac{1}{2} B \cos[c+dx]^{\frac{1}{2}+n} \cos[2(c+dx)] - \frac{1}{2} i B \cos[c+dx]^{\frac{1}{2}+n} \sin[2(c+dx)] + \sec[c+dx] \left( \left( A \cos[c+dx]^{\frac{1}{2}+n} + \frac{1}{2} C \cos[c+dx]^{\frac{1}{2}+n} \right) \right. \right.$$

$$\left. \left. \cos[2(c+dx)]^2 - \frac{1}{2} i B \cos[c+dx]^{\frac{1}{2}+n} \cos[3(c+dx)] \sin[2(c+dx)] - \frac{1}{4} i C \cos[c+dx]^{\frac{1}{2}+n} \cos[4(c+dx)] \sin[2(c+dx)] + \right. \right.$$

$$\begin{aligned}
& \frac{1}{2} B \cos [c+d x]^{\frac{1}{2}+n} \sin [c+d x] \sin [2(c+d x)] + \left( A \cos [c+d x]^{\frac{1}{2}+n} + \frac{1}{2} C \cos [c+d x]^{\frac{1}{2}+n} \right) \sin [2(c+d x)]^2 + \\
& \cos [2(c+d x)] \left( \frac{1}{4} C \cos [c+d x]^{\frac{1}{2}+n} + \frac{1}{2} B \cos [c+d x]^{\frac{1}{2}+n} \cos [3(c+d x)] + \frac{1}{4} C \cos [c+d x]^{\frac{1}{2}+n} \cos [4(c+d x)] + \right. \\
& \left. \frac{1}{2} i B \cos [c+d x]^{\frac{1}{2}+n} \sin [c+d x] + \frac{1}{2} i B \cos [c+d x]^{\frac{1}{2}+n} \sin [3(c+d x)] + \frac{1}{4} i C \cos [c+d x]^{\frac{1}{2}+n} \sin [4(c+d x)] \right) + \\
& \sin [2(c+d x)] \left( -\frac{1}{4} i C \cos [c+d x]^{\frac{1}{2}+n} + \frac{1}{2} B \cos [c+d x]^{\frac{1}{2}+n} \sin [3(c+d x)] + \frac{1}{4} C \cos [c+d x]^{\frac{1}{2}+n} \sin [4(c+d x)] \right) \Big) \Big) \\
& \tan \left[ \frac{1}{2}(c+d x) \right] \left( 1 - \tan \left[ \frac{1}{2}(c+d x) \right] \right)^{-\frac{1}{2}+n} \left( \frac{1}{1 + \tan \left[ \frac{1}{2}(c+d x) \right]^2} \right)^{\frac{5}{2}+n} \\
& \left( \left( 45(A+B+C) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}-n, \frac{5}{2}+n, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) / \right. \\
& \left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}-n, \frac{5}{2}+n, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \left( -(5+2n) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}-n, \frac{7}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + (1-2n) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}-n, \frac{5}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) + \\
& \left( 50(A-C) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}-n, \frac{5}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) / \\
& \left( 5 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}-n, \frac{5}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \\
& \left( -(5+2n) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2}-n, \frac{7}{2}+n, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \\
& \left. \left. (1-2n) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{3}{2}-n, \frac{5}{2}+n, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) - \\
& \left( 21(A-B+C) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2}-n, \frac{5}{2}+n, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \tan \left[ \frac{1}{2}(c+d x) \right]^4 \right) / \\
& \left( -7 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2}-n, \frac{5}{2}+n, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \\
& \left( (5+2n) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{1}{2}-n, \frac{7}{2}+n, \frac{9}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \\
& \left. \left. (-1+2n) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{3}{2}-n, \frac{5}{2}+n, \frac{9}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) \Big) \Big) /
\end{aligned}$$















Result (type 6, 7612 leaves) :

$$\begin{aligned}
& \left( 2 \operatorname{Cos}[c + d x]^{-n} (b \operatorname{Cos}[c + d x])^n \left( \operatorname{Sec}[c + d x] \left( \frac{1}{2} B \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} \operatorname{Cos}[2(c + d x)] - \frac{1}{2} i B \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} \operatorname{Sin}[2(c + d x)] \right) + \right. \right. \\
& \quad \operatorname{Sec}[c + d x]^2 \left( \left( A \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} + \frac{1}{2} C \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} \right) \operatorname{Cos}[2(c + d x)]^2 - \right. \\
& \quad \frac{1}{2} i B \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} \operatorname{Cos}[3(c + d x)] \operatorname{Sin}[2(c + d x)] - \frac{1}{4} i C \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} \operatorname{Cos}[4(c + d x)] \operatorname{Sin}[2(c + d x)] + \\
& \quad \frac{1}{2} B \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} \operatorname{Sin}[c + d x] \operatorname{Sin}[2(c + d x)] + \left. \left( A \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} + \frac{1}{2} C \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} \right) \operatorname{Sin}[2(c + d x)]^2 + \right. \\
& \quad \operatorname{Cos}[2(c + d x)] \left( \frac{1}{4} C \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} + \frac{1}{2} B \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} \operatorname{Cos}[3(c + d x)] + \frac{1}{4} C \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} \operatorname{Cos}[4(c + d x)] + \right. \\
& \quad \left. \left. \frac{1}{2} i B \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} \operatorname{Sin}[c + d x] + \frac{1}{2} i B \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} \operatorname{Sin}[3(c + d x)] + \frac{1}{4} i C \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} \operatorname{Sin}[4(c + d x)] \right) \right) + \\
& \quad \left. \left. \operatorname{Sin}[2(c + d x)] \left( -\frac{1}{4} i C \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} + \frac{1}{2} B \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} \operatorname{Sin}[3(c + d x)] + \frac{1}{4} C \operatorname{Cos}[c + d x]^{\frac{1}{2}+n} \operatorname{Sin}[4(c + d x)] \right) \right) \right) \\
& \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] \left( 1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] \right)^{-\frac{3}{2}+n} \left( \frac{1}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \right)^{\frac{3}{2}+n} \\
& \left( \left( 45 (A + B + C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2} - n, \frac{3}{2} + n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \right) / \right. \\
& \quad \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2} - n, \frac{3}{2} + n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] - \left( (3 + 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2} - n, \frac{5}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] + (-3 + 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2} - n, \frac{3}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) + \\
& \quad \left( 50 (A - C) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2} - n, \frac{3}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) / \\
& \quad \left( 5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2} - n, \frac{3}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] - \right. \\
& \quad \left( (3 + 2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2} - n, \frac{5}{2} + n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] + \right. \\
& \quad \left. (-3 + 2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2} - n, \frac{3}{2} + n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) - \\
& \quad \left( 21 (A - B + C) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2} - n, \frac{3}{2} + n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^4 \right) /
\end{aligned}$$



$$\begin{aligned}
& -\tan\left[\frac{1}{2}(c+dx)\right]^2 + (-3+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \Big) + \\
& \left(50(A-C) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2\right) / \\
& \left(5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \left((3+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}-n, \frac{5}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (-3+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}-n, \frac{3}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) - \\
& \left(21(A-B+C) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^4\right) / \\
& \left(-7 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \quad \left. \left((3+2n) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{3}{2}-n, \frac{5}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. (-3+2n) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{5}{2}-n, \frac{3}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \Big) - \\
& \frac{2}{15} \left(\frac{3}{2}+n\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^2 \left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{-\frac{3}{2}+n} \left(\frac{1}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}\right)^{\frac{5}{2}+n} \\
& \left(\left(45(A+B+C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) / \right. \\
& \quad \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \left((3+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-n, \frac{5}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (-3+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
& \left(50(A-C) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2\right) / \\
& \left(5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \left((3+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}-n, \frac{5}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (-3+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}-n, \frac{3}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) - \\
& \left(21(A-B+C) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^4\right) / \\
& \left(-7 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right.
\end{aligned}$$





$$\begin{aligned}
& \left. \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2 - \left(50(A-C) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right. \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \left(-\left((3+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}-n, \frac{5}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) + \right. \\
& \quad \left. (-3+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}-n, \frac{3}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \\
& 5\left(-\frac{3}{5}\left(\frac{3}{2}+n\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}-n, \frac{5}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \quad \left. \frac{3}{5}\left(\frac{3}{2}-n\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}-n, \frac{3}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right) - \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \left((3+2n) \left(-\frac{5}{7}\left(\frac{5}{2}+n\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{3}{2}-n, \frac{7}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \right. \right. \\
& \quad \left. \tan\left[\frac{1}{2}(c+dx)\right] + \frac{5}{7}\left(\frac{3}{2}-n\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{5}{2}-n, \frac{5}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \quad \left. \tan\left[\frac{1}{2}(c+dx)\right]\right) + (-3+2n) \left(-\frac{5}{7}\left(\frac{3}{2}+n\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{5}{2}-n, \frac{5}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \\
& \quad \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{5}{7}\left(\frac{5}{2}-n\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{7}{2}-n, \frac{3}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \\
& \quad \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right)\right)\right) / \left(5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \\
& \left. \left((3+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}-n, \frac{5}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (-3+2n) \operatorname{AppellF1}\left[\frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. \frac{5}{2}-n, \frac{3}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2 + \\
& \left(21(A-B+C) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}-n, \frac{3}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^4 \right. \\
& \left. \left(\left((3+2n) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{3}{2}-n, \frac{5}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \right. \\
& \quad \left. \left. (-3+2n) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{5}{2}-n, \frac{3}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - \right. \\
& 7\left(-\frac{5}{7}\left(\frac{3}{2}+n\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{3}{2}-n, \frac{5}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \quad \left. \frac{5}{7}\left(\frac{3}{2}-n\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{5}{2}-n, \frac{3}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right) + \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \left((3+2n) \left(-\frac{7}{9}\left(\frac{5}{2}+n\right) \operatorname{AppellF1}\left[\frac{9}{2}, \frac{3}{2}-n, \frac{7}{2}+n, \frac{11}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \right.
\end{aligned}$$





$$\begin{aligned}
& \frac{1}{2} i B \cos [c+d x]^{\frac{1}{2}+n} \cos [3(c+d x)] \sin [2(c+d x)] - \frac{1}{4} i C \cos [c+d x]^{\frac{1}{2}+n} \cos [4(c+d x)] \sin [2(c+d x)] + \\
& \frac{1}{2} B \cos [c+d x]^{\frac{1}{2}+n} \sin [c+d x] \sin [2(c+d x)] + \left( A \cos [c+d x]^{\frac{1}{2}+n} + \frac{1}{2} C \cos [c+d x]^{\frac{1}{2}+n} \right) \sin [2(c+d x)]^2 + \\
& \cos [2(c+d x)] \left( \frac{1}{4} C \cos [c+d x]^{\frac{1}{2}+n} + \frac{1}{2} B \cos [c+d x]^{\frac{1}{2}+n} \cos [3(c+d x)] + \frac{1}{4} C \cos [c+d x]^{\frac{1}{2}+n} \cos [4(c+d x)] + \right. \\
& \left. \frac{1}{2} i B \cos [c+d x]^{\frac{1}{2}+n} \sin [c+d x] + \frac{1}{2} i B \cos [c+d x]^{\frac{1}{2}+n} \sin [3(c+d x)] + \frac{1}{4} i C \cos [c+d x]^{\frac{1}{2}+n} \sin [4(c+d x)] \right) + \\
& \sin [2(c+d x)] \left( -\frac{1}{4} i C \cos [c+d x]^{\frac{1}{2}+n} + \frac{1}{2} B \cos [c+d x]^{\frac{1}{2}+n} \sin [3(c+d x)] + \frac{1}{4} C \cos [c+d x]^{\frac{1}{2}+n} \sin [4(c+d x)] \right) \left. \right) \tan \left[ \frac{1}{2}(c+d x) \right] \\
& \left( \frac{1 - \tan \left[ \frac{1}{2}(c+d x) \right]^2}{1 + \tan \left[ \frac{1}{2}(c+d x) \right]^2} \right)^{\frac{1}{2}+n} \left( \left( A \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}-n, \frac{1}{2}+n, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \left( -1 + \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right)^2 \right) \right) / \\
& \left( -3 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}-n, \frac{1}{2}+n, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \left( (1+2n) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, \right. \right. \right. \\
& \left. \left. -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + (-1+2n) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) - \\
& \left( B \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}-n, \frac{1}{2}+n, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \left( -1 + \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right)^2 \right) / \\
& \left( -3 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}-n, \frac{1}{2}+n, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \left( (1+2n) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, \right. \right. \right. \\
& \left. \left. -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + (-1+2n) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) + \\
& \left( C \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}-n, \frac{1}{2}+n, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \left( -1 + \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right)^2 \right) / \\
& \left( -3 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}-n, \frac{1}{2}+n, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \\
& \left( (1+2n) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \\
& \left. \left. (-1+2n) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) + \\
& \left( 4 A \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{3}{2}-n, \frac{1}{2}+n, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \left( -1 + \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) \right) / \\
& \left( -3 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{3}{2}-n, \frac{1}{2}+n, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right.
\end{aligned}$$







$$\begin{aligned}
 & \left( \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \left( (1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \\
 & \left. (-5+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \Big) - \\
 & \frac{1}{\left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^3} 6 \left(\frac{1}{2} + n\right) \tan\left[\frac{1}{2}(c+dx)\right] \left(\frac{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}\right)^{-\frac{1}{2}+n} \\
 & \left( -\frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2} - \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right) \\
 & \left( \left( \operatorname{A AppellF1}\left[\frac{1}{2}, \frac{1}{2}-n, \frac{1}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2 \right) \right) / \left( -3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-n, \frac{1}{2}+n, \right. \right. \\
 & \left. \left. \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2 \right] + \left( (1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \right. \\
 & \left. (-1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \Big) - \\
 & \left( \operatorname{B AppellF1}\left[\frac{1}{2}, \frac{1}{2}-n, \frac{1}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2 \right) / \\
 & \left( -3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-n, \frac{1}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
 & \left. \left( (1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
 & \left. \left. (-1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
 & \left( \operatorname{C AppellF1}\left[\frac{1}{2}, \frac{1}{2}-n, \frac{1}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2 \right) / \\
 & \left( -3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-n, \frac{1}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
 & \left. \left( (1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-n, \frac{3}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
 & \left. \left. (-1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
 & \left( 4 \operatorname{A AppellF1}\left[\frac{1}{2}, \frac{3}{2}-n, \frac{1}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \right) /
 \end{aligned}$$

















$$\begin{aligned}
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \left( (1+2n) \left( -\frac{3}{5} \left( \frac{3}{2} + n \right) \text{AppellF1}\left[\frac{5}{2}, \frac{5}{2} - n, \frac{5}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{3}{5} \left( \frac{5}{2} - n \right) \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2} - n, \frac{3}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) + (-5+2n) \left( -\frac{3}{5} \left( \frac{1}{2} + n \right) \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2} - n, \frac{3}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{3}{5} \left( \frac{7}{2} - n \right) \text{AppellF1}\left[\frac{5}{2}, \frac{9}{2} - n, \frac{1}{2} + n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) / \left( -3 \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2} - n, \frac{1}{2} + n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \left( (1+2n) \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2} - n, \frac{3}{2} + n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. (-5+2n) \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2} - n, \frac{1}{2} + n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right)
\end{aligned}$$

■ **Problem 382: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(b \cos[c+dx])^n (A + B \cos[c+dx] + C \cos[c+dx]^2)}{\cos[c+dx]^{7/2}} dx$$

Optimal (type 5, 223 leaves, 5 steps):

$$\begin{aligned}
& -\frac{2C(b \cos[c+dx])^n \sin[c+dx]}{d(3-2n)\cos[c+dx]^{5/2}} + \\
& \left( \frac{2(A(3-2n) + C(5-2n))(b \cos[c+dx])^n \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(-5+2n), \frac{1}{4}(-1+2n), \cos[c+dx]^2\right] \sin[c+dx]}{d(3-2n)(5-2n)\cos[c+dx]^{5/2} \sqrt{\sin[c+dx]^2}} + \right. \\
& \left. \frac{2B(b \cos[c+dx])^n \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(-3+2n), \frac{1}{4}(1+2n), \cos[c+dx]^2\right] \sin[c+dx]}{d(3-2n)\cos[c+dx]^{3/2} \sqrt{\sin[c+dx]^2}} \right)
\end{aligned}$$

Result (type 6, 7597 leaves):

$$\begin{aligned}
& \left( 2 \cos[c+dx]^{-n} (b \cos[c+dx])^n \left( \sec[c+dx]^3 \left( \frac{1}{2} B \cos[c+dx]^{\frac{1}{2}+n} \cos[2(c+dx)] - \frac{1}{2} i B \cos[c+dx]^{\frac{1}{2}+n} \sin[2(c+dx)] \right) + \right. \right. \\
& \quad \left. \left. \sec[c+dx]^4 \left( \left( A \cos[c+dx]^{\frac{1}{2}+n} + \frac{1}{2} C \cos[c+dx]^{\frac{1}{2}+n} \right) \cos[2(c+dx)]^2 - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} i B \cos [c+d x]^{\frac{1}{2}+n} \cos [3(c+d x)] \sin [2(c+d x)] - \frac{1}{4} i C \cos [c+d x]^{\frac{1}{2}+n} \cos [4(c+d x)] \sin [2(c+d x)] + \\
& \frac{1}{2} B \cos [c+d x]^{\frac{1}{2}+n} \sin [c+d x] \sin [2(c+d x)] + \left( A \cos [c+d x]^{\frac{1}{2}+n} + \frac{1}{2} C \cos [c+d x]^{\frac{1}{2}+n} \right) \sin [2(c+d x)]^2 + \\
& \cos [2(c+d x)] \left( \frac{1}{4} C \cos [c+d x]^{\frac{1}{2}+n} + \frac{1}{2} B \cos [c+d x]^{\frac{1}{2}+n} \cos [3(c+d x)] + \frac{1}{4} C \cos [c+d x]^{\frac{1}{2}+n} \cos [4(c+d x)] + \right. \\
& \left. \frac{1}{2} i B \cos [c+d x]^{\frac{1}{2}+n} \sin [c+d x] + \frac{1}{2} i B \cos [c+d x]^{\frac{1}{2}+n} \sin [3(c+d x)] + \frac{1}{4} i C \cos [c+d x]^{\frac{1}{2}+n} \sin [4(c+d x)] \right) + \\
& \sin [2(c+d x)] \left( -\frac{1}{4} i C \cos [c+d x]^{\frac{1}{2}+n} + \frac{1}{2} B \cos [c+d x]^{\frac{1}{2}+n} \sin [3(c+d x)] + \frac{1}{4} C \cos [c+d x]^{\frac{1}{2}+n} \sin [4(c+d x)] \right) \Big) \Big) \\
& \tan \left[ \frac{1}{2}(c+d x) \right] \left( 1 - \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right)^{-\frac{7}{2}+n} \left( \frac{1}{1 + \tan \left[ \frac{1}{2}(c+d x) \right]^2} \right)^{-\frac{1}{2}+n} \\
& \left( \left( 45(A+B+C) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) / \right. \\
& \left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{3}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \left( (1-2n) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{7}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + (7-2n) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{9}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) + \\
& \left( 50(A-C) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) / \\
& \left( 5 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \\
& \left( (1-2n) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{7}{2}-n, \frac{1}{2}+n, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \\
& \left. \left. (7-2n) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{9}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \right) \tan \left[ \frac{1}{2}(c+d x) \right]^2 \right) - \\
& \left( 21(A-B+C) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] \tan \left[ \frac{1}{2}(c+d x) \right]^4 \right) / \\
& \left( -7 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \\
& \left. \left( (-1+2n) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{7}{2}-n, \frac{1}{2}+n, \frac{9}{2}, \tan \left[ \frac{1}{2}(c+d x) \right]^2, -\tan \left[ \frac{1}{2}(c+d x) \right]^2 \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. (-7 + 2n) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{9}{2} - n, -\frac{1}{2} + n, \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right) \Bigg/ \\
& \left( 15d \left( -\frac{2}{15} \left( -\frac{7}{2} + n \right) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \left( 1 - \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \right)^{-\frac{9}{2} + n} \left( \frac{1}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2} \right)^{-\frac{1}{2} + n} \right. \right. \\
& \left. \left( \left( 45(A + B + C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2} - n, -\frac{1}{2} + n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \Bigg/ \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2} - n, -\frac{1}{2} + n, \frac{3}{2}, \right. \right. \right. \\
& \left. \left. \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) + \left( (1 - 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2} - n, \frac{1}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right)^2 + \right. \\
& \left. (7 - 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2} - n, -\frac{1}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \right) + \\
& \left( 50(A - C) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2} - n, -\frac{1}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \right) \Bigg/ \\
& \left( 5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2} - n, -\frac{1}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) + \\
& \left( (1 - 2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2} - n, \frac{1}{2} + n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) + \\
& \left. (7 - 2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2} - n, -\frac{1}{2} + n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \right) - \\
& \left( 21(A - B + C) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2} - n, -\frac{1}{2} + n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^4 \right) \Bigg/ \\
& \left( -7 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2} - n, -\frac{1}{2} + n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) + \\
& \left( (-1 + 2n) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{7}{2} - n, \frac{1}{2} + n, \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] + (-7 + 2n) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{9}{2} - n, -\frac{1}{2} + n, \right. \right. \\
& \left. \left. \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \right) + \frac{1}{15} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \left( 1 - \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \right)^{\frac{7}{2} + n} \\
& \left( \frac{1}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2} \right)^{\frac{1}{2} + n} \left( \left( 45(A + B + C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2} - n, -\frac{1}{2} + n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \Bigg/ \right. \\
& \left. \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2} - n, -\frac{1}{2} + n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \right) + \\
& \left( (1 - 2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2} - n, \frac{1}{2} + n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) +
\end{aligned}$$







$$\begin{aligned}
& \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\bigg)\bigg) / \left(5 \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \left. \left((1-2n) \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, \frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (7-2n) \text{AppellF1}\left[\frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. \frac{9}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2 - \right. \\
& \left. \left(42(A-B+C) \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^3\right) / \right. \\
& \left. \left(-7 \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \quad \left. \left( (-1+2n) \text{AppellF1}\left[\frac{7}{2}, \frac{7}{2}-n, \frac{1}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (-7+2n) \text{AppellF1}\left[\frac{7}{2}, \frac{9}{2}-n, \right. \right. \right. \\
& \quad \quad \left. \left. -\frac{1}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2 - \left(21(A-B+C) \tan\left[\frac{1}{2}(c+dx)\right]^4 \right. \right. \\
& \quad \left. \left. \left(-\frac{5}{7}\left(-\frac{1}{2}+n\right) \text{AppellF1}\left[\frac{7}{2}, \frac{7}{2}-n, \frac{1}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \right. \right. \\
& \quad \left. \left. \frac{5}{7}\left(\frac{7}{2}-n\right) \text{AppellF1}\left[\frac{7}{2}, \frac{9}{2}-n, -\frac{1}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right)\right) / \right. \\
& \left. \left(-7 \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \left((-1+2n) \text{AppellF1}\left[\frac{7}{2}, \frac{7}{2}-n, \frac{1}{2}+n, \frac{9}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (-7+2n) \text{AppellF1}\left[\frac{7}{2}, \frac{9}{2}-n, -\frac{1}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \right. \\
& \left. \tan\left[\frac{1}{2}(c+dx)\right]^2 - \left(45(A+B+C) \text{AppellF1}\left[\frac{1}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \right. \\
& \left. \left. \left(\left( (1-2n) \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \right. \right. \\
& \quad \left. \left. \left(7-2n\right) \text{AppellF1}\left[\frac{3}{2}, \frac{9}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \right. \\
& \quad \left. \left. 3\left(-\frac{1}{3}\left(-\frac{1}{2}+n\right) \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \right. \right. \\
& \quad \left. \left. \frac{1}{3}\left(\frac{7}{2}-n\right) \text{AppellF1}\left[\frac{3}{2}, \frac{9}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right) \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(c+dx)\right]^2 \left((1-2n) \left(-\frac{3}{5}\left(\frac{1}{2}+n\right) \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, \frac{3}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right. \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(c+dx)\right] + \frac{3}{5}\left(\frac{7}{2}-n\right) \text{AppellF1}\left[\frac{5}{2}, \frac{9}{2}-n, \frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \tan\left[\frac{1}{2}(c+dx)\right] + (7-2n) \left(-\frac{3}{5} \left(-\frac{1}{2}+n\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2}-n, \frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right. \\
& \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{3}{5} \left(\frac{9}{2}-n\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{11}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]^2 \\
& \left.\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right)\right) \Big/ \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \left. \left((1-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, \frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (7-2n) \operatorname{AppellF1}\left[\frac{3}{2}, \right.\right.\right. \\
& \left.\left.\left.\frac{9}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right)\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2 - \\
& \left(50(A-C) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2\right. \\
& \left.\left(\left((1-2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, \frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right.\right.\right. \\
& \left.\left.\left.(7-2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right)\right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \\
& 5 \left(-\frac{3}{5} \left(-\frac{1}{2}+n\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, \frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \left.\frac{3}{5} \left(\frac{7}{2}-n\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right) + \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \left((1-2n) \left(-\frac{5}{7} \left(\frac{1}{2}+n\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{7}{2}-n, \frac{3}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2\right.\right. \\
& \left.\tan\left[\frac{1}{2}(c+dx)\right] + \frac{5}{7} \left(\frac{7}{2}-n\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{9}{2}-n, \frac{1}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2\right. \\
& \left.\tan\left[\frac{1}{2}(c+dx)\right]\right) + (7-2n) \left(-\frac{5}{7} \left(-\frac{1}{2}+n\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{9}{2}-n, \frac{1}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \\
& \left.\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{5}{7} \left(\frac{9}{2}-n\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{11}{2}-n, -\frac{1}{2}+n, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]^2\right) \\
& \left.\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right)\right) \Big/ \left(5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \left. \left((1-2n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, \frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (7-2n) \operatorname{AppellF1}\left[\frac{5}{2}, \right.\right.\right. \\
& \left.\left.\left.\frac{9}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right)\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2 + \\
& \left(21(A-B+C) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}-n, -\frac{1}{2}+n, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^4\right.
\end{aligned}$$

$$\begin{aligned}
& \left( \left( (-1 + 2n) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{7}{2} - n, \frac{1}{2} + n, \frac{9}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (-7 + 2n) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{9}{2} - n, -\frac{1}{2} + n, \frac{9}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \right) \operatorname{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right] - \right. \\
& \quad 7 \left( -\frac{5}{7} \left( -\frac{1}{2} + n \right) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{7}{2} - n, \frac{1}{2} + n, \frac{9}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right] + \right. \\
& \quad \left. \frac{5}{7} \left( \frac{7}{2} - n \right) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{9}{2} - n, -\frac{1}{2} + n, \frac{9}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right] \right) + \\
& \quad \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \left( (-1 + 2n) \left( -\frac{7}{9} \left( \frac{1}{2} + n \right) \operatorname{AppellF1} \left[ \frac{9}{2}, \frac{7}{2} - n, \frac{3}{2} + n, \frac{11}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right] + \frac{7}{9} \left( \frac{7}{2} - n \right) \operatorname{AppellF1} \left[ \frac{9}{2}, \frac{9}{2} - n, \frac{1}{2} + n, \frac{11}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right] \right) + (-7 + 2n) \left( -\frac{7}{9} \left( -\frac{1}{2} + n \right) \operatorname{AppellF1} \left[ \frac{9}{2}, \frac{9}{2} - n, \frac{1}{2} + n, \right. \right. \\
& \quad \left. \left. \frac{11}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right] + \frac{7}{9} \left( \frac{9}{2} - n \right) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{9}{2}, \frac{11}{2} - n, -\frac{1}{2} + n, \frac{11}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right] \right) \right) \right) \Bigg) / \\
& \quad \left( -7 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{7}{2} - n, -\frac{1}{2} + n, \frac{7}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] + \left( (-1 + 2n) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{7}{2} - n, \right. \right. \right. \\
& \quad \left. \left. \frac{1}{2} + n, \frac{9}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] + \right. \\
& \quad \left. \left. (-7 + 2n) \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{9}{2} - n, -\frac{1}{2} + n, \frac{9}{2}, \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 383: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[e + fx])^m (A + B \cos[e + fx] + C \cos[e + fx]^2) dx$$

Optimal (type 5, 183 leaves, 4 steps):

$$\begin{aligned}
& -\frac{(C - B(2 + m))(a + a \cos[e + fx])^m \sin[e + fx]}{f(1 + m)(2 + m)} + \frac{C(a + a \cos[e + fx])^{1+m} \sin[e + fx]}{af(2 + m)} + \\
& \frac{1}{f(1 + m)(2 + m)} 2^{\frac{1}{2}+m} (Bm(2 + m) + C(1 + m + m^2) + A(2 + 3m + m^2)) (1 + \cos[e + fx])^{-\frac{1}{2}-m} \\
& (a + a \cos[e + fx])^m \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{1}{2} (1 - \cos[e + fx]) \right] \sin[e + fx]
\end{aligned}$$

Result (type 5, 376 leaves) :

$$\frac{1}{f} i 4^{-1-m} e^{i f m x} \left(1 + e^{i(e+f x)}\right)^{-2m} \left(e^{-\frac{1}{2} i(e+f x)} \left(1 + e^{i(e+f x)}\right)\right)^{2m} \cos\left[\frac{1}{2}(e+f x)\right]^{-2m} (a(1 + \cos[e+f x]))^m$$

$$\left( \frac{C e^{-i(2e+f(2+m)x)} \text{Hypergeometric2F1}[-2-m, -2m, -1-m, -e^{i(e+f x)}]}{2+m} + \frac{2 B e^{-i(e+f(1+m)x)} \text{Hypergeometric2F1}[-1-m, -2m, -m, -e^{i(e+f x)}]}{1+m} + \frac{2 B e^{i(e-f(-1+m)x)} \text{Hypergeometric2F1}[1-m, -2m, 2-m, -e^{i(e+f x)}]}{-1+m} + \frac{C e^{2i e-i f(-2+m)x} \text{Hypergeometric2F1}[2-m, -2m, 3-m, -e^{i(e+f x)}]}{-2+m} + \frac{4 A e^{-i f m x} \text{Hypergeometric2F1}[-2m, -m, 1-m, -e^{i(e+f x)}]}{m} + \frac{2 C e^{-i f m x} \text{Hypergeometric2F1}[-2m, -m, 1-m, -e^{i(e+f x)}]}{m} \right)$$

■ **Problem 384: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + a \cos[c + dx])^{2/3} (A + B \cos[c + dx] + C \cos[c + dx]^2) dx$$

Optimal (type 5, 144 leaves, 4 steps) :

$$\frac{3(8B - 3C)(a + a \cos[c + dx])^{2/3} \sin[c + dx]}{40d} + \frac{3C(a + a \cos[c + dx])^{5/3} \sin[c + dx]}{8ad} + \left( (40A + 16B + 19C)(a + a \cos[c + dx])^{2/3} \text{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1 - \cos[c + dx])\right] \sin[c + dx] \right) / (10 \times 2^{5/6} d (1 + \cos[c + dx])^{7/6})$$

Result (type 5, 137 leaves) :

$$\frac{1}{320d} 3(a(1 + \cos[c + dx]))^{2/3} \sec\left[\frac{1}{2}(c + dx)\right]^2$$

$$\left( -2i(40A + 16B + 19C) \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -e^{i(c+dx)}\right] (1 + \cos[c + dx] + i \sin[c + dx])^{2/3} + 2(40A + 32B + 28C + 2(8B + 7C) \cos[c + dx] + 5C \cos[2(c + dx)]) \sin[c + dx] \right)$$

■ **Problem 385: Unable to integrate problem.**

$$\int (a + a \cos[c + dx])^{1/3} (A + B \cos[c + dx] + C \cos[c + dx]^2) dx$$

Optimal (type 5, 144 leaves, 4 steps) :

$$\frac{3(7B - 3C)(a + a \cos[c + dx])^{1/3} \sin[c + dx]}{28d} + \frac{3C(a + a \cos[c + dx])^{4/3} \sin[c + dx]}{7ad} + \frac{(28A + 7B + 13C)(a + a \cos[c + dx])^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1 - \cos[c + dx])\right] \sin[c + dx]}{14 \times 2^{1/6} d (1 + \cos[c + dx])^{5/6}}$$

Result (type 8, 37 leaves) :

$$\int (a + a \cos [c + d x])^{1/3} (A + B \cos [c + d x] + C \cos [c + d x]^2) dx$$

■ **Problem 386: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos [c + d x] + C \cos [c + d x]^2}{(a + a \cos [c + d x])^{1/3}} dx$$

Optimal (type 5, 144 leaves, 4 steps) :

$$\frac{3 (5 B - 3 C) \sin [c + d x]}{10 d (a + a \cos [c + d x])^{1/3}} + \frac{3 C (a + a \cos [c + d x])^{2/3} \sin [c + d x]}{5 a d} +$$

$$\frac{(10 A - 5 B + 7 C) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, \frac{1}{2} (1 - \cos [c + d x])\right] \sin [c + d x]}{5 \times 2^{5/6} d (1 + \cos [c + d x])^{1/6} (a + a \cos [c + d x])^{1/3}}$$

Result (type 5, 105 leaves) :

$$\left( -3 i (10 A - 5 B + 7 C) \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -e^{i (c + d x)}\right] (1 + \cos [c + d x] + i \sin [c + d x])^{2/3} + 3 (5 B - C + 2 C \cos [c + d x]) \sin [c + d x] \right) /$$

$$(10 d (a (1 + \cos [c + d x]))^{1/3})$$

■ **Problem 387: Unable to integrate problem.**

$$\int \frac{A + B \cos [c + d x] + C \cos [c + d x]^2}{(a + a \cos [c + d x])^{2/3}} dx$$

Optimal (type 5, 144 leaves, 4 steps) :

$$\frac{3 (A - B + C) \sin [c + d x]}{d (a + a \cos [c + d x])^{2/3}} + \frac{3 C (a + a \cos [c + d x])^{1/3} \sin [c + d x]}{4 a d} -$$

$$\frac{(4 A - 8 B + 7 C) (a + a \cos [c + d x])^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2} (1 - \cos [c + d x])\right] \sin [c + d x]}{2 \times 2^{1/6} a d (1 + \cos [c + d x])^{5/6}}$$

Result (type 8, 37 leaves) :

$$\int \frac{A + B \cos [c + d x] + C \cos [c + d x]^2}{(a + a \cos [c + d x])^{2/3}} dx$$

■ **Problem 388: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^{2/3} (A + B \cos [c + d x] + C \cos [c + d x]^2) dx$$

Optimal (type 6, 290 leaves, 8 steps) :

$$\frac{3 C (a + b \cos [c + d x])^{5/3} \sin [c + d x]}{8 b d} +$$

$$\left( (a + b) (8 b B - 3 a C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{5}{3}, \frac{3}{2}, \frac{1}{2} (1 - \cos [c + d x]), \frac{b (1 - \cos [c + d x])}{a + b}\right] (a + b \cos [c + d x])^{2/3} \sin [c + d x] \right) /$$

$$\left( 4 \sqrt{2} b^2 d \sqrt{1 + \cos [c + d x]} \left( \frac{a + b \cos [c + d x]}{a + b} \right)^{2/3} \right) +$$

$$\left( (8 A b^2 - 8 a b B + 3 a^2 C + 5 b^2 C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \cos [c + d x]), \frac{b (1 - \cos [c + d x])}{a + b}\right] (a + b \cos [c + d x])^{2/3} \sin [c + d x] \right) /$$

$$\left( 4 \sqrt{2} b^2 d \sqrt{1 + \cos [c + d x]} \left( \frac{a + b \cos [c + d x]}{a + b} \right)^{2/3} \right)$$

Result (type 6, 1607 leaves):

$$-\frac{1}{2 b d} 3 a A \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, -\frac{a + b \cos [c + d x]}{(1 - \frac{a}{b}) b}, -\frac{a + b \cos [c + d x]}{(-1 - \frac{a}{b}) b}\right]$$

$$\sqrt{\frac{-b - b \cos [c + d x]}{a - b}} \sqrt{\frac{b - b \cos [c + d x]}{a + b}} (a + b \cos [c + d x])^{2/3} \operatorname{Csc}[c + d x] - \frac{1}{5 d}$$

$$3 B \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, -\frac{a + b \cos [c + d x]}{(1 - \frac{a}{b}) b}, -\frac{a + b \cos [c + d x]}{(-1 - \frac{a}{b}) b}\right] \sqrt{\frac{-b - b \cos [c + d x]}{a - b}} \sqrt{\frac{b - b \cos [c + d x]}{a + b}}$$

$$(a + b \cos [c + d x])^{2/3} \operatorname{Csc}[c + d x] - \frac{1}{80 b d} 57 a C \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, -\frac{a + b \cos [c + d x]}{(1 - \frac{a}{b}) b}, -\frac{a + b \cos [c + d x]}{(-1 - \frac{a}{b}) b}\right]$$

$$\sqrt{\frac{-b - b \cos [c + d x]}{a - b}} \sqrt{\frac{b - b \cos [c + d x]}{a + b}} (a + b \cos [c + d x])^{2/3} \operatorname{Csc}[c + d x] + \frac{1}{d}$$

$$A b \left( 1 / (2 b^2) 3 a \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, -\frac{a + b \cos [c + d x]}{(1 - \frac{a}{b}) b}, -\frac{a + b \cos [c + d x]}{(-1 - \frac{a}{b}) b}\right] \sqrt{\frac{-b - b \cos [c + d x]}{a - b}} \sqrt{\frac{b - b \cos [c + d x]}{a + b}} \right.$$

$$\left. (a + b \cos [c + d x])^{2/3} \operatorname{Csc}[c + d x] - 1 / (5 b^2) 3 \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, \frac{1}{2}, \frac{8}{3}, -\frac{a + b \cos [c + d x]}{(1 - \frac{a}{b}) b}, -\frac{a + b \cos [c + d x]}{(-1 - \frac{a}{b}) b}\right] \right.$$

$$\left. \sqrt{\frac{-b - b \cos [c + d x]}{a - b}} \sqrt{\frac{b - b \cos [c + d x]}{a + b}} (a + b \cos [c + d x])^{5/3} \operatorname{Csc}[c + d x] \right) + \frac{1}{5 d}$$



$$\begin{aligned}
& 2 a B \left( 1 / (2 b^2)^3 a \operatorname{AppellF1} \left[ \frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, -\frac{a+b \cos [c+d x]}{\left(1-\frac{a}{b}\right) b}, -\frac{a+b \cos [c+d x]}{\left(-1-\frac{a}{b}\right) b} \right] \sqrt{\frac{-b-b \cos [c+d x]}{a-b}} \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \right. \\
& \quad (a+b \cos [c+d x])^{2/3} \operatorname{Csc}[c+d x] - 1 / (5 b^2)^3 \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{2}, \frac{1}{2}, \frac{8}{3}, -\frac{a+b \cos [c+d x]}{\left(1-\frac{a}{b}\right) b}, -\frac{a+b \cos [c+d x]}{\left(-1-\frac{a}{b}\right) b} \right] \\
& \quad \left. \sqrt{\frac{-b-b \cos [c+d x]}{a-b}} \sqrt{\frac{b-b \cos [c+d x]}{a+b}} (a+b \cos [c+d x])^{5/3} \operatorname{Csc}[c+d x] \right) - \frac{1}{20 b d} \\
& 3 a^2 C \left( 1 / (2 b^2)^3 a \operatorname{AppellF1} \left[ \frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, -\frac{a+b \cos [c+d x]}{\left(1-\frac{a}{b}\right) b}, -\frac{a+b \cos [c+d x]}{\left(-1-\frac{a}{b}\right) b} \right] \sqrt{\frac{-b-b \cos [c+d x]}{a-b}} \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \right. \\
& \quad (a+b \cos [c+d x])^{2/3} \operatorname{Csc}[c+d x] - 1 / (5 b^2)^3 \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{2}, \frac{1}{2}, \frac{8}{3}, -\frac{a+b \cos [c+d x]}{\left(1-\frac{a}{b}\right) b}, -\frac{a+b \cos [c+d x]}{\left(-1-\frac{a}{b}\right) b} \right] \\
& \quad \left. \sqrt{\frac{-b-b \cos [c+d x]}{a-b}} \sqrt{\frac{b-b \cos [c+d x]}{a+b}} (a+b \cos [c+d x])^{5/3} \operatorname{Csc}[c+d x] \right) + \frac{1}{8 d} \\
& 5 b C \left( 1 / (2 b^2)^3 a \operatorname{AppellF1} \left[ \frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, -\frac{a+b \cos [c+d x]}{\left(1-\frac{a}{b}\right) b}, -\frac{a+b \cos [c+d x]}{\left(-1-\frac{a}{b}\right) b} \right] \sqrt{\frac{-b-b \cos [c+d x]}{a-b}} \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \right. \\
& \quad (a+b \cos [c+d x])^{2/3} \operatorname{Csc}[c+d x] - 1 / (5 b^2)^3 \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{2}, \frac{1}{2}, \frac{8}{3}, -\frac{a+b \cos [c+d x]}{\left(1-\frac{a}{b}\right) b}, -\frac{a+b \cos [c+d x]}{\left(-1-\frac{a}{b}\right) b} \right] \sqrt{\frac{-b-b \cos [c+d x]}{a-b}} \\
& \quad \left. \sqrt{\frac{b-b \cos [c+d x]}{a+b}} (a+b \cos [c+d x])^{5/3} \operatorname{Csc}[c+d x] \right) + \frac{(a+b \cos [c+d x])^{2/3} \left( \frac{3(4 b B+a C) \sin [c+d x]}{20 b} + \frac{3}{16} C \sin [2(c+d x)] \right)}{d}
\end{aligned}$$

■ **Problem 389: Result more than twice size of optimal antiderivative.**

$$\int (a+b \cos [c+d x])^{1/3} (A+B \cos [c+d x]+C \cos [c+d x]^2) dx$$

Optimal (type 6, 290 leaves, 8 steps):

$$\frac{3 C (a + b \cos [c + d x])^{4/3} \sin [c + d x]}{7 b d} +$$

$$\left( \sqrt{2} (a + b) (7 b B - 3 a C) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, -\frac{4}{3}, \frac{3}{2}, \frac{1}{2} (1 - \cos [c + d x]), \frac{b (1 - \cos [c + d x])}{a + b} \right] (a + b \cos [c + d x])^{1/3} \sin [c + d x] \right) /$$

$$\left( 7 b^2 d \sqrt{1 + \cos [c + d x]} \left( \frac{a + b \cos [c + d x]}{a + b} \right)^{1/3} \right) +$$

$$\left( \sqrt{2} (7 A b^2 - 7 a b B + 3 a^2 C + 4 b^2 C) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \cos [c + d x]), \frac{b (1 - \cos [c + d x])}{a + b} \right] (a + b \cos [c + d x])^{1/3} \sin [c + d x] \right) /$$

$$\left( 7 b^2 d \sqrt{1 + \cos [c + d x]} \left( \frac{a + b \cos [c + d x]}{a + b} \right)^{1/3} \right)$$

Result (type 6, 1597 leaves):

$$-\frac{1}{b d} 3 a A \operatorname{AppellF1} \left[ \frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{a + b \cos [c + d x]}{(1 - \frac{a}{b}) b}, -\frac{a + b \cos [c + d x]}{(-1 - \frac{a}{b}) b} \right]$$

$$\sqrt{\frac{-b - b \cos [c + d x]}{a - b}} \sqrt{\frac{b - b \cos [c + d x]}{a + b}} (a + b \cos [c + d x])^{1/3} \operatorname{Csc}[c + d x] - \frac{1}{4 d}$$

$$3 B \operatorname{AppellF1} \left[ \frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{a + b \cos [c + d x]}{(1 - \frac{a}{b}) b}, -\frac{a + b \cos [c + d x]}{(-1 - \frac{a}{b}) b} \right] \sqrt{\frac{-b - b \cos [c + d x]}{a - b}} \sqrt{\frac{b - b \cos [c + d x]}{a + b}}$$

$$(a + b \cos [c + d x])^{1/3} \operatorname{Csc}[c + d x] - \frac{1}{28 b d} 39 a C \operatorname{AppellF1} \left[ \frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{a + b \cos [c + d x]}{(1 - \frac{a}{b}) b}, -\frac{a + b \cos [c + d x]}{(-1 - \frac{a}{b}) b} \right]$$

$$\sqrt{\frac{-b - b \cos [c + d x]}{a - b}} \sqrt{\frac{b - b \cos [c + d x]}{a + b}} (a + b \cos [c + d x])^{1/3} \operatorname{Csc}[c + d x] + \frac{1}{d}$$

$$A b \left( 1 / b^2 3 a \operatorname{AppellF1} \left[ \frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{a + b \cos [c + d x]}{(1 - \frac{a}{b}) b}, -\frac{a + b \cos [c + d x]}{(-1 - \frac{a}{b}) b} \right] \sqrt{\frac{-b - b \cos [c + d x]}{a - b}} \sqrt{\frac{b - b \cos [c + d x]}{a + b}} \right.$$

$$\left. (a + b \cos [c + d x])^{1/3} \operatorname{Csc}[c + d x] - 1 / (4 b^2) 3 \operatorname{AppellF1} \left[ \frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{a + b \cos [c + d x]}{(1 - \frac{a}{b}) b}, -\frac{a + b \cos [c + d x]}{(-1 - \frac{a}{b}) b} \right] \right.$$

$$\left. \sqrt{\frac{-b - b \cos [c + d x]}{a - b}} \sqrt{\frac{b - b \cos [c + d x]}{a + b}} (a + b \cos [c + d x])^{4/3} \operatorname{Csc}[c + d x] \right) + \frac{1}{4 d}$$

$$\begin{aligned}
& a B \left( 1/b^2 3 a \operatorname{AppellF1} \left[ \frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{a+b \cos [c+d x]}{\left(1-\frac{a}{b}\right) b}, -\frac{a+b \cos [c+d x]}{\left(-1-\frac{a}{b}\right) b} \right] \sqrt{\frac{-b-b \cos [c+d x]}{a-b}} \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \right. \\
& \quad \left. (a+b \cos [c+d x])^{1/3} \operatorname{Csc}[c+d x] - 1 / \left(4 b^2\right) 3 \operatorname{AppellF1} \left[ \frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{a+b \cos [c+d x]}{\left(1-\frac{a}{b}\right) b}, -\frac{a+b \cos [c+d x]}{\left(-1-\frac{a}{b}\right) b} \right] \right. \\
& \quad \left. \sqrt{\frac{-b-b \cos [c+d x]}{a-b}} \sqrt{\frac{b-b \cos [c+d x]}{a+b}} (a+b \cos [c+d x])^{4/3} \operatorname{Csc}[c+d x] \right) - \frac{1}{28 b d} \\
& 3 a^2 C \left( 1/b^2 3 a \operatorname{AppellF1} \left[ \frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{a+b \cos [c+d x]}{\left(1-\frac{a}{b}\right) b}, -\frac{a+b \cos [c+d x]}{\left(-1-\frac{a}{b}\right) b} \right] \sqrt{\frac{-b-b \cos [c+d x]}{a-b}} \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \right. \\
& \quad \left. (a+b \cos [c+d x])^{1/3} \operatorname{Csc}[c+d x] - 1 / \left(4 b^2\right) 3 \operatorname{AppellF1} \left[ \frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{a+b \cos [c+d x]}{\left(1-\frac{a}{b}\right) b}, -\frac{a+b \cos [c+d x]}{\left(-1-\frac{a}{b}\right) b} \right] \right. \\
& \quad \left. \sqrt{\frac{-b-b \cos [c+d x]}{a-b}} \sqrt{\frac{b-b \cos [c+d x]}{a+b}} (a+b \cos [c+d x])^{4/3} \operatorname{Csc}[c+d x] \right) + \frac{1}{7 d} \\
& 4 b C \left( 1/b^2 3 a \operatorname{AppellF1} \left[ \frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{a+b \cos [c+d x]}{\left(1-\frac{a}{b}\right) b}, -\frac{a+b \cos [c+d x]}{\left(-1-\frac{a}{b}\right) b} \right] \sqrt{\frac{-b-b \cos [c+d x]}{a-b}} \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \right. \\
& \quad \left. (a+b \cos [c+d x])^{1/3} \operatorname{Csc}[c+d x] - 1 / \left(4 b^2\right) 3 \operatorname{AppellF1} \left[ \frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{a+b \cos [c+d x]}{\left(1-\frac{a}{b}\right) b}, -\frac{a+b \cos [c+d x]}{\left(-1-\frac{a}{b}\right) b} \right] \sqrt{\frac{-b-b \cos [c+d x]}{a-b}} \right. \\
& \quad \left. \sqrt{\frac{b-b \cos [c+d x]}{a+b}} (a+b \cos [c+d x])^{4/3} \operatorname{Csc}[c+d x] \right) + \frac{(a+b \cos [c+d x])^{1/3} \left( \frac{3(7 b B+a C) \sin [c+d x]}{28 b} + \frac{3}{14} C \sin [2(c+d x)] \right)}{d}
\end{aligned}$$

■ **Problem 392: Unable to integrate problem.**

$$\int (a+b \cos [e+f x])^m (A+(A+C) \cos [e+f x]+C \cos [e+f x]^2) dx$$

Optimal (type 6, 215 leaves, 7 steps):

$$\frac{1}{f \sqrt{1 + \cos[e + f x]}} 4 \sqrt{2} C \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}, -m, \frac{3}{2}, \frac{1}{2} (1 - \cos[e + f x]), \frac{b (1 - \cos[e + f x])}{a + b}\right]$$

$$(a + b \cos[e + f x])^m \left(\frac{a + b \cos[e + f x]}{a + b}\right)^{-m} \sin[e + f x] + \frac{1}{f \sqrt{1 + \cos[e + f x]}}$$

$$2 \sqrt{2} (A - C) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1 - \cos[e + f x]), \frac{b (1 - \cos[e + f x])}{a + b}\right] (a + b \cos[e + f x])^m \left(\frac{a + b \cos[e + f x]}{a + b}\right)^{-m} \sin[e + f x]$$

Result (type 8, 37 leaves):

$$\int (a + b \cos[e + f x])^m (A + (A + C) \cos[e + f x] + C \cos[e + f x]^2) dx$$

■ **Problem 393: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[e + f x])^m (A + B \cos[e + f x] + C \cos[e + f x]^2) dx$$

Optimal (type 6, 303 leaves, 8 steps):

$$\frac{C (a + b \cos[e + f x])^{1+m} \sin[e + f x]}{b f (2 + m)}$$

$$\left( \sqrt{2} (a + b) (a C - b B (2 + m)) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -1 - m, \frac{3}{2}, \frac{1}{2} (1 - \cos[e + f x]), \frac{b (1 - \cos[e + f x])}{a + b}\right] (a + b \cos[e + f x])^m \right.$$

$$\left. \left(\frac{a + b \cos[e + f x]}{a + b}\right)^{-m} \sin[e + f x] \right) / (b^2 f (2 + m) \sqrt{1 + \cos[e + f x]}) +$$

$$\left( \sqrt{2} (a^2 C + b^2 C (1 + m) + A b^2 (2 + m) - a b B (2 + m)) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1 - \cos[e + f x]), \frac{b (1 - \cos[e + f x])}{a + b}\right] \right.$$

$$\left. (a + b \cos[e + f x])^m \left(\frac{a + b \cos[e + f x]}{a + b}\right)^{-m} \sin[e + f x] \right) / (b^2 f (2 + m) \sqrt{1 + \cos[e + f x]})$$

Result (type 6, 16189 leaves):

$$\left( 6 (a + b) \left( A (a + b \cos[e + f x])^m + \frac{1}{2} C (a + b \cos[e + f x])^m + B \cos[e + f x] (a + b \cos[e + f x])^m + \frac{1}{2} C (a + b \cos[e + f x])^m \cos[2(e + f x)] \right) \right.$$

$$\left. \tan\left[\frac{1}{2}(e + f x)\right] \left( a + \frac{b - b \tan\left[\frac{1}{2}(e + f x)\right]^2}{1 + \tan\left[\frac{1}{2}(e + f x)\right]^2} \right)^m \right.$$

$$\left. \left( \left( A \operatorname{AppellF1}\left[\frac{1}{2}, 1 + m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e + f x)\right]^2, -\frac{(a - b) \tan\left[\frac{1}{2}(e + f x)\right]^2}{a + b}\right] \left( 1 + \tan\left[\frac{1}{2}(e + f x)\right]^2 \right)^2 \right) \right) / \right.$$



$$\begin{aligned}
& \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - \right. \\
& \left. \left. (a+b)(2+m) \operatorname{AppellF1}\left[\frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) - \\
& \left( 4 C \operatorname{AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right) \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - \right. \\
& \left. \left. (a+b)(2+m) \operatorname{AppellF1}\left[\frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& \left( 4 C \operatorname{AppellF1}\left[\frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 3+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - \right. \\
& \left. \left. (a+b)(3+m) \operatorname{AppellF1}\left[\frac{3}{2}, 4+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) / \\
& \left( f \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^3 \left( \frac{1}{\left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^3} 6 (a+b) m \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left( -\frac{b \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left(b-b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)}{\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2} \left(a+\frac{b-b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}\right)^{-1+m} \\
& \left(\left(\operatorname{A AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2\right)\right) / \\
& \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]+ \right. \\
& \left. 2\left((a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]- (a+b)(1+m) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right)\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) - \\
& \left(\operatorname{B AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2\right) / \\
& \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]+ \right. \\
& \left. 2\left((a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]- (a+b)(1+m) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right)\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& \left(\operatorname{C AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2\right) / \\
& \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]+ \right. \\
& \left. 2\left((a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]- (a+b)(1+m) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \Bigg) + \\
& \left( 2 \text{B AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \left(1 + \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right) \right) / \\
& \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
& \left. 2 \left( (a-b)m \text{AppellF1}\left[\frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - (a+b)(2+m) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{2}, 3+m, -m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) - \right. \\
& \left. \left( 4 \text{C AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \left(1 + \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right) \right) / \right. \\
& \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
& \left. 2 \left( (a-b)m \text{AppellF1}\left[\frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - (a+b)(2+m) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{2}, 3+m, -m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \right. \\
& \left. \left( 4 \text{C AppellF1}\left[\frac{1}{2}, 3+m, -m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) / \right. \\
& \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, 3+m, -m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
& \left. 2 \left( (a-b)m \text{AppellF1}\left[\frac{3}{2}, 3+m, 1-m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - (a+b)(3+m) \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \left. \text{AppellF1}\left[\frac{3}{2}, 4+m, -m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) - \\
& \frac{1}{\left(1+\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^4} 18(a+b) \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(a+\frac{b-b\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{1+\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}\right)^m \\
& \left( \left( \text{A AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \left(1+\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \right) / \\
& \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
& \left. 2\left((a-b)m \text{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - (a+b)(1+m) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) - \\
& \left( \text{B AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \left(1+\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \right) / \\
& \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
& \left. 2\left((a-b)m \text{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - (a+b)(1+m) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& \left( \text{C AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right) \left(1+\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \right) / \\
& \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (1+m) \right. \\
& \quad \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 + \\
& \left( 2 B \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& \quad \left. 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (2+m) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 - \right. \\
& \left( 4 C \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& \quad \left. 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (2+m) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 + \right. \\
& \left( 4 C \operatorname{AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - (a+b) (3+m) \right. \\
& \quad \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 4+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \Bigg) + \\
& \frac{1}{\left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^3} 3 (a+b) \operatorname{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \left( a + \frac{b - b \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{1 + \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2} \right)^m \\
& \left( \left( \operatorname{A} \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^2 \right) / \right. \\
& \quad \left. \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) + \right. \\
& \quad 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - (a+b) (1+m) \right. \\
& \quad \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \Bigg) - \\
& \left( \operatorname{B} \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^2 \right) / \\
& \quad \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) + \\
& \quad 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - (a+b) (1+m) \right. \\
& \quad \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \Bigg) + \\
& \left( \operatorname{C} \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right)^2 \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (1+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) + \\
& \left. \left( 2 B \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (2+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) - \\
& \left. \left( 4 C \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (2+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) + \\
& \left. \left( 4 C \operatorname{AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 3+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (3+m) \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 4+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \Bigg) + \\
& \frac{1}{\left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^3} 6 (a+b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right] \left( a + \frac{b - b \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2} \right)^m \\
& \left( \left( 2 A \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right] \right. \right. \\
& \left. \left. \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) \right) / \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (1+m) \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) - \\
& \left( 2 B \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right] \right. \\
& \left. \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) \right) / \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (1+m) \right. \\
& \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) +
\end{aligned}$$



$$\begin{aligned}
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (1+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) + \\
& \left( c \left( 1 / (3 (a+b)) (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right] - \frac{1}{3} (1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right] \right) \right) \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right)^2 \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (1+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) + \\
& \left( 2 B \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right] \right) / \\
& \left( 3 (a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] - (a+b) (2+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2} (e+f x) \right]^2 \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( 4 \text{C AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \text{Tan} \left[ \frac{1}{2} (e+fx) \right] \right) / \\
& \left( 3 (a+b) \text{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \text{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - (a+b) (2+m) \right. \\
& \left. \left. \text{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) + \\
& \left( 2 \text{B} \left( 1 / (3 (a+b)) (a-b) m \text{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \text{Tan} \left[ \frac{1}{2} (e+fx) \right] - \frac{1}{3} (2+m) \text{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \text{Tan} \left[ \frac{1}{2} (e+fx) \right] \right) \left( 1 + \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) \right) / \left( 3 (a+b) \text{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \text{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - (a+b) (2+m) \right. \\
& \left. \left. \text{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) - \\
& \left( 4 \text{C} \left( 1 / (3 (a+b)) (a-b) m \text{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \text{Tan} \left[ \frac{1}{2} (e+fx) \right] - \frac{1}{3} (2+m) \text{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] \text{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \text{Tan} \left[ \frac{1}{2} (e+fx) \right] \right) \left( 1 + \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2 \right) \right) / \left( 3 (a+b) \text{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] + \right. \\
& 2 \left( (a-b) m \text{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2, -\frac{(a-b) \text{Tan} \left[ \frac{1}{2} (e+fx) \right]^2}{a+b} \right] - (a+b) (2+m) \right.
\end{aligned}$$



$$\begin{aligned}
& \text{AppellF1}\left[\frac{3}{2}, 3+m, -m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \Bigg) + \\
& \left( 4 C \left( 1 / (3(a+b))(a-b) m \text{AppellF1}\left[\frac{3}{2}, 3+m, 1-m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right. \right. \\
& \quad \left. \left. \text{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3}(3+m) \text{AppellF1}\left[\frac{3}{2}, 4+m, -m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right. \right. \\
& \quad \left. \left. \frac{1}{2}(e+fx) \right) \right) \Bigg) / \left( 3(a+b) \text{AppellF1}\left[\frac{1}{2}, 3+m, -m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) + \\
& 2 \left( (a-b) m \text{AppellF1}\left[\frac{3}{2}, 3+m, 1-m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - (a+b)(3+m) \right. \\
& \quad \left. \text{AppellF1}\left[\frac{3}{2}, 4+m, -m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \Bigg) - \\
& \left( A \text{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \left( 1 + \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \right. \\
& \quad \left( 2 \left( (a-b) m \text{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - (a+b)(1+m) \text{AppellF1}\left[\frac{3}{2}, 2+m, \right. \right. \\
& \quad \left. \left. -m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + 3(a+b) \left( \frac{1}{3(a+b)} \right. \right. \\
& \quad \left. \left. (a-b) m \text{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] - \right. \right. \\
& \quad \left. \left. \frac{1}{3}(1+m) \text{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) + \\
& 2 \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left( (a-b) m \left( -\frac{1}{5(a+b)} 3(a-b)(1-m) \text{AppellF1}\left[\frac{5}{2}, 1+m, 2-m, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\frac{(a-b)\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(1+m) \text{AppellF1}\left[\frac{5}{2}, 2+m, 1-m, \frac{7}{2}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\left]\sec\left[\frac{1}{2}(e+fx)\right]^2\tan\left[\frac{1}{2}(e+fx)\right]\right] - (a+b)(1+m) \\
& \left(\frac{1}{5(a+b)}3(a-b)m\operatorname{AppellF1}\left[\frac{5}{2}, 2+m, 1-m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\sec\left[\frac{1}{2}(e+fx)\right]^2\right. \\
& \left.\tan\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(2+m)\operatorname{AppellF1}\left[\frac{5}{2}, 3+m, -m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right. \\
& \left.\sec\left[\frac{1}{2}(e+fx)\right]^2\tan\left[\frac{1}{2}(e+fx)\right]\right)\right)\bigg/\left(3(a+b)\operatorname{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left. -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + 2\left((a-b)m\operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right. \right. \\
& \left. (a+b)(1+m)\operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right)\tan\left[\frac{1}{2}(e+fx)\right]^2\right)^2 + \\
& \left(\operatorname{BAppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right)\left(1+\tan\left[\frac{1}{2}(e+fx)\right]\right)^2 \\
& \left(2\left((a-b)m\operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - (a+b)(1+m)\right. \right. \\
& \left. \left.\operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right)\sec\left[\frac{1}{2}(e+fx)\right]^2\tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \left. 3(a+b)\left(\frac{1}{3(a+b)}(a-b)m\operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right. \right. \\
& \left.\sec\left[\frac{1}{2}(e+fx)\right]^2\tan\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3}(1+m)\operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left. \left.-\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\sec\left[\frac{1}{2}(e+fx)\right]^2\tan\left[\frac{1}{2}(e+fx)\right]\right)+2\tan\left[\frac{1}{2}(e+fx)\right]^2 \\
& \left.\left((a-b)m\left(-\frac{1}{5(a+b)}3(a-b)(1-m)\operatorname{AppellF1}\left[\frac{5}{2}, 1+m, 2-m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right.\right.\right)
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(1+m) \operatorname{AppellF1}\left[\frac{5}{2}, 2+m, 1-m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right]^2, \\
& - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \left] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) - (a+b)(1+m) \left(\frac{1}{5(a+b)}\right)^3 (a-b)m \operatorname{AppellF1}\left[\frac{5}{2}, \right. \\
& \left. 2+m, 1-m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right]^2, - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \left] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(2+m) \right. \\
& \left. \operatorname{AppellF1}\left[\frac{5}{2}, 3+m, -m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right]^2, - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \left] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)\right)\right)\right) / \\
& \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right]^2, - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] + 2 \left((a-b)m \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \\
& \left. \left. 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right]^2, - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] - \\
& (a+b)(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right]^2, - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \left.\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 - \\
& \left(C \operatorname{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right]^2, - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^2 \right)^2 \\
& \left(2 \left((a-b)m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right]^2, - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] - (a+b)(1+m) \right. \\
& \left. \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right]^2, - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& 3(a+b) \left(\frac{1}{3(a+b)}(a-b)m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right]^2, - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right) \right) \\
& \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3}(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right]^2, \\
& - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \left] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) + 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2
\end{aligned}$$

$$\begin{aligned}
& \left( (a-b) m \left( -\frac{1}{5(a+b)} 3(a-b)(1-m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1+m, 2-m, \frac{7}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \right. \right. \\
& \quad \operatorname{Sec} \left[ \frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right] - \frac{3}{5}(1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 2+m, 1-m, \frac{7}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, \right. \\
& \quad \left. \left. -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right] \right) - (a+b)(1+m) \left( \frac{1}{5(a+b)} 3(a-b) m \operatorname{AppellF1} \left[ \frac{5}{2}, \right. \right. \\
& \quad \left. \left. 2+m, 1-m, \frac{7}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right] - \frac{3}{5}(2+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{5}{2}, 3+m, -m, \frac{7}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \operatorname{Sec} \left[ \frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right] \right) \right) \right) \Bigg) / \\
& \left( 3(a+b) \operatorname{AppellF1} \left[ \frac{1}{2}, 1+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] + 2 \left( (a-b) m \operatorname{AppellF1} \left[ \right. \right. \right. \\
& \quad \left. \left. \frac{3}{2}, 1+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] - \right. \\
& \quad \left. \left. (a+b)(1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2 \right)^2 - \right. \\
& \left( 2 B \operatorname{AppellF1} \left[ \frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2 \right) \right. \\
& \left. \left( 2 \left( (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] - (a+b)(2+m) \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \right) \operatorname{Sec} \left[ \frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right] + \right. \\
& \quad \left. 3(a+b) \left( \frac{1}{3(a+b)} (a-b) m \operatorname{AppellF1} \left[ \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, -\frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2}{a+b} \right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec} \left[ \frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right] - \frac{1}{3}(2+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan} \left[ \frac{1}{2}(e+fx) \right]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \left] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) + 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \\
& \left( (a-b) m \left( - \frac{1}{5(a+b)} {}_3F_2\left(\frac{5}{2}, 2+m, 2-m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right) \right. \right. \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(2+m) \operatorname{AppellF1}\left[\frac{5}{2}, 3+m, 1-m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
& \quad \left. \left. - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) - (a+b)(2+m) \left( \frac{1}{5(a+b)} {}_3F_2\left(\frac{5}{2}, \right. \right. \\
& \quad \left. \left. 3+m, 1-m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(3+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{5}{2}, 4+m, -m, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \right) \Big/ \\
& \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + 2 \left( (a-b) m \operatorname{AppellF1}\left[ \right. \right. \right. \\
& \quad \left. \left. \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - \right. \\
& \quad \left. \left. (a+b)(2+m) \operatorname{AppellF1}\left[\frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 + \\
& \left( 4 C \operatorname{AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right. \\
& \left. \left( 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - (a+b)(2+m) \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, 3+m, -m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \left. \left. 3(a+b) \left( \frac{1}{3(a+b)} (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3}(2+m) \operatorname{AppellF1}\left[\frac{3}{2}, 3+m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \quad \left. \left. - \frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \\
& \left( (a-b) m \left( -\frac{1}{5(a+b)} {}_3F_2\left[\frac{5}{2}, 2+m, 2-m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right. \right. \\
& \quad \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(2+m) \operatorname{AppellF1}\left[\frac{5}{2}, 3+m, 1-m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
& \quad \left. \left. - \frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) - (a+b)(2+m) \left( \frac{1}{5(a+b)} {}_3F_2\left[\frac{5}{2}, \right. \right. \\
& \quad \left. \left. 3+m, 1-m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{3}{5}(3+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{5}{2}, 4+m, -m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \Big/ \\
& \left( 3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 2+m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \right. \\
& \quad \left. \left. \frac{3}{2}, 2+m, 1-m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - \right. \\
& \quad \left. \left. (a+b)(2+m) \operatorname{AppellF1}\left[\frac{3}{2}, 3+m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) - \\
& \left( 4 \operatorname{AppellF1}\left[\frac{1}{2}, 3+m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \left( 2 \left( (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 3+m, 1-m, \right. \right. \right. \right. \\
& \quad \left. \left. \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] - (a+b)(3+m) \operatorname{AppellF1}\left[\frac{3}{2}, 4+m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \quad \left. \left. - \frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + 3(a+b) \left( \frac{1}{3(a+b)} (a-b) m \operatorname{AppellF1}\left[\frac{3}{2}, 3+m, 1-m, \right. \right. \right. \\
& \quad \left. \left. \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3}(3+m) \operatorname{AppellF1}\left[\frac{3}{2}, 4+m, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\left]\sec\left[\frac{1}{2}(e+fx)\right]^2\tan\left[\frac{1}{2}(e+fx)\right]\right]+2\tan\left[\frac{1}{2}(e+fx)\right]^2 \\
& \left( (a-b)m\left(-\frac{1}{5(a+b)}\right)^3(a-b)(1-m)\operatorname{AppellF1}\left[\frac{5}{2}, 3+m, 2-m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right. \\
& \quad \left. \sec\left[\frac{1}{2}(e+fx)\right]^2\tan\left[\frac{1}{2}(e+fx)\right]-\frac{3}{5}(3+m)\operatorname{AppellF1}\left[\frac{5}{2}, 4+m, 1-m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \quad \left. \left. -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\sec\left[\frac{1}{2}(e+fx)\right]^2\tan\left[\frac{1}{2}(e+fx)\right]\right)- (a+b)(3+m)\left(\frac{1}{5(a+b)}\right)^3(a-b)m\operatorname{AppellF1}\left[\frac{5}{2}, \right. \right. \\
& \quad \left. \left. 4+m, 1-m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\sec\left[\frac{1}{2}(e+fx)\right]^2\tan\left[\frac{1}{2}(e+fx)\right]-\frac{3}{5}(4+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{5}{2}, 5+m, -m, \frac{7}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\sec\left[\frac{1}{2}(e+fx)\right]^2\tan\left[\frac{1}{2}(e+fx)\right]\right]\right)\right)\right) \\
& \left( 3(a+b)\operatorname{AppellF1}\left[\frac{1}{2}, 3+m, -m, \frac{3}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]+2\left((a-b)m\operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. 3+m, 1-m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right)- \right. \\
& \quad \left. \left. (a+b)(3+m)\operatorname{AppellF1}\left[\frac{3}{2}, 4+m, -m, \frac{5}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2, -\frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right]\right)\tan\left[\frac{1}{2}(e+fx)\right]^2\right)\right)\right) \\
\end{aligned}$$

## Test results for the 1541 problems in "4.2.4.2 (a+b cos)^m (c+d cos)^n (A+B cos+C cos^2).m"

- Problem 5: Result more than twice size of optimal antiderivative.

$$\int (a + a \cos[c + dx]) (A + C \cos[c + dx]^2) \sec[c + dx]^2 dx$$

Optimal (type 3, 42 leaves, 4 steps):

$$a C x + \frac{a A \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{a C \sin[c + dx]}{d} + \frac{a A \tan[c + dx]}{d}$$

Result (type 3, 112 leaves) :

$$a C x - \frac{a A \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a A \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a C \cos[dx] \sin[c]}{d} + \frac{a C \cos[c] \sin[dx]}{d} + \frac{a A \tan[c + dx]}{d}$$

■ **Problem 6: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx]) (A + C \cos[c + dx])^2 \sec[c + dx]^3 dx$$

Optimal (type 3, 58 leaves, 4 steps) :

$$a C x + \frac{a (A + 2 C) \operatorname{ArcTanh}[\sin[c + dx]]}{2 d} + \frac{a A \tan[c + dx]}{d} + \frac{a A \sec[c + dx] \tan[c + dx]}{2 d}$$

Result (type 3, 218 leaves) :

$$a C x - \frac{a C \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a C \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} - \frac{a A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{2 d} + \frac{a A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{2 d} + \frac{a A \tan[c + dx]}{d} - \frac{a A}{4 d (\cos[\frac{1}{2}(c + dx)] - \sin[\frac{1}{2}(c + dx)])^2} - \frac{a A}{4 d (\cos[\frac{1}{2}(c + dx)] + \sin[\frac{1}{2}(c + dx)])^2}$$

■ **Problem 8: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx]) (A + C \cos[c + dx])^2 \sec[c + dx]^5 dx$$

Optimal (type 3, 117 leaves, 7 steps) :

$$\frac{a (3 A + 4 C) \operatorname{ArcTanh}[\sin[c + dx]]}{8 d} + \frac{a (2 A + 3 C) \tan[c + dx]}{3 d} + \frac{a (3 A + 4 C) \sec[c + dx] \tan[c + dx]}{8 d} + \frac{a A \sec[c + dx]^2 \tan[c + dx]}{3 d} + \frac{a A \sec[c + dx]^3 \tan[c + dx]}{4 d}$$

Result (type 3, 377 leaves) :



$$\begin{aligned}
& - \frac{3 a A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} - \frac{a C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \frac{3 a A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \\
& \frac{a C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \frac{a A}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^4} + \frac{3 a A}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^2} + \\
& \frac{a C}{4 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{a A}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^4} - \frac{3 a A}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^2} - \\
& \frac{a C}{4 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{2 a A \operatorname{Tan}[c+d x]}{3 d} + \frac{a C \operatorname{Tan}[c+d x]}{d} + \frac{a A \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{3 d}
\end{aligned}$$

■ **Problem 14: Result more than twice size of optimal antiderivative.**

$$\int (a+a \cos [c+d x])^2 (A+C \cos [c+d x])^2 \operatorname{Sec}[c+d x]^3 dx$$

Optimal (type 3, 112 leaves, 6 steps):

$$\begin{aligned}
& 2 a^2 C x + \frac{a^2 (3 A+2 C) \operatorname{ArcTanh}[\sin [c+d x]]}{2 d} - \frac{a^2 (3 A-2 C) \sin [c+d x]}{2 d} + \\
& \frac{A\left(a^2+a^2 \cos [c+d x]\right) \operatorname{Tan}[c+d x]}{d} + \frac{A(a+a \cos [c+d x])^2 \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{2 d}
\end{aligned}$$

Result (type 3, 293 leaves):

$$\begin{aligned}
& \frac{1}{16} a^2 (1+\cos [c+d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 \\
& \left( 8 C x - \frac{2(3 A+2 C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right]}{d} + \frac{2(3 A+2 C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right]}{d} + \frac{4 C \cos [d x] \sin [c]}{d} + \right. \\
& \frac{4 C \cos [c] \sin [d x]}{d} + \frac{A}{d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{8 A \sin\left[\frac{d x}{2}\right]}{d\left(\cos\left[\frac{c}{2}\right]-\sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)} - \\
& \left. \frac{A}{d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{8 A \sin\left[\frac{d x}{2}\right]}{d\left(\cos\left[\frac{c}{2}\right]+\sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)} \right)
\end{aligned}$$

■ **Problem 15: Result more than twice size of optimal antiderivative.**

$$\int (a+a \cos [c+d x])^2 (A+C \cos [c+d x])^2 \operatorname{Sec}[c+d x]^4 dx$$

Optimal (type 3, 110 leaves, 6 steps):

$$a^2 C x + \frac{a^2 (A + 2 C) \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{a^2 (A + C) \tan[c + dx]}{d} +$$

$$\frac{A (a^2 + a^2 \cos[c + dx]) \sec[c + dx] \tan[c + dx]}{3 d} + \frac{A (a + a \cos[c + dx])^2 \sec[c + dx]^2 \tan[c + dx]}{3 d}$$

Result (type 3, 748 leaves):

$$\frac{1}{4} C x (a + a \cos[c + dx])^2 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 + \frac{(-A - 2 C) (a + a \cos[c + dx])^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4}{4 d} +$$

$$\frac{(A + 2 C) (a + a \cos[c + dx])^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4}{4 d} +$$

$$\frac{A (a + a \cos[c + dx])^2 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{24 d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^3} + \frac{(a + a \cos[c + dx])^2 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (7 A \cos\left[\frac{c}{2}\right] - 5 A \sin\left[\frac{c}{2}\right])}{48 d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2} +$$

$$\frac{(a + a \cos[c + dx])^2 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (5 A \sin\left[\frac{dx}{2}\right] + 3 C \sin\left[\frac{dx}{2}\right])}{12 d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])} + \frac{A (a + a \cos[c + dx])^2 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{24 d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^3} +$$

$$\frac{(a + a \cos[c + dx])^2 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (-7 A \cos\left[\frac{c}{2}\right] - 5 A \sin\left[\frac{c}{2}\right])}{48 d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \frac{(a + a \cos[c + dx])^2 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (5 A \sin\left[\frac{dx}{2}\right] + 3 C \sin\left[\frac{dx}{2}\right])}{12 d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])}$$

■ **Problem 22: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^3 (A + C \cos[c + dx])^2 \sec[c + dx]^2 dx$$

Optimal (type 3, 145 leaves, 7 steps):

$$\frac{1}{2} a^3 (6 A + 5 C) x + \frac{3 a^3 A \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{5 a^3 C \sin[c + dx]}{2 d} -$$

$$\frac{(3 A - C) (a^2 + a^2 \cos[c + dx])^2 \sin[c + dx]}{3 a d} - \frac{(6 A - 5 C) (a^3 + a^3 \cos[c + dx]) \sin[c + dx]}{6 d} + \frac{A (a + a \cos[c + dx])^3 \tan[c + dx]}{d}$$

Result (type 3, 298 leaves):

$$\frac{1}{96} a^3 (1 + \cos [c + d x])^3 \sec \left[ \frac{1}{2} (c + d x) \right]^6$$

$$\left( \begin{aligned} & 6 (6 A + 5 C) x - \frac{36 A \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{d} + \frac{36 A \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{d} + \frac{3 (4 A + 15 C) \cos [d x] \sin [c]}{d} + \\ & \frac{9 C \cos [2 d x] \sin [2 c]}{d} + \frac{C \cos [3 d x] \sin [3 c]}{d} + \frac{3 (4 A + 15 C) \cos [c] \sin [d x]}{d} + \frac{9 C \cos [2 c] \sin [2 d x]}{d} + \frac{C \cos [3 c] \sin [3 d x]}{d} + \\ & \frac{12 A \sin \left[ \frac{d x}{2} \right]}{d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)} + \frac{12 A \sin \left[ \frac{d x}{2} \right]}{d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)} \end{aligned} \right)$$

■ **Problem 24: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^3 (A + C \cos [c + d x]^2) \sec [c + d x]^4 dx$$

Optimal (type 3, 156 leaves, 7 steps):

$$3 a^3 C x + \frac{a^3 (5 A + 6 C) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} - \frac{5 a^3 A \sin [c + d x]}{2 d} + \frac{(5 A + 3 C) (a^3 + a^3 \cos [c + d x]) \tan [c + d x]}{3 d} +$$

$$\frac{A (a^2 + a^2 \cos [c + d x])^2 \sec [c + d x] \tan [c + d x]}{2 a d} + \frac{A (a + a \cos [c + d x])^3 \sec [c + d x]^2 \tan [c + d x]}{3 d}$$

Result (type 3, 832 leaves):

$$\frac{3}{8} C x (a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 + \frac{(-5 A - 6 C) (a + a \cos [c + d x])^3 \log \left[ \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6}{16 d} +$$

$$\frac{(5 A + 6 C) (a + a \cos [c + d x])^3 \log \left[ \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6}{16 d} +$$

$$\frac{C \cos [d x] (a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sin [c]}{8 d} + \frac{C \cos [c] (a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sin [d x]}{8 d} +$$

$$\frac{A (a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sin \left[ \frac{d x}{2} \right]}{48 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^3} + \frac{(a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 (5 A \cos \left[ \frac{c}{2} \right] - 4 A \sin \left[ \frac{c}{2} \right])}{48 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^2} +$$

$$\frac{(a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 (11 A \sin \left[ \frac{d x}{2} \right] + 3 C \sin \left[ \frac{d x}{2} \right])}{24 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)} + \frac{A (a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sin \left[ \frac{d x}{2} \right]}{48 d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^3} +$$

$$\frac{(a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 (-5 A \cos \left[ \frac{c}{2} \right] - 4 A \sin \left[ \frac{c}{2} \right])}{48 d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^2} + \frac{(a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 (11 A \sin \left[ \frac{d x}{2} \right] + 3 C \sin \left[ \frac{d x}{2} \right])}{24 d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)}$$

■ **Problem 33: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^4 (A + C \cos [c + d x]^2) \sec [c + d x]^3 dx$$

Optimal (type 3, 186 leaves, 8 steps):

$$2 a^4 (2 A + 3 C) x + \frac{a^4 (13 A + 2 C) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} - \frac{5 a^4 (A - 2 C) \sin [c + d x]}{2 d} - \frac{(15 A - 2 C) (a^2 + a^2 \cos [c + d x])^2 \sin [c + d x]}{6 d} - \frac{(9 A - 4 C) (a^4 + a^4 \cos [c + d x]) \sin [c + d x]}{3 d} + \frac{2 a A (a + a \cos [c + d x])^3 \tan [c + d x]}{d} + \frac{A (a + a \cos [c + d x])^4 \sec [c + d x] \tan [c + d x]}{2 d}$$

Result (type 3, 756 leaves):

$$\begin{aligned} & \frac{1}{8} (2 A + 3 C) x (a + a \cos [c + d x])^4 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^8 + \frac{(-13 A - 2 C) (a + a \cos [c + d x])^4 \operatorname{Log} \left[ \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^8}{32 d} + \\ & \frac{(13 A + 2 C) (a + a \cos [c + d x])^4 \operatorname{Log} \left[ \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^8}{32 d} + \frac{(4 A + 27 C) \cos [d x] (a + a \cos [c + d x])^4 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^8 \sin [c]}{64 d} + \\ & \frac{C \cos [2 d x] (a + a \cos [c + d x])^4 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^8 \sin [2 c]}{16 d} + \frac{C \cos [3 d x] (a + a \cos [c + d x])^4 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^8 \sin [3 c]}{192 d} + \\ & \frac{(4 A + 27 C) \cos [c] (a + a \cos [c + d x])^4 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^8 \sin [d x]}{64 d} + \frac{C \cos [2 c] (a + a \cos [c + d x])^4 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^8 \sin [2 d x]}{16 d} + \\ & \frac{C \cos [3 c] (a + a \cos [c + d x])^4 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^8 \sin [3 d x]}{192 d} + \frac{A (a + a \cos [c + d x])^4 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^8}{64 d \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^2} + \\ & \frac{A (a + a \cos [c + d x])^4 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^8 \sin \left[ \frac{d x}{2} \right]}{4 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)} - \frac{A (a + a \cos [c + d x])^4 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^8}{64 d \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^2} + \frac{A (a + a \cos [c + d x])^4 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^8 \sin \left[ \frac{d x}{2} \right]}{4 d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)} \end{aligned}$$

■ **Problem 42: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos [c + d x]^2}{a + a \cos [c + d x]} dx$$

Optimal (type 3, 48 leaves, 3 steps):

$$-\frac{C x}{a} + \frac{C \sin [c + d x]}{a d} + \frac{(A + C) \sin [c + d x]}{a d (1 + \cos [c + d x])}$$

Result (type 3, 108 leaves):

$$\frac{1}{4 a d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \left(-2 C d x \operatorname{Cos}\left[\frac{d x}{2}\right]-2 C d x \operatorname{Cos}\left[c+\frac{d x}{2}\right]+4 A \operatorname{Sin}\left[\frac{d x}{2}\right]+5 C \operatorname{Sin}\left[\frac{d x}{2}\right]+C \operatorname{Sin}\left[c+\frac{d x}{2}\right]+C \operatorname{Sin}\left[c+\frac{3 d x}{2}\right]+C \operatorname{Sin}\left[2 c+\frac{3 d x}{2}\right]\right)$$

- **Problem 43: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+C \operatorname{Cos}[c+d x]^2) \operatorname{Sec}[c+d x]}{a+a \operatorname{Cos}[c+d x]} d x$$

Optimal (type 3, 48 leaves, 3 steps):

$$\frac{C x}{a} + \frac{A \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{a d} - \frac{(A+C) \operatorname{Sin}[c+d x]}{d(a+a \operatorname{Cos}[c+d x])}$$

Result (type 3, 114 leaves):

$$\frac{1}{a d(1+\operatorname{Cos}[c+d x])} 2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] \left(C d x-A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]+A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]\right)-(A+C) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{d x}{2}\right]\right)$$

- **Problem 44: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+C \operatorname{Cos}[c+d x]^2) \operatorname{Sec}[c+d x]^2}{a+a \operatorname{Cos}[c+d x]} d x$$

Optimal (type 3, 61 leaves, 5 steps):

$$-\frac{A \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{a d} + \frac{(2 A+C) \operatorname{Tan}[c+d x]}{a d} - \frac{(A+C) \operatorname{Tan}[c+d x]}{d(a+a \operatorname{Cos}[c+d x])}$$

Result (type 3, 229 leaves):

$$\left(4 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] \operatorname{Cos}[c+d x]^2(C+A \operatorname{Sec}[c+d x]^2)\right. \\ \left. \left((A+C) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{d x}{2}\right]+A \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]-\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]+ \right. \right. \\ \left. \left. \operatorname{Sin}[d x] / \left(\left(\operatorname{Cos}\left[\frac{c}{2}\right]-\operatorname{Sin}\left[\frac{c}{2}\right]\right)\left(\operatorname{Cos}\left[\frac{c}{2}\right]+\operatorname{Sin}\left[\frac{c}{2}\right]\right)\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)\right)\right)\right) / \\ (a d(1+\operatorname{Cos}[c+d x]) (2 A+C+C \operatorname{Cos}[2(c+d x)]))$$

- **Problem 45: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+C \operatorname{Cos}[c+d x]^2) \operatorname{Sec}[c+d x]^3}{a+a \operatorname{Cos}[c+d x]} d x$$

Optimal (type 3, 105 leaves, 6 steps):

$$\frac{(3A + 2C) \operatorname{ArcTanh}[\sin[c + dx]]}{2ad} - \frac{(2A + C) \tan[c + dx]}{ad} + \frac{(3A + 2C) \sec[c + dx] \tan[c + dx]}{2ad} - \frac{(A + C) \sec[c + dx] \tan[c + dx]}{d(a + a \cos[c + dx])}$$

Result (type 3, 284 leaves):

$$\frac{1}{2ad(1 + \cos[c + dx])} \cos\left[\frac{1}{2}(c + dx)\right] \left( -4(A + C) \sec\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] + \right. \\ \left. \cos\left[\frac{1}{2}(c + dx)\right] \left( -2(3A + 2C) \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 6A \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + \right. \right. \\ \left. \left. 4C \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + \frac{A}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{A}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} - \right. \right. \\ \left. \left. \left( 4A \sin[dx] \right) / \left( \left( \cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right) \left( \cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) \right) \right)$$

■ **Problem 46: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos[c + dx])^2 \sec[c + dx]^4}{a + a \cos[c + dx]} dx$$

Optimal (type 3, 133 leaves, 6 steps):

$$-\frac{(3A + 2C) \operatorname{ArcTanh}[\sin[c + dx]]}{2ad} + \frac{(4A + 3C) \tan[c + dx]}{ad} - \\ \frac{(3A + 2C) \sec[c + dx] \tan[c + dx]}{2ad} - \frac{(A + C) \sec[c + dx]^2 \tan[c + dx]}{d(a + a \cos[c + dx])} + \frac{(4A + 3C) \tan[c + dx]^3}{3ad}$$

Result (type 3, 765 leaves):

$$\begin{aligned}
& \frac{(3A + 2C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d(a + a \cos[c + dx])} + \frac{(-3A - 2C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d(a + a \cos[c + dx])} + \\
& \frac{2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sec\left[\frac{c}{2}\right] (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d(a + a \cos[c + dx])} + \frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{3d(a + a \cos[c + dx]) (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^3} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (-A \cos\left[\frac{c}{2}\right] + 2A \sin\left[\frac{c}{2}\right])}{3d(a + a \cos[c + dx]) (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \\
& \frac{2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (5A \sin\left[\frac{dx}{2}\right] + 3C \sin\left[\frac{dx}{2}\right])}{3d(a + a \cos[c + dx]) (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])} + \\
& \frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{dx}{2}\right]}{3d(a + a \cos[c + dx]) (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^3} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A \cos\left[\frac{c}{2}\right] + 2A \sin\left[\frac{c}{2}\right])}{3d(a + a \cos[c + dx]) (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \frac{2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (5A \sin\left[\frac{dx}{2}\right] + 3C \sin\left[\frac{dx}{2}\right])}{3d(a + a \cos[c + dx]) (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])}
\end{aligned}$$

■ **Problem 47: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^4 (A + C \cos[c + dx])^2}{(a + a \cos[c + dx])^2} dx$$

Optimal (type 3, 191 leaves, 8 steps):

$$\begin{aligned}
& \frac{(28A + 55C)x}{8a^2} - \frac{8(A + 2C) \sin[c + dx]}{a^2 d} + \frac{(28A + 55C) \cos[c + dx] \sin[c + dx]}{8a^2 d} + \frac{(28A + 55C) \cos[c + dx]^3 \sin[c + dx]}{12a^2 d} - \\
& \frac{2(A + 2C) \cos[c + dx]^4 \sin[c + dx]}{a^2 d (1 + \cos[c + dx])} - \frac{(A + C) \cos[c + dx]^5 \sin[c + dx]}{3d(a + a \cos[c + dx])^2} + \frac{8(A + 2C) \sin[c + dx]^3}{3a^2 d}
\end{aligned}$$

Result (type 3, 399 leaves):

$$\frac{1}{384 a^2 d (1 + \cos [c + d x])^2}$$

$$\cos \left[ \frac{1}{2} (c + d x) \right] \sec \left[ \frac{c}{2} \right] \left( 72 (28 A + 55 C) d x \cos \left[ \frac{d x}{2} \right] + 72 (28 A + 55 C) d x \cos \left[ c + \frac{d x}{2} \right] + 672 A d x \cos \left[ c + \frac{3 d x}{2} \right] + 1320 C d x \cos \left[ c + \frac{3 d x}{2} \right] + \right.$$

$$672 A d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 1320 C d x \cos \left[ 2 c + \frac{3 d x}{2} \right] - 3048 A \sin \left[ \frac{d x}{2} \right] - 5184 C \sin \left[ \frac{d x}{2} \right] + 1176 A \sin \left[ c + \frac{d x}{2} \right] + 1344 C \sin \left[ c + \frac{d x}{2} \right] -$$

$$1912 A \sin \left[ c + \frac{3 d x}{2} \right] - 3488 C \sin \left[ c + \frac{3 d x}{2} \right] - 504 A \sin \left[ 2 c + \frac{3 d x}{2} \right] - 1312 C \sin \left[ 2 c + \frac{3 d x}{2} \right] - 120 A \sin \left[ 2 c + \frac{5 d x}{2} \right] -$$

$$285 C \sin \left[ 2 c + \frac{5 d x}{2} \right] - 120 A \sin \left[ 3 c + \frac{5 d x}{2} \right] - 285 C \sin \left[ 3 c + \frac{5 d x}{2} \right] + 24 A \sin \left[ 3 c + \frac{7 d x}{2} \right] + 57 C \sin \left[ 3 c + \frac{7 d x}{2} \right] +$$

$$\left. 24 A \sin \left[ 4 c + \frac{7 d x}{2} \right] + 57 C \sin \left[ 4 c + \frac{7 d x}{2} \right] - 7 C \sin \left[ 4 c + \frac{9 d x}{2} \right] - 7 C \sin \left[ 5 c + \frac{9 d x}{2} \right] + 3 C \sin \left[ 5 c + \frac{11 d x}{2} \right] + 3 C \sin \left[ 6 c + \frac{11 d x}{2} \right] \right)$$

■ **Problem 48: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^3 (A + C \cos [c + d x]^2)}{(a + a \cos [c + d x])^2} dx$$

Optimal (type 3, 163 leaves, 7 steps):

$$-\frac{(2 A + 5 C) x}{a^2} + \frac{(5 A + 12 C) \sin [c + d x]}{a^2 d} - \frac{(2 A + 5 C) \cos [c + d x] \sin [c + d x]}{a^2 d} -$$

$$\frac{2 (2 A + 5 C) \cos [c + d x]^3 \sin [c + d x]}{3 a^2 d (1 + \cos [c + d x])} - \frac{(A + C) \cos [c + d x]^4 \sin [c + d x]}{3 d (a + a \cos [c + d x])^2} - \frac{(5 A + 12 C) \sin [c + d x]^3}{3 a^2 d}$$

Result (type 3, 341 leaves):

$$\frac{1}{48 a^2 d (1 + \cos [c + d x])^2}$$

$$\cos \left[ \frac{1}{2} (c + d x) \right] \sec \left[ \frac{c}{2} \right] \left( -72 (2 A + 5 C) d x \cos \left[ \frac{d x}{2} \right] - 72 (2 A + 5 C) d x \cos \left[ c + \frac{d x}{2} \right] - 48 A d x \cos \left[ c + \frac{3 d x}{2} \right] - 120 C d x \cos \left[ c + \frac{3 d x}{2} \right] - \right.$$

$$48 A d x \cos \left[ 2 c + \frac{3 d x}{2} \right] - 120 C d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 264 A \sin \left[ \frac{d x}{2} \right] + 516 C \sin \left[ \frac{d x}{2} \right] - 120 A \sin \left[ c + \frac{d x}{2} \right] - 156 C \sin \left[ c + \frac{d x}{2} \right] +$$

$$164 A \sin \left[ c + \frac{3 d x}{2} \right] + 342 C \sin \left[ c + \frac{3 d x}{2} \right] + 36 A \sin \left[ 2 c + \frac{3 d x}{2} \right] + 118 C \sin \left[ 2 c + \frac{3 d x}{2} \right] + 12 A \sin \left[ 2 c + \frac{5 d x}{2} \right] + 30 C \sin \left[ 2 c + \frac{5 d x}{2} \right] +$$

$$\left. 12 A \sin \left[ 3 c + \frac{5 d x}{2} \right] + 30 C \sin \left[ 3 c + \frac{5 d x}{2} \right] - 3 C \sin \left[ 3 c + \frac{7 d x}{2} \right] - 3 C \sin \left[ 4 c + \frac{7 d x}{2} \right] + C \sin \left[ 4 c + \frac{9 d x}{2} \right] + C \sin \left[ 5 c + \frac{9 d x}{2} \right] \right)$$

■ **Problem 50: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x] (A + C \cos [c + d x]^2)}{(a + a \cos [c + d x])^2} dx$$



Optimal (type 3, 90 leaves, 6 steps) :

$$-\frac{2 C x}{a^2} + \frac{(A + 4 C) \operatorname{Sin}[c + d x]}{3 a^2 d} + \frac{2 C \operatorname{Sin}[c + d x]}{a^2 d (1 + \operatorname{Cos}[c + d x])} - \frac{(A + C) \operatorname{Cos}[c + d x]^2 \operatorname{Sin}[c + d x]}{3 d (a + a \operatorname{Cos}[c + d x])^2}$$

Result (type 3, 195 leaves) :

$$\frac{1}{48 a^2 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^3$$

$$\left(-36 C d x \operatorname{Cos}\left[\frac{d x}{2}\right] - 36 C d x \operatorname{Cos}\left[c + \frac{d x}{2}\right] - 12 C d x \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] - 12 C d x \operatorname{Cos}\left[2 c + \frac{3 d x}{2}\right] + 12 A \operatorname{Sin}\left[\frac{d x}{2}\right] + 66 C \operatorname{Sin}\left[\frac{d x}{2}\right] - 12 A \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 30 C \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 8 A \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 41 C \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 9 C \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 3 C \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] + 3 C \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right]\right)$$

■ **Problem 51: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Cos}[c + d x]^2}{(a + a \operatorname{Cos}[c + d x])^2} dx$$

Optimal (type 3, 66 leaves, 3 steps) :

$$\frac{C x}{a^2} + \frac{(A - 5 C) \operatorname{Sin}[c + d x]}{3 a^2 d (1 + \operatorname{Cos}[c + d x])} + \frac{(A + C) \operatorname{Sin}[c + d x]}{3 d (a + a \operatorname{Cos}[c + d x])^2}$$

Result (type 3, 141 leaves) :

$$\frac{1}{24 a^2 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^3 \left(9 C d x \operatorname{Cos}\left[\frac{d x}{2}\right] + 9 C d x \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 3 C d x \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] + 3 C d x \operatorname{Cos}\left[2 c + \frac{3 d x}{2}\right] + 6 A \operatorname{Sin}\left[\frac{d x}{2}\right] - 18 C \operatorname{Sin}\left[\frac{d x}{2}\right] + 12 C \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 2 A \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - 10 C \operatorname{Sin}\left[c + \frac{3 d x}{2}\right]\right)$$

■ **Problem 52: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \operatorname{Cos}[c + d x]^2) \operatorname{Sec}[c + d x]}{(a + a \operatorname{Cos}[c + d x])^2} dx$$

Optimal (type 3, 77 leaves, 4 steps) :

$$\frac{A \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{a^2 d} - \frac{2(2A - C) \operatorname{Sin}[c + d x]}{3 a^2 d (1 + \operatorname{Cos}[c + d x])} - \frac{(A + C) \operatorname{Sin}[c + d x]}{3 d (a + a \operatorname{Cos}[c + d x])^2}$$

Result (type 3, 166 leaves) :

$$-\frac{1}{3 a^2 d (1 + \cos [c + d x])^2} \\ 2 \cos \left[ \frac{1}{2} (c + d x) \right] \left( 6 A \cos \left[ \frac{1}{2} (c + d x) \right]^3 \left( \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] - \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] \right) + \right. \\ \left. (A + C) \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + 4 (2 A - C) \cos \left[ \frac{1}{2} (c + d x) \right]^2 \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + (A + C) \cos \left[ \frac{1}{2} (c + d x) \right] \tan \left[ \frac{c}{2} \right] \right)$$

■ **Problem 53: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos [c + d x]^2) \sec [c + d x]^2}{(a + a \cos [c + d x])^2} dx$$

Optimal (type 3, 91 leaves, 6 steps):

$$-\frac{2 A \operatorname{ArcTanh}[\sin [c + d x]]}{a^2 d} + \frac{(10 A + C) \tan [c + d x]}{3 a^2 d} - \frac{2 A \tan [c + d x]}{a^2 d (1 + \cos [c + d x])} - \frac{(A + C) \tan [c + d x]}{3 d (a + a \cos [c + d x])^2}$$

Result (type 3, 288 leaves):

$$\frac{1}{3 a^2 d (1 + \cos [c + d x])^2 (2 A + C + C \cos [2 (c + d x)])} \\ 4 \cos \left[ \frac{1}{2} (c + d x) \right] \cos [c + d x]^2 (C + A \sec [c + d x]^2) \left( (A + C) \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + 2 (7 A + C) \cos \left[ \frac{1}{2} (c + d x) \right]^2 \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + 6 A \right. \\ \left. \cos \left[ \frac{1}{2} (c + d x) \right]^3 \left( 2 \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] - 2 \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \sin [d x] \right) / \left( \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \right. \right. \\ \left. \left. \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) \right) + (A + C) \cos \left[ \frac{1}{2} (c + d x) \right] \tan \left[ \frac{c}{2} \right]$$

■ **Problem 54: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos [c + d x]^2) \sec [c + d x]^3}{(a + a \cos [c + d x])^2} dx$$

Optimal (type 3, 146 leaves, 7 steps):

$$\frac{(7 A + 2 C) \operatorname{ArcTanh}[\sin [c + d x]]}{2 a^2 d} - \frac{4 (4 A + C) \tan [c + d x]}{3 a^2 d} + \\ \frac{(7 A + 2 C) \sec [c + d x] \tan [c + d x]}{2 a^2 d} - \frac{2 (4 A + C) \sec [c + d x] \tan [c + d x]}{3 a^2 d (1 + \cos [c + d x])} - \frac{(A + C) \sec [c + d x] \tan [c + d x]}{3 d (a + a \cos [c + d x])^2}$$

Result (type 3, 560 leaves):

$$\begin{aligned}
& - \frac{2 (7 A + 2 C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \cos[c + dx])^2} + \\
& \frac{2 (7 A + 2 C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \cos[c + dx])^2} + \frac{1}{48 d (a + a \cos[c + dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \\
& \operatorname{Sec}[c + dx]^2 \left( 14 A \sin\left[\frac{dx}{2}\right] + 20 C \sin\left[\frac{dx}{2}\right] - 97 A \sin\left[\frac{3 dx}{2}\right] - 22 C \sin\left[\frac{3 dx}{2}\right] + 126 A \sin\left[c - \frac{dx}{2}\right] + 36 C \sin\left[c - \frac{dx}{2}\right] - \right. \\
& 42 A \sin\left[c + \frac{dx}{2}\right] - 36 C \sin\left[c + \frac{dx}{2}\right] + 98 A \sin\left[2c + \frac{dx}{2}\right] + 20 C \sin\left[2c + \frac{dx}{2}\right] + 3 A \sin\left[c + \frac{3 dx}{2}\right] + 18 C \sin\left[c + \frac{3 dx}{2}\right] - \\
& 37 A \sin\left[2c + \frac{3 dx}{2}\right] - 22 C \sin\left[2c + \frac{3 dx}{2}\right] + 63 A \sin\left[3c + \frac{3 dx}{2}\right] + 18 C \sin\left[3c + \frac{3 dx}{2}\right] - 75 A \sin\left[c + \frac{5 dx}{2}\right] - 18 C \sin\left[c + \frac{5 dx}{2}\right] - \\
& 15 A \sin\left[2c + \frac{5 dx}{2}\right] + 6 C \sin\left[2c + \frac{5 dx}{2}\right] - 39 A \sin\left[3c + \frac{5 dx}{2}\right] - 18 C \sin\left[3c + \frac{5 dx}{2}\right] + 21 A \sin\left[4c + \frac{5 dx}{2}\right] + \\
& \left. 6 C \sin\left[4c + \frac{5 dx}{2}\right] - 32 A \sin\left[2c + \frac{7 dx}{2}\right] - 8 C \sin\left[2c + \frac{7 dx}{2}\right] - 12 A \sin\left[3c + \frac{7 dx}{2}\right] - 20 A \sin\left[4c + \frac{7 dx}{2}\right] - 8 C \sin\left[4c + \frac{7 dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 55: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos[c + dx])^2 \operatorname{Sec}[c + dx]^4}{(a + a \cos[c + dx])^2} dx$$

Optimal (type 3, 172 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(5 A + 2 C) \operatorname{ArcTanh}[\sin[c + dx]]}{a^2 d} + \frac{(12 A + 5 C) \tan[c + dx]}{a^2 d} - \frac{(5 A + 2 C) \operatorname{Sec}[c + dx] \tan[c + dx]}{a^2 d} - \\
& \frac{2 (5 A + 2 C) \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 a^2 d (1 + \cos[c + dx])} - \frac{(A + C) \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 d (a + a \cos[c + dx])^2} + \frac{(12 A + 5 C) \tan[c + dx]^3}{3 a^2 d}
\end{aligned}$$

Result (type 3, 672 leaves):

$$\frac{4(5A + 2C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d(a + a \cos[c + dx])^2} -$$

$$\frac{4(5A + 2C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d(a + a \cos[c + dx])^2} + \frac{1}{48d(a + a \cos[c + dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c] \sec[c + dx]^3$$

$$\left( -3A \sin\left[\frac{dx}{2}\right] - 24C \sin\left[\frac{dx}{2}\right] + 155A \sin\left[\frac{3dx}{2}\right] + 66C \sin\left[\frac{3dx}{2}\right] - 153A \sin\left[c - \frac{dx}{2}\right] - 60C \sin\left[c - \frac{dx}{2}\right] + 21A \sin\left[c + \frac{dx}{2}\right] + \right.$$

$$24C \sin\left[c + \frac{dx}{2}\right] - 135A \sin\left[2c + \frac{dx}{2}\right] - 60C \sin\left[2c + \frac{dx}{2}\right] + 25A \sin\left[c + \frac{3dx}{2}\right] - 4C \sin\left[c + \frac{3dx}{2}\right] + 45A \sin\left[2c + \frac{3dx}{2}\right] +$$

$$36C \sin\left[2c + \frac{3dx}{2}\right] - 85A \sin\left[3c + \frac{3dx}{2}\right] - 34C \sin\left[3c + \frac{3dx}{2}\right] + 99A \sin\left[c + \frac{5dx}{2}\right] + 42C \sin\left[c + \frac{5dx}{2}\right] + 21A \sin\left[2c + \frac{5dx}{2}\right] +$$

$$33A \sin\left[3c + \frac{5dx}{2}\right] + 24C \sin\left[3c + \frac{5dx}{2}\right] - 45A \sin\left[4c + \frac{5dx}{2}\right] - 18C \sin\left[4c + \frac{5dx}{2}\right] + 57A \sin\left[2c + \frac{7dx}{2}\right] + 24C \sin\left[2c + \frac{7dx}{2}\right] +$$

$$18A \sin\left[3c + \frac{7dx}{2}\right] + 3C \sin\left[3c + \frac{7dx}{2}\right] + 24A \sin\left[4c + \frac{7dx}{2}\right] + 15C \sin\left[4c + \frac{7dx}{2}\right] - 15A \sin\left[5c + \frac{7dx}{2}\right] - 6C \sin\left[5c + \frac{7dx}{2}\right] +$$

$$\left. 24A \sin\left[3c + \frac{9dx}{2}\right] + 10C \sin\left[3c + \frac{9dx}{2}\right] + 11A \sin\left[4c + \frac{9dx}{2}\right] + 3C \sin\left[4c + \frac{9dx}{2}\right] + 13A \sin\left[5c + \frac{9dx}{2}\right] + 7C \sin\left[5c + \frac{9dx}{2}\right] \right)$$

■ **Problem 56: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^4 (A + C \cos[c + dx]^2)}{(a + a \cos[c + dx])^3} dx$$

Optimal (type 3, 216 leaves, 8 steps):

$$-\frac{(6A + 23C)x}{2a^3} + \frac{4(9A + 34C) \sin[c + dx]}{5a^3 d} - \frac{(6A + 23C) \cos[c + dx] \sin[c + dx]}{2a^3 d} - \frac{(A + C) \cos[c + dx]^5 \sin[c + dx]}{5d(a + a \cos[c + dx])^3} -$$

$$\frac{(3A + 13C) \cos[c + dx]^4 \sin[c + dx]}{15ad(a + a \cos[c + dx])^2} - \frac{(6A + 23C) \cos[c + dx]^3 \sin[c + dx]}{3d(a^3 + a^3 \cos[c + dx])} - \frac{4(9A + 34C) \sin[c + dx]^3}{15a^3 d}$$

Result (type 3, 463 leaves):

$$\frac{1}{480 a^3 d (1 + \cos [c + d x])^3} \cos \left[ \frac{1}{2} (c + d x) \right] \sec \left[ \frac{c}{2} \right] \left( 600 (6 A + 23 C) d x \cos \left[ \frac{d x}{2} \right] + 600 (6 A + 23 C) d x \cos \left[ c + \frac{d x}{2} \right] + 1800 A d x \cos \left[ c + \frac{3 d x}{2} \right] + 6900 C d x \cos \left[ c + \frac{3 d x}{2} \right] + 1800 A d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 6900 C d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 360 A d x \cos \left[ 2 c + \frac{5 d x}{2} \right] + 1380 C d x \cos \left[ 2 c + \frac{5 d x}{2} \right] + 360 A d x \cos \left[ 3 c + \frac{5 d x}{2} \right] + 1380 C d x \cos \left[ 3 c + \frac{5 d x}{2} \right] - 7020 A \sin \left[ \frac{d x}{2} \right] - 20410 C \sin \left[ \frac{d x}{2} \right] + 4500 A \sin \left[ c + \frac{d x}{2} \right] + 11110 C \sin \left[ c + \frac{d x}{2} \right] - 4860 A \sin \left[ c + \frac{3 d x}{2} \right] - 15380 C \sin \left[ c + \frac{3 d x}{2} \right] + 900 A \sin \left[ 2 c + \frac{3 d x}{2} \right] + 380 C \sin \left[ 2 c + \frac{3 d x}{2} \right] - 1452 A \sin \left[ 2 c + \frac{5 d x}{2} \right] - 4777 C \sin \left[ 2 c + \frac{5 d x}{2} \right] - 300 A \sin \left[ 3 c + \frac{5 d x}{2} \right] - 1625 C \sin \left[ 3 c + \frac{5 d x}{2} \right] - 60 A \sin \left[ 3 c + \frac{7 d x}{2} \right] - 230 C \sin \left[ 3 c + \frac{7 d x}{2} \right] - 60 A \sin \left[ 4 c + \frac{7 d x}{2} \right] - 230 C \sin \left[ 4 c + \frac{7 d x}{2} \right] + 20 C \sin \left[ 4 c + \frac{9 d x}{2} \right] + 20 C \sin \left[ 5 c + \frac{9 d x}{2} \right] - 5 C \sin \left[ 5 c + \frac{11 d x}{2} \right] - 5 C \sin \left[ 6 c + \frac{11 d x}{2} \right] \right)$$

■ **Problem 57: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^3 (A + C \cos [c + d x]^2)}{(a + a \cos [c + d x])^3} dx$$

Optimal (type 3, 189 leaves, 4 steps):

$$\frac{(2 A + 13 C) x}{2 a^3} - \frac{2 (11 A + 76 C) \sin [c + d x]}{15 a^3 d} + \frac{(2 A + 13 C) \cos [c + d x] \sin [c + d x]}{2 a^3 d} - \frac{(A + C) \cos [c + d x]^4 \sin [c + d x]}{5 d (a + a \cos [c + d x])^3} - \frac{(A + 11 C) \cos [c + d x]^3 \sin [c + d x]}{15 a d (a + a \cos [c + d x])^2} - \frac{(11 A + 76 C) \cos [c + d x]^2 \sin [c + d x]}{15 d (a^3 + a^3 \cos [c + d x])}$$

Result (type 3, 393 leaves):

$$\frac{1}{480 a^3 d (1 + \cos [c + d x])^3} \cos \left[ \frac{1}{2} (c + d x) \right] \sec \left[ \frac{c}{2} \right] \left( 600 (2 A + 13 C) d x \cos \left[ \frac{d x}{2} \right] + 600 (2 A + 13 C) d x \cos \left[ c + \frac{d x}{2} \right] + 600 A d x \cos \left[ c + \frac{3 d x}{2} \right] + 3900 C d x \cos \left[ c + \frac{3 d x}{2} \right] + 600 A d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 3900 C d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 120 A d x \cos \left[ 2 c + \frac{5 d x}{2} \right] + 780 C d x \cos \left[ 2 c + \frac{5 d x}{2} \right] + 120 A d x \cos \left[ 3 c + \frac{5 d x}{2} \right] + 780 C d x \cos \left[ 3 c + \frac{5 d x}{2} \right] - 2960 A \sin \left[ \frac{d x}{2} \right] - 12760 C \sin \left[ \frac{d x}{2} \right] + 2160 A \sin \left[ c + \frac{d x}{2} \right] + 7560 C \sin \left[ c + \frac{d x}{2} \right] - 1840 A \sin \left[ c + \frac{3 d x}{2} \right] - 9230 C \sin \left[ c + \frac{3 d x}{2} \right] + 720 A \sin \left[ 2 c + \frac{3 d x}{2} \right] + 930 C \sin \left[ 2 c + \frac{3 d x}{2} \right] - 512 A \sin \left[ 2 c + \frac{5 d x}{2} \right] - 2782 C \sin \left[ 2 c + \frac{5 d x}{2} \right] - 750 C \sin \left[ 3 c + \frac{5 d x}{2} \right] - 105 C \sin \left[ 3 c + \frac{7 d x}{2} \right] - 105 C \sin \left[ 4 c + \frac{7 d x}{2} \right] + 15 C \sin \left[ 4 c + \frac{9 d x}{2} \right] + 15 C \sin \left[ 5 c + \frac{9 d x}{2} \right] \right)$$

■ **Problem 58: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^2 (A+C \cos [c+d x]^2)}{(a+a \cos [c+d x])^3} d x$$

Optimal (type 3, 136 leaves, 7 steps):

$$-\frac{3 C x}{a^3} + \frac{(2 A+27 C) \sin [c+d x]}{15 a^3 d} - \frac{(A+C) \cos [c+d x]^3 \sin [c+d x]}{5 d (a+a \cos [c+d x])^3} + \frac{(A-9 C) \cos [c+d x]^2 \sin [c+d x]}{15 a d (a+a \cos [c+d x])^2} + \frac{3 C \sin [c+d x]}{d (a^3+a^3 \cos [c+d x])}$$

Result (type 3, 283 leaves):

$$-\frac{1}{960 a^3 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5$$

$$\left(900 C d x \cos \left[\frac{d x}{2}\right]+900 C d x \cos \left[c+\frac{d x}{2}\right]+450 C d x \cos \left[c+\frac{3 d x}{2}\right]+450 C d x \cos \left[2 c+\frac{3 d x}{2}\right]+90 C d x \cos \left[2 c+\frac{5 d x}{2}\right]+90 C d x \cos \left[3 c+\frac{5 d x}{2}\right]-160 A \sin \left[\frac{d x}{2}\right]-1755 C \sin \left[\frac{d x}{2}\right]+120 A \sin \left[c+\frac{d x}{2}\right]+1125 C \sin \left[c+\frac{d x}{2}\right]-80 A \sin \left[c+\frac{3 d x}{2}\right]-1215 C \sin \left[c+\frac{3 d x}{2}\right]+60 A \sin \left[2 c+\frac{3 d x}{2}\right]+225 C \sin \left[2 c+\frac{3 d x}{2}\right]-28 A \sin \left[2 c+\frac{5 d x}{2}\right]-363 C \sin \left[2 c+\frac{5 d x}{2}\right]-75 C \sin \left[3 c+\frac{5 d x}{2}\right]-15 C \sin \left[3 c+\frac{7 d x}{2}\right]-15 C \sin \left[4 c+\frac{7 d x}{2}\right]\right)$$

■ **Problem 62: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+C \cos [c+d x]^2) \operatorname{Sec}[c+d x]^2}{(a+a \cos [c+d x])^3} d x$$

Optimal (type 3, 129 leaves, 7 steps):

$$-\frac{3 A \operatorname{ArcTanh}[\sin [c+d x]]}{a^3 d} + \frac{2(36 A+C) \tan [c+d x]}{15 a^3 d} - \frac{(A+C) \tan [c+d x]}{5 d (a+a \cos [c+d x])^3} - \frac{(9 A-C) \tan [c+d x]}{15 a d (a+a \cos [c+d x])^2} - \frac{3 A \tan [c+d x]}{d (a^3+a^3 \cos [c+d x])}$$

Result (type 3, 596 leaves):

$$\frac{1}{a^3} \left( \frac{48 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \cos[c + dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + A \sec[c + dx]^2)}{d (1 + \cos[c + dx])^3 (2A + C + C \cos[2c + 2dx])} - \right.$$

$$\frac{48 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \cos[c + dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + A \sec[c + dx]^2)}{d (1 + \cos[c + dx])^3 (2A + C + C \cos[2c + 2dx])} +$$

$$\frac{1}{60 d (1 + \cos[c + dx])^3 (2A + C + C \cos[2c + 2dx])} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c + dx] \sec\left[\frac{c}{2}\right] \sec[c] (C + A \sec[c + dx]^2)$$

$$\left( -255 A \sin\left[\frac{dx}{2}\right] - 20 C \sin\left[\frac{dx}{2}\right] + 567 A \sin\left[\frac{3dx}{2}\right] + 22 C \sin\left[\frac{3dx}{2}\right] - 600 A \sin\left[c - \frac{dx}{2}\right] - 10 C \sin\left[c - \frac{dx}{2}\right] + 375 A \sin\left[c + \frac{dx}{2}\right] + \right.$$

$$10 C \sin\left[c + \frac{dx}{2}\right] - 480 A \sin\left[2c + \frac{dx}{2}\right] - 20 C \sin\left[2c + \frac{dx}{2}\right] - 60 A \sin\left[c + \frac{3dx}{2}\right] + 402 A \sin\left[2c + \frac{3dx}{2}\right] + 22 C \sin\left[2c + \frac{3dx}{2}\right] -$$

$$225 A \sin\left[3c + \frac{3dx}{2}\right] + 315 A \sin\left[c + \frac{5dx}{2}\right] + 10 C \sin\left[c + \frac{5dx}{2}\right] + 30 A \sin\left[2c + \frac{5dx}{2}\right] + 240 A \sin\left[3c + \frac{5dx}{2}\right] + 10 C \sin\left[3c + \frac{5dx}{2}\right] -$$

$$\left. \left. 45 A \sin\left[4c + \frac{5dx}{2}\right] + 72 A \sin\left[2c + \frac{7dx}{2}\right] + 2 C \sin\left[2c + \frac{7dx}{2}\right] + 15 A \sin\left[3c + \frac{7dx}{2}\right] + 57 A \sin\left[4c + \frac{7dx}{2}\right] + 2 C \sin\left[4c + \frac{7dx}{2}\right] \right) \right)$$

■ **Problem 63: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos[c + dx]^2) \sec[c + dx]^3}{(a + a \cos[c + dx])^3} dx$$

Optimal (type 3, 192 leaves, 8 steps):

$$\frac{(13A + 2C) \operatorname{ArcTanh}[\sin[c + dx]]}{2a^3 d} - \frac{2(76A + 11C) \tan[c + dx]}{15a^3 d} + \frac{(13A + 2C) \sec[c + dx] \tan[c + dx]}{2a^3 d} -$$

$$\frac{(A + C) \sec[c + dx] \tan[c + dx]}{5d (a + a \cos[c + dx])^3} - \frac{(11A + C) \sec[c + dx] \tan[c + dx]}{15a d (a + a \cos[c + dx])^2} - \frac{(76A + 11C) \sec[c + dx] \tan[c + dx]}{15d (a^3 + a^3 \cos[c + dx])}$$

Result (type 3, 672 leaves):

$$\begin{aligned}
& - \frac{4 (13 A + 2 C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + dx])^3} + \\
& \frac{4 (13 A + 2 C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + dx])^3} + \frac{1}{480 d (a + a \operatorname{Cos}[c + dx])^3} \\
& \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \left( 1235 A \operatorname{Sin}\left[\frac{dx}{2}\right] + 490 C \operatorname{Sin}\left[\frac{dx}{2}\right] - 3805 A \operatorname{Sin}\left[\frac{3 dx}{2}\right] - 530 C \operatorname{Sin}\left[\frac{3 dx}{2}\right] + 4329 A \operatorname{Sin}\left[c - \frac{dx}{2}\right] + \right. \\
& 654 C \operatorname{Sin}\left[c - \frac{dx}{2}\right] - 1989 A \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 654 C \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 3575 A \operatorname{Sin}\left[2 c + \frac{dx}{2}\right] + 490 C \operatorname{Sin}\left[2 c + \frac{dx}{2}\right] + 475 A \operatorname{Sin}\left[c + \frac{3 dx}{2}\right] + \\
& 350 C \operatorname{Sin}\left[c + \frac{3 dx}{2}\right] - 2005 A \operatorname{Sin}\left[2 c + \frac{3 dx}{2}\right] - 530 C \operatorname{Sin}\left[2 c + \frac{3 dx}{2}\right] + 2275 A \operatorname{Sin}\left[3 c + \frac{3 dx}{2}\right] + 350 C \operatorname{Sin}\left[3 c + \frac{3 dx}{2}\right] - \\
& 2673 A \operatorname{Sin}\left[c + \frac{5 dx}{2}\right] - 378 C \operatorname{Sin}\left[c + \frac{5 dx}{2}\right] - 105 A \operatorname{Sin}\left[2 c + \frac{5 dx}{2}\right] + 150 C \operatorname{Sin}\left[2 c + \frac{5 dx}{2}\right] - 1593 A \operatorname{Sin}\left[3 c + \frac{5 dx}{2}\right] - \\
& 378 C \operatorname{Sin}\left[3 c + \frac{5 dx}{2}\right] + 975 A \operatorname{Sin}\left[4 c + \frac{5 dx}{2}\right] + 150 C \operatorname{Sin}\left[4 c + \frac{5 dx}{2}\right] - 1325 A \operatorname{Sin}\left[2 c + \frac{7 dx}{2}\right] - 190 C \operatorname{Sin}\left[2 c + \frac{7 dx}{2}\right] - \\
& 255 A \operatorname{Sin}\left[3 c + \frac{7 dx}{2}\right] + 30 C \operatorname{Sin}\left[3 c + \frac{7 dx}{2}\right] - 875 A \operatorname{Sin}\left[4 c + \frac{7 dx}{2}\right] - 190 C \operatorname{Sin}\left[4 c + \frac{7 dx}{2}\right] + 195 A \operatorname{Sin}\left[5 c + \frac{7 dx}{2}\right] + \\
& \left. 30 C \operatorname{Sin}\left[5 c + \frac{7 dx}{2}\right] - 304 A \operatorname{Sin}\left[3 c + \frac{9 dx}{2}\right] - 44 C \operatorname{Sin}\left[3 c + \frac{9 dx}{2}\right] - 90 A \operatorname{Sin}\left[4 c + \frac{9 dx}{2}\right] - 214 A \operatorname{Sin}\left[5 c + \frac{9 dx}{2}\right] - 44 C \operatorname{Sin}\left[5 c + \frac{9 dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 64: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \operatorname{Cos}[c + dx])^2 \operatorname{Sec}[c + dx]^4}{(a + a \operatorname{Cos}[c + dx])^3} dx$$

Optimal (type 3, 225 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(23 A + 6 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2 a^3 d} + \frac{4 (34 A + 9 C) \operatorname{Tan}[c + dx]}{5 a^3 d} - \frac{(23 A + 6 C) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2 a^3 d} - \frac{(A + C) \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{5 d (a + a \operatorname{Cos}[c + dx])^3} - \\
& \frac{(13 A + 3 C) \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{15 a d (a + a \operatorname{Cos}[c + dx])^2} - \frac{(23 A + 6 C) \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{3 d (a^3 + a^3 \operatorname{Cos}[c + dx])} + \frac{4 (34 A + 9 C) \operatorname{Tan}[c + dx]^3}{15 a^3 d}
\end{aligned}$$

Result (type 3, 798 leaves):



$$\frac{4 (23 A + 6 C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + dx])^3} -$$

$$\frac{4 (23 A + 6 C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \operatorname{Cos}[c + dx])^3} + \frac{1}{960 d (a + a \operatorname{Cos}[c + dx])^3}$$

$$\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^3 \left( -2484 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 1764 C \operatorname{Sin}\left[\frac{dx}{2}\right] + 12622 A \operatorname{Sin}\left[\frac{3 dx}{2}\right] + 3372 C \operatorname{Sin}\left[\frac{3 dx}{2}\right] - 13340 A \operatorname{Sin}\left[c - \frac{dx}{2}\right] - \right.$$

$$3480 C \operatorname{Sin}\left[c - \frac{dx}{2}\right] + 4140 A \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 2100 C \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 11684 A \operatorname{Sin}\left[2c + \frac{dx}{2}\right] - 3144 C \operatorname{Sin}\left[2c + \frac{dx}{2}\right] - 450 A \operatorname{Sin}\left[c + \frac{3 dx}{2}\right] -$$

$$960 C \operatorname{Sin}\left[c + \frac{3 dx}{2}\right] + 5022 A \operatorname{Sin}\left[2c + \frac{3 dx}{2}\right] + 2232 C \operatorname{Sin}\left[2c + \frac{3 dx}{2}\right] - 8050 A \operatorname{Sin}\left[3c + \frac{3 dx}{2}\right] - 2100 C \operatorname{Sin}\left[3c + \frac{3 dx}{2}\right] +$$

$$9230 A \operatorname{Sin}\left[c + \frac{5 dx}{2}\right] + 2460 C \operatorname{Sin}\left[c + \frac{5 dx}{2}\right] + 630 A \operatorname{Sin}\left[2c + \frac{5 dx}{2}\right] - 390 C \operatorname{Sin}\left[2c + \frac{5 dx}{2}\right] + 4230 A \operatorname{Sin}\left[3c + \frac{5 dx}{2}\right] +$$

$$1710 C \operatorname{Sin}\left[3c + \frac{5 dx}{2}\right] - 4370 A \operatorname{Sin}\left[4c + \frac{5 dx}{2}\right] - 1140 C \operatorname{Sin}\left[4c + \frac{5 dx}{2}\right] + 5347 A \operatorname{Sin}\left[2c + \frac{7 dx}{2}\right] + 1422 C \operatorname{Sin}\left[2c + \frac{7 dx}{2}\right] +$$

$$875 A \operatorname{Sin}\left[3c + \frac{7 dx}{2}\right] - 60 C \operatorname{Sin}\left[3c + \frac{7 dx}{2}\right] + 2747 A \operatorname{Sin}\left[4c + \frac{7 dx}{2}\right] + 1032 C \operatorname{Sin}\left[4c + \frac{7 dx}{2}\right] - 1725 A \operatorname{Sin}\left[5c + \frac{7 dx}{2}\right] -$$

$$450 C \operatorname{Sin}\left[5c + \frac{7 dx}{2}\right] + 2375 A \operatorname{Sin}\left[3c + \frac{9 dx}{2}\right] + 630 C \operatorname{Sin}\left[3c + \frac{9 dx}{2}\right] + 655 A \operatorname{Sin}\left[4c + \frac{9 dx}{2}\right] + 60 C \operatorname{Sin}\left[4c + \frac{9 dx}{2}\right] +$$

$$1375 A \operatorname{Sin}\left[5c + \frac{9 dx}{2}\right] + 480 C \operatorname{Sin}\left[5c + \frac{9 dx}{2}\right] - 345 A \operatorname{Sin}\left[6c + \frac{9 dx}{2}\right] - 90 C \operatorname{Sin}\left[6c + \frac{9 dx}{2}\right] + 544 A \operatorname{Sin}\left[4c + \frac{11 dx}{2}\right] +$$

$$144 C \operatorname{Sin}\left[4c + \frac{11 dx}{2}\right] + 200 A \operatorname{Sin}\left[5c + \frac{11 dx}{2}\right] + 30 C \operatorname{Sin}\left[5c + \frac{11 dx}{2}\right] + 344 A \operatorname{Sin}\left[6c + \frac{11 dx}{2}\right] + 114 C \operatorname{Sin}\left[6c + \frac{11 dx}{2}\right] \Big)$$

■ **Problem 65: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + dx]^4 (A + C \operatorname{Cos}[c + dx]^2)}{(a + a \operatorname{Cos}[c + dx])^4} dx$$

Optimal (type 3, 223 leaves, 5 steps):

$$\frac{(2A + 21C)x}{2a^4} - \frac{32(5A + 54C)\operatorname{Sin}[c + dx]}{105a^4 d} + \frac{(2A + 21C)\operatorname{Cos}[c + dx]\operatorname{Sin}[c + dx]}{2a^4 d} - \frac{(10A + 129C)\operatorname{Cos}[c + dx]^3\operatorname{Sin}[c + dx]}{105a^4 d (1 + \operatorname{Cos}[c + dx])^2} -$$

$$\frac{16(5A + 54C)\operatorname{Cos}[c + dx]^2\operatorname{Sin}[c + dx]}{105a^4 d (1 + \operatorname{Cos}[c + dx])} - \frac{(A + C)\operatorname{Cos}[c + dx]^5\operatorname{Sin}[c + dx]}{7d (a + a \operatorname{Cos}[c + dx])^4} - \frac{2C\operatorname{Cos}[c + dx]^4\operatorname{Sin}[c + dx]}{5ad (a + a \operatorname{Cos}[c + dx])^3}$$

Result (type 3, 513 leaves):

$$\frac{1}{6720 a^4 d (1 + \cos [c + d x])^4} \cos \left[ \frac{1}{2} (c + d x) \right] \sec \left[ \frac{c}{2} \right] \left( 14700 (2A + 21C) d x \cos \left[ \frac{d x}{2} \right] + 14700 (2A + 21C) d x \cos \left[ c + \frac{d x}{2} \right] + 17640 A d x \cos \left[ c + \frac{3 d x}{2} \right] + 185220 C d x \cos \left[ c + \frac{3 d x}{2} \right] + 17640 A d x \cos \left[ 2c + \frac{3 d x}{2} \right] + 185220 C d x \cos \left[ 2c + \frac{3 d x}{2} \right] + 5880 A d x \cos \left[ 2c + \frac{5 d x}{2} \right] + 61740 C d x \cos \left[ 2c + \frac{5 d x}{2} \right] + 5880 A d x \cos \left[ 3c + \frac{5 d x}{2} \right] + 61740 C d x \cos \left[ 3c + \frac{5 d x}{2} \right] + 840 A d x \cos \left[ 3c + \frac{7 d x}{2} \right] + 8820 C d x \cos \left[ 3c + \frac{7 d x}{2} \right] + 840 A d x \cos \left[ 4c + \frac{7 d x}{2} \right] + 8820 C d x \cos \left[ 4c + \frac{7 d x}{2} \right] - 79520 A \sin \left[ \frac{d x}{2} \right] - 539490 C \sin \left[ \frac{d x}{2} \right] + 66080 A \sin \left[ c + \frac{d x}{2} \right] + 386190 C \sin \left[ c + \frac{d x}{2} \right] - 57120 A \sin \left[ c + \frac{3 d x}{2} \right] - 422478 C \sin \left[ c + \frac{3 d x}{2} \right] + 30240 A \sin \left[ 2c + \frac{3 d x}{2} \right] + 132930 C \sin \left[ 2c + \frac{3 d x}{2} \right] - 22400 A \sin \left[ 2c + \frac{5 d x}{2} \right] - 181461 C \sin \left[ 2c + \frac{5 d x}{2} \right] + 6720 A \sin \left[ 3c + \frac{5 d x}{2} \right] + 3675 C \sin \left[ 3c + \frac{5 d x}{2} \right] - 4160 A \sin \left[ 3c + \frac{7 d x}{2} \right] - 36003 C \sin \left[ 3c + \frac{7 d x}{2} \right] - 9555 C \sin \left[ 4c + \frac{7 d x}{2} \right] - 945 C \sin \left[ 4c + \frac{9 d x}{2} \right] - 945 C \sin \left[ 5c + \frac{9 d x}{2} \right] + 105 C \sin \left[ 5c + \frac{11 d x}{2} \right] + 105 C \sin \left[ 6c + \frac{11 d x}{2} \right] \right)$$

■ **Problem 66: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^3 (A + C \cos [c + d x]^2)}{(a + a \cos [c + d x])^4} dx$$

Optimal (type 3, 174 leaves, 8 steps):

$$-\frac{4 C x}{a^4} + \frac{2 (3 A + 122 C) \sin [c + d x]}{105 a^4 d} + \frac{(3 A - 88 C) \cos [c + d x]^2 \sin [c + d x]}{105 a^4 d (1 + \cos [c + d x])^2} + \frac{4 C \sin [c + d x]}{a^4 d (1 + \cos [c + d x])} - \frac{(A + C) \cos [c + d x]^4 \sin [c + d x]}{7 d (a + a \cos [c + d x])^4} + \frac{2 (A - 6 C) \cos [c + d x]^3 \sin [c + d x]}{35 a d (a + a \cos [c + d x])^3}$$

Result (type 3, 371 leaves):

$$\begin{aligned}
& - \frac{1}{26\,880\,a^4\,d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^7 \\
& \left( 29\,400\,C\,d\,x \operatorname{Cos}\left[\frac{dx}{2}\right] + 29\,400\,C\,d\,x \operatorname{Cos}\left[c + \frac{dx}{2}\right] + 17\,640\,C\,d\,x \operatorname{Cos}\left[c + \frac{3dx}{2}\right] + 17\,640\,C\,d\,x \operatorname{Cos}\left[2c + \frac{3dx}{2}\right] + 5\,880\,C\,d\,x \operatorname{Cos}\left[2c + \frac{5dx}{2}\right] + \right. \\
& 5\,880\,C\,d\,x \operatorname{Cos}\left[3c + \frac{5dx}{2}\right] + 840\,C\,d\,x \operatorname{Cos}\left[3c + \frac{7dx}{2}\right] + 840\,C\,d\,x \operatorname{Cos}\left[4c + \frac{7dx}{2}\right] - 2\,520\,A \operatorname{Sin}\left[\frac{dx}{2}\right] - 60\,830\,C \operatorname{Sin}\left[\frac{dx}{2}\right] + \\
& 2\,520\,A \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 46\,130\,C \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 1\,764\,A \operatorname{Sin}\left[c + \frac{3dx}{2}\right] - 46\,116\,C \operatorname{Sin}\left[c + \frac{3dx}{2}\right] + 1\,260\,A \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] + \\
& 18\,060\,C \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] - 588\,A \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] - 19\,292\,C \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] + 420\,A \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] + 2\,100\,C \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] - \\
& \left. 144\,A \operatorname{Sin}\left[3c + \frac{7dx}{2}\right] - 3\,791\,C \operatorname{Sin}\left[3c + \frac{7dx}{2}\right] - 735\,C \operatorname{Sin}\left[4c + \frac{7dx}{2}\right] - 105\,C \operatorname{Sin}\left[4c + \frac{9dx}{2}\right] - 105\,C \operatorname{Sin}\left[5c + \frac{9dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 67: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^2 (A+C \operatorname{Cos}[c+dx])^2}{(a+a \operatorname{Cos}[c+dx])^4} dx$$

Optimal (type 3, 152 leaves, 6 steps):

$$\frac{Cx}{a^4} - \frac{(8A-55C) \operatorname{Sin}[c+dx]}{105a^4d(1+\operatorname{Cos}[c+dx])^2} + \frac{(16A-215C) \operatorname{Sin}[c+dx]}{105a^4d(1+\operatorname{Cos}[c+dx])} - \frac{(A+C) \operatorname{Cos}[c+dx]^3 \operatorname{Sin}[c+dx]}{7d(a+a \operatorname{Cos}[c+dx])^4} + \frac{2(2A-5C) \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{35ad(a+a \operatorname{Cos}[c+dx])^3}$$

Result (type 3, 315 leaves):

$$\begin{aligned}
& \frac{1}{13\,440\,a^4\,d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^7 \\
& \left( 3\,675\,C\,d\,x \operatorname{Cos}\left[\frac{dx}{2}\right] + 3\,675\,C\,d\,x \operatorname{Cos}\left[c + \frac{dx}{2}\right] + 2\,205\,C\,d\,x \operatorname{Cos}\left[c + \frac{3dx}{2}\right] + 2\,205\,C\,d\,x \operatorname{Cos}\left[2c + \frac{3dx}{2}\right] + 735\,C\,d\,x \operatorname{Cos}\left[2c + \frac{5dx}{2}\right] + \right. \\
& 735\,C\,d\,x \operatorname{Cos}\left[3c + \frac{5dx}{2}\right] + 105\,C\,d\,x \operatorname{Cos}\left[3c + \frac{7dx}{2}\right] + 105\,C\,d\,x \operatorname{Cos}\left[4c + \frac{7dx}{2}\right] + 560\,A \operatorname{Sin}\left[\frac{dx}{2}\right] - 9\,940\,C \operatorname{Sin}\left[\frac{dx}{2}\right] - \\
& 350\,A \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 8\,260\,C \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 336\,A \operatorname{Sin}\left[c + \frac{3dx}{2}\right] - 7\,140\,C \operatorname{Sin}\left[c + \frac{3dx}{2}\right] - 210\,A \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] + 3\,780\,C \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] + \\
& \left. 182\,A \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] - 2\,800\,C \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] + 840\,C \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] + 26\,A \operatorname{Sin}\left[3c + \frac{7dx}{2}\right] - 520\,C \operatorname{Sin}\left[3c + \frac{7dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 71: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+C \operatorname{Cos}[c+dx])^2 \operatorname{Sec}[c+dx]^2}{(a+a \operatorname{Cos}[c+dx])^4} dx$$

Optimal (type 3, 161 leaves, 8 steps):

$$-\frac{4 A \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{a^4 d} + \frac{2(332 A+3 C) \operatorname{Tan}[c+d x]}{105 a^4 d} - \frac{(88 A-3 C) \operatorname{Tan}[c+d x]}{105 a^4 d(1+\operatorname{Cos}[c+d x])^2} - \frac{4 A \operatorname{Tan}[c+d x]}{a^4 d(1+\operatorname{Cos}[c+d x])} - \frac{(A+C) \operatorname{Tan}[c+d x]}{7 d(a+a \operatorname{Cos}[c+d x])^4} - \frac{2(6 A-C) \operatorname{Tan}[c+d x]}{35 a d(a+a \operatorname{Cos}[c+d x])^3}$$

Result (type 3, 680 leaves):

$$\frac{1}{a^4} \left( \frac{128 A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 \operatorname{Cos}[c+d x]^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] (C+A \operatorname{Sec}[c+d x]^2)}{d(1+\operatorname{Cos}[c+d x])^4(2 A+C+C \operatorname{Cos}[2 c+2 d x])} - \frac{128 A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 \operatorname{Cos}[c+d x]^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] (C+A \operatorname{Sec}[c+d x]^2)}{d(1+\operatorname{Cos}[c+d x])^4(2 A+C+C \operatorname{Cos}[2 c+2 d x])} + \frac{1}{840 d(1+\operatorname{Cos}[c+d x])^4(2 A+C+C \operatorname{Cos}[2 c+2 d x])} \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] \operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] (C+A \operatorname{Sec}[c+d x]^2) \left( -10780 A \operatorname{Sin}\left[\frac{d x}{2}\right] - 210 C \operatorname{Sin}\left[\frac{d x}{2}\right] + 18788 A \operatorname{Sin}\left[\frac{3 d x}{2}\right] + 252 C \operatorname{Sin}\left[\frac{3 d x}{2}\right] - 20524 A \operatorname{Sin}\left[c - \frac{d x}{2}\right] - 126 C \operatorname{Sin}\left[c - \frac{d x}{2}\right] + 14644 A \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 126 C \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 16660 A \operatorname{Sin}\left[2 c + \frac{d x}{2}\right] - 210 C \operatorname{Sin}\left[2 c + \frac{d x}{2}\right] - 4690 A \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 14378 A \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 252 C \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] - 9100 A \operatorname{Sin}\left[3 c + \frac{3 d x}{2}\right] + 11668 A \operatorname{Sin}\left[c + \frac{5 d x}{2}\right] + 132 C \operatorname{Sin}\left[c + \frac{5 d x}{2}\right] - 630 A \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] + 9358 A \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] + 132 C \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] - 2940 A \operatorname{Sin}\left[4 c + \frac{5 d x}{2}\right] + 4228 A \operatorname{Sin}\left[2 c + \frac{7 d x}{2}\right] + 42 C \operatorname{Sin}\left[2 c + \frac{7 d x}{2}\right] + 315 A \operatorname{Sin}\left[3 c + \frac{7 d x}{2}\right] + 3493 A \operatorname{Sin}\left[4 c + \frac{7 d x}{2}\right] + 42 C \operatorname{Sin}\left[4 c + \frac{7 d x}{2}\right] - 420 A \operatorname{Sin}\left[5 c + \frac{7 d x}{2}\right] + 664 A \operatorname{Sin}\left[3 c + \frac{9 d x}{2}\right] + 6 C \operatorname{Sin}\left[3 c + \frac{9 d x}{2}\right] + 105 A \operatorname{Sin}\left[4 c + \frac{9 d x}{2}\right] + 559 A \operatorname{Sin}\left[5 c + \frac{9 d x}{2}\right] + 6 C \operatorname{Sin}\left[5 c + \frac{9 d x}{2}\right] \right)$$

■ **Problem 72: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+C \operatorname{Cos}[c+d x])^2 \operatorname{Sec}[c+d x]^3}{(a+a \operatorname{Cos}[c+d x])^4} dx$$

Optimal (type 3, 224 leaves, 9 steps):

$$\frac{(21 A+2 C) \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{2 a^4 d} - \frac{32(54 A+5 C) \operatorname{Tan}[c+d x]}{105 a^4 d} + \frac{(21 A+2 C) \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{2 a^4 d} - \frac{(129 A+10 C) \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{105 a^4 d(1+\operatorname{Cos}[c+d x])^2} - \frac{16(54 A+5 C) \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{105 a^4 d(1+\operatorname{Cos}[c+d x])} - \frac{(A+C) \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{7 d(a+a \operatorname{Cos}[c+d x])^4} - \frac{2 A \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{5 a d(a+a \operatorname{Cos}[c+d x])^3}$$

Result (type 3, 784 leaves):

$$\begin{aligned}
& - \frac{8 (21 A + 2 C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \cos[c + dx])^4} + \\
& \frac{8 (21 A + 2 C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (a + a \cos[c + dx])^4} + \frac{1}{6720 d (a + a \cos[c + dx])^4} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \\
& \operatorname{Sec}[c + dx]^2 \left( 73\,206 A \sin\left[\frac{dx}{2}\right] + 14\,140 C \sin\left[\frac{dx}{2}\right] - 166\,668 A \sin\left[\frac{3 dx}{2}\right] - 15\,160 C \sin\left[\frac{3 dx}{2}\right] + 183\,162 A \sin\left[c - \frac{dx}{2}\right] + \right. \\
& 17\,220 C \sin\left[c - \frac{dx}{2}\right] - 100\,842 A \sin\left[c + \frac{dx}{2}\right] - 17\,220 C \sin\left[c + \frac{dx}{2}\right] + 155\,526 A \sin\left[2c + \frac{dx}{2}\right] + 14\,140 C \sin\left[2c + \frac{dx}{2}\right] + \\
& 37\,380 A \sin\left[c + \frac{3 dx}{2}\right] + 9\,800 C \sin\left[c + \frac{3 dx}{2}\right] - 101\,148 A \sin\left[2c + \frac{3 dx}{2}\right] - 15\,160 C \sin\left[2c + \frac{3 dx}{2}\right] + 102\,900 A \sin\left[3c + \frac{3 dx}{2}\right] + \\
& 9\,800 C \sin\left[3c + \frac{3 dx}{2}\right] - 119\,364 A \sin\left[c + \frac{5 dx}{2}\right] - 10\,920 C \sin\left[c + \frac{5 dx}{2}\right] + 8\,820 A \sin\left[2c + \frac{5 dx}{2}\right] + 4\,760 C \sin\left[2c + \frac{5 dx}{2}\right] - \\
& 78\,204 A \sin\left[3c + \frac{5 dx}{2}\right] - 10\,920 C \sin\left[3c + \frac{5 dx}{2}\right] + 49\,980 A \sin\left[4c + \frac{5 dx}{2}\right] + 4\,760 C \sin\left[4c + \frac{5 dx}{2}\right] - 64\,053 A \sin\left[2c + \frac{7 dx}{2}\right] - \\
& 5\,890 C \sin\left[2c + \frac{7 dx}{2}\right] - 3\,885 A \sin\left[3c + \frac{7 dx}{2}\right] + 1\,470 C \sin\left[3c + \frac{7 dx}{2}\right] - 44\,733 A \sin\left[4c + \frac{7 dx}{2}\right] - 5\,890 C \sin\left[4c + \frac{7 dx}{2}\right] + \\
& 15\,435 A \sin\left[5c + \frac{7 dx}{2}\right] + 1\,470 C \sin\left[5c + \frac{7 dx}{2}\right] - 21\,987 A \sin\left[3c + \frac{9 dx}{2}\right] - 2\,030 C \sin\left[3c + \frac{9 dx}{2}\right] - 3\,675 A \sin\left[4c + \frac{9 dx}{2}\right] + \\
& 2\,100 C \sin\left[4c + \frac{9 dx}{2}\right] - 16\,107 A \sin\left[5c + \frac{9 dx}{2}\right] - 2\,030 C \sin\left[5c + \frac{9 dx}{2}\right] + 2\,205 A \sin\left[6c + \frac{9 dx}{2}\right] + 2\,100 C \sin\left[6c + \frac{9 dx}{2}\right] - \\
& \left. 3\,456 A \sin\left[4c + \frac{11 dx}{2}\right] - 320 C \sin\left[4c + \frac{11 dx}{2}\right] - 840 A \sin\left[5c + \frac{11 dx}{2}\right] - 2\,616 A \sin\left[6c + \frac{11 dx}{2}\right] - 320 C \sin\left[6c + \frac{11 dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 78: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos[c + dx]} (A + C \cos[c + dx]^2) \operatorname{Sec}[c + dx] dx$$

Optimal (type 3, 96 leaves, 4 steps):

$$\frac{2 \sqrt{a} A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} + \frac{2 a C \sin[c + dx]}{3 d \sqrt{a + a \cos[c + dx]}} + \frac{2 C \sqrt{a + a \cos[c + dx]} \sin[c + dx]}{3 d}$$

Result (type 3, 1487 leaves):

$$\begin{aligned}
& - \left( \left( \left( \frac{1}{4} - \frac{i}{4} \right) A (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right. \right. \\
& (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + \\
& \left. \left. 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right) - \\
& \frac{i A \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{-\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{\sqrt{2} d} - \\
& \frac{i A \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{\sqrt{2} d} - \\
& \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{2\sqrt{2} d} - \\
& \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{2\sqrt{2} d} + \\
& \frac{C \cos \left[ \frac{dx}{2} \right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] \sin \left[ \frac{c}{2} \right]}{d} - \\
& \frac{2i A \operatorname{ArcTan} \left[ \frac{2i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Cot} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{d \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} + \\
& \frac{1}{d \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right)} \\
& \sqrt{2} A \sqrt{a(1 + \cos[c + dx])} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] \\
& \left( -dx \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right) \sin \left[ \frac{c}{2} \right] + \frac{4i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} + \\
& \frac{C \cos \left[ \frac{3dx}{2} \right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] \sin \left[ \frac{3c}{2} \right]}{3d} +
\end{aligned}$$

$$\frac{C \cos\left[\frac{c}{2}\right] \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{d} +$$

$$\frac{C \cos\left[\frac{3c}{2}\right] \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{3dx}{2}\right]}{3d}$$

- **Problem 79: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+a \cos[c+dx]} (A+C \cos[c+dx])^2 \sec[c+dx]^2 dx$$

Optimal (type 3, 94 leaves, 4 steps):

$$\frac{\sqrt{a} A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} - \frac{a(A-2C) \sin[c+dx]}{d \sqrt{a+a \cos[c+dx]}} + \frac{A \sqrt{a+a \cos[c+dx]} \tan[c+dx]}{d}$$

Result (type 3, 1527 leaves):

$$-\left(\left(\left(\frac{1}{8} - \frac{i}{8}\right) A (1 + e^{ic}) \left(\sqrt{2} - (1-i) e^{\frac{ic}{2}} + (16-16i) e^{\frac{3ic}{2}+idx} + (20+20i) \sqrt{2} e^{2ic+\frac{3idx}{2}} - (34-34i) e^{\frac{5ic}{2}+2idx} - (20+20i) \sqrt{2} e^{3ic+\frac{5idx}{2}} + (16-16i) e^{\frac{7ic}{2}+3idx} + (4+4i) \sqrt{2} e^{4ic+\frac{7idx}{2}} - (1-i) e^{\frac{9ic}{2}+4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4+4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)}\right) x \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]\right) / \left(\left(\left(-1-i\right) + \sqrt{2} e^{\frac{ic}{2}}\right) \left(-1 + e^{ic}\right) \left(i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)}\right)^2\right)\right) -$$

$$\frac{i A \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]}{2\sqrt{2} d} -$$

$$\frac{i A \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]}{2\sqrt{2} d} -$$

$$\frac{A \sqrt{a(1+\cos[c+dx])} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]}{4\sqrt{2} d} -$$

$$\frac{A \sqrt{a(1+\cos[c+dx])} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]}{4\sqrt{2} d} +$$

$$\frac{2C \cos\left[\frac{dx}{2}\right] \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{d} -$$

$$\frac{i A \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right]-i\left(-\sqrt{2}+2 \sin\left[\frac{c}{2}\right]\right) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}}\right] \sqrt{a(1+\cos[c+dx])} \cot\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]}{d \sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}} +$$

$$\left( A \sqrt{a(1+\cos[c+dx])} \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}+\frac{dx}{2}\right] \right)$$

$$\left( -dx \cos\left[\frac{c}{2}\right] + 2 \operatorname{Log}\left[\sqrt{2}+2 \cos\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] \right) \sin\left[\frac{c}{2}\right] + \frac{4 i \sqrt{2} \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right]-i\left(-\sqrt{2}+2 \sin\left[\frac{c}{2}\right]\right) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}} \right) /$$

$$\left( \sqrt{2} d \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) \right) + \frac{2 C \cos\left[\frac{c}{2}\right] \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{c}{2}+\frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{d} +$$

$$\frac{A \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{c}{2}+\frac{dx}{2}\right]}{2 d \left( \cos\left[\frac{c}{2}+\frac{dx}{2}\right] - \sin\left[\frac{c}{2}+\frac{dx}{2}\right] \right)} - \frac{A \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{c}{2}+\frac{dx}{2}\right]}{2 d \left( \cos\left[\frac{c}{2}+\frac{dx}{2}\right] + \sin\left[\frac{c}{2}+\frac{dx}{2}\right] \right)}$$

■ **Problem 80: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+a \cos[c+dx]} (A+C \cos[c+dx]^2) \sec[c+dx]^3 dx$$

Optimal (type 3, 110 leaves, 4 steps):

$$\frac{\sqrt{a} (3 A + 8 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{4 d} + \frac{a A \tan[c+dx]}{4 d \sqrt{a+a \cos[c+dx]}} + \frac{A \sqrt{a+a \cos[c+dx]} \sec[c+dx] \tan[c+dx]}{2 d}$$

Result (type 3, 914 leaves):



$$\begin{aligned}
& \frac{i(-3A - 8C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{8\sqrt{2}d} + \\
& \frac{i(-3A - 8C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{8\sqrt{2}d} + \\
& \frac{(3A + 8C) \sqrt{a(1 + \cos[c + dx])} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{8\sqrt{2}d} + \\
& \frac{(-3A - 8C) \sqrt{a(1 + \cos[c + dx])} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{16\sqrt{2}d} + \\
& \frac{(-3A - 8C) \sqrt{a(1 + \cos[c + dx])} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{16\sqrt{2}d} + \\
& \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{4d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(3 \cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right)}{8d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{4d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} - \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(3 \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right)}{8d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

- **Problem 81: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos[c + dx]} (A + C \cos[c + dx])^2 \operatorname{Sec}[c + dx]^4 dx$$

Optimal (type 3, 153 leaves, 5 steps):

$$\frac{\sqrt{a} (5A + 8C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right]}{8d} + \frac{a(5A + 8C) \operatorname{Tan}[c + dx]}{8d \sqrt{a + a \cos[c + dx]}} + \frac{aA \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{12d \sqrt{a + a \cos[c + dx]}} + \frac{A \sqrt{a + a \cos[c + dx]} \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{3d}$$

Result (type 3, 1084 leaves):

$$\begin{aligned}
& \frac{i(-5A - 8C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{16\sqrt{2}d} + \\
& \frac{i(-5A - 8C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{16\sqrt{2}d} + \\
& \frac{(5A + 8C) \sqrt{a(1 + \cos[c + dx])} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{16\sqrt{2}d} + \\
& \frac{(-5A - 8C) \sqrt{a(1 + \cos[c + dx])} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{32\sqrt{2}d} + \\
& \frac{(-5A - 8C) \sqrt{a(1 + \cos[c + dx])} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{32\sqrt{2}d} + \\
& \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{12d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{8d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(5A \cos\left[\frac{c}{2}\right] + 8C \cos\left[\frac{c}{2}\right] - 3A \sin\left[\frac{c}{2}\right] - 8C \sin\left[\frac{c}{2}\right]\right)}{16d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{12d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{8d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(-5A \cos\left[\frac{c}{2}\right] - 8C \cos\left[\frac{c}{2}\right] - 3A \sin\left[\frac{c}{2}\right] - 8C \sin\left[\frac{c}{2}\right]\right)}{16d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 82: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos[c + dx]} (A + C \cos[c + dx])^2 \operatorname{Sec}[c + dx]^5 dx$$

Optimal (type 3, 196 leaves, 6 steps):

$$\begin{aligned}
& \frac{\sqrt{a} (35A + 48C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right]}{64d} + \frac{a(35A + 48C) \operatorname{Tan}[c + dx]}{64d \sqrt{a + a \cos[c + dx]}} + \\
& \frac{a(35A + 48C) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{96d \sqrt{a + a \cos[c + dx]}} + \frac{aA \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{24d \sqrt{a + a \cos[c + dx]}} + \frac{A \sqrt{a + a \cos[c + dx]} \operatorname{Sec}[c + dx]^3 \operatorname{Tan}[c + dx]}{4d}
\end{aligned}$$

Result (type 3, 1353 leaves):

$$\begin{aligned}
& \frac{i(-35A - 48C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{128\sqrt{2}d} + \\
& \frac{i(-35A - 48C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{128\sqrt{2}d} + \\
& \frac{(35A + 48C) \sqrt{a(1 + \cos[c + dx])} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{128\sqrt{2}d} + \\
& \frac{(-35A - 48C) \sqrt{a(1 + \cos[c + dx])} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{256\sqrt{2}d} + \\
& \frac{(-35A - 48C) \sqrt{a(1 + \cos[c + dx])} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]}{256\sqrt{2}d} + \\
& \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{16d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^4} + \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(7 \cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right)}{96d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(11A \sin\left[\frac{dx}{2}\right] + 16C \sin\left[\frac{dx}{2}\right]\right)}{64d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(35A \cos\left[\frac{c}{2}\right] + 48C \cos\left[\frac{c}{2}\right] - 13A \sin\left[\frac{c}{2}\right] - 16C \sin\left[\frac{c}{2}\right]\right)}{128d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{A \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{16d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^4} + \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(-7A \cos\left[\frac{c}{2}\right] - A \sin\left[\frac{c}{2}\right]\right)}{96d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(11A \sin\left[\frac{dx}{2}\right] + 16C \sin\left[\frac{dx}{2}\right]\right)}{64d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{\sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(-35A \cos\left[\frac{c}{2}\right] - 48C \cos\left[\frac{c}{2}\right] - 13A \sin\left[\frac{c}{2}\right] - 16C \sin\left[\frac{c}{2}\right]\right)}{128d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 86: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{3/2} (A + C \cos[c + dx])^2 \operatorname{Sec}[c + dx] dx$$

Optimal (type 3, 133 leaves, 5 steps) :

$$\frac{2 a^{3/2} A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{d} + \frac{2 a^2 (5 A+4 C) \sin [c+d x]}{5 d \sqrt{a+a \cos [c+d x]}} + \frac{2 a C \sqrt{a+a \cos [c+d x]} \sin [c+d x]}{5 d} + \frac{2 C (a+a \cos [c+d x])^{3/2} \sin [c+d x]}{5 d}$$

Result (type 3, 1617 leaves) :

$$\begin{aligned} & - \left( \left( \left( \frac{1}{8} - \frac{i}{8} \right) A (1 + e^{i c}) \left( \sqrt{2} - (1 - i) e^{\frac{i c}{2}} + (16 - 16 i) e^{\frac{3 i c}{2} + i d x} + (20 + 20 i) \sqrt{2} e^{2 i c + \frac{3 i d x}{2}} - (34 - 34 i) e^{\frac{5 i c}{2} + 2 i d x} - (20 + 20 i) \sqrt{2} e^{3 i c + \frac{5 i d x}{2}} + \right. \right. \\ & \quad (16 - 16 i) e^{\frac{7 i c}{2} + 3 i d x} + (4 + 4 i) \sqrt{2} e^{4 i c + \frac{7 i d x}{2}} - (1 - i) e^{\frac{9 i c}{2} + 4 i d x} + 8 i e^{\frac{1}{2} i (c+d x)} - 16 \sqrt{2} e^{i (c+d x)} - 40 i e^{\frac{3}{2} i (c+d x)} + 34 \sqrt{2} e^{2 i (c+d x)} + \\ & \quad \left. \left. 40 i e^{\frac{5}{2} i (c+d x)} - 16 \sqrt{2} e^{3 i (c+d x)} - 8 i e^{\frac{7}{2} i (c+d x)} + \sqrt{2} e^{4 i (c+d x)} - (4 + 4 i) \sqrt{2} e^{\frac{1}{2} i (2 c+d x)} \right) x (a (1 + \cos [c+d x]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^3 \right) / \\ & \quad \left( \left( (-1 - i) + \sqrt{2} e^{\frac{i c}{2}} \right) (-1 + e^{i c}) \left( i - 2 \sqrt{2} e^{\frac{1}{2} i (c+d x)} - 4 i e^{i (c+d x)} + 2 \sqrt{2} e^{\frac{3}{2} i (c+d x)} + i e^{2 i (c+d x)} \right)^2 \right) - \\ & \quad \frac{i A \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{d x}{4}\right] - \sin\left[\frac{c}{4} + \frac{d x}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{d x}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{d x}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{d x}{4}\right] - \sin\left[\frac{c}{4} + \frac{d x}{4}\right]}\right]}{2 \sqrt{2} d} (a (1 + \cos [c+d x]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^3 \\ & \quad \frac{i A \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{d x}{4}\right] + \sin\left[\frac{c}{4} + \frac{d x}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{d x}{4}\right]}{\cos\left[\frac{c}{4} + \frac{d x}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{d x}{4}\right] - \sin\left[\frac{c}{4} + \frac{d x}{4}\right]}\right]}{2 \sqrt{2} d} (a (1 + \cos [c+d x]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^3 \\ & \quad \frac{A (a (1 + \cos [c+d x]))^{3/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{d x}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^3}{4 \sqrt{2} d} \\ & \quad \frac{A (a (1 + \cos [c+d x]))^{3/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{d x}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^3}{4 \sqrt{2} d} + \\ & \quad \frac{(A + C) \cos\left[\frac{d x}{2}\right] (a (1 + \cos [c+d x]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^3 \sin\left[\frac{c}{2}\right]}{d} - \\ & \quad \frac{i A \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right] - i \left(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]\right) \tan\left[\frac{d x}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}\right]}{d \sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} (a (1 + \cos [c+d x]))^{3/2} \operatorname{Cot}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^3 \end{aligned}$$

$$\left( A (a (1 + \cos [c + dx]))^{3/2} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \right.$$

$$\left. - dx \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i (-\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right]) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) /$$

$$\left( \sqrt{2} d \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right) \right) + \frac{C \cos \left[ \frac{3dx}{2} \right] (a (1 + \cos [c + dx]))^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sin \left[ \frac{3c}{2} \right]}{4 d} +$$

$$\frac{C \cos \left[ \frac{5dx}{2} \right] (a (1 + \cos [c + dx]))^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sin \left[ \frac{5c}{2} \right]}{20 d} +$$

$$\frac{(A + C) \cos \left[ \frac{c}{2} \right] (a (1 + \cos [c + dx]))^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sin \left[ \frac{dx}{2} \right]}{d} +$$

$$\frac{C \cos \left[ \frac{3c}{2} \right] (a (1 + \cos [c + dx]))^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sin \left[ \frac{3dx}{2} \right]}{4 d} +$$

$$\frac{C \cos \left[ \frac{5c}{2} \right] (a (1 + \cos [c + dx]))^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sin \left[ \frac{5dx}{2} \right]}{20 d}$$

- **Problem 87: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + dx])^{3/2} (A + C \cos [c + dx]^2) \operatorname{Sec} [c + dx]^2 dx$$

Optimal (type 3, 136 leaves, 5 steps):

$$\frac{3 a^{3/2} A \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \sin [c+dx]}{\sqrt{a+a \cos [c+dx]}} \right]}{d} - \frac{a^2 (3 A - 8 C) \sin [c+dx]}{3 d \sqrt{a+a \cos [c+dx]}} - \frac{a (3 A - 2 C) \sqrt{a+a \cos [c+dx]} \sin [c+dx]}{3 d} + \frac{A (a + a \cos [c+dx])^{3/2} \tan [c+dx]}{d}$$

Result (type 3, 1658 leaves):

$$-\left( \left( \left( \frac{3}{16} - \frac{3 i}{16} \right) A (1 + e^{i c}) \left( \sqrt{2} - (1 - i) e^{\frac{i c}{2}} + (16 - 16 i) e^{\frac{3 i c}{2} + i dx} + (20 + 20 i) \sqrt{2} e^{2 i c + \frac{3 i dx}{2}} - (34 - 34 i) e^{\frac{5 i c}{2} + 2 i dx} - (20 + 20 i) \sqrt{2} e^{3 i c + \frac{5 i dx}{2}} + \right. \right. \right.$$

$$\left. \left. (16 - 16 i) e^{\frac{7 i c}{2} + 3 i dx} + (4 + 4 i) \sqrt{2} e^{4 i c + \frac{7 i dx}{2}} - (1 - i) e^{\frac{9 i c}{2} + 4 i dx} + 8 i e^{\frac{1}{2} i (c+dx)} - 16 \sqrt{2} e^{i (c+dx)} - 40 i e^{\frac{3}{2} i (c+dx)} + 34 \sqrt{2} e^{2 i (c+dx)} + \right. \right.$$

$$\begin{aligned}
& 40 i e^{\frac{5}{2} i (c+dx)} - 16 \sqrt{2} e^{3 i (c+dx)} - 8 i e^{\frac{7}{2} i (c+dx)} + \sqrt{2} e^{4 i (c+dx)} - (4 + 4 i) \sqrt{2} e^{\frac{1}{2} i (2c+dx)} \Big) x (a (1 + \cos [c + dx]))^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \Big) / \\
& \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2 \sqrt{2} e^{\frac{1}{2} i (c+dx)} - 4 i e^{i(c+dx)} + 2 \sqrt{2} e^{\frac{3}{2} i (c+dx)} + i e^{2 i (c+dx)} \right)^2 \right) \Big) - \\
& \frac{3 i A \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{-\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] (a (1 + \cos [c + dx]))^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{4 \sqrt{2} d} - \\
& \frac{3 i A \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] (a (1 + \cos [c + dx]))^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{4 \sqrt{2} d} - \\
& \frac{3 A (a (1 + \cos [c + dx]))^{3/2} \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{8 \sqrt{2} d} - \\
& \frac{3 A (a (1 + \cos [c + dx]))^{3/2} \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{8 \sqrt{2} d} + \\
& \frac{3 C \cos \left[ \frac{dx}{2} \right] (a (1 + \cos [c + dx]))^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sin \left[ \frac{c}{2} \right]}{2 d} - \\
& \frac{3 i A \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i (-\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right]) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] (a (1 + \cos [c + dx]))^{3/2} \cot \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{2 d \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} + \\
& \left( 3 A (a (1 + \cos [c + dx]))^{3/2} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \right. \\
& \left. \left( -dx \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right) \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i (-\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right]) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( 2 \sqrt{2} d \left( 4 \operatorname{Cos}\left[\frac{c}{2}\right]^2 + 4 \operatorname{Sin}\left[\frac{c}{2}\right]^2 \right) \right) + \frac{C \operatorname{Cos}\left[\frac{3dx}{2}\right] (a (1 + \operatorname{Cos}[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Sin}\left[\frac{3c}{2}\right]}{6 d} + \\
& \frac{3 C \operatorname{Cos}\left[\frac{c}{2}\right] (a (1 + \operatorname{Cos}[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Sin}\left[\frac{dx}{2}\right]}{2 d} + \\
& \frac{C \operatorname{Cos}\left[\frac{3c}{2}\right] (a (1 + \operatorname{Cos}[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Sin}\left[\frac{3dx}{2}\right]}{6 d} + \\
& \frac{A (a (1 + \operatorname{Cos}[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{4 d \left( \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right] \right)} - \\
& \frac{A (a (1 + \operatorname{Cos}[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{4 d \left( \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right] \right)}
\end{aligned}$$

- **Problem 88: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Cos}[c + dx])^{3/2} (A + C \operatorname{Cos}[c + dx]^2) \operatorname{Sec}[c + dx]^3 dx$$

Optimal (type 3, 147 leaves, 5 steps):

$$\begin{aligned}
& \frac{a^{3/2} (7 A + 8 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{4 d} - \frac{a^2 (5 A - 8 C) \operatorname{Sin}[c + dx]}{4 d \sqrt{a + a \operatorname{Cos}[c + dx]}} + \\
& \frac{3 a A \sqrt{a + a \operatorname{Cos}[c + dx]} \operatorname{Tan}[c + dx]}{4 d} + \frac{A (a + a \operatorname{Cos}[c + dx])^{3/2} \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2 d}
\end{aligned}$$

Result (type 3, 1028 leaves):

$$\begin{aligned}
& \frac{i(-7A - 8C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{16\sqrt{2}d} (a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 + \\
& \frac{i(-7A - 8C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{16\sqrt{2}d} (a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 + \\
& \frac{(7A + 8C)(a(1 + \cos[c + dx]))^{3/2} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{16\sqrt{2}d} + \\
& \frac{(-7A - 8C)(a(1 + \cos[c + dx]))^{3/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{32\sqrt{2}d} + \\
& \frac{(-7A - 8C)(a(1 + \cos[c + dx]))^{3/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{32\sqrt{2}d} + \\
& \frac{C \cos\left[\frac{dx}{2}\right] (a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{c}{2}\right]}{d} + \frac{C \cos\left[\frac{c}{2}\right] (a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{d} + \\
& \frac{A(a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{8d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \frac{A(a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(7 \cos\left[\frac{c}{2}\right] - 5 \sin\left[\frac{c}{2}\right]\right)}{16d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{A(a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{8d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} - \frac{A(a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(7 \cos\left[\frac{c}{2}\right] + 5 \sin\left[\frac{c}{2}\right]\right)}{16d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 89: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{3/2} (A + C \cos[c + dx])^2 \operatorname{Sec}[c + dx]^4 dx$$

Optimal (type 3, 155 leaves, 5 steps):

$$\frac{a^{3/2} (11A + 24C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right]}{8d} + \frac{a^2 (19A + 24C) \tan[c + dx]}{24d \sqrt{a + a \cos[c + dx]}} + \\
\frac{aA \sqrt{a + a \cos[c + dx]} \operatorname{Sec}[c + dx] \tan[c + dx]}{4d} + \frac{A(a + a \cos[c + dx])^{3/2} \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3d}$$

Result (type 3, 1106 leaves):



$$\begin{aligned}
& \frac{i(-11A - 24C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{32\sqrt{2}d} + \\
& \frac{i(-11A - 24C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{32\sqrt{2}d} + \\
& \frac{(11A + 24C) (a(1 + \cos[c + dx]))^{3/2} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{32\sqrt{2}d} + \\
& \frac{(-11A - 24C) (a(1 + \cos[c + dx]))^{3/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{64\sqrt{2}d} + \\
& \frac{(-11A - 24C) (a(1 + \cos[c + dx]))^{3/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{64\sqrt{2}d} + \\
& \frac{A (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{24d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \frac{3A (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{16d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{(a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(11A \cos\left[\frac{c}{2}\right] + 8C \cos\left[\frac{c}{2}\right] - 5A \sin\left[\frac{c}{2}\right] - 8C \sin\left[\frac{c}{2}\right]\right)}{32d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \\
& \frac{A (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{24d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \frac{3A (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{16d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{(a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(-11A \cos\left[\frac{c}{2}\right] - 8C \cos\left[\frac{c}{2}\right] - 5A \sin\left[\frac{c}{2}\right] - 8C \sin\left[\frac{c}{2}\right]\right)}{32d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 90: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{3/2} (A + C \cos[c + dx]^2) \sec[c + dx]^5 dx$$

Optimal (type 3, 200 leaves, 6 steps):

$$\begin{aligned}
& \frac{a^{3/2} (75A + 112C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right]}{64d} + \frac{a^2 (75A + 112C) \tan[c + dx]}{64d \sqrt{a + a \cos[c + dx]}} + \frac{a^2 (13A + 16C) \sec[c + dx] \tan[c + dx]}{32d \sqrt{a + a \cos[c + dx]}} + \\
& \frac{aA \sqrt{a + a \cos[c + dx]} \sec[c + dx]^2 \tan[c + dx]}{8d} + \frac{A (a + a \cos[c + dx])^{3/2} \sec[c + dx]^3 \tan[c + dx]}{4d}
\end{aligned}$$

Result (type 3, 1379 leaves) :

$$\begin{aligned}
& \frac{i (-75 A - 112 C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] (a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{256 \sqrt{2} d} + \\
& \frac{i (-75 A - 112 C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] (a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{256 \sqrt{2} d} + \\
& \frac{(75 A + 112 C) (a (1 + \cos[c + dx]))^{3/2} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{256 \sqrt{2} d} + \\
& \frac{(-75 A - 112 C) (a (1 + \cos[c + dx]))^{3/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{512 \sqrt{2} d} + \\
& \frac{(-75 A - 112 C) (a (1 + \cos[c + dx]))^{3/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{512 \sqrt{2} d} + \\
& \frac{A (a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{32 d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^4} + \frac{A (a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (5 \cos\left[\frac{c}{2}\right] - 3 \sin\left[\frac{c}{2}\right])}{64 d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^3} + \\
& \frac{(a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (19 A \sin\left[\frac{dx}{2}\right] + 16 C \sin\left[\frac{dx}{2}\right])}{128 d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \\
& \frac{(a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (75 A \cos\left[\frac{c}{2}\right] + 112 C \cos\left[\frac{c}{2}\right] - 37 A \sin\left[\frac{c}{2}\right] - 80 C \sin\left[\frac{c}{2}\right])}{256 d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])} + \\
& \frac{A (a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{32 d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^4} + \frac{(a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-5 A \cos\left[\frac{c}{2}\right] - 3 A \sin\left[\frac{c}{2}\right])}{64 d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^3} + \\
& \frac{(a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (19 A \sin\left[\frac{dx}{2}\right] + 16 C \sin\left[\frac{dx}{2}\right])}{128 d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \\
& \frac{(a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-75 A \cos\left[\frac{c}{2}\right] - 112 C \cos\left[\frac{c}{2}\right] - 37 A \sin\left[\frac{c}{2}\right] - 80 C \sin\left[\frac{c}{2}\right])}{256 d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])}
\end{aligned}$$

■ **Problem 91: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{3/2} (A + C \cos [c + d x]^2) \sec [c + d x]^6 dx$$

Optimal (type 3, 245 leaves, 7 steps):

$$\frac{a^{3/2} (133 A + 176 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+dx]}{\sqrt{a+a \cos [c+dx]}}\right]}{128 d} + \frac{a^2 (133 A + 176 C) \tan [c + d x]}{128 d \sqrt{a + a \cos [c + d x]}} + \frac{a^2 (133 A + 176 C) \sec [c + d x] \tan [c + d x]}{192 d \sqrt{a + a \cos [c + d x]}} +$$

$$\frac{a^2 (67 A + 80 C) \sec [c + d x]^2 \tan [c + d x]}{240 d \sqrt{a + a \cos [c + d x]}} + \frac{3 a A \sqrt{a + a \cos [c + d x]} \sec [c + d x]^3 \tan [c + d x]}{40 d} + \frac{A (a + a \cos [c + d x])^{3/2} \sec [c + d x]^4 \tan [c + d x]}{5 d}$$

Result (type 3, 1550 leaves):

$$\frac{i (-133 A - 176 C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{512 \sqrt{2} d} (a (1 + \cos [c + d x]))^{3/2} \sec \left[\frac{c}{2} + \frac{dx}{2}\right]^3 +$$

$$\frac{i (-133 A - 176 C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{512 \sqrt{2} d} (a (1 + \cos [c + d x]))^{3/2} \sec \left[\frac{c}{2} + \frac{dx}{2}\right]^3 +$$

$$\frac{(133 A + 176 C) (a (1 + \cos [c + d x]))^{3/2} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec \left[\frac{c}{2} + \frac{dx}{2}\right]^3}{512 \sqrt{2} d} +$$

$$\frac{(-133 A - 176 C) (a (1 + \cos [c + d x]))^{3/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec \left[\frac{c}{2} + \frac{dx}{2}\right]^3}{1024 \sqrt{2} d} +$$

$$\frac{(-133 A - 176 C) (a (1 + \cos [c + d x]))^{3/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec \left[\frac{c}{2} + \frac{dx}{2}\right]^3}{1024 \sqrt{2} d} +$$

$$\frac{A (a (1 + \cos [c + d x]))^{3/2} \sec \left[\frac{c}{2} + \frac{dx}{2}\right]^3}{80 d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^5} + \frac{3 A (a (1 + \cos [c + d x]))^{3/2} \sec \left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{64 d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^4} +$$

$$\frac{(a (1 + \cos [c + d x]))^{3/2} \sec \left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(29 A \cos\left[\frac{c}{2}\right] + 16 C \cos\left[\frac{c}{2}\right] - 11 A \sin\left[\frac{c}{2}\right] - 16 C \sin\left[\frac{c}{2}\right]\right)}{384 d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} +$$

$$\frac{(a (1 + \cos [c + d x]))^{3/2} \sec \left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(37 A \sin\left[\frac{dx}{2}\right] + 48 C \sin\left[\frac{dx}{2}\right]\right)}{256 d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} +$$

$$\frac{(a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(133A \cos\left[\frac{c}{2}\right] + 176C \cos\left[\frac{c}{2}\right] - 59A \sin\left[\frac{c}{2}\right] - 80C \sin\left[\frac{c}{2}\right]\right)}{512d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}$$

$$A \frac{(a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{80d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^5} + \frac{3A(a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{64d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^4} +$$

$$\frac{(a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(-29A \cos\left[\frac{c}{2}\right] - 16C \cos\left[\frac{c}{2}\right] - 11A \sin\left[\frac{c}{2}\right] - 16C \sin\left[\frac{c}{2}\right]\right)}{384d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} +$$

$$\frac{(a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(37A \sin\left[\frac{dx}{2}\right] + 48C \sin\left[\frac{dx}{2}\right]\right)}{256d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} +$$

$$\frac{(a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(-133A \cos\left[\frac{c}{2}\right] - 176C \cos\left[\frac{c}{2}\right] - 59A \sin\left[\frac{c}{2}\right] - 80C \sin\left[\frac{c}{2}\right]\right)}{512d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}$$

■ **Problem 95: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{5/2} (A + C \cos[c + dx])^2 \operatorname{Sec}[c + dx] dx$$

Optimal (type 3, 170 leaves, 6 steps):

$$\frac{2a^{5/2} A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} + \frac{2a^3 (49A + 32C) \sin[c+dx]}{21d \sqrt{a+a \cos[c+dx]}} +$$

$$\frac{2a^2 (7A + 8C) \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{21d} + \frac{2aC (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{7d} + \frac{2C (a+a \cos[c+dx])^{5/2} \sin[c+dx]}{7d}$$

Result (type 3, 1748 leaves):

$$-\left(\left(\left(\frac{1}{16} - \frac{i}{16}\right) A (1 + e^{ic}) \left(\sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} +\right.\right.\right.$$

$$\left.\left.\left(16 - 16i\right) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} +\right.\right.$$

$$\left.\left.40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)}\right) x (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5\right) /$$

$$\left(\left(\left(-1 - i\right) + \sqrt{2} e^{\frac{ic}{2}}\right) \left(-1 + e^{ic}\right) \left(i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)}\right)^2\right) -$$

$$i A \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5$$

$$4\sqrt{2} d$$

$$\frac{i A \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] (a (1 + \cos [c + dx]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{4 \sqrt{2} d} -$$

$$\frac{A (a (1 + \cos [c + dx]))^{5/2} \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{8 \sqrt{2} d} -$$

$$\frac{A (a (1 + \cos [c + dx]))^{5/2} \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{8 \sqrt{2} d} +$$

$$\frac{5 (4 A + 3 C) \cos \left[ \frac{dx}{2} \right] (a (1 + \cos [c + dx]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sin \left[ \frac{c}{2} \right]}{16 d} -$$

$$\frac{i A \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i (-\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right]) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] (a (1 + \cos [c + dx]))^{5/2} \operatorname{Cot} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{2 d \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} +$$

$$\left( A (a (1 + \cos [c + dx]))^{5/2} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \right)$$

$$\left( -dx \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right) \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i (-\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right]) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) /$$

$$\left( 2 \sqrt{2} d \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right) \right) + \frac{(4 A + 11 C) \cos \left[ \frac{3 dx}{2} \right] (a (1 + \cos [c + dx]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sin \left[ \frac{3 c}{2} \right]}{48 d} +$$

$$\frac{C \cos \left[ \frac{5 dx}{2} \right] (a (1 + \cos [c + dx]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sin \left[ \frac{5 c}{2} \right]}{16 d} +$$

$$\frac{C \cos \left[ \frac{7 dx}{2} \right] (a (1 + \cos [c + dx]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sin \left[ \frac{7 c}{2} \right]}{112 d} +$$

$$\frac{5(4A + 3C) \cos\left[\frac{c}{2}\right] (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{16d} +$$

$$\frac{(4A + 11C) \cos\left[\frac{3c}{2}\right] (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{3dx}{2}\right]}{48d} +$$

$$\frac{C \cos\left[\frac{5c}{2}\right] (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{5dx}{2}\right]}{16d} +$$

$$\frac{C \cos\left[\frac{7c}{2}\right] (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{7dx}{2}\right]}{112d}$$

- **Problem 96: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{5/2} (A + C \cos[c + dx])^2 \sec[c + dx]^2 dx$$

Optimal (type 3, 173 leaves, 6 steps):

$$\frac{5a^{5/2} A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} + \frac{a^3 (15A + 64C) \sin[c+dx]}{15d \sqrt{a+a \cos[c+dx]}} - \frac{a^2 (15A - 16C) \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{15d} -$$

$$\frac{a(5A - 2C) (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{5d} + \frac{A(a+a \cos[c+dx])^{5/2} \tan[c+dx]}{d}$$

Result (type 3, 1770 leaves):

$$- \left( \left( \left( \frac{5}{32} - \frac{5i}{32} \right) A (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + \right. \right. \right.$$

$$\left. \left. (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + \right. \right.$$

$$\left. \left. 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \Big/$$

$$\left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right) \Big)$$

$$- \frac{5i A \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{8\sqrt{2}d} (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5$$

$$- \frac{5i A \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{8\sqrt{2}d} (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5$$

$$\frac{5 A (a (1 + \cos [c + d x]))^{5/2} \log \left[ 2 - \sqrt{2} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5}{16 \sqrt{2} d}$$

$$\frac{5 A (a (1 + \cos [c + d x]))^{5/2} \log \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5}{16 \sqrt{2} d} +$$

$$\frac{(2 A + 5 C) \cos \left[ \frac{d x}{2} \right] (a (1 + \cos [c + d x]))^{5/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sin \left[ \frac{c}{2} \right]}{4 d} -$$

$$\frac{5 i A \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \tan \left[ \frac{d x}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] (a (1 + \cos [c + d x]))^{5/2} \cot \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5}{4 d \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} +$$

$$\left( 5 A (a (1 + \cos [c + d x]))^{5/2} \operatorname{Csc} \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \right)$$

$$\left( -d x \cos \left[ \frac{c}{2} \right] + 2 \log \left[ \sqrt{2} + 2 \cos \left[ \frac{d x}{2} \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \tan \left[ \frac{d x}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) /$$

$$\left( 4 \sqrt{2} d \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right) \right) + \frac{5 C \cos \left[ \frac{3 d x}{2} \right] (a (1 + \cos [c + d x]))^{5/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sin \left[ \frac{3 c}{2} \right]}{24 d} +$$

$$\frac{C \cos \left[ \frac{5 d x}{2} \right] (a (1 + \cos [c + d x]))^{5/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sin \left[ \frac{5 c}{2} \right]}{40 d} +$$

$$\frac{(2 A + 5 C) \cos \left[ \frac{c}{2} \right] (a (1 + \cos [c + d x]))^{5/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sin \left[ \frac{d x}{2} \right]}{4 d} +$$

$$\frac{5 C \cos \left[ \frac{3 c}{2} \right] (a (1 + \cos [c + d x]))^{5/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sin \left[ \frac{3 d x}{2} \right]}{24 d} +$$

$$\frac{C \cos\left[\frac{5c}{2}\right] (a (1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{5dx}{2}\right]}{40 d} +$$

$$\frac{A (a (1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8 d (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])} - \frac{A (a (1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8 d (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])}$$

- **Problem 97: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{5/2} (A + C \cos[c + dx]^2) \sec[c + dx]^3 dx$$

Optimal (type 3, 184 leaves, 6 steps):

$$\frac{a^{5/2} (19 A + 8 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{4 d} - \frac{a^3 (27 A - 56 C) \sin[c + dx]}{12 d \sqrt{a + a \cos[c + dx]}} - \frac{a^2 (21 A - 8 C) \sqrt{a + a \cos[c + dx]} \sin[c + dx]}{12 d} +$$

$$\frac{5 a A (a + a \cos[c + dx])^{3/2} \tan[c + dx]}{4 d} + \frac{A (a + a \cos[c + dx])^{5/2} \sec[c + dx] \tan[c + dx]}{2 d}$$

Result (type 3, 1134 leaves):



$$\begin{aligned}
& \frac{i(-19A - 8C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{32\sqrt{2}d} (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 + \\
& \frac{i(-19A - 8C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right]}{32\sqrt{2}d} (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 + \\
& \frac{(19A + 8C) (a(1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{32\sqrt{2}d} + \\
& \frac{(-19A - 8C) (a(1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{64\sqrt{2}d} + \\
& \frac{(-19A - 8C) (a(1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{64\sqrt{2}d} + \\
& \frac{5C \cos\left[\frac{dx}{2}\right] (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{c}{2}\right]}{4d} + \frac{C \cos\left[\frac{3dx}{2}\right] (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{3c}{2}\right]}{12d} + \\
& \frac{5C \cos\left[\frac{c}{2}\right] (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{4d} + \frac{C \cos\left[\frac{3c}{2}\right] (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{3dx}{2}\right]}{12d} + \\
& \frac{A (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{16d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \frac{A (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (11 \cos\left[\frac{c}{2}\right] - 9 \sin\left[\frac{c}{2}\right])}{32d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])} + \\
& \frac{A (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{16d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2} - \frac{A (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (11 \cos\left[\frac{c}{2}\right] + 9 \sin\left[\frac{c}{2}\right])}{32d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])}
\end{aligned}$$

■ **Problem 98: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{5/2} (A + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^4 dx$$

Optimal (type 3, 192 leaves, 6 steps):

$$\begin{aligned}
& \frac{5a^{5/2} (5A + 8C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{8d} - \frac{a^3 (49A - 24C) \sin[c + dx]}{24d \sqrt{a + a \cos[c + dx]}} + \frac{a^2 (31A + 24C) \sqrt{a + a \cos[c + dx]} \operatorname{Tan}[c + dx]}{24d} + \\
& \frac{5aA (a + a \cos[c + dx])^{3/2} \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{12d} + \frac{A (a + a \cos[c + dx])^{5/2} \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{3d}
\end{aligned}$$

Result (type 3, 1206 leaves):

$$\begin{aligned}
& \frac{5 i (5 A + 8 C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]-\sqrt{2} \sin\left[\frac{c}{4}+\frac{dx}{4}\right]}{-\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sqrt{2} \cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]}\right] (a (1+\cos [c+d x]))^{5 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^5}{64 \sqrt{2} d} \\
& - \frac{5 i (5 A + 8 C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sin\left[\frac{c}{4}+\frac{dx}{4}\right]-\sqrt{2} \sin\left[\frac{c}{4}+\frac{dx}{4}\right]}{\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sqrt{2} \cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]}\right] (a (1+\cos [c+d x]))^{5 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^5}{64 \sqrt{2} d} + \\
& \frac{5 (5 A + 8 C) (a (1+\cos [c+d x]))^{5 / 2} \operatorname{Log}\left[\sqrt{2}+2 \sin\left[\frac{c}{2}+\frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^5}{64 \sqrt{2} d} - \\
& \frac{5 (5 A + 8 C) (a (1+\cos [c+d x]))^{5 / 2} \operatorname{Log}\left[2-\sqrt{2} \cos\left[\frac{c}{2}+\frac{d x}{2}\right]-\sqrt{2} \sin\left[\frac{c}{2}+\frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^5}{128 \sqrt{2} d} - \\
& \frac{5 (5 A + 8 C) (a (1+\cos [c+d x]))^{5 / 2} \operatorname{Log}\left[2+\sqrt{2} \cos\left[\frac{c}{2}+\frac{d x}{2}\right]-\sqrt{2} \sin\left[\frac{c}{2}+\frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^5}{128 \sqrt{2} d} + \\
& \frac{C \cos\left[\frac{d x}{2}\right] (a (1+\cos [c+d x]))^{5 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^5 \sin\left[\frac{c}{2}\right]}{2 d} + \frac{C \cos\left[\frac{c}{2}\right] (a (1+\cos [c+d x]))^{5 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^5 \sin\left[\frac{d x}{2}\right]}{2 d} + \\
& \frac{A (a (1+\cos [c+d x]))^{5 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^5}{48 d\left(\cos\left[\frac{c}{2}+\frac{d x}{2}\right]-\sin\left[\frac{c}{2}+\frac{d x}{2}\right]\right)^3} + \frac{5 A (a (1+\cos [c+d x]))^{5 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^5 \sin\left[\frac{d x}{2}\right]}{32 d\left(\cos\left[\frac{c}{2}\right]-\sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{c}{2}+\frac{d x}{2}\right]-\sin\left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2} + \\
& \frac{(a (1+\cos [c+d x]))^{5 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^5\left(25 A \cos\left[\frac{c}{2}\right]+8 C \cos\left[\frac{c}{2}\right]-15 A \sin\left[\frac{c}{2}\right]-8 C \sin\left[\frac{c}{2}\right]\right)}{64 d\left(\cos\left[\frac{c}{2}\right]-\sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{c}{2}+\frac{d x}{2}\right]-\sin\left[\frac{c}{2}+\frac{d x}{2}\right]\right)} - \\
& \frac{A (a (1+\cos [c+d x]))^{5 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^5}{48 d\left(\cos\left[\frac{c}{2}+\frac{d x}{2}\right]+\sin\left[\frac{c}{2}+\frac{d x}{2}\right]\right)^3} + \frac{5 A (a (1+\cos [c+d x]))^{5 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^5 \sin\left[\frac{d x}{2}\right]}{32 d\left(\cos\left[\frac{c}{2}\right]+\sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{c}{2}+\frac{d x}{2}\right]+\sin\left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2} + \\
& \frac{(a (1+\cos [c+d x]))^{5 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^5\left(-25 A \cos\left[\frac{c}{2}\right]-8 C \cos\left[\frac{c}{2}\right]-15 A \sin\left[\frac{c}{2}\right]-8 C \sin\left[\frac{c}{2}\right]\right)}{64 d\left(\cos\left[\frac{c}{2}\right]+\sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{c}{2}+\frac{d x}{2}\right]+\sin\left[\frac{c}{2}+\frac{d x}{2}\right]\right)}
\end{aligned}$$

- **Problem 99: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c+d x])^{5 / 2} (A + C \cos [c+d x])^2 \operatorname{Sec}[c+d x]^5 dx$$

Optimal (type 3, 200 leaves, 6 steps):

$$\begin{aligned}
& \frac{a^{5/2} (163 A + 304 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{64 d} + \frac{a^3 (299 A + 432 C) \tan[c+dx]}{192 d \sqrt{a+a \cos[c+dx]}} + \frac{a^2 (17 A + 16 C) \sqrt{a+a \cos[c+dx]} \sec[c+dx] \tan[c+dx]}{32 d} + \\
& \frac{5 a A (a+a \cos[c+dx])^{3/2} \sec[c+dx]^2 \tan[c+dx]}{24 d} + \frac{A (a+a \cos[c+dx])^{5/2} \sec[c+dx]^3 \tan[c+dx]}{4 d}
\end{aligned}$$

Result (type 3, 1379 leaves):

$$\begin{aligned}
& \frac{i(-163A - 304C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{512\sqrt{2}d} + \\
& \frac{i(-163A - 304C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{512\sqrt{2}d} + \\
& \frac{(163A + 304C) (a(1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{512\sqrt{2}d} + \\
& \frac{(-163A - 304C) (a(1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{1024\sqrt{2}d} + \\
& \frac{(-163A - 304C) (a(1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{1024\sqrt{2}d} + \\
& \frac{A (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{64d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^4} + \frac{A (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (23 \cos\left[\frac{c}{2}\right] - 17 \sin\left[\frac{c}{2}\right])}{384d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^3} + \\
& \frac{(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (43A \sin\left[\frac{dx}{2}\right] + 16C \sin\left[\frac{dx}{2}\right])}{256d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \\
& \frac{(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (163A \cos\left[\frac{c}{2}\right] + 176C \cos\left[\frac{c}{2}\right] - 77A \sin\left[\frac{c}{2}\right] - 144C \sin\left[\frac{c}{2}\right])}{512d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])} + \\
& \frac{A (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{64d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^4} + \frac{(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-23A \cos\left[\frac{c}{2}\right] - 17A \sin\left[\frac{c}{2}\right])}{384d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^3} + \\
& \frac{(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (43A \sin\left[\frac{dx}{2}\right] + 16C \sin\left[\frac{dx}{2}\right])}{256d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \\
& \frac{(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-163A \cos\left[\frac{c}{2}\right] - 176C \cos\left[\frac{c}{2}\right] - 77A \sin\left[\frac{c}{2}\right] - 144C \sin\left[\frac{c}{2}\right])}{512d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])}
\end{aligned}$$

■ **Problem 100: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{5/2} (A + C \cos[c + dx]^2) \sec[c + dx]^6 dx$$

Optimal (type 3, 245 leaves, 7 steps) :

$$\frac{a^{5/2} (283 A + 400 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{128 d} + \frac{a^3 (283 A + 400 C) \tan[c+dx]}{128 d \sqrt{a+a \cos[c+dx]}} +$$

$$\frac{a^3 (787 A + 1040 C) \sec[c+dx] \tan[c+dx]}{960 d \sqrt{a+a \cos[c+dx]}} + \frac{a^2 (79 A + 80 C) \sqrt{a+a \cos[c+dx]} \sec[c+dx]^2 \tan[c+dx]}{240 d} +$$

$$\frac{a A (a+a \cos[c+dx])^{3/2} \sec[c+dx]^3 \tan[c+dx]}{8 d} + \frac{A (a+a \cos[c+dx])^{5/2} \sec[c+dx]^4 \tan[c+dx]}{5 d}$$

Result (type 3, 1550 leaves) :

$$\frac{i (-283 A - 400 C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]-\sqrt{2} \sin\left[\frac{c}{4}+\frac{dx}{4}\right]}{-\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sqrt{2} \cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]}\right]}{1024 \sqrt{2} d} +$$

$$\frac{i (-283 A - 400 C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sin\left[\frac{c}{4}+\frac{dx}{4}\right]-\sqrt{2} \sin\left[\frac{c}{4}+\frac{dx}{4}\right]}{\cos\left[\frac{c}{4}+\frac{dx}{4}\right]+\sqrt{2} \cos\left[\frac{c}{4}+\frac{dx}{4}\right]-\sin\left[\frac{c}{4}+\frac{dx}{4}\right]}\right]}{1024 \sqrt{2} d} +$$

$$\frac{(283 A + 400 C) (a (1 + \cos[c+dx]))^{5/2} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{1024 \sqrt{2} d} +$$

$$\frac{(-283 A - 400 C) (a (1 + \cos[c+dx]))^{5/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{2048 \sqrt{2} d} +$$

$$\frac{(-283 A - 400 C) (a (1 + \cos[c+dx]))^{5/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{2048 \sqrt{2} d} +$$

$$\frac{A (a (1 + \cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{160 d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^5} + \frac{5 A (a (1 + \cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{128 d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^4} +$$

$$\frac{(a (1 + \cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (59 A \cos\left[\frac{c}{2}\right] + 16 C \cos\left[\frac{c}{2}\right] - 29 A \sin\left[\frac{c}{2}\right] - 16 C \sin\left[\frac{c}{2}\right])}{768 d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} +$$

$$\frac{5 (a (1 + \cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (15 A \sin\left[\frac{dx}{2}\right] + 16 C \sin\left[\frac{dx}{2}\right])}{512 d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} +$$

$$\frac{(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (283A \cos\left[\frac{c}{2}\right] + 400C \cos\left[\frac{c}{2}\right] - 133A \sin\left[\frac{c}{2}\right] - 240C \sin\left[\frac{c}{2}\right])}{1024d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])}$$

$$+ \frac{A(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{160d (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^5} + \frac{5A(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{128d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^4}$$

$$+ \frac{(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-59A \cos\left[\frac{c}{2}\right] - 16C \cos\left[\frac{c}{2}\right] - 29A \sin\left[\frac{c}{2}\right] - 16C \sin\left[\frac{c}{2}\right])}{768d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^3}$$

$$+ \frac{5(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (15A \sin\left[\frac{dx}{2}\right] + 16C \sin\left[\frac{dx}{2}\right])}{512d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2}$$

$$+ \frac{(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-283A \cos\left[\frac{c}{2}\right] - 400C \cos\left[\frac{c}{2}\right] - 133A \sin\left[\frac{c}{2}\right] - 240C \sin\left[\frac{c}{2}\right])}{1024d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])}$$

■ **Problem 101: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{5/2} (A + C \cos[c + dx])^2 \sec[c + dx]^7 dx$$

Optimal (type 3, 290 leaves, 8 steps):

$$\frac{a^{5/2} (1015A + 1304C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{512d} + \frac{a^3 (1015A + 1304C) \tan[c + dx]}{512d \sqrt{a + a \cos[c + dx]}} + \frac{a^3 (1015A + 1304C) \sec[c + dx] \tan[c + dx]}{768d \sqrt{a + a \cos[c + dx]}}$$

$$+ \frac{a^3 (109A + 136C) \sec[c + dx]^2 \tan[c + dx]}{192d \sqrt{a + a \cos[c + dx]}} + \frac{a^2 (23A + 24C) \sqrt{a + a \cos[c + dx]} \sec[c + dx]^3 \tan[c + dx]}{96d}$$

$$+ \frac{aA(a + a \cos[c + dx])^{3/2} \sec[c + dx]^4 \tan[c + dx]}{12d} + \frac{AA(a + a \cos[c + dx])^{5/2} \sec[c + dx]^5 \tan[c + dx]}{6d}$$

Result (type 3, 791 leaves):

$$\begin{aligned}
& \frac{i(-1015A - 1304C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{4096\sqrt{2}d} + \\
& \frac{i(-1015A - 1304C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{4096\sqrt{2}d} + \\
& \frac{(1015A + 1304C) (a(1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{4096\sqrt{2}d} + \\
& \frac{(-1015A - 1304C) (a(1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8192\sqrt{2}d} + \\
& \frac{(-1015A - 1304C) (a(1 + \cos[c + dx]))^{5/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8192\sqrt{2}d} + \\
& \frac{1}{196608d} (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Sec}[c + dx]^6 \left(-9450A \sin\left[\frac{c}{2} + \frac{dx}{2}\right] - 19344C \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right) + \\
& 36898A \sin\left[\frac{3c}{2} + \frac{3dx}{2}\right] + 32848C \sin\left[\frac{3c}{2} + \frac{3dx}{2}\right] + 5655A \sin\left[\frac{5c}{2} + \frac{5dx}{2}\right] - 1512C \sin\left[\frac{5c}{2} + \frac{5dx}{2}\right] + 17661A \sin\left[\frac{7c}{2} + \frac{7dx}{2}\right] + \\
& 20232C \sin\left[\frac{7c}{2} + \frac{7dx}{2}\right] + 1015A \sin\left[\frac{9c}{2} + \frac{9dx}{2}\right] + 1304C \sin\left[\frac{9c}{2} + \frac{9dx}{2}\right] + 3045A \sin\left[\frac{11c}{2} + \frac{11dx}{2}\right] + 3912C \sin\left[\frac{11c}{2} + \frac{11dx}{2}\right]
\end{aligned}$$

■ **Problem 106: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos[c + dx])^2 \operatorname{Sec}[c + dx]}{\sqrt{a + a \cos[c + dx]}} dx$$

Optimal (type 3, 115 leaves, 6 steps):

$$\frac{2A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right]}{\sqrt{a}d} - \frac{\sqrt{2}(A + C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{2} \sqrt{a + a \cos[c + dx]}}\right]}{\sqrt{a}d} + \frac{2C \sin[c + dx]}{d \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 2199 leaves):

$$\begin{aligned}
& - \left( (1 - i) A (1 + e^{ic}) \right. \\
& \left. \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + (16 - 16i) e^{\frac{7ic}{2} + 3idx} + \right. \right. \\
& \left. \left. (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c + dx)} - 16\sqrt{2} e^{i(c + dx)} - 40i e^{\frac{3}{2}i(c + dx)} + 34\sqrt{2} e^{2i(c + dx)} + 40i e^{\frac{5}{2}i(c + dx)} - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left( 16 \sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4+4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] (C \cos[c+dx] + A \sec[c+dx]) \right) / \\
& \left( \left( (-1-i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right. \\
& \left. \sqrt{a(1+\cos[c+dx])} (2A+C+C\cos[2c+2dx]) \right) - \\
& \left( 2i\sqrt{2} A \operatorname{ArcTan}\left[ \frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] (C \cos[c+dx] + A \sec[c+dx]) \right) / \\
& \left( d \sqrt{a(1+\cos[c+dx])} (2A+C+C\cos[2c+2dx]) \right) + \\
& \frac{4(A+C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C \cos[c+dx] + A \sec[c+dx])}{d \sqrt{a(1+\cos[c+dx])} (2A+C+C\cos[2c+2dx])} - \\
& \frac{4(A+C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C \cos[c+dx] + A \sec[c+dx])}{d \sqrt{a(1+\cos[c+dx])} (2A+C+C\cos[2c+2dx])} - \\
& \left( \sqrt{2} A \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C \cos[c+dx] + A \sec[c+dx]) \right) / \\
& \left( d \sqrt{a(1+\cos[c+dx])} (2A+C+C\cos[2c+2dx]) \right) + \\
& \frac{8C \cos\left[\frac{dx}{2}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] (C \cos[c+dx] + A \sec[c+dx]) \sin\left[\frac{c}{2}\right]}{d \sqrt{a(1+\cos[c+dx])} (2A+C+C\cos[2c+2dx])} + \\
& \left( (1-i) \operatorname{ArcTan}\left[ \frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] (C \cos[c+dx] + A \sec[c+dx]) \right. \\
& \left. \left( (1+i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1-i) \sin\left[\frac{c}{4}\right] - i\sqrt{2} \sin\left[\frac{c}{4}\right] \right) \left( (-1-i) A \cos\left[\frac{c}{4}\right] + \sqrt{2} A \cos\left[\frac{c}{4}\right] + (1-i) A \sin\left[\frac{c}{4}\right] - i\sqrt{2} A \sin\left[\frac{c}{4}\right] \right) \right) / \\
& \left( \sqrt{2} d \sqrt{a(1+\cos[c+dx])} (2A+C+C\cos[2c+2dx]) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right) - \\
& \left( \left( \frac{1}{2} + \frac{i}{2} \right) \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C \cos[c+dx] + A \sec[c+dx]) \right. \\
& \left. \left( (1+i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1-i) \sin\left[\frac{c}{4}\right] - i\sqrt{2} \sin\left[\frac{c}{4}\right] \right) \left( (-1-i) A \cos\left[\frac{c}{4}\right] + \sqrt{2} A \cos\left[\frac{c}{4}\right] + (1-i) A \sin\left[\frac{c}{4}\right] - i\sqrt{2} A \sin\left[\frac{c}{4}\right] \right) \right) / \\
& \left( \sqrt{2} d \sqrt{a(1+\cos[c+dx])} (2A+C+C\cos[2c+2dx]) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right) -
\end{aligned}$$



$$\left( 8 i A \operatorname{ArcTan} \left[ \frac{2 i \operatorname{Cos} \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \operatorname{Sin} \left[ \frac{c}{2} \right] \right) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \operatorname{Cos} \left[ \frac{c}{2} \right]^2 + 4 \operatorname{Sin} \left[ \frac{c}{2} \right]^2}} \right] \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Cos} [c + dx] \operatorname{Cot} \left[ \frac{c}{2} \right] (C \operatorname{Cos} [c + dx] + A \operatorname{Sec} [c + dx]) \right) /$$

$$\left( d \sqrt{a (1 + \operatorname{Cos} [c + dx])} (2 A + C + C \operatorname{Cos} [2 c + 2 dx]) \sqrt{-2 + 4 \operatorname{Cos} \left[ \frac{c}{2} \right]^2 + 4 \operatorname{Sin} \left[ \frac{c}{2} \right]^2} \right) +$$

$$\left( 4 \sqrt{2} A \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Cos} [c + dx] \operatorname{Csc} \left[ \frac{c}{2} \right] (C \operatorname{Cos} [c + dx] + A \operatorname{Sec} [c + dx]) \right)$$

$$\left( -dx \operatorname{Cos} \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \operatorname{Cos} \left[ \frac{dx}{2} \right] \operatorname{Sin} \left[ \frac{c}{2} \right] + 2 \operatorname{Cos} \left[ \frac{c}{2} \right] \operatorname{Sin} \left[ \frac{dx}{2} \right] \right] \operatorname{Sin} \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \operatorname{Cos} \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \operatorname{Sin} \left[ \frac{c}{2} \right] \right) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \operatorname{Cos} \left[ \frac{c}{2} \right]^2 + 4 \operatorname{Sin} \left[ \frac{c}{2} \right]^2}} \right] \operatorname{Cos} \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \operatorname{Cos} \left[ \frac{c}{2} \right]^2 + 4 \operatorname{Sin} \left[ \frac{c}{2} \right]^2}} \right) /$$

$$\left( d \sqrt{a (1 + \operatorname{Cos} [c + dx])} (2 A + C + C \operatorname{Cos} [2 c + 2 dx]) \left( 4 \operatorname{Cos} \left[ \frac{c}{2} \right]^2 + 4 \operatorname{Sin} \left[ \frac{c}{2} \right]^2 \right) \right) +$$

$$\frac{8 C \operatorname{Cos} \left[ \frac{c}{2} \right] \operatorname{Cos} \left[ \frac{c}{2} + \frac{dx}{2} \right] \operatorname{Cos} [c + dx] (C \operatorname{Cos} [c + dx] + A \operatorname{Sec} [c + dx]) \operatorname{Sin} \left[ \frac{dx}{2} \right]}{d \sqrt{a (1 + \operatorname{Cos} [c + dx])} (2 A + C + C \operatorname{Cos} [2 c + 2 dx])}$$

■ **Problem 107: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \operatorname{Cos} [c + dx])^2 \operatorname{Sec} [c + dx]^2}{\sqrt{a + a \operatorname{Cos} [c + dx]}} dx$$

Optimal (type 3, 113 leaves, 6 steps):

$$-\frac{A \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \operatorname{Sin} [c + dx]}{\sqrt{a + a \operatorname{Cos} [c + dx]}} \right]}{\sqrt{a} d} + \frac{\sqrt{2} (A + C) \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \operatorname{Sin} [c + dx]}{\sqrt{2} \sqrt{a + a \operatorname{Cos} [c + dx]}} \right]}{\sqrt{a} d} + \frac{A \operatorname{Tan} [c + dx]}{d \sqrt{a + a \operatorname{Cos} [c + dx]}}$$

Result (type 3, 1968 leaves):

$$\left( \left( \frac{1}{2} - \frac{i}{2} \right) A (1 + e^{ic}) \right.$$

$$\left. \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16 i) e^{\frac{3ic}{2} + idx} + (20 + 20 i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34 i) e^{\frac{5ic}{2} + 2idx} - (20 + 20 i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + (16 - 16 i) e^{\frac{7ic}{2} + 3idx} + \right. \right.$$

$$\begin{aligned}
& (4 + 4i) \sqrt{2} e^{4i c + \frac{7i dx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4i dx} + 8i e^{\frac{1}{2} i (c+dx)} - 16 \sqrt{2} e^{i (c+dx)} - 40i e^{\frac{3}{2} i (c+dx)} + 34 \sqrt{2} e^{2i (c+dx)} + 40i e^{\frac{5}{2} i (c+dx)} - \\
& 16 \sqrt{2} e^{3i (c+dx)} - 8i e^{\frac{7}{2} i (c+dx)} + \sqrt{2} e^{4i (c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2} i (2c+dx)} \Big) x \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx]^2 (C + A \sec [c + dx]^2) \Big) / \\
& \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2 \sqrt{2} e^{\frac{1}{2} i (c+dx)} - 4i e^{i (c+dx)} + 2 \sqrt{2} e^{\frac{3}{2} i (c+dx)} + i e^{2i (c+dx)} \right)^2 \right. \\
& \left. \sqrt{a (1 + \cos [c + dx])} (2A + C + C \cos [2c + 2dx]) \right) + \\
& \frac{i \sqrt{2} A \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{-\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx]^2 (C + A \sec [c + dx]^2)}{d \sqrt{a (1 + \cos [c + dx])} (2A + C + C \cos [2c + 2dx])} + \\
& \frac{i \sqrt{2} A \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx]^2 (C + A \sec [c + dx]^2)}{d \sqrt{a (1 + \cos [c + dx])} (2A + C + C \cos [2c + 2dx])} - \\
& \frac{4 (A + C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx]^2 \operatorname{Log} \left[ \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] \right] (C + A \sec [c + dx]^2)}{d \sqrt{a (1 + \cos [c + dx])} (2A + C + C \cos [2c + 2dx])} + \\
& \frac{4 (A + C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx]^2 \operatorname{Log} \left[ \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] \right] (C + A \sec [c + dx]^2)}{d \sqrt{a (1 + \cos [c + dx])} (2A + C + C \cos [2c + 2dx])} + \\
& \frac{A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx]^2 \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] (C + A \sec [c + dx]^2)}{\sqrt{2} d \sqrt{a (1 + \cos [c + dx])} (2A + C + C \cos [2c + 2dx])} + \\
& \frac{A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx]^2 \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] (C + A \sec [c + dx]^2)}{\sqrt{2} d \sqrt{a (1 + \cos [c + dx])} (2A + C + C \cos [2c + 2dx])} + \\
& \frac{4i A \operatorname{ArcTan} \left[ \frac{2i \cos \left[ \frac{c}{2} \right] - i (-\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right]) \operatorname{Tan} \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx]^2 \operatorname{Cot} \left[ \frac{c}{2} \right] (C + A \sec [c + dx]^2)}{d \sqrt{a (1 + \cos [c + dx])} (2A + C + C \cos [2c + 2dx]) \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} - \\
& \left( 2 \sqrt{2} A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx]^2 \operatorname{Csc} \left[ \frac{c}{2} \right] (C + A \sec [c + dx]^2) \right)
\end{aligned}$$

$$\left( -dx \cos\left[\frac{c}{2}\right] + 2 \log\left[\sqrt{2} + 2 \cos\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] \right) \sin\left[\frac{c}{2}\right] + \frac{4i\sqrt{2} \operatorname{ArcTan}\left[\frac{2i\cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2\sin\left[\frac{c}{2}\right]) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}} \right) /$$

$$\frac{\left(d\sqrt{a(1+\cos[c+dx])} (2A+C+C\cos[2c+2dx]) \left(4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2\right) + 2A\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx]^2 (C+A\sec[c+dx]^2)\right)}{d\sqrt{a(1+\cos[c+dx])} (2A+C+C\cos[2c+2dx]) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right) - 2A\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx]^2 (C+A\sec[c+dx]^2)}$$

$$\frac{2A\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx]^2 (C+A\sec[c+dx]^2)}{d\sqrt{a(1+\cos[c+dx])} (2A+C+C\cos[2c+2dx]) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}$$

■ **Problem 108: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A+C\cos[c+dx])^2 \sec[c+dx]^3}{\sqrt{a+a\cos[c+dx]}} dx$$

Optimal (type 3, 159 leaves, 7 steps):

$$\frac{(7A+8C) \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{4\sqrt{a}d} - \frac{\sqrt{2}(A+C) \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{2}\sqrt{a+a\cos[c+dx]}}\right]}{\sqrt{a}d} - \frac{A \tan[c+dx]}{4d\sqrt{a+a\cos[c+dx]}} + \frac{A \sec[c+dx] \tan[c+dx]}{2d\sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 1608 leaves):

$$\frac{4(A+C)\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx]^2 \log\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C+A\sec[c+dx]^2)}{d\sqrt{a(1+\cos[c+dx])} (2A+C+C\cos[2c+2dx])} -$$

$$\frac{4(A+C)\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx]^2 \log\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C+A\sec[c+dx]^2)}{d\sqrt{a(1+\cos[c+dx])} (2A+C+C\cos[2c+2dx])} +$$

$$\frac{(7A+8C)\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx]^2 \log\left[\sqrt{2} + 2\sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C+A\sec[c+dx]^2)}{2\sqrt{2}d\sqrt{a(1+\cos[c+dx])} (2A+C+C\cos[2c+2dx])} +$$

$$\left(i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2}\sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2}\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx]^2 (C+A\sec[c+dx]^2)\right)$$

$$\left(7\sqrt{2}A + 8\sqrt{2}C - 14A\sin\left[\frac{c}{2}\right] - 16C\sin\left[\frac{c}{2}\right]\right) / \left(d\sqrt{a(1+\cos[c+dx])} (2A+C+C\cos[2c+2dx]) \left(-1 + \sqrt{2}\sin\left[\frac{c}{2}\right]\right)\right) +$$

$$\begin{aligned}
& \left( i \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx]^2 (C + A \operatorname{Sec} [c + dx]^2) \right. \\
& \quad \left. \left( 7 \sqrt{2} A + 8 \sqrt{2} C - 14 A \sin \left[ \frac{c}{2} \right] - 16 C \sin \left[ \frac{c}{2} \right] \right) \right] / \left( 4 d \sqrt{a (1 + \cos [c + dx])} (2 A + C + C \cos [2 c + 2 dx]) (-1 + \sqrt{2} \sin \left[ \frac{c}{2} \right]) \right) + \\
& \quad \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx]^2 \log \left[ 2 - \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] (C + A \operatorname{Sec} [c + dx]^2) \left( 7 \sqrt{2} A + 8 \sqrt{2} C - 14 A \sin \left[ \frac{c}{2} \right] - 16 C \sin \left[ \frac{c}{2} \right] \right) \right) / \\
& \quad \left( 8 d \sqrt{a (1 + \cos [c + dx])} (2 A + C + C \cos [2 c + 2 dx]) (-1 + \sqrt{2} \sin \left[ \frac{c}{2} \right]) \right) + \\
& \quad \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx]^2 \log \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] (C + A \operatorname{Sec} [c + dx]^2) \left( 7 \sqrt{2} A + 8 \sqrt{2} C - 14 A \sin \left[ \frac{c}{2} \right] - 16 C \sin \left[ \frac{c}{2} \right] \right) \right) / \\
& \quad \left( 8 d \sqrt{a (1 + \cos [c + dx])} (2 A + C + C \cos [2 c + 2 dx]) (-1 + \sqrt{2} \sin \left[ \frac{c}{2} \right]) \right) + \\
& \quad \frac{A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx]^2 (C + A \operatorname{Sec} [c + dx]^2) \sin \left[ \frac{dx}{2} \right]}{d \sqrt{a (1 + \cos [c + dx])} (2 A + C + C \cos [2 c + 2 dx]) (\cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right])^2} - \\
& \quad \frac{A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx]^2 (C + A \operatorname{Sec} [c + dx]^2) (\cos \left[ \frac{c}{2} \right] - 3 \sin \left[ \frac{c}{2} \right])}{2 d \sqrt{a (1 + \cos [c + dx])} (2 A + C + C \cos [2 c + 2 dx]) (\cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right])} + \\
& \quad \frac{A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx]^2 (C + A \operatorname{Sec} [c + dx]^2) \sin \left[ \frac{dx}{2} \right]}{d \sqrt{a (1 + \cos [c + dx])} (2 A + C + C \cos [2 c + 2 dx]) (\cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right])^2} + \\
& \quad \frac{A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx]^2 (C + A \operatorname{Sec} [c + dx]^2) (\cos \left[ \frac{c}{2} \right] + 3 \sin \left[ \frac{c}{2} \right])}{2 d \sqrt{a (1 + \cos [c + dx])} (2 A + C + C \cos [2 c + 2 dx]) (\cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right])}
\end{aligned}$$

■ **Problem 109: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos [c + dx])^2 \operatorname{Sec} [c + dx]^4}{\sqrt{a + a \cos [c + dx]}} dx$$

Optimal (type 3, 200 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(9 A + 8 C) \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \sin [c + dx]}{\sqrt{a + a \cos [c + dx]}} \right]}{8 \sqrt{a} d} + \frac{\sqrt{2} (A + C) \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \sin [c + dx]}{\sqrt{2} \sqrt{a + a \cos [c + dx]}} \right]}{\sqrt{a} d} + \\
& \frac{(7 A + 8 C) \tan [c + dx]}{8 d \sqrt{a + a \cos [c + dx]}} - \frac{A \operatorname{Sec} [c + dx] \tan [c + dx]}{12 d \sqrt{a + a \cos [c + dx]}} + \frac{A \operatorname{Sec} [c + dx]^2 \tan [c + dx]}{3 d \sqrt{a + a \cos [c + dx]}}
\end{aligned}$$

Result (type 3, 1212 leaves):

$$\begin{aligned}
& \frac{i (9 A + 8 C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]}{8 \sqrt{2} d \sqrt{a} (1 + \cos[c + dx])} + \\
& \frac{i (9 A + 8 C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]}{8 \sqrt{2} d \sqrt{a} (1 + \cos[c + dx])} - \frac{2 (A + C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{d \sqrt{a} (1 + \cos[c + dx])} + \\
& \frac{2 (A + C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{d \sqrt{a} (1 + \cos[c + dx])} + \frac{(-9 A - 8 C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{8 \sqrt{2} d \sqrt{a} (1 + \cos[c + dx])} + \\
& \frac{(9 A + 8 C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{16 \sqrt{2} d \sqrt{a} (1 + \cos[c + dx])} + \frac{(9 A + 8 C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{16 \sqrt{2} d \sqrt{a} (1 + \cos[c + dx])} + \\
& \frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]}{6 d \sqrt{a} (1 + \cos[c + dx])} - \frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{4 d \sqrt{a} (1 + \cos[c + dx])} - \frac{\left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3}{4 d \sqrt{a} (1 + \cos[c + dx])} - \frac{\left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2}{4 d \sqrt{a} (1 + \cos[c + dx])} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \left(7 A \cos\left[\frac{c}{2}\right] + 8 C \cos\left[\frac{c}{2}\right] - 9 A \sin\left[\frac{c}{2}\right] - 8 C \sin\left[\frac{c}{2}\right]\right)}{8 d \sqrt{a} (1 + \cos[c + dx])} - \frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]}{6 d \sqrt{a} (1 + \cos[c + dx])} - \frac{\left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3}{8 d \sqrt{a} (1 + \cos[c + dx])} + \\
& \frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{4 d \sqrt{a} (1 + \cos[c + dx])} + \frac{\left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2}{8 d \sqrt{a} (1 + \cos[c + dx])} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \left(-7 A \cos\left[\frac{c}{2}\right] - 8 C \cos\left[\frac{c}{2}\right] - 9 A \sin\left[\frac{c}{2}\right] - 8 C \sin\left[\frac{c}{2}\right]\right)}{8 d \sqrt{a} (1 + \cos[c + dx])} - \frac{\left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}{8 d \sqrt{a} (1 + \cos[c + dx])}
\end{aligned}$$

■ **Problem 110: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos[c + dx])^2 \operatorname{Sec}[c + dx]^5}{\sqrt{a + a \cos[c + dx]}} dx$$

Optimal (type 3, 243 leaves, 9 steps):

$$\begin{aligned}
& \frac{(107 A + 112 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right]}{64 \sqrt{a} d} - \frac{\sqrt{2} (A + C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{2} \sqrt{a + a \cos[c + dx]}}\right]}{\sqrt{a} d} - \frac{(21 A + 16 C) \tan[c + dx]}{64 d \sqrt{a + a \cos[c + dx]}} + \\
& \frac{(43 A + 48 C) \operatorname{Sec}[c + dx] \tan[c + dx]}{96 d \sqrt{a + a \cos[c + dx]}} - \frac{A \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{24 d \sqrt{a + a \cos[c + dx]}} + \frac{A \operatorname{Sec}[c + dx]^3 \tan[c + dx]}{4 d \sqrt{a + a \cos[c + dx]}}
\end{aligned}$$

Result (type 3, 1479 leaves):



■ **Problem 111: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^3 (A+C \cos [c+d x]^2)}{(a+a \cos [c+d x])^{3/2}} dx$$

Optimal (type 3, 259 leaves, 8 steps) :

$$\frac{(11 A+19 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{a+a \cos [c+d x]}}\right]}{2 \sqrt{2} a^{3/2} d}-\frac{(A+C) \cos [c+d x]^4 \sin [c+d x]}{2 d(a+a \cos [c+d x])^{3/2}}-\frac{(455 A+799 C) \sin [c+d x]}{105 a d \sqrt{a+a \cos [c+d x]}}-\frac{(35 A+67 C) \cos [c+d x]^2 \sin [c+d x]}{70 a d \sqrt{a+a \cos [c+d x]}}+\frac{(7 A+11 C) \cos [c+d x]^3 \sin [c+d x]}{14 a d \sqrt{a+a \cos [c+d x]}}+\frac{(245 A+397 C) \sqrt{a+a \cos [c+d x]} \sin [c+d x]}{210 a^2 d}$$

Result (type 3, 701 leaves) :

$$\frac{(-11 A-19 C) \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^3 \operatorname{Log}\left[\cos \left[\frac{c}{4}+\frac{d x}{4}\right]-\sin \left[\frac{c}{4}+\frac{d x}{4}\right]\right]}{d(a(1+\cos [c+d x]))^{3/2}}+\frac{(11 A+19 C) \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^3 \operatorname{Log}\left[\cos \left[\frac{c}{4}+\frac{d x}{4}\right]+\sin \left[\frac{c}{4}+\frac{d x}{4}\right]\right]}{d(a(1+\cos [c+d x]))^{3/2}}-\frac{(12 A+25 C) \cos \left[\frac{d x}{2}\right] \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^3 \sin \left[\frac{c}{2}\right]}{d(a(1+\cos [c+d x]))^{3/2}}+\frac{(4 A+11 C) \cos \left[\frac{3 d x}{2}\right] \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^3 \sin \left[\frac{3 c}{2}\right]}{3 d(a(1+\cos [c+d x]))^{3/2}}-\frac{3 C \cos \left[\frac{5 d x}{2}\right] \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^3 \sin \left[\frac{5 c}{2}\right]}{5 d(a(1+\cos [c+d x]))^{3/2}}+\frac{C \cos \left[\frac{7 d x}{2}\right] \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^3 \sin \left[\frac{7 c}{2}\right]}{7 d(a(1+\cos [c+d x]))^{3/2}}-\frac{(12 A+25 C) \cos \left[\frac{c}{2}\right] \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^3 \sin \left[\frac{d x}{2}\right]}{d(a(1+\cos [c+d x]))^{3/2}}+\frac{(4 A+11 C) \cos \left[\frac{3 c}{2}\right] \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^3 \sin \left[\frac{3 d x}{2}\right]}{3 d(a(1+\cos [c+d x]))^{3/2}}-\frac{3 C \cos \left[\frac{5 c}{2}\right] \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^3 \sin \left[\frac{5 d x}{2}\right]}{5 d(a(1+\cos [c+d x]))^{3/2}}+\frac{C \cos \left[\frac{7 c}{2}\right] \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^3 \sin \left[\frac{7 d x}{2}\right]}{7 d(a(1+\cos [c+d x]))^{3/2}}+\frac{(-A-C) \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^3}{2 d(a(1+\cos [c+d x]))^{3/2}\left(\cos \left[\frac{c}{4}+\frac{d x}{4}\right]-\sin \left[\frac{c}{4}+\frac{d x}{4}\right]\right)^2}+\frac{(A+C) \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^3}{2 d(a(1+\cos [c+d x]))^{3/2}\left(\cos \left[\frac{c}{4}+\frac{d x}{4}\right]+\sin \left[\frac{c}{4}+\frac{d x}{4}\right]\right)^2}$$

■ **Problem 115: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A+C \cos [c+d x]^2) \operatorname{Sec}[c+d x]}{(a+a \cos [c+d x])^{3/2}} dx$$

Optimal (type 3, 125 leaves, 6 steps) :

$$\frac{2 A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{a^{3/2} d}-\frac{(5 A-3 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{a+a \cos [c+d x]}}\right]}{2 \sqrt{2} a^{3/2} d}-\frac{(A+C) \sin [c+d x]}{2 d(a+a \cos [c+d x])^{3/2}}$$

Result (type 3, 2265 leaves) :

$$-\left(\left(2-2 i\right) A\left(1+e^{i c}\right)\right)$$

$$\begin{aligned}
& \left( \sqrt{2} - (1-i) e^{\frac{ic}{2}} + (16-16i) e^{\frac{3ic}{2}+idx} + (20+20i) \sqrt{2} e^{2ic+\frac{3idx}{2}} - (34-34i) e^{\frac{5ic}{2}+2idx} - (20+20i) \sqrt{2} e^{3ic+\frac{5idx}{2}} + (16-16i) e^{\frac{7ic}{2}+3idx} + \right. \\
& \quad (4+4i) \sqrt{2} e^{4ic+\frac{7idx}{2}} - (1-i) e^{\frac{9ic}{2}+4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + 40i e^{\frac{5}{2}i(c+dx)} - \\
& \quad \left. 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4+4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] (C \cos[c+dx] + A \sec[c+dx]) \Big/ \\
& \left( \left( (-1-i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right. \\
& \quad \left. (a(1+\cos[c+dx]))^{3/2} (2A+C+C\cos[2c+2dx]) \right) \Big/ - \\
& \left( 4i\sqrt{2} A \operatorname{ArcTan}\left[ \frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] (C \cos[c+dx] + A \sec[c+dx]) \right) \Big/ \\
& \quad (d(a(1+\cos[c+dx]))^{3/2} (2A+C+C\cos[2c+2dx])) + \\
& \quad \frac{2(5A-3C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C \cos[c+dx] + A \sec[c+dx])}{d(a(1+\cos[c+dx]))^{3/2} (2A+C+C\cos[2c+2dx])} - \\
& \quad \frac{2(5A-3C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C \cos[c+dx] + A \sec[c+dx])}{d(a(1+\cos[c+dx]))^{3/2} (2A+C+C\cos[2c+2dx])} - \\
& \quad \left( 2\sqrt{2} A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C \cos[c+dx] + A \sec[c+dx]) \right) \Big/ \\
& \quad (d(a(1+\cos[c+dx]))^{3/2} (2A+C+C\cos[2c+2dx])) + \\
& \quad \left( (1-i) \sqrt{2} \operatorname{ArcTan}\left[ \frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] (C \cos[c+dx] + A \sec[c+dx]) \right. \\
& \quad \left. \left( (1+i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1-i) \sin\left[\frac{c}{4}\right] - i\sqrt{2} \sin\left[\frac{c}{4}\right] \right) \left( (-1-i) A \cos\left[\frac{c}{4}\right] + \sqrt{2} A \cos\left[\frac{c}{4}\right] + (1-i) A \sin\left[\frac{c}{4}\right] - i\sqrt{2} A \sin\left[\frac{c}{4}\right] \right) \right) \Big/ \\
& \quad (d(a(1+\cos[c+dx]))^{3/2} (2A+C+C\cos[2c+2dx]) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)) - \\
& \quad \left( (1+i) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C \cos[c+dx] + A \sec[c+dx]) \right. \\
& \quad \left. \left( (1+i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1-i) \sin\left[\frac{c}{4}\right] - i\sqrt{2} \sin\left[\frac{c}{4}\right] \right) \left( (-1-i) A \cos\left[\frac{c}{4}\right] + \sqrt{2} A \cos\left[\frac{c}{4}\right] + (1-i) A \sin\left[\frac{c}{4}\right] - i\sqrt{2} A \sin\left[\frac{c}{4}\right] \right) \right) \Big/ \\
& \quad \left( \sqrt{2} d(a(1+\cos[c+dx]))^{3/2} (2A+C+C\cos[2c+2dx]) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right) -
\end{aligned}$$



$$\left( 16 i A \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \cos [c + dx] \cot \left[ \frac{c}{2} \right] (C \cos [c + dx] + A \sec [c + dx]) \right) /$$

$$\left( d (a (1 + \cos [c + dx]))^{3/2} (2 A + C + C \cos [2 c + 2 dx]) \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2} \right) +$$

$$\left( 8 \sqrt{2} A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \cos [c + dx] \operatorname{Csc} \left[ \frac{c}{2} \right] (C \cos [c + dx] + A \sec [c + dx]) \right)$$

$$\left( -dx \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) /$$

$$\left( d (a (1 + \cos [c + dx]))^{3/2} (2 A + C + C \cos [2 c + 2 dx]) \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right) \right) +$$

$$\frac{(-A - C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \cos [c + dx] (C \cos [c + dx] + A \sec [c + dx])}{d (a (1 + \cos [c + dx]))^{3/2} (2 A + C + C \cos [2 c + 2 dx]) \left( \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] \right)^2} +$$

$$\frac{(A + C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \cos [c + dx] (C \cos [c + dx] + A \sec [c + dx])}{d (a (1 + \cos [c + dx]))^{3/2} (2 A + C + C \cos [2 c + 2 dx]) \left( \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] \right)^2}$$

■ **Problem 116: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos [c + dx])^2 \sec [c + dx]^2}{(a + a \cos [c + dx])^{3/2}} dx$$

Optimal (type 3, 158 leaves, 7 steps):

$$-\frac{3 A \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \sin [c + dx]}{\sqrt{a + a \cos [c + dx]}} \right]}{a^{3/2} d} + \frac{(9 A + C) \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \sin [c + dx]}{\sqrt{2} \sqrt{a + a \cos [c + dx]}} \right]}{2 \sqrt{2} a^{3/2} d} - \frac{(A + C) \tan [c + dx]}{2 d (a + a \cos [c + dx])^{3/2}} + \frac{(3 A + C) \tan [c + dx]}{2 a d \sqrt{a + a \cos [c + dx]}}$$

Result (type 3, 2438 leaves):

$$\begin{aligned}
& \left( (3 - 3i) A (1 + e^{ic}) \right. \\
& \quad \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + (16 - 16i) e^{\frac{7ic}{2} + 3idx} + \right. \\
& \quad (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + 40i e^{\frac{5}{2}i(c+dx)} - \\
& \quad \left. 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 (C + A \sec[c + dx]^2) \Big/ \\
& \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right. \\
& \quad \left. (a(1 + \cos[c + dx]))^{3/2} (2A + C + C \cos[2c + 2dx]) \right) + \\
& \frac{6i\sqrt{2} A \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 (C + A \sec[c + dx]^2)}{d(a(1 + \cos[c + dx]))^{3/2} (2A + C + C \cos[2c + 2dx])} - \\
& \frac{2(9A + C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C + A \sec[c + dx]^2)}{d(a(1 + \cos[c + dx]))^{3/2} (2A + C + C \cos[2c + 2dx])} + \\
& \frac{2(9A + C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C + A \sec[c + dx]^2)}{d(a(1 + \cos[c + dx]))^{3/2} (2A + C + C \cos[2c + 2dx])} + \\
& \frac{3\sqrt{2} A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + A \sec[c + dx]^2)}{d(a(1 + \cos[c + dx]))^{3/2} (2A + C + C \cos[2c + 2dx])} - \\
& \left( (3 - 3i) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 (C + A \sec[c + dx]^2) \right. \\
& \quad \left. \left( (1 + i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1 - i) \sin\left[\frac{c}{4}\right] - i\sqrt{2} \sin\left[\frac{c}{4}\right] \right) \left( (-1 - i) A \cos\left[\frac{c}{4}\right] + \sqrt{2} A \cos\left[\frac{c}{4}\right] + (1 - i) A \sin\left[\frac{c}{4}\right] - i\sqrt{2} A \sin\left[\frac{c}{4}\right] \right) \right) \Big/ \\
& \left( \sqrt{2} d(a(1 + \cos[c + dx]))^{3/2} (2A + C + C \cos[2c + 2dx]) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right) + \\
& \left( \left( \frac{3}{2} + \frac{3i}{2} \right) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + A \sec[c + dx]^2) \right. \\
& \quad \left. \left( (1 + i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1 - i) \sin\left[\frac{c}{4}\right] - i\sqrt{2} \sin\left[\frac{c}{4}\right] \right) \left( (-1 - i) A \cos\left[\frac{c}{4}\right] + \sqrt{2} A \cos\left[\frac{c}{4}\right] + (1 - i) A \sin\left[\frac{c}{4}\right] - i\sqrt{2} A \sin\left[\frac{c}{4}\right] \right) \right) \Big/ \\
& \left( \sqrt{2} d(a(1 + \cos[c + dx]))^{3/2} (2A + C + C \cos[2c + 2dx]) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right) +
\end{aligned}$$

$$24 i A \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \cos [c + dx]^2 \cot \left[ \frac{c}{2} \right] (C + A \operatorname{Sec} [c + dx]^2)$$

$$d (a (1 + \cos [c + dx]))^{3/2} (2A + C + C \cos [2c + 2dx]) \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}$$

$$\left( 12 \sqrt{2} A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \cos [c + dx]^2 \operatorname{Csc} \left[ \frac{c}{2} \right] (C + A \operatorname{Sec} [c + dx]^2) \right)$$

$$\left( -dx \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) /$$

$$\left( d (a (1 + \cos [c + dx]))^{3/2} (2A + C + C \cos [2c + 2dx]) \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right) \right) +$$

$$\frac{(A + C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \cos [c + dx]^2 (C + A \operatorname{Sec} [c + dx]^2)}{d (a (1 + \cos [c + dx]))^{3/2} (2A + C + C \cos [2c + 2dx]) \left( \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] \right)^2} +$$

$$\frac{(-A - C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \cos [c + dx]^2 (C + A \operatorname{Sec} [c + dx]^2)}{d (a (1 + \cos [c + dx]))^{3/2} (2A + C + C \cos [2c + 2dx]) \left( \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] \right)^2} +$$

$$\frac{4A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \cos [c + dx]^2 (C + A \operatorname{Sec} [c + dx]^2)}{d (a (1 + \cos [c + dx]))^{3/2} (2A + C + C \cos [2c + 2dx]) \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)}$$

$$\frac{4A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \cos [c + dx]^2 (C + A \operatorname{Sec} [c + dx]^2)}{d (a (1 + \cos [c + dx]))^{3/2} (2A + C + C \cos [2c + 2dx]) \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)}$$

$$\frac{4A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \cos [c + dx]^2 (C + A \operatorname{Sec} [c + dx]^2)}{d (a (1 + \cos [c + dx]))^{3/2} (2A + C + C \cos [2c + 2dx]) \left( \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right)}$$

■ **Problem 117: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos [c + dx]^2) \operatorname{Sec} [c + dx]^3}{(a + a \cos [c + dx])^{3/2}} dx$$

Optimal (type 3, 217 leaves, 8 steps):

$$\frac{(19A + 8C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{4a^{3/2}d} - \frac{(13A + 5C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{2\sqrt{2}a^{3/2}d} -$$

$$\frac{(7A + 2C) \tan[c+dx]}{4ad\sqrt{a+a \cos[c+dx]}} - \frac{(A+C) \sec[c+dx] \tan[c+dx]}{2d(a+a \cos[c+dx])^{3/2}} + \frac{(2A+C) \sec[c+dx] \tan[c+dx]}{2ad\sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 1792 leaves):

$$\frac{2(13A + 5C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C + A \sec[c+dx])^2}{d(a(1 + \cos[c+dx]))^{3/2}(2A + C + C \cos[2c + 2dx])} -$$

$$\frac{2(13A + 5C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C + A \sec[c+dx])^2}{d(a(1 + \cos[c+dx]))^{3/2}(2A + C + C \cos[2c + 2dx])} +$$

$$\frac{(19A + 8C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx]^2 \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + A \sec[c+dx])^2}{\sqrt{2}d(a(1 + \cos[c+dx]))^{3/2}(2A + C + C \cos[2c + 2dx])} +$$

$$\left( i(19A + 8C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx]^2 (C + A \sec[c+dx])^2 \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right) \right) /$$

$$\left( 2d(a(1 + \cos[c+dx]))^{3/2}(2A + C + C \cos[2c + 2dx]) \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) +$$

$$\left( i(19A + 8C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx]^2 (C + A \sec[c+dx])^2 \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right) \right) /$$

$$\left( 2d(a(1 + \cos[c+dx]))^{3/2}(2A + C + C \cos[2c + 2dx]) \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) +$$

$$\left( (19A + 8C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx]^2 \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + A \sec[c+dx])^2 \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right) \right) /$$

$$\left( 4d(a(1 + \cos[c+dx]))^{3/2}(2A + C + C \cos[2c + 2dx]) \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) +$$

$$\left( (19A + 8C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx]^2 \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + A \sec[c+dx])^2 \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right) \right) /$$

$$\left( 4d(a(1 + \cos[c+dx]))^{3/2}(2A + C + C \cos[2c + 2dx]) \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) +$$

$$\frac{(-A - C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx]^2 (C + A \sec[c+dx])^2}{d(a(1 + \cos[c+dx]))^{3/2}(2A + C + C \cos[2c + 2dx]) \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2} +$$

$$\frac{(A+C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx]^2 (C+A \sec[c+dx]^2)}{d (a(1+\cos[c+dx]))^{3/2} (2A+C+C \cos[2c+2dx]) \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2} +$$

$$\frac{2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx]^2 (C+A \sec[c+dx]^2) \sin\left[\frac{dx}{2}\right]}{d (a(1+\cos[c+dx]))^{3/2} (2A+C+C \cos[2c+2dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} +$$

$$\frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx]^2 (C+A \sec[c+dx]^2) (-5A \cos\left[\frac{c}{2}\right] + 7A \sin\left[\frac{c}{2}\right])}{d (a(1+\cos[c+dx]))^{3/2} (2A+C+C \cos[2c+2dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} +$$

$$\frac{2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx]^2 (C+A \sec[c+dx]^2) \sin\left[\frac{dx}{2}\right]}{d (a(1+\cos[c+dx]))^{3/2} (2A+C+C \cos[2c+2dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} +$$

$$\frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx]^2 (C+A \sec[c+dx]^2) (5A \cos\left[\frac{c}{2}\right] + 7A \sin\left[\frac{c}{2}\right])}{d (a(1+\cos[c+dx]))^{3/2} (2A+C+C \cos[2c+2dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}$$

■ **Problem 118: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A+C \cos[c+dx]^2) \sec[c+dx]^4}{(a+a \cos[c+dx])^{3/2}} dx$$

Optimal (type 3, 266 leaves, 9 steps):

$$-\frac{(47A+24C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{8a^{3/2}d} + \frac{(17A+9C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{2\sqrt{2}a^{3/2}d} + \frac{3(7A+4C) \tan[c+dx]}{8ad\sqrt{a+a \cos[c+dx]}} -$$

$$\frac{(13A+6C) \sec[c+dx] \tan[c+dx]}{12ad\sqrt{a+a \cos[c+dx]}} - \frac{(A+C) \sec[c+dx]^2 \tan[c+dx]}{2d(a+a \cos[c+dx])^{3/2}} + \frac{(5A+3C) \sec[c+dx]^2 \tan[c+dx]}{6ad\sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 1019 leaves):

$$\begin{aligned}
& \frac{(-17A - 9C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{d(a(1 + \cos[c + dx]))^{3/2}} + \\
& \frac{(17A + 9C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right]}{d(a(1 + \cos[c + dx]))^{3/2}} + \frac{(-47A - 24C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{4\sqrt{2} d(a(1 + \cos[c + dx]))^{3/2}} + \\
& \left( i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(-47\sqrt{2}A - 24\sqrt{2}C + 94A \sin\left[\frac{c}{2}\right] + 48C \sin\left[\frac{c}{2}\right]\right) \right) / \\
& \left( 8 d(a(1 + \cos[c + dx]))^{3/2} \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) + \\
& \left( i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(-47\sqrt{2}A - 24\sqrt{2}C + 94A \sin\left[\frac{c}{2}\right] + 48C \sin\left[\frac{c}{2}\right]\right) \right) / \\
& \left( 8 d(a(1 + \cos[c + dx]))^{3/2} \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) + \\
& \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \left(-47\sqrt{2}A - 24\sqrt{2}C + 94A \sin\left[\frac{c}{2}\right] + 48C \sin\left[\frac{c}{2}\right]\right) \right) / \\
& \left( 16 d(a(1 + \cos[c + dx]))^{3/2} \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) + \\
& \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \left(-47\sqrt{2}A - 24\sqrt{2}C + 94A \sin\left[\frac{c}{2}\right] + 48C \sin\left[\frac{c}{2}\right]\right) \right) / \\
& \left( 16 d(a(1 + \cos[c + dx]))^{3/2} \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) + \\
& \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}[c + dx]^3 \left( 47A \sin\left[\frac{c}{2} + \frac{dx}{2}\right] - 12C \sin\left[\frac{c}{2} + \frac{dx}{2}\right] + 91A \sin\left[\frac{3c}{2} + \frac{3dx}{2}\right] + 60C \sin\left[\frac{3c}{2} + \frac{3dx}{2}\right] + \right. \right. \\
& \left. \left. 11A \sin\left[\frac{5c}{2} + \frac{5dx}{2}\right] + 12C \sin\left[\frac{5c}{2} + \frac{5dx}{2}\right] + 63A \sin\left[\frac{7c}{2} + \frac{7dx}{2}\right] + 36C \sin\left[\frac{7c}{2} + \frac{7dx}{2}\right] \right) \right) / (96 d(a(1 + \cos[c + dx]))^{3/2})
\end{aligned}$$

■ **Problem 123: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos[c + dx])^2 \operatorname{Sec}[c + dx]}{(a + a \cos[c + dx])^{5/2}} dx$$

Optimal (type 3, 162 leaves, 7 steps):

$$\frac{2A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{5/2} d} - \frac{(43A - 5C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{16\sqrt{2} a^{5/2} d} - \frac{(A + C) \sin[c + dx]}{4d(a + a \cos[c + dx])^{5/2}} - \frac{(11A - 5C) \sin[c + dx]}{16ad(a + a \cos[c + dx])^{3/2}}$$

Result (type 3, 2503 leaves):

$$\begin{aligned}
& - \left( (4 - 4i) A (1 + e^{ic}) \right. \\
& \quad \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + (16 - 16i) e^{\frac{7ic}{2} + 3idx} + \right. \\
& \quad (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + 40i e^{\frac{5}{2}i(c+dx)} - \\
& \quad \left. 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx] (C \cos[c + dx] + A \sec[c + dx]) \Big) / \\
& \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right. \\
& \quad \left. (a(1 + \cos[c + dx]))^{5/2} (2A + C + C \cos[2c + 2dx]) \right) \Big) - \\
& \left( 8i\sqrt{2} A \operatorname{ArcTan}\left[ \frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx] (C \cos[c + dx] + A \sec[c + dx]) \right) / \\
& \quad (d(a(1 + \cos[c + dx]))^{5/2} (2A + C + C \cos[2c + 2dx])) + \\
& \quad \frac{(43A - 5C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx] \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C \cos[c + dx] + A \sec[c + dx])}{2d(a(1 + \cos[c + dx]))^{5/2} (2A + C + C \cos[2c + 2dx])} + \\
& \quad \frac{(-43A + 5C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx] \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C \cos[c + dx] + A \sec[c + dx])}{2d(a(1 + \cos[c + dx]))^{5/2} (2A + C + C \cos[2c + 2dx])} - \\
& \quad \left( 4\sqrt{2} A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx] \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C \cos[c + dx] + A \sec[c + dx]) \right) / \\
& \quad (d(a(1 + \cos[c + dx]))^{5/2} (2A + C + C \cos[2c + 2dx])) + \\
& \quad \left( (2 - 2i) \sqrt{2} \operatorname{ArcTan}\left[ \frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx] (C \cos[c + dx] + A \sec[c + dx]) \right. \\
& \quad \left. \left( (1 + i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1 - i) \sin\left[\frac{c}{4}\right] - i\sqrt{2} \sin\left[\frac{c}{4}\right] \right) \left( (-1 - i) A \cos\left[\frac{c}{4}\right] + \sqrt{2} A \cos\left[\frac{c}{4}\right] + (1 - i) A \sin\left[\frac{c}{4}\right] - i\sqrt{2} A \sin\left[\frac{c}{4}\right] \right) \right) / \\
& \quad (d(a(1 + \cos[c + dx]))^{5/2} (2A + C + C \cos[2c + 2dx]) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)) - \\
& \quad \left( (1 + i) \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx] \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C \cos[c + dx] + A \sec[c + dx]) \right. \\
& \quad \left. \left( (1 + i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1 - i) \sin\left[\frac{c}{4}\right] - i\sqrt{2} \sin\left[\frac{c}{4}\right] \right) \left( (-1 - i) A \cos\left[\frac{c}{4}\right] + \sqrt{2} A \cos\left[\frac{c}{4}\right] + (1 - i) A \sin\left[\frac{c}{4}\right] - i\sqrt{2} A \sin\left[\frac{c}{4}\right] \right) \right) / \\
& \quad (d(a(1 + \cos[c + dx]))^{5/2} (2A + C + C \cos[2c + 2dx]) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)) -
\end{aligned}$$

$$\left( \frac{32 i A \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \cos [c + dx] \cot \left[ \frac{c}{2} \right] (C \cos [c + dx] + A \sec [c + dx])}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) /$$

$$\left( d (a (1 + \cos [c + dx]))^{5/2} (2A + C + C \cos [2c + 2dx]) \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2} \right) +$$

$$\left( 16 \sqrt{2} A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \cos [c + dx] \operatorname{Csc} \left[ \frac{c}{2} \right] (C \cos [c + dx] + A \sec [c + dx]) \right)$$

$$\left( -dx \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) /$$

$$\left( d (a (1 + \cos [c + dx]))^{5/2} (2A + C + C \cos [2c + 2dx]) \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right) \right) +$$

$$\frac{(-A - C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \cos [c + dx] (C \cos [c + dx] + A \sec [c + dx])}{4 d (a (1 + \cos [c + dx]))^{5/2} (2A + C + C \cos [2c + 2dx]) \left( \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] \right)^4} +$$

$$\frac{(-11A + 5C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \cos [c + dx] (C \cos [c + dx] + A \sec [c + dx])}{4 d (a (1 + \cos [c + dx]))^{5/2} (2A + C + C \cos [2c + 2dx]) \left( \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] \right)^2} +$$

$$\frac{(A + C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \cos [c + dx] (C \cos [c + dx] + A \sec [c + dx])}{4 d (a (1 + \cos [c + dx]))^{5/2} (2A + C + C \cos [2c + 2dx]) \left( \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] \right)^4} +$$

$$\frac{(11A - 5C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \cos [c + dx] (C \cos [c + dx] + A \sec [c + dx])}{4 d (a (1 + \cos [c + dx]))^{5/2} (2A + C + C \cos [2c + 2dx]) \left( \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] \right)^2}$$

■ **Problem 124: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos [c + dx])^2 \sec [c + dx]^2}{(a + a \cos [c + dx])^{5/2}} dx$$



Optimal (type 3, 199 leaves, 8 steps):

$$-\frac{5 A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{a^{5 / 2} d}+\frac{(115 A+3 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{a+a \cos [c+d x]}}\right]}{16 \sqrt{2} a^{5 / 2} d}-\frac{(A+C) \tan [c+d x]}{4 d(a+a \cos [c+d x])^{5 / 2}}-\frac{(15 A-C) \tan [c+d x]}{16 a d(a+a \cos [c+d x])^{3 / 2}}+\frac{(35 A+3 C) \tan [c+d x]}{16 a^2 d \sqrt{a+a \cos [c+d x]}}$$

Result (type 3, 2191 leaves):

$$\left( (10-10 i) A\left(1+e^{i c}\right)\left(\sqrt{2}-\left(1-i\right) e^{\frac{i c}{2}}+\left(16-16 i\right) e^{\frac{3 i c}{2}+i d x}+\left(20+20 i\right) \sqrt{2} e^{2 i c+\frac{3 i d x}{2}}-\left(34-34 i\right) e^{\frac{5 i c}{2}+2 i d x}-\left(20+20 i\right) \sqrt{2} e^{3 i c+\frac{5 i d x}{2}}+\left(16-16 i\right) e^{\frac{7 i c}{2}+3 i d x}+\left(4+4 i\right) \sqrt{2} e^{4 i c+\frac{7 i d x}{2}}-\left(1-i\right) e^{\frac{9 i c}{2}+4 i d x}+8 i e^{\frac{1}{2} i(c+d x)}-16 \sqrt{2} e^{i(c+d x)}-40 i e^{\frac{3}{2} i(c+d x)}+34 \sqrt{2} e^{2 i(c+d x)}+40 i e^{\frac{5}{2} i(c+d x)}-16 \sqrt{2} e^{3 i(c+d x)}-8 i e^{\frac{7}{2} i(c+d x)}+\sqrt{2} e^{4 i(c+d x)}-\left(4+4 i\right) \sqrt{2} e^{\frac{1}{2} i(2 c+d x)}\right) x \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^5 \cos [c+d x]^2\left(C+A \sec [c+d x]^2\right)\right) / \left(\left(\left(-1-i\right)+\sqrt{2} e^{\frac{i c}{2}}\right)\left(-1+e^{i c}\right)\left(i-2 \sqrt{2} e^{\frac{1}{2} i(c+d x)}-4 i e^{i(c+d x)}+2 \sqrt{2} e^{\frac{3}{2} i(c+d x)}+i e^{2 i(c+d x)}\right)^2\left(a\left(1+\cos [c+d x]\right)\right)^{5 / 2}\left(2 A+C+C \cos [2 c+2 d x]\right)\right)+\frac{20 i \sqrt{2} A \operatorname{ArcTan}\left[\frac{\cos \left[\frac{c}{4}+\frac{d x}{4}\right]-\sin \left[\frac{c}{4}+\frac{d x}{4}\right]-\sqrt{2} \sin \left[\frac{c}{4}+\frac{d x}{4}\right]}{-\cos \left[\frac{c}{4}+\frac{d x}{4}\right]+\sqrt{2} \cos \left[\frac{c}{4}+\frac{d x}{4}\right]-\sin \left[\frac{c}{4}+\frac{d x}{4}\right]}\right]}{d\left(a\left(1+\cos [c+d x]\right)\right)^{5 / 2}\left(2 A+C+C \cos [2 c+2 d x]\right)}+\frac{\left(-115 A-3 C\right) \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^5 \cos [c+d x]^2 \operatorname{Log}\left[\cos \left[\frac{c}{4}+\frac{d x}{4}\right]-\sin \left[\frac{c}{4}+\frac{d x}{4}\right]\right]\left(C+A \sec [c+d x]^2\right)}{2 d\left(a\left(1+\cos [c+d x]\right)\right)^{5 / 2}\left(2 A+C+C \cos [2 c+2 d x]\right)}+\frac{\left(115 A+3 C\right) \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^5 \cos [c+d x]^2 \operatorname{Log}\left[\cos \left[\frac{c}{4}+\frac{d x}{4}\right]+\sin \left[\frac{c}{4}+\frac{d x}{4}\right]\right]\left(C+A \sec [c+d x]^2\right)}{2 d\left(a\left(1+\cos [c+d x]\right)\right)^{5 / 2}\left(2 A+C+C \cos [2 c+2 d x]\right)}+\frac{10 \sqrt{2} A \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^5 \cos [c+d x]^2 \operatorname{Log}\left[2-\sqrt{2} \cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sqrt{2} \sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right]\left(C+A \sec [c+d x]^2\right)}{d\left(a\left(1+\cos [c+d x]\right)\right)^{5 / 2}\left(2 A+C+C \cos [2 c+2 d x]\right)}-\left(\left(5-5 i\right) \sqrt{2} \operatorname{ArcTan}\left[\frac{\cos \left[\frac{c}{4}+\frac{d x}{4}\right]+\sin \left[\frac{c}{4}+\frac{d x}{4}\right]-\sqrt{2} \sin \left[\frac{c}{4}+\frac{d x}{4}\right]}{\cos \left[\frac{c}{4}+\frac{d x}{4}\right]+\sqrt{2} \cos \left[\frac{c}{4}+\frac{d x}{4}\right]-\sin \left[\frac{c}{4}+\frac{d x}{4}\right]}\right] \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^5 \cos [c+d x]^2\left(C+A \sec [c+d x]^2\right)\right)\left(\left(1+i\right) \cos \left[\frac{c}{4}\right]+\sqrt{2} \cos \left[\frac{c}{4}\right]-\left(1-i\right) \sin \left[\frac{c}{4}\right]-i \sqrt{2} \sin \left[\frac{c}{4}\right]\right)\left(\left(-1-i\right) A \cos \left[\frac{c}{4}\right]+\sqrt{2} A \cos \left[\frac{c}{4}\right]+\left(1-i\right) A \sin \left[\frac{c}{4}\right]-i \sqrt{2} A \sin \left[\frac{c}{4}\right]\right)\right) / \left(d\left(a\left(1+\cos [c+d x]\right)\right)^{5 / 2}\left(2 A+C+C \cos [2 c+2 d x]\right)\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\right)+$$

$$\frac{\left( (5 + 5i) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx]^2 \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + A \operatorname{Sec}[c + dx]^2) \right. \\ \left. \left( (1 + i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1 - i) \sin\left[\frac{c}{4}\right] - i \sqrt{2} \sin\left[\frac{c}{4}\right] \right) \left( (-1 - i) A \cos\left[\frac{c}{4}\right] + \sqrt{2} A \cos\left[\frac{c}{4}\right] + (1 - i) A \sin\left[\frac{c}{4}\right] - i \sqrt{2} A \sin\left[\frac{c}{4}\right] \right) \right) / \\ \left( \sqrt{2} d (a (1 + \cos[c + dx]))^{5/2} (2A + C + C \cos[2c + 2dx]) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right) + \\ 80i A \operatorname{ArcTan}\left[\frac{2i \cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx]^2 \operatorname{Cot}\left[\frac{c}{2}\right] (C + A \operatorname{Sec}[c + dx]^2) \\ \left. \frac{d (a (1 + \cos[c + dx]))^{5/2} (2A + C + C \cos[2c + 2dx]) \sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}{\right.}$$

$$\left( 40 \sqrt{2} A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx]^2 \operatorname{Csc}\left[\frac{c}{2}\right] (C + A \operatorname{Sec}[c + dx]^2) \right. \\ \left. \left( -dx \cos\left[\frac{c}{2}\right] + 2 \operatorname{Log}\left[\sqrt{2} + 2 \cos\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] \right) \sin\left[\frac{c}{2}\right] + \frac{4i \sqrt{2} \operatorname{ArcTan}\left[\frac{2i \cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right) / \\ \left( d (a (1 + \cos[c + dx]))^{5/2} (2A + C + C \cos[2c + 2dx]) \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) \right) + \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c + dx] (C + A \operatorname{Sec}[c + dx]^2) \right. \\ \left. \left( 24 A \sin\left[\frac{c}{2} + \frac{dx}{2}\right] - 8 C \sin\left[\frac{c}{2} + \frac{dx}{2}\right] + 75 A \sin\left[\frac{3c}{2} + \frac{3dx}{2}\right] + 11 C \sin\left[\frac{3c}{2} + \frac{3dx}{2}\right] + 35 A \sin\left[\frac{5c}{2} + \frac{5dx}{2}\right] + 3 C \sin\left[\frac{5c}{2} + \frac{5dx}{2}\right] \right) \right) / \\ (16 d (a (1 + \cos[c + dx]))^{5/2} (2A + C + C \cos[2c + 2dx]))$$

■ **Problem 125: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos[c + dx])^2 \operatorname{Sec}[c + dx]^3}{(a + a \cos[c + dx])^{5/2}} dx$$

Optimal (type 3, 262 leaves, 9 steps):

$$\frac{(39 A + 8 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{4 a^{5/2} d} - \frac{(219 A + 43 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(63 A + 11 C) \tan[c+dx]}{16 a^2 d \sqrt{a+a \cos[c+dx]}}$$

$$\frac{(A+C) \sec[c+dx] \tan[c+dx]}{4 d (a+a \cos[c+dx])^{5/2}} - \frac{(19 A + 3 C) \sec[c+dx] \tan[c+dx]}{16 a d (a+a \cos[c+dx])^{3/2}} + \frac{(31 A + 7 C) \sec[c+dx] \tan[c+dx]}{16 a^2 d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 1250 leaves):

$$\frac{(219 A + 43 C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c+dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C+A \sec[c+dx]^2)}{2 d (a (1 + \cos[c+dx]))^{5/2} (2 A + C + C \cos[2 c + 2 d x])} +$$

$$\frac{(-219 A - 43 C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c+dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C+A \sec[c+dx]^2)}{2 d (a (1 + \cos[c+dx]))^{5/2} (2 A + C + C \cos[2 c + 2 d x])} +$$

$$\frac{\sqrt{2} (39 A + 8 C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c+dx]^2 \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C+A \sec[c+dx]^2)}{d (a (1 + \cos[c+dx]))^{5/2} (2 A + C + C \cos[2 c + 2 d x])} +$$

$$\left( i (39 A + 8 C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c+dx]^2 (C+A \sec[c+dx]^2) \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right) \right) /$$

$$\left( d (a (1 + \cos[c+dx]))^{5/2} (2 A + C + C \cos[2 c + 2 d x]) \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) +$$

$$\left( i (39 A + 8 C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c+dx]^2 (C+A \sec[c+dx]^2) \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right) \right) /$$

$$\left( d (a (1 + \cos[c+dx]))^{5/2} (2 A + C + C \cos[2 c + 2 d x]) \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) +$$

$$\left( (39 A + 8 C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c+dx]^2 \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C+A \sec[c+dx]^2) \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right) \right) /$$

$$\left( 2 d (a (1 + \cos[c+dx]))^{5/2} (2 A + C + C \cos[2 c + 2 d x]) \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) +$$

$$\left( (39 A + 8 C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c+dx]^2 \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C+A \sec[c+dx]^2) \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right) \right) /$$

$$\left( 2 d (a (1 + \cos[c+dx]))^{5/2} (2 A + C + C \cos[2 c + 2 d x]) \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) -$$

$$\left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] (C+A \sec[c+dx]^2) \left( 47 A \sin\left[\frac{c}{2} + \frac{dx}{2}\right] + 27 C \sin\left[\frac{c}{2} + \frac{dx}{2}\right] + 79 A \sin\left[\frac{3 c}{2} + \frac{3 d x}{2}\right] + 3 C \sin\left[\frac{3 c}{2} + \frac{3 d x}{2}\right] + 127 A \sin\left[\frac{5 c}{2} + \frac{5 d x}{2}\right] + \right.$$

$$\left. 19 C \sin\left[\frac{5 c}{2} + \frac{5 d x}{2}\right] + 63 A \sin\left[\frac{7 c}{2} + \frac{7 d x}{2}\right] + 11 C \sin\left[\frac{7 c}{2} + \frac{7 d x}{2}\right] \right) / \left( 32 d (a (1 + \cos[c+dx]))^{5/2} (2 A + C + C \cos[2 c + 2 d x]) \right)$$

■ **Problem 126: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^{5/2} (a + a \cos[c + dx]) (A + C \cos[c + dx]^2) dx$$

Optimal (type 4, 196 leaves, 8 steps):

$$\frac{2a(9A+7C)\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{15d} + \frac{10a(11A+9C)\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{231d} +$$

$$\frac{10a(11A+9C)\sqrt{\cos[c+dx]}\sin[c+dx]}{231d} + \frac{2a(9A+7C)\cos[c+dx]^{3/2}\sin[c+dx]}{45d} +$$

$$\frac{2a(11A+9C)\cos[c+dx]^{5/2}\sin[c+dx]}{77d} + \frac{2aC\cos[c+dx]^{7/2}\sin[c+dx]}{9d} + \frac{2aC\cos[c+dx]^{9/2}\sin[c+dx]}{11d}$$

Result (type 5, 964 leaves):

$$a \left( \sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right.$$

$$\left( -\frac{(9A+7C)\cot[c]}{15d} + \frac{(506A+435C)\cos[dx]\sin[c]}{1848d} + \frac{(18A+19C)\cos[2dx]\sin[2c]}{180d} + \frac{(44A+57C)\cos[3dx]\sin[3c]}{1232d} \right.$$

$$\left. \frac{C\cos[4dx]\sin[4c]}{72d} + \frac{C\cos[5dx]\sin[5c]}{176d} + \frac{(506A+435C)\cos[c]\sin[dx]}{1848d} + \frac{(18A+19C)\cos[2c]\sin[2dx]}{180d} \right.$$

$$\left. \frac{(44A+57C)\cos[3c]\sin[3dx]}{1232d} + \frac{C\cos[4c]\sin[4dx]}{72d} + \frac{C\cos[5c]\sin[5dx]}{176d} \right) - \frac{1}{21d\sqrt{1+\cot[c]^2}}$$

$$5A(1+\cos[c+dx])\operatorname{Csc}[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]]^2]$$

$$\sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{77d\sqrt{1 + \cot[c]^2}} 15C(1 + \cos[c+dx])\operatorname{Csc}[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]}$$

$$\sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{10d} 3A(1 + \cos[c+dx])\operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \right) \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) /$$

$$\left( \sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \left) - \frac{1}{30 d} 7 C (1 + \cos [c + d x]) \text{Csc} [c] \text{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \right) \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) /$$

$$\left( \sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \left) \right)$$

■ **Problem 127: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^{3/2} (a + a \cos [c + d x]) (A + C \cos [c + d x])^2 dx$$

Optimal (type 4, 165 leaves, 7 steps):

$$\frac{2 a (9 A + 7 C) \text{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right]}{15 d} + \frac{2 a (7 A + 5 C) \text{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right]}{21 d} + \frac{2 a (7 A + 5 C) \sqrt{\cos [c + d x]} \sin [c + d x]}{21 d} +$$

$$\frac{2 a (9 A + 7 C) \cos [c + d x]^{3/2} \sin [c + d x]}{45 d} + \frac{2 a C \cos [c + d x]^{5/2} \sin [c + d x]}{7 d} + \frac{2 a C \cos [c + d x]^{7/2} \sin [c + d x]}{9 d}$$

Result (type 5, 918 leaves):

$$\begin{aligned}
& a \left( \sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right. \\
& \left( -\frac{(9A+7C)\cot[c]}{15d} + \frac{(28A+23C)\cos[dx]\sin[c]}{84d} + \frac{(18A+19C)\cos[2dx]\sin[2c]}{180d} + \frac{C\cos[3dx]\sin[3c]}{28d} + \frac{C\cos[4dx]\sin[4c]}{72d} \right. \\
& \left. \frac{(28A+23C)\cos[c]\sin[dx]}{84d} + \frac{(18A+19C)\cos[2c]\sin[2dx]}{180d} + \frac{C\cos[3c]\sin[3dx]}{28d} + \frac{C\cos[4c]\sin[4dx]}{72d} \right) - \frac{1}{3d\sqrt{1+\cot[c]^2}} \\
& A(1+\cos[c+dx])\csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sec[dx - \operatorname{ArcTan}[\cot[c]]] \\
& \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \\
& \frac{1}{21d\sqrt{1+\cot[c]^2}} 5C(1+\cos[c+dx])\csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{10d} 3A(1+\cos[c+dx])\csc[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) - \frac{1}{30d} 7C(1+\cos[c+dx])\csc[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2
\end{aligned}$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2 \right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left( \sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) \right)$$

■ **Problem 128: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\text{Cos}[c + dx]} (a + a \text{Cos}[c + dx]) (A + C \text{Cos}[c + dx]^2) dx$$

Optimal (type 4, 134 leaves, 6 steps):

$$\frac{2 a (5 A + 3 C) \text{EllipticE}\left[\frac{1}{2} (c + dx), 2\right]}{5 d} + \frac{2 a (7 A + 5 C) \text{EllipticF}\left[\frac{1}{2} (c + dx), 2\right]}{21 d} + \\ \frac{2 a (7 A + 5 C) \sqrt{\text{Cos}[c + dx]} \text{Sin}[c + dx]}{21 d} + \frac{2 a C \text{Cos}[c + dx]^{3/2} \text{Sin}[c + dx]}{5 d} + \frac{2 a C \text{Cos}[c + dx]^{5/2} \text{Sin}[c + dx]}{7 d}$$

Result (type 5, 872 leaves):

$$a \left( \sqrt{\text{Cos}[c + dx]} (1 + \text{Cos}[c + dx]) \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left( -\frac{(5 A + 3 C) \text{Cot}[c]}{5 d} + \frac{(28 A + 23 C) \text{Cos}[dx] \text{Sin}[c]}{84 d} + \frac{C \text{Cos}[2 dx] \text{Sin}[2 c]}{10 d} + \right. \right. \\ \left. \left. \frac{C \text{Cos}[3 dx] \text{Sin}[3 c]}{28 d} + \frac{(28 A + 23 C) \text{Cos}[c] \text{Sin}[dx]}{84 d} + \frac{C \text{Cos}[2 c] \text{Sin}[2 dx]}{10 d} + \frac{C \text{Cos}[3 c] \text{Sin}[3 dx]}{28 d} \right) - \frac{1}{3 d \sqrt{1 + \text{Cot}[c]^2}} \right. \\ \left. A (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \right. \\ \left. \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} - \right. \\ \left. \frac{1}{21 d \sqrt{1 + \text{Cot}[c]^2}} 5 C (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right)$$

$$\begin{aligned}
& \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \frac{1}{2d} A (1 + \cos[c + dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left( \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]\right]^2 \right) \sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \\
& \left( \frac{\frac{\sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}} \right) - \frac{1}{10d} 3 C (1 + \cos[c + dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left( \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]\right]^2 \right) \sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \\
& \left( \frac{\frac{\sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}} \right)
\end{aligned}$$

■ **Problem 129: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx]) (A + C \cos[c + dx]^2)}{\sqrt{\cos[c + dx]}} dx$$

Optimal (type 4, 101 leaves, 5 steps):



$$\frac{2 a (5 A + 3 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{2 a (3 A + C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 d} +$$

$$\frac{2 a C \sqrt{\cos [c + d x]} \sin [c + d x]}{3 d} + \frac{2 a C \cos [c + d x]^{3/2} \sin [c + d x]}{5 d}$$

Result (type 5, 824 leaves):

$$a \left( \sqrt{\cos [c + d x]} (1 + \cos [c + d x]) \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \right.$$

$$\left. \left( -\frac{(5 A + 3 C) \cot [c]}{5 d} + \frac{C \cos [d x] \sin [c]}{3 d} + \frac{C \cos [2 d x] \sin [2 c]}{10 d} + \frac{C \cos [c] \sin [d x]}{3 d} + \frac{C \cos [2 c] \sin [2 d x]}{10 d} \right) - \frac{1}{d \sqrt{1 + \cot [c]^2}} \right.$$

$$A (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} -$$

$$\frac{1}{3 d \sqrt{1 + \cot [c]^2}} C (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} -$$

$$\sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \frac{1}{2 d} A (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right) - \frac{1}{10 d} 3 C (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]^2 \right] \text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left( \sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \left. \frac{\frac{\text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) \right)$$

■ **Problem 130: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Cos}[c + d x]) (A + C \text{Cos}[c + d x]^2)}{\text{Cos}[c + d x]^{3/2}} dx$$

Optimal (type 4, 95 leaves, 5 steps):

$$-\frac{2 a (A - C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{d} + \frac{2 a (3 A + C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} + \frac{2 a A \text{Sin}[c + d x]}{d \sqrt{\text{Cos}[c + d x]}} + \frac{2 a C \sqrt{\text{Cos}[c + d x]} \text{Sin}[c + d x]}{3 d}$$

Result (type 5, 813 leaves):

$$a \left( \sqrt{\text{Cos}[c + d x]} (1 + \text{Cos}[c + d x]) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \right. \\ \left( -\frac{(-2 A + C + C \text{Cos}[2 c]) \text{Csc}[c] \text{Sec}[c]}{2 d} + \frac{C \text{Cos}[d x] \text{Sin}[c]}{3 d} + \frac{C \text{Cos}[c] \text{Sin}[d x]}{3 d} + \frac{A \text{Sec}[c] \text{Sec}[c + d x] \text{Sin}[d x]}{d} \right) - \frac{1}{d \sqrt{1 + \text{Cot}[c]^2}} \\ A (1 + \text{Cos}[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \\ \sqrt{1 - \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} - \\ \frac{1}{3 d \sqrt{1 + \text{Cot}[c]^2}} C (1 + \text{Cos}[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \right)$$

$$\begin{aligned}
& \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} + \frac{1}{2d} A (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \\
& \left( \sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \\
& \left( \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) - \frac{1}{2d} C (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \\
& \left( \sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \\
& \left( \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right)
\end{aligned}$$

■ **Problem 131: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Cos}[c + dx]) (A + C \text{Cos}[c + dx]^2)}{\text{Cos}[c + dx]^{5/2}} dx$$

Optimal (type 4, 95 leaves, 5 steps):

$$-\frac{2a(A-C)\text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{2a(A+3C)\text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} + \frac{2aA\sin[c+dx]}{3d\cos[c+dx]^{3/2}} + \frac{2aA\sin[c+dx]}{d\sqrt{\cos[c+dx]}}$$

Result (type 5, 817 leaves):

$$a \left( \sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right. \\ \left. \left( -\frac{(-2A+C+C\cos[2c])\csc[c]\sec[c]}{2d} + \frac{A\sec[c]\sec[c+dx]^2\sin[dx]}{3d} + \frac{\sec[c]\sec[c+dx](A\sin[c]+3A\sin[dx])}{3d} \right) - \frac{1}{3d\sqrt{1+\cot[c]^2}} \right. \\ \left. A(1+\cos[c+dx])\csc[c]\text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sec[dx - \text{ArcTan}[\cot[c]]] \right. \\ \left. \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} - \right. \\ \left. \frac{1}{d\sqrt{1+\cot[c]^2}} C(1+\cos[c+dx])\csc[c]\text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right. \\ \left. \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\ \left. \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} + \frac{1}{2d} A(1+\cos[c+dx])\csc[c]\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right. \\ \left. \left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \text{ArcTan}[\tan[c]]]^2\right] \sin[dx + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \right. \\ \left. \left( \sqrt{1 - \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c]\cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right. \\ \left. \left. \frac{\frac{\sin[dx + \text{ArcTan}[\tan[c]]]\tan[c]}{\sqrt{1+\tan[c]^2}} + \frac{2\cos[c]^2\cos[dx + \text{ArcTan}[\tan[c]]]\sqrt{1+\tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c]\cos[dx + \text{ArcTan}[\tan[c]]]}\sqrt{1 + \tan[c]^2}} \right) - \frac{1}{2d} C(1+\cos[c+dx])\csc[c]\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right)$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]^2 \right] \text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left( \sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \left. \frac{\frac{\text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) \right)$$

- **Problem 132: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Cos}[c + d x]) (A + C \text{Cos}[c + d x]^2)}{\text{Cos}[c + d x]^{7/2}} dx$$

Optimal (type 4, 132 leaves, 6 steps):

$$-\frac{2 a (3 A + 5 C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{2 a (A + 3 C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} + \\ \frac{2 a A \text{Sin}[c + d x]}{5 d \text{Cos}[c + d x]^{5/2}} + \frac{2 a A \text{Sin}[c + d x]}{3 d \text{Cos}[c + d x]^{3/2}} + \frac{2 a (3 A + 5 C) \text{Sin}[c + d x]}{5 d \sqrt{\text{Cos}[c + d x]}}$$

Result (type 5, 851 leaves):

$$a \left( \sqrt{\text{Cos}[c + d x]} (1 + \text{Cos}[c + d x]) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \left( \frac{(3 A + 5 C) \text{Csc}[c] \text{Sec}[c]}{5 d} + \frac{A \text{Sec}[c] \text{Sec}[c + d x]^3 \text{Sin}[d x]}{5 d} + \right. \right. \\ \left. \left. \frac{\text{Sec}[c] \text{Sec}[c + d x]^2 (3 A \text{Sin}[c] + 5 A \text{Sin}[d x])}{15 d} + \frac{\text{Sec}[c] \text{Sec}[c + d x] (5 A \text{Sin}[c] + 9 A \text{Sin}[d x] + 15 C \text{Sin}[d x])}{15 d} \right) - \frac{1}{3 d \sqrt{1 + \text{Cot}[c]^2}} \right. \\ \left. A (1 + \text{Cos}[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \right. \\ \left. \sqrt{1 - \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} - \right.$$

$$\begin{aligned}
& \frac{1}{d \sqrt{1 + \cot [c]^2}} C (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\
& \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} + \frac{1}{10 d} 3 A (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\
& \left( \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right. \\
& \left. \left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) - \right. \\
& \left. \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right) + \frac{1}{2 d} C (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\
& \left( \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right. \\
& \left. \left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) - \right. \\
& \left. \left. \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right) \right)
\end{aligned}$$

■ **Problem 133: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x]) (A + C \cos [c + d x]^2)}{\cos [c + d x]^{9/2}} dx$$

Optimal (type 4, 165 leaves, 7 steps):

$$-\frac{2a(3A+5C)\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5d} + \frac{2a(5A+7C)\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{21d} +$$

$$\frac{2aA\sin[c+dx]}{7d\cos[c+dx]^{7/2}} + \frac{2aA\sin[c+dx]}{5d\cos[c+dx]^{5/2}} + \frac{2a(5A+7C)\sin[c+dx]}{21d\cos[c+dx]^{3/2}} + \frac{2a(3A+5C)\sin[c+dx]}{5d\sqrt{\cos[c+dx]}}$$

Result (type 5, 895 leaves):

$$a \left( \sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left( \frac{(3A+5C)\operatorname{Csc}[c]\operatorname{Sec}[c]}{5d} + \frac{A\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^4\sin[dx]}{7d} + \right. \right.$$

$$\frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^3(5A\sin[c] + 7A\sin[dx])}{35d} + \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^2(21A\sin[c] + 25A\sin[dx] + 35C\sin[dx])}{105d} +$$

$$\left. \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx](25A\sin[c] + 35C\sin[c] + 63A\sin[dx] + 105C\sin[dx])}{105d} \right) - \frac{1}{21d\sqrt{1+\cot[c]^2}}$$

$$5A(1 + \cos[c+dx])\operatorname{Csc}[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]]$$

$$\sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{3d\sqrt{1 + \cot[c]^2}} C(1 + \cos[c+dx])\operatorname{Csc}[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]}$$

$$\sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} + \frac{1}{10d} 3A(1 + \cos[c+dx])\operatorname{Csc}[c]\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -$$

$$\left( \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) + \frac{1}{2d} C (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right.$$

$$\left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2}} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) \right)$$

■ **Problem 134: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{3/2} (a + a \cos[c + d x])^2 (A + C \cos[c + d x])^2 dx$$

Optimal (type 4, 230 leaves, 9 steps):

$$\frac{4 a^2 (9 A + 7 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{8 a^2 (33 A + 25 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{231 d} +$$

$$\frac{8 a^2 (33 A + 25 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{231 d} + \frac{4 a^2 (9 A + 7 C) \cos[c + d x]^{3/2} \sin[c + d x]}{45 d} + \frac{2 a^2 (99 A + 89 C) \cos[c + d x]^{5/2} \sin[c + d x]}{693 d} +$$

$$\frac{2 C \cos[c + d x]^{5/2} (a + a \cos[c + d x])^2 \sin[c + d x]}{11 d} + \frac{8 C \cos[c + d x]^{5/2} (a^2 + a^2 \cos[c + d x]) \sin[c + d x]}{99 d}$$

Result (type 5, 982 leaves):

$$\sqrt{\cos[c + d x]} (a + a \cos[c + d x])^2 \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( -\frac{(9 A + 7 C) \cot[c]}{15 d} + \frac{(1122 A + 941 C) \cos[d x] \sin[c]}{3696 d} + \frac{(18 A + 19 C) \cos[2 d x] \sin[2 c]}{180 d} + \frac{(44 A + 101 C) \cos[3 d x] \sin[3 c]}{2464 d} + \right.$$

$$\left. \frac{C \cos[4 d x] \sin[4 c]}{72 d} + \frac{C \cos[5 d x] \sin[5 c]}{352 d} + \frac{(1122 A + 941 C) \cos[c] \sin[d x]}{3696 d} + \frac{(18 A + 19 C) \cos[2 c] \sin[2 d x]}{180 d} + \right.$$



$$\begin{aligned}
& \left. \frac{(44 A + 101 C) \cos[3 c] \sin[3 d x]}{2464 d} + \frac{C \cos[4 c] \sin[4 d x]}{72 d} + \frac{C \cos[5 c] \sin[5 d x]}{352 d} \right) - \frac{1}{7 d \sqrt{1 + \cot[c]^2}} \\
2 A (a + a \cos[c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot[c]]] & \\
\sqrt{1 - \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\cot[c]]]} - & \\
\frac{1}{231 d \sqrt{1 + \cot[c]^2}} 50 C (a + a \cos[c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\cot[c]]]^2\right] & \\
\operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\cot[c]]]} & \\
\sqrt{1 + \sin[d x - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{10 d} 3 A (a + a \cos[c + d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 & \\
\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[d x + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / & \\
\left( \sqrt{1 - \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - & \\
\frac{\frac{\sin[d x + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) - \frac{1}{30 d} 7 C (a + a \cos[c + d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 & \\
\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[d x + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / & \\
\left( \sqrt{1 - \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - &
\end{aligned}$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}$$

- **Problem 135: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c + d x]} (a + a \cos[c + d x])^2 (A + C \cos[c + d x])^2 dx$$

Optimal (type 4, 197 leaves, 8 steps):

$$\frac{16 a^2 (3 A + 2 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{4 a^2 (7 A + 5 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} +$$

$$\frac{4 a^2 (7 A + 5 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{21 d} + \frac{2 a^2 (21 A + 19 C) \cos[c + d x]^{3/2} \sin[c + d x]}{105 d} +$$

$$\frac{2 C \cos[c + d x]^{3/2} (a + a \cos[c + d x])^2 \sin[c + d x]}{9 d} + \frac{8 C \cos[c + d x]^{3/2} (a^2 + a^2 \cos[c + d x]) \sin[c + d x]}{63 d}$$

Result (type 5, 936 leaves):

$$\sqrt{\cos[c + d x]} (a + a \cos[c + d x])^2 \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( -\frac{4(3A + 2C) \cot[c]}{15d} + \frac{(28A + 23C) \cos[dx] \sin[c]}{84d} + \frac{(18A + 37C) \cos[2dx] \sin[2c]}{360d} + \frac{C \cos[3dx] \sin[3c]}{28d} + \frac{C \cos[4dx] \sin[4c]}{144d} + \right.$$

$$\left. \frac{(28A + 23C) \cos[c] \sin[dx]}{84d} + \frac{(18A + 37C) \cos[2c] \sin[2dx]}{360d} + \frac{C \cos[3c] \sin[3dx]}{28d} + \frac{C \cos[4c] \sin[4dx]}{144d} \right) - \frac{1}{3d \sqrt{1 + \cot[c]^2}}$$

$$A (a + a \cos[c + d x])^2 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \sec[dx - \text{ArcTan}[\cot[c]]]$$

$$\sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{21 d \sqrt{1 + \cot[c]^2}} 5 C (a + a \cos[c + d x])^2 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right]$$

$$\sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]}$$

$$\sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} - \frac{1}{5 d} 2 A (a + a \cos[c + d x])^2 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) - \frac{1}{15 d} 4 C (a + a \cos[c + d x])^2 \text{Csc}[c] \text{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \\ \left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right)$$

■ **Problem 136: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^2 (A + C \cos[c + d x])^2}{\sqrt{\cos[c + d x]}} dx$$

Optimal (type 4, 164 leaves, 7 steps):

$$\frac{4 a^2 (5 A + 3 C) \text{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right]}{5 d} + \frac{8 a^2 (7 A + 3 C) \text{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right]}{21 d} + \frac{2 a^2 (35 A + 33 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{105 d} + \\ \frac{2 C \sqrt{\cos[c + d x]} (a + a \cos[c + d x])^2 \sin[c + d x]}{7 d} + \frac{8 C \sqrt{\cos[c + d x]} (a^2 + a^2 \cos[c + d x]) \sin[c + d x]}{35 d}$$

Result (type 5, 890 leaves):

$$\begin{aligned}
& \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \left( -\frac{(5A+3C)\cot[c]}{5d} + \frac{(28A+51C)\cos[dx]\sin[c]}{168d} + \frac{C\cos[2dx]\sin[2c]}{10d} + \right. \\
& \quad \left. \frac{C\cos[3dx]\sin[3c]}{56d} + \frac{(28A+51C)\cos[c]\sin[dx]}{168d} + \frac{C\cos[2c]\sin[2dx]}{10d} + \frac{C\cos[3c]\sin[3dx]}{56d} \right) - \frac{1}{3d\sqrt{1+\cot[c]^2}} \\
& 2A(a+a\cos[c+dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \operatorname{Sec}[dx-\operatorname{ArcTan}[\cot[c]]] \\
& \sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]} - \\
& \frac{1}{7d\sqrt{1+\cot[c]^2}} 2C(a+a\cos[c+dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \\
& \operatorname{Sec}[dx-\operatorname{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]} - \frac{1}{2d}A(a+a\cos[c+dx])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx+\operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1-\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2} \sqrt{1+\tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1+\tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx+\operatorname{ArcTan}[\tan[c]]] \sqrt{1+\tan[c]^2}}{\cos[c]^2+\sin[c]^2}}{\sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2}} \right) - \frac{1}{10d} 3C(a+a\cos[c+dx])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx+\operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1-\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2} \sqrt{1+\tan[c]^2} \right) -
\end{aligned}$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}$$

- **Problem 137: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Cos}[c + d x])^2 (A + C \text{Cos}[c + d x])^2}{\text{Cos}[c + d x]^{3/2}} dx$$

Optimal (type 4, 160 leaves, 7 steps):

$$\frac{16 a^2 C \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^2 (3 A + C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 d} - \frac{2 a^2 (15 A - 7 C) \sqrt{\text{Cos}[c + d x]} \text{Sin}[c + d x]}{15 d} + \frac{2 A (a + a \text{Cos}[c + d x])^2 \text{Sin}[c + d x]}{d \sqrt{\text{Cos}[c + d x]}} - \frac{2 (5 A - C) \sqrt{\text{Cos}[c + d x]} (a^2 + a^2 \text{Cos}[c + d x]) \text{Sin}[c + d x]}{5 d}$$

Result (type 5, 658 leaves):

$$\begin{aligned}
& \sqrt{\cos [c+d x]} (a+a \cos [c+d x])^2 \sec \left[ \frac{c}{2}+\frac{d x}{2} \right]^4 \left( -\frac{(-5 A+8 C+5 A \cos [2 c]+8 C \cos [2 c]) \csc [c] \sec [c]}{20 d}+\frac{C \cos [d x] \sin [c]}{3 d} \right. \\
& \quad \left. +\frac{C \cos [2 d x] \sin [2 c]}{20 d}+\frac{C \cos [c] \sin [d x]}{3 d}+\frac{A \sec [c] \sec [c+d x] \sin [d x]}{2 d}+\frac{C \cos [2 c] \sin [2 d x]}{20 d} \right) -\frac{1}{d \sqrt{1+\cot [c]^2}} \\
& A(a+a \cos [c+d x])^2 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \sec \left[ \frac{c}{2}+\frac{d x}{2} \right]^4 \sec [d x-\operatorname{ArcTan}[\cot [c]]] \\
& \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \\
& \frac{1}{3 d \sqrt{1+\cot [c]^2}} C(a+a \cos [c+d x])^2 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \sec \left[ \frac{c}{2}+\frac{d x}{2} \right]^4 \\
& \sec [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} -\frac{1}{5 d} 2 C(a+a \cos [c+d x])^2 \csc [c] \sec \left[ \frac{c}{2}+\frac{d x}{2} \right]^4 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \\
& \left( \sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2} \right) - \\
& \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}}
\end{aligned}$$

■ **Problem 138: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \cos [c+d x])^2 (A+C \cos [c+d x]^2)}{\cos [c+d x]^{5/2}} d x$$

Optimal (type 4, 156 leaves, 7 steps):

$$-\frac{4a^2(A-C)\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{8a^2(A+C)\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} -$$

$$\frac{2a^2(5A-C)\sqrt{\cos[c+dx]}\sin[c+dx]}{3d} + \frac{2A(a+a\cos[c+dx])^2\sin[c+dx]}{3d\cos[c+dx]^{3/2}} + \frac{8A(a^2+a^2\cos[c+dx])\sin[c+dx]}{3d\sqrt{\cos[c+dx]}}$$

Result (type 5, 865 leaves):

$$\sqrt{\cos[c+dx]}(a+a\cos[c+dx])^2\sec\left[\frac{c}{2}+\frac{dx}{2}\right]^4\left(-\frac{(-2A+C+C\cos[2c])\csc[c]\sec[c]}{2d}+\frac{C\cos[dx]\sin[c]}{6d}+\right.$$

$$\left.\frac{C\cos[c]\sin[dx]}{6d}+\frac{A\sec[c]\sec[c+dx]^2\sin[dx]}{6d}+\frac{\sec[c]\sec[c+dx](A\sin[c]+6A\sin[dx])}{6d}\right)-\frac{1}{3d\sqrt{1+\cot[c]^2}}$$

$$2A(a+a\cos[c+dx])^2\csc[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]^2\right]\sec\left[\frac{c}{2}+\frac{dx}{2}\right]^4\sec[dx-\operatorname{ArcTan}[\cot[c]]]$$

$$\sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]}\sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]}\sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]}-$$

$$\frac{1}{3d\sqrt{1+\cot[c]^2}}2C(a+a\cos[c+dx])^2\csc[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]^2\right]\sec\left[\frac{c}{2}+\frac{dx}{2}\right]^4$$

$$\sec[dx-\operatorname{ArcTan}[\cot[c]]]\sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]}\sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]}}$$

$$\sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]}+\frac{1}{2d}A(a+a\cos[c+dx])^2\csc[c]\sec\left[\frac{c}{2}+\frac{dx}{2}\right]^4$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx+\operatorname{ArcTan}[\tan[c]]]^2\right]\sin[dx+\operatorname{ArcTan}[\tan[c]]]\tan[c]\right)/$$

$$\left(\sqrt{1-\cos[dx+\operatorname{ArcTan}[\tan[c]]]}\sqrt{1+\cos[dx+\operatorname{ArcTan}[\tan[c]]]}\sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]}\sqrt{1+\tan[c]^2}\sqrt{1+\tan[c]^2}\right)-$$

$$\frac{\frac{\sin[dx+\operatorname{ArcTan}[\tan[c]]]\tan[c]}{\sqrt{1+\tan[c]^2}}+\frac{2\cos[c]^2\cos[dx+\operatorname{ArcTan}[\tan[c]]]\sqrt{1+\tan[c]^2}}{\cos[c]^2+\sin[c]^2}}{\sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]}\sqrt{1+\tan[c]^2}}-\frac{1}{2d}C(a+a\cos[c+dx])^2\csc[c]\sec\left[\frac{c}{2}+\frac{dx}{2}\right]^4$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \left( \sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \frac{\frac{\text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right)$$

- **Problem 139: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Cos}[c + d x])^2 (A + C \text{Cos}[c + d x])^2}{\text{Cos}[c + d x]^{7/2}} dx$$

Optimal (type 4, 156 leaves, 7 steps):

$$-\frac{16 a^2 A \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^2 (A + 3 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 d} + \frac{2 a^2 (17 A + 15 C) \text{Sin}[c + d x]}{15 d \sqrt{\text{Cos}[c + d x]}} + \frac{2 A (a + a \text{Cos}[c + d x])^2 \text{Sin}[c + d x]}{5 d \text{Cos}[c + d x]^{5/2}} + \frac{8 A (a^2 + a^2 \text{Cos}[c + d x]) \text{Sin}[c + d x]}{15 d \text{Cos}[c + d x]^{3/2}}$$

Result (type 5, 656 leaves):



$$\begin{aligned}
& \sqrt{\cos [c+d x]} (a+a \cos [c+d x])^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4\left(-\frac{(-16 A-5 C+5 C \cos [2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{20 d}+\frac{A \operatorname{Sec}[c] \operatorname{Sec}[c+d x]^3 \sin [d x]}{10 d}+\right. \\
& \left.\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2(3 A \sin [c]+10 A \sin [d x])}{30 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x](10 A \sin [c]+24 A \sin [d x]+15 C \sin [d x])}{30 d}\right)-\frac{1}{3 d \sqrt{1+\cot [c]^2}} \\
& A(a+a \cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \\
& \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}- \\
& \frac{1}{d \sqrt{1+\cot [c]^2}} C(a+a \cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \\
& \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}+\frac{1}{5 d} 2 A(a+a \cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \\
& \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right)- \\
& \left.\frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}}\right)
\end{aligned}$$

- **Problem 140: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \cos [c+d x])^2 (A+C \cos [c+d x])^2}{\cos [c+d x]^{9/2}} d x$$

Optimal (type 4, 197 leaves, 8 steps):

$$\begin{aligned}
& - \frac{4 a^2 (3 A + 5 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{8 a^2 (3 A + 7 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \frac{2 a^2 (33 A + 35 C) \operatorname{Sin}[c + d x]}{105 d \operatorname{Cos}[c + d x]^{3/2}} + \\
& \frac{4 a^2 (3 A + 5 C) \operatorname{Sin}[c + d x]}{5 d \sqrt{\operatorname{Cos}[c + d x]}} + \frac{2 A (a + a \operatorname{Cos}[c + d x])^2 \operatorname{Sin}[c + d x]}{7 d \operatorname{Cos}[c + d x]^{7/2}} + \frac{8 A (a^2 + a^2 \operatorname{Cos}[c + d x]) \operatorname{Sin}[c + d x]}{35 d \operatorname{Cos}[c + d x]^{5/2}}
\end{aligned}$$

Result(type 5, 913 leaves):

$$\begin{aligned}
& \sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \left( \frac{(3 A + 5 C) \operatorname{Csc}[c] \operatorname{Sec}[c]}{5 d} + \frac{A \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^4 \operatorname{Sin}[d x]}{14 d} + \right. \\
& \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 (5 A \operatorname{Sin}[c] + 14 A \operatorname{Sin}[d x])}{70 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (42 A \operatorname{Sin}[c] + 60 A \operatorname{Sin}[d x] + 35 C \operatorname{Sin}[d x])}{210 d} + \\
& \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x] (60 A \operatorname{Sin}[c] + 35 C \operatorname{Sin}[c] + 126 A \operatorname{Sin}[d x] + 210 C \operatorname{Sin}[d x])}{210 d} \right) - \frac{1}{7 d \sqrt{1 + \operatorname{Cot}[c]^2}} \\
& 2 A (a + a \operatorname{Cos}[c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\
& \frac{1}{3 d \sqrt{1 + \operatorname{Cot}[c]^2}} 2 C (a + a \operatorname{Cos}[c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \\
& \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \frac{1}{10 d} 3 A (a + a \operatorname{Cos}[c + d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\
& \left( \sqrt{1 - \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \\
& \frac{\frac{\operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \operatorname{Cos}[c]^2 \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2}}{\sqrt{\operatorname{Cos}[c] \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{1}{2 d} C (a + a \operatorname{Cos}[c + d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4
\end{aligned}$$

$$\left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]\right) / \right. \\ \left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right. \\ \left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right)$$

■ **Problem 141: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^2 (A + C \cos[c + d x])^2}{\cos[c + d x]^{11/2}} dx$$

Optimal (type 4, 230 leaves, 9 steps):

$$-\frac{16 a^2 (2 A + 3 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{4 a^2 (5 A + 7 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \frac{2 a^2 (19 A + 21 C) \sin[c + d x]}{105 d \cos[c + d x]^{5/2}} + \\ \frac{4 a^2 (5 A + 7 C) \sin[c + d x]}{21 d \cos[c + d x]^{3/2}} + \frac{16 a^2 (2 A + 3 C) \sin[c + d x]}{15 d \sqrt{\cos[c + d x]}} + \frac{2 A (a + a \cos[c + d x])^2 \sin[c + d x]}{9 d \cos[c + d x]^{9/2}} + \frac{8 A (a^2 + a^2 \cos[c + d x]) \sin[c + d x]}{63 d \cos[c + d x]^{7/2}}$$

Result (type 5, 955 leaves):

$$\sqrt{\cos[c + d x]} (a + a \cos[c + d x])^2 \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \left( \frac{4 (2 A + 3 C) \csc[c] \sec[c]}{15 d} + \frac{A \sec[c] \sec[c + d x]^5 \sin[d x]}{18 d} + \right. \\ \frac{\sec[c] \sec[c + d x]^4 (7 A \sin[c] + 18 A \sin[d x])}{126 d} + \frac{\sec[c] \sec[c + d x]^3 (90 A \sin[c] + 112 A \sin[d x] + 63 C \sin[d x])}{630 d} + \\ \frac{\sec[c] \sec[c + d x] (25 A \sin[c] + 35 C \sin[c] + 56 A \sin[d x] + 84 C \sin[d x])}{105 d} + \\ \left. \frac{\sec[c] \sec[c + d x]^2 (112 A \sin[c] + 63 C \sin[c] + 150 A \sin[d x] + 210 C \sin[d x])}{630 d} \right) - \\ \frac{1}{21 d \sqrt{1 + \cot[c]^2}} 5 A (a + a \cos[c + d x])^2 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \\ \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]}$$

$$\begin{aligned}
& \sqrt{-\sqrt{1+\cot^2[c]} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{3d\sqrt{1+\cot^2[c]}} \\
& C (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\
& \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot^2[c]} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} + \frac{1}{15d} 4A (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos^2[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos^2[c] + \sin^2[c]}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} + \frac{1}{5d} 2C (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos^2[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos^2[c] + \sin^2[c]}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}}
\end{aligned}$$

■ **Problem 142: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^{3 / 2} (a+a \cos [c+d x])^3 (A+C \cos [c+d x])^2 d x$$

Optimal (type 4, 279 leaves, 10 steps):

$$\frac{4 a^3 (221 A+175 C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{195 d} + \frac{4 a^3 (121 A+95 C) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{231 d} +$$

$$\frac{4 a^3 (121 A+95 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{231 d} + \frac{4 a^3 (221 A+175 C) \cos [c+d x]^{3 / 2} \sin [c+d x]}{585 d} +$$

$$\frac{40 a^3 (143 A+118 C) \cos [c+d x]^{5 / 2} \sin [c+d x]}{9009 d} + \frac{2 C \cos [c+d x]^{5 / 2} (a+a \cos [c+d x])^3 \sin [c+d x]}{13 d} +$$

$$\frac{12 C \cos [c+d x]^{5 / 2} (a^2+a^2 \cos [c+d x])^2 \sin [c+d x]}{143 a d} + \frac{2 (143 A+145 C) \cos [c+d x]^{5 / 2} (a^3+a^3 \cos [c+d x]) \sin [c+d x]}{1287 d}$$

Result (type 5, 1028 leaves):

$$\sqrt{\cos [c+d x]} (a+a \cos [c+d x])^3 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6$$

$$\left(-\frac{(221 A+175 C) \cot [c]}{390 d}+\frac{(2134 A+1811 C) \cos [d x] \sin [c]}{7392 d}+\frac{(7592 A+7825 C) \cos [2 d x] \sin [2 c]}{74880 d}+\right.$$

$$\frac{(132 A+215 C) \cos [3 d x] \sin [3 c]}{4928 d}+\frac{(13 A+59 C) \cos [4 d x] \sin [4 c]}{3744 d}+\frac{3 C \cos [5 d x] \sin [5 c]}{704 d}+$$

$$\frac{C \cos [6 d x] \sin [6 c]}{1664 d}+\frac{(2134 A+1811 C) \cos [c] \sin [d x]}{7392 d}+\frac{(7592 A+7825 C) \cos [2 c] \sin [2 d x]}{74880 d}+$$

$$\left.\frac{(132 A+215 C) \cos [3 c] \sin [3 d x]}{4928 d}+\frac{(13 A+59 C) \cos [4 c] \sin [4 d x]}{3744 d}+\frac{3 C \cos [5 c] \sin [5 d x]}{704 d}+\frac{C \cos [6 c] \sin [6 d x]}{1664 d}\right)-$$

$$\frac{1}{42 d \sqrt{1+\cot [c]^2}} 11 A (a+a \cos [c+d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \frac{1}{462 d \sqrt{1+\cot [c]^2}}$$

$$95 C (a+a \cos [c+d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6$$

$$\operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \frac{1}{60 d} 17 A (a+a \cos [c+d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2 \right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \\
\left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right. \\
\left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) - \frac{1}{156 d} 35 C (a + a \cos[c + d x])^3 \text{Csc}[c] \text{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \\
\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2 \right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \\
\left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right. \\
\left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right)$$

■ **Problem 143: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c + d x]} (a + a \cos[c + d x])^3 (A + C \cos[c + d x]^2) dx$$

Optimal (type 4, 246 leaves, 9 steps):

$$\frac{4 a^3 (7 A + 5 C) \text{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right]}{5 d} + \frac{4 a^3 (143 A + 105 C) \text{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right]}{231 d} + \frac{4 a^3 (143 A + 105 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{231 d} + \\
\frac{8 a^3 (44 A + 35 C) \cos[c + d x]^{3/2} \sin[c + d x]}{385 d} + \frac{2 C \cos[c + d x]^{3/2} (a + a \cos[c + d x])^3 \sin[c + d x]}{11 d} + \\
\frac{4 C \cos[c + d x]^{3/2} (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{33 a d} + \frac{2 (33 A + 35 C) \cos[c + d x]^{3/2} (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{231 d}$$

Result (type 5, 982 leaves) :

$$\begin{aligned}
 & \sqrt{\cos [c+d x]} (a+a \cos [c+d x])^3 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 \\
 & \left( -\frac{(7 A+5 C) \cot [c]}{10 d} + \frac{(2354 A+1953 C) \cos [d x] \sin [c]}{7392 d} + \frac{(18 A+25 C) \cos [2 d x] \sin [2 c]}{240 d} + \frac{(44 A+189 C) \cos [3 d x] \sin [3 c]}{4928 d} + \right. \\
 & \frac{C \cos [4 d x] \sin [4 c]}{96 d} + \frac{C \cos [5 d x] \sin [5 c]}{704 d} + \frac{(2354 A+1953 C) \cos [c] \sin [d x]}{7392 d} + \frac{(18 A+25 C) \cos [2 c] \sin [2 d x]}{240 d} + \\
 & \left. \frac{(44 A+189 C) \cos [3 c] \sin [3 d x]}{4928 d} + \frac{C \cos [4 c] \sin [4 d x]}{96 d} + \frac{C \cos [5 c] \sin [5 d x]}{704 d} \right) - \frac{1}{42 d \sqrt{1+\cot [c]^2}} \\
 & 13 A (a+a \cos [c+d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \\
 & \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \\
 & \frac{1}{22 d \sqrt{1+\cot [c]^2}} 5 C (a+a \cos [c+d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \\
 & \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\
 & \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \frac{1}{20 d} 7 A (a+a \cos [c+d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 \\
 & \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \\
 & \left( \sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2} \right) - \\
 & \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}} \right) - \frac{1}{4 d} C (a+a \cos [c+d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6
 \end{aligned}$$

$$\left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \\ \left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right. \\ \left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right)$$

- **Problem 144: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^3 (A + C \cos[c + d x])^2}{\sqrt{\cos[c + d x]}} dx$$

Optimal (type 4, 213 leaves, 8 steps):

$$\frac{4 a^3 (27 A + 17 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{4 a^3 (21 A + 11 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \\ \frac{8 a^3 (21 A + 16 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{105 d} + \frac{2 C \sqrt{\cos[c + d x]} (a + a \cos[c + d x])^3 \sin[c + d x]}{9 d} + \\ \frac{4 C \sqrt{\cos[c + d x]} (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{21 a d} + \frac{2 (63 A + 73 C) \sqrt{\cos[c + d x]} (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{315 d}$$

Result (type 5, 936 leaves):

$$\sqrt{\cos[c + d x]} (a + a \cos[c + d x])^3 \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\ \left( -\frac{(27 A + 17 C) \cot[c]}{30 d} + \frac{(84 A + 97 C) \cos[d x] \sin[c]}{336 d} + \frac{(18 A + 73 C) \cos[2 d x] \sin[2 c]}{720 d} + \frac{3 C \cos[3 d x] \sin[3 c]}{112 d} + \frac{C \cos[4 d x] \sin[4 c]}{288 d} + \right. \\ \left. \frac{(84 A + 97 C) \cos[c] \sin[d x]}{336 d} + \frac{(18 A + 73 C) \cos[2 c] \sin[2 d x]}{720 d} + \frac{3 C \cos[3 c] \sin[3 d x]}{112 d} + \frac{C \cos[4 c] \sin[4 d x]}{288 d} \right) - \frac{1}{2 d \sqrt{1 + \cot[c]^2}}$$

$$A (a + a \cos[c + d x])^3 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \sec[d x - \text{ArcTan}[\cot[c]]]$$

$$\sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -$$



$$\frac{1}{42 d \sqrt{1 + \cot [c]^2}} {}_{11}C (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \frac{1}{20 d} {}_9A (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} - \frac{1}{60 d} {}_{17}C (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}}$$

■ **Problem 145: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^3 (A + C \cos [c + d x]^2)}{\cos [c + d x]^{3/2}} dx$$

Optimal (type 4, 217 leaves, 8 steps):

$$\frac{4 a^3 (5 A + 7 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^3 (35 A + 13 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} -$$

$$\frac{4 a^3 (35 A - 41 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{105 d} + \frac{2 A (a + a \cos[c + d x])^3 \sin[c + d x]}{d \sqrt{\cos[c + d x]}} -$$

$$\frac{2 (7 A - C) \sqrt{\cos[c + d x]} (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{7 a d} - \frac{2 (35 A - 11 C) \sqrt{\cos[c + d x]} (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{35 d}$$

Result (type 5, 926 leaves):

$$\sqrt{\cos[c + d x]} (a + a \cos[c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( -\frac{(5 A + 14 C + 15 A \cos[2 c] + 14 C \cos[2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{40 d} + \frac{(28 A + 107 C) \cos[d x] \sin[c]}{336 d} + \frac{3 C \cos[2 d x] \sin[2 c]}{40 d} + \frac{C \cos[3 d x] \sin[3 c]}{112 d} + \right.$$

$$\left. \frac{(28 A + 107 C) \cos[c] \sin[d x]}{336 d} + \frac{A \operatorname{Sec}[c] \operatorname{Sec}[c + d x] \sin[d x]}{4 d} + \frac{3 C \cos[2 c] \sin[2 d x]}{40 d} + \frac{C \cos[3 c] \sin[3 d x]}{112 d} \right) - \frac{1}{6 d \sqrt{1 + \cot[c]^2}}$$

$$5 A (a + a \cos[c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot[c]]]$$

$$\sqrt{1 - \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{42 d \sqrt{1 + \cot[c]^2}} 13 C (a + a \cos[c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\cot[c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\cot[c]]]}$$

$$\sqrt{1 + \sin[d x - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{4 d} A (a + a \cos[c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[d x + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left( \sqrt{1 - \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -$$

$$\left( \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) - \frac{1}{20 d} 7 C (a + a \cos[c + d x])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \sqrt{1 + \tan[c]^2} \right) -$$

$$\left( \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right)$$

■ **Problem 146: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^3 (A + C \cos[c + d x])^2}{\cos[c + d x]^{5/2}} dx$$

Optimal (type 4, 211 leaves, 8 steps):

$$-\frac{4 a^3 (5 A - 9 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^3 (5 A + 3 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 d} - \frac{8 a^3 (10 A - 3 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{15 d} +$$

$$\frac{2 A (a + a \cos[c + d x])^3 \sin[c + d x]}{3 d \cos[c + d x]^{3/2}} + \frac{4 A (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{a d \sqrt{\cos[c + d x]}} - \frac{2 (35 A - 3 C) \sqrt{\cos[c + d x]} (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{15 d}$$

Result (type 5, 909 leaves):

$$\sqrt{\cos[c + d x]} (a + a \cos[c + d x])^3 \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( -\frac{(-25 A + 18 C + 5 A \cos[2 c] + 18 C \cos[2 c]) \text{Csc}[c] \text{Sec}[c]}{40 d} + \frac{C \cos[d x] \sin[c]}{4 d} + \frac{C \cos[2 d x] \sin[2 c]}{40 d} + \frac{C \cos[c] \sin[d x]}{4 d} + \right.$$

$$\left. \frac{A \text{Sec}[c] \text{Sec}[c + d x]^2 \sin[d x]}{12 d} + \frac{\text{Sec}[c] \text{Sec}[c + d x] (A \sin[c] + 9 A \sin[d x])}{12 d} + \frac{C \cos[2 c] \sin[2 d x]}{40 d} \right) - \frac{1}{6 d \sqrt{1 + \cot[c]^2}}$$

$$\begin{aligned}
& 5 A (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\
& \frac{1}{2 d \sqrt{1 + \operatorname{Cot}[c]^2}} C (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \frac{1}{4 d} A (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\
& \left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \\
& \frac{\frac{\sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}} - \frac{1}{20 d} 9 C (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\
& \left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \\
& \frac{\frac{\sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}}
\end{aligned}$$

**Problem 147: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx])^3 (A + C \cos[c + dx])^2}{\cos[c + dx]^{7/2}} dx$$

Optimal (type 4, 213 leaves, 8 steps):

$$\begin{aligned} & -\frac{4a^3(9A-5C)\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5d} + \frac{4a^3(3A+5C)\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} - \frac{4a^3(21A+5C)\sqrt{\cos[c+dx]}\sin[c+dx]}{15d} + \\ & \frac{2A(a+a\cos[c+dx])^3\sin[c+dx]}{5d\cos[c+dx]^{5/2}} + \frac{4A(a^2+a^2\cos[c+dx])^2\sin[c+dx]}{5ad\cos[c+dx]^{3/2}} + \frac{2(11A+5C)(a^3+a^3\cos[c+dx])\sin[c+dx]}{5d\sqrt{\cos[c+dx]}} \end{aligned}$$

Result (type 5, 905 leaves):

$$\begin{aligned} & \sqrt{\cos[c+dx]}(a+a\cos[c+dx])^3\sec\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \\ & \left( -\frac{(-36A+5C+15C\cos[2c])\csc[c]\sec[c]}{40d} + \frac{C\cos[dx]\sin[c]}{12d} + \frac{C\cos[c]\sin[dx]}{12d} + \frac{A\sec[c]\sec[c+dx]^3\sin[dx]}{20d} + \right. \\ & \left. \frac{\sec[c]\sec[c+dx]^2(A\sin[c]+5A\sin[dx])}{20d} + \frac{\sec[c]\sec[c+dx](5A\sin[c]+18A\sin[dx]+5C\sin[dx])}{20d} \right) - \frac{1}{2d\sqrt{1+\cot[c]^2}} \\ & A(a+a\cos[c+dx])^3\csc[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]^2\right]\sec\left[\frac{c}{2}+\frac{dx}{2}\right]^6\sec[dx-\operatorname{ArcTan}[\cot[c]]] \\ & \sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]}\sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]}\sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]} - \\ & \frac{1}{6d\sqrt{1+\cot[c]^2}}5C(a+a\cos[c+dx])^3\csc[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]^2\right]\sec\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \\ & \sec[dx-\operatorname{ArcTan}[\cot[c]]]\sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]}\sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \\ & \sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]} + \frac{1}{20d}9A(a+a\cos[c+dx])^3\csc[c]\sec\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \\ & \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx+\operatorname{ArcTan}[\tan[c]]]^2\right]\sin[dx+\operatorname{ArcTan}[\tan[c]]]\tan[c] \right) / \\ & \left( \sqrt{1-\cos[dx+\operatorname{ArcTan}[\tan[c]]]}\sqrt{1+\cos[dx+\operatorname{ArcTan}[\tan[c]]]}\sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]}\sqrt{1+\tan[c]^2}\sqrt{1+\tan[c]^2} \right) - \end{aligned}$$

$$\left( \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) - \frac{1}{4 d} C (a + a \cos[c + d x])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right.$$

$$\left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2}} \right) - \right.$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right)$$

■ **Problem 148: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^3 (A + C \cos[c + d x])^2}{\cos[c + d x]^{9/2}} dx$$

Optimal (type 4, 213 leaves, 8 steps):

$$-\frac{4 a^3 (7 A + 5 C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{4 a^3 (13 A + 35 C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{21 d} + \frac{8 a^3 (53 A + 70 C) \sin[c + d x]}{105 d \sqrt{\cos[c + d x]}} +$$

$$\frac{2 A (a + a \cos[c + d x])^3 \sin[c + d x]}{7 d \cos[c + d x]^{7/2}} + \frac{12 A (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{35 a d \cos[c + d x]^{5/2}} + \frac{2 (7 A + 5 C) (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{15 d \cos[c + d x]^{3/2}}$$

Result (type 5, 920 leaves):

$$\sqrt{\cos[c + d x]} (a + a \cos[c + d x])^3 \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \left( -\frac{(-28 A - 25 C + 5 C \cos[2 c]) \text{Csc}[c] \text{Sec}[c]}{40 d} + \frac{A \text{Sec}[c] \text{Sec}[c + d x]^4 \sin[d x]}{28 d} \right. +$$

$$\frac{\text{Sec}[c] \text{Sec}[c + d x]^3 (5 A \sin[c] + 21 A \sin[d x])}{140 d} + \frac{\text{Sec}[c] \text{Sec}[c + d x]^2 (63 A \sin[c] + 130 A \sin[d x] + 35 C \sin[d x])}{420 d} +$$

$$\left. \frac{\text{Sec}[c] \text{Sec}[c + d x] (130 A \sin[c] + 35 C \sin[c] + 294 A \sin[d x] + 315 C \sin[d x])}{420 d} \right) - \frac{1}{42 d \sqrt{1 + \cot[c]^2}}$$

$$\begin{aligned}
& 13 A (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\
& \frac{1}{6 d \sqrt{1 + \operatorname{Cot}[c]^2}} 5 C (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \frac{1}{20 d} 7 A (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\
& \left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \\
& \frac{\frac{\sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{1}{4 d} C (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\
& \left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \\
& \frac{\frac{\sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}}
\end{aligned}$$

**Problem 149: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx])^3 (A + C \cos[c + dx])^2}{\cos[c + dx]^{11/2}} dx$$

Optimal (type 4, 246 leaves, 9 steps):

$$\begin{aligned} & - \frac{4 a^3 (17 A + 27 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{15 d} + \frac{4 a^3 (11 A + 21 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{21 d} + \\ & \frac{8 a^3 (16 A + 21 C) \sin[c + dx]}{105 d \cos[c + dx]^{3/2}} + \frac{4 a^3 (17 A + 27 C) \sin[c + dx]}{15 d \sqrt{\cos[c + dx]}} + \frac{2 A (a + a \cos[c + dx])^3 \sin[c + dx]}{9 d \cos[c + dx]^{9/2}} + \\ & \frac{4 A (a^2 + a^2 \cos[c + dx])^2 \sin[c + dx]}{21 a d \cos[c + dx]^{7/2}} + \frac{2 (73 A + 63 C) (a^3 + a^3 \cos[c + dx]) \sin[c + dx]}{315 d \cos[c + dx]^{5/2}} \end{aligned}$$

Result (type 5, 955 leaves):

$$\begin{aligned} & \sqrt{\cos[c + dx]} (a + a \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \left( \frac{(17 A + 27 C) \operatorname{Csc}[c] \operatorname{Sec}[c]}{30 d} + \frac{A \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^5 \sin[dx]}{36 d} + \right. \\ & \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + dx]^4 (7 A \sin[c] + 27 A \sin[dx])}{252 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + dx]^3 (135 A \sin[c] + 238 A \sin[dx] + 63 C \sin[dx])}{1260 d} + \\ & \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 (238 A \sin[c] + 63 C \sin[c] + 330 A \sin[dx] + 315 C \sin[dx])}{1260 d} + \\ & \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + dx] (110 A \sin[c] + 105 C \sin[c] + 238 A \sin[dx] + 378 C \sin[dx])}{420 d} \right) - \\ & \frac{1}{42 d \sqrt{1 + \cot[c]^2}} 11 A (a + a \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \\ & \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\ & \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{2 d \sqrt{1 + \cot[c]^2}} \\ & C (a + a \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \\ & \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\ & \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} + \frac{1}{60 d} 17 A (a + a \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \end{aligned}$$



$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) / \right. \\
\left. \left( \sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) - \right. \\
\left. \frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \right) + \frac{1}{20 d} 9 C (a + a \cos [c + d x])^3 \text{Csc} [c] \text{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \\
\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) / \right. \\
\left. \left( \sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) - \right. \\
\left. \frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \right)$$

■ **Problem 150: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^3 (A + C \cos [c + d x]^2)}{\cos [c + d x]^{13/2}} dx$$

Optimal (type 4, 279 leaves, 10 steps):

$$\begin{aligned}
& - \frac{4 a^3 (5 A + 7 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^3 (105 A + 143 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{231 d} + \\
& \frac{8 a^3 (35 A + 44 C) \operatorname{Sin}[c + d x]}{385 d \operatorname{Cos}[c + d x]^{5/2}} + \frac{4 a^3 (105 A + 143 C) \operatorname{Sin}[c + d x]}{231 d \operatorname{Cos}[c + d x]^{3/2}} + \frac{4 a^3 (5 A + 7 C) \operatorname{Sin}[c + d x]}{5 d \sqrt{\operatorname{Cos}[c + d x]}} + \\
& \frac{2 A (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Sin}[c + d x]}{11 d \operatorname{Cos}[c + d x]^{11/2}} + \frac{4 A (a^2 + a^2 \operatorname{Cos}[c + d x])^2 \operatorname{Sin}[c + d x]}{33 a d \operatorname{Cos}[c + d x]^{9/2}} + \frac{2 (35 A + 33 C) (a^3 + a^3 \operatorname{Cos}[c + d x]) \operatorname{Sin}[c + d x]}{231 d \operatorname{Cos}[c + d x]^{7/2}}
\end{aligned}$$

Result (type 5, 997 leaves):

$$\begin{aligned}
& \sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \left( \frac{(5 A + 7 C) \operatorname{Csc}[c] \operatorname{Sec}[c]}{10 d} + \frac{A \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^6 \operatorname{Sin}[d x]}{44 d} + \right. \\
& \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^5 (3 A \operatorname{Sin}[c] + 11 A \operatorname{Sin}[d x])}{132 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^4 (77 A \operatorname{Sin}[c] + 126 A \operatorname{Sin}[d x] + 33 C \operatorname{Sin}[d x])}{924 d} + \\
& \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 (630 A \operatorname{Sin}[c] + 165 C \operatorname{Sin}[c] + 770 A \operatorname{Sin}[d x] + 693 C \operatorname{Sin}[d x])}{4620 d} + \\
& \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (770 A \operatorname{Sin}[c] + 693 C \operatorname{Sin}[c] + 1050 A \operatorname{Sin}[d x] + 1430 C \operatorname{Sin}[d x])}{4620 d} + \\
& \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x] (525 A \operatorname{Sin}[c] + 715 C \operatorname{Sin}[c] + 1155 A \operatorname{Sin}[d x] + 1617 C \operatorname{Sin}[d x])}{2310 d} \right) - \\
& \frac{1}{22 d \sqrt{1 + \operatorname{Cot}[c]^2}} 5 A (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \frac{1}{42 d \sqrt{1 + \operatorname{Cot}[c]^2}} \\
& 13 C (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \frac{1}{4 d} A (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /
\end{aligned}$$

$$\left( \frac{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}} \right) + \frac{1}{20 d} 7 C (a + a \cos [c + d x])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \text{ArcTan}[\text{Tan}[c]]]^2\right\} \sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c] \right) / \right.$$

$$\left. \frac{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}} \right)$$

■ **Problem 151: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^{5/2} (A + C \cos [c + d x]^2)}{a + a \cos [c + d x]} dx$$

Optimal (type 4, 192 leaves, 7 steps):

$$-\frac{3(5A+7C) \text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5ad} + \frac{5(7A+9C) \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{21ad} + \frac{5(7A+9C) \sqrt{\cos [c+dx]} \sin [c+dx]}{21ad} - \frac{(5A+7C) \cos [c+dx]^{3/2} \sin [c+dx]}{5ad} + \frac{(7A+9C) \cos [c+dx]^{5/2} \sin [c+dx]}{7ad} - \frac{(A+C) \cos [c+dx]^{7/2} \sin [c+dx]}{d(a+a \cos [c+dx])}$$

Result (type 5, 1219 leaves):

$$-\frac{1}{4(a+a \cos [c+dx])} 3iA \cos \left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos [c] + i \sin [c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos [c] + 2i(-1+e^{2idx}) \sin [c])} \right) \right.$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(3 i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] - 3 d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right) -} \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(-i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right) - \right. \\
& \frac{1}{20 \left(a + a \operatorname{Cos}[c + d x]\right)} 21 i C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left. \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right. \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(3 i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] - 3 d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right) -} \right. \right. \\
& \left. \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right. \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(-i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right) + \frac{1}{a + a \operatorname{Cos}[c + d x]} \right. \\
& \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sqrt{\operatorname{Cos}[c + d x]} \left(\frac{2 \left(5 A + 5 C + 10 A \operatorname{Cos}[c] + 16 C \operatorname{Cos}[c]\right) \operatorname{Csc}[c]}{5 d} + \frac{\left(28 A + 51 C\right) \operatorname{Cos}[d x] \operatorname{Sin}[c]}{21 d} - \right. \\
& \frac{2 C \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{5 d} + \frac{C \operatorname{Cos}[3 d x] \operatorname{Sin}[3 c]}{7 d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] \left(A \operatorname{Sin}\left[\frac{d x}{2}\right] + C \operatorname{Sin}\left[\frac{d x}{2}\right]\right)}{d} + \\
& \left. \frac{\left(28 A + 51 C\right) \operatorname{Cos}[c] \operatorname{Sin}[d x]}{21 d} - \frac{2 C \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x]}{5 d} + \frac{C \operatorname{Cos}[3 c] \operatorname{Sin}[3 d x]}{7 d}\right) - \\
& \left(5 A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \left. \frac{\sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}}{\left(3 d \left(a + a \operatorname{Cos}[c + d x]\right) \sqrt{1 + \operatorname{Cot}[c]^2}\right)}\right) - \\
& \left(15 C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \left. \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}\right)
\end{aligned}$$

$$\sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \Bigg/ \left( 7d (a + a \cos[c + dx]) \sqrt{1 + \text{Cot}[c]^2} \right)$$

- **Problem 152: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{3/2} (A + C \cos[c + dx]^2)}{a + a \cos[c + dx]} dx$$

Optimal (type 4, 159 leaves, 6 steps):

$$\frac{3(5A + 7C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5ad} - \frac{(3A + 5C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3ad} - \frac{(3A + 5C) \sqrt{\cos[c + dx]} \sin[c + dx]}{3ad} + \frac{(5A + 7C) \cos[c + dx]^{3/2} \sin[c + dx]}{5ad} - \frac{(A + C) \cos[c + dx]^{5/2} \sin[c + dx]}{d(a + a \cos[c + dx])}$$

Result (type 5, 1170 leaves):

$$\frac{1}{4(a + a \cos[c + dx])} - 3iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \\ \left( \left( 2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) \Bigg/ (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ \left. \left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) \Bigg/ (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \frac{1}{20(a + a \cos[c + dx])} 21iC \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\ \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) \Bigg/ (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ \left. \left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) \Bigg/ (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \frac{1}{a + a \cos[c + dx]} \\ \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \left( -\frac{2(5A + 5C + 10A \cos[c] + 16C \cos[c]) \text{Csc}[c]}{5d} - \frac{4C \cos[dx] \sin[c]}{3d} + \frac{2C \cos[2dx] \sin[2c]}{5d} - \right.$$

$$\left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(A \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right]\right)}{d} - \frac{4 C \operatorname{Cos}[c] \operatorname{Sin}[dx]}{3 d} + \frac{2 C \operatorname{Cos}[2 c] \operatorname{Sin}[2 dx]}{5 d} \right) +$$

$$\left( A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]$$

$$\left. \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) /$$

$$\left( d (a + a \operatorname{Cos}[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \left( 5 C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right)$$

$$\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}$$

$$\left. \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \operatorname{Cos}[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right)$$

- **Problem 153: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Cos}[c + dx]} (A + C \operatorname{Cos}[c + dx]^2)}{a + a \operatorname{Cos}[c + dx]} dx$$

Optimal (type 4, 122 leaves, 5 steps):

$$- \frac{(A + 3 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{a d} + \frac{(3 A + 5 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3 a d} +$$

$$\frac{(3 A + 5 C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{3 a d} - \frac{(A + C) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{d (a + a \operatorname{Cos}[c + dx])}$$

Result (type 5, 1126 leaves):

$$- \frac{1}{4 (a + a \operatorname{Cos}[c + dx])} i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \right.$$

$$\left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right) \right)$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{(-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c])} \right) - \frac{1}{4 (a + a \cos[c + d x])} 3 i C \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\
& \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{(3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c])} - \right. \right. \\
& \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{(-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c])} \right) \right) + \\
& \frac{1}{a + a \cos[c + d x]} \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sqrt{\cos[c + d x]} \left( \frac{2 (A + C + 2 C \cos[c]) \operatorname{Csc}[c]}{d} + \frac{4 C \cos[d x] \sin[c]}{3 d} + \right. \\
& \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (A \sin\left[\frac{d x}{2}\right] + C \sin\left[\frac{d x}{2}\right])}{d} + \frac{4 C \cos[c] \sin[d x]}{3 d} \right) - \\
& \left( A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right) \\
& \frac{\sqrt{1 - \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \left. \frac{\sqrt{1 + \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) \left/ \left( d (a + a \cos[c + d x]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \right. \\
& \left( 5 C \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right) \\
& \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \left. \frac{\sqrt{1 + \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) \left/ \left( 3 d (a + a \cos[c + d x]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) \right)
\end{aligned}$$

■ **Problem 154: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c + d x]^2}{\sqrt{\cos[c + d x]} (a + a \cos[c + d x])} dx$$

Optimal (type 4, 83 leaves, 4 steps):

$$\frac{(A+3C) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{ad} + \frac{(A-C) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{ad} - \frac{(A+C) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{d(a+a \operatorname{Cos}[c+dx])}$$

Result (type 5, 1095 leaves):

$$\frac{1}{4(a+a \operatorname{Cos}[c+dx])} i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]}\right) / (3id(1+e^{2idx}) \operatorname{Cos}[c] - 3d(-1+e^{2idx}) \operatorname{Sin}[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]}\right) / (-id(1+e^{2idx}) \operatorname{Cos}[c] + d(-1+e^{2idx}) \operatorname{Sin}[c]) \right) + \frac{1}{4(a+a \operatorname{Cos}[c+dx])} 3iC \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]}\right) / (3id(1+e^{2idx}) \operatorname{Cos}[c] - 3d(-1+e^{2idx}) \operatorname{Sin}[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]}\right) / (-id(1+e^{2idx}) \operatorname{Cos}[c] + d(-1+e^{2idx}) \operatorname{Sin}[c]) \right) + \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\operatorname{Cos}[c+dx]} \left( -\frac{2(A+C+2C \operatorname{Cos}[c]) \operatorname{Csc}[c]}{d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c+dx}{2}\right] (A \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right])}{d} \right) \right) - \left( A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / (d(a+a \operatorname{Cos}[c+dx]) \sqrt{1 + \operatorname{Cot}[c]^2}) + \left( C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)$$



$$\frac{\sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]}}{\left( d (a + a \cos[c + dx]) \sqrt{1 + \text{Cot}[c]^2} \right)}$$

■ **Problem 155: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c + dx]^2}{\cos[c + dx]^{3/2} (a + a \cos[c + dx])} dx$$

Optimal (type 4, 113 leaves, 5 steps):

$$-\frac{(3A + C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} - \frac{(A - C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{(3A + C) \sin[c + dx]}{ad \sqrt{\cos[c + dx]}} - \frac{(A + C) \sin[c + dx]}{d \sqrt{\cos[c + dx]} (a + a \cos[c + dx])}$$

Result (type 5, 1128 leaves):

$$\begin{aligned} & -\frac{1}{4(a + a \cos[c + dx])} 3iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \\ & \left( \left( 2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ & \left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) - \frac{1}{4(a + a \cos[c + dx])} iC \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\ & \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ & \left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \\ & \frac{1}{a + a \cos[c + dx]} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \left( \frac{(2A + A \cos[c] + C \cos[c]) \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c]}{d} + \right. \end{aligned}$$

$$\frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(A \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right]\right) + \frac{4 A \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \operatorname{Sin}[dx]}{d}}{d} +$$

$$\left( A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right.$$

$$\left. \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right.$$

$$\left. \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( d (a + a \operatorname{Cos}[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) -$$

$$\left( C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right.$$

$$\left. \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right.$$

$$\left. \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( d (a + a \operatorname{Cos}[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right)$$

■ **Problem 156: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Cos}[c + dx]^2}{\operatorname{Cos}[c + dx]^{5/2} (a + a \operatorname{Cos}[c + dx])} dx$$

Optimal (type 4, 150 leaves, 6 steps):

$$\frac{(3A + C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{(5A + 3C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3ad} +$$

$$\frac{(5A + 3C) \operatorname{Sin}[c + dx]}{3ad \operatorname{Cos}[c + dx]^{3/2}} - \frac{(3A + C) \operatorname{Sin}[c + dx]}{ad \sqrt{\operatorname{Cos}[c + dx]}} - \frac{(A + C) \operatorname{Sin}[c + dx]}{d \operatorname{Cos}[c + dx]^{3/2} (a + a \operatorname{Cos}[c + dx])}$$

Result (type 5, 1163 leaves):

$$\frac{1}{4(a + a \operatorname{Cos}[c + dx])} - 3i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2(1 + e^{2i dx}) \operatorname{Cos}[c] + 2i(-1 + e^{2i dx}) \operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2i dx} \operatorname{Cos}[2c] + i e^{2i dx} \operatorname{Sin}[2c]} \right) \right) / \left( 3id(1 + e^{2i dx}) \operatorname{Cos}[c] - 3d(-1 + e^{2i dx}) \operatorname{Sin}[c] \right) -$$

$$\begin{aligned}
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \right. \\
& \quad \left. \sqrt{1+e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( -id(1+e^{2idx}) \cos[c] + d(-1+e^{2idx}) \sin[c] \right) + \frac{1}{4(a+a \cos[c+dx])} i C \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \\
& \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \left( \left( 2 e^{2idx} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1+e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( 3id(1+e^{2idx}) \cos[c] - 3d(-1+e^{2idx}) \sin[c] \right) - \right. \\
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \right. \\
& \quad \left. \sqrt{1+e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( -id(1+e^{2idx}) \cos[c] + d(-1+e^{2idx}) \sin[c] \right) + \frac{1}{a+a \cos[c+dx]} \\
& \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \sqrt{\cos[c+dx]} \left( -\frac{(2A+A \cos[c] + C \cos[c]) \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[c]}{d} - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] (A \sin \left[ \frac{dx}{2} \right] + C \sin \left[ \frac{dx}{2} \right])}{d} \right) + \\
& \quad \left. \frac{4A \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 \sin[dx]}{3d} + \frac{4 \operatorname{Sec}[c] \operatorname{Sec}[c+dx] (A \sin[c] - 3A \sin[dx])}{3d} \right) - \\
& \left( 5A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \right) \\
& \quad \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left( 3d(a+a \cos[c+dx]) \sqrt{1 + \cot[c]^2} \right) - \left( C \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] \right) \\
& \quad \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \quad \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( d(a+a \cos[c+dx]) \sqrt{1 + \cot[c]^2} \right)
\end{aligned}$$

■ **Problem 157: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c+dx]^2}{\cos[c+dx]^{7/2} (a+a \cos[c+dx])} dx$$

Optimal (type 4, 192 leaves, 7 steps):

$$\begin{aligned}
& \frac{3(7A+5C)\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] - (5A+3C)\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{5ad} + \\
& \frac{(7A+5C)\operatorname{Sin}[c+dx]}{5ad\operatorname{Cos}[c+dx]^{5/2}} - \frac{(5A+3C)\operatorname{Sin}[c+dx]}{3ad\operatorname{Cos}[c+dx]^{3/2}} + \frac{3(7A+5C)\operatorname{Sin}[c+dx]}{5ad\sqrt{\operatorname{Cos}[c+dx]}} - \frac{(A+C)\operatorname{Sin}[c+dx]}{d\operatorname{Cos}[c+dx]^{5/2}(a+a\operatorname{Cos}[c+dx])}
\end{aligned}$$

Result (type 5, 1207 leaves):

$$\begin{aligned}
& -\frac{1}{20(a+a\operatorname{Cos}[c+dx])} 21iA\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\operatorname{Cos}[c] + i\operatorname{Sin}[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\operatorname{Cos}[c] + 2i(-1+e^{2idx})\operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1+e^{2idx}\operatorname{Cos}[2c] + ie^{2idx}\operatorname{Sin}[2c]} \right) / (3id(1+e^{2idx})\operatorname{Cos}[c] - 3d(-1+e^{2idx})\operatorname{Sin}[c]) - \right. \\
& \quad \left( 2\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\operatorname{Cos}[c] + i\operatorname{Sin}[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\operatorname{Cos}[c] + 2i(-1+e^{2idx})\operatorname{Sin}[c])} \right. \\
& \quad \left. \left. \sqrt{1+e^{2idx}\operatorname{Cos}[2c] + ie^{2idx}\operatorname{Sin}[2c]} \right) / (-id(1+e^{2idx})\operatorname{Cos}[c] + d(-1+e^{2idx})\operatorname{Sin}[c]) \right) - \frac{1}{4(a+a\operatorname{Cos}[c+dx])} 3iC\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\operatorname{Cos}[c] + i\operatorname{Sin}[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\operatorname{Cos}[c] + 2i(-1+e^{2idx})\operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1+e^{2idx}\operatorname{Cos}[2c] + ie^{2idx}\operatorname{Sin}[2c]} \right) / (3id(1+e^{2idx})\operatorname{Cos}[c] - 3d(-1+e^{2idx})\operatorname{Sin}[c]) - \right. \\
& \quad \left( 2\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\operatorname{Cos}[c] + i\operatorname{Sin}[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\operatorname{Cos}[c] + 2i(-1+e^{2idx})\operatorname{Sin}[c])} \right. \\
& \quad \left. \left. \sqrt{1+e^{2idx}\operatorname{Cos}[2c] + ie^{2idx}\operatorname{Sin}[2c]} \right) / (-id(1+e^{2idx})\operatorname{Cos}[c] + d(-1+e^{2idx})\operatorname{Sin}[c]) \right) + \\
& \frac{1}{a+a\operatorname{Cos}[c+dx]} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\operatorname{Cos}[c+dx]} \left( \frac{(16A+10C+5A\operatorname{Cos}[c] + 5C\operatorname{Cos}[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{5d} + \right. \\
& \quad \frac{2\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A\operatorname{Sin}\left[\frac{dx}{2}\right] + C\operatorname{Sin}\left[\frac{dx}{2}\right])}{d} + \frac{4A\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^3 \operatorname{Sin}[dx]}{5d} + \\
& \quad \left. \frac{4\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (3A\operatorname{Sin}[c] - 5A\operatorname{Sin}[dx])}{15d} - \frac{4\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (5A\operatorname{Sin}[c] - 24A\operatorname{Sin}[dx] - 15C\operatorname{Sin}[dx])}{15d} \right) + \\
& \left( 5A\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right)
\end{aligned}$$

$$\left. \frac{\sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]}}{\left(3 d (a + a \cos[c + d x]) \sqrt{1 + \text{Cot}[c]^2}\right) + \left( c \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \right)} \right) \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \left. \frac{\sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]}}{\left(d (a + a \cos[c + d x]) \sqrt{1 + \text{Cot}[c]^2}\right)} \right)$$

■ **Problem 158: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^{5/2} (A + C \cos[c + d x]^2)}{(a + a \cos[c + d x])^2} dx$$

Optimal (type 4, 196 leaves, 7 steps):

$$\frac{4 (5 A + 14 C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 a^2 d} - \frac{5 (A + 3 C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 a^2 d} - \frac{5 (A + 3 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{3 a^2 d} + \frac{4 (5 A + 14 C) \cos[c + d x]^{3/2} \sin[c + d x]}{15 a^2 d} - \frac{(A + 3 C) \cos[c + d x]^{5/2} \sin[c + d x]}{a^2 d (1 + \cos[c + d x])} - \frac{(A + C) \cos[c + d x]^{7/2} \sin[c + d x]}{3 d (a + a \cos[c + d x])^2}$$

Result (type 5, 1248 leaves):

$$\frac{1}{(a + a \cos[c + d x])^2} {}_2F_1\left[\frac{c}{2} + \frac{d x}{2}, 4, \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right]\right] \left( \left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) \right) / \left( 3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c] \right) - \right. \\ \left. \left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) \right) / \left( -i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c] \right) \right) + \frac{1}{5 (a + a \cos[c + d x])^2} {}_2F_1\left[\frac{c}{2} + \frac{d x}{2}, 4, \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right]\right] \left( \left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) \right) / \left( 3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c] \right) - \right.$$

$$\begin{aligned}
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) + \\
& \left( 10 A \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \right) \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \quad \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \operatorname{Cos}[c + d x])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \\
& \left( 10 C \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \right) \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \quad \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left( d (a + a \operatorname{Cos}[c + d x])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a + a \operatorname{Cos}[c + d x])^2} \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\operatorname{Cos}[c + d x]} \\
& \left( -\frac{8 (5 A + 10 C + 5 A \operatorname{Cos}[c] + 18 C \operatorname{Cos}[c]) \operatorname{Csc}[c]}{5 d} - \frac{16 C \operatorname{Cos}[d x] \operatorname{Sin}[c]}{3 d} + \frac{4 C \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{5 d} + \right. \\
& \quad \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (A \operatorname{Sin} \left[ \frac{d x}{2} \right] + C \operatorname{Sin} \left[ \frac{d x}{2} \right])}{3 d} - \frac{8 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (A \operatorname{Sin} \left[ \frac{d x}{2} \right] + 2 C \operatorname{Sin} \left[ \frac{d x}{2} \right])}{d} - \\
& \quad \left. \frac{16 C \operatorname{Cos}[c] \operatorname{Sin}[d x]}{3 d} + \frac{4 C \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x]}{5 d} + \frac{2 (A + C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Tan} \left[ \frac{c}{2} \right]}{3 d} \right)
\end{aligned}$$

■ **Problem 159: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^{3/2} (A + C \operatorname{Cos}[c + d x]^2)}{(a + a \operatorname{Cos}[c + d x])^2} dx$$

Optimal (type 4, 161 leaves, 6 steps):

$$-\frac{(A+7C)\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} + \frac{2(A+5C)\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3a^2 d} +$$

$$\frac{2(A+5C)\sqrt{\cos[c+dx]}\sin[c+dx]}{3a^2 d} - \frac{(A+7C)\cos[c+dx]^{3/2}\sin[c+dx]}{3a^2 d(1+\cos[c+dx])} - \frac{(A+C)\cos[c+dx]^{5/2}\sin[c+dx]}{3d(a+a\cos[c+dx])^2}$$

Result (type 5, 1209 leaves):

$$-\frac{1}{2(a+a\cos[c+dx])^2} i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right.$$

$$\left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) - \frac{1}{2(a+a\cos[c+dx])^2} 7 i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4$$

$$\operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right.$$

$$\left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) -$$

$$\left( 4 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right.$$

$$\left. \frac{\sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}}{\sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}} \right) / \left( 3 d (a + a \cos[c + dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) -$$

$$\left( 20 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)$$

$$\left. \begin{aligned} & \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \Big/ \\ & \left( 3 d (a + a \cos[c + d x])^2 \sqrt{1 + \text{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + d x])^2} \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \sqrt{\cos[c + d x]} \\ & \left( \frac{4 (A + 3 C + 4 C \cos[c]) \text{Csc}[c]}{d} + \frac{8 C \cos[d x] \sin[c]}{3 d} - \frac{2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^3 (A \sin\left[\frac{d x}{2}\right] + C \sin\left[\frac{d x}{2}\right])}{3 d} \right) + \\ & \left. \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (A \sin\left[\frac{d x}{2}\right] + 3 C \sin\left[\frac{d x}{2}\right])}{d} + \frac{8 C \cos[c] \sin[d x]}{3 d} - \frac{2 (A + C) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Tan}\left[\frac{c}{2}\right]}{3 d} \right) \end{aligned} \right)$$

- **Problem 160: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c + d x]} (A + C \cos[c + d x]^2)}{(a + a \cos[c + d x])^2} dx$$

Optimal (type 4, 126 leaves, 5 steps):

$$\frac{4 C \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{a^2 d} + \frac{(A - 5 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 a^2 d} + \frac{(A - 5 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{3 a^2 d (1 + \cos[c + d x])} - \frac{(A + C) \cos[c + d x]^{3/2} \sin[c + d x]}{3 d (a + a \cos[c + d x])^2}$$

Result (type 5, 814 leaves):



$$\begin{aligned}
& \frac{1}{(a + a \cos[c + dx])^2} 2 i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\
& \quad \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) - \\
& \left( 2 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / (3 d (a + a \cos[c + dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2}) + \right. \\
& \left( 10 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \right. \\
& \quad \left( 3 d (a + a \cos[c + dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \\
& \left( -\frac{8 C \operatorname{Cot}\left[\frac{c}{2}\right]}{d} - \frac{8 C \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3 d} + \frac{2 (A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right)
\end{aligned}$$

■ **Problem 161: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c + dx]^2}{\sqrt{\cos[c + dx]} (a + a \cos[c + dx])^2} dx$$

Optimal (type 4, 125 leaves, 5 steps):

$$\frac{(A - C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{a^2 d} + \frac{2(A + C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3 a^2 d} - \frac{(A - C) \sqrt{\cos[c + dx]} \sin[c + dx]}{a^2 d (1 + \cos[c + dx])} - \frac{(A + C) \sqrt{\cos[c + dx]} \sin[c + dx]}{3 d (a + a \cos[c + dx])^2}$$

Result (type 5, 1176 leaves):

$$\begin{aligned}
& \frac{1}{2 (a + a \cos [c + d x])^2} i A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \\
& \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \left. \right) - \frac{1}{2 (a + a \cos [c + d x])^2} i C \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \\
& \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \left. \right) - \\
& \left( 4 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} [d x - \operatorname{ArcTan}[\cot [c]]] \right. \\
& \quad \left. \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right. \\
& \quad \left. \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right) / \left( 3 d (a + a \cos [c + d x])^2 \sqrt{1 + \cot [c]^2} \right) - \\
& \left( 4 C \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} [d x - \operatorname{ArcTan}[\cot [c]]] \right. \\
& \quad \left. \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right) / \\
& \quad \left( 3 d (a + a \cos [c + d x])^2 \sqrt{1 + \cot [c]^2} \right) + \frac{1}{(a + a \cos [c + d x])^2} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\cos [c + d x]}
\end{aligned}$$

$$\left( -\frac{4(A-C)\operatorname{Csc}[c]}{d} - \frac{4\operatorname{Sec}\left[\frac{c}{2}\right]\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right](A\operatorname{Sin}\left[\frac{dx}{2}\right] - C\operatorname{Sin}\left[\frac{dx}{2}\right])}{d} - \frac{2\operatorname{Sec}\left[\frac{c}{2}\right]\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3(A\operatorname{Sin}\left[\frac{dx}{2}\right] + C\operatorname{Sin}\left[\frac{dx}{2}\right])}{3d} - \frac{2(A+C)\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2\operatorname{Tan}\left[\frac{c}{2}\right]}{3d} \right)$$

- **Problem 162: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Cos}[c + dx]^2}{\operatorname{Cos}[c + dx]^{3/2} (a + a \operatorname{Cos}[c + dx])^2} dx$$

Optimal (type 4, 155 leaves, 6 steps):

$$-\frac{4A\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2d} - \frac{(5A-C)\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3a^2d} + \frac{4A\operatorname{Sin}[c+dx]}{a^2d\sqrt{\operatorname{Cos}[c+dx]}} - \frac{(5A-C)\operatorname{Sin}[c+dx]}{3a^2d\sqrt{\operatorname{Cos}[c+dx]}(1+\operatorname{Cos}[c+dx])} - \frac{(A+C)\operatorname{Sin}[c+dx]}{3d\sqrt{\operatorname{Cos}[c+dx]}(a+a\operatorname{Cos}[c+dx])^2}$$

Result (type 5, 834 leaves):

$$\begin{aligned}
& - \frac{1}{(a + a \cos[c + dx])^2} 2i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left( \left( 2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\
& \quad \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \\
& \left( 10 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3d(a + a \cos[c + dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left( 2 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left( 3d(a + a \cos[c + dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \\
& \left( \frac{8 A \operatorname{Cot}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{d} + \frac{8 A \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{d} + \right. \\
& \quad \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} + \frac{8 A \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \sin[dx]}{d} + \frac{2(A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} \right)
\end{aligned}$$

- **Problem 163: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c + dx]^2}{\cos[c + dx]^{5/2} (a + a \cos[c + dx])^2} dx$$

Optimal (type 4, 189 leaves, 7 steps):

$$\frac{(7A+C) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} + \frac{2(5A+C) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3a^2 d} + \frac{2(5A+C) \operatorname{Sin}[c+dx]}{3a^2 d \operatorname{Cos}[c+dx]^{3/2}} - \frac{(7A+C) \operatorname{Sin}[c+dx]}{a^2 d \sqrt{\operatorname{Cos}[c+dx]}} - \frac{(7A+C) \operatorname{Sin}[c+dx]}{3a^2 d \operatorname{Cos}[c+dx]^{3/2} (1+\operatorname{Cos}[c+dx])} - \frac{(A+C) \operatorname{Sin}[c+dx]}{3d \operatorname{Cos}[c+dx]^{3/2} (a+a \operatorname{Cos}[c+dx])^2}$$

Result (type 5, 1245 leaves):

$$\frac{1}{2(a+a \operatorname{Cos}[c+dx])^2} 7iA \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]}} \right) / (3id(1+e^{2idx}) \operatorname{Cos}[c] - 3d(-1+e^{2idx}) \operatorname{Sin}[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]}} \right) / (-id(1+e^{2idx}) \operatorname{Cos}[c] + d(-1+e^{2idx}) \operatorname{Sin}[c]) \right) + \frac{1}{2(a+a \operatorname{Cos}[c+dx])^2} iC \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]}} \right) / (3id(1+e^{2idx}) \operatorname{Cos}[c] - 3d(-1+e^{2idx}) \operatorname{Sin}[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]}} \right) / (-id(1+e^{2idx}) \operatorname{Cos}[c] + d(-1+e^{2idx}) \operatorname{Sin}[c]) \right) - \left( 20A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3d(a+a \operatorname{Cos}[c+dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \left( 4C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)$$

$$\left. \begin{aligned} & \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \Big/ \\ & \left( 3d (a + a \cos[c + dx])^2 \sqrt{1 + \text{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \\ & \left( -\frac{2(4A + 3A \cos[c] + C \cos[c]) \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c] - 2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} - \frac{2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} \right) \\ & \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (3A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} + \frac{8A \text{Sec}[c] \text{Sec}[c + dx]^2 \sin[dx]}{3d} + \\ & \left. \frac{8 \text{Sec}[c] \text{Sec}[c + dx] (A \sin[c] - 6A \sin[dx])}{3d} - \frac{2(A + C) \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Tan}\left[\frac{c}{2}\right]}{3d} \right) \end{aligned} \right)$$

■ **Problem 164: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{7/2} (A + C \cos[c + dx])^2}{(a + a \cos[c + dx])^3} dx$$

Optimal (type 4, 250 leaves, 8 steps):

$$\frac{7(7A + 33C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] - (13A + 63C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{10a^3d} - \frac{6a^3d}{(13A + 63C) \sqrt{\cos[c + dx]} \sin[c + dx]} + \frac{7(7A + 33C) \cos[c + dx]^{3/2} \sin[c + dx]}{30a^3d} - \frac{(A + C) \cos[c + dx]^{9/2} \sin[c + dx]}{5d(a + a \cos[c + dx])^3} - \frac{2(A + 6C) \cos[c + dx]^{7/2} \sin[c + dx]}{15ad(a + a \cos[c + dx])^2} - \frac{(13A + 63C) \cos[c + dx]^{5/2} \sin[c + dx]}{10d(a^3 + a^3 \cos[c + dx])}$$

Result (type 5, 1333 leaves):

$$\frac{1}{10(a + a \cos[c + dx])^3} 49iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) \Big/ (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ \left. \left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) \Big/ (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) +$$

$$\begin{aligned}
& \frac{1}{10 (a + a \cos [c + d x])^3} {}^{231} i C \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \\
& \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \left( 26 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \right. \\
& \quad \left. \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot} [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right. \\
& \quad \left. \left. \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right) / \left( 3 d (a + a \cos [c + d x])^3 \sqrt{1 + \operatorname{Cot} [c]^2} \right) + \\
& \left( 42 C \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \right. \\
& \quad \left. \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot} [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right) / \\
& \quad \left( d (a + a \cos [c + d x])^3 \sqrt{1 + \operatorname{Cot} [c]^2} \right) + \frac{1}{(a + a \cos [c + d x])^3} \\
& \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \left( -\frac{4 (29 A + 99 C + 20 A \cos [c] + 132 C \cos [c]) \operatorname{Csc} [c]}{5 d} - \right. \\
& \quad \frac{16 C \cos [d x] \sin [c]}{d} + \frac{8 C \cos [2 d x] \sin [2 c]}{5 d} - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 (A \sin \left[ \frac{d x}{2} \right] + C \sin \left[ \frac{d x}{2} \right])}{5 d} + \\
& \quad \frac{8 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (7 A \sin \left[ \frac{d x}{2} \right] + 12 C \sin \left[ \frac{d x}{2} \right])}{15 d} - \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (29 A \sin \left[ \frac{d x}{2} \right] + 99 C \sin \left[ \frac{d x}{2} \right])}{5 d} - \\
& \quad \left. \frac{16 C \cos [c] \sin [d x]}{d} + \frac{8 C \cos [2 c] \sin [2 d x]}{5 d} + \frac{8 (7 A + 12 C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Tan} \left[ \frac{c}{2} \right]}{15 d} - \frac{2 (A + C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Tan} \left[ \frac{c}{2} \right]}{5 d} \right)
\end{aligned}$$

**Problem 165: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{5/2} (A + C \cos[c + dx]^2)}{(a + a \cos[c + dx])^3} dx$$

Optimal (type 4, 209 leaves, 7 steps):

$$-\frac{(9A + 119C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10a^3d} + \frac{(A + 11C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{2a^3d} + \frac{(A + 11C) \sqrt{\cos[c + dx]} \sin[c + dx]}{2a^3d} - \frac{(A + C) \cos[c + dx]^{7/2} \sin[c + dx]}{5d(a + a \cos[c + dx])^3} - \frac{2C \cos[c + dx]^{5/2} \sin[c + dx]}{3ad(a + a \cos[c + dx])^2} - \frac{(9A + 119C) \cos[c + dx]^{3/2} \sin[c + dx]}{30d(a^3 + a^3 \cos[c + dx])^2}$$

Result (type 5, 1296 leaves):

$$-\frac{1}{10(a + a \cos[c + dx])^3} 9iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) - \\ \frac{1}{10(a + a \cos[c + dx])^3} 119iC \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) - \\ \left( 2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\ \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right)$$



$$\left. \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \left( d (a + a \cos[c + d x])^3 \sqrt{1 + \text{Cot}[c]^2} \right) -$$

$$\left( 22 C \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \right.$$

$$\left. \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) /$$

$$\left( d (a + a \cos[c + d x])^3 \sqrt{1 + \text{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + d x])^3} \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\sqrt{\cos[c + d x]}$$

$$\left( \frac{4 (9 A + 59 C + 60 C \cos[c]) \text{Csc}[c]}{5 d} + \frac{16 C \cos[d x] \sin[c]}{3 d} + \frac{2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^5 (A \sin\left[\frac{d x}{2}\right] + C \sin\left[\frac{d x}{2}\right])}{5 d} - \right.$$

$$\frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^3 (9 A \sin\left[\frac{d x}{2}\right] + 19 C \sin\left[\frac{d x}{2}\right])}{15 d} + \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (9 A \sin\left[\frac{d x}{2}\right] + 59 C \sin\left[\frac{d x}{2}\right])}{5 d} +$$

$$\left. \frac{16 C \cos[c] \sin[d x]}{3 d} - \frac{4 (9 A + 19 C) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Tan}\left[\frac{c}{2}\right]}{15 d} + \frac{2 (A + C) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \text{Tan}\left[\frac{c}{2}\right]}{5 d} \right)$$

■ **Problem 166: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^{3/2} (A + C \cos[c + d x]^2)}{(a + a \cos[c + d x])^3} dx$$

Optimal (type 4, 178 leaves, 6 steps):

$$-\frac{(A - 49 C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{10 a^3 d} + \frac{(A - 13 C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{6 a^3 d} -$$

$$\frac{(A + C) \cos[c + d x]^{5/2} \sin[c + d x]}{5 d (a + a \cos[c + d x])^3} + \frac{2 (A - 4 C) \cos[c + d x]^{3/2} \sin[c + d x]}{15 a d (a + a \cos[c + d x])^2} + \frac{(A - 13 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{6 d (a^3 + a^3 \cos[c + d x])}$$

Result (type 5, 1271 leaves):

$$-\frac{1}{10 (a + a \cos[c + d x])^3} i A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right.$$

$$\begin{aligned}
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) + \\
& \frac{1}{10 (a + a \operatorname{Cos}[c + d x])^3} 49 i c \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \\
& \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left( 3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \right. \\
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \\
& \left( 2 A \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \quad \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \operatorname{Cos}[c + d x])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \\
& \left( 26 C \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \quad \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left( 3 d (a + a \operatorname{Cos}[c + d x])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a + a \operatorname{Cos}[c + d x])^3} \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \\
& \sqrt{\operatorname{Cos}[c + d x]} \\
& \left( -\frac{4 (-A + 29 C + 20 C \operatorname{Cos}[c]) \operatorname{Csc}[c]}{5 d} + \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (A \operatorname{Sin} \left[ \frac{d x}{2} \right] - 29 C \operatorname{Sin} \left[ \frac{d x}{2} \right])}{5 d} - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 (A \operatorname{Sin} \left[ \frac{d x}{2} \right] + C \operatorname{Sin} \left[ \frac{d x}{2} \right])}{5 d} \right) + \\
& \left. \frac{8 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (2 A \operatorname{Sin} \left[ \frac{d x}{2} \right] + 7 C \operatorname{Sin} \left[ \frac{d x}{2} \right])}{15 d} + \frac{8 (2 A + 7 C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Tan} \left[ \frac{c}{2} \right]}{15 d} - \frac{2 (A + C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Tan} \left[ \frac{c}{2} \right]}{5 d} \right)
\end{aligned}$$

**Problem 167: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]} (A+C \cos[c+dx]^2)}{(a+a \cos[c+dx])^3} dx$$

Optimal (type 4, 180 leaves, 6 steps):

$$\frac{(A-9C) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{10a^3d} + \frac{(A+3C) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{6a^3d} -$$

$$\frac{(A+C) \cos[c+dx]^{3/2} \sin[c+dx]}{5d(a+a \cos[c+dx])^3} + \frac{2(2A-3C) \sqrt{\cos[c+dx]} \sin[c+dx]}{15ad(a+a \cos[c+dx])^2} - \frac{(A-9C) \sqrt{\cos[c+dx]} \sin[c+dx]}{10d(a^3+a^3 \cos[c+dx])}$$

Result (type 5, 1259 leaves):

$$\frac{1}{10(a+a \cos[c+dx])^3} i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1+e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1+e^{2idx}) \cos[c] - 3d(-1+e^{2idx}) \sin[c]) - \right.$$

$$\left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1+e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1+e^{2idx}) \cos[c] + d(-1+e^{2idx}) \sin[c]) \right) - \frac{1}{10(a+a \cos[c+dx])^3} 9i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6$$

$$\operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1+e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1+e^{2idx}) \cos[c] - 3d(-1+e^{2idx}) \sin[c]) - \right.$$

$$\left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1+e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1+e^{2idx}) \cos[c] + d(-1+e^{2idx}) \sin[c]) \right) -$$

$$\left( 2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right.$$

$$\left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right)$$

$$\left. \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \left( 3d (a + a \cos[c + dx])^3 \sqrt{1 + \cot[c]^2} \right) -$$

$$\left( 2C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \right.$$

$$\left. \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) /$$

$$\left( d (a + a \cos[c + dx])^3 \sqrt{1 + \cot[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]}$$

$$\left( -\frac{4(A - 9C) \text{Csc}[c]}{5d} - \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - 9C \sin\left[\frac{dx}{2}\right])}{5d} + \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - 9C \sin\left[\frac{dx}{2}\right])}{15d} + \right.$$

$$\left. \frac{2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} + \frac{4(A - 9C) \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Tan}\left[\frac{c}{2}\right]}{15d} + \frac{2(A + C) \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Tan}\left[\frac{c}{2}\right]}{5d} \right)$$

■ **Problem 168: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c + dx]^2}{\sqrt{\cos[c + dx]} (a + a \cos[c + dx])^3} dx$$

Optimal (type 4, 184 leaves, 6 steps):

$$\frac{(9A - C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10a^3d} + \frac{(3A + C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6a^3d} -$$

$$\frac{(A + C) \sqrt{\cos[c + dx]} \sin[c + dx]}{5d(a + a \cos[c + dx])^3} - \frac{2(3A - 2C) \sqrt{\cos[c + dx]} \sin[c + dx]}{15ad(a + a \cos[c + dx])^2} - \frac{(9A - C) \sqrt{\cos[c + dx]} \sin[c + dx]}{10d(a^3 + a^3 \cos[c + dx])}$$

Result (type 5, 1265 leaves):

$$\frac{1}{10(a + a \cos[c + dx])^3} 9iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( 3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c] \right) - \right.$$

$$\left. \left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right) \right)$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]}}{(-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c])} - \frac{1}{10(a + a \cos[c + dx])^3} i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \right. \\
& \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]}}{(3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c])} - \right. \right. \\
& \left. \left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]}}{(-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c])} \right) - \right. \\
& \left. \left( 2 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \right. \right. \\
& \left. \frac{\sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \left. \left. \frac{\sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]}}{d(a + a \cos[c + dx])^3 \sqrt{1 + \text{Cot}[c]^2}} - \right. \right. \\
& \left. \left( 2 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \right. \right. \\
& \left. \left. \frac{\sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) \right. \\
& \left. \left( 3 d (a + a \cos[c + dx])^3 \sqrt{1 + \text{Cot}[c]^2} + \frac{1}{(a + a \cos[c + dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]} \right. \right. \\
& \left. \left( -\frac{4(9A - C) \text{Csc}[c]}{5d} - \frac{8 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (3A \sin\left[\frac{dx}{2}\right] - 2C \sin\left[\frac{dx}{2}\right])}{15d} - \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (9A \sin\left[\frac{dx}{2}\right] - C \sin\left[\frac{dx}{2}\right])}{5d} \right. \right. \\
& \left. \left. \frac{2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} - \frac{8(3A - 2C) \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Tan}\left[\frac{c}{2}\right]}{15d} - \frac{2(A + C) \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Tan}\left[\frac{c}{2}\right]}{5d} \right) \right)
\end{aligned}$$

- **Problem 169: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c + dx]^2}{\cos[c + dx]^{3/2} (a + a \cos[c + dx])^3} dx$$

Optimal (type 4, 219 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(49 A - C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10 a^3 d} - \frac{(13 A - C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6 a^3 d} + \frac{(49 A - C) \text{Sin}[c + dx]}{10 a^3 d \sqrt{\text{Cos}[c + dx]}} - \\
& \frac{(A + C) \text{Sin}[c + dx]}{5 d \sqrt{\text{Cos}[c + dx]} (a + a \text{Cos}[c + dx])^3} - \frac{2(4 A - C) \text{Sin}[c + dx]}{15 a d \sqrt{\text{Cos}[c + dx]} (a + a \text{Cos}[c + dx])^2} - \frac{(13 A - C) \text{Sin}[c + dx]}{6 d \sqrt{\text{Cos}[c + dx]} (a^3 + a^3 \text{Cos}[c + dx])}
\end{aligned}$$

Result (type 5, 1301 leaves):

$$\begin{aligned}
& - \frac{1}{10 (a + a \text{Cos}[c + dx])^3} 49 i A \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \\
& \left( \left( 2 e^{2 i dx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\text{Cos}[c] + i \text{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \text{Cos}[c] + 2 i (-1 + e^{2 i dx}) \text{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \text{Cos}[2 c] + i e^{2 i dx} \text{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \text{Cos}[c] - 3 d (-1 + e^{2 i dx}) \text{Sin}[c]) - \right. \\
& \quad \left. \left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\text{Cos}[c] + i \text{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \text{Cos}[c] + 2 i (-1 + e^{2 i dx}) \text{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \text{Cos}[2 c] + i e^{2 i dx} \text{Sin}[2 c]} \right) / (-i d (1 + e^{2 i dx}) \text{Cos}[c] + d (-1 + e^{2 i dx}) \text{Sin}[c]) \right) + \frac{1}{10 (a + a \text{Cos}[c + dx])^3} i C \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \\
& \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i dx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\text{Cos}[c] + i \text{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \text{Cos}[c] + 2 i (-1 + e^{2 i dx}) \text{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \text{Cos}[2 c] + i e^{2 i dx} \text{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \text{Cos}[c] - 3 d (-1 + e^{2 i dx}) \text{Sin}[c]) - \right. \\
& \quad \left. \left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\text{Cos}[c] + i \text{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \text{Cos}[c] + 2 i (-1 + e^{2 i dx}) \text{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \text{Cos}[2 c] + i e^{2 i dx} \text{Sin}[2 c]} \right) / (-i d (1 + e^{2 i dx}) \text{Cos}[c] + d (-1 + e^{2 i dx}) \text{Sin}[c]) \right) + \\
& \left( 26 A \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]\right]^2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \left( 3 d (a + a \text{Cos}[c + dx])^3 \sqrt{1 + \text{Cot}[c]^2} \right) - \\
& \left( 2 C \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]\right]^2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \right.
\end{aligned}$$

$$\left. \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) /$$

$$\left( 3 d (a + a \cos[c + d x])^3 \sqrt{1 + \text{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + d x])^3} \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \sqrt{\cos[c + d x]}$$

$$\left( \frac{2 (20 A + 29 A \cos[c] - C \cos[c]) \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c]}{5 d} + \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (29 A \sin\left[\frac{d x}{2}\right] - C \sin\left[\frac{d x}{2}\right])}{5 d} + \right.$$

$$\left. \frac{2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^5 (A \sin\left[\frac{d x}{2}\right] + C \sin\left[\frac{d x}{2}\right])}{5 d} + \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^3 (11 A \sin\left[\frac{d x}{2}\right] + C \sin\left[\frac{d x}{2}\right])}{15 d} + \right.$$

$$\left. \frac{16 A \text{Sec}[c] \text{Sec}[c + d x] \sin[d x]}{d} + \frac{4 (11 A + C) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Tan}\left[\frac{c}{2}\right]}{15 d} + \frac{2 (A + C) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \text{Tan}\left[\frac{c}{2}\right]}{5 d} \right)$$

■ **Problem 170: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c + d x]^2}{\cos[c + d x]^{5/2} (a + a \cos[c + d x])^3} dx$$

Optimal (type 4, 242 leaves, 8 steps):

$$\frac{(119 A + 9 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{10 a^3 d} + \frac{(11 A + C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{2 a^3 d} + \frac{(11 A + C) \sin[c + d x]}{2 a^3 d \cos[c + d x]^{3/2}} - \frac{(119 A + 9 C) \sin[c + d x]}{10 a^3 d \sqrt{\cos[c + d x]}}$$

$$\frac{(A + C) \sin[c + d x]}{5 d \cos[c + d x]^{3/2} (a + a \cos[c + d x])^3} - \frac{2 A \sin[c + d x]}{3 a d \cos[c + d x]^{3/2} (a + a \cos[c + d x])^2} - \frac{(119 A + 9 C) \sin[c + d x]}{30 d \cos[c + d x]^{3/2} (a^3 + a^3 \cos[c + d x])}$$

Result (type 5, 1331 leaves):

$$\frac{1}{10 (a + a \cos[c + d x])^3} 119 i A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right.$$

$$\left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) + \frac{1}{10 (a + a \cos[c + d x])^3} 9 i C \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\begin{aligned}
& \left. \sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]} \right) / \left( 3 i d (1 + e^{2ix}) \cos[c] - 3 d (-1 + e^{2ix}) \sin[c] \right) - \\
& \left( 2 \text{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2 i (-1 + e^{2ix}) \sin[c])} \right. \\
& \left. \sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]} \right) / \left( -i d (1 + e^{2ix}) \cos[c] + d (-1 + e^{2ix}) \sin[c] \right) - \\
& \left( 22 A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \csc \left[ \frac{c}{2} \right] \text{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \text{ArcTan}[\cot[c]]] \right]^2 \sec \left[ \frac{c}{2} \right] \sec[dx - \text{ArcTan}[\cot[c]]] \right. \\
& \left. \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \left( d (a + a \cos[c + dx])^3 \sqrt{1 + \cot[c]^2} \right) - \\
& \left( 2 C \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \csc \left[ \frac{c}{2} \right] \text{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \text{ArcTan}[\cot[c]]] \right]^2 \sec \left[ \frac{c}{2} \right] \sec[dx - \text{ArcTan}[\cot[c]]] \right. \\
& \left. \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left( d (a + a \cos[c + dx])^3 \sqrt{1 + \cot[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^3} \\
& \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \sqrt{\cos[c + dx]} \left( -\frac{2 (60 A + 59 A \cos[c] + 9 C \cos[c]) \csc \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} \right] \sec[c]}{5 d} - \right. \\
& \frac{2 \sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 (A \sin \left[ \frac{dx}{2} \right] + C \sin \left[ \frac{dx}{2} \right])}{5 d} - \frac{8 \sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 (8 A \sin \left[ \frac{dx}{2} \right] + 3 C \sin \left[ \frac{dx}{2} \right])}{15 d} - \\
& \frac{4 \sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right] (59 A \sin \left[ \frac{dx}{2} \right] + 9 C \sin \left[ \frac{dx}{2} \right])}{5 d} + \frac{16 A \sec[c] \sec[c + dx]^2 \sin[dx]}{3 d} + \\
& \left. \frac{16 \sec[c] \sec[c + dx] (A \sin[c] - 9 A \sin[dx])}{3 d} - \frac{8 (8 A + 3 C) \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \tan \left[ \frac{c}{2} \right]}{15 d} - \frac{2 (A + C) \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \tan \left[ \frac{c}{2} \right]}{5 d} \right)
\end{aligned}$$

■ **Problem 171: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^{3/2} \sqrt{a + a \cos[c + dx]} (A + C \cos[c + dx]^2) dx$$

Optimal (type 3, 214 leaves, 6 steps):



$$\frac{\sqrt{a} (48 A + 35 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{64 d} + \frac{a (48 A + 35 C) \sqrt{\cos[c+dx]} \sin[c+dx]}{64 d \sqrt{a+a \cos[c+dx]}} +$$

$$\frac{a (48 A + 35 C) \cos[c+dx]^{3/2} \sin[c+dx]}{96 d \sqrt{a+a \cos[c+dx]}} + \frac{a C \cos[c+dx]^{5/2} \sin[c+dx]}{24 d \sqrt{a+a \cos[c+dx]}} + \frac{C \cos[c+dx]^{5/2} \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{4 d}$$

Result (type 3, 991 leaves) :

$$\frac{1}{128} (48 A + 35 C) \sqrt{a (1 + \cos[c+dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]$$

$$\left( \frac{1}{2} i \sin\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]}\right)\right] \right) \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right.$$

$$\left. \frac{\sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c] \right)}}{\left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]} \right)} - \right.$$

$$\left. \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]}\right] \right) \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right.$$

$$\left. \frac{\sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c] \right)}}{\left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]} \right)} \right) +$$

$$\frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]}\right)\right] \right) \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right.$$

$$\left. \frac{\sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c] \right)}}{\left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]} \right)} + \right.$$

$$\left. \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]}\right] \right) \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right.$$

$$\left. \frac{\sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c] \right)}}{\left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]} \right)} \right) \right) +$$

$$\sqrt{\cos[c+dx]} \sqrt{a (1 + \cos[c+dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left( \frac{(6 A + 5 C) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{12 d} + \frac{(16 A + 13 C) \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{64 d} + \right.$$

$$\frac{C \cos\left[\frac{5dx}{2}\right] \sin\left[\frac{5c}{2}\right]}{24 d} + \frac{C \cos\left[\frac{7dx}{2}\right] \sin\left[\frac{7c}{2}\right]}{32 d} +$$

$$\frac{(6 A + 5 C) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{12 d} + \frac{(16 A + 13 C) \cos\left[\frac{3c}{2}\right] \sin\left[\frac{3dx}{2}\right]}{64 d} +$$

$$\left. \frac{C \cos\left[\frac{5c}{2}\right] \sin\left[\frac{5dx}{2}\right]}{24 d} + \frac{C \cos\left[\frac{7c}{2}\right] \sin\left[\frac{7dx}{2}\right]}{32 d} \right)$$

■ **Problem 172: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]} (A+C \cos[c+dx]^2) dx$$

Optimal (type 3, 169 leaves, 5 steps) :

$$\frac{\sqrt{a} (8A + 5C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{8d} + \frac{a(8A + 5C) \sqrt{\cos[c+dx]} \sin[c+dx]}{8d \sqrt{a+a\cos[c+dx]}} + \frac{aC \cos[c+dx]^{3/2} \sin[c+dx]}{12d \sqrt{a+a\cos[c+dx]}} + \frac{C \cos[c+dx]^{3/2} \sqrt{a+a\cos[c+dx]} \sin[c+dx]}{3d}$$

Result (type 3, 578 leaves) :

$$\frac{1}{48d \sqrt{(1+e^{2ix}) \cos[c] + i(-1+e^{2ix}) \sin[c]}} \sqrt{\cos[c+dx]} \sqrt{a(1+\cos[c+dx])} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \\ \left( -3i(8A+5C) \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2\left(e^{ix} \cos\left[\frac{c}{2}\right] + i e^{ix} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2ix}) \cos[c] + i(-1+e^{2ix}) \sin[c]}\right)\right] + \right. \\ \left. 3i(8A+5C) \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2ix}) \cos[c] + i(-1+e^{2ix}) \sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) + \right. \\ \left. 24A \operatorname{Log}\left[2\left(e^{ix} \cos\left[\frac{c}{2}\right] + i e^{ix} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2ix}) \cos[c] + i(-1+e^{2ix}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] + \right. \\ \left. 15C \operatorname{Log}\left[2\left(e^{ix} \cos\left[\frac{c}{2}\right] + i e^{ix} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2ix}) \cos[c] + i(-1+e^{2ix}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] + \right. \\ \left. 48\sqrt{2} A \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + 28\sqrt{2} C \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + \right. \\ \left. 6\sqrt{2} C \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] + 4\sqrt{2} C \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{5}{2}(c+dx)\right] \right)$$

■ **Problem 173: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+a\cos[c+dx]} (A+C\cos[c+dx])^2}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 3, 124 leaves, 4 steps) :

$$\frac{\sqrt{a} (8A + 3C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{4d} + \frac{aC \sqrt{\cos[c+dx]} \sin[c+dx]}{4d \sqrt{a+a\cos[c+dx]}} + \frac{C \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]} \sin[c+dx]}{2d}$$

Result (type 3, 494 leaves) :

$$\frac{1}{8 d \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]}} \sqrt{\cos[c + d x]} \sqrt{a (1 + \cos[c + d x])} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]$$

$$\left( -i (8 A + 3 C) \cos\left[\frac{d x}{2}\right] \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \right) \right] + \right.$$

$$i (8 A + 3 C) \operatorname{ArcTanh}\left[\left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \right] \left( \cos\left[\frac{d x}{2}\right] + i \sin\left[\frac{d x}{2}\right] \right) +$$

$$8 A \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \right) \right] \sin\left[\frac{d x}{2}\right] +$$

$$3 C \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \right) \right] \sin\left[\frac{d x}{2}\right] +$$

$$4 \sqrt{2} C \sqrt{\cos[c + d x]} (\cos[d x] + i \sin[d x]) \sin\left[\frac{1}{2} (c + d x)\right] + 2 \sqrt{2} C \sqrt{\cos[c + d x]} (\cos[d x] + i \sin[d x]) \sin\left[\frac{3}{2} (c + d x)\right] \Big)$$

- **Problem 174: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \cos[c + d x]} (A + C \cos[c + d x])^2}{\cos[c + d x]^{3/2}} dx$$

Optimal (type 3, 117 leaves, 4 steps):

$$\frac{\sqrt{a} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c + d x]}{\sqrt{a + a \cos[c + d x]}}\right]}{d} - \frac{a (2 A - C) \sqrt{\cos[c + d x]} \sin[c + d x]}{d \sqrt{a + a \cos[c + d x]}} + \frac{2 A \sqrt{a + a \cos[c + d x]} \sin[c + d x]}{d \sqrt{\cos[c + d x]}}$$

Result (type 3, 615 leaves):

$$\begin{aligned}
& \frac{1}{4 \sqrt{2} d \sqrt{\cos[c+dx]} \sqrt{\cos[c+dx] (\cos[dx] + i \sin[dx])}} \\
& \sqrt{a(1+\cos[c+dx])} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left( -i C \cos\left[c + \frac{dx}{2}\right] \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right) \right] - \right. \\
& \quad \left. i C \cos\left[c + \frac{3 dx}{2}\right] \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right) \right] + \right. \\
& \quad \left. 2 i C \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right] \cos[c+dx] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) - \right. \\
& \quad \left. C \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right) \right] \sin\left[c + \frac{dx}{2}\right] + \right. \\
& \quad \left. 8 \sqrt{2} A \sqrt{\cos[c+dx] (\cos[dx] + i \sin[dx])} \sin\left[\frac{1}{2}(c+dx)\right] - \right. \\
& \quad \left. 2 \sqrt{2} C \sqrt{\cos[c+dx] (\cos[dx] + i \sin[dx])} \sin\left[\frac{1}{2}(c+dx)\right] + 2 \sqrt{2} C \sqrt{\cos[c+dx] (\cos[dx] + i \sin[dx])} \sin\left[\frac{3}{2}(c+dx)\right] + \right. \\
& \quad \left. C \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right) \right] \sin\left[c + \frac{3 dx}{2}\right] \right)
\end{aligned}$$

- **Problem 175: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+a \cos[c+dx]} (A+C \cos[c+dx])^2}{\cos[c+dx]^{5/2}} dx$$

Optimal (type 3, 116 leaves, 4 steps):

$$\frac{2 \sqrt{a} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} + \frac{2 a A \sin[c+dx]}{3 d \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}} + \frac{2 A \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{3 d \cos[c+dx]^{3/2}}$$

Result (type 3, 468 leaves):

$$\frac{1}{6\sqrt{2}d\cos[c+dx]^{3/2}\sqrt{\cos[c+dx]}\sqrt{\cos[dx]+i\sin[dx]}} i\sqrt{a(1+\cos[c+dx])}\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]$$

$$\left(\cos\left[\frac{dx}{2}\right]+i\sin\left[\frac{dx}{2}\right]\right)\left(6C\operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right]+i\sin\left[\frac{c}{2}\right]\right)\sqrt{(1+e^{2idx})\cos[c]+i(-1+e^{2idx})\sin[c]}\right]\cos[c+dx]^2-\right.$$

$$3C\cos[2(c+dx)]\operatorname{Log}\left[2\left(e^{idx}\cos\left[\frac{c}{2}\right]+ie^{idx}\sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2idx})\cos[c]+i(-1+e^{2idx})\sin[c]}\right)\right]-$$

$$\left.\left(\cos\left[\frac{dx}{2}\right]-i\sin\left[\frac{dx}{2}\right]\right)\left(3C\cos\left[\frac{dx}{2}\right]\operatorname{Log}\left[2\left(e^{idx}\cos\left[\frac{c}{2}\right]+ie^{idx}\sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2idx})\cos[c]+i(-1+e^{2idx})\sin[c]}\right)\right]\right)+$$

$$3iC\operatorname{Log}\left[2\left(e^{idx}\cos\left[\frac{c}{2}\right]+ie^{idx}\sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2idx})\cos[c]+i(-1+e^{2idx})\sin[c]}\right)\right]\sin\left[\frac{dx}{2}\right]+$$

$$\left.4i\sqrt{2}A\sqrt{\cos[c+dx]}\sqrt{\cos[dx]+i\sin[dx]}\sin\left[\frac{3}{2}(c+dx)\right]\right)$$

■ **Problem 179: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos[c+dx]^{3/2}(a+a\cos[c+dx])^{3/2}(A+C\cos[c+dx]^2)dx$$

Optimal (type 3, 265 leaves, 7 steps):

$$\frac{a^{3/2}(176A+133C)\operatorname{ArcSin}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{128d} + \frac{a^2(176A+133C)\sqrt{\cos[c+dx]}\sin[c+dx]}{128d\sqrt{a+a\cos[c+dx]}} +$$

$$\frac{a^2(176A+133C)\cos[c+dx]^{3/2}\sin[c+dx]}{192d\sqrt{a+a\cos[c+dx]}} + \frac{a^2(80A+67C)\cos[c+dx]^{5/2}\sin[c+dx]}{240d\sqrt{a+a\cos[c+dx]}} +$$

$$\frac{3aC\cos[c+dx]^{5/2}\sqrt{a+a\cos[c+dx]}\sin[c+dx]}{40d} + \frac{C\cos[c+dx]^{5/2}(a+a\cos[c+dx])^{3/2}\sin[c+dx]}{5d}$$

Result (type 3, 367 leaves):

$$-\frac{1}{7680d\sqrt{2(1+e^{2idx})\cos[c]+2i(-1+e^{2idx})\sin[c]}}(a(1+\cos[c+dx]))^{3/2}\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3$$

$$\left(-15i(176A+133C)e^{\frac{idx}{2}}\left(\operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right]+i\sin\left[\frac{c}{2}\right]\right)\sqrt{(1+e^{2idx})\cos[c]+i(-1+e^{2idx})\sin[c]}\right]-\right.\right.$$

$$\left.\left.\operatorname{Log}\left[2\left(e^{idx}\cos\left[\frac{c}{2}\right]+ie^{idx}\sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2idx})\cos[c]+i(-1+e^{2idx})\sin[c]}\right)\right]\right)\right)$$

$$\sqrt{e^{-idx}\left((1+e^{2idx})\cos[c]+i(-1+e^{2idx})\sin[c]\right)}-4\sqrt{\cos[c+dx]}\left(2960A+2671C+2(880A+1007C)\cos[c+dx]+$$

$$4(80A+181C)\cos[2(c+dx)]+228C\cos[3(c+dx)]+48C\cos[4(c+dx)]\right)\sqrt{\cos[c+dx]}\sqrt{\cos[dx]+i\sin[dx]}\sin\left[\frac{1}{2}(c+dx)\right]\right)$$

- **Problem 180: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^{3/2} (A+C\cos[c+dx]^2) dx$$

Optimal (type 3, 218 leaves, 6 steps):

$$\frac{a^{3/2} (112 A + 75 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{64 d} + \frac{a^2 (112 A + 75 C) \sqrt{\cos[c+dx]} \sin[c+dx]}{64 d \sqrt{a+a\cos[c+dx]}} + \frac{a^2 (16 A + 13 C) \cos[c+dx]^{3/2} \sin[c+dx]}{32 d \sqrt{a+a\cos[c+dx]}} +$$

$$\frac{a C \cos[c+dx]^{3/2} \sqrt{a+a\cos[c+dx]} \sin[c+dx]}{8 d} + \frac{C \cos[c+dx]^{3/2} (a+a\cos[c+dx])^{3/2} \sin[c+dx]}{4 d}$$

Result (type 3, 995 leaves):

$$\begin{aligned}
& \frac{1}{256} (112 A + 75 C) (a (1 + \cos [c + d x]))^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \\
& \left( \frac{1}{2} i \sin \left[ \frac{c}{2} \right] \left( - \left( 2 i e^{\frac{i d x}{2}} \operatorname{Log} \left[ 2 \left( e^{i d x} \cos \left[ \frac{c}{2} \right] + i e^{i d x} \sin \left[ \frac{c}{2} \right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) \right] \right) \left( \cos \left[ \frac{c}{2} \right] - i \sin \left[ \frac{c}{2} \right] \right) \right. \\
& \quad \left. \sqrt{e^{-i d x} \left( (1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c]} \right) - \\
& \quad \left( 2 i e^{\frac{i d x}{2}} \operatorname{ArcTanh} \left[ \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right] \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \right. \\
& \quad \left. \sqrt{e^{-i d x} \left( (1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c]} \right) \right) + \\
& \frac{1}{2} \cos \left[ \frac{c}{2} \right] \left( - \left( 2 i e^{\frac{i d x}{2}} \operatorname{Log} \left[ 2 \left( e^{i d x} \cos \left[ \frac{c}{2} \right] + i e^{i d x} \sin \left[ \frac{c}{2} \right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) \right] \right) \left( \cos \left[ \frac{c}{2} \right] - i \sin \left[ \frac{c}{2} \right] \right) \right. \\
& \quad \left. \sqrt{e^{-i d x} \left( (1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c]} \right) + \\
& \quad \left( 2 i e^{\frac{i d x}{2}} \operatorname{ArcTanh} \left[ \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right] \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \right. \\
& \quad \left. \sqrt{e^{-i d x} \left( (1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c]} \right) \right) \right) + \\
& \sqrt{\cos [c + d x]} (a (1 + \cos [c + d x]))^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \left( \frac{(3 A + 2 C) \cos \left[ \frac{d x}{2} \right] \sin \left[ \frac{c}{2} \right]}{4 d} + \frac{(16 A + 21 C) \cos \left[ \frac{3 d x}{2} \right] \sin \left[ \frac{3 c}{2} \right]}{128 d} + \right. \\
& \quad \frac{C \cos \left[ \frac{5 d x}{2} \right] \sin \left[ \frac{5 c}{2} \right]}{16 d} + \frac{C \cos \left[ \frac{7 d x}{2} \right] \sin \left[ \frac{7 c}{2} \right]}{64 d} + \\
& \quad \frac{(3 A + 2 C) \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right]}{4 d} + \frac{(16 A + 21 C) \cos \left[ \frac{3 c}{2} \right] \sin \left[ \frac{3 d x}{2} \right]}{128 d} + \\
& \quad \left. \frac{C \cos \left[ \frac{5 c}{2} \right] \sin \left[ \frac{5 d x}{2} \right]}{16 d} + \frac{C \cos \left[ \frac{7 c}{2} \right] \sin \left[ \frac{7 d x}{2} \right]}{64 d} \right)
\end{aligned}$$

- **Problem 181: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^{3/2} (A + C \cos [c + d x]^2)}{\sqrt{\cos [c + d x]}} dx$$

Optimal (type 3, 171 leaves, 5 steps):

$$\frac{a^{3/2} (24 A + 11 C) \operatorname{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right]}{8 d} + \frac{a^2 (24 A + 19 C) \sqrt{\cos [c + d x]} \sin [c + d x]}{24 d \sqrt{a + a \cos [c + d x]}} + \\
\frac{a C \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{4 d} + \frac{C \sqrt{\cos [c + d x]} (a + a \cos [c + d x])^{3/2} \sin [c + d x]}{3 d}$$

Result (type 3, 579 leaves) :

$$\frac{1}{48 d \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]}} a \sqrt{\cos[c + d x]} \sqrt{a (1 + \cos[c + d x])} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \\ \left( -3 i (24 A + 11 C) \cos\left[\frac{d x}{2}\right] \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \right)\right] \right) + \\ 3 i (24 A + 11 C) \operatorname{ArcTanh}\left[\left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \right] \left( \cos\left[\frac{d x}{2}\right] + i \sin\left[\frac{d x}{2}\right] \right) + \\ 72 A \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \right)\right] \sin\left[\frac{d x}{2}\right] + \\ 33 C \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \right)\right] \sin\left[\frac{d x}{2}\right] + \\ 48 \sqrt{2} A \sqrt{\cos[c + d x]} (\cos[d x] + i \sin[d x]) \sin\left[\frac{1}{2} (c + d x)\right] + 52 \sqrt{2} C \sqrt{\cos[c + d x]} (\cos[d x] + i \sin[d x]) \sin\left[\frac{1}{2} (c + d x)\right] + \\ 18 \sqrt{2} C \sqrt{\cos[c + d x]} (\cos[d x] + i \sin[d x]) \sin\left[\frac{3}{2} (c + d x)\right] + 4 \sqrt{2} C \sqrt{\cos[c + d x]} (\cos[d x] + i \sin[d x]) \sin\left[\frac{5}{2} (c + d x)\right] \Bigg)$$

■ **Problem 182: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^{3/2} (A + C \cos[c + d x]^2)}{\cos[c + d x]^{3/2}} dx$$

Optimal (type 3, 175 leaves, 5 steps) :

$$\frac{a^{3/2} (8 A + 7 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c + d x]}{\sqrt{a + a \cos[c + d x]}}\right]}{4 d} - \frac{a^2 (8 A - 5 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{4 d \sqrt{a + a \cos[c + d x]}} - \\ \frac{a (4 A - C) \sqrt{\cos[c + d x]} \sqrt{a + a \cos[c + d x]} \sin[c + d x]}{2 d} + \frac{2 A (a + a \cos[c + d x])^{3/2} \sin[c + d x]}{d \sqrt{\cos[c + d x]}}$$

Result (type 3, 911 leaves) :



$$\frac{1}{16} (8A + 7C) (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3$$

$$\left(\frac{1}{2} i \sin\left[\frac{c}{2}\right] \left(-\left(2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2\left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]}\right]\right)\right) \left(\cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right]\right)\right.\right.$$

$$\left.\frac{\sqrt{e^{-idx} \left((1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]\right)}}{\left(d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]}\right)} - \right.$$

$$\left.\left(2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]}\right] \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right)\right.\right.$$

$$\left.\frac{\sqrt{e^{-idx} \left((1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]\right)}}{\left(d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]}\right)}\right) +$$

$$\frac{1}{2} \cos\left[\frac{c}{2}\right] \left(-\left(2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2\left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]}\right]\right)\right) \left(\cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right]\right)\right.$$

$$\left.\frac{\sqrt{e^{-idx} \left((1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]\right)}}{\left(d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]}\right)} + \right.$$

$$\left.\left(2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]}\right] \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right)\right.\right.$$

$$\left.\frac{\sqrt{e^{-idx} \left((1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]\right)}}{\left(d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]}\right)}\right) +$$

$$\sqrt{\cos[c + dx]} (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(\frac{3C \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{4d} + \frac{C \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{8d} + \right.$$

$$\frac{3C \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{4d} + \frac{C \cos\left[\frac{3c}{2}\right] \sin\left[\frac{3dx}{2}\right]}{8d} +$$

$$\left.\frac{A \sec[c + dx] \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{d}\right)$$

■ **Problem 183: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx])^{3/2} (A + C \cos[c + dx]^2)}{\cos[c + dx]^{5/2}} dx$$

Optimal (type 3, 161 leaves, 5 steps):

$$\frac{3 a^{3/2} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} - \frac{a^2 (8A - 3C) \sqrt{\cos[c+dx]} \sin[c+dx]}{3 d \sqrt{a+a \cos[c+dx]}} +$$

$$\frac{2 a A \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}} + \frac{2 A (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{3 d \cos[c+dx]^{3/2}}$$

Result (type 3, 895 leaves):

$$\begin{aligned}
& \frac{3}{4} C (a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \\
& \left( \frac{1}{2} i \operatorname{Sin}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left( e^{idx} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right)} \right] \right) \left( \operatorname{Cos}\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) - \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \right) \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \Big) + \\
& \frac{1}{2} \operatorname{Cos}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left( e^{idx} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right)} \right] \right) \left( \operatorname{Cos}\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) + \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \right) \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \Big) \Big) + \\
& \sqrt{\operatorname{Cos}[c + dx]} (a (1 + \operatorname{Cos}[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left( \frac{C \operatorname{Cos}\left[\frac{dx}{2}\right] \operatorname{Sin}\left[\frac{c}{2}\right]}{2 d} + \frac{C \operatorname{Cos}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{2 d} + \right. \\
& \quad \left. \frac{5 A \operatorname{Sec}[c + dx] \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]}{3 d} + \frac{A \operatorname{Sec}[c + dx]^2 \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]}{3 d} \right)
\end{aligned}$$

■ **Problem 184: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Cos}[c + dx])^{3/2} (A + C \operatorname{Cos}[c + dx]^2)}{\operatorname{Cos}[c + dx]^{7/2}} dx$$

Optimal (type 3, 163 leaves, 5 steps):

$$\frac{2 a^{3/2} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{d} + \frac{2 a^2 (4 A + 5 C) \operatorname{Sin}[c + dx]}{5 d \sqrt{\operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Cos}[c + dx]}} + \\
\frac{2 a A \sqrt{a + a \operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{5 d \operatorname{Cos}[c + dx]^{3/2}} + \frac{2 A (a + a \operatorname{Cos}[c + dx])^{3/2} \operatorname{Sin}[c + dx]}{5 d \operatorname{Cos}[c + dx]^{5/2}}$$

Result (type 3, 747 leaves):

$$\frac{1}{20 \sqrt{2} d \cos [c+d x]^{5/2} \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])}} (a (1+\cos [c+d x]))^{3/2} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3$$

$$\left(\frac{5}{4} C e^{-\frac{5}{2} i d x} \cos \left[\frac{C}{2}\right]^2 \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{C}{2}\right]+i e^{i d x} \sin \left[\frac{C}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right]\right)$$

$$\left(i\left(1+e^{2 i d x}\right) \cos [c]-\left(-1+e^{2 i d x}\right) \sin [c]\right)^3+\frac{5}{4} C e^{-\frac{5}{2} i d x} \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{C}{2}\right]+i e^{i d x} \sin \left[\frac{C}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right] \sin \left[\frac{C}{2}\right]^2\left(i\left(1+e^{2 i d x}\right) \cos [c]-\left(-1+e^{2 i d x}\right) \sin [c]\right)^3+$$

$$\frac{5}{4} i C e^{-\frac{5}{2} i d x} \operatorname{ArcTanh}\left[\left(\cos \left[\frac{C}{2}\right]+i \sin \left[\frac{C}{2}\right]\right) \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right] \cos \left[\frac{C}{2}\right]^2$$

$$\left(\left(1+e^{2 i d x}\right) \cos [c]+i(-1+e^{2 i d x}) \sin [c]\right)^3+\frac{5}{4} i C e^{-\frac{5}{2} i d x} \operatorname{ArcTanh}\left[\left(\cos \left[\frac{C}{2}\right]+i \sin \left[\frac{C}{2}\right]\right) \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right] \sin \left[\frac{C}{2}\right]^2\left(\left(1+e^{2 i d x}\right) \cos [c]+i(-1+e^{2 i d x}) \sin [c]\right)^3+4 A \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i(-1+e^{2 i d x}) \sin [c]} \sin \left[\frac{1}{2}(c+d x)\right]+$$

$$12 A \cos [c+d x] \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i(-1+e^{2 i d x}) \sin [c]} \sin \left[\frac{1}{2}(c+d x)\right]+$$

$$24 \sqrt{2} A \cos [c+d x]^2 \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])} \sin \left[\frac{1}{2}(c+d x)\right]+$$

$$20 \sqrt{2} C \cos [c+d x]^2 \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])} \sin \left[\frac{1}{2}(c+d x)\right]$$

■ **Problem 188: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^{3/2} (a+a \cos [c+d x])^{5/2} (A+C \cos [c+d x]^2) dx$$

Optimal (type 3, 312 leaves, 8 steps):

$$\frac{a^{5/2} (1304 A+1015 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{512 d}+\frac{a^3 (1304 A+1015 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{512 d \sqrt{a+a \cos [c+d x]}}+\frac{a^3 (1304 A+1015 C) \cos [c+d x]^{3/2} \sin [c+d x]}{768 d \sqrt{a+a \cos [c+d x]}}+$$

$$\frac{a^3 (136 A+109 C) \cos [c+d x]^{5/2} \sin [c+d x]}{192 d \sqrt{a+a \cos [c+d x]}}+\frac{a^2 (24 A+23 C) \cos [c+d x]^{5/2} \sqrt{a+a \cos [c+d x]} \sin [c+d x]}{96 d}+$$

$$\frac{a C \cos [c+d x]^{5/2} (a+a \cos [c+d x])^{3/2} \sin [c+d x]}{12 d}+\frac{C \cos [c+d x]^{5/2} (a+a \cos [c+d x])^{5/2} \sin [c+d x]}{6 d}$$

Result (type 3, 1103 leaves):

$$\frac{1}{4096} (1304 A+1015 C) (a (1+\cos [c+d x]))^{5/2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^5$$

$$\left(\frac{1}{2} i \sin \left[\frac{C}{2}\right] \left(-\left(2 i e^{\frac{i d x}{2}} \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{C}{2}\right]+i e^{i d x} \sin \left[\frac{C}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i(-1+e^{2 i d x}) \sin [c]}\right)\right]\right)\left(\cos \left[\frac{C}{2}\right]-i \sin \left[\frac{C}{2}\right]\right)\right)$$

$$\begin{aligned}
& \left. \frac{\sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c] \right)}}{\left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]} \right)} - \right. \\
& \left. \left( 2i e^{\frac{idx}{2}} \operatorname{ArcTanh} \left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right) \right. \\
& \left. \frac{\sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c] \right)}}{\left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]} \right)} \right) + \\
& \frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2i e^{\frac{idx}{2}} \operatorname{Log} \left[ 2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right) \right. \\
& \left. \frac{\sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c] \right)}}{\left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]} \right)} + \right. \\
& \left. \left( 2i e^{\frac{idx}{2}} \operatorname{ArcTanh} \left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right) \right. \\
& \left. \frac{\sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c] \right)}}{\left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]} \right)} \right) \right) + \\
& \sqrt{\cos[c + dx]} \left( a(1 + \cos[c + dx]) \right)^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \left( \frac{(200A + 161C) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{384d} + \right. \\
& \frac{(360A + 329C) \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{2048d} + \\
& \frac{(10A + 13C) \cos\left[\frac{5dx}{2}\right] \sin\left[\frac{5c}{2}\right]}{192d} + \\
& \frac{(24A + 79C) \cos\left[\frac{7dx}{2}\right] \sin\left[\frac{7c}{2}\right]}{3072d} + \\
& \frac{C \cos\left[\frac{9dx}{2}\right] \sin\left[\frac{9c}{2}\right]}{128d} + \\
& \frac{C \cos\left[\frac{11dx}{2}\right] \sin\left[\frac{11c}{2}\right]}{768d} + \\
& \frac{(200A + 161C) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{384d} + \\
& \frac{(360A + 329C) \cos\left[\frac{3c}{2}\right] \sin\left[\frac{3dx}{2}\right]}{2048d} + \\
& \frac{(10A + 13C) \cos\left[\frac{5c}{2}\right] \sin\left[\frac{5dx}{2}\right]}{192d} + \\
& \left. \frac{(24A + 79C) \cos\left[\frac{7c}{2}\right] \sin\left[\frac{7dx}{2}\right]}{3072d} \right)
\end{aligned}$$

$$\frac{C \operatorname{Cos}\left[\frac{9c}{2}\right] \operatorname{Sin}\left[\frac{9dx}{2}\right]}{128d} + \frac{C \operatorname{Cos}\left[\frac{11c}{2}\right] \operatorname{Sin}\left[\frac{11dx}{2}\right]}{768d}$$

- **Problem 189: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\operatorname{Cos}[c+dx]} (a+a \operatorname{Cos}[c+dx])^{5/2} (A+C \operatorname{Cos}[c+dx]^2) dx$$

Optimal (type 3, 265 leaves, 7 steps):

$$\frac{a^{5/2} (400A + 283C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{128d} + \frac{a^3 (400A + 283C) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{128d \sqrt{a+a \operatorname{Cos}[c+dx]}} +$$

$$\frac{a^3 (1040A + 787C) \operatorname{Cos}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{960d \sqrt{a+a \operatorname{Cos}[c+dx]}} + \frac{a^2 (80A + 79C) \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+a \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{240d} +$$

$$\frac{aC \operatorname{Cos}[c+dx]^{3/2} (a+a \operatorname{Cos}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{8d} + \frac{C \operatorname{Cos}[c+dx]^{3/2} (a+a \operatorname{Cos}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{5d}$$

Result (type 3, 366 leaves):

$$-\frac{1}{15360d \sqrt{2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c]}} (a(1+\operatorname{Cos}[c+dx]))^{5/2} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5$$

$$\left(-15i(400A+283C) e^{\frac{idx}{2}} \left(\operatorname{ArcTanh}\left[\left(\operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2idx}) \operatorname{Cos}[c] + i(-1+e^{2idx}) \operatorname{Sin}[c]}\right] - \right.$$

$$\left.\operatorname{Log}\left[2\left(e^{idx} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \operatorname{Cos}[c] + i(-1+e^{2idx}) \operatorname{Sin}[c]}\right]\right)\right]$$

$$\sqrt{e^{-idx} \left((1+e^{2idx}) \operatorname{Cos}[c] + i(-1+e^{2idx}) \operatorname{Sin}[c]\right)} - 4 \sqrt{\operatorname{Cos}[c+dx]} (6320A + 5521C + (2720A + 3874C) \operatorname{Cos}[c+dx] +$$

$$4(80A + 331C) \operatorname{Cos}[2(c+dx)] + 348C \operatorname{Cos}[3(c+dx)] + 48C \operatorname{Cos}[4(c+dx)]) \sqrt{\operatorname{Cos}[c+dx]} (\operatorname{Cos}[dx] + i \operatorname{Sin}[dx]) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \Bigg)$$

- **Problem 190: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \operatorname{Cos}[c+dx])^{5/2} (A+C \operatorname{Cos}[c+dx]^2)}{\sqrt{\operatorname{Cos}[c+dx]}} dx$$

Optimal (type 3, 218 leaves, 6 steps):

$$\frac{a^{5/2} (304 A + 163 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{64 d} +$$

$$\frac{a^3 (432 A + 299 C) \sqrt{\cos[c+dx]} \sin[c+dx]}{192 d \sqrt{a+a \cos[c+dx]}} + \frac{a^2 (16 A + 17 C) \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{32 d} +$$

$$\frac{5 a C \sqrt{\cos[c+dx]} (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{24 d} + \frac{C \sqrt{\cos[c+dx]} (a+a \cos[c+dx])^{5/2} \sin[c+dx]}{4 d}$$

Result (type 3, 995 leaves):

$$\frac{1}{512} (304 A + 163 C) (a (1 + \cos[c+dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5$$

$$\left(\frac{1}{2} i \sin\left[\frac{c}{2}\right] \left(-\left(2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right]\right)\right) \left(\cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right]\right)\right.\right.$$

$$\left.\frac{\sqrt{e^{-idx} \left((1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]\right)}}{\left(d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]}\right)} - \right.$$

$$\left.2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right] \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right)\right.$$

$$\left.\frac{\sqrt{e^{-idx} \left((1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]\right)}}{\left(d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]}\right)}\right) +$$

$$\frac{1}{2} \cos\left[\frac{c}{2}\right] \left(-\left(2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right]\right)\right) \left(\cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right]\right)\right.$$

$$\left.\frac{\sqrt{e^{-idx} \left((1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]\right)}}{\left(d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]}\right)} + \right.$$

$$\left.2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right] \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right)\right.$$

$$\left.\frac{\sqrt{e^{-idx} \left((1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]\right)}}{\left(d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]}\right)}\right) +$$

$$\sqrt{\cos[c+dx]} (a (1 + \cos[c+dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left(\frac{5 (6 A + 5 C) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{48 d} + \frac{(16 A + 45 C) \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{256 d} + \right.$$

$$\frac{5 C \cos\left[\frac{5dx}{2}\right] \sin\left[\frac{5c}{2}\right]}{96 d} + \frac{C \cos\left[\frac{7dx}{2}\right] \sin\left[\frac{7c}{2}\right]}{128 d} +$$

$$\frac{5 (6 A + 5 C) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{48 d} + \frac{(16 A + 45 C) \cos\left[\frac{3c}{2}\right] \sin\left[\frac{3dx}{2}\right]}{256 d} +$$

$$\left.\frac{5 C \cos\left[\frac{5c}{2}\right] \sin\left[\frac{5dx}{2}\right]}{96 d} + \frac{C \cos\left[\frac{7c}{2}\right] \sin\left[\frac{7dx}{2}\right]}{128 d}\right)$$

- **Problem 191: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx])^{5/2} (A + C \cos[c + dx]^2)}{\cos[c + dx]^{3/2}} dx$$

Optimal (type 3, 222 leaves, 6 steps) :

$$\frac{5 a^{5/2} (8 A + 5 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{8 d} - \frac{a^3 (24 A - 49 C) \sqrt{\cos[c+dx]} \sin[c+dx]}{24 d \sqrt{a+a \cos[c+dx]}} -$$

$$\frac{a^2 (8 A - 3 C) \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{4 d} -$$

$$\frac{a (6 A - C) \sqrt{\cos[c+dx]} (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{3 d} + \frac{2 A (a+a \cos[c+dx])^{5/2} \sin[c+dx]}{d \sqrt{\cos[c+dx]}}$$

Result (type 3, 968 leaves) :

$$\begin{aligned}
& \frac{5}{64} (8A + 5C) (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \\
& \left( \frac{1}{2} i \operatorname{Sin}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right)} \right] \right) \left( \operatorname{Cos}\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) - \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right) + \\
& \quad \frac{1}{2} \operatorname{Cos}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right)} \right] \right) \left( \operatorname{Cos}\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) + \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right) \Bigg) + \\
& \sqrt{\operatorname{Cos}[c + dx]} (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \frac{(12A + 31C) \operatorname{Cos}\left[\frac{dx}{2}\right] \operatorname{Sin}\left[\frac{c}{2}\right]}{48d} + \frac{5C \operatorname{Cos}\left[\frac{3dx}{2}\right] \operatorname{Sin}\left[\frac{3c}{2}\right]}{32d} + \right. \\
& \quad \frac{C \operatorname{Cos}\left[\frac{5dx}{2}\right] \operatorname{Sin}\left[\frac{5c}{2}\right]}{48d} + \frac{(12A + 31C) \operatorname{Cos}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{48d} + \\
& \quad \left. \frac{5C \operatorname{Cos}\left[\frac{3c}{2}\right] \operatorname{Sin}\left[\frac{3dx}{2}\right]}{32d} + \frac{C \operatorname{Cos}\left[\frac{5c}{2}\right] \operatorname{Sin}\left[\frac{5dx}{2}\right]}{48d} + \frac{A \operatorname{Sec}[c + dx] \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]}{2d} \right)
\end{aligned}$$

■ **Problem 192: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Cos}[c + dx])^{5/2} (A + C \operatorname{Cos}[c + dx]^2)}{\operatorname{Cos}[c + dx]^{5/2}} dx$$

Optimal (type 3, 218 leaves, 6 steps):

$$\begin{aligned}
& \frac{a^{5/2} (8A + 19C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c + dx]}{\sqrt{a + a \operatorname{Cos}[c + dx]}}\right]}{4d} - \frac{a^3 (56A - 27C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{12d \sqrt{a + a \operatorname{Cos}[c + dx]}} - \\
& \frac{a^2 (8A - C) \sqrt{\operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{2d} + \frac{10aA (a + a \operatorname{Cos}[c + dx])^{3/2} \operatorname{Sin}[c + dx]}{3d \sqrt{\operatorname{Cos}[c + dx]}} + \frac{2A (a + a \operatorname{Cos}[c + dx])^{5/2} \operatorname{Sin}[c + dx]}{3d \operatorname{Cos}[c + dx]^{3/2}}
\end{aligned}$$

Result (type 3, 943 leaves):



$$\begin{aligned}
& \frac{1}{32} (8A + 19C) (a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \\
& \left( \frac{1}{2} i \operatorname{Sin}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right)} \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) - \right. \\
& \quad \left. \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right) + \\
& \frac{1}{2} \operatorname{Cos}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right)} \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) + \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right) \right) + \\
& \sqrt{\operatorname{Cos}[c + dx]} (a (1 + \operatorname{Cos}[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \frac{5C \operatorname{Cos}\left[\frac{dx}{2}\right] \operatorname{Sin}\left[\frac{c}{2}\right]}{8d} + \frac{C \operatorname{Cos}\left[\frac{3dx}{2}\right] \operatorname{Sin}\left[\frac{3c}{2}\right]}{16d} + \right. \\
& \quad \left. \frac{5C \operatorname{Cos}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{8d} + \frac{C \operatorname{Cos}\left[\frac{3c}{2}\right] \operatorname{Sin}\left[\frac{3dx}{2}\right]}{16d} + \right. \\
& \quad \left. \frac{4A \operatorname{Sec}[c + dx] \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]}{3d} + \frac{A \operatorname{Sec}[c + dx]^2 \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]}{6d} \right)
\end{aligned}$$

■ **Problem 193: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Cos}[c + dx])^{5/2} (A + C \operatorname{Cos}[c + dx]^2)}{\operatorname{Cos}[c + dx]^{7/2}} dx$$

Optimal (type 3, 210 leaves, 6 steps):

$$\begin{aligned}
& \frac{5 a^{5/2} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{d} - \frac{a^3 (64 A + 15 C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{15 d \sqrt{a + a \operatorname{Cos}[c + dx]}} + \\
& \frac{2 a^2 (8 A + 5 C) \sqrt{a + a \operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{5 d \sqrt{\operatorname{Cos}[c + dx]}} + \frac{2 a A (a + a \operatorname{Cos}[c + dx])^{3/2} \operatorname{Sin}[c + dx]}{3 d \operatorname{Cos}[c + dx]^{3/2}} + \frac{2 A (a + a \operatorname{Cos}[c + dx])^{5/2} \operatorname{Sin}[c + dx]}{5 d \operatorname{Cos}[c + dx]^{5/2}}
\end{aligned}$$

Result (type 3, 853 leaves):

$$\begin{aligned}
& \frac{1}{240 \sqrt{2} d \cos [c+d x]^{5/2} \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])}} (a(1+\cos [c+d x]))^{5/2} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 \\
& \left(\frac{75}{4} C e^{-\frac{5}{2} i d x} \cos \left[\frac{c}{2}\right]^2 \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right)\right]\right. \\
& \quad \left.\left(i\left(1+e^{2 i d x}\right) \cos [c]-\left(-1+e^{2 i d x}\right) \sin [c]\right)^3+\frac{75}{4} C e^{-\frac{5}{2} i d x} \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right)\right]\right. \\
& \quad \left.\left(i\left(1+e^{2 i d x}\right) \cos [c]-\left(-1+e^{2 i d x}\right) \sin [c]\right)^3+\frac{75}{4} i C e^{-\frac{5}{2} i d x} \operatorname{ArcTan h}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right] \cos \left[\frac{c}{2}\right]^2\right. \\
& \quad \left.\left(\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]\right)^3+\frac{75}{4} i C e^{-\frac{5}{2} i d x} \operatorname{ArcTan h}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right]\right. \\
& \quad \left.\sin \left[\frac{c}{2}\right]^2\left(\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]\right)^3+60 \sqrt{2} C \cos \left[\frac{d x}{2}\right] \cos [c+d x]^3 \sin \left[\frac{c}{2}\right] \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])}+\right. \\
& \quad \left.60 \sqrt{2} C \cos \left[\frac{c}{2}\right] \cos [c+d x]^3 \sin \left[\frac{d x}{2}\right] \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])}+\right. \\
& \quad \left.24 A \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]} \sin \left[\frac{1}{2}(c+d x)\right]+\right. \\
& \quad \left.112 A \cos [c+d x] \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]} \sin \left[\frac{1}{2}(c+d x)\right]+\right. \\
& \quad \left.344 \sqrt{2} A \cos [c+d x]^2 \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])} \sin \left[\frac{1}{2}(c+d x)\right]+\right. \\
& \quad \left.120 \sqrt{2} C \cos [c+d x]^2 \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])} \sin \left[\frac{1}{2}(c+d x)\right]\right)
\end{aligned}$$

■ **Problem 194: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \cos [c+d x])^{5/2} (A+C \cos [c+d x]^2)}{\cos [c+d x]^{9/2}} dx$$

Optimal (type 3, 210 leaves, 6 steps):

$$\begin{aligned}
& \frac{2 a^{5/2} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{d} + \frac{2 a^3 (32 A+49 C) \sin [c+d x]}{21 d \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}} + \\
& \frac{2 a^2 (8 A+7 C) \sqrt{a+a \cos [c+d x]} \sin [c+d x]}{21 d \cos [c+d x]^{3/2}} + \frac{2 a A (a+a \cos [c+d x])^{3/2} \sin [c+d x]}{7 d \cos [c+d x]^{5/2}} + \frac{2 A (a+a \cos [c+d x])^{5/2} \sin [c+d x]}{7 d \cos [c+d x]^{7/2}}
\end{aligned}$$

Result (type 3, 949 leaves):

$$\begin{aligned}
& \frac{1}{4} C (a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \\
& \left( \frac{1}{2} i \operatorname{Sin}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right)\right] \right) \left( \cos\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) - \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \Big) + \\
& \frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right)\right] \right) \left( \cos\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) + \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \Big) \Big) + \\
& \sqrt{\cos[c + dx]} (a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \frac{2 A \operatorname{Sec}[c + dx]^3 \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]}{7 d} + \frac{A \operatorname{Sec}[c + dx]^4 \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]}{14 d} + \right. \\
& \quad \left. \frac{\operatorname{Sec}[c + dx]^2 \left( 23 A \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right] + 7 C \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right] \right)}{42 d} + \right. \\
& \quad \left. \frac{\operatorname{Sec}[c + dx] \left( 23 A \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right] + 28 C \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right] \right)}{21 d} \right)
\end{aligned}$$

■ **Problem 198: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c + dx]^{3/2} (A + C \cos[c + dx])^2}{\sqrt{a + a \cos[c + dx]}} dx$$

Optimal (type 3, 226 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(8 A + 9 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{8 \sqrt{a} d} + \frac{\sqrt{2} (A + C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} + \\
& \frac{(8 A + 7 C) \sqrt{\cos[c + dx]} \operatorname{Sin}[c + dx]}{8 d \sqrt{a + a \cos[c + dx]}} - \frac{C \cos[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{12 d \sqrt{a + a \cos[c + dx]}} + \frac{C \cos[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{3 d \sqrt{a + a \cos[c + dx]}}
\end{aligned}$$

Result (type 3, 349 leaves):

$$\frac{1}{48 d \sqrt{a (1 + \cos [c + d x])}}$$

$$\cos \left[ \frac{1}{2} (c + d x) \right] \left( -\frac{1}{\sqrt{1 + e^{2 i (c + d x)}}} 3 i \sqrt{2} e^{\frac{1}{2} i (c + d x)} \sqrt{e^{-i (c + d x)} (1 + e^{2 i (c + d x)})} \left( -8 i A d x - 9 i C d x - (8 A + 9 C) \operatorname{ArcSinh} \left[ e^{i (c + d x)} \right] + \right. \right.$$

$$16 \sqrt{2} (A + C) \operatorname{Log} \left[ 1 + e^{i (c + d x)} \right] + 8 A \operatorname{Log} \left[ 1 + \sqrt{1 + e^{2 i (c + d x)}} \right] + 9 C \operatorname{Log} \left[ 1 + \sqrt{1 + e^{2 i (c + d x)}} \right] -$$

$$16 \sqrt{2} A \operatorname{Log} \left[ 1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}} \right] - 16 \sqrt{2} C \operatorname{Log} \left[ 1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}} \right] \left. \right) +$$

$$4 \sqrt{\cos [c + d x]} (24 A + 25 C - 2 C \cos [c + d x] + 4 C \cos [2 (c + d x)]) \sin \left[ \frac{1}{2} (c + d x) \right]$$

■ **Problem 199: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{\cos [c + d x]} (A + C \cos [c + d x]^2)}{\sqrt{a + a \cos [c + d x]}} dx$$

Optimal (type 3, 183 leaves, 7 steps):

$$\frac{(8 A + 7 C) \operatorname{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right]}{4 \sqrt{a} d} - \frac{\sqrt{2} (A + C) \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{2} \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]}} \right]}{\sqrt{a} d} - \frac{C \sqrt{\cos [c + d x]} \sin [c + d x]}{4 d \sqrt{a + a \cos [c + d x]}} + \frac{C \cos [c + d x]^{3/2} \sin [c + d x]}{2 d \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 344 leaves):

$$\frac{1}{8 \sqrt{a} (1 + \cos [c + d x])}$$

$$\cos \left[ \frac{1}{2} (c + d x) \right] \left( 1 / \left( d \sqrt{1 + e^{2 i (c + d x)}} \right) \sqrt{2} e^{\frac{1}{2} i (c + d x)} \sqrt{e^{-i (c + d x)} (1 + e^{2 i (c + d x)})} \left( 8 A d x + 7 C d x - i (8 A + 7 C) \operatorname{ArcSinh} \left[ e^{i (c + d x)} \right] + 8 i \sqrt{2} (A + C) \right. \right.$$

$$\operatorname{Log} \left[ 1 + e^{i (c + d x)} \right] + 8 i A \operatorname{Log} \left[ 1 + \sqrt{1 + e^{2 i (c + d x)}} \right] + 7 i C \operatorname{Log} \left[ 1 + \sqrt{1 + e^{2 i (c + d x)}} \right] - 8 i \sqrt{2} A \operatorname{Log} \left[ 1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}} \right] -$$

$$8 i \sqrt{2} C \operatorname{Log} \left[ 1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}} \right] \left. \right) + \frac{4 C \sqrt{\cos [c + d x]} \left( -2 \sin \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{3}{2} (c + d x) \right] \right)}{d}$$

■ **Problem 200: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos [c + d x]^2}{\sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]}} dx$$

Optimal (type 3, 133 leaves, 6 steps):

$$-\frac{C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} + \frac{\sqrt{2} (A+C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} + \frac{C \sqrt{\cos[c+dx]} \sin[c+dx]}{d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 289 leaves):

$$\frac{1}{2 \sqrt{a} (1 + \cos[c+dx])} \cos\left[\frac{1}{2} (c+dx)\right] \left( -1 / \left( d \sqrt{1 + e^{2i(c+dx)}} \right) i \sqrt{2} e^{\frac{1}{2} i (c+dx)} \right. \\ \left. \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( -i C dx - C \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] + 2 \sqrt{2} (A+C) \operatorname{Log}\left[1 + e^{i(c+dx)}\right] + C \operatorname{Log}\left[1 + \sqrt{1 + e^{2i(c+dx)}}\right] - \right. \right. \\ \left. \left. 2 \sqrt{2} A \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] - 2 \sqrt{2} C \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] \right) + \frac{4 C \sqrt{\cos[c+dx]} \sin\left[\frac{1}{2} (c+dx)\right]}{d} \right)$$

■ **Problem 201: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c+dx]^2}{\cos[c+dx]^{3/2} \sqrt{a+a \cos[c+dx]}} dx$$

Optimal (type 3, 135 leaves, 6 steps):

$$\frac{2 C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} - \frac{\sqrt{2} (A+C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} + \frac{2 A \sin[c+dx]}{d \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 288 leaves):

$$\frac{1}{d \sqrt{a} (1 + \cos[c+dx])} \cos\left[\frac{1}{2} (c+dx)\right] \left( 1 / \left( \sqrt{1 + e^{2i(c+dx)}} \right) \sqrt{2} e^{\frac{1}{2} i (c+dx)} \right. \\ \left. \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( C dx - i C \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] + i \sqrt{2} (A+C) \operatorname{Log}\left[1 + e^{i(c+dx)}\right] + i C \operatorname{Log}\left[1 + \sqrt{1 + e^{2i(c+dx)}}\right] - \right. \right. \\ \left. \left. i \sqrt{2} A \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] - i \sqrt{2} C \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] \right) + \frac{4 A \sin\left[\frac{1}{2} (c+dx)\right]}{\sqrt{\cos[c+dx]}} \right)$$

■ **Problem 202: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + C \cos[c+dx]^2}{\cos[c+dx]^{5/2} \sqrt{a+a \cos[c+dx]}} dx$$

Optimal (type 3, 136 leaves, 5 steps):

$$\frac{\sqrt{2} (A+C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} + \frac{2 A \sin[c+dx]}{3 d \cos[c+dx]^{3/2} \sqrt{a+a \cos[c+dx]}} - \frac{2 A \sin[c+dx]}{3 d \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 181 leaves) :

$$\frac{1}{\sqrt{a (1 + \cos [c + d x])}} \cos \left[ \frac{1}{2} (c + d x) \right]$$

$$\left( -1 / \left( d \sqrt{1 + e^{2 i (c+d x)}} \right) 2 i (A + C) e^{\frac{1}{2} i (c+d x)} \sqrt{e^{-i (c+d x)} (1 + e^{2 i (c+d x)})} \left( \log [1 + e^{i (c+d x)}] - \log [1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}] \right) \right) +$$

$$\frac{8 A \sin \left[ \frac{1}{2} (c + d x) \right]^3}{3 d \cos [c + d x]^{3/2}}$$

■ **Problem 203: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + C \cos [c + d x]^2}{\cos [c + d x]^{7/2} \sqrt{a + a \cos [c + d x]}} dx$$

Optimal (type 3, 181 leaves, 6 steps) :

$$-\frac{\sqrt{2} (A + C) \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}} \right]}{\sqrt{a} d} + \frac{2 A \sin [c + d x]}{5 d \cos [c + d x]^{5/2} \sqrt{a + a \cos [c + d x]}} -$$

$$\frac{2 A \sin [c + d x]}{15 d \cos [c + d x]^{3/2} \sqrt{a + a \cos [c + d x]}} + \frac{2 (13 A + 15 C) \sin [c + d x]}{15 d \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 229 leaves) :

$$\left( 2 \cos \left[ \frac{1}{2} (c + d x) \right] \right.$$

$$\left. \left( 1 / \left( \sqrt{1 + e^{2 i (c+d x)}} \right) 15 i (A + C) e^{\frac{1}{2} i (c+d x)} \sqrt{e^{-i (c+d x)} (1 + e^{2 i (c+d x)})} \left( \log [1 + e^{i (c+d x)}] - \log [1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}] \right) \right) + \right.$$

$$\left. \left. \frac{6 A \sin \left[ \frac{1}{2} (c + d x) \right]}{\cos [c + d x]^{5/2}} - \frac{2 A \sin \left[ \frac{1}{2} (c + d x) \right]}{\cos [c + d x]^{3/2}} + \frac{2 (13 A + 15 C) \sin \left[ \frac{1}{2} (c + d x) \right]}{\sqrt{\cos [c + d x]}} \right) \right) / \left( 15 d \sqrt{a (1 + \cos [c + d x])} \right)$$

■ **Problem 204: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + C \cos [c + d x]^2}{\cos [c + d x]^{9/2} \sqrt{a + a \cos [c + d x]}} dx$$

Optimal (type 3, 224 leaves, 7 steps) :

$$\frac{\sqrt{2} (A + C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} + \frac{2 A \sin[c+dx]}{7 d \cos[c+dx]^{7/2} \sqrt{a+a \cos[c+dx]}} -$$

$$\frac{2 A \sin[c+dx]}{35 d \cos[c+dx]^{5/2} \sqrt{a+a \cos[c+dx]}} + \frac{2 (31 A + 35 C) \sin[c+dx]}{105 d \cos[c+dx]^{3/2} \sqrt{a+a \cos[c+dx]}} - \frac{2 (43 A + 35 C) \sin[c+dx]}{105 d \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 212 leaves):

$$\frac{1}{\sqrt{a} (1 + \cos[c+dx])} \cos\left[\frac{1}{2} (c+dx)\right]$$

$$\left( -1 / \left( d \sqrt{1 + e^{2i(c+dx)}} \right) 2 i (A + C) e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( \operatorname{Log}[1 + e^{i(c+dx)}] - \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) \right) +$$

$$\left. \frac{4 (73 A + 35 C + 24 A \cos[c+dx] + (43 A + 35 C) \cos[2(c+dx)]) \sin\left[\frac{1}{2} (c+dx)\right]^3}{105 d \cos[c+dx]^{7/2}} \right)$$

■ **Problem 205: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^{3/2} (A + C \cos[c+dx]^2)}{(a + a \cos[c+dx])^{3/2}} dx$$

Optimal (type 3, 245 leaves, 8 steps):

$$\frac{(8 A + 19 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{4 a^{3/2} d} - \frac{(5 A + 13 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} -$$

$$\frac{(A + C) \cos[c+dx]^{5/2} \sin[c+dx]}{2 d (a + a \cos[c+dx])^{3/2}} - \frac{(2 A + 7 C) \sqrt{\cos[c+dx]} \sin[c+dx]}{4 a d \sqrt{a + a \cos[c+dx]}} + \frac{(A + 2 C) \cos[c+dx]^{3/2} \sin[c+dx]}{2 a d \sqrt{a + a \cos[c+dx]}}$$

Result (type 3, 370 leaves):

$$\frac{1}{4 d (a (1 + \cos [c + d x]))^{3/2}} \cos \left[ \frac{1}{2} (c + d x) \right]^3 \left( \frac{1}{\sqrt{1 + e^{2 i (c + d x)}}} \sqrt{2} e^{\frac{1}{2} i (c + d x)} \sqrt{e^{-i (c + d x)} (1 + e^{2 i (c + d x)})} (8 A d x + 19 C d x - i (8 A + 19 C) \operatorname{ArcSinh}[e^{i (c + d x)}] + 2 i \sqrt{2} (5 A + 13 C) \operatorname{Log}[1 + e^{i (c + d x)}] + 8 i A \operatorname{Log}[1 + \sqrt{1 + e^{2 i (c + d x)}}] + 19 i C \operatorname{Log}[1 + \sqrt{1 + e^{2 i (c + d x)}}] - 10 i \sqrt{2} A \operatorname{Log}[1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}] - 26 i \sqrt{2} C \operatorname{Log}[1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}]) - 2 \sqrt{\cos [c + d x]} (2 A + 6 C + 3 C \cos [c + d x] - C \cos [2 (c + d x)]) \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \right)$$

■ **Problem 206: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{\cos [c + d x]} (A + C \cos [c + d x]^2)}{(a + a \cos [c + d x])^{3/2}} dx$$

Optimal (type 3, 188 leaves, 7 steps):

$$-\frac{3 C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}}\right]}{a^{3/2} d} + \frac{(A + 9 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c + d x]}{\sqrt{2} \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{(A + C) \cos [c + d x]^{3/2} \sin [c + d x]}{2 d (a + a \cos [c + d x])^{3/2}} + \frac{(A + 3 C) \sqrt{\cos [c + d x]} \sin [c + d x]}{2 a d \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 316 leaves):

$$\frac{1}{2 (a (1 + \cos [c + d x]))^{3/2}} \cos \left[ \frac{1}{2} (c + d x) \right]^3 \left( -1 / \left( d \sqrt{1 + e^{2 i (c + d x)}} \right) i \sqrt{2} e^{\frac{1}{2} i (c + d x)} \sqrt{e^{-i (c + d x)} (1 + e^{2 i (c + d x)})} \left( -6 i C d x - 6 C \operatorname{ArcSinh}[e^{i (c + d x)}] + \sqrt{2} (A + 9 C) \operatorname{Log}[1 + e^{i (c + d x)}] + 6 C \operatorname{Log}[1 + \sqrt{1 + e^{2 i (c + d x)}}] - \sqrt{2} A \operatorname{Log}[1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}] - 9 \sqrt{2} C \operatorname{Log}[1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}] \right) + \frac{2 \sqrt{\cos [c + d x]} (A + 3 C + 2 C \cos [c + d x]) \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{d} \right)$$

■ **Problem 207: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos [c + d x]^2}{\sqrt{\cos [c + d x]} (a + a \cos [c + d x])^{3/2}} dx$$



Optimal (type 3, 145 leaves, 6 steps) :

$$\frac{2 C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{a^{3 / 2} d} + \frac{(3 A-5 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right]}{2 \sqrt{2} a^{3 / 2} d} - \frac{(A+C) \sqrt{\cos [c+d x]} \sin [c+d x]}{2 d(a+a \cos [c+d x])^{3 / 2}}$$

Result (type 3, 313 leaves) :

$$\frac{1}{2(a(1+\cos [c+d x]))^{3 / 2}} \cos \left[\frac{1}{2}(c+d x)\right]^3$$

$$\left(1 / \left(d \sqrt{1+e^{2 i(c+d x)}}\right) \sqrt{2} e^{\frac{1}{2} i(c+d x)} \sqrt{e^{-i(c+d x)}\left(1+e^{2 i(c+d x)}\right)}\left(4 C d x-4 i C \operatorname{ArcSinh}\left[e^{i(c+d x)}\right]-i \sqrt{2}(3 A-5 C) \operatorname{Log}\left[1+e^{i(c+d x)}\right]+4 i C \operatorname{Log}\left[1+\sqrt{1+e^{2 i(c+d x)}}\right]+3 i \sqrt{2} A \operatorname{Log}\left[1-e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]-5 i \sqrt{2} C \operatorname{Log}\left[1-e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]\right)-\frac{2(A+C) \sqrt{\cos [c+d x]} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{d}\right)$$

■ **Problem 208: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+C \cos [c+d x]^2}{\cos [c+d x]^{3 / 2}(a+a \cos [c+d x])^{3 / 2}} d x$$

Optimal (type 3, 152 leaves, 5 steps) :

$$-\frac{(7 A-C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right]}{2 \sqrt{2} a^{3 / 2} d}-\frac{(A+C) \sin [c+d x]}{2 d \sqrt{\cos [c+d x]}(a+a \cos [c+d x])^{3 / 2}}+\frac{(5 A+C) \sin [c+d x]}{2 a d \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}$$

Result (type 3, 207 leaves) :

$$\frac{1}{(a(1+\cos [c+d x]))^{3 / 2}} \cos \left[\frac{1}{2}(c+d x)\right]^3$$

$$\left(1 / \left(d \sqrt{1+e^{2 i(c+d x)}}\right) i(7 A-C) e^{\frac{1}{2} i(c+d x)} \sqrt{e^{-i(c+d x)}\left(1+e^{2 i(c+d x)}\right)}\left(\operatorname{Log}\left[1+e^{i(c+d x)}\right]-\operatorname{Log}\left[1-e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]\right)+\frac{(4 A+(5 A+C) \cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{d \sqrt{\cos [c+d x]}}\right)$$

■ **Problem 209: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+C \cos [c+d x]^2}{\cos [c+d x]^{5 / 2}(a+a \cos [c+d x])^{3 / 2}} d x$$

Optimal (type 3, 201 leaves, 6 steps) :

$$\frac{(11A + 3C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right]}{2\sqrt{2} a^{3/2} d} - \frac{(A+C) \sin[c+dx]}{2d \cos[c+dx]^{3/2} (a+a\cos[c+dx])^{3/2}} +$$

$$\frac{(7A + 3C) \sin[c+dx]}{6ad \cos[c+dx]^{3/2} \sqrt{a+a\cos[c+dx]}} - \frac{(19A + 3C) \sin[c+dx]}{6ad \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 226 leaves):

$$\frac{1}{(a(1+\cos[c+dx]))^{3/2}} \cos\left[\frac{1}{2}(c+dx)\right]^3$$

$$\left( -1 / \left( d \sqrt{1+e^{2i(c+dx)}} \right) i (11A + 3C) e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) -$$

$$\left. \frac{(11A + 3C + 24A \cos[c+dx] + (19A + 3C) \cos[2(c+dx)]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{6d \cos[c+dx]^{3/2}} \right)$$

■ **Problem 210: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + C \cos[c+dx]^2}{\cos[c+dx]^{7/2} (a+a\cos[c+dx])^{3/2}} dx$$

Optimal (type 3, 248 leaves, 7 steps):

$$-\frac{(15A + 7C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right]}{2\sqrt{2} a^{3/2} d} - \frac{(A+C) \sin[c+dx]}{2d \cos[c+dx]^{5/2} (a+a\cos[c+dx])^{3/2}} +$$

$$\frac{(9A + 5C) \sin[c+dx]}{10ad \cos[c+dx]^{5/2} \sqrt{a+a\cos[c+dx]}} - \frac{(13A + 5C) \sin[c+dx]}{10ad \cos[c+dx]^{3/2} \sqrt{a+a\cos[c+dx]}} + \frac{(49A + 25C) \sin[c+dx]}{10ad \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 254 leaves):

$$\frac{1}{(a(1+\cos[c+dx]))^{3/2}} \cos\left[\frac{1}{2}(c+dx)\right]^3$$

$$\left( 1 / \left( d \sqrt{1+e^{2i(c+dx)}} \right) i (15A + 7C) e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) +$$

$$1 / \left( 20d \cos[c+dx]^{5/2} (88A + 40C + (131A + 75C) \cos[c+dx] + 8(9A + 5C) \cos[2(c+dx)] + 49A \cos[3(c+dx)] + 25C \cos[3(c+dx)]) \right)$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]$$

■ **Problem 211: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^{3/2} (A + C \cos[c+dx]^2)}{(a+a\cos[c+dx])^{5/2}} dx$$

Optimal (type 3, 237 leaves, 8 steps) :

$$-\frac{5 C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{a^{5/2} d} + \frac{(3 A+115 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right]}{16 \sqrt{2} a^{5/2} d} -$$

$$\frac{(A+C) \cos [c+d x]^{5/2} \sin [c+d x]}{4 d(a+a \cos [c+d x])^{5/2}} + \frac{(A-15 C) \cos [c+d x]^{3/2} \sin [c+d x]}{16 a d(a+a \cos [c+d x])^{3/2}} + \frac{(3 A+35 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{16 a^2 d \sqrt{a+a \cos [c+d x]}}$$

Result (type 3, 334 leaves) :

$$\frac{1}{8 d(a(1+\cos [c+d x]))^{5/2}} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^5$$

$$\left(-\frac{1}{\sqrt{1+e^{2 i(c+d x)}}} i \sqrt{2} e^{\frac{1}{2} i(c+d x)} \sqrt{e^{-i(c+d x)}(1+e^{2 i(c+d x)})} \left(-80 i C d x-80 C \operatorname{ArcSinh}\left[e^{i(c+d x)}\right]+\sqrt{2}(3 A+115 C) \operatorname{Log}\left[1+e^{i(c+d x)}\right]+\right.\right.$$

$$\left.80 C \operatorname{Log}\left[1+\sqrt{1+e^{2 i(c+d x)}}\right]-3 \sqrt{2} A \operatorname{Log}\left[1-e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]-115 \sqrt{2} C \operatorname{Log}\left[1-e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]\right)+$$

$$\left.\sqrt{\cos [c+d x]}(3 A+43 C+(7 A+55 C) \cos [c+d x]+8 C \cos [2(c+d x)]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)$$

■ **Problem 212: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{\cos [c+d x]}(A+C \cos [c+d x]^2)}{(a+a \cos [c+d x])^{5/2}} d x$$

Optimal (type 3, 192 leaves, 7 steps) :

$$\frac{2 C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{a^{5/2} d} + \frac{(5 A-43 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right]}{16 \sqrt{2} a^{5/2} d} -$$

$$\frac{(A+C) \cos [c+d x]^{3/2} \sin [c+d x]}{4 d(a+a \cos [c+d x])^{5/2}} + \frac{(5 A-11 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{16 a d(a+a \cos [c+d x])^{3/2}}$$

Result (type 3, 327 leaves) :

$$\frac{1}{8 d (a (1 + \operatorname{Cos}[c + d x]))^{5/2}} \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]^5$$

$$\left( \frac{1}{\sqrt{1 + e^{2i(c+dx)}}} \sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( 32 C d x - 32 i C \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] - i \sqrt{2} (5 A - 43 C) \operatorname{Log}\left[1 + e^{i(c+dx)}\right] + \right. \right.$$

$$\left. 32 i C \operatorname{Log}\left[1 + \sqrt{1 + e^{2i(c+dx)}}\right] + 5 i \sqrt{2} A \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] - 43 i \sqrt{2} C \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] \right) +$$

$$\left. \sqrt{\operatorname{Cos}[c + d x]} (5 A - 11 C + (A - 15 C) \operatorname{Cos}[c + d x]) \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^3 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \right)$$

■ **Problem 213: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + C \operatorname{Cos}[c + d x]^2}{\sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Cos}[c + d x])^{5/2}} dx$$

Optimal (type 3, 154 leaves, 5 steps):

$$\frac{(19 A + 3 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A + C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{4 d (a + a \operatorname{Cos}[c + d x])^{5/2}} - \frac{(9 A - 7 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{16 a d (a + a \operatorname{Cos}[c + d x])^{3/2}}$$

Result (type 3, 217 leaves):

$$\left( \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]^5 \right.$$

$$\left. \left( -1 / \left( \sqrt{1 + e^{2i(c+dx)}} \right) i (19 A + 3 C) e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( \operatorname{Log}\left[1 + e^{i(c+dx)}\right] - \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] \right) \right) - \right.$$

$$\left. \frac{1}{2} \sqrt{\operatorname{Cos}[c + d x]} (13 A - 3 C + (9 A - 7 C) \operatorname{Cos}[c + d x]) \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^3 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \right) / (4 d (a (1 + \operatorname{Cos}[c + d x]))^{5/2})$$

■ **Problem 214: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + C \operatorname{Cos}[c + d x]^2}{\operatorname{Cos}[c + d x]^{3/2} (a + a \operatorname{Cos}[c + d x])^{5/2}} dx$$

Optimal (type 3, 199 leaves, 6 steps):

$$-\frac{5 (15 A - C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A + C) \operatorname{Sin}[c + d x]}{4 d \sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Cos}[c + d x])^{5/2}} -$$

$$\frac{(13 A - 3 C) \operatorname{Sin}[c + d x]}{16 a d \sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Cos}[c + d x])^{3/2}} + \frac{(49 A + C) \operatorname{Sin}[c + d x]}{16 a^2 d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + a \operatorname{Cos}[c + d x]}}$$

Result (type 3, 228 leaves):

$$\left( \cos \left[ \frac{1}{2} (c + dx) \right] \right)^5$$

$$\left( \frac{1}{\sqrt{1 + e^{2i(c+dx)}}} \right)^5 i (15A - C) e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( \log[1 + e^{i(c+dx)}] - \log[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) +$$

$$\frac{1}{\left( 4 \sqrt{\cos[c + dx]} \right)} \left( 113A + C + 10(17A + C) \cos[c + dx] + (49A + C) \cos[2(c + dx)] \right)$$

$$\sec \left[ \frac{1}{2} (c + dx) \right]^3 \tan \left[ \frac{1}{2} (c + dx) \right] \Bigg) / \left( 4d (a(1 + \cos[c + dx]))^{5/2} \right)$$

■ **Problem 215: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + C \cos[c + dx]^2}{\cos[c + dx]^{5/2} (a + a \cos[c + dx])^{5/2}} dx$$

Optimal (type 3, 246 leaves, 7 steps):

$$\frac{(163A + 19C) \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin[c + dx]}{\sqrt{2} \sqrt{\cos[c + dx]} \sqrt{a + a \cos[c + dx]}} \right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A + C) \sin[c + dx]}{4d \cos[c + dx]^{3/2} (a + a \cos[c + dx])^{5/2}} -$$

$$\frac{(17A + C) \sin[c + dx]}{16ad \cos[c + dx]^{3/2} (a + a \cos[c + dx])^{3/2}} + \frac{5(19A + 3C) \sin[c + dx]}{48a^2 d \cos[c + dx]^{3/2} \sqrt{a + a \cos[c + dx]}} - \frac{(299A + 27C) \sin[c + dx]}{48a^2 d \sqrt{\cos[c + dx]} \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 256 leaves):

$$- \left( \cos \left[ \frac{1}{2} (c + dx) \right] \right)^5$$

$$\left( \frac{1}{\sqrt{1 + e^{2i(c+dx)}}} \right)^3 i (163A + 19C) e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( \log[1 + e^{i(c+dx)}] - \log[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) +$$

$$\frac{1}{\left( 8 \cos[c + dx]^{3/2} \right)} \left( 878A + 78C + (1537A + 81C) \cos[c + dx] + 2(503A + 39C) \cos[2(c + dx)] + \right.$$

$$\left. 299A \cos[3(c + dx)] + 27C \cos[3(c + dx)] \right) \sec \left[ \frac{1}{2} (c + dx) \right]^3 \tan \left[ \frac{1}{2} (c + dx) \right] \Bigg) / \left( 12d (a(1 + \cos[c + dx]))^{5/2} \right)$$

■ **Problem 221: Result more than twice size of optimal antiderivative.**

$$\int (B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^2 dx$$

Optimal (type 3, 16 leaves, 3 steps):

$$Cx + \frac{B \operatorname{ArcTanh}[\sin[c + dx]]}{d}$$

Result (type 3, 73 leaves):

$$Cx - \frac{B \log \left[ \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{B \log \left[ \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right]}{d}$$

■ **Problem 222: Result more than twice size of optimal antiderivative.**

$$\int (B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^3 dx$$

Optimal (type 3, 24 leaves, 5 steps):

$$\frac{C \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{B \tan[c + dx]}{d}$$

Result (type 3, 81 leaves):

$$-\frac{C \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{C \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{B \tan[c + dx]}{d}$$

■ **Problem 225: Result more than twice size of optimal antiderivative.**

$$\int (B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^6 dx$$

Optimal (type 3, 85 leaves, 7 steps):

$$\frac{3 B \operatorname{ArcTanh}[\sin[c + dx]]}{8 d} + \frac{C \tan[c + dx]}{d} + \frac{3 B \sec[c + dx] \tan[c + dx]}{8 d} + \frac{B \sec[c + dx]^3 \tan[c + dx]}{4 d} + \frac{C \tan[c + dx]^3}{3 d}$$

Result (type 3, 227 leaves):

$$-\frac{3 B \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8 d} + \frac{3 B \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8 d} + \frac{B}{3 B} - \frac{16 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4}{3 B} + \frac{16 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{3 d} - \frac{16 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4}{3 d} + \frac{2 C \tan[c + dx]}{3 d} + \frac{C \sec[c + dx]^2 \tan[c + dx]}{3 d}$$

■ **Problem 230: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx]) (B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^2 dx$$

Optimal (type 3, 32 leaves, 5 steps):

$$a(B + C)x + \frac{a B \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{a C \sin[c + dx]}{d}$$

Result (type 3, 104 leaves):

$$a B x + a C x - \frac{a B \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a B \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a C \cos[dx] \sin[c]}{d} + \frac{a C \cos[c] \sin[dx]}{d}$$

■ **Problem 231: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x]) (B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^3 dx$$

Optimal (type 3, 32 leaves, 5 steps):

$$a C x + \frac{a (B + C) \operatorname{ArcTanh}[\sin [c + d x]]}{d} + \frac{a B \tan [c + d x]}{d}$$

Result (type 3, 159 leaves):

$$a C x - \frac{a B \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} - \frac{a C \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} +$$

$$\frac{a B \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a C \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a B \tan [c + d x]}{d}$$

■ **Problem 232: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x]) (B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^4 dx$$

Optimal (type 3, 56 leaves, 7 steps):

$$\frac{a (B + 2 C) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{a (B + C) \tan [c + d x]}{d} + \frac{a B \sec [c + d x] \tan [c + d x]}{2 d}$$

Result (type 3, 154 leaves):

$$\frac{1}{4 d} a \left( -2 (B + 2 C) \operatorname{Log}\left[\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right] + \right.$$

$$2 B \operatorname{Log}\left[\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right] + 4 C \operatorname{Log}\left[\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right] +$$

$$\left. \frac{B}{\left(\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right)^2} - \frac{B}{\left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right)^2} + 4 (B + C) \tan [c + d x] \right)$$

■ **Problem 234: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x]) (B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^6 dx$$

Optimal (type 3, 106 leaves, 8 steps):

$$\frac{a (3 B + 4 C) \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} + \frac{a (B + C) \tan [c + d x]}{d} +$$

$$\frac{a (3 B + 4 C) \sec [c + d x] \tan [c + d x]}{8 d} + \frac{a B \sec [c + d x]^3 \tan [c + d x]}{4 d} + \frac{a (B + C) \tan [c + d x]^3}{3 d}$$

Result (type 3, 403 leaves):

$$\begin{aligned}
& - \frac{3 a B \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right]}{8 d}-\frac{a C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right]}{2 d}+\frac{3 a B \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right]}{8 d}+ \\
& \frac{a C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right]}{2 d}+\frac{a B}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^4}+\frac{3 a B}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}+ \\
& \frac{a C}{4 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}-\frac{a B}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^4}-\frac{3 a B}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}- \\
& \frac{a C}{4 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}+\frac{2 a B \tan [c+d x]}{3 d}+\frac{2 a C \tan [c+d x]}{3 d}+\frac{a B \sec [c+d x]^2 \tan [c+d x]}{3 d}+\frac{a C \sec [c+d x]^2 \tan [c+d x]}{3 d}
\end{aligned}$$

■ **Problem 240: Result more than twice size of optimal antiderivative.**

$$\int (a+a \cos [c+d x])^2 (B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]^4 dx$$

Optimal (type 3, 88 leaves, 6 steps):

$$a^2 C x+\frac{a^2(3 B+4 C) \operatorname{ArcTanh}[\sin [c+d x]]}{2 d}+\frac{a^2(3 B+2 C) \tan [c+d x]}{2 d}+\frac{B\left(a^2+a^2 \cos [c+d x]\right) \sec [c+d x] \tan [c+d x]}{2 d}$$

Result (type 3, 277 leaves):

$$\begin{aligned}
& \frac{1}{16} a^2(1+\cos [c+d x])^2 \sec \left[\frac{1}{2}(c+d x)\right]^4 \\
& \left(4 C x-\frac{2(3 B+4 C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right]}{d}+\frac{2(3 B+4 C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right]}{d}+\right. \\
& \left.\frac{B}{d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}+\frac{4(2 B+C) \sin\left[\frac{d x}{2}\right]}{d\left(\cos\left[\frac{c}{2}\right]-\sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)}-\right. \\
& \left.\frac{B}{d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}+\frac{4(2 B+C) \sin\left[\frac{d x}{2}\right]}{d\left(\cos\left[\frac{c}{2}\right]+\sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)}\right)
\end{aligned}$$

■ **Problem 241: Result more than twice size of optimal antiderivative.**

$$\int (a+a \cos [c+d x])^2 (B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]^5 dx$$

Optimal (type 3, 113 leaves, 8 steps):



$$\frac{a^2 (2B + 3C) \operatorname{ArcTanh}[\sin[c + dx]]}{2d} + \frac{a^2 (5B + 6C) \tan[c + dx]}{3d} +$$

$$\frac{a^2 (4B + 3C) \sec[c + dx] \tan[c + dx]}{6d} + \frac{B (a^2 + a^2 \cos[c + dx]) \sec[c + dx]^2 \tan[c + dx]}{3d}$$

Result (type 3, 753 leaves):

$$\frac{(-2B - 3C) (a + a \cos[c + dx])^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4}{8d} +$$

$$\frac{(2B + 3C) (a + a \cos[c + dx])^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4}{8d} + \frac{B (a + a \cos[c + dx])^2 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{24d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} +$$

$$\frac{(a + a \cos[c + dx])^2 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(7B \cos\left[\frac{c}{2}\right] + 3C \cos\left[\frac{c}{2}\right] - 5B \sin\left[\frac{c}{2}\right] - 3C \sin\left[\frac{c}{2}\right]\right)}{48d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} +$$

$$\frac{(a + a \cos[c + dx])^2 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(5B \sin\left[\frac{dx}{2}\right] + 6C \sin\left[\frac{dx}{2}\right]\right)}{12d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \frac{B (a + a \cos[c + dx])^2 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{dx}{2}\right]}{24d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} +$$

$$\frac{(a + a \cos[c + dx])^2 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(-7B \cos\left[\frac{c}{2}\right] - 3C \cos\left[\frac{c}{2}\right] - 5B \sin\left[\frac{c}{2}\right] - 3C \sin\left[\frac{c}{2}\right]\right)}{48d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} +$$

$$\frac{(a + a \cos[c + dx])^2 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(5B \sin\left[\frac{dx}{2}\right] + 6C \sin\left[\frac{dx}{2}\right]\right)}{12d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}$$

■ **Problem 248: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^3 (B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^3 dx$$

Optimal (type 3, 110 leaves, 7 steps):

$$\frac{1}{2} a^3 (6B + 7C) x + \frac{a^3 (3B + C) \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{5a^3 C \sin[c + dx]}{2d} -$$

$$\frac{(2B - C) (a^3 + a^3 \cos[c + dx]) \sin[c + dx]}{2d} + \frac{aB (a + a \cos[c + dx])^2 \tan[c + dx]}{d}$$

Result (type 3, 272 leaves):

$$\frac{1}{32} a^3 (1 + \cos [c + d x])^3 \sec \left[ \frac{1}{2} (c + d x) \right]^6$$

$$\left( 2 (6 B + 7 C) x - \frac{4 (3 B + C) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{d} + \frac{4 (3 B + C) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{d} + \right.$$

$$\frac{4 (B + 3 C) \cos [d x] \sin [c]}{d} + \frac{C \cos [2 d x] \sin [2 c]}{d} + \frac{4 (B + 3 C) \cos [c] \sin [d x]}{d} + \frac{C \cos [2 c] \sin [2 d x]}{d} +$$

$$\left. \frac{4 B \sin \left[ \frac{d x}{2} \right]}{d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)} + \frac{4 B \sin \left[ \frac{d x}{2} \right]}{d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)} \right)$$

■ **Problem 250: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^3 (B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^5 dx$$

Optimal (type 3, 125 leaves, 7 steps):

$$a^3 C x + \frac{a^3 (5 B + 7 C) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{5 a^3 (B + C) \tan [c + d x]}{2 d} +$$

$$\frac{(5 B + 3 C) (a^3 + a^3 \cos [c + d x]) \sec [c + d x] \tan [c + d x]}{6 d} + \frac{a B (a + a \cos [c + d x])^2 \sec [c + d x]^2 \tan [c + d x]}{3 d}$$

Result (type 3, 786 leaves):

$$\frac{1}{8} C x (a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 + \frac{(-5 B - 7 C) (a + a \cos [c + d x])^3 \operatorname{Log} \left[ \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6}{16 d} +$$

$$\frac{(5 B + 7 C) (a + a \cos [c + d x])^3 \operatorname{Log} \left[ \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6}{16 d} + \frac{B (a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sin \left[ \frac{d x}{2} \right]}{48 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^3} +$$

$$\frac{(a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \left( 10 B \cos \left[ \frac{c}{2} \right] + 3 C \cos \left[ \frac{c}{2} \right] - 8 B \sin \left[ \frac{c}{2} \right] - 3 C \sin \left[ \frac{c}{2} \right] \right)}{96 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^2} +$$

$$\frac{(a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \left( 11 B \sin \left[ \frac{d x}{2} \right] + 9 C \sin \left[ \frac{d x}{2} \right] \right)}{24 d \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)} + \frac{B (a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sin \left[ \frac{d x}{2} \right]}{48 d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^3} +$$

$$\frac{(a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \left( -10 B \cos \left[ \frac{c}{2} \right] - 3 C \cos \left[ \frac{c}{2} \right] - 8 B \sin \left[ \frac{c}{2} \right] - 3 C \sin \left[ \frac{c}{2} \right] \right)}{96 d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)^2} +$$

$$\frac{(a + a \cos [c + d x])^3 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \left( 11 B \sin \left[ \frac{d x}{2} \right] + 9 C \sin \left[ \frac{d x}{2} \right] \right)}{24 d \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right)}$$

■ **Problem 253: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^2 (B \cos[c + dx] + C \cos[c + dx]^2)}{a + a \cos[c + dx]} dx$$

Optimal (type 3, 122 leaves, 7 steps):

$$\frac{3(B-C)x}{2a} - \frac{(3B-4C)\sin[c+dx]}{ad} + \frac{3(B-C)\cos[c+dx]\sin[c+dx]}{2ad} + \frac{(B-C)\cos[c+dx]^3\sin[c+dx]}{d(a+a\cos[c+dx])} + \frac{(3B-4C)\sin[c+dx]^3}{3ad}$$

Result (type 3, 249 leaves):

$$\frac{1}{24ad(1+\cos[c+dx])} \cos\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( 36(B-C)dx \cos\left[\frac{dx}{2}\right] + 36(B-C)dx \cos\left[c + \frac{dx}{2}\right] - 60B \sin\left[\frac{dx}{2}\right] + 69C \sin\left[\frac{dx}{2}\right] - 12B \sin\left[c + \frac{dx}{2}\right] + 21C \sin\left[c + \frac{dx}{2}\right] - 9B \sin\left[c + \frac{3dx}{2}\right] + 18C \sin\left[c + \frac{3dx}{2}\right] - 9B \sin\left[2c + \frac{3dx}{2}\right] + 18C \sin\left[2c + \frac{3dx}{2}\right] + 3B \sin\left[2c + \frac{5dx}{2}\right] - 2C \sin\left[2c + \frac{5dx}{2}\right] + 3B \sin\left[3c + \frac{5dx}{2}\right] - 2C \sin\left[3c + \frac{5dx}{2}\right] + C \sin\left[3c + \frac{7dx}{2}\right] + C \sin\left[4c + \frac{7dx}{2}\right] \right)$$

■ **Problem 255: Result more than twice size of optimal antiderivative.**

$$\int \frac{B \cos[c + dx] + C \cos[c + dx]^2}{a + a \cos[c + dx]} dx$$

Optimal (type 3, 54 leaves, 4 steps):

$$\frac{(B-C)x}{a} + \frac{C \sin[c+dx]}{ad} - \frac{(B-C)\sin[c+dx]}{ad(1+\cos[c+dx])}$$

Result (type 3, 126 leaves):

$$\frac{1}{2ad(1+\cos[c+dx])} \cos\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( 2(B-C)dx \cos\left[\frac{dx}{2}\right] + 2(B-C)dx \cos\left[c + \frac{dx}{2}\right] - 4B \sin\left[\frac{dx}{2}\right] + 5C \sin\left[\frac{dx}{2}\right] + C \sin\left[c + \frac{dx}{2}\right] + C \sin\left[c + \frac{3dx}{2}\right] + C \sin\left[2c + \frac{3dx}{2}\right] \right)$$

■ **Problem 256: Result more than twice size of optimal antiderivative.**

$$\int \frac{(B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]}{a + a \cos[c + dx]} dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{Cx}{a} + \frac{(B-C)\sin[c+dx]}{d(a+a\cos[c+dx])}$$

Result (type 3, 72 leaves):

$$\frac{\cos\left[\frac{1}{2}(c+dx)\right] \sec\left[\frac{c}{2}\right] \left(C dx \cos\left[\frac{dx}{2}\right] + C dx \cos\left[c + \frac{dx}{2}\right] + 2(B-C) \sin\left[\frac{dx}{2}\right]\right)}{a d (1 + \cos[c+dx])}$$

■ **Problem 257: Result more than twice size of optimal antiderivative.**

$$\int \frac{(B \cos[c+dx] + C \cos[c+dx]^2) \sec[c+dx]^2}{a + a \cos[c+dx]} dx$$

Optimal (type 3, 44 leaves, 4 steps) :

$$\frac{B \operatorname{ArcTanh}[\sin[c+dx]]}{a d} - \frac{(B-C) \sin[c+dx]}{d (a + a \cos[c+dx])}$$

Result (type 3, 109 leaves) :

$$\frac{1}{a d (1 + \cos[c+dx])} 2 \cos\left[\frac{1}{2}(c+dx)\right] \left( B \cos\left[\frac{1}{2}(c+dx)\right] \left( -\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) + (-B+C) \sec\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] \right)$$

■ **Problem 258: Result more than twice size of optimal antiderivative.**

$$\int \frac{(B \cos[c+dx] + C \cos[c+dx]^2) \sec[c+dx]^3}{a + a \cos[c+dx]} dx$$

Optimal (type 3, 69 leaves, 6 steps) :

$$-\frac{(B-C) \operatorname{ArcTanh}[\sin[c+dx]]}{a d} + \frac{(2B-C) \tan[c+dx]}{a d} - \frac{(B-C) \tan[c+dx]}{d (a + a \cos[c+dx])}$$

Result (type 3, 201 leaves) :

$$\frac{1}{a d (1 + \cos[c+dx])} 2 \cos\left[\frac{1}{2}(c+dx)\right] \left( (B-C) \sec\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] + \cos\left[\frac{1}{2}(c+dx)\right] \left( (B-C) \left( \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) + (B \sin[dx]) / \left( \left( \cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left( \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right) \left( \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) \right)$$

■ **Problem 259: Result more than twice size of optimal antiderivative.**

$$\int \frac{(B \cos[c+dx] + C \cos[c+dx]^2) \sec[c+dx]^4}{a + a \cos[c+dx]} dx$$

Optimal (type 3, 107 leaves, 7 steps) :

$$\frac{(3B-2C) \operatorname{ArcTanh}[\sin[c+dx]]}{2 a d} - \frac{2(B-C) \tan[c+dx]}{a d} + \frac{(3B-2C) \sec[c+dx] \tan[c+dx]}{2 a d} - \frac{(B-C) \sec[c+dx] \tan[c+dx]}{d (a + a \cos[c+dx])}$$

Result (type 3, 289 leaves) :

$$\frac{1}{2 a d (1 + \cos [c + d x])} \cos \left[ \frac{1}{2} (c + d x) \right] \left( 4 (-B + C) \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + \right.$$

$$\cos \left[ \frac{1}{2} (c + d x) \right] \left( (-6 B + 4 C) \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] + 6 B \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) - \right.$$

$$4 C \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \frac{B}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \frac{B}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} - (4 (B - C) \sin [d x]) / \left( \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) \right)$$

■ **Problem 260: Result more than twice size of optimal antiderivative.**

$$\int \frac{(B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^5}{a + a \cos [c + d x]} dx$$

Optimal (type 3, 131 leaves, 7 steps) :

$$-\frac{3 (B - C) \operatorname{ArcTanh}[\sin [c + d x]]}{2 a d} + \frac{(4 B - 3 C) \tan [c + d x]}{a d} -$$

$$\frac{3 (B - C) \sec [c + d x] \tan [c + d x]}{2 a d} - \frac{(B - C) \sec [c + d x]^2 \tan [c + d x]}{d (a + a \cos [c + d x])} + \frac{(4 B - 3 C) \tan [c + d x]^3}{3 a d}$$

Result (type 3, 567 leaves) :

$$\frac{1}{a} \left( \frac{3 (B - C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (1 + \cos[c + dx])} - \right.$$

$$\frac{3 (B - C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (1 + \cos[c + dx])} + \frac{1}{48 d (1 + \cos[c + dx])} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c] \sec[c + dx]^3$$

$$\left( 6 B \sin\left[\frac{dx}{2}\right] + 6 C \sin\left[\frac{dx}{2}\right] + 39 B \sin\left[\frac{3 dx}{2}\right] - 27 C \sin\left[\frac{3 dx}{2}\right] - 24 B \sin\left[c - \frac{dx}{2}\right] + 12 C \sin\left[c - \frac{dx}{2}\right] - 6 B \sin\left[c + \frac{dx}{2}\right] + \right.$$

$$6 C \sin\left[c + \frac{dx}{2}\right] - 24 B \sin\left[2c + \frac{dx}{2}\right] + 24 C \sin\left[2c + \frac{dx}{2}\right] + 21 B \sin\left[c + \frac{3 dx}{2}\right] - 9 C \sin\left[c + \frac{3 dx}{2}\right] + 9 B \sin\left[2c + \frac{3 dx}{2}\right] -$$

$$9 C \sin\left[2c + \frac{3 dx}{2}\right] - 9 B \sin\left[3c + \frac{3 dx}{2}\right] + 9 C \sin\left[3c + \frac{3 dx}{2}\right] + 7 B \sin\left[c + \frac{5 dx}{2}\right] - 3 C \sin\left[c + \frac{5 dx}{2}\right] + B \sin\left[2c + \frac{5 dx}{2}\right] +$$

$$3 C \sin\left[2c + \frac{5 dx}{2}\right] - 3 B \sin\left[3c + \frac{5 dx}{2}\right] + 3 C \sin\left[3c + \frac{5 dx}{2}\right] - 9 B \sin\left[4c + \frac{5 dx}{2}\right] + 9 C \sin\left[4c + \frac{5 dx}{2}\right] + 16 B \sin\left[2c + \frac{7 dx}{2}\right] -$$

$$\left. \left. 12 C \sin\left[2c + \frac{7 dx}{2}\right] + 10 B \sin\left[3c + \frac{7 dx}{2}\right] - 6 C \sin\left[3c + \frac{7 dx}{2}\right] + 6 B \sin\left[4c + \frac{7 dx}{2}\right] - 6 C \sin\left[4c + \frac{7 dx}{2}\right] \right) \right)$$

■ **Problem 261: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^3 (B \cos[c + dx] + C \cos[c + dx]^2)}{(a + a \cos[c + dx])^2} dx$$

Optimal (type 3, 170 leaves, 8 steps):

$$\frac{(7B - 10C)x}{2a^2} - \frac{4(2B - 3C)\sin[c + dx]}{a^2 d} + \frac{(7B - 10C)\cos[c + dx]\sin[c + dx]}{2a^2 d} +$$

$$\frac{(7B - 10C)\cos[c + dx]^3 \sin[c + dx]}{3a^2 d (1 + \cos[c + dx])} + \frac{(B - C)\cos[c + dx]^4 \sin[c + dx]}{3d (a + a \cos[c + dx])^2} + \frac{4(2B - 3C)\sin[c + dx]^3}{3a^2 d}$$

Result (type 3, 369 leaves):

$$\frac{1}{48 a^2 d (1 + \cos [c + d x])^2} \cos \left[ \frac{1}{2} (c + d x) \right] \sec \left[ \frac{c}{2} \right] \left( 36 (7 B - 10 C) d x \cos \left[ \frac{d x}{2} \right] + 36 (7 B - 10 C) d x \cos \left[ c + \frac{d x}{2} \right] + 84 B d x \cos \left[ c + \frac{3 d x}{2} \right] - 120 C d x \cos \left[ c + \frac{3 d x}{2} \right] + 84 B d x \cos \left[ 2 c + \frac{3 d x}{2} \right] - 120 C d x \cos \left[ 2 c + \frac{3 d x}{2} \right] - 381 B \sin \left[ \frac{d x}{2} \right] + 516 C \sin \left[ \frac{d x}{2} \right] + 147 B \sin \left[ c + \frac{d x}{2} \right] - 156 C \sin \left[ c + \frac{d x}{2} \right] - 239 B \sin \left[ c + \frac{3 d x}{2} \right] + 342 C \sin \left[ c + \frac{3 d x}{2} \right] - 63 B \sin \left[ 2 c + \frac{3 d x}{2} \right] + 118 C \sin \left[ 2 c + \frac{3 d x}{2} \right] - 15 B \sin \left[ 2 c + \frac{5 d x}{2} \right] + 30 C \sin \left[ 2 c + \frac{5 d x}{2} \right] - 15 B \sin \left[ 3 c + \frac{5 d x}{2} \right] + 30 C \sin \left[ 3 c + \frac{5 d x}{2} \right] + 3 B \sin \left[ 3 c + \frac{7 d x}{2} \right] - 3 C \sin \left[ 3 c + \frac{7 d x}{2} \right] + 3 B \sin \left[ 4 c + \frac{7 d x}{2} \right] - 3 C \sin \left[ 4 c + \frac{7 d x}{2} \right] + C \sin \left[ 4 c + \frac{9 d x}{2} \right] + C \sin \left[ 5 c + \frac{9 d x}{2} \right] \right)$$

■ **Problem 262: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^2 (B \cos [c + d x] + C \cos [c + d x])^2}{(a + a \cos [c + d x])^2} dx$$

Optimal (type 3, 147 leaves, 4 steps):

$$-\frac{(4 B - 7 C) x}{2 a^2} + \frac{2 (5 B - 8 C) \sin [c + d x]}{3 a^2 d} - \frac{(4 B - 7 C) \cos [c + d x] \sin [c + d x]}{2 a^2 d} + \frac{(5 B - 8 C) \cos [c + d x]^2 \sin [c + d x]}{3 a^2 d (1 + \cos [c + d x])} + \frac{(B - C) \cos [c + d x]^3 \sin [c + d x]}{3 d (a + a \cos [c + d x])^2}$$

Result (type 3, 315 leaves):

$$\frac{1}{48 a^2 d (1 + \cos [c + d x])^2} \cos \left[ \frac{1}{2} (c + d x) \right] \sec \left[ \frac{c}{2} \right] \left( -36 (4 B - 7 C) d x \cos \left[ \frac{d x}{2} \right] - 36 (4 B - 7 C) d x \cos \left[ c + \frac{d x}{2} \right] - 48 B d x \cos \left[ c + \frac{3 d x}{2} \right] + 84 C d x \cos \left[ c + \frac{3 d x}{2} \right] - 48 B d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 84 C d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 264 B \sin \left[ \frac{d x}{2} \right] - 381 C \sin \left[ \frac{d x}{2} \right] - 120 B \sin \left[ c + \frac{d x}{2} \right] + 147 C \sin \left[ c + \frac{d x}{2} \right] + 164 B \sin \left[ c + \frac{3 d x}{2} \right] - 239 C \sin \left[ c + \frac{3 d x}{2} \right] + 36 B \sin \left[ 2 c + \frac{3 d x}{2} \right] - 63 C \sin \left[ 2 c + \frac{3 d x}{2} \right] + 12 B \sin \left[ 2 c + \frac{5 d x}{2} \right] - 15 C \sin \left[ 2 c + \frac{5 d x}{2} \right] + 12 B \sin \left[ 3 c + \frac{5 d x}{2} \right] - 15 C \sin \left[ 3 c + \frac{5 d x}{2} \right] + 3 C \sin \left[ 3 c + \frac{7 d x}{2} \right] + 3 C \sin \left[ 4 c + \frac{7 d x}{2} \right] \right)$$

■ **Problem 264: Result more than twice size of optimal antiderivative.**

$$\int \frac{B \cos [c + d x] + C \cos [c + d x]^2}{(a + a \cos [c + d x])^2} dx$$

Optimal (type 3, 70 leaves, 3 steps):

$$\frac{C x}{a^2} + \frac{(2B - 5C) \sin[c + dx]}{3 a^2 d (1 + \cos[c + dx])} - \frac{(B - C) \sin[c + dx]}{3 d (a + a \cos[c + dx])^2}$$

Result (type 3, 153 leaves):

$$\frac{1}{24 a^2 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 \left(9 C dx \cos\left[\frac{dx}{2}\right] + 9 C dx \cos\left[c + \frac{dx}{2}\right] + 3 C dx \cos\left[c + \frac{3dx}{2}\right] + 3 C dx \cos\left[2c + \frac{3dx}{2}\right] + 6 B \sin\left[\frac{dx}{2}\right] - 18 C \sin\left[\frac{dx}{2}\right] - 6 B \sin\left[c + \frac{dx}{2}\right] + 12 C \sin\left[c + \frac{dx}{2}\right] + 4 B \sin\left[c + \frac{3dx}{2}\right] - 10 C \sin\left[c + \frac{3dx}{2}\right]\right)$$

■ **Problem 266: Result more than twice size of optimal antiderivative.**

$$\int \frac{(B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^2}{(a + a \cos[c + dx])^2} dx$$

Optimal (type 3, 79 leaves, 5 steps):

$$\frac{B \operatorname{ArcTanh}[\sin[c + dx]]}{a^2 d} - \frac{(4B - C) \sin[c + dx]}{3 a^2 d (1 + \cos[c + dx])} - \frac{(B - C) \sin[c + dx]}{3 d (a + a \cos[c + dx])^2}$$

Result (type 3, 170 leaves):

$$-\frac{1}{3 a^2 d (1 + \cos[c + dx])^2} + 2 \cos\left[\frac{1}{2}(c + dx)\right] \left(6 B \cos\left[\frac{1}{2}(c + dx)\right]^3 \left(\log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]\right) + (B - C) \operatorname{Sec}\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] + 2(4B - C) \cos\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] + (B - C) \cos\left[\frac{1}{2}(c + dx)\right] \tan\left[\frac{c}{2}\right]\right)$$

■ **Problem 267: Result more than twice size of optimal antiderivative.**

$$\int \frac{(B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^3}{(a + a \cos[c + dx])^2} dx$$

Optimal (type 3, 107 leaves, 7 steps):

$$-\frac{(2B - C) \operatorname{ArcTanh}[\sin[c + dx]]}{a^2 d} + \frac{2(5B - 2C) \tan[c + dx]}{3 a^2 d} - \frac{(2B - C) \tan[c + dx]}{a^2 d (1 + \cos[c + dx])} - \frac{(B - C) \tan[c + dx]}{3 d (a + a \cos[c + dx])^2}$$

Result (type 3, 264 leaves):



$$\frac{1}{3 a^2 d (1 + \cos [c + d x])^2} 2 \cos \left[ \frac{1}{2} (c + d x) \right] \left( (B - C) \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + 2 (7 B - 4 C) \cos \left[ \frac{1}{2} (c + d x) \right]^2 \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + 6 \cos \left[ \frac{1}{2} (c + d x) \right]^3 \right. \\ \left. \left( (2 B - C) \left( \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] - \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] \right) + (B \sin [d x]) / \left( \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \right. \right. \right. \\ \left. \left. \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) \right) + (B - C) \cos \left[ \frac{1}{2} (c + d x) \right] \tan \left[ \frac{c}{2} \right] \right)$$

■ **Problem 268: Result more than twice size of optimal antiderivative.**

$$\int \frac{(B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^4}{(a + a \cos [c + d x])^2} dx$$

Optimal (type 3, 152 leaves, 8 steps):

$$\frac{(7 B - 4 C) \operatorname{ArcTanh}[\sin [c + d x]]}{2 a^2 d} - \frac{2 (8 B - 5 C) \tan [c + d x]}{3 a^2 d} + \\ \frac{(7 B - 4 C) \sec [c + d x] \tan [c + d x]}{2 a^2 d} - \frac{(8 B - 5 C) \sec [c + d x] \tan [c + d x]}{3 a^2 d (1 + \cos [c + d x])} - \frac{(B - C) \sec [c + d x] \tan [c + d x]}{3 d (a + a \cos [c + d x])^2}$$

Result (type 3, 572 leaves):

$$\frac{1}{a^2} \left( - \frac{2 (7 B - 4 C) \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \log \left[ \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] - \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right]}{d (1 + \cos [c + d x])^2} + \right. \\ \left. \frac{2 (7 B - 4 C) \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \log \left[ \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] + \sin \left[ \frac{c}{2} + \frac{d x}{2} \right] \right]}{d (1 + \cos [c + d x])^2} + \frac{1}{48 d (1 + \cos [c + d x])^2} \right. \\ \left. \cos \left[ \frac{c}{2} + \frac{d x}{2} \right] \sec \left[ \frac{c}{2} \right] \sec [c] \sec [c + d x]^2 \left( 14 B \sin \left[ \frac{d x}{2} \right] - 14 C \sin \left[ \frac{d x}{2} \right] - 97 B \sin \left[ \frac{3 d x}{2} \right] + 64 C \sin \left[ \frac{3 d x}{2} \right] + 126 B \sin \left[ c - \frac{d x}{2} \right] - \right. \right. \\ \left. 84 C \sin \left[ c - \frac{d x}{2} \right] - 42 B \sin \left[ c + \frac{d x}{2} \right] + 42 C \sin \left[ c + \frac{d x}{2} \right] + 98 B \sin \left[ 2 c + \frac{d x}{2} \right] - 56 C \sin \left[ 2 c + \frac{d x}{2} \right] + 3 B \sin \left[ c + \frac{3 d x}{2} \right] - 6 C \sin \left[ c + \frac{3 d x}{2} \right] - \right. \\ \left. 37 B \sin \left[ 2 c + \frac{3 d x}{2} \right] + 34 C \sin \left[ 2 c + \frac{3 d x}{2} \right] + 63 B \sin \left[ 3 c + \frac{3 d x}{2} \right] - 36 C \sin \left[ 3 c + \frac{3 d x}{2} \right] - 75 B \sin \left[ c + \frac{5 d x}{2} \right] + 48 C \sin \left[ c + \frac{5 d x}{2} \right] - \right. \\ \left. 15 B \sin \left[ 2 c + \frac{5 d x}{2} \right] + 6 C \sin \left[ 2 c + \frac{5 d x}{2} \right] - 39 B \sin \left[ 3 c + \frac{5 d x}{2} \right] + 30 C \sin \left[ 3 c + \frac{5 d x}{2} \right] + 21 B \sin \left[ 4 c + \frac{5 d x}{2} \right] - 12 C \sin \left[ 4 c + \frac{5 d x}{2} \right] - \right. \\ \left. 32 B \sin \left[ 2 c + \frac{7 d x}{2} \right] + 20 C \sin \left[ 2 c + \frac{7 d x}{2} \right] - 12 B \sin \left[ 3 c + \frac{7 d x}{2} \right] + 6 C \sin \left[ 3 c + \frac{7 d x}{2} \right] - 20 B \sin \left[ 4 c + \frac{7 d x}{2} \right] + 14 C \sin \left[ 4 c + \frac{7 d x}{2} \right] \right) \right)$$

■ **Problem 269: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^3 (B \cos [c + d x] + C \cos [c + d x]^2)}{(a + a \cos [c + d x])^3} dx$$

Optimal (type 3, 193 leaves, 5 steps) :

$$-\frac{(6B-13C)x}{2a^3} + \frac{8(9B-19C)\sin[c+dx]}{15a^3d} - \frac{(6B-13C)\cos[c+dx]\sin[c+dx]}{2a^3d} +$$

$$\frac{(B-C)\cos[c+dx]^4\sin[c+dx]}{5d(a+a\cos[c+dx])^3} + \frac{(6B-11C)\cos[c+dx]^3\sin[c+dx]}{15ad(a+a\cos[c+dx])^2} + \frac{4(9B-19C)\cos[c+dx]^2\sin[c+dx]}{15d(a^3+a^3\cos[c+dx])}$$

Result (type 3, 435 leaves) :

$$\frac{1}{480a^3d(1+\cos[c+dx])^3}$$

$$\cos\left[\frac{1}{2}(c+dx)\right]\sec\left[\frac{c}{2}\right]\left(-600(6B-13C)dx\cos\left[\frac{dx}{2}\right]-600(6B-13C)dx\cos\left[c+\frac{dx}{2}\right]-1800Bdx\cos\left[c+\frac{3dx}{2}\right]+3900Cdx\cos\left[c+\frac{3dx}{2}\right]-\right.$$

$$1800Bdx\cos\left[2c+\frac{3dx}{2}\right]+3900Cdx\cos\left[2c+\frac{3dx}{2}\right]-360Bdx\cos\left[2c+\frac{5dx}{2}\right]+780Cdx\cos\left[2c+\frac{5dx}{2}\right]-$$

$$360Bdx\cos\left[3c+\frac{5dx}{2}\right]+780Cdx\cos\left[3c+\frac{5dx}{2}\right]+7020B\sin\left[\frac{dx}{2}\right]-12760C\sin\left[\frac{dx}{2}\right]-4500B\sin\left[c+\frac{dx}{2}\right]+$$

$$7560C\sin\left[c+\frac{dx}{2}\right]+4860B\sin\left[c+\frac{3dx}{2}\right]-9230C\sin\left[c+\frac{3dx}{2}\right]-900B\sin\left[2c+\frac{3dx}{2}\right]+930C\sin\left[2c+\frac{3dx}{2}\right]+$$

$$1452B\sin\left[2c+\frac{5dx}{2}\right]-2782C\sin\left[2c+\frac{5dx}{2}\right]+300B\sin\left[3c+\frac{5dx}{2}\right]-750C\sin\left[3c+\frac{5dx}{2}\right]+60B\sin\left[3c+\frac{7dx}{2}\right]-$$

$$\left.105C\sin\left[3c+\frac{7dx}{2}\right]+60B\sin\left[4c+\frac{7dx}{2}\right]-105C\sin\left[4c+\frac{7dx}{2}\right]+15C\sin\left[4c+\frac{9dx}{2}\right]+15C\sin\left[5c+\frac{9dx}{2}\right]\right)$$

■ **Problem 270: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^2(B\cos[c+dx]+C\cos[c+dx]^2)}{(a+a\cos[c+dx])^3} dx$$

Optimal (type 3, 147 leaves, 8 steps) :

$$\frac{(B-3C)x}{a^3} - \frac{(7B-27C)\sin[c+dx]}{15a^3d} + \frac{(B-C)\cos[c+dx]^3\sin[c+dx]}{5d(a+a\cos[c+dx])^3} + \frac{(4B-9C)\cos[c+dx]^2\sin[c+dx]}{15ad(a+a\cos[c+dx])^2} - \frac{(B-3C)\sin[c+dx]}{d(a^3+a^3\cos[c+dx])}$$

Result (type 3, 361 leaves) :

$$\frac{1}{120 a^3 d (1 + \cos [c + d x])^3}$$

$$\cos \left[ \frac{1}{2} (c + d x) \right] \sec \left[ \frac{c}{2} \right] \left( 300 (B - 3 C) d x \cos \left[ \frac{d x}{2} \right] + 300 (B - 3 C) d x \cos \left[ c + \frac{d x}{2} \right] + 150 B d x \cos \left[ c + \frac{3 d x}{2} \right] - 450 C d x \cos \left[ c + \frac{3 d x}{2} \right] + \right.$$

$$150 B d x \cos \left[ 2 c + \frac{3 d x}{2} \right] - 450 C d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 30 B d x \cos \left[ 2 c + \frac{5 d x}{2} \right] - 90 C d x \cos \left[ 2 c + \frac{5 d x}{2} \right] +$$

$$30 B d x \cos \left[ 3 c + \frac{5 d x}{2} \right] - 90 C d x \cos \left[ 3 c + \frac{5 d x}{2} \right] - 740 B \sin \left[ \frac{d x}{2} \right] + 1755 C \sin \left[ \frac{d x}{2} \right] + 540 B \sin \left[ c + \frac{d x}{2} \right] -$$

$$1125 C \sin \left[ c + \frac{d x}{2} \right] - 460 B \sin \left[ c + \frac{3 d x}{2} \right] + 1215 C \sin \left[ c + \frac{3 d x}{2} \right] + 180 B \sin \left[ 2 c + \frac{3 d x}{2} \right] - 225 C \sin \left[ 2 c + \frac{3 d x}{2} \right] -$$

$$\left. 128 B \sin \left[ 2 c + \frac{5 d x}{2} \right] + 363 C \sin \left[ 2 c + \frac{5 d x}{2} \right] + 75 C \sin \left[ 3 c + \frac{5 d x}{2} \right] + 15 C \sin \left[ 3 c + \frac{7 d x}{2} \right] + 15 C \sin \left[ 4 c + \frac{7 d x}{2} \right] \right)$$

■ **Problem 271: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x] (B \cos [c + d x] + C \cos [c + d x]^2)}{(a + a \cos [c + d x])^3} dx$$

Optimal (type 3, 116 leaves, 6 steps):

$$\frac{C x}{a^3} + \frac{(B - C) \cos [c + d x]^2 \sin [c + d x]}{5 d (a + a \cos [c + d x])^3} - \frac{(2 B - 7 C) \sin [c + d x]}{15 a d (a + a \cos [c + d x])^2} + \frac{(4 B - 29 C) \sin [c + d x]}{15 d (a^3 + a^3 \cos [c + d x])}$$

Result (type 3, 241 leaves):

$$\frac{1}{480 a^3 d}$$

$$\sec \left[ \frac{c}{2} \right] \sec \left[ \frac{1}{2} (c + d x) \right]^5 \left( 150 C d x \cos \left[ \frac{d x}{2} \right] + 150 C d x \cos \left[ c + \frac{d x}{2} \right] + 75 C d x \cos \left[ c + \frac{3 d x}{2} \right] + 75 C d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 15 C d x \cos \left[ 2 c + \frac{5 d x}{2} \right] + \right.$$

$$15 C d x \cos \left[ 3 c + \frac{5 d x}{2} \right] + 80 B \sin \left[ \frac{d x}{2} \right] - 370 C \sin \left[ \frac{d x}{2} \right] - 60 B \sin \left[ c + \frac{d x}{2} \right] + 270 C \sin \left[ c + \frac{d x}{2} \right] + 40 B \sin \left[ c + \frac{3 d x}{2} \right] -$$

$$\left. 230 C \sin \left[ c + \frac{3 d x}{2} \right] - 30 B \sin \left[ 2 c + \frac{3 d x}{2} \right] + 90 C \sin \left[ 2 c + \frac{3 d x}{2} \right] + 14 B \sin \left[ 2 c + \frac{5 d x}{2} \right] - 64 C \sin \left[ 2 c + \frac{5 d x}{2} \right] \right)$$

■ **Problem 275: Result more than twice size of optimal antiderivative.**

$$\int \frac{(B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^3}{(a + a \cos [c + d x])^3} dx$$

Optimal (type 3, 145 leaves, 8 steps):

$$-\frac{(3 B - C) \operatorname{ArcTanh}[\sin [c + d x]]}{a^3 d} + \frac{2 (36 B - 11 C) \tan [c + d x]}{15 a^3 d} - \frac{(B - C) \tan [c + d x]}{5 d (a + a \cos [c + d x])^3} - \frac{(9 B - 4 C) \tan [c + d x]}{15 a d (a + a \cos [c + d x])^2} - \frac{(3 B - C) \tan [c + d x]}{d (a^3 + a^3 \cos [c + d x])}$$

Result (type 3, 556 leaves):

$$\frac{1}{a^3} \left( \frac{8 (3B - C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (1 + \cos[c + dx])^3} - \frac{8 (3B - C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (1 + \cos[c + dx])^3} + \frac{1}{120 d (1 + \cos[c + dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \left( -255 B \sin\left[\frac{dx}{2}\right] + 160 C \sin\left[\frac{dx}{2}\right] + 567 B \sin\left[\frac{3dx}{2}\right] - 167 C \sin\left[\frac{3dx}{2}\right] - 600 B \sin\left[c - \frac{dx}{2}\right] + 170 C \sin\left[c - \frac{dx}{2}\right] + 375 B \sin\left[c + \frac{dx}{2}\right] - 170 C \sin\left[c + \frac{dx}{2}\right] - 480 B \sin\left[2c + \frac{dx}{2}\right] + 160 C \sin\left[2c + \frac{dx}{2}\right] - 60 B \sin\left[c + \frac{3dx}{2}\right] + 75 C \sin\left[c + \frac{3dx}{2}\right] + 402 B \sin\left[2c + \frac{3dx}{2}\right] - 167 C \sin\left[2c + \frac{3dx}{2}\right] - 225 B \sin\left[3c + \frac{3dx}{2}\right] + 75 C \sin\left[3c + \frac{3dx}{2}\right] + 315 B \sin\left[c + \frac{5dx}{2}\right] - 95 C \sin\left[c + \frac{5dx}{2}\right] + 30 B \sin\left[2c + \frac{5dx}{2}\right] + 15 C \sin\left[2c + \frac{5dx}{2}\right] + 240 B \sin\left[3c + \frac{5dx}{2}\right] - 95 C \sin\left[3c + \frac{5dx}{2}\right] - 45 B \sin\left[4c + \frac{5dx}{2}\right] + 15 C \sin\left[4c + \frac{5dx}{2}\right] + 72 B \sin\left[2c + \frac{7dx}{2}\right] - 22 C \sin\left[2c + \frac{7dx}{2}\right] + 15 B \sin\left[3c + \frac{7dx}{2}\right] + 57 B \sin\left[4c + \frac{7dx}{2}\right] - 22 C \sin\left[4c + \frac{7dx}{2}\right] \right)$$

■ **Problem 276: Result more than twice size of optimal antiderivative.**

$$\int \frac{(B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^4}{(a + a \cos[c + dx])^3} dx$$

Optimal (type 3, 196 leaves, 9 steps):

$$\frac{(13B - 6C) \operatorname{ArcTanh}[\sin[c + dx]]}{2a^3 d} - \frac{8(19B - 9C) \tan[c + dx]}{15a^3 d} + \frac{(13B - 6C) \operatorname{Sec}[c + dx] \tan[c + dx]}{2a^3 d} - \frac{(B - C) \operatorname{Sec}[c + dx] \tan[c + dx]}{5d (a + a \cos[c + dx])^3} - \frac{(11B - 6C) \operatorname{Sec}[c + dx] \tan[c + dx]}{15ad (a + a \cos[c + dx])^2} - \frac{4(19B - 9C) \operatorname{Sec}[c + dx] \tan[c + dx]}{15d (a^3 + a^3 \cos[c + dx])}$$

Result (type 3, 684 leaves):

$$\frac{1}{a^3} \left( -\frac{4 (13 B - 6 C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (1 + \cos[c + dx])^3} + \frac{4 (13 B - 6 C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d (1 + \cos[c + dx])^3} + \frac{1}{480 d (1 + \cos[c + dx])^3} \right. \\ \left. \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \left( 1235 B \sin\left[\frac{dx}{2}\right] - 870 C \sin\left[\frac{dx}{2}\right] - 3805 B \sin\left[\frac{3 dx}{2}\right] + 1830 C \sin\left[\frac{3 dx}{2}\right] + 4329 B \sin\left[c - \frac{dx}{2}\right] - \right. \right. \\ \left. 2094 C \sin\left[c - \frac{dx}{2}\right] - 1989 B \sin\left[c + \frac{dx}{2}\right] + 1314 C \sin\left[c + \frac{dx}{2}\right] + 3575 B \sin\left[2c + \frac{dx}{2}\right] - 1650 C \sin\left[2c + \frac{dx}{2}\right] + 475 B \sin\left[c + \frac{3 dx}{2}\right] - \right. \\ \left. 450 C \sin\left[c + \frac{3 dx}{2}\right] - 2005 B \sin\left[2c + \frac{3 dx}{2}\right] + 1230 C \sin\left[2c + \frac{3 dx}{2}\right] + 2275 B \sin\left[3c + \frac{3 dx}{2}\right] - 1050 C \sin\left[3c + \frac{3 dx}{2}\right] - \right. \\ \left. 2673 B \sin\left[c + \frac{5 dx}{2}\right] + 1278 C \sin\left[c + \frac{5 dx}{2}\right] - 105 B \sin\left[2c + \frac{5 dx}{2}\right] - 90 C \sin\left[2c + \frac{5 dx}{2}\right] - 1593 B \sin\left[3c + \frac{5 dx}{2}\right] + \right. \\ \left. 918 C \sin\left[3c + \frac{5 dx}{2}\right] + 975 B \sin\left[4c + \frac{5 dx}{2}\right] - 450 C \sin\left[4c + \frac{5 dx}{2}\right] - 1325 B \sin\left[2c + \frac{7 dx}{2}\right] + 630 C \sin\left[2c + \frac{7 dx}{2}\right] - \right. \\ \left. 255 B \sin\left[3c + \frac{7 dx}{2}\right] + 60 C \sin\left[3c + \frac{7 dx}{2}\right] - 875 B \sin\left[4c + \frac{7 dx}{2}\right] + 480 C \sin\left[4c + \frac{7 dx}{2}\right] + 195 B \sin\left[5c + \frac{7 dx}{2}\right] - 90 C \sin\left[5c + \frac{7 dx}{2}\right] - \right. \\ \left. 304 B \sin\left[3c + \frac{9 dx}{2}\right] + 144 C \sin\left[3c + \frac{9 dx}{2}\right] - 90 B \sin\left[4c + \frac{9 dx}{2}\right] + 30 C \sin\left[4c + \frac{9 dx}{2}\right] - 214 B \sin\left[5c + \frac{9 dx}{2}\right] + 114 C \sin\left[5c + \frac{9 dx}{2}\right] \right)$$

■ **Problem 295: Result more than twice size of optimal antiderivative.**

$$\int (A + B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx] dx$$

Optimal (type 3, 27 leaves, 3 steps):

$$Bx + \frac{A \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{C \sin[c + dx]}{d}$$

Result (type 3, 95 leaves):

$$Bx - \frac{A \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{A \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{C \cos[dx] \sin[c]}{d} + \frac{C \cos[c] \sin[dx]}{d}$$

■ **Problem 296: Result more than twice size of optimal antiderivative.**

$$\int (A + B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^2 dx$$

Optimal (type 3, 27 leaves, 3 steps):

$$Cx + \frac{B \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{A \tan[c + dx]}{d}$$

Result (type 3, 84 leaves) :

$$C x - \frac{B \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{B \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{A \operatorname{Tan}[c + dx]}{d}$$

■ **Problem 297: Result more than twice size of optimal antiderivative.**

$$\int (A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^3 dx$$

Optimal (type 3, 51 leaves, 5 steps) :

$$\frac{(A + 2C) \operatorname{ArcTanh}[\sin[c + dx]]}{2d} + \frac{B \operatorname{Tan}[c + dx]}{d} + \frac{A \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2d}$$

Result (type 3, 151 leaves) :

$$\frac{1}{4d} \left( -2(A + 2C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 2A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \left( 4C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + \frac{A}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{A}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + 4B \operatorname{Tan}[c + dx] \right)$$

■ **Problem 299: Result more than twice size of optimal antiderivative.**

$$\int (A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^5 dx$$

Optimal (type 3, 97 leaves, 6 steps) :

$$\frac{(3A + 4C) \operatorname{ArcTanh}[\sin[c + dx]]}{8d} + \frac{B \operatorname{Tan}[c + dx]}{d} + \frac{(3A + 4C) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{8d} + \frac{A \operatorname{Sec}[c + dx]^3 \operatorname{Tan}[c + dx]}{4d} + \frac{B \operatorname{Tan}[c + dx]^3}{3d}$$

Result (type 3, 353 leaves) :

$$\begin{aligned} & - \frac{3A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d} - \frac{C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{2d} + \\ & \frac{3A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d} + \frac{C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{2d} + \frac{A}{16d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \\ & \frac{3A}{16d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{C}{4d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{A}{16d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} - \\ & \frac{3A}{16d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{C}{4d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{2B \operatorname{Tan}[c + dx]}{3d} + \frac{B \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{3d} \end{aligned}$$

■ **Problem 300: Result more than twice size of optimal antiderivative.**

$$\int (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^6 dx$$

Optimal (type 3, 122 leaves, 7 steps):

$$\frac{3 B \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} + \frac{(4 A + 5 C) \tan [c + d x]}{5 d} + \frac{3 B \sec [c + d x] \tan [c + d x]}{8 d} +$$

$$\frac{B \sec [c + d x]^3 \tan [c + d x]}{4 d} + \frac{A \sec [c + d x]^4 \tan [c + d x]}{5 d} + \frac{(4 A + 5 C) \tan [c + d x]^3}{15 d}$$

Result (type 3, 285 leaves):

$$-\frac{3 B \operatorname{Log}\left[\cos \left[\frac{1}{2}(c + d x)\right] - \sin \left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{3 B \operatorname{Log}\left[\cos \left[\frac{1}{2}(c + d x)\right] + \sin \left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{B}{16 d \left(\cos \left[\frac{1}{2}(c + d x)\right] - \sin \left[\frac{1}{2}(c + d x)\right]\right)^4} +$$

$$\frac{3 B}{16 d \left(\cos \left[\frac{1}{2}(c + d x)\right] - \sin \left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{B}{16 d \left(\cos \left[\frac{1}{2}(c + d x)\right] + \sin \left[\frac{1}{2}(c + d x)\right]\right)^4} - \frac{3 B}{16 d \left(\cos \left[\frac{1}{2}(c + d x)\right] + \sin \left[\frac{1}{2}(c + d x)\right]\right)^2} +$$

$$\frac{8 A \tan [c + d x]}{15 d} + \frac{2 C \tan [c + d x]}{3 d} + \frac{4 A \sec [c + d x]^2 \tan [c + d x]}{15 d} + \frac{C \sec [c + d x]^2 \tan [c + d x]}{3 d} + \frac{A \sec [c + d x]^4 \tan [c + d x]}{5 d}$$

■ **Problem 305: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x]) (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^2 dx$$

Optimal (type 3, 46 leaves, 4 steps):

$$a (B + C) x + \frac{a (A + B) \operatorname{ArcTanh}[\sin [c + d x]]}{d} + \frac{a C \sin [c + d x]}{d} + \frac{a A \tan [c + d x]}{d}$$

Result (type 3, 187 leaves):

$$a B x + a C x - \frac{a A \operatorname{Log}\left[\cos \left[\frac{c}{2} + \frac{d x}{2}\right] - \sin \left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} - \frac{a B \operatorname{Log}\left[\cos \left[\frac{c}{2} + \frac{d x}{2}\right] - \sin \left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a A \operatorname{Log}\left[\cos \left[\frac{c}{2} + \frac{d x}{2}\right] + \sin \left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} +$$

$$\frac{a B \operatorname{Log}\left[\cos \left[\frac{c}{2} + \frac{d x}{2}\right] + \sin \left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a C \cos [d x] \sin [c]}{d} + \frac{a C \cos [c] \sin [d x]}{d} + \frac{a A \tan [c + d x]}{d}$$

■ **Problem 306: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x]) (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^3 dx$$

Optimal (type 3, 62 leaves, 4 steps):

$$a C x + \frac{a (A + 2 (B + C)) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{a (A + B) \tan [c + d x]}{d} + \frac{a A \sec [c + d x] \tan [c + d x]}{2 d}$$

Result (type 3, 203 leaves):

$$\frac{1}{4} a \left( 4 C x - \frac{2 (A + 2 (B + C)) \operatorname{Log}\left[\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right]}{d} + \frac{2 A \operatorname{Log}\left[\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right]}{d} + \frac{4 B \operatorname{Log}\left[\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right]}{d} + \frac{4 C \operatorname{Log}\left[\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right]}{d} + \frac{A}{d \left(\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right)^2} - \frac{A}{d \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right)^2} + \frac{4 (A + B) \operatorname{Tan}[c + d x]}{d} \right)$$

■ **Problem 308: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + d x]) (A + B \cos[c + d x] + C \cos[c + d x]^2) \operatorname{Sec}[c + d x]^5 dx$$

Optimal (type 3, 125 leaves, 7 steps):

$$\frac{a (3 A + 4 (B + C)) \operatorname{ArcTanh}[\sin[c + d x]]}{8 d} - \frac{a (A + B - 3 (A + B + C)) \operatorname{Tan}[c + d x]}{3 d} + \frac{a (3 A + 4 (B + C)) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{a (A + B) \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d} + \frac{a A \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d}$$

Result (type 3, 545 leaves):

$$\begin{aligned} & - \frac{3 a A \operatorname{Log}\left[\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right]}{8 d} - \frac{a B \operatorname{Log}\left[\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right]}{2 d} - \\ & \frac{a C \operatorname{Log}\left[\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right]}{2 d} + \frac{3 a A \operatorname{Log}\left[\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right]}{8 d} + \frac{a B \operatorname{Log}\left[\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right]}{2 d} + \\ & \frac{a C \operatorname{Log}\left[\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right]}{2 d} + \frac{a A}{16 d \left(\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right)^4} + \frac{3 a A}{16 d \left(\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right)^2} + \\ & \frac{a B}{4 d \left(\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right)^2} + \frac{a C}{4 d \left(\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right)^2} - \frac{a A}{16 d \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right)^4} - \\ & \frac{3 a A}{16 d \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right)^2} - \frac{a B}{4 d \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right)^2} - \frac{a C}{4 d \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right)^2} + \\ & \frac{2 a A \operatorname{Tan}[c + d x]}{3 d} + \frac{2 a B \operatorname{Tan}[c + d x]}{3 d} + \frac{a C \operatorname{Tan}[c + d x]}{d} + \frac{a A \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d} + \frac{a B \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d} \end{aligned}$$

■ **Problem 314: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + d x])^2 (A + B \cos[c + d x] + C \cos[c + d x]^2) \operatorname{Sec}[c + d x]^3 dx$$

Optimal (type 3, 123 leaves, 6 steps):



$$a^2 (B + 2 C) x + \frac{a^2 (3 A + 4 B + 2 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} - \frac{a^2 (3 A + 2 B - 2 C) \operatorname{Sin}[c + d x]}{2 d} +$$

$$\frac{(A + B) (a^2 + a^2 \operatorname{Cos}[c + d x]) \operatorname{Tan}[c + d x]}{d} + \frac{A (a + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 d}$$

Result (type 3, 259 leaves):

$$\frac{1}{16 d} a^2 (1 + \operatorname{Cos}[c + d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \left( 4 (B + 2 C) (c + d x) - 2 (3 A + 4 B + 2 C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) +$$

$$2 (3 A + 4 B + 2 C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \frac{A}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} +$$

$$\left( \frac{4 (2 A + B) \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]} - \frac{A}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{4 (2 A + B) \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]} + 4 C \operatorname{Sin}[c + d x] \right)$$

■ **Problem 315: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Cos}[c + d x])^2 (A + B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2) \operatorname{Sec}[c + d x]^4 dx$$

Optimal (type 3, 134 leaves, 6 steps):

$$a^2 C x + \frac{a^2 (2 A + 3 B + 4 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{a^2 (2 A + 3 B + 2 C) \operatorname{Tan}[c + d x]}{2 d} +$$

$$\frac{(2 A + 3 B) (a^2 + a^2 \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{6 d} + \frac{A (a + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d}$$

Result (type 3, 639 leaves):

$$\begin{aligned}
& \frac{C (c + dx) (a + a \cos [c + dx])^2 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4}{4d} + \frac{(-2A - 3B - 4C) (a + a \cos [c + dx])^2 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4}{8d} + \\
& \frac{(2A + 3B + 4C) (a + a \cos [c + dx])^2 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4}{8d} + \\
& \frac{(7A + 3B) (a + a \cos [c + dx])^2 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4}{48d \left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right)^2} + \frac{A (a + a \cos [c + dx])^2 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \sin \left[ \frac{1}{2} (c + dx) \right]}{24d \left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right)^3} + \\
& \frac{A (a + a \cos [c + dx])^2 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \sin \left[ \frac{1}{2} (c + dx) \right]}{24d \left( \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right)^3} + \frac{(-7A - 3B) (a + a \cos [c + dx])^2 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4}{48d \left( \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right)^2} + \\
& \frac{(a + a \cos [c + dx])^2 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \left( 5A \sin \left[ \frac{1}{2} (c + dx) \right] + 6B \sin \left[ \frac{1}{2} (c + dx) \right] + 3C \sin \left[ \frac{1}{2} (c + dx) \right] \right)}{12d \left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right)} + \\
& \frac{(a + a \cos [c + dx])^2 \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \left( 5A \sin \left[ \frac{1}{2} (c + dx) \right] + 6B \sin \left[ \frac{1}{2} (c + dx) \right] + 3C \sin \left[ \frac{1}{2} (c + dx) \right] \right)}{12d \left( \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right)}
\end{aligned}$$

■ **Problem 316: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + dx])^2 (A + B \cos [c + dx] + C \cos [c + dx]^2) \operatorname{Sec} [c + dx]^5 dx$$

Optimal (type 3, 160 leaves, 8 steps):

$$\begin{aligned}
& \frac{a^2 (7A + 8B + 12C) \operatorname{ArcTanh}[\sin [c + dx]]}{8d} + \frac{a^2 (4A + 5B + 6C) \tan [c + dx]}{3d} + \frac{a^2 (11A + 16B + 12C) \operatorname{Sec} [c + dx] \tan [c + dx]}{24d} + \\
& \frac{(A + 2B) (a^2 + a^2 \cos [c + dx]) \operatorname{Sec} [c + dx]^2 \tan [c + dx]}{6d} + \frac{A (a + a \cos [c + dx])^2 \operatorname{Sec} [c + dx]^3 \tan [c + dx]}{4d}
\end{aligned}$$

Result (type 3, 404 leaves):

$$\frac{1}{192 d} a^2 (1 + \cos[c + dx])^2 \sec\left[\frac{1}{2}(c + dx)\right]^4$$

$$\left( -6 (7A + 8B + 12C) \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 6 (7A + 8B + 12C) \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) +$$

$$\frac{3A}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{29A + 28B + 12C}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{8(2A + B) \sin\left[\frac{1}{2}(c + dx)\right]}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} +$$

$$\frac{16(4A + 5B + 6C) \sin\left[\frac{1}{2}(c + dx)\right]}{\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]} - \frac{3A}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} +$$

$$\frac{8(2A + B) \sin\left[\frac{1}{2}(c + dx)\right]}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{-29A - 4(7B + 3C)}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{16(4A + 5B + 6C) \sin\left[\frac{1}{2}(c + dx)\right]}{\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]}$$

■ **Problem 317: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^2 (A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^6 dx$$

Optimal (type 3, 196 leaves, 9 steps):

$$\frac{a^2 (6A + 7B + 8C) \operatorname{ArcTanh}[\sin[c + dx]]}{8d} + \frac{a^2 (18A + 20B + 25C) \tan[c + dx]}{15d} +$$

$$\frac{a^2 (6A + 7B + 8C) \sec[c + dx] \tan[c + dx]}{8d} + \frac{a^2 (18A + 25B + 20C) \sec[c + dx]^2 \tan[c + dx]}{60d} +$$

$$\frac{(2A + 5B) (a^2 + a^2 \cos[c + dx]) \sec[c + dx]^3 \tan[c + dx]}{20d} + \frac{A (a + a \cos[c + dx])^2 \sec[c + dx]^4 \tan[c + dx]}{5d}$$

Result (type 3, 931 leaves):

$$\begin{aligned}
& \frac{(-6A - 7B - 8C)(a + a \cos[c + dx])^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4}{32d} + \\
& \frac{(6A + 7B + 8C)(a + a \cos[c + dx])^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4}{32d} + \\
& \frac{(12A + 5B)(a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4}{320d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{(129A + 145B + 140C)(a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4}{960d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{A(a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{1}{2}(c + dx)\right]}{80d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^5} + \frac{A(a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sin\left[\frac{1}{2}(c + dx)\right]}{80d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^5} + \\
& \frac{(-12A - 5B)(a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4}{320d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{(-129A - 145B - 140C)(a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4}{960d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{(a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(39A \sin\left[\frac{1}{2}(c + dx)\right] + 40B \sin\left[\frac{1}{2}(c + dx)\right] + 20C \sin\left[\frac{1}{2}(c + dx)\right]\right)}{480d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(39A \sin\left[\frac{1}{2}(c + dx)\right] + 40B \sin\left[\frac{1}{2}(c + dx)\right] + 20C \sin\left[\frac{1}{2}(c + dx)\right]\right)}{480d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(18A \sin\left[\frac{1}{2}(c + dx)\right] + 20B \sin\left[\frac{1}{2}(c + dx)\right] + 25C \sin\left[\frac{1}{2}(c + dx)\right]\right)}{60d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \\
& \frac{(a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(18A \sin\left[\frac{1}{2}(c + dx)\right] + 20B \sin\left[\frac{1}{2}(c + dx)\right] + 25C \sin\left[\frac{1}{2}(c + dx)\right]\right)}{60d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)}
\end{aligned}$$

■ **Problem 324: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^3 (A + B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^4 dx$$

Optimal (type 3, 169 leaves, 7 steps):

$$\begin{aligned}
& a^3 (B + 3C)x + \frac{a^3 (5A + 7B + 6C) \operatorname{ArcTanh}[\sin[c + dx]]}{2d} - \frac{5a^3 (A + B) \sin[c + dx]}{2d} + \frac{(5A + 6B + 3C)(a^3 + a^3 \cos[c + dx]) \tan[c + dx]}{3d} + \\
& \frac{(A + B)(a^2 + a^2 \cos[c + dx])^2 \operatorname{Sec}[c + dx] \tan[c + dx]}{2ad} + \frac{A(a + a \cos[c + dx])^3 \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3d}
\end{aligned}$$

Result (type 3, 684 leaves):

$$\begin{aligned}
& \frac{(B+3C)(c+dx)(a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{8d} + \frac{(-5A-7B-6C)(a+a\cos[c+dx])^3 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{16d} + \\
& \frac{(5A+7B+6C)(a+a\cos[c+dx])^3 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{16d} + \\
& \frac{(10A+3B)(a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{96d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{A(a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{1}{2}(c+dx)\right]}{48d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} + \\
& \frac{A(a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{1}{2}(c+dx)\right]}{48d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{(-10A-3B)(a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{96d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \\
& \frac{(a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \left(11A \sin\left[\frac{1}{2}(c+dx)\right] + 9B \sin\left[\frac{1}{2}(c+dx)\right] + 3C \sin\left[\frac{1}{2}(c+dx)\right]\right)}{24d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)} + \\
& \frac{(a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \left(11A \sin\left[\frac{1}{2}(c+dx)\right] + 9B \sin\left[\frac{1}{2}(c+dx)\right] + 3C \sin\left[\frac{1}{2}(c+dx)\right]\right)}{24d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)} + \\
& \frac{C(a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin[c+dx]}{8d}
\end{aligned}$$

■ **Problem 325: Result more than twice size of optimal antiderivative.**

$$\int (a+a\cos[c+dx])^3 (A+B\cos[c+dx]+C\cos[c+dx]^2) \operatorname{Sec}[c+dx]^5 dx$$

Optimal (type 3, 183 leaves, 7 steps):

$$\begin{aligned}
& a^3 C x + \frac{a^3 (15A+20B+28C) \operatorname{ArcTanh}[\sin[c+dx]]}{8d} + \\
& \frac{5a^3 (3A+4(B+C)) \tan[c+dx]}{8d} + \frac{(15A+20B+12C) (a^3+a^3\cos[c+dx]) \operatorname{Sec}[c+dx] \tan[c+dx]}{24d} + \\
& \frac{(3A+4B) (a^2+a^2\cos[c+dx])^2 \operatorname{Sec}[c+dx]^2 \tan[c+dx]}{12ad} + \frac{A(a+a\cos[c+dx])^3 \operatorname{Sec}[c+dx]^3 \tan[c+dx]}{4d}
\end{aligned}$$

Result (type 3, 793 leaves):

$$\begin{aligned}
& \frac{C (c + d x) (a + a \cos [c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{8 d} + \frac{(-15 A - 20 B - 28 C) (a + a \cos [c + d x])^3 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{64 d} + \\
& \frac{(15 A + 20 B + 28 C) (a + a \cos [c + d x])^3 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{64 d} + \frac{A (a + a \cos [c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{128 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^4} + \\
& \frac{(57 A + 40 B + 12 C) (a + a \cos [c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{384 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{A (a + a \cos [c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{128 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^4} + \\
& \frac{(-57 A - 40 B - 12 C) (a + a \cos [c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{384 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{(a + a \cos [c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \left(3 A \sin\left[\frac{1}{2}(c + d x)\right] + B \sin\left[\frac{1}{2}(c + d x)\right]\right)}{48 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^3} + \\
& \frac{(a + a \cos [c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \left(3 A \sin\left[\frac{1}{2}(c + d x)\right] + B \sin\left[\frac{1}{2}(c + d x)\right]\right)}{48 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^3} + \\
& \frac{(a + a \cos [c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \left(9 A \sin\left[\frac{1}{2}(c + d x)\right] + 11 B \sin\left[\frac{1}{2}(c + d x)\right] + 9 C \sin\left[\frac{1}{2}(c + d x)\right]\right)}{24 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)} + \\
& \frac{(a + a \cos [c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \left(9 A \sin\left[\frac{1}{2}(c + d x)\right] + 11 B \sin\left[\frac{1}{2}(c + d x)\right] + 9 C \sin\left[\frac{1}{2}(c + d x)\right]\right)}{24 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)}
\end{aligned}$$

■ **Problem 326: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^3 (A + B \cos [c + d x] + C \cos [c + d x]^2) \operatorname{Sec}[c + d x]^6 dx$$

Optimal (type 3, 212 leaves, 9 steps):

$$\begin{aligned}
& \frac{a^3 (13 A + 15 B + 20 C) \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} + \frac{a^3 (38 A + 45 B + 55 C) \tan [c + d x]}{15 d} + \\
& \frac{a^3 (109 A + 135 B + 140 C) \operatorname{Sec}[c + d x] \tan [c + d x]}{120 d} + \frac{(11 A + 15 B + 10 C) (a^3 + a^3 \cos [c + d x]) \operatorname{Sec}[c + d x]^2 \tan [c + d x]}{30 d} + \\
& \frac{(3 A + 5 B) (a^2 + a^2 \cos [c + d x])^2 \operatorname{Sec}[c + d x]^3 \tan [c + d x]}{20 a d} + \frac{A (a + a \cos [c + d x])^3 \operatorname{Sec}[c + d x]^4 \tan [c + d x]}{5 d}
\end{aligned}$$

Result (type 3, 931 leaves):

$$\begin{aligned}
& \frac{(-13A - 15B - 20C)(a + a \cos[c + dx])^3 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{64d} + \\
& \frac{(13A + 15B + 20C)(a + a \cos[c + dx])^3 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{64d} + \\
& \frac{(17A + 5B)(a + a \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{640d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{(274A + 285B + 200C)(a + a \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{1920d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{A(a + a \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{1}{2}(c + dx)\right]}{160d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^5} + \frac{A(a + a \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sin\left[\frac{1}{2}(c + dx)\right]}{160d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^5} + \\
& \frac{(-17A - 5B)(a + a \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{640d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{(-274A - 285B - 200C)(a + a \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6}{1920d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{(a + a \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \left(79A \sin\left[\frac{1}{2}(c + dx)\right] + 60B \sin\left[\frac{1}{2}(c + dx)\right] + 20C \sin\left[\frac{1}{2}(c + dx)\right]\right)}{960d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(a + a \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \left(79A \sin\left[\frac{1}{2}(c + dx)\right] + 60B \sin\left[\frac{1}{2}(c + dx)\right] + 20C \sin\left[\frac{1}{2}(c + dx)\right]\right)}{960d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(a + a \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \left(38A \sin\left[\frac{1}{2}(c + dx)\right] + 45B \sin\left[\frac{1}{2}(c + dx)\right] + 55C \sin\left[\frac{1}{2}(c + dx)\right]\right)}{120d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \\
& \frac{(a + a \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \left(38A \sin\left[\frac{1}{2}(c + dx)\right] + 45B \sin\left[\frac{1}{2}(c + dx)\right] + 55C \sin\left[\frac{1}{2}(c + dx)\right]\right)}{120d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)}
\end{aligned}$$

■ **Problem 333: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^4 (A + B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^3 dx$$

Optimal (type 3, 206 leaves, 8 steps):

$$\begin{aligned}
& \frac{1}{2} a^4 (8A + 13B + 12C) x + \frac{a^4 (13A + 8B + 2C) \operatorname{ArcTanh}[\sin[c + dx]]}{2d} - \frac{5a^4 (A - B - 2C) \sin[c + dx]}{2d} - \\
& \frac{(15A + 6B - 2C)(a^2 + a^2 \cos[c + dx])^2 \sin[c + dx]}{6d} - \frac{(18A + 3B - 8C)(a^4 + a^4 \cos[c + dx]) \sin[c + dx]}{6d} + \\
& \frac{a(2A + B)(a + a \cos[c + dx])^3 \tan[c + dx]}{d} + \frac{A(a + a \cos[c + dx])^4 \operatorname{Sec}[c + dx] \tan[c + dx]}{2d}
\end{aligned}$$

Result (type 3, 610 leaves) :

$$\begin{aligned}
 & \frac{(8A + 13B + 12C) (c + dx) (a + a \cos[c + dx])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8}{32d} + \\
 & \frac{(-13A - 8B - 2C) (a + a \cos[c + dx])^4 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8}{32d} + \\
 & \frac{(13A + 8B + 2C) (a + a \cos[c + dx])^4 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8}{32d} + \frac{A (a + a \cos[c + dx])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8}{64d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} - \\
 & \frac{A (a + a \cos[c + dx])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8}{64d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{(a + a \cos[c + dx])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (4A \sin\left[\frac{1}{2}(c + dx)\right] + B \sin\left[\frac{1}{2}(c + dx)\right])}{16d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \\
 & \frac{(a + a \cos[c + dx])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (4A \sin\left[\frac{1}{2}(c + dx)\right] + B \sin\left[\frac{1}{2}(c + dx)\right])}{16d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \frac{(4A + 16B + 27C) (a + a \cos[c + dx])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \sin[c + dx]}{64d} + \\
 & \frac{(B + 4C) (a + a \cos[c + dx])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \sin[2(c + dx)]}{64d} + \frac{C (a + a \cos[c + dx])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \sin[3(c + dx)]}{192d}
 \end{aligned}$$

■ **Problem 334: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^4 (A + B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^4 dx$$

Optimal (type 3, 219 leaves, 8 steps) :

$$\begin{aligned}
 & \frac{1}{2} a^4 (2A + 8B + 13C) x + \frac{a^4 (12A + 13B + 8C) \operatorname{ArcTanh}[\sin[c + dx]]}{2d} - \frac{5a^4 (2A + B - C) \sin[c + dx]}{2d} - \\
 & \frac{(22A + 18B + 3C) (a^4 + a^4 \cos[c + dx]) \sin[c + dx]}{6d} + \frac{(16A + 15B + 6C) (a^2 + a^2 \cos[c + dx])^2 \tan[c + dx]}{6d} + \\
 & \frac{a (4A + 3B) (a + a \cos[c + dx])^3 \operatorname{Sec}[c + dx] \tan[c + dx]}{6d} + \frac{A (a + a \cos[c + dx])^4 \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3d}
 \end{aligned}$$

Result (type 3, 440 leaves) :



$$\begin{aligned}
& a^4 \left( \frac{(2A + 8B + 13C)(c + dx)}{2d} + \frac{(-12A - 13B - 8C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{2d} + \right. \\
& \frac{(12A + 13B + 8C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{2d} + \frac{13A + 3B}{12d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{A \sin\left[\frac{1}{2}(c + dx)\right]}{6d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{A \sin\left[\frac{1}{2}(c + dx)\right]}{6d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{-13A - 3B}{12d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{20A \sin\left[\frac{1}{2}(c + dx)\right] + 12B \sin\left[\frac{1}{2}(c + dx)\right] + 3C \sin\left[\frac{1}{2}(c + dx)\right]}{3d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \\
& \left. \frac{20A \sin\left[\frac{1}{2}(c + dx)\right] + 12B \sin\left[\frac{1}{2}(c + dx)\right] + 3C \sin\left[\frac{1}{2}(c + dx)\right]}{3d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \frac{(B + 4C) \sin[c + dx]}{d} + \frac{C \sin[2(c + dx)]}{4d} \right)
\end{aligned}$$

■ **Problem 335: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^4 (A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^5 dx$$

Optimal (type 3, 217 leaves, 8 steps):

$$\begin{aligned}
& a^4 (B + 4C) x + \frac{a^4 (35A + 48B + 52C) \operatorname{ArcTanh}[\sin[c + dx]]}{8d} - \frac{5a^4 (7A + 8B + 4C) \sin[c + dx]}{8d} + \\
& \frac{(35A + 44B + 36C) (a^4 + a^4 \cos[c + dx]) \tan[c + dx]}{12d} + \frac{(7A + 8B + 4C) (a^2 + a^2 \cos[c + dx])^2 \sec[c + dx] \tan[c + dx]}{8d} + \\
& \frac{a(A + B) (a + a \cos[c + dx])^3 \sec[c + dx]^2 \tan[c + dx]}{3d} + \frac{A (a + a \cos[c + dx])^4 \sec[c + dx]^3 \tan[c + dx]}{4d}
\end{aligned}$$

Result (type 3, 838 leaves):

$$\begin{aligned}
& \frac{(B + 4C)(c + dx)(a + a \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8}{16d} + \\
& \frac{(-35A - 48B - 52C)(a + a \cos[c + dx])^4 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8}{128d} + \\
& \frac{(35A + 48B + 52C)(a + a \cos[c + dx])^4 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8}{128d} + \frac{A(a + a \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8}{256d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \\
& \frac{(97A + 52B + 12C)(a + a \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8}{768d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{A(a + a \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8}{256d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \\
& \frac{(-97A - 52B - 12C)(a + a \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8}{768d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{(a + a \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (4A \sin\left[\frac{1}{2}(c + dx)\right] + B \sin\left[\frac{1}{2}(c + dx)\right])}{96d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(a + a \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (4A \sin\left[\frac{1}{2}(c + dx)\right] + B \sin\left[\frac{1}{2}(c + dx)\right])}{96d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(a + a \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (5A \sin\left[\frac{1}{2}(c + dx)\right] + 5B \sin\left[\frac{1}{2}(c + dx)\right] + 3C \sin\left[\frac{1}{2}(c + dx)\right])}{12d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \\
& \frac{(a + a \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (5A \sin\left[\frac{1}{2}(c + dx)\right] + 5B \sin\left[\frac{1}{2}(c + dx)\right] + 3C \sin\left[\frac{1}{2}(c + dx)\right])}{12d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \\
& \frac{C(a + a \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \sin[c + dx]}{16d}
\end{aligned}$$

■ **Problem 336: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^4 (A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^6 dx$$

Optimal (type 3, 225 leaves, 8 steps):

$$\begin{aligned}
& a^4 C x + \frac{a^4 (28A + 35B + 48C) \operatorname{ArcTanh}[\sin[c + dx]]}{8d} + \frac{a^4 (28A + 35B + 40C) \tan[c + dx]}{8d} + \\
& \frac{(28A + 35B + 32C)(a^4 + a^4 \cos[c + dx]) \sec[c + dx] \tan[c + dx]}{24d} + \frac{(28A + 35B + 20C)(a^2 + a^2 \cos[c + dx])^2 \sec[c + dx]^2 \tan[c + dx]}{60d} + \\
& \frac{a(4A + 5B)(a + a \cos[c + dx])^3 \sec[c + dx]^3 \tan[c + dx]}{20d} + \frac{A(a + a \cos[c + dx])^4 \sec[c + dx]^4 \tan[c + dx]}{5d}
\end{aligned}$$

Result (type 3, 971 leaves):

$$\begin{aligned}
& \frac{C (c + d x) (a + a \cos [c + d x])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8}{16 d} + \frac{(-28 A - 35 B - 48 C) (a + a \cos [c + d x])^4 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8}{128 d} + \\
& \frac{(28 A + 35 B + 48 C) (a + a \cos [c + d x])^4 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8}{128 d} + \\
& \frac{(22 A + 5 B) (a + a \cos [c + d x])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8}{1280 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{(559 A + 485 B + 260 C) (a + a \cos [c + d x])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8}{3840 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^2} + \\
& \frac{A (a + a \cos [c + d x])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 \sin\left[\frac{1}{2}(c + d x)\right]}{320 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^5} + \frac{A (a + a \cos [c + d x])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 \sin\left[\frac{1}{2}(c + d x)\right]}{320 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^5} + \\
& \frac{(-22 A - 5 B) (a + a \cos [c + d x])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8}{1280 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{(-559 A - 485 B - 260 C) (a + a \cos [c + d x])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8}{3840 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^2} + \\
& \frac{(a + a \cos [c + d x])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 \left(139 A \sin\left[\frac{1}{2}(c + d x)\right] + 80 B \sin\left[\frac{1}{2}(c + d x)\right] + 20 C \sin\left[\frac{1}{2}(c + d x)\right]\right)}{1920 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^3} + \\
& \frac{(a + a \cos [c + d x])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 \left(139 A \sin\left[\frac{1}{2}(c + d x)\right] + 80 B \sin\left[\frac{1}{2}(c + d x)\right] + 20 C \sin\left[\frac{1}{2}(c + d x)\right]\right)}{1920 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^3} + \\
& \left( \frac{(a + a \cos [c + d x])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 \left(83 A \sin\left[\frac{1}{2}(c + d x)\right] + 100 B \sin\left[\frac{1}{2}(c + d x)\right] + 100 C \sin\left[\frac{1}{2}(c + d x)\right]\right)}{240 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)} \right) / \\
& \left( \frac{(a + a \cos [c + d x])^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 \left(83 A \sin\left[\frac{1}{2}(c + d x)\right] + 100 B \sin\left[\frac{1}{2}(c + d x)\right] + 100 C \sin\left[\frac{1}{2}(c + d x)\right]\right)}{240 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)} \right) /
\end{aligned}$$

■ **Problem 339: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^3 (A + B \cos [c + d x] + C \cos [c + d x]^2)}{a + a \cos [c + d x]} dx$$

Optimal (type 3, 174 leaves, 7 steps):

$$\frac{3 (4 A - 4 B + 5 C) x}{8 a} - \frac{(3 A - 4 B + 4 C) \sin [c + d x]}{a d} + \frac{3 (4 A - 4 B + 5 C) \cos [c + d x] \sin [c + d x]}{8 a d} + \\
\frac{(4 A - 4 B + 5 C) \cos [c + d x]^3 \sin [c + d x]}{4 a d} - \frac{(A - B + C) \cos [c + d x]^4 \sin [c + d x]}{d (a + a \cos [c + d x])} + \frac{(3 A - 4 B + 4 C) \sin [c + d x]^3}{3 a d}$$

Result (type 3, 393 leaves) :

$$\frac{1}{192 a d (1 + \operatorname{Cos}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( 72 (4 A - 4 B + 5 C) d x \operatorname{Cos}\left[\frac{d x}{2}\right] + 72 (4 A - 4 B + 5 C) d x \operatorname{Cos}\left[c + \frac{d x}{2}\right] - 480 A \operatorname{Sin}\left[\frac{d x}{2}\right] + 552 B \operatorname{Sin}\left[\frac{d x}{2}\right] - 552 C \operatorname{Sin}\left[\frac{d x}{2}\right] - 96 A \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 168 B \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 168 C \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 72 A \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 144 B \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - 120 C \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - 72 A \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 144 B \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] - 120 C \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 24 A \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] - 16 B \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] + 40 C \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] + 24 A \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] - 16 B \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] + 40 C \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] + 8 B \operatorname{Sin}\left[3 c + \frac{7 d x}{2}\right] - 5 C \operatorname{Sin}\left[3 c + \frac{7 d x}{2}\right] + 8 B \operatorname{Sin}\left[4 c + \frac{7 d x}{2}\right] - 5 C \operatorname{Sin}\left[4 c + \frac{7 d x}{2}\right] + 3 C \operatorname{Sin}\left[4 c + \frac{9 d x}{2}\right] + 3 C \operatorname{Sin}\left[5 c + \frac{9 d x}{2}\right] \right)$$

■ **Problem 340: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^2 (A + B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2)}{a + a \operatorname{Cos}[c + d x]} dx$$

Optimal (type 3, 139 leaves, 6 steps) :

$$\frac{(2 A - 3 B + 3 C) x}{2 a} + \frac{(3 A - 3 B + 4 C) \operatorname{Sin}[c + d x]}{a d} - \frac{(2 A - 3 B + 3 C) \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{2 a d} - \frac{(A - B + C) \operatorname{Cos}[c + d x]^3 \operatorname{Sin}[c + d x]}{d (a + a \operatorname{Cos}[c + d x])} - \frac{(3 A - 3 B + 4 C) \operatorname{Sin}[c + d x]^3}{3 a d}$$

Result (type 3, 307 leaves) :

$$\frac{1}{24 a d (1 + \operatorname{Cos}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( -12 (2 A - 3 B + 3 C) d x \operatorname{Cos}\left[\frac{d x}{2}\right] - 12 (2 A - 3 B + 3 C) d x \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 60 A \operatorname{Sin}\left[\frac{d x}{2}\right] - 60 B \operatorname{Sin}\left[\frac{d x}{2}\right] + 69 C \operatorname{Sin}\left[\frac{d x}{2}\right] + 12 A \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 12 B \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 21 C \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 12 A \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - 9 B \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 18 C \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 12 A \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] - 9 B \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 18 C \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 3 B \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] - 2 C \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] + 3 B \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] - 2 C \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] + C \operatorname{Sin}\left[3 c + \frac{7 d x}{2}\right] + C \operatorname{Sin}\left[4 c + \frac{7 d x}{2}\right] \right)$$

■ **Problem 342: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2}{a + a \operatorname{Cos}[c + d x]} dx$$

Optimal (type 3, 54 leaves, 3 steps):

$$\frac{(B-C)x}{a} + \frac{C \operatorname{Sin}[c+dx]}{ad} + \frac{(A-B+C) \operatorname{Sin}[c+dx]}{ad(1+\operatorname{Cos}[c+dx])}$$

Result (type 3, 136 leaves):

$$\frac{1}{2ad(1+\operatorname{Cos}[c+dx])} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( 2(B-C)dx \operatorname{Cos}\left[\frac{dx}{2}\right] + 2(B-C)dx \operatorname{Cos}\left[c + \frac{dx}{2}\right] + 4A \operatorname{Sin}\left[\frac{dx}{2}\right] - 4B \operatorname{Sin}\left[\frac{dx}{2}\right] + 5C \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[c + \frac{dx}{2}\right] + C \operatorname{Sin}\left[c + \frac{3dx}{2}\right] + C \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] \right)$$

■ **Problem 343: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2) \operatorname{Sec}[c+dx]}{a+a \operatorname{Cos}[c+dx]} dx$$

Optimal (type 3, 51 leaves, 3 steps):

$$\frac{Cx}{a} + \frac{A \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{ad} - \frac{(A-B+C) \operatorname{Sin}[c+dx]}{d(a+a \operatorname{Cos}[c+dx])}$$

Result (type 3, 163 leaves):

$$\left( 4 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] (A+B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2) \left( \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \left( Cdx - A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) - (A-B+C) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right] \right) \right) / (ad(1+\operatorname{Cos}[c+dx]) (2A+C+2B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[2(c+dx)]))$$

■ **Problem 344: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2) \operatorname{Sec}[c+dx]^2}{a+a \operatorname{Cos}[c+dx]} dx$$

Optimal (type 3, 71 leaves, 5 steps):

$$-\frac{(A-B) \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{ad} + \frac{(2A-B+C) \operatorname{Tan}[c+dx]}{ad} - \frac{(A-B+C) \operatorname{Tan}[c+dx]}{d(a+a \operatorname{Cos}[c+dx])}$$

Result (type 3, 256 leaves):

$$\frac{1}{a d (1 + \cos [c + d x]) (2 A + C + 2 B \cos [c + d x] + C \cos [2 (c + d x)])} 4 \cos \left[ \frac{1}{2} (c + d x) \right] \cos [c + d x]^2 (C + B \sec [c + d x] + A \sec [c + d x]^2) \\ \left( (A - B + C) \sec \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right] + \cos \left[ \frac{1}{2} (c + d x) \right] \left( (A - B) \left( \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] - \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] \right) \right) + \\ (A \sin [d x]) / \left( \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right) \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) \right)$$

■ **Problem 345: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^3}{a + a \cos [c + d x]} dx$$

Optimal (type 3, 117 leaves, 6 steps):

$$\frac{(3 A - 2 B + 2 C) \operatorname{ArcTanh}[\sin [c + d x]]}{2 a d} - \frac{(2 A - 2 B + C) \tan [c + d x]}{a d} + \frac{(3 A - 2 B + 2 C) \sec [c + d x] \tan [c + d x]}{2 a d} - \frac{(A - B + C) \sec [c + d x] \tan [c + d x]}{d (a + a \cos [c + d x])}$$

Result (type 3, 256 leaves):

$$\frac{1}{2 a d (1 + \cos [c + d x])} \\ \cos \left[ \frac{1}{2} (c + d x) \right]^2 \left( -2 (3 A - 2 B + 2 C) \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] + 2 (3 A - 2 B + 2 C) \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] \right) + \\ \frac{A}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{4 (-A + B) \sin \left[ \frac{1}{2} (c + d x) \right]}{\cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right]} - \\ \frac{A}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{4 (-A + B) \sin \left[ \frac{1}{2} (c + d x) \right]}{\cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right]} - 4 (A - B + C) \tan \left[ \frac{1}{2} (c + d x) \right] \right)$$

■ **Problem 346: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^4}{a + a \cos [c + d x]} dx$$

Optimal (type 3, 148 leaves, 6 steps):

$$- \frac{(3 A - 3 B + 2 C) \operatorname{ArcTanh}[\sin [c + d x]]}{2 a d} + \frac{(4 A - 3 B + 3 C) \tan [c + d x]}{a d} - \\ \frac{(3 A - 3 B + 2 C) \sec [c + d x] \tan [c + d x]}{2 a d} - \frac{(A - B + C) \sec [c + d x]^2 \tan [c + d x]}{d (a + a \cos [c + d x])} + \frac{(4 A - 3 B + 3 C) \tan [c + d x]^3}{3 a d}$$

Result (type 3, 673 leaves):

$$\begin{aligned}
& \frac{(3A - 3B + 2C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{d(a + a \cos[c + dx])} + \frac{(-3A + 3B - 2C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{d(a + a \cos[c + dx])} + \\
& \frac{(-2A + 3B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2}{6d(a + a \cos[c + dx]) \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{1}{2}(c + dx)\right]}{3d(a + a \cos[c + dx]) \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sin\left[\frac{1}{2}(c + dx)\right]}{3d(a + a \cos[c + dx]) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{(2A - 3B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2}{6d(a + a \cos[c + dx]) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \left(A \sin\left[\frac{1}{2}(c + dx)\right] - B \sin\left[\frac{1}{2}(c + dx)\right] + C \sin\left[\frac{1}{2}(c + dx)\right]\right)}{d(a + a \cos[c + dx])} + \\
& \frac{2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left(5A \sin\left[\frac{1}{2}(c + dx)\right] - 3B \sin\left[\frac{1}{2}(c + dx)\right] + 3C \sin\left[\frac{1}{2}(c + dx)\right]\right)}{3d(a + a \cos[c + dx]) \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \\
& \frac{2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left(5A \sin\left[\frac{1}{2}(c + dx)\right] - 3B \sin\left[\frac{1}{2}(c + dx)\right] + 3C \sin\left[\frac{1}{2}(c + dx)\right]\right)}{3d(a + a \cos[c + dx]) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)}
\end{aligned}$$

■ **Problem 347: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^3 (A + B \cos[c + dx] + C \cos[c + dx]^2)}{(a + a \cos[c + dx])^2} dx$$

Optimal (type 3, 185 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(4A - 7B + 10C)x}{2a^2} + \frac{(5A - 8B + 12C) \sin[c + dx]}{a^2 d} - \frac{(4A - 7B + 10C) \cos[c + dx] \sin[c + dx]}{2a^2 d} - \\
& \frac{(4A - 7B + 10C) \cos[c + dx]^3 \sin[c + dx]}{3a^2 d (1 + \cos[c + dx])} - \frac{(A - B + C) \cos[c + dx]^4 \sin[c + dx]}{3d(a + a \cos[c + dx])^2} - \frac{(5A - 8B + 12C) \sin[c + dx]^3}{3a^2 d}
\end{aligned}$$

Result (type 3, 481 leaves):

$$\frac{1}{48 a^2 d (1 + \cos [c + d x])^2} \cos \left[ \frac{1}{2} (c + d x) \right] \sec \left[ \frac{c}{2} \right] \left( -36 (4 A - 7 B + 10 C) d x \cos \left[ \frac{d x}{2} \right] - 36 (4 A - 7 B + 10 C) d x \cos \left[ c + \frac{d x}{2} \right] - 48 A d x \cos \left[ c + \frac{3 d x}{2} \right] + 84 B d x \cos \left[ c + \frac{3 d x}{2} \right] - 120 C d x \cos \left[ c + \frac{3 d x}{2} \right] - 48 A d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 84 B d x \cos \left[ 2 c + \frac{3 d x}{2} \right] - 120 C d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 264 A \sin \left[ \frac{d x}{2} \right] - 381 B \sin \left[ \frac{d x}{2} \right] + 516 C \sin \left[ \frac{d x}{2} \right] - 120 A \sin \left[ c + \frac{d x}{2} \right] + 147 B \sin \left[ c + \frac{d x}{2} \right] - 156 C \sin \left[ c + \frac{d x}{2} \right] + 164 A \sin \left[ c + \frac{3 d x}{2} \right] - 239 B \sin \left[ c + \frac{3 d x}{2} \right] + 342 C \sin \left[ c + \frac{3 d x}{2} \right] + 36 A \sin \left[ 2 c + \frac{3 d x}{2} \right] - 63 B \sin \left[ 2 c + \frac{3 d x}{2} \right] + 118 C \sin \left[ 2 c + \frac{3 d x}{2} \right] + 12 A \sin \left[ 2 c + \frac{5 d x}{2} \right] - 15 B \sin \left[ 2 c + \frac{5 d x}{2} \right] + 30 C \sin \left[ 2 c + \frac{5 d x}{2} \right] + 12 A \sin \left[ 3 c + \frac{5 d x}{2} \right] - 15 B \sin \left[ 3 c + \frac{5 d x}{2} \right] + 30 C \sin \left[ 3 c + \frac{5 d x}{2} \right] + 3 B \sin \left[ 3 c + \frac{7 d x}{2} \right] - 3 C \sin \left[ 3 c + \frac{7 d x}{2} \right] + 3 B \sin \left[ 4 c + \frac{7 d x}{2} \right] - 3 C \sin \left[ 4 c + \frac{7 d x}{2} \right] + C \sin \left[ 4 c + \frac{9 d x}{2} \right] + C \sin \left[ 5 c + \frac{9 d x}{2} \right] \right)$$

■ **Problem 348: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^2 (A + B \cos [c + d x] + C \cos [c + d x]^2)}{(a + a \cos [c + d x])^2} dx$$

Optimal (type 3, 160 leaves, 3 steps):

$$\frac{(2 A - 4 B + 7 C) x}{2 a^2} - \frac{2 (2 A - 5 B + 8 C) \sin [c + d x]}{3 a^2 d} + \frac{(2 A - 4 B + 7 C) \cos [c + d x] \sin [c + d x]}{2 a^2 d} - \frac{(2 A - 5 B + 8 C) \cos [c + d x]^2 \sin [c + d x]}{3 a^2 d (1 + \cos [c + d x])} - \frac{(A - B + C) \cos [c + d x]^3 \sin [c + d x]}{3 d (a + a \cos [c + d x])^2}$$

Result (type 3, 385 leaves):

$$\frac{1}{48 a^2 d (1 + \cos [c + d x])^2} \cos \left[ \frac{1}{2} (c + d x) \right] \sec \left[ \frac{c}{2} \right] \left( 36 (2 A - 4 B + 7 C) d x \cos \left[ \frac{d x}{2} \right] + 36 (2 A - 4 B + 7 C) d x \cos \left[ c + \frac{d x}{2} \right] + 24 A d x \cos \left[ c + \frac{3 d x}{2} \right] - 48 B d x \cos \left[ c + \frac{3 d x}{2} \right] + 84 C d x \cos \left[ c + \frac{3 d x}{2} \right] + 24 A d x \cos \left[ 2 c + \frac{3 d x}{2} \right] - 48 B d x \cos \left[ 2 c + \frac{3 d x}{2} \right] + 84 C d x \cos \left[ 2 c + \frac{3 d x}{2} \right] - 144 A \sin \left[ \frac{d x}{2} \right] + 264 B \sin \left[ \frac{d x}{2} \right] - 381 C \sin \left[ \frac{d x}{2} \right] + 96 A \sin \left[ c + \frac{d x}{2} \right] - 120 B \sin \left[ c + \frac{d x}{2} \right] + 147 C \sin \left[ c + \frac{d x}{2} \right] - 80 A \sin \left[ c + \frac{3 d x}{2} \right] + 164 B \sin \left[ c + \frac{3 d x}{2} \right] - 239 C \sin \left[ c + \frac{3 d x}{2} \right] + 36 B \sin \left[ 2 c + \frac{3 d x}{2} \right] - 63 C \sin \left[ 2 c + \frac{3 d x}{2} \right] + 12 B \sin \left[ 2 c + \frac{5 d x}{2} \right] - 15 C \sin \left[ 2 c + \frac{5 d x}{2} \right] + 12 B \sin \left[ 3 c + \frac{5 d x}{2} \right] - 15 C \sin \left[ 3 c + \frac{5 d x}{2} \right] + 3 C \sin \left[ 3 c + \frac{7 d x}{2} \right] + 3 C \sin \left[ 4 c + \frac{7 d x}{2} \right] \right)$$



■ **Problem 349: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x] \left(A+B \cos [c+d x]+C \cos [c+d x]^2\right)}{\left(a+a \cos [c+d x]\right)^2} d x$$

Optimal (type 3, 103 leaves, 6 steps):

$$\frac{(B-2 C) x}{a^2} + \frac{(A-B+4 C) \sin [c+d x]}{3 a^2 d} - \frac{(B-2 C) \sin [c+d x]}{a^2 d (1+\cos [c+d x])} - \frac{(A-B+C) \cos [c+d x]^2 \sin [c+d x]}{3 d (a+a \cos [c+d x])^2}$$

Result (type 3, 275 leaves):

$$\frac{1}{12 a^2 d (1+\cos [c+d x])^2} \cos \left[\frac{1}{2}(c+d x)\right] \sec \left[\frac{c}{2}\right] \left(18(B-2 C) d x \cos \left[\frac{d x}{2}\right]+18(B-2 C) d x \cos \left[c+\frac{d x}{2}\right]+6 B d x \cos \left[c+\frac{3 d x}{2}\right]-12 C d x \cos \left[c+\frac{3 d x}{2}\right]+6 B d x \cos \left[2 c+\frac{3 d x}{2}\right]-12 C d x \cos \left[2 c+\frac{3 d x}{2}\right]+12 A \sin \left[\frac{d x}{2}\right]-36 B \sin \left[\frac{d x}{2}\right]+66 C \sin \left[\frac{d x}{2}\right]-12 A \sin \left[c+\frac{d x}{2}\right]+24 B \sin \left[c+\frac{d x}{2}\right]-30 C \sin \left[c+\frac{d x}{2}\right]+8 A \sin \left[c+\frac{3 d x}{2}\right]-20 B \sin \left[c+\frac{3 d x}{2}\right]+41 C \sin \left[c+\frac{3 d x}{2}\right]+9 C \sin \left[2 c+\frac{3 d x}{2}\right]+3 C \sin \left[2 c+\frac{5 d x}{2}\right]+3 C \sin \left[3 c+\frac{5 d x}{2}\right]\right)$$

■ **Problem 350: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+B \cos [c+d x]+C \cos [c+d x]^2}{\left(a+a \cos [c+d x]\right)^2} d x$$

Optimal (type 3, 72 leaves, 3 steps):

$$\frac{C x}{a^2} + \frac{(A+2 B-5 C) \sin [c+d x]}{3 a^2 d (1+\cos [c+d x])} + \frac{(A-B+C) \sin [c+d x]}{3 d (a+a \cos [c+d x])^2}$$

Result (type 3, 175 leaves):

$$\frac{1}{24 a^2 d} \sec \left[\frac{c}{2}\right] \sec \left[\frac{1}{2}(c+d x)\right]^3 \left(9 C d x \cos \left[\frac{d x}{2}\right]+9 C d x \cos \left[c+\frac{d x}{2}\right]+3 C d x \cos \left[c+\frac{3 d x}{2}\right]+3 C d x \cos \left[2 c+\frac{3 d x}{2}\right]+6 A \sin \left[\frac{d x}{2}\right]+6 B \sin \left[\frac{d x}{2}\right]-18 C \sin \left[\frac{d x}{2}\right]-6 B \sin \left[c+\frac{d x}{2}\right]+12 C \sin \left[c+\frac{d x}{2}\right]+2 A \sin \left[c+\frac{3 d x}{2}\right]+4 B \sin \left[c+\frac{3 d x}{2}\right]-10 C \sin \left[c+\frac{3 d x}{2}\right]\right)$$

■ **Problem 351: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(A+B \cos [c+d x]+C \cos [c+d x]^2\right) \sec [c+d x]}{\left(a+a \cos [c+d x]\right)^2} d x$$

Optimal (type 3, 83 leaves, 4 steps):

$$\frac{A \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{a^2 d} - \frac{(4 A - B - 2 C) \operatorname{Sin}[c + d x]}{3 a^2 d (1 + \operatorname{Cos}[c + d x])} - \frac{(A - B + C) \operatorname{Sin}[c + d x]}{3 d (a + a \operatorname{Cos}[c + d x])^2}$$

Result (type 3, 221 leaves):

$$\begin{aligned} & - \left( \left( 4 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \right) (A + B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2) \right. \\ & \quad \left( 6 A \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^3 \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + \right. \\ & \quad \left. (A - B + C) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{d x}{2}\right] + 2(4 A - B - 2 C) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{d x}{2}\right] + (A - B + C) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Tan}\left[\frac{c}{2}\right] \right) \Bigg) / \\ & \quad \left( 3 a^2 d (1 + \operatorname{Cos}[c + d x])^2 (2 A + C + 2 B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[2(c + d x)]) \right) \end{aligned}$$

■ **Problem 352: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2) \operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Cos}[c + d x])^2} dx$$

Optimal (type 3, 109 leaves, 6 steps):

$$-\frac{(2 A - B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{a^2 d} + \frac{(10 A - 4 B + C) \operatorname{Tan}[c + d x]}{3 a^2 d} - \frac{(2 A - B) \operatorname{Tan}[c + d x]}{a^2 d (1 + \operatorname{Cos}[c + d x])} - \frac{(A - B + C) \operatorname{Tan}[c + d x]}{3 d (a + a \operatorname{Cos}[c + d x])^2}$$

Result (type 3, 321 leaves):

$$\begin{aligned} & \frac{1}{3 a^2 d (1 + \operatorname{Cos}[c + d x])^2 (2 A + C + 2 B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[2(c + d x)])} 4 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Cos}[c + d x]^2 \\ & (C + B \operatorname{Sec}[c + d x] + A \operatorname{Sec}[c + d x]^2) \left( (A - B + C) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{d x}{2}\right] + 2(7 A - 4 B + C) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{d x}{2}\right] + \right. \\ & \quad \left. 6 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^3 \left( (2 A - B) \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) \right) + \right. \\ & \quad \left. (A \operatorname{Sin}[d x]) / \left( \left( \operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left( \operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left( \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \left( \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \right) \right) \Bigg) + \\ & (A - B + C) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Tan}\left[\frac{c}{2}\right] \end{aligned}$$

■ **Problem 353: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2) \operatorname{Sec}[c + d x]^3}{(a + a \operatorname{Cos}[c + d x])^2} dx$$

Optimal (type 3, 165 leaves, 7 steps):

$$\frac{(7A - 4B + 2C) \operatorname{ArcTanh}[\sin[c + dx]]}{2a^2d} - \frac{2(8A - 5B + 2C) \tan[c + dx]}{3a^2d} +$$

$$\frac{(7A - 4B + 2C) \sec[c + dx] \tan[c + dx]}{2a^2d} - \frac{(8A - 5B + 2C) \sec[c + dx] \tan[c + dx]}{3a^2d(1 + \cos[c + dx])} - \frac{(A - B + C) \sec[c + dx] \tan[c + dx]}{3d(a + a \cos[c + dx])^2}$$

Result (type 3, 578 leaves):

$$-\frac{2(7A - 4B + 2C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{d(a + a \cos[c + dx])^2} + \frac{2(7A - 4B + 2C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{d(a + a \cos[c + dx])^2} +$$

$$\frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4}{d(a + a \cos[c + dx])^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4}{d(a + a \cos[c + dx])^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} -$$

$$\frac{4 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(2A \sin\left[\frac{1}{2}(c + dx)\right] - B \sin\left[\frac{1}{2}(c + dx)\right]\right)}{d(a + a \cos[c + dx])^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} - \frac{4 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(2A \sin\left[\frac{1}{2}(c + dx)\right] - B \sin\left[\frac{1}{2}(c + dx)\right]\right)}{d(a + a \cos[c + dx])^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)} -$$

$$\frac{2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec\left[\frac{1}{2}(c + dx)\right]^3 \left(A \sin\left[\frac{1}{2}(c + dx)\right] - B \sin\left[\frac{1}{2}(c + dx)\right] + C \sin\left[\frac{1}{2}(c + dx)\right]\right)}{3d(a + a \cos[c + dx])^2} -$$

$$\frac{4 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec\left[\frac{1}{2}(c + dx)\right] \left(10A \sin\left[\frac{1}{2}(c + dx)\right] - 7B \sin\left[\frac{1}{2}(c + dx)\right] + 4C \sin\left[\frac{1}{2}(c + dx)\right]\right)}{3d(a + a \cos[c + dx])^2}$$

■ **Problem 354: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^4}{(a + a \cos[c + dx])^2} dx$$

Optimal (type 3, 194 leaves, 7 steps):

$$-\frac{(10A - 7B + 4C) \operatorname{ArcTanh}[\sin[c + dx]]}{2a^2d} + \frac{(12A - 8B + 5C) \tan[c + dx]}{a^2d} - \frac{(10A - 7B + 4C) \sec[c + dx] \tan[c + dx]}{2a^2d} -$$

$$\frac{(10A - 7B + 4C) \sec[c + dx]^2 \tan[c + dx]}{3a^2d(1 + \cos[c + dx])} - \frac{(A - B + C) \sec[c + dx]^2 \tan[c + dx]}{3d(a + a \cos[c + dx])^2} + \frac{(12A - 8B + 5C) \tan[c + dx]^3}{3a^2d}$$

Result (type 3, 763 leaves):

$$\begin{aligned}
& \frac{2 (10 A - 7 B + 4 C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{d (a + a \operatorname{Cos}[c+dx])^2} - \\
& \frac{2 (10 A - 7 B + 4 C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{d (a + a \operatorname{Cos}[c+dx])^2} + \\
& \frac{(-5 A + 3 B) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4}{3 d (a + a \operatorname{Cos}[c+dx])^2 (\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^2} + \frac{2 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{3 d (a + a \operatorname{Cos}[c+dx])^2 (\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^3} + \\
& \frac{2 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{3 d (a + a \operatorname{Cos}[c+dx])^2 (\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^3} + \frac{(5 A - 3 B) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4}{3 d (a + a \operatorname{Cos}[c+dx])^2 (\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^2} + \\
& \frac{2 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (A \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - B \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + C \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])}{3 d (a + a \operatorname{Cos}[c+dx])^2} + \\
& \frac{4 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (11 A \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - 6 B \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 3 C \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])}{3 d (a + a \operatorname{Cos}[c+dx])^2 (\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])} + \\
& \frac{4 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (11 A \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - 6 B \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 3 C \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])}{3 d (a + a \operatorname{Cos}[c+dx])^2 (\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])} + \\
& \frac{4 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] (13 A \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - 10 B \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 7 C \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])}{3 d (a + a \operatorname{Cos}[c+dx])^2}
\end{aligned}$$

■ **Problem 355: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^4 (A + B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2)}{(a + a \operatorname{Cos}[c+dx])^3} dx$$

Optimal (type 3, 237 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(6 A - 13 B + 23 C) x}{2 a^3} + \frac{4 (9 A - 19 B + 34 C) \operatorname{Sin}[c+dx]}{5 a^3 d} - \frac{(6 A - 13 B + 23 C) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{2 a^3 d} - \frac{(A - B + C) \operatorname{Cos}[c+dx]^5 \operatorname{Sin}[c+dx]}{5 d (a + a \operatorname{Cos}[c+dx])^3} - \\
& \frac{(3 A - 8 B + 13 C) \operatorname{Cos}[c+dx]^4 \operatorname{Sin}[c+dx]}{15 a d (a + a \operatorname{Cos}[c+dx])^2} - \frac{(6 A - 13 B + 23 C) \operatorname{Cos}[c+dx]^3 \operatorname{Sin}[c+dx]}{3 d (a^3 + a^3 \operatorname{Cos}[c+dx])} - \frac{4 (9 A - 19 B + 34 C) \operatorname{Sin}[c+dx]^3}{15 a^3 d}
\end{aligned}$$

Result (type 3, 663 leaves):

1

$$480 a^3 d (1 + \cos[c + dx])^3$$

$$\begin{aligned} & \cos\left[\frac{1}{2}(c + dx)\right] \sec\left[\frac{c}{2}\right] \left( -600(6A - 13B + 23C) dx \cos\left[\frac{dx}{2}\right] - 600(6A - 13B + 23C) dx \cos\left[c + \frac{dx}{2}\right] - 1800A dx \cos\left[c + \frac{3dx}{2}\right] + \right. \\ & 3900B dx \cos\left[c + \frac{3dx}{2}\right] - 6900C dx \cos\left[c + \frac{3dx}{2}\right] - 1800A dx \cos\left[2c + \frac{3dx}{2}\right] + 3900B dx \cos\left[2c + \frac{3dx}{2}\right] - 6900C dx \cos\left[2c + \frac{3dx}{2}\right] - \\ & 360A dx \cos\left[2c + \frac{5dx}{2}\right] + 780B dx \cos\left[2c + \frac{5dx}{2}\right] - 1380C dx \cos\left[2c + \frac{5dx}{2}\right] - 360A dx \cos\left[3c + \frac{5dx}{2}\right] + 780B dx \cos\left[3c + \frac{5dx}{2}\right] - \\ & 1380C dx \cos\left[3c + \frac{5dx}{2}\right] + 7020A \sin\left[\frac{dx}{2}\right] - 12760B \sin\left[\frac{dx}{2}\right] + 20410C \sin\left[\frac{dx}{2}\right] - 4500A \sin\left[c + \frac{dx}{2}\right] + 7560B \sin\left[c + \frac{dx}{2}\right] - \\ & 11110C \sin\left[c + \frac{dx}{2}\right] + 4860A \sin\left[c + \frac{3dx}{2}\right] - 9230B \sin\left[c + \frac{3dx}{2}\right] + 15380C \sin\left[c + \frac{3dx}{2}\right] - 900A \sin\left[2c + \frac{3dx}{2}\right] + \\ & 930B \sin\left[2c + \frac{3dx}{2}\right] - 380C \sin\left[2c + \frac{3dx}{2}\right] + 1452A \sin\left[2c + \frac{5dx}{2}\right] - 2782B \sin\left[2c + \frac{5dx}{2}\right] + 4777C \sin\left[2c + \frac{5dx}{2}\right] + \\ & 300A \sin\left[3c + \frac{5dx}{2}\right] - 750B \sin\left[3c + \frac{5dx}{2}\right] + 1625C \sin\left[3c + \frac{5dx}{2}\right] + 60A \sin\left[3c + \frac{7dx}{2}\right] - 105B \sin\left[3c + \frac{7dx}{2}\right] + \\ & 230C \sin\left[3c + \frac{7dx}{2}\right] + 60A \sin\left[4c + \frac{7dx}{2}\right] - 105B \sin\left[4c + \frac{7dx}{2}\right] + 230C \sin\left[4c + \frac{7dx}{2}\right] + 15B \sin\left[4c + \frac{9dx}{2}\right] - \\ & \left. 20C \sin\left[4c + \frac{9dx}{2}\right] + 15B \sin\left[5c + \frac{9dx}{2}\right] - 20C \sin\left[5c + \frac{9dx}{2}\right] + 5C \sin\left[5c + \frac{11dx}{2}\right] + 5C \sin\left[6c + \frac{11dx}{2}\right] \right) \end{aligned}$$

■ **Problem 356: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^3 (A + B \cos[c + dx] + C \cos[c + dx]^2)}{(a + a \cos[c + dx])^3} dx$$

Optimal (type 3, 207 leaves, 4 steps):

$$\begin{aligned} & \frac{(2A - 6B + 13C)x}{2a^3} - \frac{2(11A - 36B + 76C) \sin[c + dx]}{15a^3 d} + \frac{(2A - 6B + 13C) \cos[c + dx] \sin[c + dx]}{2a^3 d} + \\ & \frac{(A - B + C) \cos[c + dx]^4 \sin[c + dx]}{5d (a + a \cos[c + dx])^3} - \frac{(A - 6B + 11C) \cos[c + dx]^3 \sin[c + dx]}{15ad (a + a \cos[c + dx])^2} - \frac{(11A - 36B + 76C) \cos[c + dx]^2 \sin[c + dx]}{15d (a^3 + a^3 \cos[c + dx])} \end{aligned}$$

Result (type 3, 565 leaves):

$$\frac{1}{480 a^3 d (1 + \cos [c + dx])^3} \cos \left[ \frac{1}{2} (c + dx) \right] \sec \left[ \frac{c}{2} \right] \left( 600 (2A - 6B + 13C) dx \cos \left[ \frac{dx}{2} \right] + 600 (2A - 6B + 13C) dx \cos \left[ c + \frac{dx}{2} \right] + 600 A dx \cos \left[ c + \frac{3dx}{2} \right] - 1800 B dx \cos \left[ c + \frac{3dx}{2} \right] + 3900 C dx \cos \left[ c + \frac{3dx}{2} \right] + 600 A dx \cos \left[ 2c + \frac{3dx}{2} \right] - 1800 B dx \cos \left[ 2c + \frac{3dx}{2} \right] + 3900 C dx \cos \left[ 2c + \frac{3dx}{2} \right] + 120 A dx \cos \left[ 2c + \frac{5dx}{2} \right] - 360 B dx \cos \left[ 2c + \frac{5dx}{2} \right] + 780 C dx \cos \left[ 2c + \frac{5dx}{2} \right] + 120 A dx \cos \left[ 3c + \frac{5dx}{2} \right] - 360 B dx \cos \left[ 3c + \frac{5dx}{2} \right] + 780 C dx \cos \left[ 3c + \frac{5dx}{2} \right] - 2960 A \sin \left[ \frac{dx}{2} \right] + 7020 B \sin \left[ \frac{dx}{2} \right] - 12760 C \sin \left[ \frac{dx}{2} \right] + 2160 A \sin \left[ c + \frac{dx}{2} \right] - 4500 B \sin \left[ c + \frac{dx}{2} \right] + 7560 C \sin \left[ c + \frac{dx}{2} \right] - 1840 A \sin \left[ c + \frac{3dx}{2} \right] + 4860 B \sin \left[ c + \frac{3dx}{2} \right] - 9230 C \sin \left[ c + \frac{3dx}{2} \right] + 720 A \sin \left[ 2c + \frac{3dx}{2} \right] - 900 B \sin \left[ 2c + \frac{3dx}{2} \right] + 930 C \sin \left[ 2c + \frac{3dx}{2} \right] - 512 A \sin \left[ 2c + \frac{5dx}{2} \right] + 1452 B \sin \left[ 2c + \frac{5dx}{2} \right] - 2782 C \sin \left[ 2c + \frac{5dx}{2} \right] + 300 B \sin \left[ 3c + \frac{5dx}{2} \right] - 750 C \sin \left[ 3c + \frac{5dx}{2} \right] + 60 B \sin \left[ 3c + \frac{7dx}{2} \right] - 105 C \sin \left[ 3c + \frac{7dx}{2} \right] + 60 B \sin \left[ 4c + \frac{7dx}{2} \right] - 105 C \sin \left[ 4c + \frac{7dx}{2} \right] + 15 C \sin \left[ 4c + \frac{9dx}{2} \right] + 15 C \sin \left[ 5c + \frac{9dx}{2} \right] \right)$$

■ **Problem 357: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + dx]^2 (A + B \cos [c + dx] + C \cos [c + dx]^2)}{(a + a \cos [c + dx])^3} dx$$

Optimal (type 3, 152 leaves, 7 steps):

$$\frac{(B - 3C)x}{a^3} + \frac{(2A - 7B + 27C) \sin [c + dx]}{15 a^3 d} - \frac{(A - B + C) \cos [c + dx]^3 \sin [c + dx]}{5 d (a + a \cos [c + dx])^3} + \frac{(A + 4B - 9C) \cos [c + dx]^2 \sin [c + dx]}{15 a d (a + a \cos [c + dx])^2} - \frac{(B - 3C) \sin [c + dx]}{d (a^3 + a^3 \cos [c + dx])}$$

Result (type 3, 423 leaves):

$$\frac{1}{120 a^3 d (1 + \cos [c + dx])^3} \cos \left[ \frac{1}{2} (c + dx) \right] \sec \left[ \frac{c}{2} \right] \left( 300 (B - 3C) dx \cos \left[ \frac{dx}{2} \right] + 300 (B - 3C) dx \cos \left[ c + \frac{dx}{2} \right] + 150 B dx \cos \left[ c + \frac{3dx}{2} \right] - 450 C dx \cos \left[ c + \frac{3dx}{2} \right] + 150 B dx \cos \left[ 2c + \frac{3dx}{2} \right] - 450 C dx \cos \left[ 2c + \frac{3dx}{2} \right] + 30 B dx \cos \left[ 2c + \frac{5dx}{2} \right] - 90 C dx \cos \left[ 2c + \frac{5dx}{2} \right] + 30 B dx \cos \left[ 3c + \frac{5dx}{2} \right] - 90 C dx \cos \left[ 3c + \frac{5dx}{2} \right] + 160 A \sin \left[ \frac{dx}{2} \right] - 740 B \sin \left[ \frac{dx}{2} \right] + 1755 C \sin \left[ \frac{dx}{2} \right] - 120 A \sin \left[ c + \frac{dx}{2} \right] + 540 B \sin \left[ c + \frac{dx}{2} \right] - 1125 C \sin \left[ c + \frac{dx}{2} \right] + 80 A \sin \left[ c + \frac{3dx}{2} \right] - 460 B \sin \left[ c + \frac{3dx}{2} \right] + 1215 C \sin \left[ c + \frac{3dx}{2} \right] - 60 A \sin \left[ 2c + \frac{3dx}{2} \right] + 180 B \sin \left[ 2c + \frac{3dx}{2} \right] - 225 C \sin \left[ 2c + \frac{3dx}{2} \right] + 28 A \sin \left[ 2c + \frac{5dx}{2} \right] - 128 B \sin \left[ 2c + \frac{5dx}{2} \right] + 363 C \sin \left[ 2c + \frac{5dx}{2} \right] + 75 C \sin \left[ 3c + \frac{5dx}{2} \right] + 15 C \sin \left[ 3c + \frac{7dx}{2} \right] + 15 C \sin \left[ 4c + \frac{7dx}{2} \right] \right)$$

■ **Problem 358: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x] \left(A+B \cos [c+d x]+C \cos [c+d x]^2\right)}{\left(a+a \cos [c+d x]\right)^3} d x$$

Optimal (type 3, 123 leaves, 5 steps):

$$\frac{C x}{a^3} - \frac{(A-B+C) \cos [c+d x]^2 \sin [c+d x]}{5 d\left(a+a \cos [c+d x]\right)^3} - \frac{(3 A+2 B-7 C) \sin [c+d x]}{15 a d\left(a+a \cos [c+d x]\right)^2} + \frac{(6 A+4 B-29 C) \sin [c+d x]}{15 d\left(a^3+a^3 \cos [c+d x]\right)}$$

Result (type 3, 289 leaves):

$$\frac{1}{480 a^3 d} \sec \left[\frac{c}{2}\right] \sec \left[\frac{1}{2}(c+d x)\right]^5 \left(150 C d x \cos \left[\frac{d x}{2}\right]+150 C d x \cos \left[c+\frac{d x}{2}\right]+75 C d x \cos \left[c+\frac{3 d x}{2}\right]+75 C d x \cos \left[2 c+\frac{3 d x}{2}\right]+15 C d x \cos \left[2 c+\frac{5 d x}{2}\right]+15 C d x \cos \left[3 c+\frac{5 d x}{2}\right]+30 A \sin \left[\frac{d x}{2}\right]+80 B \sin \left[\frac{d x}{2}\right]-370 C \sin \left[\frac{d x}{2}\right]-30 A \sin \left[c+\frac{d x}{2}\right]-60 B \sin \left[c+\frac{d x}{2}\right]+270 C \sin \left[c+\frac{d x}{2}\right]+30 A \sin \left[c+\frac{3 d x}{2}\right]+40 B \sin \left[c+\frac{3 d x}{2}\right]-230 C \sin \left[c+\frac{3 d x}{2}\right]-30 B \sin \left[2 c+\frac{3 d x}{2}\right]+90 C \sin \left[2 c+\frac{3 d x}{2}\right]+6 A \sin \left[2 c+\frac{5 d x}{2}\right]+14 B \sin \left[2 c+\frac{5 d x}{2}\right]-64 C \sin \left[2 c+\frac{5 d x}{2}\right]\right)$$

■ **Problem 360: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(A+B \cos [c+d x]+C \cos [c+d x]^2\right) \sec [c+d x]}{\left(a+a \cos [c+d x]\right)^3} d x$$

Optimal (type 3, 124 leaves, 5 steps):

$$\frac{A \operatorname{ArcTanh}[\sin [c+d x]]}{a^3 d} - \frac{(A-B+C) \sin [c+d x]}{5 d\left(a+a \cos [c+d x]\right)^3} - \frac{(7 A-2 B-3 C) \sin [c+d x]}{15 a d\left(a+a \cos [c+d x]\right)^2} - \frac{(22 A-2 B-3 C) \sin [c+d x]}{15 d\left(a^3+a^3 \cos [c+d x]\right)}$$

Result (type 3, 276 leaves):

$$-\left(\left(\left(A+B \cos [c+d x]+C \cos [c+d x]^2\right)\left(240 A \cos \left[\frac{1}{2}(c+d x)\right]^6\left(\operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]\right)-\operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]\right)+\cos \left[\frac{1}{2}(c+d x)\right] \sec \left[\frac{c}{2}\right]\left(5(29 A-4 B-3 C) \sin \left[\frac{d x}{2}\right]+15(-5 A+C) \sin \left[c+\frac{d x}{2}\right]+95 A \sin \left[c+\frac{3 d x}{2}\right]-10 B \sin \left[c+\frac{3 d x}{2}\right]-15 C \sin \left[c+\frac{3 d x}{2}\right]-15 A \sin \left[2 c+\frac{3 d x}{2}\right]+22 A \sin \left[2 c+\frac{5 d x}{2}\right]-2 B \sin \left[2 c+\frac{5 d x}{2}\right]-3 C \sin \left[2 c+\frac{5 d x}{2}\right]\right)\right)\right) / \left(15 a^3 d\left(1+\cos [c+d x]\right)^3\left(2 A+C+2 B \cos [c+d x]+C \cos [2(c+d x)]\right)\right)$$

■ **Problem 361: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^2}{(a + a \cos[c + dx])^3} dx$$

Optimal (type 3, 150 leaves, 7 steps):

$$-\frac{(3A - B) \operatorname{ArcTanh}[\sin[c + dx]]}{a^3 d} + \frac{2(36A - 11B + C) \tan[c + dx]}{15a^3 d} - \frac{(A - B + C) \tan[c + dx]}{5d(a + a \cos[c + dx])^3} - \frac{(9A - 4B - C) \tan[c + dx]}{15ad(a + a \cos[c + dx])^2} - \frac{(3A - B) \tan[c + dx]}{d(a^3 + a^3 \cos[c + dx])}$$

Result (type 3, 839 leaves):

$$\begin{aligned} & \frac{1}{a^3} \left( \left( 16(3A - B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \cos[c + dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + B \sec[c + dx] + A \sec[c + dx]^2) \right) / \right. \\ & \quad \left. (d(1 + \cos[c + dx])^3 (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx])) - \right. \\ & \quad \left. \left( 16(3A - B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \cos[c + dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + B \sec[c + dx] + A \sec[c + dx]^2) \right) / \right. \\ & \quad \left. (d(1 + \cos[c + dx])^3 (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx])) + \right. \\ & \quad \frac{1}{60d(1 + \cos[c + dx])^3 (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx])} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c + dx] \sec\left[\frac{c}{2}\right] \sec[c] \\ & \quad (C + B \sec[c + dx] + A \sec[c + dx]^2) \left( -255A \sin\left[\frac{dx}{2}\right] + 160B \sin\left[\frac{dx}{2}\right] - 20C \sin\left[\frac{dx}{2}\right] + 567A \sin\left[\frac{3dx}{2}\right] - 167B \sin\left[\frac{3dx}{2}\right] + \right. \\ & \quad 22C \sin\left[\frac{3dx}{2}\right] - 600A \sin\left[c - \frac{dx}{2}\right] + 170B \sin\left[c - \frac{dx}{2}\right] - 10C \sin\left[c - \frac{dx}{2}\right] + 375A \sin\left[c + \frac{dx}{2}\right] - 170B \sin\left[c + \frac{dx}{2}\right] + \\ & \quad 10C \sin\left[c + \frac{dx}{2}\right] - 480A \sin\left[2c + \frac{dx}{2}\right] + 160B \sin\left[2c + \frac{dx}{2}\right] - 20C \sin\left[2c + \frac{dx}{2}\right] - 60A \sin\left[c + \frac{3dx}{2}\right] + 75B \sin\left[c + \frac{3dx}{2}\right] + \\ & \quad 402A \sin\left[2c + \frac{3dx}{2}\right] - 167B \sin\left[2c + \frac{3dx}{2}\right] + 22C \sin\left[2c + \frac{3dx}{2}\right] - 225A \sin\left[3c + \frac{3dx}{2}\right] + 75B \sin\left[3c + \frac{3dx}{2}\right] + \\ & \quad 315A \sin\left[c + \frac{5dx}{2}\right] - 95B \sin\left[c + \frac{5dx}{2}\right] + 10C \sin\left[c + \frac{5dx}{2}\right] + 30A \sin\left[2c + \frac{5dx}{2}\right] + 15B \sin\left[2c + \frac{5dx}{2}\right] + 240A \sin\left[3c + \frac{5dx}{2}\right] - \\ & \quad 95B \sin\left[3c + \frac{5dx}{2}\right] + 10C \sin\left[3c + \frac{5dx}{2}\right] - 45A \sin\left[4c + \frac{5dx}{2}\right] + 15B \sin\left[4c + \frac{5dx}{2}\right] + 72A \sin\left[2c + \frac{7dx}{2}\right] - \\ & \quad \left. \left. 22B \sin\left[2c + \frac{7dx}{2}\right] + 2C \sin\left[2c + \frac{7dx}{2}\right] + 15A \sin\left[3c + \frac{7dx}{2}\right] + 57A \sin\left[4c + \frac{7dx}{2}\right] - 22B \sin\left[4c + \frac{7dx}{2}\right] + 2C \sin\left[4c + \frac{7dx}{2}\right] \right) \right) \end{aligned}$$

■ **Problem 365: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^3 (A + B \cos[c + dx] + C \cos[c + dx]^2)}{(a + a \cos[c + dx])^4} dx$$

Optimal (type 3, 195 leaves, 8 steps):



$$\frac{(B-4C)x}{a^4} + \frac{(6A-55B+244C)\sin[c+dx]}{105a^4d} + \frac{(3A+25B-88C)\cos[c+dx]^2\sin[c+dx]}{105a^4d(1+\cos[c+dx])^2} -$$

$$\frac{(B-4C)\sin[c+dx]}{a^4d(1+\cos[c+dx])} - \frac{(A-B+C)\cos[c+dx]^4\sin[c+dx]}{7d(a+a\cos[c+dx])^4} + \frac{(2A+5B-12C)\cos[c+dx]^3\sin[c+dx]}{35ad(a+a\cos[c+dx])^3}$$

Result (type 3, 571 leaves):

$$\frac{1}{1680a^4d(1+\cos[c+dx])^4}$$

$$\cos\left[\frac{1}{2}(c+dx)\right] \sec\left[\frac{c}{2}\right] \left( 7350(B-4C)dx\cos\left[\frac{dx}{2}\right] + 7350(B-4C)dx\cos\left[c+\frac{dx}{2}\right] + 4410Bdx\cos\left[c+\frac{3dx}{2}\right] - 17640Cdx\cos\left[c+\frac{3dx}{2}\right] + \right.$$

$$4410Bdx\cos\left[2c+\frac{3dx}{2}\right] - 17640Cdx\cos\left[2c+\frac{3dx}{2}\right] + 1470Bdx\cos\left[2c+\frac{5dx}{2}\right] - 5880Cdx\cos\left[2c+\frac{5dx}{2}\right] +$$

$$1470Bdx\cos\left[3c+\frac{5dx}{2}\right] - 5880Cdx\cos\left[3c+\frac{5dx}{2}\right] + 210Bdx\cos\left[3c+\frac{7dx}{2}\right] - 840Cdx\cos\left[3c+\frac{7dx}{2}\right] +$$

$$210Bdx\cos\left[4c+\frac{7dx}{2}\right] - 840Cdx\cos\left[4c+\frac{7dx}{2}\right] + 2520A\sin\left[\frac{dx}{2}\right] - 19880B\sin\left[\frac{dx}{2}\right] + 60830C\sin\left[\frac{dx}{2}\right] - 2520A\sin\left[c+\frac{dx}{2}\right] +$$

$$16520B\sin\left[c+\frac{dx}{2}\right] - 46130C\sin\left[c+\frac{dx}{2}\right] + 1764A\sin\left[c+\frac{3dx}{2}\right] - 14280B\sin\left[c+\frac{3dx}{2}\right] + 46116C\sin\left[c+\frac{3dx}{2}\right] -$$

$$1260A\sin\left[2c+\frac{3dx}{2}\right] + 7560B\sin\left[2c+\frac{3dx}{2}\right] - 18060C\sin\left[2c+\frac{3dx}{2}\right] + 588A\sin\left[2c+\frac{5dx}{2}\right] - 5600B\sin\left[2c+\frac{5dx}{2}\right] +$$

$$19292C\sin\left[2c+\frac{5dx}{2}\right] - 420A\sin\left[3c+\frac{5dx}{2}\right] + 1680B\sin\left[3c+\frac{5dx}{2}\right] - 2100C\sin\left[3c+\frac{5dx}{2}\right] + 144A\sin\left[3c+\frac{7dx}{2}\right] -$$

$$\left. 1040B\sin\left[3c+\frac{7dx}{2}\right] + 3791C\sin\left[3c+\frac{7dx}{2}\right] + 735C\sin\left[4c+\frac{7dx}{2}\right] + 105C\sin\left[4c+\frac{9dx}{2}\right] + 105C\sin\left[5c+\frac{9dx}{2}\right] \right)$$

■ **Problem 366: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^2 (A+B\cos[c+dx]+C\cos[c+dx]^2)}{(a+a\cos[c+dx])^4} dx$$

Optimal (type 3, 164 leaves, 6 steps):

$$\frac{Cx}{a^4} - \frac{(8A+6B-55C)\sin[c+dx]}{105a^4d(1+\cos[c+dx])^2} + \frac{(16A+12B-215C)\sin[c+dx]}{105a^4d(1+\cos[c+dx])} -$$

$$\frac{(A-B+C)\cos[c+dx]^3\sin[c+dx]}{7d(a+a\cos[c+dx])^4} + \frac{(4A+3B-10C)\cos[c+dx]^2\sin[c+dx]}{35ad(a+a\cos[c+dx])^3}$$

Result (type 3, 405 leaves):

$$\frac{1}{13440 a^4 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^7$$

$$\left(3675 C dx \cos\left[\frac{dx}{2}\right] + 3675 C dx \cos\left[c + \frac{dx}{2}\right] + 2205 C dx \cos\left[c + \frac{3dx}{2}\right] + 2205 C dx \cos\left[2c + \frac{3dx}{2}\right] + 735 C dx \cos\left[2c + \frac{5dx}{2}\right] + \right.$$

$$\left.735 C dx \cos\left[3c + \frac{5dx}{2}\right] + 105 C dx \cos\left[3c + \frac{7dx}{2}\right] + 105 C dx \cos\left[4c + \frac{7dx}{2}\right] + 560 A \sin\left[\frac{dx}{2}\right] + 1260 B \sin\left[\frac{dx}{2}\right] - 9940 C \sin\left[\frac{dx}{2}\right] - \right.$$

$$\left.350 A \sin\left[c + \frac{dx}{2}\right] - 1260 B \sin\left[c + \frac{dx}{2}\right] + 8260 C \sin\left[c + \frac{dx}{2}\right] + 336 A \sin\left[c + \frac{3dx}{2}\right] + 882 B \sin\left[c + \frac{3dx}{2}\right] - 7140 C \sin\left[c + \frac{3dx}{2}\right] - \right.$$

$$\left.210 A \sin\left[2c + \frac{3dx}{2}\right] - 630 B \sin\left[2c + \frac{3dx}{2}\right] + 3780 C \sin\left[2c + \frac{3dx}{2}\right] + 182 A \sin\left[2c + \frac{5dx}{2}\right] + 294 B \sin\left[2c + \frac{5dx}{2}\right] - \right.$$

$$\left.2800 C \sin\left[2c + \frac{5dx}{2}\right] - 210 B \sin\left[3c + \frac{5dx}{2}\right] + 840 C \sin\left[3c + \frac{5dx}{2}\right] + 26 A \sin\left[3c + \frac{7dx}{2}\right] + 72 B \sin\left[3c + \frac{7dx}{2}\right] - 520 C \sin\left[3c + \frac{7dx}{2}\right]\right)$$

■ **Problem 369: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]}{(a + a \cos[c + dx])^4} dx$$

Optimal (type 3, 157 leaves, 6 steps):

$$\frac{A \operatorname{ArcTanh}[\sin[c + dx]]}{a^4 d} - \frac{(55A - 6B - 8C) \sin[c + dx]}{105 a^4 d (1 + \cos[c + dx])^2} - \frac{2(80A - 3B - 4C) \sin[c + dx]}{105 a^4 d (1 + \cos[c + dx])} - \frac{(A - B + C) \sin[c + dx]}{7 d (a + a \cos[c + dx])^4} - \frac{(10A - 3B - 4C) \sin[c + dx]}{35 a d (a + a \cos[c + dx])^3}$$

Result (type 3, 334 leaves):

$$- \left( \left( (A + B \cos[c + dx] + C \cos[c + dx]^2) \right. \right.$$

$$\left. \left( 6720 A \cos\left[\frac{1}{2}(c + dx)\right]^8 \left( \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \right.$$

$$\left. \cos\left[\frac{1}{2}(c + dx)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( 70(49A - 3B - 2C) \sin\left[\frac{dx}{2}\right] - 70(31A - 2C) \sin\left[c + \frac{dx}{2}\right] + 2625 A \sin\left[c + \frac{3dx}{2}\right] - \right.$$

$$\left. 126 B \sin\left[c + \frac{3dx}{2}\right] - 168 C \sin\left[c + \frac{3dx}{2}\right] - 735 A \sin\left[2c + \frac{3dx}{2}\right] + 1015 A \sin\left[2c + \frac{5dx}{2}\right] - 42 B \sin\left[2c + \frac{5dx}{2}\right] - \right.$$

$$\left. 56 C \sin\left[2c + \frac{5dx}{2}\right] - 105 A \sin\left[3c + \frac{5dx}{2}\right] + 160 A \sin\left[3c + \frac{7dx}{2}\right] - 6 B \sin\left[3c + \frac{7dx}{2}\right] - 8 C \sin\left[3c + \frac{7dx}{2}\right] \right) \right) /$$

$$\left( 210 a^4 d (1 + \cos[c + dx])^4 (2A + C + 2B \cos[c + dx] + C \cos[2(c + dx)]) \right)$$

■ **Problem 370: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^2}{(a + a \cos[c + dx])^4} dx$$

Optimal (type 3, 185 leaves, 8 steps):

$$- \frac{(4A - B) \operatorname{ArcTanh}[\sin[c + dx]]}{a^4 d} + \frac{2(332A - 80B + 3C) \tan[c + dx]}{105 a^4 d (1 + \cos[c + dx])^2} - \frac{(4A - B) \tan[c + dx]}{a^4 d (1 + \cos[c + dx])} - \frac{(A - B + C) \tan[c + dx]}{7 d (a + a \cos[c + dx])^4} - \frac{(12A - 5B - 2C) \tan[c + dx]}{35 a d (a + a \cos[c + dx])^3}$$

Result (type 3, 1190 leaves):

$$\frac{1}{a^4} \left( \left( 32(4A - B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \cos[c + dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + B \sec[c + dx] + A \sec[c + dx]^2) \right) / \right. \\ \left. (d(1 + \cos[c + dx])^4 (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx])) - \right. \\ \left( 32(4A - B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \cos[c + dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + B \sec[c + dx] + A \sec[c + dx]^2) \right) / \\ \left. (d(1 + \cos[c + dx])^4 (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx])) + \right. \\ \left( 4 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx]^2 \sec\left[\frac{c}{2}\right] (C + B \sec[c + dx] + A \sec[c + dx]^2) \left( A \sin\left[\frac{c}{2}\right] - B \sin\left[\frac{c}{2}\right] + C \sin\left[\frac{c}{2}\right] \right) \right) / \\ \left. (7 d (1 + \cos[c + dx])^4 (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx])) + \right. \\ \left( 8 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \cos[c + dx]^2 \sec\left[\frac{c}{2}\right] (C + B \sec[c + dx] + A \sec[c + dx]^2) \left( 17A \sin\left[\frac{c}{2}\right] - 10B \sin\left[\frac{c}{2}\right] + 3C \sin\left[\frac{c}{2}\right] \right) \right) / \\ \left. (35 d (1 + \cos[c + dx])^4 (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx])) + \right. \\ \left( 16 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \cos[c + dx]^2 \sec\left[\frac{c}{2}\right] (C + B \sec[c + dx] + A \sec[c + dx]^2) \left( 139A \sin\left[\frac{c}{2}\right] - 55B \sin\left[\frac{c}{2}\right] + 6C \sin\left[\frac{c}{2}\right] \right) \right) / \\ \left. (105 d (1 + \cos[c + dx])^4 (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx])) + \right. \\ \left( 4 \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c + dx]^2 \sec\left[\frac{c}{2}\right] (C + B \sec[c + dx] + A \sec[c + dx]^2) \left( A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right] \right) \right) / \\ \left. (7 d (1 + \cos[c + dx])^4 (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx])) + \right. \\ \left( 8 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 \sec\left[\frac{c}{2}\right] (C + B \sec[c + dx] + A \sec[c + dx]^2) \left( 17A \sin\left[\frac{dx}{2}\right] - 10B \sin\left[\frac{dx}{2}\right] + 3C \sin\left[\frac{dx}{2}\right] \right) \right) / \\ \left. (35 d (1 + \cos[c + dx])^4 (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx])) + \right. \\ \left( 32 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \cos[c + dx]^2 \sec\left[\frac{c}{2}\right] (C + B \sec[c + dx] + A \sec[c + dx]^2) \left( 559A \sin\left[\frac{dx}{2}\right] - 160B \sin\left[\frac{dx}{2}\right] + 6C \sin\left[\frac{dx}{2}\right] \right) \right) / \\ \left. (105 d (1 + \cos[c + dx])^4 (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx])) + \right. \\ \left( 16 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx]^2 \sec\left[\frac{c}{2}\right] (C + B \sec[c + dx] + A \sec[c + dx]^2) \left( 139A \sin\left[\frac{dx}{2}\right] - 55B \sin\left[\frac{dx}{2}\right] + 6C \sin\left[\frac{dx}{2}\right] \right) \right) / \\ \left. (105 d (1 + \cos[c + dx])^4 (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx])) + \right. \\ \left. \frac{32A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \cos[c + dx] \sec[c] (C + B \sec[c + dx] + A \sec[c + dx]^2) \sin[dx]}{d(1 + \cos[c + dx])^4 (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx])} \right)$$

- **Problem 377: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos[c + dx]} (A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx] dx$$

Optimal (type 3, 100 leaves, 4 steps):

$$\frac{2\sqrt{a} A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} + \frac{2a(3B+C) \sin[c+dx]}{3d\sqrt{a+a \cos[c+dx]}} + \frac{2C\sqrt{a+a \cos[c+dx]} \sin[c+dx]}{3d}$$

Result (type 3, 1495 leaves):

$$\begin{aligned} & - \left( \left( \left( \frac{1}{4} - \frac{i}{4} \right) A (1 + e^{ic}) \left( \sqrt{2} - (1-i) e^{\frac{ic}{2}} + (16-16i) e^{\frac{3ic}{2}+idx} + (20+20i) \sqrt{2} e^{2ic+\frac{3idx}{2}} - (34-34i) e^{\frac{5ic}{2}+2idx} - (20+20i) \sqrt{2} e^{3ic+\frac{5idx}{2}} + \right. \right. \right. \\ & \quad (16-16i) e^{\frac{7ic}{2}+3idx} + (4+4i) \sqrt{2} e^{4ic+\frac{7idx}{2}} - (1-i) e^{\frac{9ic}{2}+4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + \\ & \quad \left. \left. 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4+4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \right) / \\ & \quad \left( \left( (-1-i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right) - \\ & \quad \frac{i A \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]}{\sqrt{2} d} - \\ & \quad \frac{i A \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]}{\sqrt{2} d} - \\ & \quad \frac{A \sqrt{a(1+\cos[c+dx])} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]}{2\sqrt{2} d} - \\ & \quad \frac{A \sqrt{a(1+\cos[c+dx])} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]}{2\sqrt{2} d} + \\ & \quad \frac{(2B+C) \cos\left[\frac{dx}{2}\right] \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{d} - \\ & \quad \frac{2i A \operatorname{ArcTan}\left[\frac{2i \cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}\right] \sqrt{a(1+\cos[c+dx])} \operatorname{Cot}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} + \end{aligned}$$

$$\frac{1}{d \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right)}$$

$$\sqrt{2} A \sqrt{a (1 + \cos[c + dx])} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]$$

$$\left( -dx \cos\left[\frac{c}{2}\right] + 2 \operatorname{Log}\left[\sqrt{2} + 2 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + \frac{4 i \sqrt{2} \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right] - i (-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right) +$$

$$\frac{C \cos\left[\frac{3dx}{2}\right] \sqrt{a (1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{3d} +$$

$$\frac{(2B + C) \cos\left[\frac{c}{2}\right] \sqrt{a (1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{d} +$$

$$\frac{C \cos\left[\frac{3c}{2}\right] \sqrt{a (1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{3dx}{2}\right]}{3d}$$

- **Problem 378: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos[c + dx]} (A + B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^2 dx$$

Optimal (type 3, 98 leaves, 4 steps):

$$\frac{\sqrt{a} (A + 2B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right]}{d} - \frac{a (A - 2C) \sin[c + dx]}{d \sqrt{a + a \cos[c + dx]}} + \frac{A \sqrt{a + a \cos[c + dx]} \tan[c + dx]}{d}$$

Result (type 3, 527 leaves):

$$\frac{1}{d} \left( \frac{1}{32} + \frac{i}{32} \right) \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec} \left[ \frac{1}{2} (c + dx) \right] \left( \frac{2i\sqrt{2} \left( (-3 + i) + \sqrt{2} \right) \left( (1 + i) + \sqrt{2} \right) (A + 2B) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c + dx) \right] - (-1 + \sqrt{2}) \sin \left[ \frac{1}{4} (c + dx) \right]}{(1 + \sqrt{2}) \cos \left[ \frac{1}{4} (c + dx) \right] - \sin \left[ \frac{1}{4} (c + dx) \right]} \right]}{i + \sqrt{2}} - \right.$$

$$\frac{2\sqrt{2} \left( (-1 + i) + \sqrt{2} \right) \left( (3 + i) + \sqrt{2} \right) (A + 2B) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c + dx) \right] - (1 + \sqrt{2}) \sin \left[ \frac{1}{4} (c + dx) \right]}{(-1 + \sqrt{2}) \cos \left[ \frac{1}{4} (c + dx) \right] - \sin \left[ \frac{1}{4} (c + dx) \right]} \right]}{i + \sqrt{2}} +$$

$$\frac{(4 + 4i) \left( -2i + \sqrt{2} \right) (A + 2B) \operatorname{Log} \left[ \sqrt{2} + 2 \sin \left[ \frac{1}{2} (c + dx) \right] \right]}{i + \sqrt{2}} +$$

$$\frac{i\sqrt{2} \left( (-1 + i) + \sqrt{2} \right) \left( (3 + i) + \sqrt{2} \right) (A + 2B) \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + dx) \right] \right]}{i + \sqrt{2}} +$$

$$\frac{\sqrt{2} \left( (-3 + i) + \sqrt{2} \right) \left( (1 + i) + \sqrt{2} \right) (A + 2B) \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + dx) \right] \right]}{i + \sqrt{2}} +$$

$$\left. \frac{(8 - 8i)A}{\cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right]} + (32 - 32i)C \sin \left[ \frac{1}{2} (c + dx) \right] - \frac{(8 - 8i)A}{\cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right]} \right)$$

- **Problem 379: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos[c + dx]} (A + B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^3 dx$$

Optimal (type 3, 117 leaves, 4 steps):

$$\frac{\sqrt{a} (3A + 4B + 8C) \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}} \right]}{4d} + \frac{a(A + 4B) \tan[c + dx]}{4d \sqrt{a + a \cos[c + dx]}} + \frac{A \sqrt{a + a \cos[c + dx]} \operatorname{Sec}[c + dx] \tan[c + dx]}{2d}$$

Result (type 3, 627 leaves):

$$\frac{1}{d} \left( \frac{1}{128} + \frac{i}{128} \right) \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec} \left[ \frac{1}{2} (c + dx) \right]$$

$$\left( \frac{2i\sqrt{2} \left( (-3 + i) + \sqrt{2} \right) \left( (1 + i) + \sqrt{2} \right) (3A + 4B + 8C) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c + dx) \right] - (-1 + \sqrt{2}) \sin \left[ \frac{1}{4} (c + dx) \right]}{(1 + \sqrt{2}) \cos \left[ \frac{1}{4} (c + dx) \right] - \sin \left[ \frac{1}{4} (c + dx) \right]} \right]}{i + \sqrt{2}} - \right.$$

$$\frac{2\sqrt{2} \left( (-1 + i) + \sqrt{2} \right) \left( (3 + i) + \sqrt{2} \right) (3A + 4B + 8C) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c + dx) \right] - (1 + \sqrt{2}) \sin \left[ \frac{1}{4} (c + dx) \right]}{(-1 + \sqrt{2}) \cos \left[ \frac{1}{4} (c + dx) \right] - \sin \left[ \frac{1}{4} (c + dx) \right]} \right]}{i + \sqrt{2}} +$$

$$\frac{(4 + 4i) \left( -2i + \sqrt{2} \right) (3A + 4B + 8C) \operatorname{Log} \left[ \sqrt{2} + 2 \sin \left[ \frac{1}{2} (c + dx) \right] \right]}{i + \sqrt{2}} + \frac{1}{i + \sqrt{2}}$$

$$i\sqrt{2} \left( (-1 + i) + \sqrt{2} \right) \left( (3 + i) + \sqrt{2} \right) (3A + 4B + 8C) \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + dx) \right] \right] + \frac{1}{i + \sqrt{2}}$$

$$\sqrt{2} \left( (-3 + i) + \sqrt{2} \right) \left( (1 + i) + \sqrt{2} \right) (3A + 4B + 8C) \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + dx) \right] \right] + \frac{(8 - 8i)(3A + 4B)}{\cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right]} +$$

$$\left. \frac{(16 - 16i)A \sin \left[ \frac{1}{2} (c + dx) \right]}{\left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right)^2} + \frac{(16 - 16i)A \sin \left[ \frac{1}{2} (c + dx) \right]}{\left( \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right)^2} - \frac{(8 - 8i)(3A + 4B)}{\cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right]} \right)$$

■ **Problem 380: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos[c + dx]} (A + B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^4 dx$$

Optimal (type 3, 163 leaves, 5 steps):

$$\frac{\sqrt{a} (5A + 6B + 8C) \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}} \right]}{8d} + \frac{a (5A + 6B + 8C) \tan[c + dx]}{8d \sqrt{a + a \cos[c + dx]}} +$$

$$\frac{a (A + 6B) \operatorname{Sec}[c + dx] \tan[c + dx]}{12d \sqrt{a + a \cos[c + dx]}} + \frac{A \sqrt{a + a \cos[c + dx]} \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3d}$$

Result (type 3, 1178 leaves):

$$\begin{aligned}
& - \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{64} + \frac{i}{64} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (15 + 5i) A + 5\sqrt{2} A + (18 + 6i) B + 6\sqrt{2} B + (24 + 8i) C + 8\sqrt{2} C \right) \\
& \quad \text{ArcTan} \left[ \frac{\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c + dx)\right]}{-\cos\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]} \right] \sqrt{a(1 + \cos[c + dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right] - \\
& \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{64} - \frac{i}{64} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-15 + 5i) A + 5\sqrt{2} A - (18 - 6i) B + 6\sqrt{2} B - (24 - 8i) C + 8\sqrt{2} C \right) \\
& \quad \text{ArcTan} \left[ \frac{\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c + dx)\right]}{\cos\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]} \right] \sqrt{a(1 + \cos[c + dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right] + \frac{1}{32(i + \sqrt{2}) d} \\
& \left( 10A + 5i\sqrt{2}A + 12B + 6i\sqrt{2}B + 16C + 8i\sqrt{2}C \right) \sqrt{a(1 + \cos[c + dx])} \text{Log} \left[ \sqrt{2} + 2 \sin\left[\frac{1}{2}(c + dx)\right] \right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] - \\
& \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{128} - \frac{i}{128} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (15 + 5i) A + 5\sqrt{2} A + (18 + 6i) B + 6\sqrt{2} B + (24 + 8i) C + 8\sqrt{2} C \right) \\
& \quad \sqrt{a(1 + \cos[c + dx])} \text{Log} \left[ 2 - \sqrt{2} \cos\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c + dx)\right] \right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] + \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \\
& \left( \frac{1}{128} + \frac{i}{128} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-15 + 5i) A + 5\sqrt{2} A - (18 - 6i) B + 6\sqrt{2} B - (24 - 8i) C + 8\sqrt{2} C \right) \sqrt{a(1 + \cos[c + dx])} \\
& \quad \text{Log} \left[ 2 + \sqrt{2} \cos\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c + dx)\right] \right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] + \frac{A \sqrt{a(1 + \cos[c + dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]}{12d \left( \cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \frac{(5A + 6B + 8C) \sqrt{a(1 + \cos[c + dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]}{16d \left( \cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)} - \frac{A \sqrt{a(1 + \cos[c + dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]}{12d \left( \cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \frac{(-5A - 6B - 8C) \sqrt{a(1 + \cos[c + dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]}{16d \left( \cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)} + \frac{\sqrt{a(1 + \cos[c + dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{1}{2}(c + dx)\right] + 2B \sin\left[\frac{1}{2}(c + dx)\right])}{8d \left( \cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^2} + \\
& \frac{\sqrt{a(1 + \cos[c + dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{1}{2}(c + dx)\right] + 2B \sin\left[\frac{1}{2}(c + dx)\right])}{8d \left( \cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}
\end{aligned}$$

■ **Problem 381: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos[c + dx]} (A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^5 dx$$

Optimal (type 3, 209 leaves, 6 steps):



$$\frac{\sqrt{a} (35 A + 40 B + 48 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{64 d} + \frac{a (35 A + 40 B + 48 C) \tan[c+dx]}{64 d \sqrt{a+a \cos[c+dx]}} +$$

$$\frac{a (35 A + 40 B + 48 C) \sec[c+dx] \tan[c+dx]}{96 d \sqrt{a+a \cos[c+dx]}} + \frac{a (A + 8 B) \sec[c+dx]^2 \tan[c+dx]}{24 d \sqrt{a+a \cos[c+dx]}} + \frac{A \sqrt{a+a \cos[c+dx]} \sec[c+dx]^3 \tan[c+dx]}{4 d}$$

Result (type 3, 1356 leaves):

$$\begin{aligned}
& - \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{512} + \frac{i}{512} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (105 + 35 i) A + 35 \sqrt{2} A + (120 + 40 i) B + 40 \sqrt{2} B + (144 + 48 i) C + 48 \sqrt{2} C \right) \\
& \quad \text{ArcTan} \left[ \frac{\text{Cos} \left[ \frac{1}{4} (c + dx) \right] - \text{Sin} \left[ \frac{1}{4} (c + dx) \right] - \sqrt{2} \text{Sin} \left[ \frac{1}{4} (c + dx) \right]}{-\text{Cos} \left[ \frac{1}{4} (c + dx) \right] + \sqrt{2} \text{Cos} \left[ \frac{1}{4} (c + dx) \right] - \text{Sin} \left[ \frac{1}{4} (c + dx) \right]} \right] \sqrt{a (1 + \text{Cos}[c + dx])} \text{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] - \\
& \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{512} - \frac{i}{512} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-105 + 35 i) A + 35 \sqrt{2} A - (120 - 40 i) B + 40 \sqrt{2} B - (144 - 48 i) C + 48 \sqrt{2} C \right) \\
& \quad \text{ArcTan} \left[ \frac{\text{Cos} \left[ \frac{1}{4} (c + dx) \right] + \text{Sin} \left[ \frac{1}{4} (c + dx) \right] - \sqrt{2} \text{Sin} \left[ \frac{1}{4} (c + dx) \right]}{\text{Cos} \left[ \frac{1}{4} (c + dx) \right] + \sqrt{2} \text{Cos} \left[ \frac{1}{4} (c + dx) \right] - \text{Sin} \left[ \frac{1}{4} (c + dx) \right]} \right] \sqrt{a (1 + \text{Cos}[c + dx])} \text{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] + \frac{1}{256 (i + \sqrt{2}) d} \\
& \left( 70 A + 35 i \sqrt{2} A + 80 B + 40 i \sqrt{2} B + 96 C + 48 i \sqrt{2} C \right) \sqrt{a (1 + \text{Cos}[c + dx])} \text{Log} \left[ \sqrt{2} + 2 \text{Sin} \left[ \frac{1}{2} (c + dx) \right] \right] \text{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] - \\
& \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{1024} - \frac{i}{1024} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (105 + 35 i) A + 35 \sqrt{2} A + (120 + 40 i) B + 40 \sqrt{2} B + (144 + 48 i) C + 48 \sqrt{2} C \right) \\
& \quad \sqrt{a (1 + \text{Cos}[c + dx])} \text{Log} \left[ 2 - \sqrt{2} \text{Cos} \left[ \frac{1}{2} (c + dx) \right] - \sqrt{2} \text{Sin} \left[ \frac{1}{2} (c + dx) \right] \right] \text{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] + \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \\
& \left( \frac{1}{1024} + \frac{i}{1024} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-105 + 35 i) A + 35 \sqrt{2} A - (120 - 40 i) B + 40 \sqrt{2} B - (144 - 48 i) C + 48 \sqrt{2} C \right) \\
& \quad \sqrt{a (1 + \text{Cos}[c + dx])} \text{Log} \left[ 2 + \sqrt{2} \text{Cos} \left[ \frac{1}{2} (c + dx) \right] - \sqrt{2} \text{Sin} \left[ \frac{1}{2} (c + dx) \right] \right] \text{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] + \\
& \frac{(7 A + 8 B) \sqrt{a (1 + \text{Cos}[c + dx])} \text{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{96 d (\text{Cos} \left[ \frac{1}{2} (c + dx) \right] - \text{Sin} \left[ \frac{1}{2} (c + dx) \right])^3} + \frac{(35 A + 40 B + 48 C) \sqrt{a (1 + \text{Cos}[c + dx])} \text{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{128 d (\text{Cos} \left[ \frac{1}{2} (c + dx) \right] - \text{Sin} \left[ \frac{1}{2} (c + dx) \right])} + \\
& \frac{A \sqrt{a (1 + \text{Cos}[c + dx])} \text{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] \text{Sin} \left[ \frac{1}{2} (c + dx) \right]}{16 d (\text{Cos} \left[ \frac{1}{2} (c + dx) \right] - \text{Sin} \left[ \frac{1}{2} (c + dx) \right])^4} + \frac{A \sqrt{a (1 + \text{Cos}[c + dx])} \text{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] \text{Sin} \left[ \frac{1}{2} (c + dx) \right]}{16 d (\text{Cos} \left[ \frac{1}{2} (c + dx) \right] + \text{Sin} \left[ \frac{1}{2} (c + dx) \right])^4} + \\
& \frac{(-7 A - 8 B) \sqrt{a (1 + \text{Cos}[c + dx])} \text{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{96 d (\text{Cos} \left[ \frac{1}{2} (c + dx) \right] + \text{Sin} \left[ \frac{1}{2} (c + dx) \right])^3} + \frac{(-35 A - 40 B - 48 C) \sqrt{a (1 + \text{Cos}[c + dx])} \text{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]}{128 d (\text{Cos} \left[ \frac{1}{2} (c + dx) \right] + \text{Sin} \left[ \frac{1}{2} (c + dx) \right])} + \\
& \frac{\sqrt{a (1 + \text{Cos}[c + dx])} \text{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] (11 A \text{Sin} \left[ \frac{1}{2} (c + dx) \right] + 8 B \text{Sin} \left[ \frac{1}{2} (c + dx) \right] + 16 C \text{Sin} \left[ \frac{1}{2} (c + dx) \right])}{64 d (\text{Cos} \left[ \frac{1}{2} (c + dx) \right] - \text{Sin} \left[ \frac{1}{2} (c + dx) \right])^2} + \\
& \frac{\sqrt{a (1 + \text{Cos}[c + dx])} \text{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] (11 A \text{Sin} \left[ \frac{1}{2} (c + dx) \right] + 8 B \text{Sin} \left[ \frac{1}{2} (c + dx) \right] + 16 C \text{Sin} \left[ \frac{1}{2} (c + dx) \right])}{64 d (\text{Cos} \left[ \frac{1}{2} (c + dx) \right] + \text{Sin} \left[ \frac{1}{2} (c + dx) \right])^2}
\end{aligned}$$

■ **Problem 385: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{3/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x] dx$$

Optimal (type 3, 142 leaves, 5 steps):

$$\frac{2 a^{3/2} A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{d} + \frac{2 a^2 (15 A + 20 B + 12 C) \sin [c+d x]}{15 d \sqrt{a+a \cos [c+d x]}} +$$

$$\frac{2 a (5 B + 3 C) \sqrt{a+a \cos [c+d x]} \sin [c+d x]}{15 d} + \frac{2 C (a+a \cos [c+d x])^{3/2} \sin [c+d x]}{5 d}$$

Result (type 3, 1649 leaves):

$$-\left(\left(\left(\frac{1}{8} - \frac{i}{8}\right) A (1 + e^{i c}) \left(\sqrt{2} - (1 - i) e^{\frac{i c}{2}} + (16 - 16 i) e^{\frac{3 i c}{2} + i d x} + (20 + 20 i) \sqrt{2} e^{2 i c + \frac{3 i d x}{2}} - (34 - 34 i) e^{\frac{5 i c}{2} + 2 i d x} - (20 + 20 i) \sqrt{2} e^{3 i c + \frac{5 i d x}{2}} +\right.\right.\right.$$

$$\left.\left.\left(16 - 16 i\right) e^{\frac{7 i c}{2} + 3 i d x} + (4 + 4 i) \sqrt{2} e^{4 i c + \frac{7 i d x}{2}} - (1 - i) e^{\frac{9 i c}{2} + 4 i d x} + 8 i e^{\frac{1}{2} i (c+d x)} - 16 \sqrt{2} e^{i (c+d x)} - 40 i e^{\frac{3}{2} i (c+d x)} + 34 \sqrt{2} e^{2 i (c+d x)} +\right.\right.$$

$$\left.\left.40 i e^{\frac{5}{2} i (c+d x)} - 16 \sqrt{2} e^{3 i (c+d x)} - 8 i e^{\frac{7}{2} i (c+d x)} + \sqrt{2} e^{4 i (c+d x)} - (4 + 4 i) \sqrt{2} e^{\frac{1}{2} i (2 c+d x)}\right) x (a (1 + \cos [c + d x]))^{3/2} \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^3\right) /$$

$$\left(\left(\left(-1 - i\right) + \sqrt{2} e^{\frac{i c}{2}}\right) (-1 + e^{i c}) \left(i - 2 \sqrt{2} e^{\frac{1}{2} i (c+d x)} - 4 i e^{i (c+d x)} + 2 \sqrt{2} e^{\frac{3}{2} i (c+d x)} + i e^{2 i (c+d x)}\right)^2\right) -$$

$$\frac{i A \operatorname{ArcTan}\left[\frac{\cos \left[\frac{c}{4} + \frac{d x}{4}\right] - \sin \left[\frac{c}{4} + \frac{d x}{4}\right] - \sqrt{2} \sin \left[\frac{c}{4} + \frac{d x}{4}\right]}{-\cos \left[\frac{c}{4} + \frac{d x}{4}\right] + \sqrt{2} \cos \left[\frac{c}{4} + \frac{d x}{4}\right] - \sin \left[\frac{c}{4} + \frac{d x}{4}\right]}\right] (a (1 + \cos [c + d x]))^{3/2} \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^3}{2 \sqrt{2} d} -$$

$$\frac{i A \operatorname{ArcTan}\left[\frac{\cos \left[\frac{c}{4} + \frac{d x}{4}\right] + \sin \left[\frac{c}{4} + \frac{d x}{4}\right] - \sqrt{2} \sin \left[\frac{c}{4} + \frac{d x}{4}\right]}{\cos \left[\frac{c}{4} + \frac{d x}{4}\right] + \sqrt{2} \cos \left[\frac{c}{4} + \frac{d x}{4}\right] - \sin \left[\frac{c}{4} + \frac{d x}{4}\right]}\right] (a (1 + \cos [c + d x]))^{3/2} \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^3}{2 \sqrt{2} d} -$$

$$\frac{A (a (1 + \cos [c + d x]))^{3/2} \operatorname{Log}\left[2 - \sqrt{2} \cos \left[\frac{c}{2} + \frac{d x}{2}\right] - \sqrt{2} \sin \left[\frac{c}{2} + \frac{d x}{2}\right]\right] \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^3}{4 \sqrt{2} d} -$$

$$\frac{A (a (1 + \cos [c + d x]))^{3/2} \operatorname{Log}\left[2 + \sqrt{2} \cos \left[\frac{c}{2} + \frac{d x}{2}\right] - \sqrt{2} \sin \left[\frac{c}{2} + \frac{d x}{2}\right]\right] \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^3}{4 \sqrt{2} d} +$$

$$\frac{(2 A + 3 B + 2 C) \cos \left[\frac{d x}{2}\right] (a (1 + \cos [c + d x]))^{3/2} \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^3 \sin \left[\frac{c}{2}\right]}{2 d} -$$

$$\begin{aligned}
& \frac{i A \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \left( a \left( 1 + \cos [c + dx] \right) \right)^{3/2} \cot \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{d \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} + \\
& \left( A \left( a \left( 1 + \cos [c + dx] \right) \right)^{3/2} \csc \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \right. \\
& \left. \left( -dx \cos \left[ \frac{c}{2} \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + \frac{4 i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2 i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) \right) / \\
& \left( \sqrt{2} d \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right) \right) + \frac{(2 B + 3 C) \cos \left[ \frac{3 dx}{2} \right] \left( a \left( 1 + \cos [c + dx] \right) \right)^{3/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sin \left[ \frac{3 c}{2} \right]}{12 d} + \\
& \frac{C \cos \left[ \frac{5 dx}{2} \right] \left( a \left( 1 + \cos [c + dx] \right) \right)^{3/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sin \left[ \frac{5 c}{2} \right]}{20 d} + \\
& \frac{(2 A + 3 B + 2 C) \cos \left[ \frac{c}{2} \right] \left( a \left( 1 + \cos [c + dx] \right) \right)^{3/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sin \left[ \frac{dx}{2} \right]}{2 d} + \\
& \frac{(2 B + 3 C) \cos \left[ \frac{3 c}{2} \right] \left( a \left( 1 + \cos [c + dx] \right) \right)^{3/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sin \left[ \frac{3 dx}{2} \right]}{12 d} + \\
& \frac{C \cos \left[ \frac{5 c}{2} \right] \left( a \left( 1 + \cos [c + dx] \right) \right)^{3/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sin \left[ \frac{5 dx}{2} \right]}{20 d}
\end{aligned}$$

■ **Problem 386: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + dx])^{3/2} (A + B \cos [c + dx] + C \cos [c + dx]^2) \sec [c + dx]^2 dx$$

Optimal (type 3, 144 leaves, 5 steps):

$$\frac{a^{3/2} (3A + 2B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} - \frac{a^2 (3A - 6B - 8C) \sin[c+dx]}{3d \sqrt{a+a \cos[c+dx]}} -$$

$$\frac{a (3A - 2C) \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{3d} + \frac{A (a+a \cos[c+dx])^{3/2} \tan[c+dx]}{d}$$

Result (type 3, 561 leaves):

$$\frac{1}{d} \left( \frac{1}{192} + \frac{i}{192} \right) (a (1 + \cos[c+dx]))^{3/2} \operatorname{Sec}\left[\frac{1}{2} (c+dx)\right]^3$$

$$\left( \frac{6i\sqrt{2} \left( (-3+i) + \sqrt{2} \right) \left( (1+i) + \sqrt{2} \right) (3A+2B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right]}{i + \sqrt{2}} \right) -$$

$$\frac{6\sqrt{2} \left( (-1+i) + \sqrt{2} \right) \left( (3+i) + \sqrt{2} \right) (3A+2B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right]}{i + \sqrt{2}} +$$

$$\frac{(12+12i) \left( -2i + \sqrt{2} \right) (3A+2B) \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{1}{2} (c+dx)\right]\right]}{i + \sqrt{2}} + \frac{1}{i + \sqrt{2}}$$

$$3i\sqrt{2} \left( (-1+i) + \sqrt{2} \right) \left( (3+i) + \sqrt{2} \right) (3A+2B) \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2} (c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2} (c+dx)\right]\right] + \frac{1}{i + \sqrt{2}}$$

$$3\sqrt{2} \left( (-3+i) + \sqrt{2} \right) \left( (1+i) + \sqrt{2} \right) (3A+2B) \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2} (c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2} (c+dx)\right]\right] + \frac{(24-24i)A}{\cos\left[\frac{1}{2} (c+dx)\right] - \sin\left[\frac{1}{2} (c+dx)\right]} +$$

$$\left. \frac{(48-48i) (2B+3C) \sin\left[\frac{1}{2} (c+dx)\right] - \frac{(24-24i)A}{\cos\left[\frac{1}{2} (c+dx)\right] + \sin\left[\frac{1}{2} (c+dx)\right]} + (16-16i) C \sin\left[\frac{3}{2} (c+dx)\right]}{d} \right)$$

■ **Problem 387: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c+dx])^{3/2} (A + B \cos[c+dx] + C \cos[c+dx]^2) \operatorname{Sec}[c+dx]^3 dx$$

Optimal (type 3, 159 leaves, 5 steps):

$$\frac{a^{3/2} (7A + 12B + 8C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{4d} - \frac{a^2 (5A + 4B - 8C) \sin[c+dx]}{4d\sqrt{a+a\cos[c+dx]}} +$$

$$\frac{a(3A + 4B)\sqrt{a+a\cos[c+dx]}\tan[c+dx]}{4d} + \frac{A(a+a\cos[c+dx])^{3/2}\sec[c+dx]\tan[c+dx]}{2d}$$

Result (type 3, 644 leaves):

$$\frac{1}{d} \left( \frac{1}{256} + \frac{i}{256} \right) (a(1 + \cos[c+dx]))^{3/2} \sec\left[\frac{1}{2}(c+dx)\right]^3$$

$$\left( \frac{2i\sqrt{2} \left( (-3+i) + \sqrt{2} \right) \left( (1+i) + \sqrt{2} \right) (7A + 12B + 8C) \operatorname{ArcTan}\left[ \frac{\cos\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2})\sin\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2})\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right]}{i + \sqrt{2}} \right) -$$

$$\frac{2\sqrt{2} \left( (-1+i) + \sqrt{2} \right) \left( (3+i) + \sqrt{2} \right) (7A + 12B + 8C) \operatorname{ArcTan}\left[ \frac{\cos\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2})\sin\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2})\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right]}{i + \sqrt{2}} +$$

$$\frac{(4+4i) \left( -2i + \sqrt{2} \right) (7A + 12B + 8C) \operatorname{Log}\left[ \sqrt{2} + 2\sin\left[\frac{1}{2}(c+dx)\right] \right]}{i + \sqrt{2}} + \frac{1}{i + \sqrt{2}}$$

$$i\sqrt{2} \left( (-1+i) + \sqrt{2} \right) \left( (3+i) + \sqrt{2} \right) (7A + 12B + 8C) \operatorname{Log}\left[ 2 - \sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right] \right] +$$

$$\frac{1}{i + \sqrt{2}} \sqrt{2} \left( (-3+i) + \sqrt{2} \right) \left( (1+i) + \sqrt{2} \right) (7A + 12B + 8C) \operatorname{Log}\left[ 2 + \sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right] \right] +$$

$$\frac{(8-8i)(7A+4B)}{\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]} + (128-128i)C \sin\left[\frac{1}{2}(c+dx)\right] + \frac{(16-16i)A \sin\left[\frac{1}{2}(c+dx)\right]}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} +$$

$$\left( \frac{(16-16i)A \sin\left[\frac{1}{2}(c+dx)\right]}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{(8-8i)(7A+4B)}{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]} \right)$$

■ **Problem 388: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c+dx])^{3/2} (A + B \cos[c+dx] + C \cos[c+dx]^2) \sec[c+dx]^4 dx$$

Optimal (type 3, 165 leaves, 5 steps):

$$\frac{a^{3/2} (11 A + 14 B + 24 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{8 d} + \frac{a^2 (19 A + 30 B + 24 C) \tan[c+dx]}{24 d \sqrt{a+a \cos[c+dx]}} +$$

$$\frac{a (A + 2 B) \sqrt{a+a \cos[c+dx]} \sec[c+dx] \tan[c+dx]}{4 d} + \frac{A (a+a \cos[c+dx])^{3/2} \sec[c+dx]^2 \tan[c+dx]}{3 d}$$

Result (type 3, 1202 leaves):

$$\begin{aligned}
& - \frac{1}{\sqrt{2} (i + \sqrt{2})} \frac{1}{d} \left( \frac{1}{128} + \frac{i}{128} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (33 + 11i) A + 11\sqrt{2} A + (42 + 14i) B + 14\sqrt{2} B + (72 + 24i) C + 24\sqrt{2} C \right) \\
& \quad \text{ArcTan} \left[ \frac{\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c + dx)\right]}{-\cos\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]} \right] (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 - \\
& \frac{1}{\sqrt{2} (i + \sqrt{2})} \frac{1}{d} \left( \frac{1}{128} - \frac{i}{128} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-33 + 11i) A + 11\sqrt{2} A - (42 - 14i) B + 14\sqrt{2} B - (72 - 24i) C + 24\sqrt{2} C \right) \\
& \quad \text{ArcTan} \left[ \frac{\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c + dx)\right]}{\cos\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]} \right] (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 + \frac{1}{64 (i + \sqrt{2})} \frac{1}{d} \\
& \left( 22A + 11i\sqrt{2} A + 28B + 14i\sqrt{2} B + 48C + 24i\sqrt{2} C \right) (a(1 + \cos[c + dx]))^{3/2} \text{Log} \left[ \sqrt{2} + 2 \sin\left[\frac{1}{2}(c + dx)\right] \right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 - \\
& \frac{1}{\sqrt{2} (i + \sqrt{2})} \frac{1}{d} \left( \frac{1}{256} - \frac{i}{256} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (33 + 11i) A + 11\sqrt{2} A + (42 + 14i) B + 14\sqrt{2} B + (72 + 24i) C + 24\sqrt{2} C \right) \\
& \quad (a(1 + \cos[c + dx]))^{3/2} \text{Log} \left[ 2 - \sqrt{2} \cos\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c + dx)\right] \right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 + \frac{1}{\sqrt{2} (i + \sqrt{2})} \frac{1}{d} \\
& \left( \frac{1}{256} + \frac{i}{256} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-33 + 11i) A + 11\sqrt{2} A - (42 - 14i) B + 14\sqrt{2} B - (72 - 24i) C + 24\sqrt{2} C \right) (a(1 + \cos[c + dx]))^{3/2} \\
& \quad \text{Log} \left[ 2 + \sqrt{2} \cos\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c + dx)\right] \right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 + \frac{A(a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{24d (\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right])^3} + \\
& \frac{(11A + 14B + 8C) (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{32d (\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right])} - \frac{A(a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{24d (\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])^3} + \\
& \frac{(-11A - 14B - 8C) (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{32d (\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])} + \frac{(a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (3A \sin\left[\frac{1}{2}(c + dx)\right] + 2B \sin\left[\frac{1}{2}(c + dx)\right])}{16d (\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right])^2} + \\
& \frac{(a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (3A \sin\left[\frac{1}{2}(c + dx)\right] + 2B \sin\left[\frac{1}{2}(c + dx)\right])}{16d (\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])^2}
\end{aligned}$$

■ **Problem 389: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{3/2} (A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^5 dx$$

Optimal (type 3, 215 leaves, 6 steps):



$$\frac{a^{3/2} (75 A + 88 B + 112 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{64 d} + \frac{a^2 (75 A + 88 B + 112 C) \tan[c+dx]}{64 d \sqrt{a+a \cos[c+dx]}} + \frac{a^2 (39 A + 56 B + 48 C) \sec[c+dx] \tan[c+dx]}{96 d \sqrt{a+a \cos[c+dx]}} +$$

$$\frac{a (3 A + 8 B) \sqrt{a+a \cos[c+dx]} \sec[c+dx]^2 \tan[c+dx]}{24 d} + \frac{A (a+a \cos[c+dx])^{3/2} \sec[c+dx]^3 \tan[c+dx]}{4 d}$$

Result (type 3, 1382 leaves):

$$-\frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{1024} + \frac{i}{1024} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (225 + 75 i) A + 75 \sqrt{2} A + (264 + 88 i) B + 88 \sqrt{2} B + (336 + 112 i) C + 112 \sqrt{2} C \right)$$

$$\operatorname{ArcTan}\left[ \frac{\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c+dx)\right]}{-\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] (a (1 + \cos[c+dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 -$$

$$\frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{1024} - \frac{i}{1024} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-225 + 75 i) A + 75 \sqrt{2} A - (264 - 88 i) B + 88 \sqrt{2} B - (336 - 112 i) C + 112 \sqrt{2} C \right)$$

$$\operatorname{ArcTan}\left[ \frac{\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] (a (1 + \cos[c+dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 + \frac{1}{512 (i + \sqrt{2}) d}$$

$$\left( 150 A + 75 i \sqrt{2} A + 176 B + 88 i \sqrt{2} B + 224 C + 112 i \sqrt{2} C \right) (a (1 + \cos[c+dx]))^{3/2} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(c+dx)\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 -$$

$$\frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{2048} - \frac{i}{2048} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (225 + 75 i) A + 75 \sqrt{2} A + (264 + 88 i) B + 88 \sqrt{2} B + (336 + 112 i) C + 112 \sqrt{2} C \right)$$

$$(a (1 + \cos[c+dx]))^{3/2} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 + \frac{1}{\sqrt{2} (i + \sqrt{2}) d}$$

$$\left( \frac{1}{2048} + \frac{i}{2048} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-225 + 75 i) A + 75 \sqrt{2} A - (264 - 88 i) B + 88 \sqrt{2} B - (336 - 112 i) C + 112 \sqrt{2} C \right)$$

$$(a (1 + \cos[c+dx]))^{3/2} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 +$$

$$\frac{(15 A + 8 B) (a (1 + \cos[c+dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{192 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{(75 A + 88 B + 112 C) (a (1 + \cos[c+dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{256 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)}$$

$$\frac{A (a (1 + \cos[c+dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{1}{2}(c+dx)\right]}{32 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{A (a (1 + \cos[c+dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{1}{2}(c+dx)\right]}{32 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} +$$

$$\frac{(-15A - 8B)(a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{192d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{(-75A - 88B - 112C)(a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{256d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)} +$$

$$\left( (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left( 19A \sin\left[\frac{1}{2}(c + dx)\right] + 24B \sin\left[\frac{1}{2}(c + dx)\right] + 16C \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) /$$

$$\left( 128d \left( \cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right) \right)^2 +$$

$$\left( (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left( 19A \sin\left[\frac{1}{2}(c + dx)\right] + 24B \sin\left[\frac{1}{2}(c + dx)\right] + 16C \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) /$$

$$\left( 128d \left( \cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \right)^2$$

■ **Problem 390: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{3/2} (A + B \cos[c + dx] + C \cos^2[c + dx]) \sec[c + dx]^6 dx$$

Optimal (type 3, 263 leaves, 7 steps):

$$\frac{a^{3/2} (133A + 150B + 176C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{128d} + \frac{a^2 (133A + 150B + 176C) \tan[c + dx]}{128d \sqrt{a + a \cos[c + dx]}} +$$

$$\frac{a^2 (133A + 150B + 176C) \sec[c + dx] \tan[c + dx]}{192d \sqrt{a + a \cos[c + dx]}} + \frac{a^2 (67A + 90B + 80C) \sec[c + dx]^2 \tan[c + dx]}{240d \sqrt{a + a \cos[c + dx]}} +$$

$$\frac{a (3A + 10B) \sqrt{a + a \cos[c + dx]} \sec[c + dx]^3 \tan[c + dx]}{40d} + \frac{A (a + a \cos[c + dx])^{3/2} \sec[c + dx]^4 \tan[c + dx]}{5d}$$

Result (type 3, 1542 leaves):

$$-\frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{2048} + \frac{i}{2048} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (399 + 133i) A + 133\sqrt{2} A + (450 + 150i) B + 150\sqrt{2} B + (528 + 176i) C + 176\sqrt{2} C \right)$$

$$\operatorname{ArcTan}\left[ \frac{\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c + dx)\right]}{-\cos\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]} \right] (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 - \frac{1}{\sqrt{2} (i + \sqrt{2}) d}$$

$$\left( \frac{1}{2048} - \frac{i}{2048} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-399 + 133i) A + 133\sqrt{2} A - (450 - 150i) B + 150\sqrt{2} B - (528 - 176i) C + 176\sqrt{2} C \right)$$

$$\operatorname{ArcTan}\left[ \frac{\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c + dx)\right]}{\cos\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]} \right] (a(1 + \cos[c + dx]))^{3/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 + \frac{1}{1024 (i + \sqrt{2}) d}$$

$$\begin{aligned}
& \left( 266 A + 133 i \sqrt{2} A + 300 B + 150 i \sqrt{2} B + 352 C + 176 i \sqrt{2} C \right) \left( a (1 + \cos [c + d x]) \right)^{3/2} \operatorname{Log} \left[ \sqrt{2} + 2 \sin \left[ \frac{1}{2} (c + d x) \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 - \\
& \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{4096} - \frac{i}{4096} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (399 + 133 i) A + 133 \sqrt{2} A + (450 + 150 i) B + 150 \sqrt{2} B + (528 + 176 i) C + 176 \sqrt{2} C \right) \\
& \left( a (1 + \cos [c + d x]) \right)^{3/2} \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{1}{2} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + d x) \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 + \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \\
& \left( \frac{1}{4096} + \frac{i}{4096} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-399 + 133 i) A + 133 \sqrt{2} A - (450 - 150 i) B + 150 \sqrt{2} B - (528 - 176 i) C + 176 \sqrt{2} C \right) \\
& \left( a (1 + \cos [c + d x]) \right)^{3/2} \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{1}{2} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + d x) \right] \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 + \\
& \frac{A \left( a (1 + \cos [c + d x]) \right)^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3}{80 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^5} + \frac{(29 A + 30 B + 16 C) \left( a (1 + \cos [c + d x]) \right)^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3}{384 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^3} + \\
& \frac{(133 A + 150 B + 176 C) \left( a (1 + \cos [c + d x]) \right)^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3}{512 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)} - \frac{A \left( a (1 + \cos [c + d x]) \right)^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3}{80 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^5} + \\
& \frac{(-29 A - 30 B - 16 C) \left( a (1 + \cos [c + d x]) \right)^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3}{384 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^3} + \frac{(-133 A - 150 B - 176 C) \left( a (1 + \cos [c + d x]) \right)^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3}{512 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)} + \\
& \frac{\left( a (1 + \cos [c + d x]) \right)^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \left( 3 A \sin \left[ \frac{1}{2} (c + d x) \right] + 2 B \sin \left[ \frac{1}{2} (c + d x) \right] \right)}{64 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^4} + \\
& \frac{\left( a (1 + \cos [c + d x]) \right)^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \left( 3 A \sin \left[ \frac{1}{2} (c + d x) \right] + 2 B \sin \left[ \frac{1}{2} (c + d x) \right] \right)}{64 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^4} + \\
& \left( \left( a (1 + \cos [c + d x]) \right)^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \left( 37 A \sin \left[ \frac{1}{2} (c + d x) \right] + 38 B \sin \left[ \frac{1}{2} (c + d x) \right] + 48 C \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) / \\
& \left( 256 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2 \right) + \\
& \left( \left( a (1 + \cos [c + d x]) \right)^{3/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \left( 37 A \sin \left[ \frac{1}{2} (c + d x) \right] + 38 B \sin \left[ \frac{1}{2} (c + d x) \right] + 48 C \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) / \\
& \left( 256 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2 \right)
\end{aligned}$$

■ **Problem 394:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (a + a \cos [c + dx])^{5/2} (A + B \cos [c + dx] + C \cos [c + dx]^2) \sec [c + dx] dx$$

Optimal (type 3, 182 leaves, 6 steps):

$$\frac{2 a^{5/2} A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+dx]}{\sqrt{a+a \cos [c+dx]}}\right]}{d} + \frac{2 a^3 (245 A + 224 B + 160 C) \sin [c+dx]}{105 d \sqrt{a+a \cos [c+dx]}} + \frac{2 a^2 (35 A + 56 B + 40 C) \sqrt{a+a \cos [c+dx]} \sin [c+dx]}{105 d} +$$

$$\frac{2 a (7 B + 5 C) (a + a \cos [c+dx])^{3/2} \sin [c+dx]}{35 d} + \frac{2 C (a + a \cos [c+dx])^{5/2} \sin [c+dx]}{7 d}$$

Result (type 3, 1772 leaves):

$$- \left( \left( \left( \frac{1}{16} - \frac{i}{16} \right) A (1 + e^{ic}) \left( \sqrt{2} - (1-i) e^{\frac{ic}{2}} + (16-16i) e^{\frac{3ic}{2}+idx} + (20+20i) \sqrt{2} e^{2ic+\frac{3idx}{2}} - (34-34i) e^{\frac{5ic}{2}+2idx} - (20+20i) \sqrt{2} e^{3ic+\frac{5idx}{2}} + \right. \right. \right.$$

$$\left. \left. (16-16i) e^{\frac{7ic}{2}+3idx} + (4+4i) \sqrt{2} e^{4ic+\frac{7idx}{2}} - (1-i) e^{\frac{9ic}{2}+4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + \right. \right.$$

$$\left. \left. 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4+4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) x (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \right) /$$

$$\left( \left( (-1-i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 \right) -$$

$$\frac{i A \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{-\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{4 \sqrt{2} d} -$$

$$\frac{i A \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sin \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sqrt{2} \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]}{\cos \left[ \frac{c}{4} + \frac{dx}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} + \frac{dx}{4} \right] - \sin \left[ \frac{c}{4} + \frac{dx}{4} \right]} \right] (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{4 \sqrt{2} d} -$$

$$\frac{A (a (1 + \cos [c + dx]))^{5/2} \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{8 \sqrt{2} d} -$$

$$\frac{A (a (1 + \cos [c + dx]))^{5/2} \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5}{8 \sqrt{2} d} +$$

$$\frac{5 (4 A + 4 B + 3 C) \cos \left[ \frac{dx}{2} \right] (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sin \left[ \frac{c}{2} \right]}{16 d} -$$

$$\begin{aligned}
& \frac{i A \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right]-i\left(-\sqrt{2}+2 \sin\left[\frac{c}{2}\right]\right) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}}\right] (a(1+\cos[c+dx]))^{5/2} \cot\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^5}{2 d \sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}} + \\
& \left( A (a(1+\cos[c+dx]))^{5/2} \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \right. \\
& \left. \left( -dx \cos\left[\frac{c}{2}\right] + 2 \operatorname{Log}\left[\sqrt{2}+2 \cos\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] \right) \sin\left[\frac{c}{2}\right] + \frac{4 i \sqrt{2} \operatorname{ArcTan}\left[\frac{2 i \cos\left[\frac{c}{2}\right]-i\left(-\sqrt{2}+2 \sin\left[\frac{c}{2}\right]\right) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2+4 \cos\left[\frac{c}{2}\right]^2+4 \sin\left[\frac{c}{2}\right]^2}} \right) \right) / \\
& \left( 2 \sqrt{2} d \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) \right) + \frac{(4 A + 10 B + 11 C) \cos\left[\frac{3 dx}{2}\right] (a(1+\cos[c+dx]))^{5/2} \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \sin\left[\frac{3 c}{2}\right]}{48 d} + \\
& \frac{(2 B + 5 C) \cos\left[\frac{5 dx}{2}\right] (a(1+\cos[c+dx]))^{5/2} \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \sin\left[\frac{5 c}{2}\right]}{80 d} + \\
& \frac{C \cos\left[\frac{7 dx}{2}\right] (a(1+\cos[c+dx]))^{5/2} \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \sin\left[\frac{7 c}{2}\right]}{112 d} + \\
& \frac{5(4 A + 4 B + 3 C) \cos\left[\frac{c}{2}\right] (a(1+\cos[c+dx]))^{5/2} \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \sin\left[\frac{dx}{2}\right]}{16 d} + \\
& \frac{(4 A + 10 B + 11 C) \cos\left[\frac{3 c}{2}\right] (a(1+\cos[c+dx]))^{5/2} \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \sin\left[\frac{3 dx}{2}\right]}{48 d} + \\
& \frac{(2 B + 5 C) \cos\left[\frac{5 c}{2}\right] (a(1+\cos[c+dx]))^{5/2} \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \sin\left[\frac{5 dx}{2}\right]}{80 d} + \\
& \frac{C \cos\left[\frac{7 c}{2}\right] (a(1+\cos[c+dx]))^{5/2} \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \sin\left[\frac{7 dx}{2}\right]}{112 d}
\end{aligned}$$

■ **Problem 395:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^2 dx$$

Optimal (type 3, 184 leaves, 6 steps):

$$\frac{a^{5/2} (5 A + 2 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{d} + \frac{a^3 (15 A + 70 B + 64 C) \sin [c+d x]}{15 d \sqrt{a+a \cos [c+d x]}} - \frac{a^2 (15 A - 10 B - 16 C) \sqrt{a+a \cos [c+d x]} \sin [c+d x]}{15 d} - \frac{a (5 A - 2 C) (a + a \cos [c+d x])^{3/2} \sin [c+d x]}{5 d} + \frac{A (a + a \cos [c+d x])^{5/2} \tan [c+d x]}{d}$$

Result (type 3, 584 leaves):

$$\frac{1}{d} \left( \frac{1}{1920} + \frac{i}{1920} \right) (a (1 + \cos [c + d x]))^{5/2} \sec \left[ \frac{1}{2} (c + d x) \right]^5$$

$$\left( \frac{30 i \sqrt{2} \left( (-3 + i) + \sqrt{2} \right) \left( (1 + i) + \sqrt{2} \right) (5 A + 2 B) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c+d x) \right] - (-1+\sqrt{2}) \sin \left[ \frac{1}{4} (c+d x) \right]}{(1+\sqrt{2}) \cos \left[ \frac{1}{4} (c+d x) \right] - \sin \left[ \frac{1}{4} (c+d x) \right]} \right]}{i + \sqrt{2}} - \frac{30 \sqrt{2} \left( (-1 + i) + \sqrt{2} \right) \left( (3 + i) + \sqrt{2} \right) (5 A + 2 B) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c+d x) \right] - (1+\sqrt{2}) \sin \left[ \frac{1}{4} (c+d x) \right]}{(-1+\sqrt{2}) \cos \left[ \frac{1}{4} (c+d x) \right] - \sin \left[ \frac{1}{4} (c+d x) \right]} \right]}{i + \sqrt{2}} + \frac{(60 + 60 i) \left( -2 i + \sqrt{2} \right) (5 A + 2 B) \operatorname{Log} \left[ \sqrt{2} + 2 \sin \left[ \frac{1}{2} (c + d x) \right] \right]}{i + \sqrt{2}} + \frac{1}{i + \sqrt{2}} \right.$$

$$15 i \sqrt{2} \left( (-1 + i) + \sqrt{2} \right) \left( (3 + i) + \sqrt{2} \right) (5 A + 2 B) \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{1}{2} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \frac{1}{i + \sqrt{2}} 15 \sqrt{2} \left( (-3 + i) + \sqrt{2} \right) \left( (1 + i) + \sqrt{2} \right) (5 A + 2 B) \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{1}{2} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \frac{(120 - 120 i) A}{\cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right]} + (240 - 240 i) (2 A + 5 (B + C)) \sin \left[ \frac{1}{2} (c + d x) \right] - \frac{(120 - 120 i) A}{\cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right]} + (40 - 40 i) (2 B + 5 C) \sin \left[ \frac{3}{2} (c + d x) \right] + (24 - 24 i) C \sin \left[ \frac{5}{2} (c + d x) \right] \left. \right)$$

■ **Problem 396: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^3 dx$$

Optimal (type 3, 199 leaves, 6 steps) :

$$\frac{a^{5/2} (19 A + 20 B + 8 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{4 d} - \frac{a^3 (27 A - 12 B - 56 C) \sin[c+dx]}{12 d \sqrt{a+a \cos[c+dx]}} - \frac{a^2 (21 A + 12 B - 8 C) \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{12 d} + \frac{a (5 A + 4 B) (a+a \cos[c+dx])^{3/2} \tan[c+dx]}{4 d} + \frac{A (a+a \cos[c+dx])^{5/2} \sec[c+dx] \tan[c+dx]}{2 d}$$

Result (type 3, 666 leaves) :

$$\frac{1}{d} \left( \frac{1}{1536} + \frac{i}{1536} \right) (a (1 + \cos[c+dx]))^{5/2} \sec\left[\frac{1}{2} (c+dx)\right]^5$$

$$\left( \frac{6 i \sqrt{2} \left( (-3+i) + \sqrt{2} \right) \left( (1+i) + \sqrt{2} \right) (19 A + 20 B + 8 C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4} (c+dx)\right] - (-1+\sqrt{2}) \sin\left[\frac{1}{4} (c+dx)\right]}{(1+\sqrt{2}) \cos\left[\frac{1}{4} (c+dx)\right] - \sin\left[\frac{1}{4} (c+dx)\right]}\right]}{i + \sqrt{2}} - \frac{6 \sqrt{2} \left( (-1+i) + \sqrt{2} \right) \left( (3+i) + \sqrt{2} \right) (19 A + 20 B + 8 C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4} (c+dx)\right] - (1+\sqrt{2}) \sin\left[\frac{1}{4} (c+dx)\right]}{(-1+\sqrt{2}) \cos\left[\frac{1}{4} (c+dx)\right] - \sin\left[\frac{1}{4} (c+dx)\right]}\right]}{i + \sqrt{2}} + \frac{(12 + 12 i) \left( -2 i + \sqrt{2} \right) (19 A + 20 B + 8 C) \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{1}{2} (c+dx)\right]\right]}{i + \sqrt{2}} + \frac{1}{i + \sqrt{2}} + \frac{3 i \sqrt{2} \left( (-1+i) + \sqrt{2} \right) \left( (3+i) + \sqrt{2} \right) (19 A + 20 B + 8 C) \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2} (c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2} (c+dx)\right]\right]}{i + \sqrt{2}} + \frac{1}{i + \sqrt{2}} \frac{3 \sqrt{2} \left( (-3+i) + \sqrt{2} \right) \left( (1+i) + \sqrt{2} \right) (19 A + 20 B + 8 C) \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2} (c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2} (c+dx)\right]\right]}{i + \sqrt{2}} + \frac{(24 - 24 i) (11 A + 4 B)}{\cos\left[\frac{1}{2} (c+dx)\right] - \sin\left[\frac{1}{2} (c+dx)\right]} + (192 - 192 i) (2 B + 5 C) \sin\left[\frac{1}{2} (c+dx)\right] + \frac{(48 - 48 i) A \sin\left[\frac{1}{2} (c+dx)\right]}{\left(\cos\left[\frac{1}{2} (c+dx)\right] - \sin\left[\frac{1}{2} (c+dx)\right]\right)^2} + \left. \frac{(48 - 48 i) A \sin\left[\frac{1}{2} (c+dx)\right]}{\left(\cos\left[\frac{1}{2} (c+dx)\right] + \sin\left[\frac{1}{2} (c+dx)\right]\right)^2} - \frac{(24 - 24 i) (11 A + 4 B)}{\cos\left[\frac{1}{2} (c+dx)\right] + \sin\left[\frac{1}{2} (c+dx)\right]} + (64 - 64 i) C \sin\left[\frac{3}{2} (c+dx)\right] \right)$$

- **Problem 397: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c+dx])^{5/2} (A + B \cos[c+dx] + C \cos[c+dx]^2) \sec[c+dx]^4 dx$$

Optimal (type 3, 207 leaves, 6 steps) :

$$\frac{a^{5/2} (25 A + 38 B + 40 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{8 d} - \frac{a^3 (49 A + 54 B - 24 C) \sin[c+dx]}{24 d \sqrt{a+a \cos[c+dx]}} + \frac{a^2 (31 A + 42 B + 24 C) \sqrt{a+a \cos[c+dx]} \tan[c+dx]}{24 d} +$$

$$\frac{a (5 A + 6 B) (a+a \cos[c+dx])^{3/2} \sec[c+dx] \tan[c+dx]}{12 d} + \frac{A (a+a \cos[c+dx])^{5/2} \sec[c+dx]^2 \tan[c+dx]}{3 d}$$

Result (type 3, 1249 leaves):



$$\begin{aligned}
& - \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{256} + \frac{i}{256} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (75 + 25i) A + 25\sqrt{2} A + (114 + 38i) B + 38\sqrt{2} B + (120 + 40i) C + 40\sqrt{2} C \right) \\
& \quad \text{ArcTan} \left[ \frac{\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c + dx)\right]}{-\cos\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]} \right] (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 - \\
& \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{256} - \frac{i}{256} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-75 + 25i) A + 25\sqrt{2} A - (114 - 38i) B + 38\sqrt{2} B - (120 - 40i) C + 40\sqrt{2} C \right) \\
& \quad \text{ArcTan} \left[ \frac{\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c + dx)\right]}{\cos\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]} \right] (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 + \frac{1}{128 (i + \sqrt{2}) d} \\
& \left( 50A + 25i\sqrt{2} A + 76B + 38i\sqrt{2} B + 80C + 40i\sqrt{2} C \right) (a(1 + \cos[c + dx]))^{5/2} \log\left[\sqrt{2} + 2\sin\left[\frac{1}{2}(c + dx)\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 - \\
& \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{512} - \frac{i}{512} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (75 + 25i) A + 25\sqrt{2} A + (114 + 38i) B + 38\sqrt{2} B + (120 + 40i) C + 40\sqrt{2} C \right) \\
& \quad (a(1 + \cos[c + dx]))^{5/2} \log\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c + dx)\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 + \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \\
& \left( \frac{1}{512} + \frac{i}{512} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-75 + 25i) A + 25\sqrt{2} A - (114 - 38i) B + 38\sqrt{2} B - (120 - 40i) C + 40\sqrt{2} C \right) (a(1 + \cos[c + dx]))^{5/2} \\
& \quad \log\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c + dx)\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 + \frac{A(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{48d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(25A + 22B + 8C) (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{64d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \frac{C(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sin\left[\frac{1}{2}(c + dx)\right]}{2d} - \\
& \frac{A(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{48d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{(-25A - 22B - 8C) (a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{64d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \\
& \frac{(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (5A \sin\left[\frac{1}{2}(c + dx)\right] + 2B \sin\left[\frac{1}{2}(c + dx)\right])}{32d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{(a(1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (5A \sin\left[\frac{1}{2}(c + dx)\right] + 2B \sin\left[\frac{1}{2}(c + dx)\right])}{32d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}
\end{aligned}$$

■ **Problem 398: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^5 dx$$

Optimal (type 3, 215 leaves, 6 steps):

$$\frac{a^{5/2} (163 A + 200 B + 304 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{64 d} +$$

$$\frac{a^3 (299 A + 392 B + 432 C) \tan [c+d x]}{192 d \sqrt{a+a \cos [c+d x]}} + \frac{a^2 (17 A + 24 B + 16 C) \sqrt{a+a \cos [c+d x]} \sec [c+d x] \tan [c+d x]}{32 d} +$$

$$\frac{a (5 A + 8 B) (a + a \cos [c+d x])^{3/2} \sec [c+d x]^2 \tan [c+d x]}{24 d} + \frac{A (a + a \cos [c+d x])^{5/2} \sec [c+d x]^3 \tan [c+d x]}{4 d}$$

Result (type 3, 1382 leaves):

$$-\frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{2048} + \frac{i}{2048} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (489 + 163 i) A + 163 \sqrt{2} A + (600 + 200 i) B + 200 \sqrt{2} B + (912 + 304 i) C + 304 \sqrt{2} C \right)$$

$$\operatorname{ArcTan}\left[ \frac{\cos \left[ \frac{1}{4} (c + d x) \right] - \sin \left[ \frac{1}{4} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{4} (c + d x) \right]}{-\cos \left[ \frac{1}{4} (c + d x) \right] + \sqrt{2} \cos \left[ \frac{1}{4} (c + d x) \right] - \sin \left[ \frac{1}{4} (c + d x) \right]} \right] (a (1 + \cos [c + d x]))^{5/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 - \frac{1}{\sqrt{2} (i + \sqrt{2}) d}$$

$$\left( \frac{1}{2048} - \frac{i}{2048} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-489 + 163 i) A + 163 \sqrt{2} A - (600 - 200 i) B + 200 \sqrt{2} B - (912 - 304 i) C + 304 \sqrt{2} C \right)$$

$$\operatorname{ArcTan}\left[ \frac{\cos \left[ \frac{1}{4} (c + d x) \right] + \sin \left[ \frac{1}{4} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{4} (c + d x) \right]}{\cos \left[ \frac{1}{4} (c + d x) \right] + \sqrt{2} \cos \left[ \frac{1}{4} (c + d x) \right] - \sin \left[ \frac{1}{4} (c + d x) \right]} \right] (a (1 + \cos [c + d x]))^{5/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 + \frac{1}{1024 (i + \sqrt{2}) d}$$

$$\left( 326 A + 163 i \sqrt{2} A + 400 B + 200 i \sqrt{2} B + 608 C + 304 i \sqrt{2} C \right) (a (1 + \cos [c + d x]))^{5/2} \operatorname{Log}\left[ \sqrt{2} + 2 \sin \left[ \frac{1}{2} (c + d x) \right] \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 -$$

$$\frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{4096} - \frac{i}{4096} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (489 + 163 i) A + 163 \sqrt{2} A + (600 + 200 i) B + 200 \sqrt{2} B + (912 + 304 i) C + 304 \sqrt{2} C \right)$$

$$(a (1 + \cos [c + d x]))^{5/2} \operatorname{Log}\left[ 2 - \sqrt{2} \cos \left[ \frac{1}{2} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + d x) \right] \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 + \frac{1}{\sqrt{2} (i + \sqrt{2}) d}$$

$$\left( \frac{1}{4096} + \frac{i}{4096} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-489 + 163 i) A + 163 \sqrt{2} A - (600 - 200 i) B + 200 \sqrt{2} B - (912 - 304 i) C + 304 \sqrt{2} C \right)$$

$$(a (1 + \cos [c + d x]))^{5/2} \operatorname{Log}\left[ 2 + \sqrt{2} \cos \left[ \frac{1}{2} (c + d x) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + d x) \right] \right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 +$$

$$\frac{(23 A + 8 B) (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^5}{384 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^3} + \frac{(163 A + 200 B + 176 C) (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^5}{512 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)} +$$

$$\frac{A (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^5 \sin\left[\frac{1}{2}(c + d x)\right]}{64 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{A (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^5 \sin\left[\frac{1}{2}(c + d x)\right]}{64 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^4} +$$

$$\frac{(-23 A - 8 B) (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^5}{384 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^3} + \frac{(-163 A - 200 B - 176 C) (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^5}{512 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)} +$$

$$\left( (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^5 \left( 43 A \sin\left[\frac{1}{2}(c + d x)\right] + 40 B \sin\left[\frac{1}{2}(c + d x)\right] + 16 C \sin\left[\frac{1}{2}(c + d x)\right] \right) \right) /$$

$$\left( 256 d \left( \cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right] \right)^2 \right) +$$

$$\left( (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^5 \left( 43 A \sin\left[\frac{1}{2}(c + d x)\right] + 40 B \sin\left[\frac{1}{2}(c + d x)\right] + 16 C \sin\left[\frac{1}{2}(c + d x)\right] \right) \right) /$$

$$\left( 256 d \left( \cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^2 \right)$$

■ **Problem 399: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) \operatorname{Sec}[c + d x]^6 dx$$

Optimal (type 3, 261 leaves, 7 steps):

$$\frac{a^{5/2} (283 A + 326 B + 400 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}}\right]}{128 d} + \frac{a^3 (283 A + 326 B + 400 C) \tan [c + d x]}{128 d \sqrt{a + a \cos [c + d x]}} +$$

$$\frac{a^3 (787 A + 950 B + 1040 C) \operatorname{Sec}[c + d x] \tan [c + d x]}{960 d \sqrt{a + a \cos [c + d x]}} + \frac{a^2 (79 A + 110 B + 80 C) \sqrt{a + a \cos [c + d x]} \operatorname{Sec}[c + d x]^2 \tan [c + d x]}{240 d} +$$

$$\frac{a (A + 2 B) (a + a \cos [c + d x])^{3/2} \operatorname{Sec}[c + d x]^3 \tan [c + d x]}{8 d} + \frac{A (a + a \cos [c + d x])^{5/2} \operatorname{Sec}[c + d x]^4 \tan [c + d x]}{5 d}$$

Result (type 3, 1542 leaves):

$$-\frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{4096} + \frac{i}{4096} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (849 + 283 i) A + 283 \sqrt{2} A + (978 + 326 i) B + 326 \sqrt{2} B + (1200 + 400 i) C + 400 \sqrt{2} C \right)$$

$$\operatorname{ArcTan}\left[ \frac{\cos\left[\frac{1}{4}(c + d x)\right] - \sin\left[\frac{1}{4}(c + d x)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c + d x)\right]}{-\cos\left[\frac{1}{4}(c + d x)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c + d x)\right] - \sin\left[\frac{1}{4}(c + d x)\right]} \right] (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^5 - \frac{1}{\sqrt{2} (i + \sqrt{2}) d}$$

$$\begin{aligned}
& \left( \frac{1}{4096} - \frac{i}{4096} \right) \left( (1+i) + \sqrt{2} \right) \left( (-849 + 283i) A + 283\sqrt{2} A - (978 - 326i) B + 326\sqrt{2} B - (1200 - 400i) C + 400\sqrt{2} C \right) \\
& \text{ArcTan} \left[ \frac{\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \left( a(1 + \cos[c+dx]) \right)^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 + \frac{1}{2048(i + \sqrt{2})d} \\
& \left( 566A + 283i\sqrt{2}A + 652B + 326i\sqrt{2}B + 800C + 400i\sqrt{2}C \right) \left( a(1 + \cos[c+dx]) \right)^{5/2} \log\left[\sqrt{2} + 2\sin\left[\frac{1}{2}(c+dx)\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 - \\
& \frac{1}{\sqrt{2}(i + \sqrt{2})d} \left( \frac{1}{8192} - \frac{i}{8192} \right) \left( (-1+i) + \sqrt{2} \right) \left( (849 + 283i) A + 283\sqrt{2} A + (978 + 326i) B + 326\sqrt{2} B + (1200 + 400i) C + 400\sqrt{2} C \right) \\
& \left( a(1 + \cos[c+dx]) \right)^{5/2} \log\left[2 - \sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 + \frac{1}{\sqrt{2}(i + \sqrt{2})d} \\
& \left( \frac{1}{8192} + \frac{i}{8192} \right) \left( (1+i) + \sqrt{2} \right) \left( (-849 + 283i) A + 283\sqrt{2} A - (978 - 326i) B + 326\sqrt{2} B - (1200 - 400i) C + 400\sqrt{2} C \right) \\
& \left( a(1 + \cos[c+dx]) \right)^{5/2} \log\left[2 + \sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 + \\
& \frac{A \left( a(1 + \cos[c+dx]) \right)^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{160d \left( \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^5} + \frac{(59A + 46B + 16C) \left( a(1 + \cos[c+dx]) \right)^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{768d \left( \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^3} + \\
& \frac{(283A + 326B + 400C) \left( a(1 + \cos[c+dx]) \right)^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{1024d \left( \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)} - \frac{A \left( a(1 + \cos[c+dx]) \right)^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{160d \left( \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5} + \\
& \frac{(-59A - 46B - 16C) \left( a(1 + \cos[c+dx]) \right)^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{768d \left( \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3} + \frac{(-283A - 326B - 400C) \left( a(1 + \cos[c+dx]) \right)^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{1024d \left( \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)} + \\
& \frac{\left( a(1 + \cos[c+dx]) \right)^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( 5A \sin\left[\frac{1}{2}(c+dx)\right] + 2B \sin\left[\frac{1}{2}(c+dx)\right] \right)}{128d \left( \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^4} + \\
& \frac{\left( a(1 + \cos[c+dx]) \right)^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( 5A \sin\left[\frac{1}{2}(c+dx)\right] + 2B \sin\left[\frac{1}{2}(c+dx)\right] \right)}{128d \left( \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4} + \\
& \left( \left( a(1 + \cos[c+dx]) \right)^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( 75A \sin\left[\frac{1}{2}(c+dx)\right] + 86B \sin\left[\frac{1}{2}(c+dx)\right] + 80C \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) / \\
& \left( 512d \left( \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 \right) +
\end{aligned}$$

$$\left( (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \left( 75 A \sin \left[ \frac{1}{2} (c + d x) \right] + 86 B \sin \left[ \frac{1}{2} (c + d x) \right] + 80 C \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) / \left( 512 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2 \right)$$

- **Problem 400: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) \operatorname{Sec} [c + d x]^7 dx$$

Optimal (type 3, 311 leaves, 8 steps):

$$\frac{a^{5/2} (1015 A + 1132 B + 1304 C) \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right]}{512 d} + \frac{a^3 (1015 A + 1132 B + 1304 C) \tan [c + d x]}{512 d \sqrt{a + a \cos [c + d x]}} +$$

$$\frac{a^3 (1015 A + 1132 B + 1304 C) \operatorname{Sec} [c + d x] \tan [c + d x]}{768 d \sqrt{a + a \cos [c + d x]}} + \frac{a^3 (545 A + 628 B + 680 C) \operatorname{Sec} [c + d x]^2 \tan [c + d x]}{960 d \sqrt{a + a \cos [c + d x]}} +$$

$$\frac{a^2 (115 A + 156 B + 120 C) \sqrt{a + a \cos [c + d x]} \operatorname{Sec} [c + d x]^3 \tan [c + d x]}{480 d} +$$

$$\frac{a (5 A + 12 B) (a + a \cos [c + d x])^{3/2} \operatorname{Sec} [c + d x]^4 \tan [c + d x]}{60 d} + \frac{A (a + a \cos [c + d x])^{5/2} \operatorname{Sec} [c + d x]^5 \tan [c + d x]}{6 d}$$

Result (type 3, 1045 leaves):

$$\begin{aligned}
& - \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{16384} + \frac{i}{16384} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (3045 + 1015 i) A + 1015 \sqrt{2} A + (3396 + 1132 i) B + 1132 \sqrt{2} B + (3912 + 1304 i) C + 1304 \sqrt{2} C \right) \\
& \text{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c + dx) \right] - \sin \left[ \frac{1}{4} (c + dx) \right] - \sqrt{2} \sin \left[ \frac{1}{4} (c + dx) \right]}{-\cos \left[ \frac{1}{4} (c + dx) \right] + \sqrt{2} \cos \left[ \frac{1}{4} (c + dx) \right] - \sin \left[ \frac{1}{4} (c + dx) \right]} \right] (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 - \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \\
& \left( \frac{1}{16384} - \frac{i}{16384} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-3045 + 1015 i) A + 1015 \sqrt{2} A - (3396 - 1132 i) B + 1132 \sqrt{2} B - (3912 - 1304 i) C + 1304 \sqrt{2} C \right) \\
& \text{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c + dx) \right] + \sin \left[ \frac{1}{4} (c + dx) \right] - \sqrt{2} \sin \left[ \frac{1}{4} (c + dx) \right]}{\cos \left[ \frac{1}{4} (c + dx) \right] + \sqrt{2} \cos \left[ \frac{1}{4} (c + dx) \right] - \sin \left[ \frac{1}{4} (c + dx) \right]} \right] (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 + \frac{1}{8192 (i + \sqrt{2}) d} \\
& (2030 A + 1015 i \sqrt{2} A + 2264 B + 1132 i \sqrt{2} B + 2608 C + 1304 i \sqrt{2} C) (a (1 + \cos [c + dx]))^{5/2} \log \left[ \sqrt{2} + 2 \sin \left[ \frac{1}{2} (c + dx) \right] \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 - \\
& \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \left( \frac{1}{32768} - \frac{i}{32768} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (3045 + 1015 i) A + 1015 \sqrt{2} A + (3396 + 1132 i) B + 1132 \sqrt{2} B + (3912 + 1304 i) C + 1304 \sqrt{2} C \right) \\
& (a (1 + \cos [c + dx]))^{5/2} \log \left[ 2 - \sqrt{2} \cos \left[ \frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + dx) \right] \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 + \frac{1}{\sqrt{2} (i + \sqrt{2}) d} \\
& \left( \frac{1}{32768} + \frac{i}{32768} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-3045 + 1015 i) A + 1015 \sqrt{2} A - (3396 - 1132 i) B + 1132 \sqrt{2} B - (3912 - 1304 i) C + 1304 \sqrt{2} C \right) \\
& (a (1 + \cos [c + dx]))^{5/2} \log \left[ 2 + \sqrt{2} \cos \left[ \frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + dx) \right] \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 + \\
& \frac{1}{983040 d} (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sec [c + dx]^6 \left( -47250 A \sin \left[ \frac{1}{2} (c + dx) \right] - 78120 B \sin \left[ \frac{1}{2} (c + dx) \right] - 96720 C \sin \left[ \frac{1}{2} (c + dx) \right] + \right. \\
& 184490 A \sin \left[ \frac{3}{2} (c + dx) \right] + 167944 B \sin \left[ \frac{3}{2} (c + dx) \right] + 164240 C \sin \left[ \frac{3}{2} (c + dx) \right] + 28275 A \sin \left[ \frac{5}{2} (c + dx) \right] + 13980 B \sin \left[ \frac{5}{2} (c + dx) \right] - \\
& 7560 C \sin \left[ \frac{5}{2} (c + dx) \right] + 88305 A \sin \left[ \frac{7}{2} (c + dx) \right] + 98484 B \sin \left[ \frac{7}{2} (c + dx) \right] + 101160 C \sin \left[ \frac{7}{2} (c + dx) \right] + 5075 A \sin \left[ \frac{9}{2} (c + dx) \right] + \\
& \left. 5660 B \sin \left[ \frac{9}{2} (c + dx) \right] + 6520 C \sin \left[ \frac{9}{2} (c + dx) \right] + 15225 A \sin \left[ \frac{11}{2} (c + dx) \right] + 16980 B \sin \left[ \frac{11}{2} (c + dx) \right] + 19560 C \sin \left[ \frac{11}{2} (c + dx) \right] \right)
\end{aligned}$$

■ **Problem 405: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + dx] + C \cos [c + dx]^2) \sec [c + dx]}{\sqrt{a + a \cos [c + dx]}} dx$$

Optimal (type 3, 118 leaves, 6 steps):

$$\frac{2 A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{\sqrt{a} d}-\frac{\sqrt{2}(A-B+C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{a+a \cos [c+d x]}}\right]}{\sqrt{a} d}+\frac{2 C \sin [c+d x]}{d \sqrt{a+a \cos [c+d x]}}$$

Result (type 3, 2315 leaves):

$$\begin{aligned} & -\left(\left((1-i) A\left(1+e^{i c}\right)\left(\sqrt{2}-\left(1-i\right) e^{\frac{i c}{2}}+\left(16-16 i\right) e^{\frac{3 i c}{2}+i d x}+\left(20+20 i\right) \sqrt{2} e^{2 i c+\frac{3 i d x}{2}}-\left(34-34 i\right) e^{\frac{5 i c}{2}+2 i d x}-\left(20+20 i\right) \sqrt{2} e^{3 i c+\frac{5 i d x}{2}}+\right.\right.\right. \\ & \quad \left.\left.\left(16-16 i\right) e^{\frac{7 i c}{2}+3 i d x}+\left(4+4 i\right) \sqrt{2} e^{4 i c+\frac{7 i d x}{2}}-\left(1-i\right) e^{\frac{9 i c}{2}+4 i d x}+8 i e^{\frac{1}{2} i(c+d x)}-16 \sqrt{2} e^{i(c+d x)}-40 i e^{\frac{3}{2} i(c+d x)}+\right.\right. \\ & \quad \left.\left.34 \sqrt{2} e^{2 i(c+d x)}+40 i e^{\frac{5}{2} i(c+d x)}-16 \sqrt{2} e^{3 i(c+d x)}-8 i e^{\frac{7}{2} i(c+d x)}+\sqrt{2} e^{4 i(c+d x)}-\left(4+4 i\right) \sqrt{2} e^{\frac{1}{2} i(2 c+d x)}\right) \right. \\ & \quad \left. x \cos \left[\frac{c}{2}+\frac{d x}{2}\right] \cos [c+d x]\left(B+C \cos [c+d x]+A \sec [c+d x]\right)\right) / \left(\left(\left(-1-i\right)+\sqrt{2} e^{\frac{i c}{2}}\right)\left(-1+e^{i c}\right)\right. \\ & \quad \left.\left(i-2 \sqrt{2} e^{\frac{1}{2} i(c+d x)}-4 i e^{i(c+d x)}+2 \sqrt{2} e^{\frac{3}{2} i(c+d x)}+i e^{2 i(c+d x)}\right)^2 \sqrt{a(1+\cos [c+d x])}\left(2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]\right)\right) \Bigg) - \\ & \left(\frac{2 i \sqrt{2} A \operatorname{ArcTan}\left[\frac{\cos \left[\frac{c}{4}+\frac{d x}{4}\right]-\sin \left[\frac{c}{4}+\frac{d x}{4}\right]-\sqrt{2} \sin \left[\frac{c}{4}+\frac{d x}{4}\right]}{-\cos \left[\frac{c}{4}+\frac{d x}{4}\right]+\sqrt{2} \cos \left[\frac{c}{4}+\frac{d x}{4}\right]-\sin \left[\frac{c}{4}+\frac{d x}{4}\right]}\right] \cos \left[\frac{c}{2}+\frac{d x}{2}\right] \cos [c+d x]\left(B+C \cos [c+d x]+A \sec [c+d x]\right)}{\left(d \sqrt{a(1+\cos [c+d x])}\left(2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]\right)\right)}\right) / \\ & \frac{4(A-B+C) \cos \left[\frac{c}{2}+\frac{d x}{2}\right] \cos [c+d x] \operatorname{Log}\left[\cos \left[\frac{c}{4}+\frac{d x}{4}\right]-\sin \left[\frac{c}{4}+\frac{d x}{4}\right]\right]\left(B+C \cos [c+d x]+A \sec [c+d x]\right)}{d \sqrt{a(1+\cos [c+d x])}\left(2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]\right)} - \\ & \frac{4(A-B+C) \cos \left[\frac{c}{2}+\frac{d x}{2}\right] \cos [c+d x] \operatorname{Log}\left[\cos \left[\frac{c}{4}+\frac{d x}{4}\right]+\sin \left[\frac{c}{4}+\frac{d x}{4}\right]\right]\left(B+C \cos [c+d x]+A \sec [c+d x]\right)}{d \sqrt{a(1+\cos [c+d x])}\left(2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]\right)} - \\ & \left(\frac{\sqrt{2} A \cos \left[\frac{c}{2}+\frac{d x}{2}\right] \cos [c+d x] \operatorname{Log}\left[2-\sqrt{2} \cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sqrt{2} \sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right]\left(B+C \cos [c+d x]+A \sec [c+d x]\right)}{\left(d \sqrt{a(1+\cos [c+d x])}\left(2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]\right)\right)}\right) / \\ & \frac{8 C \cos \left[\frac{d x}{2}\right] \cos \left[\frac{c}{2}+\frac{d x}{2}\right] \cos [c+d x]\left(B+C \cos [c+d x]+A \sec [c+d x]\right) \sin \left[\frac{c}{2}\right]}{d \sqrt{a(1+\cos [c+d x])}\left(2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]\right)} + \\ & \left(\frac{(1-i) \operatorname{ArcTan}\left[\frac{\cos \left[\frac{c}{4}+\frac{d x}{4}\right]+\sin \left[\frac{c}{4}+\frac{d x}{4}\right]-\sqrt{2} \sin \left[\frac{c}{4}+\frac{d x}{4}\right]}{\cos \left[\frac{c}{4}+\frac{d x}{4}\right]+\sqrt{2} \cos \left[\frac{c}{4}+\frac{d x}{4}\right]-\sin \left[\frac{c}{4}+\frac{d x}{4}\right]}\right] \cos \left[\frac{c}{2}+\frac{d x}{2}\right] \cos [c+d x]\left(B+C \cos [c+d x]+A \sec [c+d x]\right)}{\left(\left(1+i\right) \cos \left[\frac{c}{4}\right]+\sqrt{2} \cos \left[\frac{c}{4}\right]-\left(1-i\right) \sin \left[\frac{c}{4}\right]-i \sqrt{2} \sin \left[\frac{c}{4}\right]\right)\left(\left(-1-i\right) A \cos \left[\frac{c}{4}\right]+\sqrt{2} A \cos \left[\frac{c}{4}\right]+\left(1-i\right) A \sin \left[\frac{c}{4}\right]-i \sqrt{2} A \sin \left[\frac{c}{4}\right]\right)}\right) / \\ & \left(\sqrt{2} d \sqrt{a(1+\cos [c+d x])}\left(2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]\right)\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\right) - \end{aligned}$$

$$\begin{aligned}
& \left( \left( \frac{1}{2} + \frac{i}{2} \right) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx] \log \left[ 2 + \sqrt{2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sqrt{2} \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right] (B + C \cos [c + dx] + A \sec [c + dx]) \right. \\
& \quad \left. \left( (1 + i) \cos \left[ \frac{c}{4} \right] + \sqrt{2} \cos \left[ \frac{c}{4} \right] - (1 - i) \sin \left[ \frac{c}{4} \right] - i \sqrt{2} \sin \left[ \frac{c}{4} \right] \right) \left( (-1 - i) A \cos \left[ \frac{c}{4} \right] + \sqrt{2} A \cos \left[ \frac{c}{4} \right] + (1 - i) A \sin \left[ \frac{c}{4} \right] - i \sqrt{2} A \sin \left[ \frac{c}{4} \right] \right) \right) / \\
& \quad \left( \sqrt{2} d \sqrt{a (1 + \cos [c + dx])} (2A + C + 2B \cos [c + dx] + C \cos [2c + 2dx]) \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right) \right) - \\
& \quad \left( 8i A \operatorname{ArcTan} \left[ \frac{2i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx] \cot \left[ \frac{c}{2} \right] (B + C \cos [c + dx] + A \sec [c + dx]) \right) / \\
& \quad \left( d \sqrt{a (1 + \cos [c + dx])} (2A + C + 2B \cos [c + dx] + C \cos [2c + 2dx]) \sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2} \right) + \\
& \quad \left( 4 \sqrt{2} A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx] \operatorname{Csc} \left[ \frac{c}{2} \right] (B + C \cos [c + dx] + A \sec [c + dx]) \right) \\
& \quad \left( -dx \cos \left[ \frac{c}{2} \right] + 2 \log \left[ \sqrt{2} + 2 \cos \left[ \frac{dx}{2} \right] \sin \left[ \frac{c}{2} \right] + 2 \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{dx}{2} \right] \right] \sin \left[ \frac{c}{2} \right] + \frac{4i \sqrt{2} \operatorname{ArcTan} \left[ \frac{2i \cos \left[ \frac{c}{2} \right] - i \left( -\sqrt{2} + 2 \sin \left[ \frac{c}{2} \right] \right) \tan \left[ \frac{dx}{4} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right] \cos \left[ \frac{c}{2} \right]}{\sqrt{-2 + 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2}} \right) / \\
& \quad \left( d \sqrt{a (1 + \cos [c + dx])} (2A + C + 2B \cos [c + dx] + C \cos [2c + 2dx]) \left( 4 \cos \left[ \frac{c}{2} \right]^2 + 4 \sin \left[ \frac{c}{2} \right]^2 \right) \right) + \\
& \quad \frac{8C \cos \left[ \frac{c}{2} \right] \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \cos [c + dx] (B + C \cos [c + dx] + A \sec [c + dx]) \sin \left[ \frac{dx}{2} \right]}{d \sqrt{a (1 + \cos [c + dx])} (2A + C + 2B \cos [c + dx] + C \cos [2c + 2dx])}
\end{aligned}$$

■ **Problem 406: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + dx] + C \cos [c + dx]^2) \sec [c + dx]^2}{\sqrt{a + a \cos [c + dx]}} dx$$

Optimal (type 3, 120 leaves, 6 steps):



$$-\frac{(A-2B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} + \frac{\sqrt{2} (A-B+C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} + \frac{A \tan[c+dx]}{d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 588 leaves):

$$\left( \cos\left[\frac{1}{2}(c+dx)\right] \cos[c+dx]^2 (C+B \sec[c+dx] + A \sec[c+dx]^2) \right.$$

$$\left. -8(A-B+C) \log\left[\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right] + 8(A-B+C) \log\left[\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right] - \right.$$

$$2\sqrt{2}(A-2B) \log\left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(c+dx)\right]\right] - \frac{2i(A-2B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right)}{-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]} -$$

$$\frac{2i(A-2B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right)}{-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]} -$$

$$\frac{(A-2B) \log\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right)}{-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]} -$$

$$\frac{(A-2B) \log\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right)}{-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]} +$$

$$\left. \left. \left. \frac{4A}{\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]} - \frac{4A}{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]} \right) \right) \right)$$

$$\left(2d\sqrt{a(1+\cos[c+dx])} (2A+C+2B\cos[c+dx] + C\cos[2(c+dx)])\right)$$

■ **Problem 407: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^3}{\sqrt{a + a \cos[c + dx]}} dx$$

Optimal (type 3, 169 leaves, 7 steps):

$$\frac{(7A - 4B + 8C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{4\sqrt{a}d} - \frac{\sqrt{2}(A - B + C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2}\sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a}d} - \frac{(A - 4B) \tan[c + dx]}{4d\sqrt{a + a \cos[c + dx]}} + \frac{A \sec[c + dx] \tan[c + dx]}{2d\sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 689 leaves):

$$\frac{1}{d\sqrt{a(1 + \cos[c + dx])}}$$

$$\left( \frac{1}{64} + \frac{i}{64} \right) \cos\left[\frac{1}{2}(c + dx)\right] \left( \frac{2i\sqrt{2}((-3 + i) + \sqrt{2})(1 + i) + \sqrt{2}}{i + \sqrt{2}} (7A - 4B + 8C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2})\sin\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2})\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right]}{i + \sqrt{2}} - \right.$$

$$\frac{2\sqrt{2}((-1 + i) + \sqrt{2})(3 + i) + \sqrt{2}}{i + \sqrt{2}} (7A - 4B + 8C) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2})\sin\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2})\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] +$$

$$(64 - 64i)(A - B + C) \operatorname{Log}\left[\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]\right] - (64 - 64i)(A - B + C) \operatorname{Log}\left[\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right]\right] +$$

$$\frac{(4 + 4i)(-2i + \sqrt{2})(7A - 4B + 8C) \operatorname{Log}\left[\sqrt{2} + 2\sin\left[\frac{1}{2}(c + dx)\right]\right]}{i + \sqrt{2}} + \frac{1}{i + \sqrt{2}}$$

$$i\sqrt{2}((-1 + i) + \sqrt{2})(3 + i) + \sqrt{2} (7A - 4B + 8C) \operatorname{Log}\left[2 - \sqrt{2}\cos\left[\frac{1}{2}(c + dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c + dx)\right]\right] + \frac{1}{i + \sqrt{2}}$$

$$\sqrt{2}((-3 + i) + \sqrt{2})(1 + i) + \sqrt{2} (7A - 4B + 8C) \operatorname{Log}\left[2 + \sqrt{2}\cos\left[\frac{1}{2}(c + dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c + dx)\right]\right] - \frac{(8 - 8i)(A - 4B)}{\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]} +$$

$$\left. \frac{(16 - 16i)A \sin\left[\frac{1}{2}(c + dx)\right]}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{(16 - 16i)A \sin\left[\frac{1}{2}(c + dx)\right]}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{(8 - 8i)(A - 4B)}{\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]} \right)$$

■ **Problem 408: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^4}{\sqrt{a + a \cos[c + dx]}} dx$$

Optimal (type 3, 213 leaves, 8 steps):

$$\begin{aligned} & - \frac{(9A - 14B + 8C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{8\sqrt{a}d} + \frac{\sqrt{2}(A - B + C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2}\sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a}d} \\ & + \frac{(7A - 2B + 8C) \tan[c + dx]}{8d\sqrt{a + a \cos[c + dx]}} - \frac{(A - 6B) \sec[c + dx] \tan[c + dx]}{12d\sqrt{a + a \cos[c + dx]}} + \frac{A \sec[c + dx]^2 \tan[c + dx]}{3d\sqrt{a + a \cos[c + dx]}} \end{aligned}$$

Result (type 3, 1310 leaves):

$$\begin{aligned} & - \left( \left( \frac{1}{32} - \frac{i}{32} \right) \left( (1+i) - i\sqrt{2} \right) \left( (27+9i)A + 9\sqrt{2}A - (42+14i)B - 14\sqrt{2}B + (24+8i)C + 8\sqrt{2}C \right) \right. \\ & \quad \left. \operatorname{ArcTan}\left[ \frac{\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{-\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \right) / \left( \sqrt{2}(i+\sqrt{2})d\sqrt{a(1+\cos[c+dx])} \right) + \\ & \left( \left( \frac{1}{32} - \frac{i}{32} \right) \left( (1+i) + \sqrt{2} \right) \left( (-27+9i)A + 9\sqrt{2}A + (42-14i)B - 14\sqrt{2}B - (24-8i)C + 8\sqrt{2}C \right) \right. \\ & \quad \left. \operatorname{ArcTan}\left[ \frac{\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \right) / \left( \sqrt{2}(i+\sqrt{2})d\sqrt{a(1+\cos[c+dx])} \right) - \\ & \frac{2(A-B+C)\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right]}{d\sqrt{a(1+\cos[c+dx])}} + \frac{2(A-B+C)\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right]}{d\sqrt{a(1+\cos[c+dx])}} + \\ & \frac{(-18A - 9i\sqrt{2}A + 28B + 14i\sqrt{2}B - 16C - 8i\sqrt{2}C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Log}\left[\sqrt{2} + 2\sin\left[\frac{1}{2}(c+dx)\right]\right]}{16(i+\sqrt{2})d\sqrt{a(1+\cos[c+dx])}} + \\ & \left( \left( \frac{1}{64} + \frac{i}{64} \right) \left( (1+i) - i\sqrt{2} \right) \left( (27+9i)A + 9\sqrt{2}A - (42+14i)B - 14\sqrt{2}B + (24+8i)C + 8\sqrt{2}C \right) \right. \\ & \quad \left. \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Log}\left[2 - \sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right] \right) / \left( \sqrt{2}(i+\sqrt{2})d\sqrt{a(1+\cos[c+dx])} \right) - \\ & \left( \left( \frac{1}{64} + \frac{i}{64} \right) \left( (1+i) + \sqrt{2} \right) \left( (-27+9i)A + 9\sqrt{2}A + (42-14i)B - 14\sqrt{2}B - (24-8i)C + 8\sqrt{2}C \right) \right. \\ & \quad \left. \operatorname{Log}\left[2 + \sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right] \right) / \left( \sqrt{2}(i+\sqrt{2})d\sqrt{a(1+\cos[c+dx])} \right) + \end{aligned}$$

$$\frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]}{6d\sqrt{a(1+\cos[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{(7A - 2B + 8C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]}{8d\sqrt{a(1+\cos[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)} -$$

$$\frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]}{6d\sqrt{a(1+\cos[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} +$$

$$\frac{(-7A + 2B - 8C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]}{8d\sqrt{a(1+\cos[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)} +$$

$$\frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \left(-A \sin\left[\frac{1}{2}(c+dx)\right] + 2B \sin\left[\frac{1}{2}(c+dx)\right]\right)}{4d\sqrt{a(1+\cos[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} +$$

$$\frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \left(-A \sin\left[\frac{1}{2}(c+dx)\right] + 2B \sin\left[\frac{1}{2}(c+dx)\right]\right)}{4d\sqrt{a(1+\cos[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}$$

■ **Problem 409: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^5}{\sqrt{a + a \cos[c + dx]}} dx$$

Optimal (type 3, 259 leaves, 9 steps):

$$\frac{(107A - 72B + 112C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{64\sqrt{a}d} - \frac{\sqrt{2}(A - B + C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2}\sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a}d} - \frac{(21A - 56B + 16C) \tan[c + dx]}{64d\sqrt{a+a \cos[c + dx]}} +$$

$$\frac{(43A - 8B + 48C) \sec[c + dx] \tan[c + dx]}{96d\sqrt{a+a \cos[c + dx]}} - \frac{(A - 8B) \sec[c + dx]^2 \tan[c + dx]}{24d\sqrt{a+a \cos[c + dx]}} + \frac{A \sec[c + dx]^3 \tan[c + dx]}{4d\sqrt{a+a \cos[c + dx]}}$$

Result (type 3, 1476 leaves):

$$-\left(\frac{1}{256} + \frac{i}{256}\right) \left((-1 + i) + \sqrt{2}\right) \left((321 + 107i)A + 107\sqrt{2}A - (216 + 72i)B - 72\sqrt{2}B + (336 + 112i)C + 112\sqrt{2}C\right)$$

$$\operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{-\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \Big/ \left(\sqrt{2}(i + \sqrt{2})d\sqrt{a(1+\cos[c+dx])}\right) -$$

$$\left(\frac{1}{256} - \frac{i}{256}\right) \left((1 + i) + \sqrt{2}\right) \left((-321 + 107i)A + 107\sqrt{2}A + (216 - 72i)B - 72\sqrt{2}B - (336 - 112i)C + 112\sqrt{2}C\right)$$

$$\begin{aligned}
& \text{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c + dx) \right] + \sin \left[ \frac{1}{4} (c + dx) \right] - \sqrt{2} \sin \left[ \frac{1}{4} (c + dx) \right]}{\cos \left[ \frac{1}{4} (c + dx) \right] + \sqrt{2} \cos \left[ \frac{1}{4} (c + dx) \right] - \sin \left[ \frac{1}{4} (c + dx) \right]} \right] \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \Big/ \left( \sqrt{2} (i + \sqrt{2}) d \sqrt{a (1 + \cos [c + dx])} \right) + \\
& \frac{2 (A - B + C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \log \left[ \cos \left[ \frac{1}{4} (c + dx) \right] - \sin \left[ \frac{1}{4} (c + dx) \right] \right]}{d \sqrt{a (1 + \cos [c + dx])}} - \frac{2 (A - B + C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \log \left[ \cos \left[ \frac{1}{4} (c + dx) \right] + \sin \left[ \frac{1}{4} (c + dx) \right] \right]}{d \sqrt{a (1 + \cos [c + dx])}} + \\
& \frac{\left( 214 A + 107 i \sqrt{2} A - 144 B - 72 i \sqrt{2} B + 224 C + 112 i \sqrt{2} C \right) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \log \left[ \sqrt{2} + 2 \sin \left[ \frac{1}{2} (c + dx) \right] \right]}{128 (i + \sqrt{2}) d \sqrt{a (1 + \cos [c + dx])}} - \\
& \left( \left( \frac{1}{512} - \frac{i}{512} \right) \left( (-1 + i) + \sqrt{2} \right) \left( (321 + 107 i) A + 107 \sqrt{2} A - (216 + 72 i) B - 72 \sqrt{2} B + (336 + 112 i) C + 112 \sqrt{2} C \right) \right. \\
& \quad \left. \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \log \left[ 2 - \sqrt{2} \cos \left[ \frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + dx) \right] \right] \right) \Big/ \left( \sqrt{2} (i + \sqrt{2}) d \sqrt{a (1 + \cos [c + dx])} \right) + \\
& \left( \left( \frac{1}{512} + \frac{i}{512} \right) \left( (1 + i) + \sqrt{2} \right) \left( (-321 + 107 i) A + 107 \sqrt{2} A + (216 - 72 i) B - 72 \sqrt{2} B - (336 - 112 i) C + 112 \sqrt{2} C \right) \right. \\
& \quad \left. \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \log \left[ 2 + \sqrt{2} \cos \left[ \frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + dx) \right] \right] \right) \Big/ \left( \sqrt{2} (i + \sqrt{2}) d \sqrt{a (1 + \cos [c + dx])} \right) + \\
& \frac{(-A + 8 B) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]}{48 d \sqrt{a (1 + \cos [c + dx])} \left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right)^3} + \frac{(-21 A + 56 B - 16 C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]}{64 d \sqrt{a (1 + \cos [c + dx])} \left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right)} + \\
& \frac{A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \sin \left[ \frac{1}{2} (c + dx) \right]}{8 d \sqrt{a (1 + \cos [c + dx])} \left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right)^4} + \\
& \frac{A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \sin \left[ \frac{1}{2} (c + dx) \right]}{8 d \sqrt{a (1 + \cos [c + dx])} \left( \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right)^4} + \\
& \frac{(A - 8 B) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]}{48 d \sqrt{a (1 + \cos [c + dx])} \left( \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right)^3} + \\
& \frac{(21 A - 56 B + 16 C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]}{64 d \sqrt{a (1 + \cos [c + dx])} \left( \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right)} + \\
& \frac{\cos \left[ \frac{c}{2} + \frac{dx}{2} \right] \left( 19 A \sin \left[ \frac{1}{2} (c + dx) \right] - 8 B \sin \left[ \frac{1}{2} (c + dx) \right] + 16 C \sin \left[ \frac{1}{2} (c + dx) \right] \right)}{32 d \sqrt{a (1 + \cos [c + dx])} \left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right)^2} +
\end{aligned}$$

$$\frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \left(19 A \sin\left[\frac{1}{2}(c+dx)\right] - 8 B \sin\left[\frac{1}{2}(c+dx)\right] + 16 C \sin\left[\frac{1}{2}(c+dx)\right]\right)}{32 d \sqrt{a(1+\cos[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}$$

- **Problem 414: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c+dx] + C \cos[c+dx]^2) \sec[c+dx]}{(a + a \cos[c+dx])^{3/2}} dx$$

Optimal (type 3, 131 leaves, 6 steps):

$$\frac{2 A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{3/2} d} - \frac{(5 A - B - 3 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{(A - B + C) \sin[c+dx]}{2 d (a + a \cos[c+dx])^{3/2}}$$

Result (type 3, 2385 leaves):

$$\begin{aligned} & - \left( \left( (2 - 2i) A (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - \right. \right. \right. \\ & \quad (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16 \sqrt{2} e^{i(c+dx)} - \\ & \quad \left. \left. 40i e^{\frac{3}{2}i(c+dx)} + 34 \sqrt{2} e^{2i(c+dx)} + 40i e^{\frac{5}{2}i(c+dx)} - 16 \sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) \right. \\ & \quad \left. \times \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] (B + C \cos[c+dx] + A \sec[c+dx]) \right) / \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \right. \\ & \quad \left. \left( i - 2 \sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2 \sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 (a(1 + \cos[c+dx]))^{3/2} (2A + C + 2B \cos[c+dx] + C \cos[2c + 2dx]) \right) \Big) - \\ & \left( 4i \sqrt{2} A \operatorname{ArcTan}\left[ \frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] (B + C \cos[c+dx] + A \sec[c+dx]) \right) / \\ & \quad (d (a(1 + \cos[c+dx]))^{3/2} (2A + C + 2B \cos[c+dx] + C \cos[2c + 2dx])) + \\ & \left( 2(5A - B - 3C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (B + C \cos[c+dx] + A \sec[c+dx]) \right) / \\ & \quad (d (a(1 + \cos[c+dx]))^{3/2} (2A + C + 2B \cos[c+dx] + C \cos[2c + 2dx])) - \\ & \left( 2(5A - B - 3C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (B + C \cos[c+dx] + A \sec[c+dx]) \right) / \\ & \quad (d (a(1 + \cos[c+dx]))^{3/2} (2A + C + 2B \cos[c+dx] + C \cos[2c + 2dx])) - \\ & \left( 2 \sqrt{2} A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (B + C \cos[c+dx] + A \sec[c+dx]) \right) / \\ & \quad (d (a(1 + \cos[c+dx]))^{3/2} (2A + C + 2B \cos[c+dx] + C \cos[2c + 2dx])) + \\ & \left( (1 - i) \sqrt{2} \operatorname{ArcTan}\left[ \frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c+dx] (B + C \cos[c+dx] + A \sec[c+dx]) \right) \end{aligned}$$

$$\begin{aligned}
& \left( (1+i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1-i) \sin\left[\frac{c}{4}\right] - i\sqrt{2} \sin\left[\frac{c}{4}\right] \right) \left( (-1-i) A \cos\left[\frac{c}{4}\right] + \sqrt{2} A \cos\left[\frac{c}{4}\right] + (1-i) A \sin\left[\frac{c}{4}\right] - i\sqrt{2} A \sin\left[\frac{c}{4}\right] \right) \Big/ \\
& \left( d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right) - \\
& \left( (1+i) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx] \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (B + C \cos[c + dx] + A \operatorname{Sec}[c + dx]) \right. \\
& \quad \left. \left( (1+i) \cos\left[\frac{c}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4}\right] - (1-i) \sin\left[\frac{c}{4}\right] - i\sqrt{2} \sin\left[\frac{c}{4}\right] \right) \left( (-1-i) A \cos\left[\frac{c}{4}\right] + \sqrt{2} A \cos\left[\frac{c}{4}\right] + (1-i) A \sin\left[\frac{c}{4}\right] - i\sqrt{2} A \sin\left[\frac{c}{4}\right] \right) \right) \Big/ \\
& \left( \sqrt{2} d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right) - \\
& \left( 16 i A \operatorname{ArcTan}\left[ \frac{2 i \cos\left[\frac{c}{2}\right] - i (-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx] \operatorname{Cot}\left[\frac{c}{2}\right] (B + C \cos[c + dx] + A \operatorname{Sec}[c + dx]) \right) \Big/ \\
& \left( d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2} \right) + \\
& \left( 8 \sqrt{2} A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] (B + C \cos[c + dx] + A \operatorname{Sec}[c + dx]) \right) \\
& \left( -dx \cos\left[\frac{c}{2}\right] + 2 \operatorname{Log}\left[\sqrt{2} + 2 \cos\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] \right) \sin\left[\frac{c}{2}\right] + \frac{4 i \sqrt{2} \operatorname{ArcTan}\left[ \frac{2 i \cos\left[\frac{c}{2}\right] - i (-\sqrt{2} + 2 \sin\left[\frac{c}{2}\right]) \operatorname{Tan}\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2}} \right) \Big/ \\
& \left( d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left( 4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2 \right) \right) + \\
& \frac{(-A + B - C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx] (B + C \cos[c + dx] + A \operatorname{Sec}[c + dx])}{d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left( \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] \right)^2} +
\end{aligned}$$

$$\frac{(A - B + C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx] (B + C \cos[c + dx] + A \sec[c + dx])}{d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2}$$

■ **Problem 415: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^2}{(a + a \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 173 leaves, 7 steps):

$$-\frac{(3A - 2B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{3/2} d} + \frac{(9A - 5B + C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{2\sqrt{2} a^{3/2} d} - \frac{(A - B + C) \tan[c + dx]}{2d (a + a \cos[c + dx])^{3/2}} + \frac{(3A - B + C) \tan[c + dx]}{2ad \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 1661 leaves):

$$\begin{aligned} & - \left( 2 (9A - 5B + C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C + B \sec[c + dx] + A \sec[c + dx]^2) \right) / \\ & \quad \left( d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \right) + \\ & \left( 2 (9A - 5B + C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C + B \sec[c + dx] + A \sec[c + dx]^2) \right) / \\ & \quad \left( d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \right) - \\ & \left( 2\sqrt{2} (3A - 2B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + B \sec[c + dx] + A \sec[c + dx]^2) \right) / \\ & \quad \left( d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \right) - \\ & \left( 2i (3A - 2B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 (C + B \sec[c + dx] + A \sec[c + dx]^2) \right. \\ & \quad \left. \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right) \right) / \left( d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) - \\ & \left( 2i (3A - 2B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 (C + B \sec[c + dx] + A \sec[c + dx]^2) \right. \\ & \quad \left. \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right) \right) / \left( d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) - \\ & \left( (3A - 2B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + B \sec[c + dx] + A \sec[c + dx]^2) \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right) \right) / \\ & \quad \left( d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) - \end{aligned}$$



$$\begin{aligned}
& \left( (3A - 2B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + B \operatorname{Sec}[c + dx] + A \operatorname{Sec}[c + dx]^2) \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right) \right) / \\
& \left( d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) + \\
& \frac{(A - B + C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 (C + B \operatorname{Sec}[c + dx] + A \operatorname{Sec}[c + dx]^2)}{d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2} + \\
& \frac{(-A + B - C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 (C + B \operatorname{Sec}[c + dx] + A \operatorname{Sec}[c + dx]^2)}{d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left(\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right)^2} + \\
& \frac{4A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 (C + B \operatorname{Sec}[c + dx] + A \operatorname{Sec}[c + dx]^2)}{d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \\
& \frac{4A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cos[c + dx]^2 (C + B \operatorname{Sec}[c + dx] + A \operatorname{Sec}[c + dx]^2)}{d (a (1 + \cos[c + dx]))^{3/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 416: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^3}{(a + a \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 232 leaves, 8 steps):

$$\frac{(19A - 12B + 8C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right]}{4a^{3/2}d} - \frac{(13A - 9B + 5C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{2} \sqrt{a + a \cos[c + dx]}}\right]}{2\sqrt{2}a^{3/2}d} - \\
\frac{(7A - 6B + 2C) \tan[c + dx]}{4ad\sqrt{a + a \cos[c + dx]}} - \frac{(A - B + C) \operatorname{Sec}[c + dx] \tan[c + dx]}{2d(a + a \cos[c + dx])^{3/2}} + \frac{(2A - B + C) \operatorname{Sec}[c + dx] \tan[c + dx]}{2ad\sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 1302 leaves):

$$-\left(\frac{1}{8} + \frac{i}{8}\right) \left((-1 + i) + \sqrt{2}\right) \left((57 + 19i)A + 19\sqrt{2}A - (36 + 12i)B - 12\sqrt{2}B + (24 + 8i)C + 8\sqrt{2}C\right) \\
\operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c + dx)\right]}{-\cos\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 / \left(\sqrt{2} (i + \sqrt{2}) d (a (1 + \cos[c + dx]))^{3/2}\right) - \\
\left(\frac{1}{8} - \frac{i}{8}\right) \left((1 + i) + \sqrt{2}\right) \left((-57 + 19i)A + 19\sqrt{2}A + (36 - 12i)B - 12\sqrt{2}B - (24 - 8i)C + 8\sqrt{2}C\right)$$

$$\begin{aligned}
& \text{ArcTan} \left[ \frac{\cos \left[ \frac{1}{4} (c + dx) \right] + \sin \left[ \frac{1}{4} (c + dx) \right] - \sqrt{2} \sin \left[ \frac{1}{4} (c + dx) \right]}{\cos \left[ \frac{1}{4} (c + dx) \right] + \sqrt{2} \cos \left[ \frac{1}{4} (c + dx) \right] - \sin \left[ \frac{1}{4} (c + dx) \right]} \right] \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \Big/ \left( \sqrt{2} (i + \sqrt{2}) d (a (1 + \cos [c + dx]))^{3/2} \right) + \\
& \frac{(13A - 9B + 5C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \text{Log} \left[ \cos \left[ \frac{1}{4} (c + dx) \right] - \sin \left[ \frac{1}{4} (c + dx) \right] \right]}{d (a (1 + \cos [c + dx]))^{3/2}} + \frac{(-13A + 9B - 5C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \text{Log} \left[ \cos \left[ \frac{1}{4} (c + dx) \right] + \sin \left[ \frac{1}{4} (c + dx) \right] \right]}{d (a (1 + \cos [c + dx]))^{3/2}} + \\
& \frac{(38A + 19i\sqrt{2}A - 24B - 12i\sqrt{2}B + 16C + 8i\sqrt{2}C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \text{Log} \left[ \sqrt{2} + 2 \sin \left[ \frac{1}{2} (c + dx) \right] \right]}{4 (i + \sqrt{2}) d (a (1 + \cos [c + dx]))^{3/2}} - \\
& \left( \left( \frac{1}{16} - \frac{i}{16} \right) ((-1 + i) + \sqrt{2}) \left( (57 + 19i)A + 19\sqrt{2}A - (36 + 12i)B - 12\sqrt{2}B + (24 + 8i)C + 8\sqrt{2}C \right) \right. \\
& \left. \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \text{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + dx) \right] \right] \right) \Big/ \left( \sqrt{2} (i + \sqrt{2}) d (a (1 + \cos [c + dx]))^{3/2} \right) + \\
& \left( \left( \frac{1}{16} + \frac{i}{16} \right) ((1 + i) + \sqrt{2}) \left( (-57 + 19i)A + 19\sqrt{2}A + (36 - 12i)B - 12\sqrt{2}B - (24 - 8i)C + 8\sqrt{2}C \right) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \right. \\
& \left. \text{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[ \frac{1}{2} (c + dx) \right] \right] \right) \Big/ \left( \sqrt{2} (i + \sqrt{2}) d (a (1 + \cos [c + dx]))^{3/2} \right) + \\
& \frac{(-A + B - C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{2 d (a (1 + \cos [c + dx]))^{3/2} \left( \cos \left[ \frac{1}{4} (c + dx) \right] - \sin \left[ \frac{1}{4} (c + dx) \right] \right)^2} + \frac{(A - B + C) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{2 d (a (1 + \cos [c + dx]))^{3/2} \left( \cos \left[ \frac{1}{4} (c + dx) \right] + \sin \left[ \frac{1}{4} (c + dx) \right] \right)^2} + \\
& \frac{(-5A + 4B) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{2 d (a (1 + \cos [c + dx]))^{3/2} \left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right)} + \\
& \frac{A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sin \left[ \frac{1}{2} (c + dx) \right]}{d (a (1 + \cos [c + dx]))^{3/2} \left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right)^2} + \\
& \frac{A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 \sin \left[ \frac{1}{2} (c + dx) \right]}{d (a (1 + \cos [c + dx]))^{3/2} \left( \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right)^2} + \\
& \frac{(5A - 4B) \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^3}{2 d (a (1 + \cos [c + dx]))^{3/2} \left( \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right)}
\end{aligned}$$

■ **Problem 417: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + dx] + C \cos [c + dx]^2) \sec [c + dx]^4}{(a + a \cos [c + dx])^{3/2}} dx$$

Optimal (type 3, 284 leaves, 9 steps):

$$\begin{aligned}
& - \frac{(47 A - 38 B + 24 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{8 a^{3/2} d} + \frac{(17 A - 13 B + 9 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} + \frac{(21 A - 14 B + 12 C) \tan[c+dx]}{8 a d \sqrt{a+a \cos[c+dx]}} - \\
& \frac{(13 A - 12 B + 6 C) \sec[c+dx] \tan[c+dx]}{12 a d \sqrt{a+a \cos[c+dx]}} - \frac{(A - B + C) \sec[c+dx]^2 \tan[c+dx]}{2 d (a+a \cos[c+dx])^{3/2}} + \frac{(5 A - 3 B + 3 C) \sec[c+dx]^2 \tan[c+dx]}{6 a d \sqrt{a+a \cos[c+dx]}}
\end{aligned}$$

Result (type 3, 1476 leaves):

$$\begin{aligned}
& - \left( \left( \frac{1}{16} - \frac{i}{16} \right) \left( (1+i) - i\sqrt{2} \right) \left( (141+47i)A + 47\sqrt{2}A - (114+38i)B - 38\sqrt{2}B + (72+24i)C + 24\sqrt{2}C \right) \right. \\
& \quad \left. \operatorname{ArcTan}\left[ \frac{\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{-\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \right) / \left( \sqrt{2} (i+\sqrt{2}) d (a(1+\cos[c+dx]))^{3/2} \right) + \\
& \left( \left( \frac{1}{16} - \frac{i}{16} \right) \left( (1+i) + \sqrt{2} \right) \left( (-141+47i)A + 47\sqrt{2}A + (114-38i)B - 38\sqrt{2}B - (72-24i)C + 24\sqrt{2}C \right) \right. \\
& \quad \left. \operatorname{ArcTan}\left[ \frac{\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \right) / \\
& \quad \left( \sqrt{2} (i+\sqrt{2}) d (a(1+\cos[c+dx]))^{3/2} \right) + \frac{(-17A+13B-9C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right]}{d (a(1+\cos[c+dx]))^{3/2}} + \\
& \frac{(17A-13B+9C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right]}{d (a(1+\cos[c+dx]))^{3/2}} + \\
& \frac{(-94A-47i\sqrt{2}A+76B+38i\sqrt{2}B-48C-24i\sqrt{2}C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[\sqrt{2}+2\sin\left[\frac{1}{2}(c+dx)\right]\right]}{8 (i+\sqrt{2}) d (a(1+\cos[c+dx]))^{3/2}} + \\
& \left( \left( \frac{1}{32} + \frac{i}{32} \right) \left( (1+i) - i\sqrt{2} \right) \left( (141+47i)A + 47\sqrt{2}A - (114+38i)B - 38\sqrt{2}B + (72+24i)C + 24\sqrt{2}C \right) \right. \\
& \quad \left. \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Log}\left[2-\sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right] \right) / \left( \sqrt{2} (i+\sqrt{2}) d (a(1+\cos[c+dx]))^{3/2} \right) - \\
& \left( \left( \frac{1}{32} + \frac{i}{32} \right) \left( (1+i) + \sqrt{2} \right) \left( (-141+47i)A + 47\sqrt{2}A + (114-38i)B - 38\sqrt{2}B - (72-24i)C + 24\sqrt{2}C \right) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \right. \\
& \quad \left. \operatorname{Log}\left[2+\sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right] \right) / \left( \sqrt{2} (i+\sqrt{2}) d (a(1+\cos[c+dx]))^{3/2} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{(A - B + C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2d (a (1 + \operatorname{Cos}[c + dx]))^{3/2} \left(\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]\right)^2} + \frac{(-A + B - C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{2d (a (1 + \operatorname{Cos}[c + dx]))^{3/2} \left(\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]\right)^2} + \\
& \frac{A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{3d (a (1 + \operatorname{Cos}[c + dx]))^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(17A - 10B + 8C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{4d (a (1 + \operatorname{Cos}[c + dx]))^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)} - \\
& \frac{A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{3d (a (1 + \operatorname{Cos}[c + dx]))^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(-17A + 10B - 8C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{4d (a (1 + \operatorname{Cos}[c + dx]))^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)} + \\
& \frac{\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(-3A \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + 2B \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)}{2d (a (1 + \operatorname{Cos}[c + dx]))^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(-3A \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + 2B \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)}{2d (a (1 + \operatorname{Cos}[c + dx]))^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2}
\end{aligned}$$

■ **Problem 422: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \operatorname{Cos}[c + dx] + C \operatorname{Cos}[c + dx]^2) \operatorname{Sec}[c + dx]}{(a + a \operatorname{Cos}[c + dx])^{5/2}} dx$$

Optimal (type 3, 171 leaves, 7 steps):

$$\frac{2A \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{a^{5/2} d} - \frac{(43A - 3B - 5C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A - B + C) \operatorname{Sin}[c + dx]}{4d (a + a \operatorname{Cos}[c + dx])^{5/2}} - \frac{(11A - 3B - 5C) \operatorname{Sin}[c + dx]}{16ad (a + a \operatorname{Cos}[c + dx])^{3/2}}$$

Result (type 3, 2329 leaves):

$$\begin{aligned}
& - \left( (4 - 4i) A (1 + e^{ic}) \left( \sqrt{2} - (1 - i) e^{\frac{ic}{2}} + (16 - 16i) e^{\frac{3ic}{2} + idx} + (20 + 20i) \sqrt{2} e^{2ic + \frac{3idx}{2}} - (34 - 34i) e^{\frac{5ic}{2} + 2idx} - \right. \right. \\
& \quad (20 + 20i) \sqrt{2} e^{3ic + \frac{5idx}{2}} + (16 - 16i) e^{\frac{7ic}{2} + 3idx} + (4 + 4i) \sqrt{2} e^{4ic + \frac{7idx}{2}} - (1 - i) e^{\frac{9ic}{2} + 4idx} + 8i e^{\frac{1}{2}i(c+dx)} - 16\sqrt{2} e^{i(c+dx)} - \\
& \quad \left. 40i e^{\frac{3}{2}i(c+dx)} + 34\sqrt{2} e^{2i(c+dx)} + 40i e^{\frac{5}{2}i(c+dx)} - 16\sqrt{2} e^{3i(c+dx)} - 8i e^{\frac{7}{2}i(c+dx)} + \sqrt{2} e^{4i(c+dx)} - (4 + 4i) \sqrt{2} e^{\frac{1}{2}i(2c+dx)} \right) \\
& \quad \left. x \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Cos}[c + dx] (B + C \operatorname{Cos}[c + dx] + A \operatorname{Sec}[c + dx]) \right) / \left( \left( (-1 - i) + \sqrt{2} e^{\frac{ic}{2}} \right) (-1 + e^{ic}) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( i - 2\sqrt{2} e^{\frac{1}{2}i(c+dx)} - 4i e^{i(c+dx)} + 2\sqrt{2} e^{\frac{3}{2}i(c+dx)} + i e^{2i(c+dx)} \right)^2 (a(1+\cos[c+dx]))^{5/2} (2A+C+2B\cos[c+dx]+C\cos[2c+2dx]) \Big) - \\
& \left( 8i\sqrt{2} A \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{c}{4}+\frac{dx}{4}\right] - \sin\left[\frac{c}{4}+\frac{dx}{4}\right] - \sqrt{2}\sin\left[\frac{c}{4}+\frac{dx}{4}\right]}{-\cos\left[\frac{c}{4}+\frac{dx}{4}\right] + \sqrt{2}\cos\left[\frac{c}{4}+\frac{dx}{4}\right] - \sin\left[\frac{c}{4}+\frac{dx}{4}\right]} \right] \cos\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \cos[c+dx] (B+C\cos[c+dx]+A\sec[c+dx]) \right) / \\
& (d(a(1+\cos[c+dx]))^{5/2} (2A+C+2B\cos[c+dx]+C\cos[2c+2dx])) + \\
& \left( (43A-3B-5C) \cos\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \cos[c+dx] \operatorname{Log}\left[\cos\left[\frac{c}{4}+\frac{dx}{4}\right] - \sin\left[\frac{c}{4}+\frac{dx}{4}\right]\right] (B+C\cos[c+dx]+A\sec[c+dx]) \right) / \\
& (2d(a(1+\cos[c+dx]))^{5/2} (2A+C+2B\cos[c+dx]+C\cos[2c+2dx])) + \\
& \left( (-43A+3B+5C) \cos\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \cos[c+dx] \operatorname{Log}\left[\cos\left[\frac{c}{4}+\frac{dx}{4}\right] + \sin\left[\frac{c}{4}+\frac{dx}{4}\right]\right] (B+C\cos[c+dx]+A\sec[c+dx]) \right) / \\
& (2d(a(1+\cos[c+dx]))^{5/2} (2A+C+2B\cos[c+dx]+C\cos[2c+2dx])) - \\
& \left( 4\sqrt{2} A \cos\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \cos[c+dx] \operatorname{Log}\left[2-\sqrt{2}\cos\left[\frac{c}{2}+\frac{dx}{2}\right] - \sqrt{2}\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right] (B+C\cos[c+dx]+A\sec[c+dx]) \right) / \\
& (d(a(1+\cos[c+dx]))^{5/2} (2A+C+2B\cos[c+dx]+C\cos[2c+2dx])) + \\
& \left( (2-2i)\sqrt{2} \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{c}{4}+\frac{dx}{4}\right] + \sin\left[\frac{c}{4}+\frac{dx}{4}\right] - \sqrt{2}\sin\left[\frac{c}{4}+\frac{dx}{4}\right]}{\cos\left[\frac{c}{4}+\frac{dx}{4}\right] + \sqrt{2}\cos\left[\frac{c}{4}+\frac{dx}{4}\right] - \sin\left[\frac{c}{4}+\frac{dx}{4}\right]} \right] \cos\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \cos[c+dx] (B+C\cos[c+dx]+A\sec[c+dx]) \right) \\
& \left( (1+i)\cos\left[\frac{c}{4}\right] + \sqrt{2}\cos\left[\frac{c}{4}\right] - (1-i)\sin\left[\frac{c}{4}\right] - i\sqrt{2}\sin\left[\frac{c}{4}\right] \right) \left( (-1-i)A\cos\left[\frac{c}{4}\right] + \sqrt{2}A\cos\left[\frac{c}{4}\right] + (1-i)A\sin\left[\frac{c}{4}\right] - i\sqrt{2}A\sin\left[\frac{c}{4}\right] \right) / \\
& (d(a(1+\cos[c+dx]))^{5/2} (2A+C+2B\cos[c+dx]+C\cos[2c+2dx])) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) - \\
& (1+i)\sqrt{2}\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \cos[c+dx] \operatorname{Log}\left[2+\sqrt{2}\cos\left[\frac{c}{2}+\frac{dx}{2}\right] - \sqrt{2}\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right] (B+C\cos[c+dx]+A\sec[c+dx]) \\
& \left( (1+i)\cos\left[\frac{c}{4}\right] + \sqrt{2}\cos\left[\frac{c}{4}\right] - (1-i)\sin\left[\frac{c}{4}\right] - i\sqrt{2}\sin\left[\frac{c}{4}\right] \right) \left( (-1-i)A\cos\left[\frac{c}{4}\right] + \sqrt{2}A\cos\left[\frac{c}{4}\right] + (1-i)A\sin\left[\frac{c}{4}\right] - i\sqrt{2}A\sin\left[\frac{c}{4}\right] \right) / \\
& (d(a(1+\cos[c+dx]))^{5/2} (2A+C+2B\cos[c+dx]+C\cos[2c+2dx])) \left( \cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) - \\
& \left( 32iA \operatorname{ArcTan} \left[ \frac{2i\cos\left[\frac{c}{2}\right] - i(-\sqrt{2}+2\sin\left[\frac{c}{2}\right])\tan\left[\frac{dx}{4}\right]}{\sqrt{-2+4\cos\left[\frac{c}{2}\right]^2+4\sin\left[\frac{c}{2}\right]^2}} \right] \cos\left[\frac{c}{2}+\frac{dx}{2}\right]^5 \cos[c+dx] \cot\left[\frac{c}{2}\right] (B+C\cos[c+dx]+A\sec[c+dx]) \right) / \\
& (d(a(1+\cos[c+dx]))^{5/2} (2A+C+2B\cos[c+dx]+C\cos[2c+2dx])) \sqrt{-2+4\cos\left[\frac{c}{2}\right]^2+4\sin\left[\frac{c}{2}\right]^2} +
\end{aligned}$$

$$\left( 16 \sqrt{2} A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c+dx] \csc\left[\frac{c}{2}\right] (B + C \cos[c+dx] + A \sec[c+dx]) \right. \\
\left. - dx \cos\left[\frac{c}{2}\right] + 2 \log\left[\sqrt{2} + 2 \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right] + 2 \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]\right] \sin\left[\frac{c}{2}\right] + \frac{4i\sqrt{2} \operatorname{ArcTan}\left[\frac{2i\cos\left[\frac{c}{2}\right] - i(-\sqrt{2} + 2\sin\left[\frac{c}{2}\right]) \tan\left[\frac{dx}{4}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}}\right] \cos\left[\frac{c}{2}\right]}{\sqrt{-2 + 4\cos\left[\frac{c}{2}\right]^2 + 4\sin\left[\frac{c}{2}\right]^2}} \right) / \\
\left( d (a (1 + \cos[c+dx]))^{5/2} (2A + C + 2B \cos[c+dx] + C \cos[2c + 2dx]) (4 \cos\left[\frac{c}{2}\right]^2 + 4 \sin\left[\frac{c}{2}\right]^2) \right) - \\
\left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c+dx] (B + C \cos[c+dx] + A \sec[c+dx]) \right. \\
\left. \left( 19A \sin\left[\frac{c}{2} + \frac{dx}{2}\right] - 11B \sin\left[\frac{c}{2} + \frac{dx}{2}\right] + 3C \sin\left[\frac{c}{2} + \frac{dx}{2}\right] + 11A \sin\left[\frac{3c}{2} + \frac{3dx}{2}\right] - 3B \sin\left[\frac{3c}{2} + \frac{3dx}{2}\right] - 5C \sin\left[\frac{3c}{2} + \frac{3dx}{2}\right] \right) \right) / \\
(8d (a (1 + \cos[c+dx]))^{5/2} (2A + C + 2B \cos[c+dx] + C \cos[2c + 2dx]))$$

■ **Problem 423: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c+dx] + C \cos[c+dx]^2) \sec[c+dx]^2}{(a + a \cos[c+dx])^{5/2}} dx$$

Optimal (type 3, 217 leaves, 8 steps):

$$-\frac{(5A - 2B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{5/2} d} + \frac{(115A - 43B + 3C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \cos[c+dx]}}\right]}{16\sqrt{2} a^{5/2} d} - \\
\frac{(A - B + C) \tan[c+dx]}{4d (a + a \cos[c+dx])^{5/2}} - \frac{(15A - 7B - C) \tan[c+dx]}{16ad (a + a \cos[c+dx])^{3/2}} + \frac{(35A - 11B + 3C) \tan[c+dx]}{16a^2 d \sqrt{a + a \cos[c+dx]}}$$

Result (type 3, 1411 leaves):

$$\begin{aligned}
& \left( (-115A + 43B - 3C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C + B \operatorname{Sec}[c + dx] + A \operatorname{Sec}[c + dx]^2) \right) / \\
& \left( 2d (a(1 + \cos[c + dx]))^{5/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \right) + \\
& \left( (115A - 43B + 3C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right]\right] (C + B \operatorname{Sec}[c + dx] + A \operatorname{Sec}[c + dx]^2) \right) / \\
& \left( 2d (a(1 + \cos[c + dx]))^{5/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \right) - \\
& \left( 4\sqrt{2} (5A - 2B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx]^2 \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + B \operatorname{Sec}[c + dx] + A \operatorname{Sec}[c + dx]^2) \right) / \\
& \left( d (a(1 + \cos[c + dx]))^{5/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \right) - \\
& \left( 4i (5A - 2B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{-\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx]^2 (C + B \operatorname{Sec}[c + dx] + A \operatorname{Sec}[c + dx]^2) \right. \\
& \left. \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right) \right) / \left( d (a(1 + \cos[c + dx]))^{5/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) - \\
& \left( 4i (5A - 2B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sin\left[\frac{c}{4} + \frac{dx}{4}\right] - \sqrt{2} \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}{\cos\left[\frac{c}{4} + \frac{dx}{4}\right] + \sqrt{2} \cos\left[\frac{c}{4} + \frac{dx}{4}\right] - \sin\left[\frac{c}{4} + \frac{dx}{4}\right]}\right] \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx]^2 (C + B \operatorname{Sec}[c + dx] + A \operatorname{Sec}[c + dx]^2) \right. \\
& \left. \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right) \right) / \left( d (a(1 + \cos[c + dx]))^{5/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) - \\
& \left( 2(5A - 2B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx]^2 \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + B \operatorname{Sec}[c + dx] + A \operatorname{Sec}[c + dx]^2) \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right) \right) / \\
& \left( d (a(1 + \cos[c + dx]))^{5/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) - \\
& \left( 2(5A - 2B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \cos[c + dx]^2 \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sqrt{2} \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (C + B \operatorname{Sec}[c + dx] + A \operatorname{Sec}[c + dx]^2) \left(\sqrt{2} - 2 \sin\left[\frac{c}{2}\right]\right) \right) / \\
& \left( d (a(1 + \cos[c + dx]))^{5/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \left(-1 + \sqrt{2} \sin\left[\frac{c}{2}\right]\right) \right) + \\
& \left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c + dx] (C + B \operatorname{Sec}[c + dx] + A \operatorname{Sec}[c + dx]^2) \left( 24A \sin\left[\frac{c}{2} + \frac{dx}{2}\right] + 8B \sin\left[\frac{c}{2} + \frac{dx}{2}\right] - 8C \sin\left[\frac{c}{2} + \frac{dx}{2}\right] + 75A \sin\left[\frac{3c}{2} + \frac{3dx}{2}\right] - \right. \right. \\
& \left. \left. 19B \sin\left[\frac{3c}{2} + \frac{3dx}{2}\right] + 11C \sin\left[\frac{3c}{2} + \frac{3dx}{2}\right] + 35A \sin\left[\frac{5c}{2} + \frac{5dx}{2}\right] - 11B \sin\left[\frac{5c}{2} + \frac{5dx}{2}\right] + 3C \sin\left[\frac{5c}{2} + \frac{5dx}{2}\right] \right) \right) / \\
& \left( 16d (a(1 + \cos[c + dx]))^{5/2} (2A + C + 2B \cos[c + dx] + C \cos[2c + 2dx]) \right)
\end{aligned}$$

■ **Problem 424: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^3}{(a + a \cos[c + dx])^{5/2}} dx$$

Optimal (type 3, 280 leaves, 9 steps) :

$$\frac{(39 A - 20 B + 8 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{4 a^{5/2} d} - \frac{(219 A - 115 B + 43 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(63 A - 35 B + 11 C) \operatorname{Tan}[c+dx]}{16 a^2 d \sqrt{a+a \operatorname{Cos}[c+dx]}}$$

$$\frac{(A - B + C) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{4 d (a+a \operatorname{Cos}[c+dx])^{5/2}} - \frac{(19 A - 11 B + 3 C) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{16 a d (a+a \operatorname{Cos}[c+dx])^{3/2}} + \frac{(31 A - 15 B + 7 C) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{16 a^2 d \sqrt{a+a \operatorname{Cos}[c+dx]}}$$

Result (type 3, 1444 leaves) :

$$-\left(\frac{1}{4} + \frac{i}{4}\right) \left((-1+i) + \sqrt{2}\right) \left((117+39i) A + 39\sqrt{2} A - (60+20i) B - 20\sqrt{2} B + (24+8i) C + 8\sqrt{2} C\right)$$

$$\operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{-\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right] \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \Big/ \left(\sqrt{2} (i + \sqrt{2}) d (a(1 + \operatorname{Cos}[c+dx]))^{5/2}\right) -$$

$$\left(\frac{1}{4} - \frac{i}{4}\right) \left((1+i) + \sqrt{2}\right) \left((-117+39i) A + 39\sqrt{2} A + (60-20i) B - 20\sqrt{2} B - (24-8i) C + 8\sqrt{2} C\right)$$

$$\operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right] \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \Big/ \left(\sqrt{2} (i + \sqrt{2}) d (a(1 + \operatorname{Cos}[c+dx]))^{5/2}\right) +$$

$$\frac{(219 A - 115 B + 43 C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right]}{4 d (a(1 + \operatorname{Cos}[c+dx]))^{5/2}} +$$

$$\frac{(-219 A + 115 B - 43 C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right]}{4 d (a(1 + \operatorname{Cos}[c+dx]))^{5/2}} +$$

$$\frac{(78 A + 39 i \sqrt{2} A - 40 B - 20 i \sqrt{2} B + 16 C + 8 i \sqrt{2} C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{2 (i + \sqrt{2}) d (a(1 + \operatorname{Cos}[c+dx]))^{5/2}} -$$

$$\left(\frac{1}{8} - \frac{i}{8}\right) \left((-1+i) + \sqrt{2}\right) \left((117+39i) A + 39\sqrt{2} A - (60+20i) B - 20\sqrt{2} B + (24+8i) C + 8\sqrt{2} C\right)$$

$$\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \Big/ \left(\sqrt{2} (i + \sqrt{2}) d (a(1 + \operatorname{Cos}[c+dx]))^{5/2}\right) +$$

$$\left(\frac{1}{8} + \frac{i}{8}\right) \left((1+i) + \sqrt{2}\right) \left((-117+39i) A + 39\sqrt{2} A + (60-20i) B - 20\sqrt{2} B - (24-8i) C + 8\sqrt{2} C\right) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^5$$

$$\operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \Big/ \left(\sqrt{2} (i + \sqrt{2}) d (a(1 + \operatorname{Cos}[c+dx]))^{5/2}\right) +$$



$$\begin{aligned}
& \frac{(-A+B-C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8 d (a (1 + \operatorname{Cos}[c + dx]))^{5/2} \left(\operatorname{Cos}\left[\frac{1}{4} (c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + dx)\right]\right)^4} + \frac{(-27 A + 19 B - 11 C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8 d (a (1 + \operatorname{Cos}[c + dx]))^{5/2} \left(\operatorname{Cos}\left[\frac{1}{4} (c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + dx)\right]\right)^2} + \\
& \frac{(A - B + C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8 d (a (1 + \operatorname{Cos}[c + dx]))^{5/2} \left(\operatorname{Cos}\left[\frac{1}{4} (c + dx)\right] + \operatorname{Sin}\left[\frac{1}{4} (c + dx)\right]\right)^4} + \\
& \frac{(27 A - 19 B + 11 C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{8 d (a (1 + \operatorname{Cos}[c + dx]))^{5/2} \left(\operatorname{Cos}\left[\frac{1}{4} (c + dx)\right] + \operatorname{Sin}\left[\frac{1}{4} (c + dx)\right]\right)^2} + \\
& \frac{(-9 A + 4 B) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{d (a (1 + \operatorname{Cos}[c + dx]))^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2} (c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + dx)\right]\right)} + \\
& \frac{2 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Sin}\left[\frac{1}{2} (c + dx)\right]}{d (a (1 + \operatorname{Cos}[c + dx]))^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2} (c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + dx)\right]\right)^2} + \\
& \frac{2 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Sin}\left[\frac{1}{2} (c + dx)\right]}{d (a (1 + \operatorname{Cos}[c + dx]))^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2} (c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + dx)\right]\right)^2} + \\
& \frac{(9 A - 4 B) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^5}{d (a (1 + \operatorname{Cos}[c + dx]))^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2} (c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + dx)\right]\right)}
\end{aligned}$$

■ **Problem 431: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + dx]^{5/2} (a + a \operatorname{Cos}[c + dx]) (A + B \operatorname{Cos}[c + dx] + C \operatorname{Cos}[c + dx]^2) dx$$

Optimal (type 4, 211 leaves, 8 steps):

$$\begin{aligned}
& \frac{2 a (9 A + 7 (B + C)) \operatorname{EllipticE}\left[\frac{1}{2} (c + dx), 2\right]}{15 d} + \frac{10 a (11 A + 11 B + 9 C) \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), 2\right]}{231 d} + \\
& \frac{10 a (11 A + 11 B + 9 C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{231 d} + \frac{2 a (9 A + 7 (B + C)) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{45 d} + \\
& \frac{2 a (11 A + 11 B + 9 C) \operatorname{Cos}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{77 d} + \frac{2 a (B + C) \operatorname{Cos}[c + dx]^{7/2} \operatorname{Sin}[c + dx]}{9 d} + \frac{2 a C \operatorname{Cos}[c + dx]^{9/2} \operatorname{Sin}[c + dx]}{11 d}
\end{aligned}$$

Result (type 5, 1344 leaves):

$$\begin{aligned}
& a \left( \sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right. \\
& \left( -\frac{(9A+7B+7C)\cot[c]}{15d} + \frac{(506A+506B+435C)\cos[dx]\sin[c]}{1848d} + \frac{(18A+19B+19C)\cos[2dx]\sin[2c]}{180d} + \right. \\
& \frac{(44A+44B+57C)\cos[3dx]\sin[3c]}{1232d} + \frac{(B+C)\cos[4dx]\sin[4c]}{72d} + \frac{C\cos[5dx]\sin[5c]}{176d} + \frac{(506A+506B+435C)\cos[c]\sin[dx]}{1848d} \\
& \left. \frac{(18A+19B+19C)\cos[2c]\sin[2dx]}{180d} + \frac{(44A+44B+57C)\cos[3c]\sin[3dx]}{1232d} + \frac{(B+C)\cos[4c]\sin[4dx]}{72d} + \frac{C\cos[5c]\sin[5dx]}{176d} \right) - \\
& \frac{1}{21d\sqrt{1+\cot[c]^2}} 5A(1+\cos[c+dx])\operatorname{Csc}[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]}} - \frac{1}{21d\sqrt{1+\cot[c]^2}} \\
& 5B(1+\cos[c+dx])\operatorname{Csc}[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \\
& \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]}} - \\
& \frac{1}{77d\sqrt{1+\cot[c]^2}} 15C(1+\cos[c+dx])\operatorname{Csc}[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]}} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{10d} 3A(1+\cos[c+dx])\operatorname{Csc}[c]\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c]\cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \sqrt{1 + \tan[c]^2} \right) -
\end{aligned}$$

$$\left( \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) - \frac{1}{30 d} 7 B (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right.$$

$$\left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \sqrt{1 + \tan[c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) - \frac{1}{30 d} 7 C (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right.$$

$$\left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \sqrt{1 + \tan[c]^2} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) \right)$$

■ **Problem 432: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{3/2} (a + a \cos[c + d x]) (A + B \cos[c + d x] + C \cos[c + d x]^2) dx$$

Optimal (type 4, 177 leaves, 7 steps):

$$\frac{2 a (9 A+9 B+7 C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{15 d} + \frac{2 a (7 A+5(B+C)) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{21 d} + \frac{2 a (7 A+5(B+C)) \sqrt{\cos [c+d x]} \sin [c+d x]}{21 d} +$$

$$\frac{2 a (9 A+9 B+7 C) \cos [c+d x]^{3 / 2} \sin [c+d x]}{45 d} + \frac{2 a (B+C) \cos [c+d x]^{5 / 2} \sin [c+d x]}{7 d} + \frac{2 a C \cos [c+d x]^{7 / 2} \sin [c+d x]}{9 d}$$

Result (type 5, 1292 leaves):

$$a \left( \sqrt{\cos [c+d x]} (1+\cos [c+d x]) \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^2 \left( -\frac{(9 A+9 B+7 C) \cot [c]}{15 d} + \frac{(28 A+23 B+23 C) \cos [d x] \sin [c]}{84 d} + \right. \right.$$

$$\left. \frac{(18 A+18 B+19 C) \cos [2 d x] \sin [2 c]}{180 d} + \frac{(B+C) \cos [3 d x] \sin [3 c]}{28 d} + \frac{C \cos [4 d x] \sin [4 c]}{72 d} + \frac{(28 A+23 B+23 C) \cos [c] \sin [d x]}{84 d} + \right.$$

$$\left. \frac{(18 A+18 B+19 C) \cos [2 c] \sin [2 d x]}{180 d} + \frac{(B+C) \cos [3 c] \sin [3 d x]}{28 d} + \frac{C \cos [4 c] \sin [4 d x]}{72 d} \right) - \frac{1}{3 d \sqrt{1+\cot [c]^2}}$$

$$A (1+\cos [c+d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^2 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \frac{1}{21 d \sqrt{1+\cot [c]^2}}$$

$$5 B (1+\cos [c+d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^2 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} -$$

$$\frac{1}{21 d \sqrt{1+\cot [c]^2}} 5 C (1+\cos [c+d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^2$$

$$\operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \frac{1}{10 d} 3 A (1+\cos [c+d x]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^2$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left( \frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) - \frac{1}{10 d} 3 B (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left( \frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) - \frac{1}{30 d} 7 C (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left( \frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right)$$

■ **Problem 433: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c + d x]} (a + a \cos[c + d x]) (A + B \cos[c + d x] + C \cos[c + d x]^2) dx$$

Optimal (type 4, 144 leaves, 6 steps) :

$$\frac{2 a (5 A + 3 (B + C)) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{2 a (7 A + 7 B + 5 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} +$$

$$\frac{2 a (7 A + 7 B + 5 C) \sqrt{\cos [c + d x]} \sin [c + d x]}{21 d} + \frac{2 a (B + C) \cos [c + d x]^{3/2} \sin [c + d x]}{5 d} + \frac{2 a C \cos [c + d x]^{5/2} \sin [c + d x]}{7 d}$$

Result (type 5, 1240 leaves) :

$$a \left( \sqrt{\cos [c + d x]} (1 + \cos [c + d x]) \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \left( -\frac{(5 A + 3 B + 3 C) \cot [c]}{5 d} + \frac{(28 A + 28 B + 23 C) \cos [d x] \sin [c]}{84 d} + \frac{(B + C) \cos [2 d x] \sin [2 c]}{10 d} \right. \right.$$

$$\left. \left. \frac{C \cos [3 d x] \sin [3 c]}{28 d} + \frac{(28 A + 28 B + 23 C) \cos [c] \sin [d x]}{84 d} + \frac{(B + C) \cos [2 c] \sin [2 d x]}{10 d} + \frac{C \cos [3 c] \sin [3 d x]}{28 d} \right) - \frac{1}{3 d \sqrt{1 + \cot [c]^2}} \right.$$

$$A (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \frac{1}{3 d \sqrt{1 + \cot [c]^2}}$$

$$B (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} -$$

$$\frac{1}{21 d \sqrt{1 + \cot [c]^2}} 5 C (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \frac{1}{2 d} A (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left( \frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) - \frac{1}{10 d} 3 B (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left( \frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) - \frac{1}{10 d} 3 C (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left( \frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right)$$

■ **Problem 434:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + a \cos[c + dx]) (A + B \cos[c + dx] + C \cos[c + dx]^2)}{\sqrt{\cos[c + dx]}} dx$$

Optimal (type 4, 107 leaves, 5 steps):

$$\frac{2a(5A + 5B + 3C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5d} + \frac{2a(3A + B + C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3d} + \frac{2a(B + C) \sqrt{\cos[c + dx]} \sin[c + dx]}{3d} + \frac{2aC \cos[c + dx]^{3/2} \sin[c + dx]}{5d}$$

Result (type 5, 1186 leaves):

$$a \left( \sqrt{\cos[c + dx]} (1 + \cos[c + dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right. \\ \left. \left( -\frac{(5A + 5B + 3C) \cot[c]}{5d} + \frac{(B + C) \cos[dx] \sin[c]}{3d} + \frac{C \cos[2dx] \sin[2c]}{10d} + \frac{(B + C) \cos[c] \sin[dx]}{3d} + \frac{C \cos[2c] \sin[2dx]}{10d} \right) - \right. \\ \frac{1}{d \sqrt{1 + \cot[c]^2}} A (1 + \cos[c + dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \\ \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\ \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{3d \sqrt{1 + \cot[c]^2}} \\ B (1 + \cos[c + dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \\ \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \\ \frac{1}{3d \sqrt{1 + \cot[c]^2}} C (1 + \cos[c + dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\ \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\ \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{2d} A (1 + \cos[c + dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right.$$



$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \right) \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) /$$

$$\left( \sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} - \frac{1}{2 d} B (1 + \cos [c + d x]) \text{Csc} [c] \text{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \right) \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) /$$

$$\left( \sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} - \frac{1}{10 d} 3 C (1 + \cos [c + d x]) \text{Csc} [c] \text{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \right) \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) /$$

$$\left( \sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) -$$

$$\left( \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right)$$

- **Problem 435: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x]) (A + B \cos[c + d x] + C \cos[c + d x]^2)}{\cos[c + d x]^{3/2}} dx$$

Optimal (type 4, 101 leaves, 5 steps):

$$-\frac{2 a (A - B - C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{d} + \frac{2 a (3 A + 3 B + C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} + \frac{2 a A \sin[c + d x]}{d \sqrt{\cos[c + d x]}} + \frac{2 a C \sqrt{\cos[c + d x]} \sin[c + d x]}{3 d}$$

Result (type 5, 1173 leaves):

$$a \left( \sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \right. \\ \left( -\frac{(-2 A + B + C + B \cos[2 c] + C \cos[2 c]) \csc[c] \sec[c]}{2 d} + \frac{C \cos[d x] \sin[c]}{3 d} + \frac{C \cos[c] \sin[d x]}{3 d} + \frac{A \sec[c] \sec[c + d x] \sin[d x]}{d} \right) - \\ \frac{1}{d \sqrt{1 + \cot[c]^2}} A (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \\ \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \\ \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \frac{1}{d \sqrt{1 + \cot[c]^2}} \\ B (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sec[d x - \text{ArcTan}[\cot[c]]] \\ \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \\ \frac{1}{3 d \sqrt{1 + \cot[c]^2}} C (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\ \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \right)$$

$$\sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} + \frac{1}{2d} A (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} - \frac{1}{2d} B (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} - \frac{1}{2d} C (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}}{\right)}$$

- **Problem 436: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x]) (A + B \cos[c + d x] + C \cos[c + d x]^2)}{\cos[c + d x]^{5/2}} dx$$

Optimal (type 4, 100 leaves, 5 steps):

$$-\frac{2 a (A + B - C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{d} + \frac{2 a (A + 3 (B + C)) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} + \frac{2 a A \sin[c + d x]}{3 d \cos[c + d x]^{3/2}} + \frac{2 a (A + B) \sin[c + d x]}{d \sqrt{\cos[c + d x]}}$$

Result (type 5, 1180 leaves):

$$a \left( \sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \right. \\ \left. - \frac{(-2 A - 2 B + C + C \cos[2 c]) \csc[c] \sec[c]}{2 d} + \frac{A \sec[c] \sec[c + d x]^2 \sin[d x]}{3 d} + \frac{\sec[c] \sec[c + d x] (A \sin[c] + 3 A \sin[d x] + 3 B \sin[d x])}{3 d} \right) - \\ \frac{1}{3 d \sqrt{1 + \cot[c]^2}} A (1 + \cos[c + d x]) \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \\ \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \\ \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \frac{1}{d \sqrt{1 + \cot[c]^2}}} \\ B (1 + \cos[c + d x]) \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sec[d x - \text{ArcTan}[\cot[c]]] \\ \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]] \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \\ \frac{1}{d \sqrt{1 + \cot[c]^2}} C (1 + \cos[c + d x]) \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\ \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]] \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]}}$$

$$\sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} + \frac{1}{2d} A (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} + \frac{1}{2d} B (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} - \frac{1}{2d} C (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left( \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right)$$

- **Problem 437: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x]) (A + B \cos[c + d x] + C \cos[c + d x]^2)}{\cos[c + d x]^{7/2}} dx$$

Optimal (type 4, 139 leaves, 6 steps):

$$-\frac{2 a (3 A + 5 (B + C)) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{2 a (A + B + 3 C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} + \frac{2 a A \sin[c + d x]}{5 d \cos[c + d x]^{5/2}} + \frac{2 a (A + B) \sin[c + d x]}{3 d \cos[c + d x]^{3/2}} + \frac{2 a (3 A + 5 (B + C)) \sin[c + d x]}{5 d \sqrt{\cos[c + d x]}}$$

Result (type 5, 1228 leaves):

$$a \left( \sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \left( \frac{(3 A + 5 B + 5 C) \csc[c] \sec[c]}{5 d} + \frac{A \sec[c] \sec[c + d x]^3 \sin[d x]}{5 d} + \frac{\sec[c] \sec[c + d x]^2 (3 A \sin[c] + 5 A \sin[d x] + 5 B \sin[d x])}{15 d} + \frac{\sec[c] \sec[c + d x] (5 A \sin[c] + 5 B \sin[c] + 9 A \sin[d x] + 15 B \sin[d x] + 15 C \sin[d x])}{15 d} \right) - \frac{1}{3 d \sqrt{1 + \cot[c]^2}} \right. \\ A (1 + \cos[c + d x]) \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sec[d x - \text{ArcTan}[\cot[c]]] \\ \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \frac{1}{3 d \sqrt{1 + \cot[c]^2}} \\ B (1 + \cos[c + d x]) \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sec[d x - \text{ArcTan}[\cot[c]]] \\ \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \\ \frac{1}{d \sqrt{1 + \cot[c]^2}} C (1 + \cos[c + d x]) \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \right)$$

$$\begin{aligned}
& \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{1 + \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} + \frac{1}{10 d} 3 A (1 + \text{Cos}[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\
& \left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \\
& \left( \sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \\
& \frac{\frac{\text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) + \frac{1}{2 d} B (1 + \text{Cos}[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\
& \left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \\
& \left( \sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \\
& \frac{\frac{\text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) + \frac{1}{2 d} C (1 + \text{Cos}[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\
& \left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /
\end{aligned}$$

$$\left( \frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) - \frac{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2}}$$

- **Problem 438: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x]) (A + B \cos[c + d x] + C \cos[c + d x]^2)}{\cos[c + d x]^{9/2}} dx$$

Optimal (type 4, 177 leaves, 7 steps):

$$-\frac{2 a (3 A + 3 B + 5 C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{2 a (5 A + 7 (B + C)) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{21 d} + \frac{2 a A \sin[c + d x]}{7 d \cos[c + d x]^{7/2}} + \frac{2 a (A + B) \sin[c + d x]}{5 d \cos[c + d x]^{5/2}} + \frac{2 a (5 A + 7 (B + C)) \sin[c + d x]}{21 d \cos[c + d x]^{3/2}} + \frac{2 a (3 A + 3 B + 5 C) \sin[c + d x]}{5 d \sqrt{\cos[c + d x]}}$$

Result (type 5, 1284 leaves):

$$a \left( \sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \left( \frac{(3 A + 3 B + 5 C) \text{Csc}[c] \text{Sec}[c]}{5 d} + \frac{A \text{Sec}[c] \text{Sec}[c + d x]^4 \sin[d x]}{7 d} + \frac{\text{Sec}[c] \text{Sec}[c + d x]^3 (5 A \sin[c] + 7 A \sin[d x] + 7 B \sin[d x])}{35 d} + \frac{\text{Sec}[c] \text{Sec}[c + d x]^2 (21 A \sin[c] + 21 B \sin[c] + 25 A \sin[d x] + 35 B \sin[d x] + 35 C \sin[d x])}{105 d} + \frac{1}{105 d} \text{Sec}[c] \text{Sec}[c + d x] (25 A \sin[c] + 35 B \sin[c] + 35 C \sin[c] + 63 A \sin[d x] + 63 B \sin[d x] + 105 C \sin[d x]) \right) - \frac{1}{21 d \sqrt{1 + \cot[c]^2}} \right. \\ \left. 5 A (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]\right]^2 \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Sec}[d x - \text{ArcTan}[\cot[c]]] \right. \\ \left. \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \frac{1}{3 d \sqrt{1 + \cot[c]^2}} \right. \\ \left. B (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]\right]^2 \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Sec}[d x - \text{ArcTan}[\cot[c]]] \right)$$



$$\begin{aligned}
& \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} - \\
& \frac{1}{3 d \sqrt{1 + \text{Cot}[c]^2}} C (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\
& \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} + \frac{1}{10 d} 3 A (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\
& \left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\
& \left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\
& \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) + \frac{1}{10 d} 3 B (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\
& \left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\
& \left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\
& \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) + \frac{1}{2 d} C (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2
\end{aligned}$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \left( \sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right)$$

- **Problem 439: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + dx]^{3/2} (a + a \text{Cos}[c + dx])^2 (A + B \text{Cos}[c + dx] + C \text{Cos}[c + dx]^2) dx$$

Optimal (type 4, 251 leaves, 9 steps):

$$\frac{4 a^2 (9 A + 8 B + 7 C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{15 d} + \frac{4 a^2 (66 A + 55 B + 50 C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{231 d} + \frac{4 a^2 (66 A + 55 B + 50 C) \sqrt{\text{Cos}[c + dx]} \text{Sin}[c + dx]}{231 d} + \frac{4 a^2 (9 A + 8 B + 7 C) \text{Cos}[c + dx]^{3/2} \text{Sin}[c + dx]}{45 d} + \frac{2 a^2 (99 A + 121 B + 89 C) \text{Cos}[c + dx]^{5/2} \text{Sin}[c + dx]}{693 d} + \frac{2 C \text{Cos}[c + dx]^{5/2} (a + a \text{Cos}[c + dx])^2 \text{Sin}[c + dx]}{11 d} + \frac{2 (11 B + 4 C) \text{Cos}[c + dx]^{5/2} (a^2 + a^2 \text{Cos}[c + dx]) \text{Sin}[c + dx]}{99 d}$$

Result (type 5, 1374 leaves):

$$\sqrt{\text{Cos}[c + dx]} (a + a \text{Cos}[c + dx])^2 \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left( -\frac{(9 A + 8 B + 7 C) \text{Cot}[c]}{15 d} + \frac{(1122 A + 1012 B + 941 C) \text{Cos}[dx] \text{Sin}[c]}{3696 d} + \frac{(36 A + 37 B + 38 C) \text{Cos}[2 dx] \text{Sin}[2 c]}{360 d} + \frac{(44 A + 88 B + 101 C) \text{Cos}[3 dx] \text{Sin}[3 c]}{2464 d} + \frac{(B + 2 C) \text{Cos}[4 dx] \text{Sin}[4 c]}{144 d} + \frac{C \text{Cos}[5 dx] \text{Sin}[5 c]}{352 d} + \frac{(1122 A + 1012 B + 941 C) \text{Cos}[c] \text{Sin}[dx]}{3696 d} + \frac{(36 A + 37 B + 38 C) \text{Cos}[2 c] \text{Sin}[2 dx]}{360 d} + \frac{(44 A + 88 B + 101 C) \text{Cos}[3 c] \text{Sin}[3 dx]}{2464 d} + \frac{(B + 2 C) \text{Cos}[4 c] \text{Sin}[4 dx]}{144 d} + \frac{C \text{Cos}[5 c] \text{Sin}[5 dx]}{352 d} \right) -$$

$$\frac{1}{7 d \sqrt{1 + \cot [c]^2}} 2 A (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]\right]^2$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \frac{1}{21 d \sqrt{1 + \cot [c]^2}}$$

$$5 B (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} -$$

$$\frac{1}{231 d \sqrt{1 + \cot [c]^2}} 50 C (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]\right]^2$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \frac{1}{10 d} 3 A (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]\right]^2 \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}} \right) -$$

$$\frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}} - \frac{1}{15 d} 4 B (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]\right]^2 \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left( \frac{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\frac{\frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2}}} - \frac{1}{30 d} 7 C (a + a \cos [c + d x])^2 \csc [c] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \right.$$

$$\left. \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c] \right) / \right.$$

$$\left( \frac{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\frac{\frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2}}} - \right.$$

■ **Problem 440: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos [c + d x]} (a + a \cos [c + d x])^2 (A + B \cos [c + d x] + C \cos [c + d x]^2) dx$$

Optimal (type 4, 215 leaves, 8 steps):

$$\frac{4 a^2 (12 A + 9 B + 8 C) \text{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right]}{15 d} + \frac{4 a^2 (7 A + 6 B + 5 C) \text{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right]}{21 d} +$$

$$\frac{4 a^2 (7 A + 6 B + 5 C) \sqrt{\cos [c + d x]} \sin [c + d x]}{21 d} + \frac{2 a^2 (21 A + 27 B + 19 C) \cos [c + d x]^{3/2} \sin [c + d x]}{105 d} +$$

$$\frac{2 C \cos [c + d x]^{3/2} (a + a \cos [c + d x])^2 \sin [c + d x]}{9 d} + \frac{2 (9 B + 4 C) \cos [c + d x]^{3/2} (a^2 + a^2 \cos [c + d x]) \sin [c + d x]}{63 d}$$

Result (type 5, 1322 leaves):

$$\sqrt{\cos [c + d x]} (a + a \cos [c + d x])^2 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \left( -\frac{(12 A + 9 B + 8 C) \cot [c]}{15 d} + \frac{(56 A + 51 B + 46 C) \cos [d x] \sin [c]}{168 d} + \right.$$

$$\left. \frac{(18 A + 36 B + 37 C) \cos [2 d x] \sin [2 c]}{360 d} + \frac{(B + 2 C) \cos [3 d x] \sin [3 c]}{56 d} + \frac{C \cos [4 d x] \sin [4 c]}{144 d} + \frac{(56 A + 51 B + 46 C) \cos [c] \sin [d x]}{168 d} + \right.$$

$$\begin{aligned}
& \left. \frac{(18A + 36B + 37C) \cos[2c] \sin[2dx]}{360d} + \frac{(B + 2C) \cos[3c] \sin[3dx]}{56d} + \frac{C \cos[4c] \sin[4dx]}{144d} \right) - \\
& \frac{1}{3d \sqrt{1 + \cot[c]^2}} A (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{7d \sqrt{1 + \cot[c]^2}} \\
& 2B (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \\
& \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \\
& \frac{1}{21d \sqrt{1 + \cot[c]^2}} 5C (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{5d} 2A (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \sqrt{1 + \tan[c]^2} \right) - \\
& \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) - \frac{1}{10d} 3B (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4
\end{aligned}$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) / \right. \\
\left. \left( \sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) - \right. \\
\left. \frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \right) - \frac{1}{15 d} 4 C (a + a \cos [c + d x])^2 \text{Csc} [c] \text{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \\
\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) / \right. \\
\left. \left( \sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) - \right. \\
\left. \frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \right)$$

■ **Problem 441: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^2 (A + B \cos [c + d x] + C \cos [c + d x]^2)}{\sqrt{\cos [c + d x]}} dx$$

Optimal (type 4, 179 leaves, 7 steps):

$$\frac{4 a^2 (5 A + 4 B + 3 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} +$$

$$\frac{4 a^2 (14 A + 7 B + 6 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \frac{2 a^2 (35 A + 49 B + 33 C) \sqrt{\cos [c + d x]} \sin [c + d x]}{105 d} +$$

$$\frac{2 C \sqrt{\cos [c + d x]} (a + a \cos [c + d x])^2 \sin [c + d x]}{7 d} + \frac{2 (7 B + 4 C) \sqrt{\cos [c + d x]} (a^2 + a^2 \cos [c + d x]) \sin [c + d x]}{35 d}$$

Result (type 5, 1270 leaves):

$$\sqrt{\cos [c + d x]} (a + a \cos [c + d x])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( -\frac{(5 A + 4 B + 3 C) \cot [c]}{5 d} + \frac{(28 A + 56 B + 51 C) \cos [d x] \sin [c]}{168 d} + \frac{(B + 2 C) \cos [2 d x] \sin [2 c]}{20 d} + \frac{C \cos [3 d x] \sin [3 c]}{56 d} + \right.$$

$$\left. \frac{(28 A + 56 B + 51 C) \cos [c] \sin [d x]}{168 d} + \frac{(B + 2 C) \cos [2 c] \sin [2 d x]}{20 d} + \frac{C \cos [3 c] \sin [3 d x]}{56 d} \right) - \frac{1}{3 d \sqrt{1 + \cot [c]^2}}$$

$$2 A (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \frac{1}{3 d \sqrt{1 + \cot [c]^2}}$$

$$B (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} -$$

$$\frac{1}{7 d \sqrt{1 + \cot [c]^2}} 2 C (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \frac{1}{2 d} A (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left( \frac{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}} \right) - \frac{1}{5 d} 2 B (a + a \cos [c + d x])^2 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( \frac{\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c]}{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}} \right) - \frac{1}{10 d} 3 C (a + a \cos [c + d x])^2 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( \frac{\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c]}{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}} \right) - \frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2}}$$

■ **Problem 442:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.



$$\int \frac{(a + a \cos[c + dx])^2 (A + B \cos[c + dx] + C \cos[c + dx]^2)}{\cos[c + dx]^{3/2}} dx$$

Optimal (type 4, 172 leaves, 7 steps):

$$\frac{4 a^2 (5 B + 4 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5 d} + \frac{4 a^2 (3 A + 2 B + C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3 d} - \frac{2 a^2 (15 A - 5 B - 7 C) \sqrt{\cos[c + dx]} \sin[c + dx]}{15 d} + \frac{2 A (a + a \cos[c + dx])^2 \sin[c + dx]}{d \sqrt{\cos[c + dx]}} - \frac{2 (5 A - C) \sqrt{\cos[c + dx]} (a^2 + a^2 \cos[c + dx]) \sin[c + dx]}{5 d}$$

Result (type 5, 1039 leaves):

$$\begin{aligned} & \sqrt{\cos[c + dx]} (a + a \cos[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\ & \left( -\frac{(-5 A + 10 B + 8 C + 5 A \cos[2 c] + 10 B \cos[2 c] + 8 C \cos[2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{20 d} + \frac{(B + 2 C) \cos[dx] \sin[c]}{6 d} + \frac{C \cos[2 dx] \sin[2 c]}{20 d} + \frac{(B + 2 C) \cos[c] \sin[dx]}{6 d} + \frac{A \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \sin[dx]}{2 d} + \frac{C \cos[2 c] \sin[2 dx]}{20 d} \right) - \frac{1}{d \sqrt{1 + \cot[c]^2}} \\ & A (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \\ & \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{3 d \sqrt{1 + \cot[c]^2}} \\ & 2 B (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \\ & \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \\ & \frac{1}{3 d \sqrt{1 + \cot[c]^2}} C (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\ & \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\ & \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{2 d} B (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\ & \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \end{aligned}$$

$$\left( \frac{\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}} \right) - \frac{1}{5 d} 2 C (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]\right]^2 \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left( \frac{\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}} \right) -$$

■ **Problem 443: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^2 (A + B \cos [c + d x] + C \cos [c + d x]^2)}{\cos [c + d x]^{5/2}} dx$$

Optimal (type 4, 172 leaves, 7 steps):

$$-\frac{4 a^2 (A - C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{d} + \frac{4 a^2 (2 A + 3 B + 2 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 d} -$$

$$\frac{2 a^2 (5 A + 3 B - C) \sqrt{\cos [c + d x]} \sin [c + d x]}{3 d} + \frac{2 A (a + a \cos [c + d x])^2 \sin [c + d x]}{3 d \cos [c + d x]^{3/2}} + \frac{2 (4 A + 3 B) (a^2 + a^2 \cos [c + d x]) \sin [c + d x]}{3 d \sqrt{\cos [c + d x]}}$$

Result (type 5, 1025 leaves):

$$\sqrt{\cos [c + d x]} (a + a \cos [c + d x])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \left( -\frac{(-4 A - B + 2 C + B \cos [2 c] + 2 C \cos [2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{4 d} + \frac{C \cos [d x] \sin [c]}{6 d} + \right.$$

$$\left. \frac{C \cos [c] \sin [d x]}{6 d} + \frac{A \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 \sin [d x]}{6 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x] (A \sin [c] + 6 A \sin [d x] + 3 B \sin [d x])}{6 d} \right) -$$

$$\frac{1}{3 d \sqrt{1 + \cot [c]^2}} 2 A (a + a \cos [c + d x])^2 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right]$$

$$\sec \left[\frac{c}{2} + \frac{d x}{2}\right]^4 \sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \frac{1}{d \sqrt{1 + \cot [c]^2}}}$$

$$B (a + a \cos [c + d x])^2 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^4 \sec [d x - \operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} -$$

$$\frac{1}{3 d \sqrt{1 + \cot [c]^2}} 2 C (a + a \cos [c + d x])^2 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} + \frac{1}{2 d} A (a + a \cos [c + d x])^2 \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}} \sqrt{1 + \tan [c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}} - \frac{1}{2 d} C (a + a \cos [c + d x])^2 \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left( \frac{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2}} \right) - \frac{\frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2}}$$

■ **Problem 444: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^2 (A + B \cos [c + d x] + C \cos [c + d x]^2)}{\cos [c + d x]^{7/2}} dx$$

Optimal (type 4, 174 leaves, 7 steps):

$$-\frac{4 a^2 (4 A + 5 B) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{4 a^2 (A + 2 B + 3 C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} + \frac{2 a^2 (17 A + 25 B + 15 C) \sin [c + d x]}{15 d \sqrt{\cos [c + d x]}} + \frac{2 A (a + a \cos [c + d x])^2 \sin [c + d x]}{5 d \cos [c + d x]^{5/2}} + \frac{2 (4 A + 5 B) (a^2 + a^2 \cos [c + d x]) \sin [c + d x]}{15 d \cos [c + d x]^{3/2}}$$

Result (type 5, 1041 leaves):

$$\sqrt{\cos [c + d x]} (a + a \cos [c + d x])^2 \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \left( -\frac{(-16 A - 20 B - 5 C + 5 C \cos [2 c]) \csc [c] \sec [c]}{20 d} + \frac{A \sec [c] \sec [c + d x]^3 \sin [d x]}{10 d} + \frac{\sec [c] \sec [c + d x]^2 (3 A \sin [c] + 10 A \sin [d x] + 5 B \sin [d x])}{30 d} + \frac{\sec [c] \sec [c + d x] (10 A \sin [c] + 5 B \sin [c] + 24 A \sin [d x] + 30 B \sin [d x] + 15 C \sin [d x])}{30 d} \right) - \frac{1}{3 d \sqrt{1 + \cot [c]^2}}$$

$$A (a + a \cos [c + d x])^2 \csc [c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \text{ArcTan}[\cot [c]]]^2\right] \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \sec [d x - \text{ArcTan}[\cot [c]]]$$

$$\sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\cot [c]]]} - \frac{1}{3 d \sqrt{1 + \cot [c]^2}} 2 B (a + a \cos [c + d x])^2 \csc [c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \text{ArcTan}[\cot [c]]]^2\right]$$

$$\sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \sec [d x - \text{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]}$$

$$\begin{aligned}
& \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{d \sqrt{1+\cot[c]^2}} \\
& C (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\
& \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} + \frac{1}{5d} 2A (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) + \frac{1}{2d} B (a + a \cos[c + dx])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right)
\end{aligned}$$

■ **Problem 445: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^2 (A + B \cos [c + d x] + C \cos [c + d x]^2)}{\cos [c + d x]^{9/2}} dx$$

Optimal (type 4, 215 leaves, 8 steps):

$$\begin{aligned} & - \frac{4 a^2 (3 A + 4 B + 5 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^2 (6 A + 7 B + 14 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \frac{2 a^2 (33 A + 49 B + 35 C) \sin [c + d x]}{105 d \cos [c + d x]^{3/2}} + \\ & \frac{4 a^2 (3 A + 4 B + 5 C) \sin [c + d x]}{5 d \sqrt{\cos [c + d x]}} + \frac{2 A (a + a \cos [c + d x])^2 \sin [c + d x]}{7 d \cos [c + d x]^{7/2}} + \frac{2 (4 A + 7 B) (a^2 + a^2 \cos [c + d x]) \sin [c + d x]}{35 d \cos [c + d x]^{5/2}} \end{aligned}$$

Result (type 5, 1310 leaves):

$$\begin{aligned} & \sqrt{\cos [c + d x]} (a + a \cos [c + d x])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \\ & \left( \frac{(3 A + 4 B + 5 C) \operatorname{Csc}[c] \operatorname{Sec}[c]}{5 d} + \frac{A \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^4 \sin [d x]}{14 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 (5 A \sin [c] + 14 A \sin [d x] + 7 B \sin [d x])}{70 d} \right. \\ & \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (42 A \sin [c] + 21 B \sin [c] + 60 A \sin [d x] + 70 B \sin [d x] + 35 C \sin [d x])}{210 d} + \frac{1}{210 d} \right. \\ & \left. \operatorname{Sec}[c] \operatorname{Sec}[c + d x] (60 A \sin [c] + 70 B \sin [c] + 35 C \sin [c] + 126 A \sin [d x] + 168 B \sin [d x] + 210 C \sin [d x]) \right) - \\ & \frac{1}{7 d \sqrt{1 + \cot [c]^2}} 2 A (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \\ & \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \\ & \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \frac{1}{3 d \sqrt{1 + \cot [c]^2}} \\ & B (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \\ & \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \\ & \frac{1}{3 d \sqrt{1 + \cot [c]^2}} 2 C (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \\ & \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \\ & \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} + \frac{1}{10 d} 3 A (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \end{aligned}$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2 \right\} \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left( \sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) + \frac{1}{5d} 2B (a + a \text{Cos}[c + dx])^2 \text{Csc}[c] \text{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2 \right\} \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left( \sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) + \frac{1}{2d} C (a + a \text{Cos}[c + dx])^2 \text{Csc}[c] \text{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2 \right\} \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left( \sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right.$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}$$

- **Problem 446: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^2 (A + B \cos[c + d x] + C \cos[c + d x]^2)}{\cos[c + d x]^{11/2}} dx$$

Optimal (type 4, 251 leaves, 9 steps):

$$\begin{aligned} & - \frac{4 a^2 (8 A + 9 B + 12 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{4 a^2 (5 A + 6 B + 7 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \\ & \frac{2 a^2 (19 A + 27 B + 21 C) \sin[c + d x]}{105 d \cos[c + d x]^{5/2}} + \frac{4 a^2 (5 A + 6 B + 7 C) \sin[c + d x]}{21 d \cos[c + d x]^{3/2}} + \frac{4 a^2 (8 A + 9 B + 12 C) \sin[c + d x]}{15 d \sqrt{\cos[c + d x]}} + \\ & \frac{2 A (a + a \cos[c + d x])^2 \sin[c + d x]}{9 d \cos[c + d x]^{9/2}} + \frac{2 (4 A + 9 B) (a^2 + a^2 \cos[c + d x]) \sin[c + d x]}{63 d \cos[c + d x]^{7/2}} \end{aligned}$$

Result (type 5, 1364 leaves):

$$\begin{aligned} & \sqrt{\cos[c + d x]} (a + a \cos[c + d x])^2 \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \\ & \left( \frac{(8 A + 9 B + 12 C) \text{Csc}[c] \text{Sec}[c]}{15 d} + \frac{A \text{Sec}[c] \text{Sec}[c + d x]^5 \sin[d x]}{18 d} + \frac{\text{Sec}[c] \text{Sec}[c + d x]^4 (7 A \sin[c] + 18 A \sin[d x] + 9 B \sin[d x])}{126 d} \right. \\ & \left. + \frac{\text{Sec}[c] \text{Sec}[c + d x]^3 (90 A \sin[c] + 45 B \sin[c] + 112 A \sin[d x] + 126 B \sin[d x] + 63 C \sin[d x])}{630 d} + \frac{1}{105 d} \right. \\ & \left. + \frac{\text{Sec}[c] \text{Sec}[c + d x] (25 A \sin[c] + 30 B \sin[c] + 35 C \sin[c] + 56 A \sin[d x] + 63 B \sin[d x] + 84 C \sin[d x])}{630 d} + \frac{1}{630 d} \right. \\ & \left. + \frac{\text{Sec}[c] \text{Sec}[c + d x]^2 (112 A \sin[c] + 126 B \sin[c] + 63 C \sin[c] + 150 A \sin[d x] + 180 B \sin[d x] + 210 C \sin[d x])}{630 d} \right) - \\ & \frac{1}{21 d \sqrt{1 + \text{Cot}[c]^2}} 5 A (a + a \cos[c + d x])^2 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \\ & \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \\ & \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} - \frac{1}{7 d \sqrt{1 + \text{Cot}[c]^2}} \end{aligned}$$



$$2 B (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]$$

$$\frac{\sqrt{1 - \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \frac{1}{3 d \sqrt{1 + \operatorname{Cot}[c]^2}} C (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \frac{1}{15 d} 4 A (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /$$

$$\left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{1}{10 d} 3 B (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /$$

$$\left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{1}{5 d} 2 C (a + a \cos [c + d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right)$$

- **Problem 447: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{3/2} (a + a \cos[c + d x])^3 (A + B \cos[c + d x] + C \cos[c + d x]^2) dx$$

Optimal (type 4, 303 leaves, 10 steps):

$$\frac{4 a^3 (221 A + 195 B + 175 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{195 d} + \frac{4 a^3 (121 A + 105 B + 95 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{231 d} + \\ \frac{4 a^3 (121 A + 105 B + 95 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{231 d} + \frac{4 a^3 (221 A + 195 B + 175 C) \cos[c + d x]^{3/2} \sin[c + d x]}{585 d} + \\ \frac{20 a^3 (286 A + 273 B + 236 C) \cos[c + d x]^{5/2} \sin[c + d x]}{9009 d} + \frac{2 C \cos[c + d x]^{5/2} (a + a \cos[c + d x])^3 \sin[c + d x]}{13 d} + \\ \frac{2 (13 B + 6 C) \cos[c + d x]^{5/2} (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{143 a d} + \frac{2 (143 A + 195 B + 145 C) \cos[c + d x]^{5/2} (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{1287 d}$$

Result (type 5, 1426 leaves):

$$\sqrt{\cos[c + d x]} (a + a \cos[c + d x])^3 \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\ \left( -\frac{(221 A + 195 B + 175 C) \cot[c]}{390 d} + \frac{(2134 A + 1953 B + 1811 C) \cos[d x] \sin[c]}{7392 d} + \frac{(7592 A + 7800 B + 7825 C) \cos[2 d x] \sin[2 c]}{74880 d} + \right. \\ \frac{(132 A + 189 B + 215 C) \cos[3 d x] \sin[3 c]}{4928 d} + \frac{(13 A + 39 B + 59 C) \cos[4 d x] \sin[4 c]}{3744 d} + \frac{(B + 3 C) \cos[5 d x] \sin[5 c]}{704 d} + \\ \frac{C \cos[6 d x] \sin[6 c]}{1664 d} + \frac{(2134 A + 1953 B + 1811 C) \cos[c] \sin[d x]}{7392 d} + \frac{(7592 A + 7800 B + 7825 C) \cos[2 c] \sin[2 d x]}{74880 d} + \\ \left. \frac{(132 A + 189 B + 215 C) \cos[3 c] \sin[3 d x]}{4928 d} + \frac{(13 A + 39 B + 59 C) \cos[4 c] \sin[4 d x]}{3744 d} + \frac{(B + 3 C) \cos[5 c] \sin[5 d x]}{704 d} + \frac{C \cos[6 c] \sin[6 d x]}{1664 d} \right) -$$

$$\frac{1}{42 d \sqrt{1 + \cot [c]^2}} 11 A (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]\right]^2$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \frac{1}{22 d \sqrt{1 + \cot [c]^2}}$$

$$5 B (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} -$$

$$\frac{1}{462 d \sqrt{1 + \cot [c]^2}} 95 C (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]\right]^2$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \frac{1}{60 d} 17 A (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]\right]^2 \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}} \right) -$$

$$\frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}} - \frac{1}{4 d} B (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]\right]^2 \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left( \frac{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2}} \right) - \frac{1}{156 d} 35 C (a + a \cos [c + d x])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c] \right) / \left( \sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) - \frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \right)$$

■ **Problem 448: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos [c + d x]} (a + a \cos [c + d x])^3 (A + B \cos [c + d x] + C \cos [c + d x]^2) dx$$

Optimal (type 4, 267 leaves, 9 steps):

$$\frac{4 a^3 (21 A + 17 B + 15 C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{15 d} + \frac{4 a^3 (143 A + 121 B + 105 C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{231 d} + \frac{4 a^3 (143 A + 121 B + 105 C) \sqrt{\cos [c + d x]} \sin [c + d x]}{231 d} + \frac{4 a^3 (264 A + 253 B + 210 C) \cos [c + d x]^{3/2} \sin [c + d x]}{1155 d} + \frac{2 C \cos [c + d x]^{3/2} (a + a \cos [c + d x])^3 \sin [c + d x]}{11 d} + \frac{2 (11 B + 6 C) \cos [c + d x]^{3/2} (a^2 + a^2 \cos [c + d x])^2 \sin [c + d x]}{99 a d} + \frac{2 (99 A + 143 B + 105 C) \cos [c + d x]^{3/2} (a^3 + a^3 \cos [c + d x]) \sin [c + d x]}{693 d}$$

Result (type 5, 1374 leaves):

$$\begin{aligned}
& \sqrt{\cos[c+dx]} (a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \\
& \left( -\frac{(21A+17B+15C)\cot[c]}{30d} + \frac{(2354A+2134B+1953C)\cos[dx]\sin[c]}{7392d} + \frac{(54A+73B+75C)\cos[2dx]\sin[2c]}{720d} + \right. \\
& \frac{(44A+132B+189C)\cos[3dx]\sin[3c]}{4928d} + \frac{(B+3C)\cos[4dx]\sin[4c]}{288d} + \frac{C\cos[5dx]\sin[5c]}{704d} + \frac{(2354A+2134B+1953C)\cos[c]\sin[dx]}{7392d} + \\
& \left. \frac{(54A+73B+75C)\cos[2c]\sin[2dx]}{720d} + \frac{(44A+132B+189C)\cos[3c]\sin[3dx]}{4928d} + \frac{(B+3C)\cos[4c]\sin[4dx]}{288d} + \frac{C\cos[5c]\sin[5dx]}{704d} \right) - \\
& \frac{1}{42d\sqrt{1+\cot[c]^2}} 13A (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \operatorname{Sec}[dx-\operatorname{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]} - \frac{1}{42d\sqrt{1+\cot[c]^2}} \\
& 11B (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \operatorname{Sec}[dx-\operatorname{ArcTan}[\cot[c]]] \\
& \sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]} - \\
& \frac{1}{22d\sqrt{1+\cot[c]^2}} 5C (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \operatorname{Sec}[dx-\operatorname{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]} - \frac{1}{20d} 7A (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx+\operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1-\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2} \sqrt{1+\tan[c]^2} \right) -
\end{aligned}$$

$$\left( \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) - \frac{1}{60 d} 17 B (a + a \cos[c + d x])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left( \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) - \frac{1}{4 d} C (a + a \cos[c + d x])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left( \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right)$$

■ **Problem 449: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^3 (A + B \cos[c + d x] + C \cos[c + d x]^2)}{\sqrt{\cos[c + d x]}} dx$$

Optimal (type 4, 231 leaves, 8 steps):

$$\frac{4 a^3 (27 A + 21 B + 17 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{15 d} + \frac{4 a^3 (21 A + 13 B + 11 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{21 d} +$$

$$\frac{4 a^3 (42 A + 41 B + 32 C) \sqrt{\cos[c + dx]} \sin[c + dx]}{105 d} + \frac{2 C \sqrt{\cos[c + dx]} (a + a \cos[c + dx])^3 \sin[c + dx]}{9 d} +$$

$$\frac{2 (3 B + 2 C) \sqrt{\cos[c + dx]} (a^2 + a^2 \cos[c + dx])^2 \sin[c + dx]}{21 a d} + \frac{2 (63 A + 99 B + 73 C) \sqrt{\cos[c + dx]} (a^3 + a^3 \cos[c + dx]) \sin[c + dx]}{315 d}$$

Result (type 5, 1322 leaves):

$$\sqrt{\cos[c + dx]} (a + a \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6$$

$$\left( -\frac{(27 A + 21 B + 17 C) \cot[c]}{30 d} + \frac{(84 A + 107 B + 97 C) \cos[dx] \sin[c]}{336 d} + \frac{(18 A + 54 B + 73 C) \cos[2 dx] \sin[2 c]}{720 d} + \right.$$

$$\frac{(B + 3 C) \cos[3 dx] \sin[3 c]}{112 d} + \frac{C \cos[4 dx] \sin[4 c]}{288 d} + \frac{(84 A + 107 B + 97 C) \cos[c] \sin[dx]}{336 d} +$$

$$\left. \frac{(18 A + 54 B + 73 C) \cos[2 c] \sin[2 dx]}{720 d} + \frac{(B + 3 C) \cos[3 c] \sin[3 dx]}{112 d} + \frac{C \cos[4 c] \sin[4 dx]}{288 d} \right) - \frac{1}{2 d \sqrt{1 + \cot[c]^2}}$$

$$A (a + a \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]]$$

$$\sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{42 d \sqrt{1 + \cot[c]^2}} 13 B (a + a \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]}$$

$$\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{42 d \sqrt{1 + \cot[c]^2}}$$

$$11 C (a + a \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6$$

$$\operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]}$$

$$\sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{20 d} 9 A (a + a \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \right) \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) /$$

$$\left( \sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \right) - \frac{1}{20 d} 7 B (a + a \cos [c + d x])^3 \text{Csc} [c] \text{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^6$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \right) \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) /$$

$$\left( \sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \right) - \frac{1}{60 d} 17 C (a + a \cos [c + d x])^3 \text{Csc} [c] \text{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^6$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \right) \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) /$$

$$\left( \sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) -$$



$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}$$

- **Problem 450: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^3 (A + B \cos[c + d x] + C \cos[c + d x]^2)}{\cos[c + d x]^{3/2}} dx$$

Optimal (type 4, 229 leaves, 8 steps):

$$\frac{4 a^3 (5 A + 9 B + 7 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^3 (35 A + 21 B + 13 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} -$$

$$\frac{4 a^3 (35 A - 42 B - 41 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{105 d} + \frac{2 A (a + a \cos[c + d x])^3 \sin[c + d x]}{d \sqrt{\cos[c + d x]}} -$$

$$\frac{2 (7 A - C) \sqrt{\cos[c + d x]} (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{7 a d} - \frac{2 (35 A - 7 B - 11 C) \sqrt{\cos[c + d x]} (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{35 d}$$

Result (type 5, 1313 leaves):

$$\sqrt{\cos[c + d x]} (a + a \cos[c + d x])^3 \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( - \frac{(5 A + 18 B + 14 C + 15 A \cos[2 c] + 18 B \cos[2 c] + 14 C \cos[2 c]) \text{Csc}[c] \text{Sec}[c]}{40 d} + \frac{(28 A + 84 B + 107 C) \cos[d x] \sin[c]}{336 d} + \right.$$

$$\frac{(B + 3 C) \cos[2 d x] \sin[2 c]}{40 d} + \frac{C \cos[3 d x] \sin[3 c]}{112 d} + \frac{(28 A + 84 B + 107 C) \cos[c] \sin[d x]}{336 d} +$$

$$\left. \frac{A \text{Sec}[c] \text{Sec}[c + d x] \sin[d x]}{4 d} + \frac{(B + 3 C) \cos[2 c] \sin[2 d x]}{40 d} + \frac{C \cos[3 c] \sin[3 d x]}{112 d} \right) - \frac{1}{6 d \sqrt{1 + \cot[c]^2}}$$

$$5 A (a + a \cos[c + d x])^3 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \text{Sec}[d x - \text{ArcTan}[\cot[c]]]$$

$$\sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \frac{1}{2 d \sqrt{1 + \cot[c]^2}}$$

$$B (a + a \cos[c + d x])^3 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \text{Sec}[d x - \text{ArcTan}[\cot[c]]]$$

$$\sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$\begin{aligned}
& \frac{1}{42 d \sqrt{1 + \cot [c]^2}} 13 C (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \frac{1}{4 d} A (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \\
& \left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) - \\
& \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \left) - \frac{1}{20 d} 9 B (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \\
& \left( \sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) - \\
& \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \left) - \frac{1}{20 d} 7 C (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6
\end{aligned}$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \left( \sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \frac{\frac{\text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right)$$

■ **Problem 451: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Cos}[c + d x])^3 (A + B \text{Cos}[c + d x] + C \text{Cos}[c + d x]^2)}{\text{Cos}[c + d x]^{5/2}} dx$$

Optimal (type 4, 227 leaves, 8 steps):

$$\begin{aligned} & - \frac{4 a^3 (5 A - 5 B - 9 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^3 (5 A + 5 B + 3 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 d} - \\ & \frac{4 a^3 (20 A + 5 B - 6 C) \sqrt{\text{Cos}[c + d x]} \text{Sin}[c + d x]}{15 d} + \frac{2 A (a + a \text{Cos}[c + d x])^3 \text{Sin}[c + d x]}{3 d \text{Cos}[c + d x]^{3/2}} + \\ & \frac{2 (2 A + B) (a^2 + a^2 \text{Cos}[c + d x])^2 \text{Sin}[c + d x]}{a d \sqrt{\text{Cos}[c + d x]}} - \frac{2 (35 A + 15 B - 3 C) \sqrt{\text{Cos}[c + d x]} (a^3 + a^3 \text{Cos}[c + d x]) \text{Sin}[c + d x]}{15 d} \end{aligned}$$

Result (type 5, 1297 leaves):

$$\begin{aligned} & \sqrt{\text{Cos}[c + d x]} (a + a \text{Cos}[c + d x])^3 \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \left( - \frac{(-25 A + 5 B + 18 C + 5 A \text{Cos}[2 c] + 15 B \text{Cos}[2 c] + 18 C \text{Cos}[2 c]) \text{Csc}[c] \text{Sec}[c]}{40 d} + \right. \\ & \frac{(B + 3 C) \text{Cos}[d x] \text{Sin}[c]}{12 d} + \frac{C \text{Cos}[2 d x] \text{Sin}[2 c]}{40 d} + \frac{(B + 3 C) \text{Cos}[c] \text{Sin}[d x]}{12 d} + \frac{A \text{Sec}[c] \text{Sec}[c + d x]^2 \text{Sin}[d x]}{12 d} + \\ & \left. \frac{\text{Sec}[c] \text{Sec}[c + d x] (A \text{Sin}[c] + 9 A \text{Sin}[d x] + 3 B \text{Sin}[d x])}{12 d} + \frac{C \text{Cos}[2 c] \text{Sin}[2 d x]}{40 d} \right) - \frac{1}{6 d \sqrt{1 + \text{Cot}[c]^2}} \end{aligned}$$

$$5 A (a + a \text{Cos}[c + d x])^3 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]]$$

$$\sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} - \frac{1}{6 d \sqrt{1 + \text{Cot}[c]^2}}$$

$$5 B (a + a \cos[c + d x])^3 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]]$$

$$\sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} - \frac{1}{2 d \sqrt{1 + \text{Cot}[c]^2}} C (a + a \cos[c + d x])^3 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]}$$

$$\sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} + \frac{1}{4 d} A (a + a \cos[c + d x])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} - \frac{1}{4 d} B (a + a \cos[c + d x])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left( \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) - \frac{1}{20 d} 9 C (a + a \cos[c + d x])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right.$$

$$\left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2}} \right) - \right.$$

$$\left. \left( \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) \right)$$

■ **Problem 452: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^3 (A + B \cos[c + d x] + C \cos[c + d x]^2)}{\cos[c + d x]^{7/2}} dx$$

Optimal (type 4, 230 leaves, 8 steps):

$$-\frac{4 a^3 (9 A + 5 B - 5 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^3 (3 A + 5 (B + C)) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 d} -$$

$$\frac{4 a^3 (21 A + 20 B + 5 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{15 d} + \frac{2 A (a + a \cos[c + d x])^3 \sin[c + d x]}{5 d \cos[c + d x]^{5/2}} +$$

$$\frac{2 (6 A + 5 B) (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{15 a d \cos[c + d x]^{3/2}} + \frac{2 (33 A + 35 B + 15 C) (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{15 d \sqrt{\cos[c + d x]}}$$

Result (type 5, 1298 leaves):

$$\sqrt{\cos[c + d x]} (a + a \cos[c + d x])^3 \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( -\frac{(-36 A - 25 B + 5 C + 5 B \cos[2 c] + 15 C \cos[2 c]) \text{Csc}[c] \text{Sec}[c]}{40 d} + \frac{C \cos[d x] \sin[c]}{12 d} + \frac{C \cos[c] \sin[d x]}{12 d} + \right.$$

$$\left. \frac{A \text{Sec}[c] \text{Sec}[c + d x]^3 \sin[d x]}{20 d} + \frac{\text{Sec}[c] \text{Sec}[c + d x]^2 (3 A \sin[c] + 15 A \sin[d x] + 5 B \sin[d x])}{60 d} + \right)$$

$$\begin{aligned}
& \left. \frac{\text{Sec}[c] \text{Sec}[c + dx] (15 A \text{Sin}[c] + 5 B \text{Sin}[c] + 54 A \text{Sin}[dx] + 45 B \text{Sin}[dx] + 15 C \text{Sin}[dx])}{60 d} \right) - \frac{1}{2 d \sqrt{1 + \text{Cot}[c]^2}} \\
& A (a + a \text{Cos}[c + dx])^3 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \\
& \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} - \frac{1}{6 d \sqrt{1 + \text{Cot}[c]^2}} \\
& 5 B (a + a \text{Cos}[c + dx])^3 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \\
& \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} - \\
& \frac{1}{6 d \sqrt{1 + \text{Cot}[c]^2}} 5 C (a + a \text{Cos}[c + dx])^3 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \\
& \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} + \frac{1}{20 d} 9 A (a + a \text{Cos}[c + dx])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \\
& \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2\right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \\
& \left( \sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \\
& \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) + \frac{1}{4 d} B (a + a \text{Cos}[c + dx])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \\
& \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2\right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /
\end{aligned}$$

$$\left( \frac{\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}} \right) - \frac{1}{4 d} C (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right\] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left( \frac{\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}} \right) -$$

■ **Problem 453: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^3 (A + B \cos [c + d x] + C \cos [c + d x]^2)}{\cos [c + d x]^{9/2}} dx$$

Optimal (type 4, 231 leaves, 8 steps):

$$-\frac{4 a^3 (7 A + 9 B + 5 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^3 (13 A + 21 B + 35 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \frac{4 a^3 (106 A + 147 B + 140 C) \sin [c + d x]}{105 d \sqrt{\cos [c + d x]}}$$

$$+\frac{2 A (a + a \cos [c + d x])^3 \sin [c + d x]}{7 d \cos [c + d x]^{7/2}} + \frac{2 (6 A + 7 B) (a^2 + a^2 \cos [c + d x])^2 \sin [c + d x]}{35 a d \cos [c + d x]^{5/2}} + \frac{2 (7 A + 9 B + 5 C) (a^3 + a^3 \cos [c + d x]) \sin [c + d x]}{15 d \cos [c + d x]^{3/2}}$$

Result (type 5, 1317 leaves):

$$\sqrt{\cos [c + d x]} (a + a \cos [c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \left( -\frac{(-28 A - 36 B - 25 C + 5 C \cos [2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{40 d} + \right.$$

$$\left. \frac{A \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^4 \sin [d x]}{28 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 (5 A \sin [c] + 21 A \sin [d x] + 7 B \sin [d x])}{140 d} + \right.$$

$$\begin{aligned}
& \frac{\text{Sec}[c] \text{Sec}[c + dx]^2 (63 A \text{Sin}[c] + 21 B \text{Sin}[c] + 130 A \text{Sin}[dx] + 105 B \text{Sin}[dx] + 35 C \text{Sin}[dx])}{420 d} + \frac{1}{420 d} \\
& \left. \text{Sec}[c] \text{Sec}[c + dx] (130 A \text{Sin}[c] + 105 B \text{Sin}[c] + 35 C \text{Sin}[c] + 294 A \text{Sin}[dx] + 378 B \text{Sin}[dx] + 315 C \text{Sin}[dx]) \right) - \\
& \frac{1}{42 d \sqrt{1 + \text{Cot}[c]^2}} 13 A (a + a \text{Cos}[c + dx])^3 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \\
& \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} - \frac{1}{2 d \sqrt{1 + \text{Cot}[c]^2}} \\
& B (a + a \text{Cos}[c + dx])^3 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \\
& \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} - \\
& \frac{1}{6 d \sqrt{1 + \text{Cot}[c]^2}} 5 C (a + a \text{Cos}[c + dx])^3 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \\
& \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} + \frac{1}{20 d} 7 A (a + a \text{Cos}[c + dx])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \\
& \left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2\right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \\
& \left( \sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \\
& \left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) + \frac{1}{20 d} 9 B (a + a \text{Cos}[c + dx])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6
\end{aligned}$$



$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) / \right. \\
\left. \left( \sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) - \right. \\
\left. \frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \right) + \frac{1}{4 d} C (a + a \cos [c + d x])^3 \text{Csc} [c] \text{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \\
\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) / \right. \\
\left. \left( \sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) - \right. \\
\left. \frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \right)$$

■ **Problem 454: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^3 (A + B \cos [c + d x] + C \cos [c + d x]^2)}{\cos [c + d x]^{11/2}} dx$$

Optimal (type 4, 267 leaves, 9 steps):

$$\begin{aligned}
& - \frac{4 a^3 (17 A + 21 B + 27 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{4 a^3 (11 A + 13 B + 21 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} \\
& + \frac{4 a^3 (32 A + 41 B + 42 C) \operatorname{Sin}[c + d x]}{105 d \operatorname{Cos}[c + d x]^{3/2}} + \frac{4 a^3 (17 A + 21 B + 27 C) \operatorname{Sin}[c + d x]}{15 d \sqrt{\operatorname{Cos}[c + d x]}} + \frac{2 A (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Sin}[c + d x]}{9 d \operatorname{Cos}[c + d x]^{9/2}} \\
& + \frac{2 (2 A + 3 B) (a^2 + a^2 \operatorname{Cos}[c + d x])^2 \operatorname{Sin}[c + d x]}{21 a d \operatorname{Cos}[c + d x]^{7/2}} + \frac{2 (73 A + 99 B + 63 C) (a^3 + a^3 \operatorname{Cos}[c + d x]) \operatorname{Sin}[c + d x]}{315 d \operatorname{Cos}[c + d x]^{5/2}}
\end{aligned}$$

Result (type 5, 1364 leaves):

$$\begin{aligned}
& \sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \left( \frac{(17 A + 21 B + 27 C) \operatorname{Csc}[c] \operatorname{Sec}[c]}{30 d} + \frac{A \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^5 \operatorname{Sin}[d x]}{36 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^4 (7 A \operatorname{Sin}[c] + 27 A \operatorname{Sin}[d x] + 9 B \operatorname{Sin}[d x])}{252 d} \right. \\
& \left. + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 (135 A \operatorname{Sin}[c] + 45 B \operatorname{Sin}[c] + 238 A \operatorname{Sin}[d x] + 189 B \operatorname{Sin}[d x] + 63 C \operatorname{Sin}[d x])}{1260 d} + \frac{1}{1260 d} \right. \\
& \left. + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (238 A \operatorname{Sin}[c] + 189 B \operatorname{Sin}[c] + 63 C \operatorname{Sin}[c] + 330 A \operatorname{Sin}[d x] + 390 B \operatorname{Sin}[d x] + 315 C \operatorname{Sin}[d x])}{420 d} \right. \\
& \left. + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x] (110 A \operatorname{Sin}[c] + 130 B \operatorname{Sin}[c] + 105 C \operatorname{Sin}[c] + 238 A \operatorname{Sin}[d x] + 294 B \operatorname{Sin}[d x] + 378 C \operatorname{Sin}[d x])}{42 d \sqrt{1 + \operatorname{Cot}[c]^2}} \right) - \\
& \frac{1}{42 d \sqrt{1 + \operatorname{Cot}[c]^2}} 11 A (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \frac{1}{42 d \sqrt{1 + \operatorname{Cot}[c]^2}} \\
& 13 B (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\
& \frac{1}{2 d \sqrt{1 + \operatorname{Cot}[c]^2}} C (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \frac{1}{60 d} 17 A (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6
\end{aligned}$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2 \right\} \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) + \frac{1}{20 d} 7 B (a + a \cos[c + d x])^3 \text{Csc}[c] \text{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^6$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2 \right\} \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) + \frac{1}{20 d} 9 C (a + a \cos[c + d x])^3 \text{Csc}[c] \text{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^6$$

$$\left( \left( \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2 \right\} \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left( \sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right.$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}$$

- **Problem 455: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + d x])^3 (A + B \cos[c + d x] + C \cos[c + d x]^2)}{\cos[c + d x]^{13/2}} dx$$

Optimal (type 4, 303 leaves, 10 steps):

$$\begin{aligned} & - \frac{4 a^3 (15 A + 17 B + 21 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{4 a^3 (105 A + 121 B + 143 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{231 d} + \\ & \frac{4 a^3 (210 A + 253 B + 264 C) \sin[c + d x]}{1155 d \cos[c + d x]^{5/2}} + \frac{4 a^3 (105 A + 121 B + 143 C) \sin[c + d x]}{231 d \cos[c + d x]^{3/2}} + \frac{4 a^3 (15 A + 17 B + 21 C) \sin[c + d x]}{15 d \sqrt{\cos[c + d x]}} + \\ & \frac{2 A (a + a \cos[c + d x])^3 \sin[c + d x]}{11 d \cos[c + d x]^{11/2}} + \frac{2 (6 A + 11 B) (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{99 a d \cos[c + d x]^{9/2}} + \frac{2 (105 A + 143 B + 99 C) (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{693 d \cos[c + d x]^{7/2}} \end{aligned}$$

Result (type 5, 1418 leaves):

$$\begin{aligned} & \sqrt{\cos[c + d x]} (a + a \cos[c + d x])^3 \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\ & \left( \frac{(15 A + 17 B + 21 C) \text{Csc}[c] \text{Sec}[c]}{30 d} + \frac{A \text{Sec}[c] \text{Sec}[c + d x]^6 \sin[d x]}{44 d} + \frac{\text{Sec}[c] \text{Sec}[c + d x]^5 (9 A \sin[c] + 33 A \sin[d x] + 11 B \sin[d x])}{396 d} + \right. \\ & \frac{\text{Sec}[c] \text{Sec}[c + d x]^4 (231 A \sin[c] + 77 B \sin[c] + 378 A \sin[d x] + 297 B \sin[d x] + 99 C \sin[d x])}{2772 d} + \frac{1}{2310 d} \\ & \text{Sec}[c] \text{Sec}[c + d x] (525 A \sin[c] + 605 B \sin[c] + 715 C \sin[c] + 1155 A \sin[d x] + 1309 B \sin[d x] + 1617 C \sin[d x]) + \frac{1}{13860 d} \\ & \text{Sec}[c] \text{Sec}[c + d x]^3 (1890 A \sin[c] + 1485 B \sin[c] + 495 C \sin[c] + 2310 A \sin[d x] + 2618 B \sin[d x] + 2079 C \sin[d x]) + \frac{1}{13860 d} \\ & \left. \text{Sec}[c] \text{Sec}[c + d x]^2 (2310 A \sin[c] + 2618 B \sin[c] + 2079 C \sin[c] + 3150 A \sin[d x] + 3630 B \sin[d x] + 4290 C \sin[d x]) \right) - \\ & \frac{1}{22 d \sqrt{1 + \cot[c]^2}} 5 A (a + a \cos[c + d x])^3 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \\ & \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \text{Sec}[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \end{aligned}$$

$$\begin{aligned}
& \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{42 d \sqrt{1+\cot[c]^2}} \\
11 B (a + a \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \\
& \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \\
& \frac{1}{42 d \sqrt{1+\cot[c]^2}} 13 C (a + a \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1+\sin[dx - \operatorname{ArcTan}[\cot[c]]]} + \frac{1}{4 d} A (a + a \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} + \frac{1}{60 d} 17 B (a + a \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \\
& \left( \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left( \sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -
\end{aligned}$$

$$\frac{\frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} + \frac{1}{20 d} 7 C (a + a \cos[c + d x])^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6$$

$$\left( \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left( \sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \sqrt{1 + \tan[c]^2} \right) -$$

$$\frac{\frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}$$

■ **Problem 456: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^{5/2} (A + B \cos[c + d x] + C \cos[c + d x]^2)}{a + a \cos[c + d x]} dx$$

Optimal (type 4, 210 leaves, 7 steps):

$$-\frac{3(5A - 7B + 7C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5ad} + \frac{5(7A - 7B + 9C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{21ad} + \frac{5(7A - 7B + 9C) \sqrt{\cos[c + dx]} \sin[c + dx]}{21ad} -$$

$$\frac{(5A - 7B + 7C) \cos[c + dx]^{3/2} \sin[c + dx]}{5ad} + \frac{(7A - 7B + 9C) \cos[c + dx]^{5/2} \sin[c + dx]}{7ad} - \frac{(A - B + C) \cos[c + dx]^{7/2} \sin[c + dx]}{d(a + a \cos[c + dx])}$$

Result (type 5, 1752 leaves):

$$-\frac{1}{4(a + a \cos[c + dx])} 3 i A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right.$$

$$\left. \left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}\right. \right.$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{(-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c])} \right) + \\
& \frac{1}{20 (a + a \cos[c + d x])} 21 i B \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \right. \right. \\
& \left. \left. \frac{\sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}}{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}} \right) \right) / \\
& (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \right. \\
& \left. \frac{\sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}}{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}} \right) / \\
& \left. (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) - \frac{1}{20 (a + a \cos[c + d x])} 21 i C \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}} \right) \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \\
& \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \\
& \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) + \frac{1}{a + a \cos[c + d x]} \\
& \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sqrt{\cos[c + d x]} \left( \frac{2 (5 A - 5 B + 5 C + 10 A \cos[c] - 16 B \cos[c] + 16 C \cos[c]) \operatorname{Csc}[c]}{5 d} + \frac{(28 A - 28 B + 51 C) \cos[d x] \sin[c]}{21 d} + \right. \\
& \frac{2 (B - C) \cos[2 d x] \sin[2 c]}{5 d} + \frac{C \cos[3 d x] \sin[3 c]}{7 d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (A \sin\left[\frac{d x}{2}\right] - B \sin\left[\frac{d x}{2}\right] + C \sin\left[\frac{d x}{2}\right])}{d} + \\
& \left. \frac{(28 A - 28 B + 51 C) \cos[c] \sin[d x]}{21 d} + \frac{2 (B - C) \cos[2 c] \sin[2 d x]}{5 d} + \frac{C \cos[3 c] \sin[3 d x]}{7 d} \right) - \\
& \left( 5 A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \left. \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \frac{\sqrt{1 + \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]}}{\sqrt{1 + \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]}} \right) / \left( 3 d (a + a \cos[c + d x]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) +
\end{aligned}$$

$$\left( 5 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \csc\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\ \left. \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\ \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( 3 d (a + a \cos[c + dx]) \sqrt{1 + \cot[c]^2} \right) - \\ \left( 15 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \csc\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\ \left. \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\ \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( 7 d (a + a \cos[c + dx]) \sqrt{1 + \cot[c]^2} \right)$$

■ **Problem 457: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{3/2} (A + B \cos[c + dx] + C \cos[c + dx]^2)}{a + a \cos[c + dx]} dx$$

Optimal (type 4, 174 leaves, 6 steps):

$$\frac{3(5A - 5B + 7C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] - (3A - 5B + 5C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{5ad} - \frac{3ad}{(3A - 5B + 5C) \sqrt{\cos[c + dx]} \sin[c + dx]} + \frac{(5A - 5B + 7C) \cos[c + dx]^{3/2} \sin[c + dx]}{5ad} - \frac{(A - B + C) \cos[c + dx]^{5/2} \sin[c + dx]}{d(a + a \cos[c + dx])}$$

Result (type 5, 1697 leaves):

$$\frac{1}{4(a + a \cos[c + dx])} - 3iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] \\ \left( \left( 2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( 3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c] \right) - \right. \\ \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( -id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c] \right) \right) - \frac{1}{4(a + a \cos[c + dx])} - 3iB \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$



$$\begin{aligned}
& \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) \right) / \left( 3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \\
& \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) \left. \right) + \frac{1}{20 (a + a \operatorname{Cos}[c + d x])} 21 i c \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\
& \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) \right) / \left( 3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \\
& \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) \left. \right) + \frac{1}{a + a \operatorname{Cos}[c + d x]} \\
& \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sqrt{\operatorname{Cos}[c + d x]} \left( -\frac{2 (5 A - 5 B + 5 C + 10 A \operatorname{Cos}[c] - 10 B \operatorname{Cos}[c] + 16 C \operatorname{Cos}[c]) \operatorname{Csc}[c]}{5 d} + \frac{4 (B - C) \operatorname{Cos}[d x] \operatorname{Sin}[c]}{3 d} + \right. \\
& \quad \left. \frac{2 C \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{5 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (A \operatorname{Sin}\left[\frac{d x}{2}\right] - B \operatorname{Sin}\left[\frac{d x}{2}\right] + C \operatorname{Sin}\left[\frac{d x}{2}\right])}{d} + \frac{4 (B - C) \operatorname{Cos}[c] \operatorname{Sin}[d x]}{3 d} + \frac{2 C \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x]}{5 d} \right) + \\
& \left( A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( d (a + a \operatorname{Cos}[c + d x]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left( 5 B \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) /
\end{aligned}$$

$$\left( 3 d (a + a \cos [c + d x]) \sqrt{1 + \cot [c]^2} \right) + \left( 5 C \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \csc \left[ \frac{c}{2} \right] \text{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan} [\cot [c]]]^2 \right] \right. \\ \left. \sec \left[ \frac{c}{2} \right] \sec [d x - \text{ArcTan} [\cot [c]]] \sqrt{1 - \sin [d x - \text{ArcTan} [\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan} [\cot [c]]]} \right. \\ \left. \sqrt{1 + \sin [d x - \text{ArcTan} [\cot [c]]]} \right) / \left( 3 d (a + a \cos [c + d x]) \sqrt{1 + \cot [c]^2} \right)$$

■ **Problem 458: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos [c + d x]} (A + B \cos [c + d x] + C \cos [c + d x]^2)}{a + a \cos [c + d x]} dx$$

Optimal (type 4, 134 leaves, 5 steps):

$$-\frac{(A - 3 B + 3 C) \text{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right]}{a d} + \frac{(3 A - 3 B + 5 C) \text{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right]}{3 a d} + \\ \frac{(3 A - 3 B + 5 C) \sqrt{\cos [c + d x]} \sin [c + d x]}{3 a d} - \frac{(A - B + C) \cos [c + d x]^{3/2} \sin [c + d x]}{d (a + a \cos [c + d x])}$$

Result (type 5, 1644 leaves):

$$-\frac{1}{4 (a + a \cos [c + d x])} i A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \csc \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} \right] \\ \left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\ \left. \left( 2 \text{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\ \frac{1}{4 (a + a \cos [c + d x])} 3 i B \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \csc \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} \right] \left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \right. \right. \\ \left. \left. \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / \right. \\ \left. (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \left( 2 \text{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \right. \right.$$

$$\begin{aligned}
& \left( \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \sqrt{1 + e^{2 i dx} \cos[2 c] + i e^{2 i dx} \sin[2 c]} \right) / \\
& \left( -i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c] \right) - \frac{1}{4 (a + a \cos[c + dx])} 3 i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2 i dx} \cos[2 c] + i e^{2 i dx} \sin[2 c]} \right) / \left( 3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c] \right) - \right. \\
& \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \\
& \left. \left. \sqrt{1 + e^{2 i dx} \cos[2 c] + i e^{2 i dx} \sin[2 c]} \right) / \left( -i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c] \right) \right) + \\
& \frac{1}{a + a \cos[c + dx]} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \left( -\frac{2 (-A + B - C + 2 B \cos[c] - 2 C \cos[c]) \operatorname{Csc}[c]}{d} + \frac{4 C \cos[dx] \sin[c]}{3 d} + \right. \\
& \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} + \frac{4 C \cos[c] \sin[dx]}{3 d} \right) - \\
& \left( A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( d (a + a \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \\
& \left( B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( d (a + a \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left( 5 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right.
\end{aligned}$$

$$\frac{\operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \Bigg) / \left( 3d (a + a \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right)$$

■ **Problem 459: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx] + C \cos[c + dx]^2}{\sqrt{\cos[c + dx]} (a + a \cos[c + dx])} dx$$

Optimal (type 4, 90 leaves, 4 steps):

$$\frac{(A - B + 3C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{(A + B - C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{ad} - \frac{(A - B + C) \sqrt{\cos[c + dx]} \sin[c + dx]}{d(a + a \cos[c + dx])}$$

Result (type 5, 1607 leaves):

$$\frac{1}{4(a + a \cos[c + dx])} - i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) - \frac{1}{4(a + a \cos[c + dx])} - i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \frac{1}{4(a + a \cos[c + dx])} - 3i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \frac{1}{4(a + a \cos[c + dx])} - 3i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \frac{1}{4(a + a \cos[c + dx])} - 3i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\frac{\left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])}\right.}{\left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( -i d (1 + e^{2idx}) \cos[c] + d (-1 + e^{2idx}) \sin[c] \right) + \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \left( -\frac{2(A-B+C+2C \cos[c]) \operatorname{Csc}[c]}{d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} \right)}{a + a \cos[c + dx]}$$

$$\frac{\left( A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( d (a + a \cos[c + dx]) \sqrt{1 + \cot[c]^2} \right) - \left( B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( d (a + a \cos[c + dx]) \sqrt{1 + \cot[c]^2} \right) + \left( C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( d (a + a \cos[c + dx]) \sqrt{1 + \cot[c]^2} \right)$$

- **Problem 460: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx] + C \cos[c + dx]^2}{\cos[c + dx]^{3/2} (a + a \cos[c + dx])} dx$$

Optimal (type 4, 125 leaves, 5 steps):

$$-\frac{(3A - B + C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} - \frac{(A - B - C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{(3A - B + C) \sin[c + dx]}{ad \sqrt{\cos[c + dx]}} - \frac{(A - B + C) \sin[c + dx]}{d \sqrt{\cos[c + dx]} (a + a \cos[c + dx])}$$

Result (type 5, 1642 leaves):

$$\begin{aligned}
& - \frac{1}{4(a + a \cos[c + dx])} 3 i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\
& \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) + \\
& \frac{1}{4(a + a \cos[c + dx])} i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \right. \right. \\
& \left. \left. \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / \right. \\
& \left. (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \right. \right. \\
& \left. \left. \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / \right. \\
& \left. (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) - \frac{1}{4(a + a \cos[c + dx])} i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\
& \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) + \\
& \frac{1}{a + a \cos[c + dx]} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \left( \frac{(2 A + A \cos[c] - B \cos[c] + C \cos[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{d} + \right. \\
& \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} + \frac{4 A \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \sin[dx]}{d} \right) +
\end{aligned}$$

$$\left( A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \csc\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \sec[dx - \operatorname{ArcTan}[\cot[c]]] \right. \\ \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\ \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( d (a + a \cos[c + dx]) \sqrt{1 + \cot[c]^2} \right) -$$

$$\left( B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \csc\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \sec[dx - \operatorname{ArcTan}[\cot[c]]] \right. \\ \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\ \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( d (a + a \cos[c + dx]) \sqrt{1 + \cot[c]^2} \right) -$$

$$\left( C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \csc\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \sec[dx - \operatorname{ArcTan}[\cot[c]]] \right. \\ \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\ \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( d (a + a \cos[c + dx]) \sqrt{1 + \cot[c]^2} \right) -$$

■ **Problem 461: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx] + C \cos[c + dx]^2}{\cos[c + dx]^{5/2} (a + a \cos[c + dx])} dx$$

Optimal (type 4, 165 leaves, 6 steps):

$$\frac{(3A - 3B + C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{(5A - 3B + 3C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3ad} + \\ \frac{(5A - 3B + 3C) \sin[c + dx]}{3ad \cos[c + dx]^{3/2}} - \frac{(3A - 3B + C) \sin[c + dx]}{ad \sqrt{\cos[c + dx]}} - \frac{(A - B + C) \sin[c + dx]}{d \cos[c + dx]^{3/2} (a + a \cos[c + dx])}$$

Result (type 5, 1686 leaves):

$$\frac{1}{4(a + a \cos[c + dx])} - 3i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right]$$

$$\begin{aligned}
& \left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / \left( 3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c] \right) - \\
& \left( 2 \text{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c] \right) - \frac{1}{4 (a + a \cos [c + d x])} 3 i B \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \\
\text{Csc} \left[ \frac{c}{2} \right] \text{Sec} \left[ \frac{c}{2} \right] & \left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / \left( 3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c] \right) - \\
& \left( 2 \text{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c] \right) + \frac{1}{4 (a + a \cos [c + d x])} i C \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \\
\text{Csc} \left[ \frac{c}{2} \right] \text{Sec} \left[ \frac{c}{2} \right] & \left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / \left( 3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c] \right) - \\
& \left( 2 \text{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c] \right) + \\
& \frac{1}{a + a \cos [c + d x]} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \sqrt{\cos [c + d x]} \left( -\frac{(2 A - 2 B + A \cos [c] - B \cos [c] + C \cos [c]) \text{Csc} \left[ \frac{c}{2} \right] \text{Sec} \left[ \frac{c}{2} \right] \text{Sec} [c]}{d} - \right. \\
& \quad \frac{2 \text{Sec} \left[ \frac{c}{2} \right] \text{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (A \sin \left[ \frac{d x}{2} \right] - B \sin \left[ \frac{d x}{2} \right] + C \sin \left[ \frac{d x}{2} \right])}{d} + \frac{4 A \text{Sec} [c] \text{Sec} [c + d x]^2 \sin [d x]}{3 d} + \\
& \quad \left. \frac{4 \text{Sec} [c] \text{Sec} [c + d x] (A \sin [c] - 3 A \sin [d x] + 3 B \sin [d x])}{3 d} \right) - \\
& \left( 5 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \text{Csc} \left[ \frac{c}{2} \right] \text{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan} [\text{Cot} [c]]] \right]^2 \right) \text{Sec} \left[ \frac{c}{2} \right] \\
& \text{Sec} [d x - \text{ArcTan} [\text{Cot} [c]]] \sqrt{1 - \sin [d x - \text{ArcTan} [\text{Cot} [c]]]} \sqrt{-\sqrt{1 + \text{Cot} [c]^2} \sin [c] \sin [d x - \text{ArcTan} [\text{Cot} [c]]]}
\end{aligned}$$



$$\begin{aligned}
& \left. \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \left( 3 d (a + a \cos[c + d x]) \sqrt{1 + \text{Cot}[c]^2} \right) + \\
& \left( B \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \right. \\
& \left. \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left( d (a + a \cos[c + d x]) \sqrt{1 + \text{Cot}[c]^2} \right) - \left( C \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \right. \\
& \left. \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \left. \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \left( d (a + a \cos[c + d x]) \sqrt{1 + \text{Cot}[c]^2} \right)
\end{aligned}$$

- **Problem 462: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + d x] + C \cos[c + d x]^2}{\cos[c + d x]^{7/2} (a + a \cos[c + d x])} dx$$

Optimal (type 4, 210 leaves, 7 steps):

$$\begin{aligned}
& - \frac{3 (7 A - 5 B + 5 C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] - (5 A - 5 B + 3 C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{5 a d} + \\
& \frac{(7 A - 5 B + 5 C) \sin[c + d x]}{5 a d \cos[c + d x]^{5/2}} - \frac{(5 A - 5 B + 3 C) \sin[c + d x]}{3 a d \cos[c + d x]^{3/2}} + \frac{3 (7 A - 5 B + 5 C) \sin[c + d x]}{5 a d \sqrt{\cos[c + d x]}} - \frac{(A - B + C) \sin[c + d x]}{d \cos[c + d x]^{5/2} (a + a \cos[c + d x])}
\end{aligned}$$

Result (type 5, 1745 leaves):

$$\begin{aligned}
& - \frac{1}{20 (a + a \cos[c + d x])} 21 i A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \\
& \left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / \left( 3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c] \right) - \right. \\
& \left. \left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(-i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} + \\
& \frac{1}{4 \left(a + a \operatorname{Cos}[c + d x]\right)} 3 i B \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} \left(\operatorname{Cos}[c] + i \operatorname{Sin}[c]\right)^2\right]\right.\right. \\
& \left.\left.\sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}\right)\right) / \\
& \left(3 i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] - 3 d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right) - \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} \left(\operatorname{Cos}[c] + i \operatorname{Sin}[c]\right)^2\right]\right. \\
& \left.\sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}\right) / \\
& \left(-i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right) - \frac{1}{4 \left(a + a \operatorname{Cos}[c + d x]\right)} 3 i C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} \left(\operatorname{Cos}[c] + i \operatorname{Sin}[c]\right)^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right.\right. \\
& \left.\left.\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}\right) / \left(3 i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] - 3 d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right) - \right. \\
& \left.\left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} \left(\operatorname{Cos}[c] + i \operatorname{Sin}[c]\right)^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right.\right. \\
& \left.\left.\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}\right) / \left(-i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right) + \frac{1}{a + a \operatorname{Cos}[c + d x]} \right. \\
& \left. \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sqrt{\operatorname{Cos}[c + d x]} \left(\frac{\left(16 A - 10 B + 10 C + 5 A \operatorname{Cos}[c] - 5 B \operatorname{Cos}[c] + 5 C \operatorname{Cos}[c]\right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{5 d} + \right. \\
& \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] \left(A \operatorname{Sin}\left[\frac{d x}{2}\right] - B \operatorname{Sin}\left[\frac{d x}{2}\right] + C \operatorname{Sin}\left[\frac{d x}{2}\right]\right)}{d} + \frac{4 A \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 \operatorname{Sin}[d x]}{5 d} + \\
& \frac{4 \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 \left(3 A \operatorname{Sin}[c] - 5 A \operatorname{Sin}[d x] + 5 B \operatorname{Sin}[d x]\right)}{15 d} - \\
& \left.\frac{4 \operatorname{Sec}[c] \operatorname{Sec}[c + d x] \left(5 A \operatorname{Sin}[c] - 5 B \operatorname{Sin}[c] - 24 A \operatorname{Sin}[d x] + 15 B \operatorname{Sin}[d x] - 15 C \operatorname{Sin}[d x]\right)}{15 d}\right) + \\
& \left(5 A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \left. \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}\right)
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \left( 3 d (a + a \cos[c + d x]) \sqrt{1 + \text{Cot}[c]^2} \right) - \\
& \left( 5 B \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2}\right] \right. \\
& \left. \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \left. \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \left( 3 d (a + a \cos[c + d x]) \sqrt{1 + \text{Cot}[c]^2} \right) + \\
& \left( C \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \right. \\
& \left. \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \left. \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \left( d (a + a \cos[c + d x]) \sqrt{1 + \text{Cot}[c]^2} \right)
\end{aligned}$$

■ **Problem 463: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^{5/2} (A + B \cos[c + d x] + C \cos[c + d x]^2)}{(a + a \cos[c + d x])^2} dx$$

Optimal (type 4, 214 leaves, 7 steps):

$$\begin{aligned}
& \frac{(20 A - 35 B + 56 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 a^2 d} - \frac{5 (A - 2 B + 3 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 a^2 d} - \frac{5 (A - 2 B + 3 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{3 a^2 d} + \\
& \frac{(20 A - 35 B + 56 C) \cos[c + d x]^{3/2} \sin[c + d x]}{15 a^2 d} - \frac{(A - 2 B + 3 C) \cos[c + d x]^{5/2} \sin[c + d x]}{a^2 d (1 + \cos[c + d x])} - \frac{(A - B + C) \cos[c + d x]^{7/2} \sin[c + d x]}{3 d (a + a \cos[c + d x])^2}
\end{aligned}$$

Result (type 5, 1789 leaves):

$$\begin{aligned}
& \frac{1}{(a + a \cos[c + d x])^2} {}_2F_1\left[\frac{c}{2} + \frac{d x}{2}, 4, \frac{c}{2}\right] \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \\
& \left( \left( 2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) \right) / \left( 3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c] \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \frac{1}{2 (a + a \operatorname{Cos}[c + d x])^2} 7 i B \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \\
& \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left( 3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \right. \\
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) + \frac{1}{5 (a + a \operatorname{Cos}[c + d x])^2} 28 i C \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \\
& \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left( 3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \right. \\
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) + \\
& \left( 10 A \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \right) \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \quad \left. \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \operatorname{Cos}[c + d x])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left( 20 B \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \right) \operatorname{Sec} \left[ \frac{c}{2} \right] \\
& \quad \left. \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \operatorname{Cos}[c + d x])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) +
\end{aligned}$$

$$\left( 10 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\ \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\ \left( d (a + a \cos[c + dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \\ \left( -\frac{4(10A - 15B + 20C + 10A \cos[c] - 20B \cos[c] + 36C \cos[c]) \operatorname{Csc}[c]}{5d} + \frac{8(B - 2C) \cos[dx] \sin[c]}{3d} + \frac{4C \cos[2dx] \sin[2c]}{5d} + \right. \\ \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (2A \sin\left[\frac{dx}{2}\right] - 3B \sin\left[\frac{dx}{2}\right] + 4C \sin\left[\frac{dx}{2}\right])}{d} + \right. \\ \left. \frac{8(B - 2C) \cos[c] \sin[dx]}{3d} + \frac{4C \cos[2c] \sin[2dx]}{5d} + \frac{2(A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} \right)$$

- **Problem 464: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{3/2} (A + B \cos[c + dx] + C \cos[c + dx]^2)}{(a + a \cos[c + dx])^2} dx$$

Optimal (type 4, 180 leaves, 6 steps):

$$-\frac{(A - 4B + 7C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{a^2 d} + \frac{(2A - 5B + 10C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3a^2 d} + \\ \frac{(2A - 5B + 10C) \sqrt{\cos[c + dx]} \sin[c + dx]}{3a^2 d} - \frac{(A - 4B + 7C) \cos[c + dx]^{3/2} \sin[c + dx]}{3a^2 d (1 + \cos[c + dx])} - \frac{(A - B + C) \cos[c + dx]^{5/2} \sin[c + dx]}{3d (a + a \cos[c + dx])^2}$$

Result (type 5, 1741 leaves):

$$-\frac{1}{2(a + a \cos[c + dx])^2} i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ \left( \left( 2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) +$$

$$\begin{aligned}
& \frac{1}{(a + a \cos[c + dx])^2} 2 i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \right. \right. \\
& \quad \left. \left. \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / \right. \\
& \quad \left. (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \right. \right. \\
& \quad \left. \left. \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / \right. \\
& \quad \left. (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) - \frac{1}{2 (a + a \cos[c + dx])^2} 7 i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \quad \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\
& \quad \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) - \\
& \quad \left( 4 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \cos[c + dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \\
& \quad \left( 10 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \cos[c + dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \quad \left( 20 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right.
\end{aligned}$$

$$\left. \begin{aligned} & \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \Big/ \\ & \left( 3d (a + a \cos[c + dx])^2 \sqrt{1 + \text{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \\ & \left( -\frac{4(-A + 2B - 3C + 2B \cos[c] - 4C \cos[c]) \text{Csc}[c]}{d} + \frac{8C \cos[dx] \sin[c]}{3d} - \frac{2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} \right) + \\ & \left. \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - 2B \sin\left[\frac{dx}{2}\right] + 3C \sin\left[\frac{dx}{2}\right])}{d} + \frac{8C \cos[c] \sin[dx]}{3d} - \frac{2(A - B + C) \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Tan}\left[\frac{c}{2}\right]}{3d} \right) \end{aligned} \right)$$

■ **Problem 465: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c + dx]} (A + B \cos[c + dx] + C \cos[c + dx]^2)}{(a + a \cos[c + dx])^2} dx$$

Optimal (type 4, 139 leaves, 5 steps):

$$\begin{aligned} & -\frac{(B - 4C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{a^2 d} + \frac{(A + 2B - 5C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3a^2 d} + \\ & \frac{(A + 2B - 5C) \sqrt{\cos[c + dx]} \sin[c + dx]}{3a^2 d (1 + \cos[c + dx])} - \frac{(A - B + C) \cos[c + dx]^{3/2} \sin[c + dx]}{3d (a + a \cos[c + dx])^2} \end{aligned}$$

Result (type 5, 1347 leaves):

$$\begin{aligned} & -\frac{1}{2(a + a \cos[c + dx])^2} i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \\ & \left( \left( 2 e^{2i dx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1 + e^{2i dx}) \cos[c] + 2i(-1 + e^{2i dx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2i dx} \cos[2c] + i e^{2i dx} \sin[2c]} \right) \Big/ (3i d (1 + e^{2i dx}) \cos[c] - 3d(-1 + e^{2i dx}) \sin[c]) - \right. \\ & \left( 2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1 + e^{2i dx}) \cos[c] + 2i(-1 + e^{2i dx}) \sin[c])} \right. \\ & \quad \left. \sqrt{1 + e^{2i dx} \cos[2c] + i e^{2i dx} \sin[2c]} \right) \Big/ (-i d (1 + e^{2i dx}) \cos[c] + d(-1 + e^{2i dx}) \sin[c]) \Big) + \frac{1}{(a + a \cos[c + dx])^2} 2i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\ & \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2i dx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1 + e^{2i dx}) \cos[c] + 2i(-1 + e^{2i dx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2i dx} \cos[2c] + i e^{2i dx} \sin[2c]} \right) \Big/ (3i d (1 + e^{2i dx}) \cos[c] - 3d(-1 + e^{2i dx}) \sin[c]) - \right. \end{aligned}$$

$$\begin{aligned}
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( -id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c] \right) - \\
& \left( 2A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \right. \\
& \quad \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \quad \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( 3d(a + a \cos[c + dx])^2 \sqrt{1 + \cot[c]^2} \right) - \\
& \left( 4B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \right. \\
& \quad \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \quad \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( 3d(a + a \cos[c + dx])^2 \sqrt{1 + \cot[c]^2} \right) + \\
& \left( 10C \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \right. \\
& \quad \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left( 3d(a + a \cos[c + dx])^2 \sqrt{1 + \cot[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \sqrt{\cos[c + dx]} \\
& \left( -\frac{4(-B + 2C + 2C \cos[c]) \operatorname{Csc}[c]}{d} + \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] (B \sin \left[ \frac{dx}{2} \right] - 2C \sin \left[ \frac{dx}{2} \right])}{d} + \right. \\
& \quad \left. \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 (A \sin \left[ \frac{dx}{2} \right] - B \sin \left[ \frac{dx}{2} \right] + C \sin \left[ \frac{dx}{2} \right])}{3d} + \frac{2(A - B + C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Tan} \left[ \frac{c}{2} \right]}{3d} \right)
\end{aligned}$$

■ **Problem 466: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**



$$\int \frac{A + B \cos[c + dx] + C \cos[c + dx]^2}{\sqrt{\cos[c + dx]} (a + a \cos[c + dx])^2} dx$$

Optimal (type 4, 133 leaves, 5 steps):

$$\frac{(A - C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{a^2 d} + \frac{(2A + B + 2C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3 a^2 d} - \frac{(A - C) \sqrt{\cos[c + dx]} \operatorname{Sin}[c + dx]}{a^2 d (1 + \cos[c + dx])} - \frac{(A - B + C) \sqrt{\cos[c + dx]} \operatorname{Sin}[c + dx]}{3 d (a + a \cos[c + dx])^2}$$

Result (type 5, 1342 leaves):

$$\frac{1}{2 (a + a \cos[c + dx])^2} i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \right. \\ \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\ \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) - \frac{1}{2 (a + a \cos[c + dx])^2} i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \\ \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\ \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) - \\ \left( 4 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\ \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\ \left. \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \cos[c + dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) -$$

$$\begin{aligned}
& \left( 2 B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \right. \\
& \quad \left. \operatorname{Sec} [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \sqrt{1 - \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot} [c]^2} \sin [c] \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right. \\
& \quad \left. \sqrt{1 + \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right) / \left( 3 d (a + a \cos [c + dx])^2 \sqrt{1 + \operatorname{Cot} [c]^2} \right) - \\
& \left( 4 C \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \right. \\
& \quad \left. \sqrt{1 - \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot} [c]^2} \sin [c] \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \sqrt{1 + \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right) / \\
& \left( 3 d (a + a \cos [c + dx])^2 \sqrt{1 + \operatorname{Cot} [c]^2} \right) + \frac{1}{(a + a \cos [c + dx])^2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \sqrt{\cos [c + dx]} \\
& \left( -\frac{4 (A - C) \operatorname{Csc} [c]}{d} - \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] (A \sin \left[ \frac{dx}{2} \right] - C \sin \left[ \frac{dx}{2} \right])}{d} - \right. \\
& \quad \left. \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 (A \sin \left[ \frac{dx}{2} \right] - B \sin \left[ \frac{dx}{2} \right] + C \sin \left[ \frac{dx}{2} \right])}{3 d} - \frac{2 (A - B + C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Tan} \left[ \frac{c}{2} \right]}{3 d} \right)
\end{aligned}$$

- **Problem 467: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos [c + dx] + C \cos [c + dx]^2}{\cos [c + dx]^{3/2} (a + a \cos [c + dx])^2} dx$$

Optimal (type 4, 175 leaves, 6 steps):

$$\begin{aligned}
& \frac{(4 A - B) \operatorname{EllipticE} \left[ \frac{1}{2} (c + dx), 2 \right]}{a^2 d} - \frac{(5 A - 2 B - C) \operatorname{EllipticF} \left[ \frac{1}{2} (c + dx), 2 \right]}{3 a^2 d} + \\
& \frac{(4 A - B) \sin [c + dx]}{a^2 d \sqrt{\cos [c + dx]}} - \frac{(5 A - 2 B - C) \sin [c + dx]}{3 a^2 d \sqrt{\cos [c + dx]} (1 + \cos [c + dx])} - \frac{(A - B + C) \sin [c + dx]}{3 d \sqrt{\cos [c + dx]} (a + a \cos [c + dx])^2}
\end{aligned}$$

Result (type 5, 1380 leaves):

$$\begin{aligned}
& -\frac{1}{(a + a \cos [c + dx])^2} 2 i A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \\
& \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos [c] + 2 i (-1 + e^{2 i dx}) \sin [c])} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]} \right) / \left( 3 i d (1 + e^{2ix}) \cos[c] - 3 d (-1 + e^{2ix}) \sin[c] \right) - \\
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2 i (-1 + e^{2ix}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]} \right) / \left( -i d (1 + e^{2ix}) \cos[c] + d (-1 + e^{2ix}) \sin[c] \right) \right) + \frac{1}{2 (a + a \cos[c + dx])^2} i B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \\
& \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \left( \left( 2 e^{2ix} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2 i (-1 + e^{2ix}) \sin[c])} \right. \right. \right. \\
& \left. \left. \sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]} \right) / \left( 3 i d (1 + e^{2ix}) \cos[c] - 3 d (-1 + e^{2ix}) \sin[c] \right) - \right. \\
& \left. \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2 i (-1 + e^{2ix}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]} \right) / \left( -i d (1 + e^{2ix}) \cos[c] + d (-1 + e^{2ix}) \sin[c] \right) \right) + \\
& \left( 10 A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right) \\
& \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \cos[c + dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left( 4 B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \right. \\
& \left. \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \cos[c + dx])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left( 2 C \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right) \\
& \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) /
\end{aligned}$$

$$\left( 3 d (a + a \cos [c + d x])^2 \sqrt{1 + \cot [c]^2} \right) + \frac{1}{(a + a \cos [c + d x])^2} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\cos [c + d x]}$$

$$\left( \frac{2 (2 A + 2 A \cos [c] - B \cos [c]) \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} [c]}{d} + \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (2 A \sin \left[ \frac{d x}{2} \right] - B \sin \left[ \frac{d x}{2} \right])}{d} + \right.$$

$$\left. \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (A \sin \left[ \frac{d x}{2} \right] - B \sin \left[ \frac{d x}{2} \right] + C \sin \left[ \frac{d x}{2} \right])}{3 d} + \frac{8 A \operatorname{Sec} [c] \operatorname{Sec} [c + d x] \sin [d x]}{d} + \frac{2 (A - B + C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \tan \left[ \frac{c}{2} \right]}{3 d} \right)$$

■ **Problem 468: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos [c + d x] + C \cos [c + d x]^2}{\cos [c + d x]^{5/2} (a + a \cos [c + d x])^2} dx$$

Optimal (type 4, 211 leaves, 7 steps):

$$\frac{(7 A - 4 B + C) \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right]}{a^2 d} + \frac{(10 A - 5 B + 2 C) \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right]}{3 a^2 d} + \frac{(10 A - 5 B + 2 C) \sin [c + d x]}{3 a^2 d \cos [c + d x]^{3/2}} -$$

$$\frac{(7 A - 4 B + C) \sin [c + d x]}{a^2 d \sqrt{\cos [c + d x]}} - \frac{(7 A - 4 B + C) \sin [c + d x]}{3 a^2 d \cos [c + d x]^{3/2} (1 + \cos [c + d x])} - \frac{(A - B + C) \sin [c + d x]}{3 d \cos [c + d x]^{3/2} (a + a \cos [c + d x])^2}$$

Result (type 5, 1782 leaves):

$$\frac{1}{2 (a + a \cos [c + d x])^2} 7 i A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]$$

$$\left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right.$$

$$\left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) - \frac{1}{(a + a \cos [c + d x])^2} 2 i B \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4$$

$$\operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right.$$

$$\left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \frac{1}{2 (a + a \cos [c + d x])^2} i C \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4$$

$$\begin{aligned}
& \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) \right) / \left( 3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \\
& \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) \Big) - \\
& \left( 20 A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \operatorname{Cos}[c + d x])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \\
& \left( 10 B \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \operatorname{Cos}[c + d x])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left( 4 C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left( 3 d (a + a \operatorname{Cos}[c + d x])^2 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a + a \operatorname{Cos}[c + d x])^2} \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \sqrt{\operatorname{Cos}[c + d x]} \\
& \left( -\frac{2 (4 A - 2 B + 3 A \operatorname{Cos}[c] - 2 B \operatorname{Cos}[c] + C \operatorname{Cos}[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (3 A \operatorname{Sin}\left[\frac{d x}{2}\right] - 2 B \operatorname{Sin}\left[\frac{d x}{2}\right] + C \operatorname{Sin}\left[\frac{d x}{2}\right])}{d} \right) -
\end{aligned}$$

$$\frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \operatorname{Sin}\left[\frac{dx}{2}\right] - B \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right])}{3 d} + \frac{8 A \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \operatorname{Sin}[dx]}{3 d} + \left. \frac{8 \operatorname{Sec}[c] \operatorname{Sec}[c + dx] (A \operatorname{Sin}[c] - 6 A \operatorname{Sin}[dx] + 3 B \operatorname{Sin}[dx])}{3 d} - \frac{2 (A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right)$$

- **Problem 469: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + dx]^{7/2} (A + B \operatorname{Cos}[c + dx] + C \operatorname{Cos}[c + dx]^2)}{(a + a \operatorname{Cos}[c + dx])^3} dx$$

Optimal (type 4, 273 leaves, 8 steps):

$$\frac{7 (7 A - 17 B + 33 C) \operatorname{EllipticE}\left[\frac{1}{2} (c + dx), 2\right]}{10 a^3 d} - \frac{(13 A - 33 B + 63 C) \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), 2\right]}{6 a^3 d} - \frac{(13 A - 33 B + 63 C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{6 a^3 d} + \frac{7 (7 A - 17 B + 33 C) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{30 a^3 d} - \frac{(A - B + C) \operatorname{Cos}[c + dx]^{9/2} \operatorname{Sin}[c + dx]}{5 d (a + a \operatorname{Cos}[c + dx])^3} - \frac{(2 A - 7 B + 12 C) \operatorname{Cos}[c + dx]^{7/2} \operatorname{Sin}[c + dx]}{15 a d (a + a \operatorname{Cos}[c + dx])^2} - \frac{(13 A - 33 B + 63 C) \operatorname{Cos}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{10 d (a^3 + a^3 \operatorname{Cos}[c + dx])}$$

Result (type 5, 1888 leaves):

$$\frac{1}{10 (a + a \operatorname{Cos}[c + dx])^3} 49 i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) - \frac{1}{10 (a + a \operatorname{Cos}[c + dx])^3} 119 i B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) /$$

$$\begin{aligned}
& \left( -i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c] \right) + \frac{1}{10 (a + a \cos [c + d x])^3} 231 i C \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \\
& \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \left( 26 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \right. \\
& \quad \left. \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot} [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right. \\
& \quad \left. \left. \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right) / \left( 3 d (a + a \cos [c + d x])^3 \sqrt{1 + \operatorname{Cot} [c]^2} \right) - \\
& \left( 22 B \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \right. \\
& \quad \left. \operatorname{Sec} [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot} [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right. \\
& \quad \left. \left. \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right) / \left( d (a + a \cos [c + d x])^3 \sqrt{1 + \operatorname{Cot} [c]^2} \right) + \\
& \left( 42 C \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \right]^2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \right. \\
& \quad \left. \left. \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot} [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right) / \right. \\
& \quad \left( d (a + a \cos [c + d x])^3 \sqrt{1 + \operatorname{Cot} [c]^2} \right) + \frac{1}{(a + a \cos [c + d x])^3} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \\
& \left( -\frac{4 (29 A - 59 B + 99 C + 20 A \cos [c] - 60 B \cos [c] + 132 C \cos [c]) \operatorname{Csc} [c]}{5 d} + \frac{16 (B - 3 C) \cos [d x] \sin [c]}{3 d} + \frac{8 C \cos [2 d x] \sin [2 c]}{5 d} - \right.
\end{aligned}$$

$$\frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left(A \operatorname{Sin}\left[\frac{dx}{2}\right] - B \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right]\right)}{5 d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(14 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 19 B \operatorname{Sin}\left[\frac{dx}{2}\right] + 24 C \operatorname{Sin}\left[\frac{dx}{2}\right]\right)}{15 d}$$

$$\frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(29 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 59 B \operatorname{Sin}\left[\frac{dx}{2}\right] + 99 C \operatorname{Sin}\left[\frac{dx}{2}\right]\right)}{5 d} + \frac{16 (B - 3 C) \operatorname{Cos}[c] \operatorname{Sin}[dx]}{3 d} +$$

$$\left. \frac{8 C \operatorname{Cos}[2 c] \operatorname{Sin}[2 dx]}{5 d} + \frac{4 (14 A - 19 B + 24 C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} - \frac{2 (A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right)$$

■ **Problem 470: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + dx]^{5/2} (A + B \operatorname{Cos}[c + dx] + C \operatorname{Cos}[c + dx]^2)}{(a + a \operatorname{Cos}[c + dx])^3} dx$$

Optimal (type 4, 232 leaves, 7 steps):

$$-\frac{(9 A - 49 B + 119 C) \operatorname{EllipticE}\left[\frac{1}{2} (c + dx), 2\right]}{10 a^3 d} + \frac{(3 A - 13 B + 33 C) \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), 2\right]}{6 a^3 d} + \frac{(3 A - 13 B + 33 C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{6 a^3 d}$$

$$-\frac{(A - B + C) \operatorname{Cos}[c + dx]^{7/2} \operatorname{Sin}[c + dx]}{5 d (a + a \operatorname{Cos}[c + dx])^3} + \frac{(B - 2 C) \operatorname{Cos}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{3 a d (a + a \operatorname{Cos}[c + dx])^2} - \frac{(9 A - 49 B + 119 C) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{30 d (a^3 + a^3 \operatorname{Cos}[c + dx])}$$

Result (type 5, 1841 leaves):

$$-\frac{1}{10 (a + a \operatorname{Cos}[c + dx])^3} 9 i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \right.$$

$$\left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) +$$

$$\frac{1}{10 (a + a \operatorname{Cos}[c + dx])^3} 49 i B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \right. \right.$$

$$\left. \left. \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])}\right. \right. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) /$$

$$(3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \right.$$

$$\left. \left. \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])}\right. \right. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) /$$



$$\begin{aligned}
& \left( -i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \frac{1}{10 (a + a \operatorname{Cos}[c + d x])^3} 119 i C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) - \right. \\
& \quad \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) \right) - \\
& \left( 2 A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \left. \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( d (a + a \operatorname{Cos}[c + d x])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \\
& \left( 26 B \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \left. \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \operatorname{Cos}[c + d x])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left( 22 C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \left. \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \right. \\
& \quad \left( d (a + a \operatorname{Cos}[c + d x])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a + a \operatorname{Cos}[c + d x])^3} \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \sqrt{\operatorname{Cos}[c + d x]} \\
& \left( -\frac{4 (-9 A + 29 B - 59 C + 20 B \operatorname{Cos}[c] - 60 C \operatorname{Cos}[c]) \operatorname{Csc}[c]}{5 d} + \frac{16 C \operatorname{Cos}[d x] \operatorname{Sin}[c]}{3 d} + \right.
\end{aligned}$$

$$\frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left(A \operatorname{Sin}\left[\frac{dx}{2}\right] - B \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right]\right)}{5 d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(9 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 14 B \operatorname{Sin}\left[\frac{dx}{2}\right] + 19 C \operatorname{Sin}\left[\frac{dx}{2}\right]\right)}{15 d} +$$

$$\frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(9 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 29 B \operatorname{Sin}\left[\frac{dx}{2}\right] + 59 C \operatorname{Sin}\left[\frac{dx}{2}\right]\right)}{5 d} + \frac{16 C \operatorname{Cos}[c] \operatorname{Sin}[dx]}{3 d} -$$

$$\left( \frac{4 (9 A - 14 B + 19 C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} + \frac{2 (A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right)$$

■ **Problem 471: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + dx]^{3/2} (A + B \operatorname{Cos}[c + dx] + C \operatorname{Cos}[c + dx]^2)}{(a + a \operatorname{Cos}[c + dx])^3} dx$$

Optimal (type 4, 195 leaves, 6 steps):

$$-\frac{(A + 9 B - 49 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10 a^3 d} + \frac{(A + 3 B - 13 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6 a^3 d} -$$

$$\frac{(A - B + C) \operatorname{Cos}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{5 d (a + a \operatorname{Cos}[c + dx])^3} + \frac{(2 A + 3 B - 8 C) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{15 a d (a + a \operatorname{Cos}[c + dx])^2} + \frac{(A + 3 B - 13 C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{6 d (a^3 + a^3 \operatorname{Cos}[c + dx])}$$

Result (type 5, 1809 leaves):

$$-\frac{1}{10 (a + a \operatorname{Cos}[c + dx])^3} i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \right.$$

$$\left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) -$$

$$\frac{1}{10 (a + a \operatorname{Cos}[c + dx])^3} 9 i B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \right. \right.$$

$$\left. \left. \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])}\right. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) /$$

$$(3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \right.$$

$$\left. \left. \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])}\right. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) /$$

$$\begin{aligned}
& \left( -i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) + \frac{1}{10 (a + a \operatorname{Cos}[c + d x])^3} 49 i C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left( \left( 2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) - \right. \\
& \quad \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) \right) - \\
& \left( 2 A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \left. \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \operatorname{Cos}[c + d x])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left( 2 B \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \left. \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( d (a + a \operatorname{Cos}[c + d x])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \\
& \left( 26 C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left( 3 d (a + a \operatorname{Cos}[c + d x])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a + a \operatorname{Cos}[c + d x])^3} \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \sqrt{\operatorname{Cos}[c + d x]} \\
& \left( -\frac{4 (-A - 9 B + 29 C + 20 C \operatorname{Cos}[c]) \operatorname{Csc}[c]}{5 d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (A \operatorname{Sin}\left[\frac{d x}{2}\right] + 9 B \operatorname{Sin}\left[\frac{d x}{2}\right] - 29 C \operatorname{Sin}\left[\frac{d x}{2}\right])}{5 d} \right) -
\end{aligned}$$

$$\frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \operatorname{Sin}\left[\frac{dx}{2}\right] - B \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right])}{5 d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (4 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 9 B \operatorname{Sin}\left[\frac{dx}{2}\right] + 14 C \operatorname{Sin}\left[\frac{dx}{2}\right])}{15 d} + \frac{4 (4 A - 9 B + 14 C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} - \frac{2 (A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d}$$

■ **Problem 472: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Cos}[c + dx]} (A + B \operatorname{Cos}[c + dx] + C \operatorname{Cos}[c + dx]^2)}{(a + a \operatorname{Cos}[c + dx])^3} dx$$

Optimal (type 4, 191 leaves, 6 steps):

$$\frac{(A - B - 9 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10 a^3 d} + \frac{(A + B + 3 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6 a^3 d} - \frac{(A - B + C) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{5 d (a + a \operatorname{Cos}[c + dx])^3} + \frac{(4 A + B - 6 C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{15 a d (a + a \operatorname{Cos}[c + dx])^2} - \frac{(A - B - 9 C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{10 d (a^3 + a^3 \operatorname{Cos}[c + dx])}$$

Result (type 5, 1799 leaves):

$$\frac{1}{10 (a + a \operatorname{Cos}[c + dx])^3} i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) - \frac{1}{10 (a + a \operatorname{Cos}[c + dx])^3} i B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) - \frac{1}{10 (a + a \operatorname{Cos}[c + dx])^3} 9 i C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right)$$

$$\begin{aligned}
& \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( 3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c] \right) - \\
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \\
& \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( -id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c] \right) - \\
& \left( 2A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3d(a + a \cos[c + dx])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left( 2B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \right. \\
& \left. \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3d(a + a \cos[c + dx])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left( 2C \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left( d(a + a \cos[c + dx])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^3} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \sqrt{\cos[c + dx]} \\
& \left( -\frac{4(A - B - 9C) \operatorname{Csc}[c]}{5d} - \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] (A \sin \left[ \frac{dx}{2} \right] - B \sin \left[ \frac{dx}{2} \right] - 9C \sin \left[ \frac{dx}{2} \right])}{5d} \right) + \\
& \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 (A \sin \left[ \frac{dx}{2} \right] + 4B \sin \left[ \frac{dx}{2} \right] - 9C \sin \left[ \frac{dx}{2} \right])}{15d} + \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 (A \sin \left[ \frac{dx}{2} \right] - B \sin \left[ \frac{dx}{2} \right] + C \sin \left[ \frac{dx}{2} \right])}{5d} +
\end{aligned}$$

$$\frac{4(A + 4B - 9C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15d} + \frac{2(A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5d}$$

■ **Problem 473: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx] + C \cos[c + dx]^2}{\sqrt{\cos[c + dx]} (a + a \cos[c + dx])^3} dx$$

Optimal (type 4, 193 leaves, 6 steps):

$$\frac{(9A + B - C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10a^3d} + \frac{(3A + B + C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6a^3d} - \frac{(A - B + C) \sqrt{\cos[c + dx]} \operatorname{Sin}[c + dx]}{5d(a + a \cos[c + dx])^3} - \frac{(6A - B - 4C) \sqrt{\cos[c + dx]} \operatorname{Sin}[c + dx]}{15ad(a + a \cos[c + dx])^2} - \frac{(9A + B - C) \sqrt{\cos[c + dx]} \operatorname{Sin}[c + dx]}{10d(a^3 + a^3 \cos[c + dx])}$$

Result (type 5, 1802 leaves):

$$\frac{1}{10(a + a \cos[c + dx])^3} {}_9F_9 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \sqrt{1 + e^{2idx}\cos[2c] + i e^{2idx}\sin[2c]} \right) / (3id(1 + e^{2idx})\cos[c] - 3d(-1 + e^{2idx})\sin[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \sqrt{1 + e^{2idx}\cos[2c] + i e^{2idx}\sin[2c]} \right) / (-id(1 + e^{2idx})\cos[c] + d(-1 + e^{2idx})\sin[c]) \right) + \frac{1}{10(a + a \cos[c + dx])^3} i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \sqrt{1 + e^{2idx}\cos[2c] + i e^{2idx}\sin[2c]} \right) / (3id(1 + e^{2idx})\cos[c] - 3d(-1 + e^{2idx})\sin[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \sqrt{1 + e^{2idx}\cos[2c] + i e^{2idx}\sin[2c]} \right) / (-id(1 + e^{2idx})\cos[c] + d(-1 + e^{2idx})\sin[c]) \right) - \frac{1}{10(a + a \cos[c + dx])^3} i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \sqrt{1 + e^{2idx}\cos[2c] + i e^{2idx}\sin[2c]} \right) / (3id(1 + e^{2idx})\cos[c] - 3d(-1 + e^{2idx})\sin[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \sqrt{1 + e^{2idx}\cos[2c] + i e^{2idx}\sin[2c]} \right) / (-id(1 + e^{2idx})\cos[c] + d(-1 + e^{2idx})\sin[c]) \right)$$

$$\begin{aligned}
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( -id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c] \right) - \\
& \left( 2A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \right. \\
& \quad \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \quad \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( d(a + a \cos[c + dx])^3 \sqrt{1 + \cot[c]^2} \right) - \\
& \left( 2B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \right. \\
& \quad \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \quad \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( 3d(a + a \cos[c + dx])^3 \sqrt{1 + \cot[c]^2} \right) - \\
& \left( 2C \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \right. \\
& \quad \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left( 3d(a + a \cos[c + dx])^3 \sqrt{1 + \cot[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^3} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \sqrt{\cos[c + dx]} \\
& \left( -\frac{4(9A + B - C) \operatorname{Csc}[c]}{5d} - \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 (6A \sin \left[ \frac{dx}{2} \right] - B \sin \left[ \frac{dx}{2} \right] - 4C \sin \left[ \frac{dx}{2} \right])}{15d} \right. \\
& \quad \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] (9A \sin \left[ \frac{dx}{2} \right] + B \sin \left[ \frac{dx}{2} \right] - C \sin \left[ \frac{dx}{2} \right])}{5d} - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 (A \sin \left[ \frac{dx}{2} \right] - B \sin \left[ \frac{dx}{2} \right] + C \sin \left[ \frac{dx}{2} \right])}{5d} \\
& \quad \left. \frac{4(6A - B - 4C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Tan} \left[ \frac{c}{2} \right]}{15d} - \frac{2(A - B + C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Tan} \left[ \frac{c}{2} \right]}{5d} \right)
\end{aligned}$$

**Problem 474: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx] + C \cos[c + dx]^2}{\cos[c + dx]^{3/2} (a + a \cos[c + dx])^3} dx$$

Optimal (type 4, 237 leaves, 7 steps):

$$\begin{aligned} & - \frac{(49A - 9B - C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10a^3d} - \frac{(13A - 3B - C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6a^3d} + \frac{(49A - 9B - C) \sin[c + dx]}{10a^3d \sqrt{\cos[c + dx]}} - \\ & \frac{(A - B + C) \sin[c + dx]}{5d \sqrt{\cos[c + dx]} (a + a \cos[c + dx])^3} - \frac{(8A - 3B - 2C) \sin[c + dx]}{15ad \sqrt{\cos[c + dx]} (a + a \cos[c + dx])^2} - \frac{(13A - 3B - C) \sin[c + dx]}{6d \sqrt{\cos[c + dx]} (a^3 + a^3 \cos[c + dx])} \end{aligned}$$

Result (type 5, 1841 leaves):

$$\begin{aligned} & - \frac{1}{10(a + a \cos[c + dx])^3} 49iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ & \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx}\cos[2c] + ie^{2idx}\sin[2c]} \right) / (3id(1 + e^{2idx})\cos[c] - 3d(-1 + e^{2idx})\sin[c]) - \right. \\ & \left. \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx}\cos[2c] + ie^{2idx}\sin[2c]} \right) / (-id(1 + e^{2idx})\cos[c] + d(-1 + e^{2idx})\sin[c]) \right) + \\ & \frac{1}{10(a + a \cos[c + dx])^3} 9iB \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \right. \right. \\ & \quad \left. \left. \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \sqrt{1 + e^{2idx}\cos[2c] + ie^{2idx}\sin[2c]} \right) / \right. \\ & \left. (3id(1 + e^{2idx})\cos[c] - 3d(-1 + e^{2idx})\sin[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \right. \right. \\ & \quad \left. \left. \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \sqrt{1 + e^{2idx}\cos[2c] + ie^{2idx}\sin[2c]} \right) / \right. \\ & \left. (-id(1 + e^{2idx})\cos[c] + d(-1 + e^{2idx})\sin[c]) \right) + \frac{1}{10(a + a \cos[c + dx])^3} iC \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ & \left( \left( 2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx}\cos[2c] + ie^{2idx}\sin[2c]} \right) / (3id(1 + e^{2idx})\cos[c] - 3d(-1 + e^{2idx})\sin[c]) - \right. \end{aligned}$$



$$\begin{aligned}
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left( -i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) + \\
& \left( 26 A \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( 3 d (a + a \operatorname{Cos}[c + d x])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left( 2 B \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \right. \\
& \quad \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left( d (a + a \operatorname{Cos}[c + d x])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left( 2 C \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left( 3 d (a + a \operatorname{Cos}[c + d x])^3 \sqrt{1 + \operatorname{Cot}[c]^2} \right) + \frac{1}{(a + a \operatorname{Cos}[c + d x])^3} \operatorname{Cos} \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\operatorname{Cos}[c + d x]} \\
& \left( \frac{2 (20 A + 29 A \operatorname{Cos}[c] - 9 B \operatorname{Cos}[c] - C \operatorname{Cos}[c]) \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[c]}{5 d} + \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (29 A \operatorname{Sin} \left[ \frac{d x}{2} \right] - 9 B \operatorname{Sin} \left[ \frac{d x}{2} \right] - C \operatorname{Sin} \left[ \frac{d x}{2} \right])}{5 d} + \right. \\
& \quad \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (11 A \operatorname{Sin} \left[ \frac{d x}{2} \right] - 6 B \operatorname{Sin} \left[ \frac{d x}{2} \right] + C \operatorname{Sin} \left[ \frac{d x}{2} \right])}{15 d} + \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 (A \operatorname{Sin} \left[ \frac{d x}{2} \right] - B \operatorname{Sin} \left[ \frac{d x}{2} \right] + C \operatorname{Sin} \left[ \frac{d x}{2} \right])}{5 d} + \\
& \quad \left. \frac{16 A \operatorname{Sec}[c] \operatorname{Sec}[c + d x] \operatorname{Sin}[d x]}{d} + \frac{4 (11 A - 6 B + C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Tan} \left[ \frac{c}{2} \right]}{15 d} + \frac{2 (A - B + C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Tan} \left[ \frac{c}{2} \right]}{5 d} \right)
\end{aligned}$$

**Problem 475: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx] + C \cos[c + dx]^2}{\cos[c + dx]^{5/2} (a + a \cos[c + dx])^3} dx$$

Optimal (type 4, 270 leaves, 8 steps):

$$\frac{(119 A - 49 B + 9 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10 a^3 d} + \frac{(33 A - 13 B + 3 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6 a^3 d} +$$

$$\frac{(33 A - 13 B + 3 C) \sin[c + dx]}{6 a^3 d \cos[c + dx]^{3/2}} - \frac{(119 A - 49 B + 9 C) \sin[c + dx]}{10 a^3 d \sqrt{\cos[c + dx]}} - \frac{(A - B + C) \sin[c + dx]}{5 d \cos[c + dx]^{3/2} (a + a \cos[c + dx])^3} -$$

$$\frac{(2 A - B) \sin[c + dx]}{3 a d \cos[c + dx]^{3/2} (a + a \cos[c + dx])^2} - \frac{(119 A - 49 B + 9 C) \sin[c + dx]}{30 d \cos[c + dx]^{3/2} (a^3 + a^3 \cos[c + dx])}$$

Result (type 5, 1883 leaves):

$$\frac{1}{10 (a + a \cos[c + dx])^3} 119 i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right.$$

$$\left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) - \frac{1}{10 (a + a \cos[c + dx])^3}$$

$$49 i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \right. \right.$$

$$\left. \left. \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / \right.$$

$$(3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \left( 2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \right. \right.$$

$$\left. \left. \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / \right.$$

$$\left. (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) + \frac{1}{10 (a + a \cos[c + dx])^3} 9 i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\left( \left( 2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right.$$

$$\begin{aligned}
& \left( 2 \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left( -id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c] \right) - \\
& \left( 22 A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \right. \\
& \quad \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \quad \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( d(a + a \cos[c + dx])^3 \sqrt{1 + \cot[c]^2} \right) + \\
& \left( 26 B \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \right. \\
& \quad \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \quad \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \left( 3d(a + a \cos[c + dx])^3 \sqrt{1 + \cot[c]^2} \right) - \\
& \left( 2 C \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \right. \\
& \quad \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left( d(a + a \cos[c + dx])^3 \sqrt{1 + \cot[c]^2} \right) + \frac{1}{(a + a \cos[c + dx])^3} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^6 \sqrt{\cos[c + dx]} \\
& \left( -\frac{2(60A - 20B + 59A \cos[c] - 29B \cos[c] + 9C \cos[c]) \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec}[c]}{5d} - \right. \\
& \quad \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 (A \sin \left[ \frac{dx}{2} \right] - B \sin \left[ \frac{dx}{2} \right] + C \sin \left[ \frac{dx}{2} \right])}{5d} - \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 (16A \sin \left[ \frac{dx}{2} \right] - 11B \sin \left[ \frac{dx}{2} \right] + 6C \sin \left[ \frac{dx}{2} \right])}{15d} - \\
& \quad \left. \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] (59A \sin \left[ \frac{dx}{2} \right] - 29B \sin \left[ \frac{dx}{2} \right] + 9C \sin \left[ \frac{dx}{2} \right])}{5d} + \frac{16A \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \sin[dx]}{3d} + \right.
\end{aligned}$$

$$\frac{16 \operatorname{Sec}[c] \operatorname{Sec}[c+dx] (A \operatorname{Sin}[c] - 9 A \operatorname{Sin}[dx] + 3 B \operatorname{Sin}[dx])}{3 d} - \frac{4 (16 A - 11 B + 6 C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} - \frac{2 (A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d}$$

■ **Problem 476: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+a \operatorname{Cos}[c+dx]} (A+B \operatorname{Cos}[c+dx]+C \operatorname{Cos}[c+dx]^2) dx$$

Optimal (type 3, 227 leaves, 6 steps):

$$\frac{\sqrt{a} (48 A + 40 B + 35 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{64 d} + \frac{a (48 A + 40 B + 35 C) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{64 d \sqrt{a+a \operatorname{Cos}[c+dx]}} +$$

$$\frac{a (48 A + 40 B + 35 C) \operatorname{Cos}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{96 d \sqrt{a+a \operatorname{Cos}[c+dx]}} + \frac{a (8 B + C) \operatorname{Cos}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{24 d \sqrt{a+a \operatorname{Cos}[c+dx]}} + \frac{C \operatorname{Cos}[c+dx]^{5/2} \sqrt{a+a \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{4 d}$$

Result (type 3, 339 leaves):

$$\frac{1}{768 d} \sqrt{a(1+\operatorname{Cos}[c+dx])} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]$$

$$\left( \left( 3 i (48 A + 40 B + 35 C) e^{\frac{id x}{2}} \left( \operatorname{ArcTanh}\left[\left(\operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right]\right)\right] \sqrt{(1+e^{2 i d x}) \operatorname{Cos}[c] + i(-1+e^{2 i d x}) \operatorname{Sin}[c]} \right) - \right. \right.$$

$$\left. \left. \operatorname{Log}\left[2\left(e^{i d x} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{i d x} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1+e^{2 i d x}) \operatorname{Cos}[c] + i(-1+e^{2 i d x}) \operatorname{Sin}[c]}\right)\right]\right] \right)$$

$$\sqrt{e^{-i d x} (2(1+e^{2 i d x}) \operatorname{Cos}[c] + 2 i(-1+e^{2 i d x}) \operatorname{Sin}[c])} \Big/ \left( \sqrt{(1+e^{2 i d x}) \operatorname{Cos}[c] + i(-1+e^{2 i d x}) \operatorname{Sin}[c]} \right) + 4 \sqrt{\operatorname{Cos}[c+dx]} \right)$$

$$(144 A + 152 B + 133 C + 2(48 A + 40 B + 53 C) \operatorname{Cos}[c+dx] + 4(8 B + 7 C) \operatorname{Cos}[2(c+dx)] + 12 C \operatorname{Cos}[3(c+dx)]) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]$$

■ **Problem 477: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Cos}[c+dx]} (A+B \operatorname{Cos}[c+dx]+C \operatorname{Cos}[c+dx]^2) dx$$

Optimal (type 3, 179 leaves, 5 steps):

$$\frac{\sqrt{a} (8 A + 6 B + 5 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{8 d} + \frac{a (8 A + 6 B + 5 C) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{8 d \sqrt{a+a \operatorname{Cos}[c+dx]}} +$$

$$\frac{a (6 B + C) \operatorname{Cos}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{12 d \sqrt{a+a \operatorname{Cos}[c+dx]}} + \frac{C \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+a \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{3 d}$$

Result (type 3, 319 leaves):

$$\frac{1}{96 d} \sqrt{a (1 + \cos [c + d x])} \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right] \left( \left( 3 i (8 A + 6 B + 5 C) e^{\frac{i d x}{2}} \left( \operatorname{ArcTanh} \left[ \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right] \right) - \right. \right. \\ \left. \left. \operatorname{Log} \left[ 2 \left( e^{i d x} \cos \left[ \frac{c}{2} \right] + i e^{i d x} \sin \left[ \frac{c}{2} \right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) \right] \right) \right) \\ \left. \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right) \Big/ \left( \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) + \\ 4 \sqrt{\cos [c + d x]} (24 A + 18 B + 19 C + 2 (6 B + 5 C) \cos [c + d x] + 4 C \cos [2 (c + d x)]) \sin \left[ \frac{1}{2} (c + d x) \right] \Big)$$

- **Problem 478: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \cos [c + d x]} (A + B \cos [c + d x] + C \cos [c + d x]^2)}{\sqrt{\cos [c + d x]}} dx$$

Optimal (type 3, 131 leaves, 4 steps):

$$\frac{\sqrt{a} (8 A + 4 B + 3 C) \operatorname{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right]}{4 d} + \frac{a (4 B + C) \sqrt{\cos [c + d x]} \sin [c + d x]}{4 d \sqrt{a + a \cos [c + d x]}} + \frac{C \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{2 d}$$

Result (type 3, 299 leaves):

$$\frac{1}{16 d} \sqrt{a (1 + \cos [c + d x])} \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right] \left( \left( i (8 A + 4 B + 3 C) e^{\frac{i d x}{2}} \left( \operatorname{ArcTanh} \left[ \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right] - \operatorname{Log} \left[ 2 \left( e^{i d x} \cos \left[ \frac{c}{2} \right] + i e^{i d x} \sin \left[ \frac{c}{2} \right] + \right. \right. \right. \right. \\ \left. \left. \left. \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) \right] \right) \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right) \Big/ \\ \left( \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) + 4 \sqrt{\cos [c + d x]} (4 B + 3 C + 2 C \cos [c + d x]) \sin \left[ \frac{1}{2} (c + d x) \right] \Big)$$

- **Problem 479: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \cos [c + d x]} (A + B \cos [c + d x] + C \cos [c + d x]^2)}{\cos [c + d x]^{3/2}} dx$$

Optimal (type 3, 121 leaves, 4 steps):

$$\frac{\sqrt{a} (2 B + C) \operatorname{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right]}{d} - \frac{a (2 A - C) \sqrt{\cos [c + d x]} \sin [c + d x]}{d \sqrt{a + a \cos [c + d x]}} + \frac{2 A \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{d \sqrt{\cos [c + d x]}}$$

Result (type 3, 858 leaves):

$$\begin{aligned}
& \frac{1}{2} (2B + C) \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \\
& \left( \frac{1}{2} i \operatorname{Sin}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left( e^{i dx} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{i dx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i dx}) \operatorname{Cos}[c] + i(-1 + e^{2 i dx}) \operatorname{Sin}[c]} \right)} \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-i dx} \left( (1 + e^{2 i dx}) \operatorname{Cos}[c] + i(-1 + e^{2 i dx}) \operatorname{Sin}[c] \right)} \right] \right) / \left( d \sqrt{2(1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i(-1 + e^{2 i dx}) \operatorname{Sin}[c]} \right) - \right. \\
& \quad \left. \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2 i dx}) \operatorname{Cos}[c] + i(-1 + e^{2 i dx}) \operatorname{Sin}[c]} \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-i dx} \left( (1 + e^{2 i dx}) \operatorname{Cos}[c] + i(-1 + e^{2 i dx}) \operatorname{Sin}[c] \right)} \right] \right) / \left( d \sqrt{2(1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i(-1 + e^{2 i dx}) \operatorname{Sin}[c]} \right) \right) \right) + \\
& \frac{1}{2} \operatorname{Cos}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left( e^{i dx} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{i dx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i dx}) \operatorname{Cos}[c] + i(-1 + e^{2 i dx}) \operatorname{Sin}[c]} \right)} \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-i dx} \left( (1 + e^{2 i dx}) \operatorname{Cos}[c] + i(-1 + e^{2 i dx}) \operatorname{Sin}[c] \right)} \right] \right) / \left( d \sqrt{2(1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i(-1 + e^{2 i dx}) \operatorname{Sin}[c]} \right) + \right. \\
& \quad \left. \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2 i dx}) \operatorname{Cos}[c] + i(-1 + e^{2 i dx}) \operatorname{Sin}[c]} \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-i dx} \left( (1 + e^{2 i dx}) \operatorname{Cos}[c] + i(-1 + e^{2 i dx}) \operatorname{Sin}[c] \right)} \right] \right) / \left( d \sqrt{2(1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i(-1 + e^{2 i dx}) \operatorname{Sin}[c]} \right) \right) \right) \right) + \\
& \sqrt{\operatorname{Cos}[c + dx]} \sqrt{a(1 + \operatorname{Cos}[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left( \frac{C \operatorname{Cos}\left[\frac{dx}{2}\right] \operatorname{Sin}\left[\frac{c}{2}\right]}{d} + \frac{C \operatorname{Cos}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{d} + \right. \\
& \quad \left. \frac{2A \operatorname{Sec}[c + dx] \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]}{d} \right)
\end{aligned}$$

■ **Problem 480: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \operatorname{Cos}[c + dx]} (A + B \operatorname{Cos}[c + dx] + C \operatorname{Cos}[c + dx]^2)}{\operatorname{Cos}[c + dx]^{5/2}} dx$$

Optimal (type 3, 120 leaves, 4 steps):

$$\frac{2\sqrt{a} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c + dx]}{\sqrt{a + a \operatorname{Cos}[c + dx]}}\right]}{d} + \frac{2a(A + 3B) \operatorname{Sin}[c + dx]}{3d\sqrt{\operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Cos}[c + dx]}} + \frac{2A\sqrt{a + a \operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{3d \operatorname{Cos}[c + dx]^{3/2}}$$

Result (type 3, 700 leaves):

$$\begin{aligned}
& \frac{1}{6 \sqrt{2} d \cos [c+d x]^{3/2} \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])}} \\
& \sqrt{a(1+\cos [c+d x])} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \left(\frac{3}{2} i C e^{-\frac{3}{2} i d x} \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right]\right. \\
& \left.\cos \left[\frac{c}{2}\right]^2\left(\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]\right)^2-\frac{3}{2} i C e^{-\frac{3}{2} i d x} \cos \left[\frac{c}{2}\right]^2\right. \\
& \left.\log \left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right)\right]\left(\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]\right)^2+\frac{3}{2} i C\right. \\
& \left.e^{-\frac{3}{2} i d x} \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right]\right) \sin \left[\frac{c}{2}\right]^2\left(\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]\right)^2- \\
& \frac{3}{2} i C e^{-\frac{3}{2} i d x} \log \left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right)\right] \sin \left[\frac{c}{2}\right]^2 \\
& \left.\left(\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]\right)^2+4 A \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right) \sin \left[\frac{1}{2}(c+d x)\right]+ \\
& 8 A \cos [c+d x] \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]} \sin \left[\frac{1}{2}(c+d x)\right]+ \\
& 12 B \cos [c+d x] \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]} \sin \left[\frac{1}{2}(c+d x)\right]
\end{aligned}$$

■ **Problem 484: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos [c+d x]^{3/2} (a+a \cos [c+d x])^{3/2} (A+B \cos [c+d x]+C \cos [c+d x]^2) d x$$

Optimal (type 3, 283 leaves, 7 steps):

$$\begin{aligned}
& \frac{a^{3/2} (176 A+150 B+133 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{128 d} + \frac{a^2 (176 A+150 B+133 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{128 d \sqrt{a+a \cos [c+d x]}} + \\
& \frac{a^2 (176 A+150 B+133 C) \cos [c+d x]^{3/2} \sin [c+d x]}{192 d \sqrt{a+a \cos [c+d x]}} + \frac{a^2 (80 A+90 B+67 C) \cos [c+d x]^{5/2} \sin [c+d x]}{240 d \sqrt{a+a \cos [c+d x]}} + \\
& \frac{a (10 B+3 C) \cos [c+d x]^{5/2} \sqrt{a+a \cos [c+d x]} \sin [c+d x]}{40 d} + \frac{C \cos [c+d x]^{5/2} (a+a \cos [c+d x])^{3/2} \sin [c+d x]}{5 d}
\end{aligned}$$

Result (type 3, 390 leaves):

$$\begin{aligned}
& - \frac{1}{7680 d \sqrt{2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c]}} (a (1 + \cos [c + d x]))^{3/2} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^3 \\
& \left( -15 i (176 A + 150 B + 133 C) e^{\frac{i d x}{2}} \left( \operatorname{ArcTanh}\left[\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right] \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) - \right. \\
& \quad \left. \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) \right] \right) \\
& \frac{\sqrt{e^{-i d x} \left( (1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c] \right)} - 4 \sqrt{\cos [c + d x]}}{(2960 A + 2850 B + 2671 C + 2 (880 A + 930 B + 1007 C) \cos [c + d x] + 4 (80 A + 150 B + 181 C) \cos [2 (c + d x)] +} \\
& \quad \left. 120 B \cos [3 (c + d x)] + 228 C \cos [3 (c + d x)] + 48 C \cos [4 (c + d x)] \right) \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin\left[\frac{1}{2} (c + d x)\right] \Big)
\end{aligned}$$

■ **Problem 485: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\cos [c + d x]} (a + a \cos [c + d x])^{3/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) dx$$

Optimal (type 3, 233 leaves, 6 steps):

$$\begin{aligned}
& \frac{a^{3/2} (112 A + 88 B + 75 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}}\right]}{64 d} + \\
& \frac{a^2 (112 A + 88 B + 75 C) \sqrt{\cos [c + d x]} \sin [c + d x]}{64 d \sqrt{a + a \cos [c + d x]}} + \frac{a^2 (48 A + 56 B + 39 C) \cos [c + d x]^{3/2} \sin [c + d x]}{96 d \sqrt{a + a \cos [c + d x]}} + \\
& \frac{a (8 B + 3 C) \cos [c + d x]^{3/2} \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{24 d} + \frac{C \cos [c + d x]^{3/2} (a + a \cos [c + d x])^{3/2} \sin [c + d x]}{4 d}
\end{aligned}$$

Result (type 3, 365 leaves):

$$\begin{aligned}
& - \frac{1}{768 d \sqrt{2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c]}} (a (1 + \cos [c + d x]))^{3/2} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^3 \\
& \left( -3 i (112 A + 88 B + 75 C) e^{\frac{i d x}{2}} \left( \operatorname{ArcTanh}\left[\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right] \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) - \right. \\
& \quad \left. \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) \right] \right) \\
& \frac{\sqrt{e^{-i d x} \left( (1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c] \right)} - 4 \sqrt{\cos [c + d x]}}{(336 A + 296 B + 285 C + 2 (48 A + 88 B + 93 C) \cos [c + d x] +} \\
& \quad \left. 4 (8 B + 15 C) \cos [2 (c + d x)] + 12 C \cos [3 (c + d x)] \right) \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin\left[\frac{1}{2} (c + d x)\right] \Big)
\end{aligned}$$

■ **Problem 486: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \cos [c + d x])^{3/2} (A + B \cos [c + d x] + C \cos [c + d x]^2)}{\sqrt{\cos [c + d x]}} dx$$



Optimal (type 3, 181 leaves, 5 steps):

$$\frac{a^{3/2} (24 A + 14 B + 11 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{8 d} + \frac{a^2 (24 A + 30 B + 19 C) \sqrt{\cos[c+dx]} \sin[c+dx]}{24 d \sqrt{a+a \cos[c+dx]}} +$$

$$\frac{a (2 B + C) \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{4 d} + \frac{C \sqrt{\cos[c+dx]} (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{3 d}$$

Result (type 3, 345 leaves):

$$-\frac{1}{96 d \sqrt{2 (1+e^{2 i d x}) \cos[c] + 2 i (-1+e^{2 i d x}) \sin[c]}} (a (1+\cos[c+dx]))^{3/2} \operatorname{Sec}\left[\frac{1}{2} (c+dx)\right]^3$$

$$\left( -3 i (24 A + 14 B + 11 C) e^{\frac{i d x}{2}} \left( \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right)\right] \sqrt{(1+e^{2 i d x}) \cos[c] + i (-1+e^{2 i d x}) \sin[c]} \right) - \right.$$

$$\left. \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2 i d x}) \cos[c] + i (-1+e^{2 i d x}) \sin[c]} \right) \right] \right)$$

$$\sqrt{e^{-i d x} \left( (1+e^{2 i d x}) \cos[c] + i (-1+e^{2 i d x}) \sin[c] \right)} - 4 \sqrt{\cos[c+dx]} (24 A + 42 B + 37 C + 2 (6 B + 11 C) \cos[c+dx] + 4 C \cos[2 (c+dx)])$$

$$\sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2} (c+dx)\right]$$

■ **Problem 487: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a+a \cos[c+dx])^{3/2} (A+B \cos[c+dx]+C \cos[c+dx]^2)}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 3, 181 leaves, 5 steps):

$$\frac{a^{3/2} (8 A + 12 B + 7 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{4 d} - \frac{a^2 (8 A - 4 B - 5 C) \sqrt{\cos[c+dx]} \sin[c+dx]}{4 d \sqrt{a+a \cos[c+dx]}} -$$

$$\frac{a (4 A - C) \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{2 d} + \frac{2 A (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{d \sqrt{\cos[c+dx]}}$$

Result (type 3, 338 leaves):

$$\begin{aligned}
& - \frac{1}{16 d \sqrt{2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c]}} (a (1 + \cos [c + d x]))^{3/2} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^3 \\
& \left( -i (8 A + 12 B + 7 C) e^{\frac{i d x}{2}} \left( \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right] - \right. \right. \\
& \quad \left. \left. \operatorname{Log}\left[2 \left(e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right]\right) \right) \\
& \quad \frac{\sqrt{e^{-i d x} ((1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c])}}{\sqrt{\cos [c + d x]}} - \frac{1}{\sqrt{\cos [c + d x]}} \\
& \quad \left. 4 (8 A + C + (4 B + 7 C) \cos [c + d x] + C \cos [2 (c + d x)]) \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin\left[\frac{1}{2} (c + d x)\right] \right)
\end{aligned}$$

- **Problem 488: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^{3/2} (A + B \cos [c + d x] + C \cos [c + d x]^2)}{\cos [c + d x]^{5/2}} dx$$

Optimal (type 3, 171 leaves, 5 steps):

$$\begin{aligned}
& \frac{a^{3/2} (2 B + 3 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}}\right]}{d} - \frac{a^2 (8 A + 6 B - 3 C) \sqrt{\cos [c + d x]} \sin [c + d x]}{3 d \sqrt{a + a \cos [c + d x]}} + \\
& \frac{2 a (A + B) \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{d \sqrt{\cos [c + d x]}} + \frac{2 A (a + a \cos [c + d x])^{3/2} \sin [c + d x]}{3 d \cos [c + d x]^{3/2}}
\end{aligned}$$

Result (type 3, 920 leaves):

$$\begin{aligned}
& \frac{1}{4} (2B + 3C) (a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \\
& \left( \frac{1}{2} i \operatorname{Sin}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \operatorname{Sin}[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-i dx} \left( (1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c]} \right) - \right. \\
& \quad \left. \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \operatorname{Sin}[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-i dx} \left( (1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c]} \right) \right) \right) + \\
& \frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \operatorname{Sin}[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-i dx} \left( (1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c]} \right) + \right. \\
& \quad \left. \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \operatorname{Sin}[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-i dx} \left( (1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c]} \right) \right) \right) \right) + \\
& \sqrt{\cos[c + dx]} (a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left( \frac{C \cos\left[\frac{dx}{2}\right] \operatorname{Sin}\left[\frac{c}{2}\right]}{2 d} + \frac{C \cos\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{2 d} + \right. \\
& \quad \frac{A \operatorname{Sec}[c + dx]^2 \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]}{3 d} + \\
& \quad \left. \frac{\operatorname{Sec}[c + dx] \left( 5 A \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right] + 3 B \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right] \right)}{3 d} \right)
\end{aligned}$$

■ **Problem 489: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx])^{3/2} (A + B \cos[c + dx] + C \cos[c + dx]^2)}{\cos[c + dx]^{7/2}} dx$$

Optimal (type 3, 172 leaves, 5 steps):

$$\begin{aligned}
& \frac{2 a^{3/2} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{d} + \frac{2 a^2 (12 A + 20 B + 15 C) \operatorname{Sin}[c + dx]}{15 d \sqrt{\cos[c + dx]} \sqrt{a + a \cos[c + dx]}} + \\
& \frac{2 a (3 A + 5 B) \sqrt{a + a \cos[c + dx]} \operatorname{Sin}[c + dx]}{15 d \cos[c + dx]^{3/2}} + \frac{2 A (a + a \cos[c + dx])^{3/2} \operatorname{Sin}[c + dx]}{5 d \cos[c + dx]^{5/2}}
\end{aligned}$$

Result (type 3, 850 leaves):

$$\begin{aligned}
& \frac{1}{60 \sqrt{2} d \cos [c+d x]^{5/2} \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])}} (a (1+\cos [c+d x]))^{3/2} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 \\
& \left( \frac{15}{4} C e^{-\frac{5}{2} i d x} \cos \left[\frac{c}{2}\right]^2 \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right)\right] \right) \\
& \left( i\left(1+e^{2 i d x}\right) \cos [c]-\left(-1+e^{2 i d x}\right) \sin [c]\right)^3 + \frac{15}{4} C e^{-\frac{5}{2} i d x} \operatorname{Log}\left[ \right. \\
& \left. 2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right)\right] \sin \left[\frac{c}{2}\right]^2 \left(i\left(1+e^{2 i d x}\right) \cos [c]-\left(-1+e^{2 i d x}\right) \sin [c]\right)^3 + \right. \\
& \left. \frac{15}{4} i C e^{-\frac{5}{2} i d x} \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right] \cos \left[\frac{c}{2}\right]^2 \right. \\
& \left. \left(\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]\right)^3 + \frac{15}{4} i C e^{-\frac{5}{2} i d x} \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right] \right) \\
& \sin \left[\frac{c}{2}\right]^2 \left(\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]\right)^3 + 12 A \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]} \sin \left[\frac{1}{2}(c+d x)\right] + \\
& 36 A \cos [c+d x] \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]} \sin \left[\frac{1}{2}(c+d x)\right] + \\
& 20 B \cos [c+d x] \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]} \sin \left[\frac{1}{2}(c+d x)\right] + \\
& 72 \sqrt{2} A \cos [c+d x]^2 \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])} \sin \left[\frac{1}{2}(c+d x)\right] + \\
& 100 \sqrt{2} B \cos [c+d x]^2 \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])} \sin \left[\frac{1}{2}(c+d x)\right] + \\
& \left. 60 \sqrt{2} C \cos [c+d x]^2 \sqrt{\cos [c+d x] (\cos [d x]+i \sin [d x])} \sin \left[\frac{1}{2}(c+d x)\right] \right)
\end{aligned}$$

- **Problem 493: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^{3/2} (a+a \cos [c+d x])^{5/2} (A+B \cos [c+d x]+C \cos [c+d x]^2) d x$$

Optimal (type 3, 333 leaves, 8 steps):

$$\begin{aligned}
& \frac{a^{5/2} (1304 A + 1132 B + 1015 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{512 d} + \frac{a^3 (1304 A + 1132 B + 1015 C) \sqrt{\cos[c+dx]} \sin[c+dx]}{512 d \sqrt{a+a \cos[c+dx]}} + \\
& \frac{a^3 (1304 A + 1132 B + 1015 C) \cos[c+dx]^{3/2} \sin[c+dx]}{768 d \sqrt{a+a \cos[c+dx]}} + \frac{a^3 (680 A + 628 B + 545 C) \cos[c+dx]^{5/2} \sin[c+dx]}{960 d \sqrt{a+a \cos[c+dx]}} + \\
& \frac{a^2 (120 A + 156 B + 115 C) \cos[c+dx]^{5/2} \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{480 d} + \\
& \frac{a (12 B + 5 C) \cos[c+dx]^{5/2} (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{60 d} + \frac{C \cos[c+dx]^{5/2} (a+a \cos[c+dx])^{5/2} \sin[c+dx]}{6 d}
\end{aligned}$$

Result (type 3, 1142 leaves):

$$\begin{aligned}
& \frac{1}{4096} (1304 A + 1132 B + 1015 C) (a (1 + \cos[c+dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \\
& \left( \frac{1}{2} i \sin\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{id x}{2}} \operatorname{Log}\left[ 2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c]}\right)\right] \right) \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \frac{\sqrt{e^{-i dx} ((1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c])}}{\left( d \sqrt{2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c]} \right)} - \right. \right. \\
& \quad \left. \left( 2 i e^{\frac{id x}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c]} \right] \right) \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \frac{\sqrt{e^{-i dx} ((1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c])}}{\left( d \sqrt{2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c]} \right)} \right) \right) + \\
& \frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{id x}{2}} \operatorname{Log}\left[ 2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c]} \right)\right] \right) \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \frac{\sqrt{e^{-i dx} ((1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c])}}{\left( d \sqrt{2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c]} \right)} + \right. \right. \\
& \quad \left. \left( 2 i e^{\frac{id x}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c]} \right] \right) \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \frac{\sqrt{e^{-i dx} ((1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c])}}{\left( d \sqrt{2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c]} \right)} \right) \right) \right) + \\
& \sqrt{\cos[c+dx]} (a (1 + \cos[c+dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \frac{(1000 A + 896 B + 805 C) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{1920 d} + \right. \\
& \quad \frac{(360 A + 340 B + 329 C) \cos\left[\frac{3 dx}{2}\right] \sin\left[\frac{3 c}{2}\right]}{2048 d} + \\
& \quad \frac{(50 A + 61 B + 65 C) \cos\left[\frac{5 dx}{2}\right] \sin\left[\frac{5 c}{2}\right]}{960 d} + \\
& \quad \left. \frac{(24 A + 60 B + 79 C) \cos\left[\frac{7 dx}{2}\right] \sin\left[\frac{7 c}{2}\right]}{3072 d} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{(2B + 5C) \cos\left[\frac{9dx}{2}\right] \sin\left[\frac{9c}{2}\right]}{640d} + \\
& \frac{C \cos\left[\frac{11dx}{2}\right] \sin\left[\frac{11c}{2}\right]}{768d} + \\
& \frac{(1000A + 896B + 805C) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{1920d} + \\
& \frac{(360A + 340B + 329C) \cos\left[\frac{3c}{2}\right] \sin\left[\frac{3dx}{2}\right]}{2048d} + \\
& \frac{(50A + 61B + 65C) \cos\left[\frac{5c}{2}\right] \sin\left[\frac{5dx}{2}\right]}{960d} + \\
& \frac{(24A + 60B + 79C) \cos\left[\frac{7c}{2}\right] \sin\left[\frac{7dx}{2}\right]}{3072d} + \\
& \left. \frac{(2B + 5C) \cos\left[\frac{9c}{2}\right] \sin\left[\frac{9dx}{2}\right]}{640d} + \frac{C \cos\left[\frac{11c}{2}\right] \sin\left[\frac{11dx}{2}\right]}{768d} \right)
\end{aligned}$$

■ **Problem 494: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c + dx]} (a + a \cos[c + dx])^{5/2} (A + B \cos[c + dx] + C \cos[c + dx]^2) dx$$

Optimal (type 3, 281 leaves, 7 steps):

$$\begin{aligned}
& \frac{a^{5/2} (400A + 326B + 283C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{128d} + \frac{a^3 (400A + 326B + 283C) \sqrt{\cos[c+dx]} \sin[c+dx]}{128d \sqrt{a+a\cos[c+dx]}} + \\
& \frac{a^3 (1040A + 950B + 787C) \cos[c+dx]^{3/2} \sin[c+dx]}{960d \sqrt{a+a\cos[c+dx]}} + \frac{a^2 (80A + 110B + 79C) \cos[c+dx]^{3/2} \sqrt{a+a\cos[c+dx]} \sin[c+dx]}{240d} + \\
& \frac{a (2B + C) \cos[c+dx]^{3/2} (a+a\cos[c+dx])^{3/2} \sin[c+dx]}{8d} + \frac{C \cos[c+dx]^{3/2} (a+a\cos[c+dx])^{5/2} \sin[c+dx]}{5d}
\end{aligned}$$

Result (type 3, 1082 leaves):

$$\begin{aligned}
& \frac{1}{1024} (400A + 326B + 283C) (a (1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \\
& \left(\frac{1}{2} i \sin\left[\frac{c}{2}\right] \left(-\left(2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]}\right]\right)\right) \left(\cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right]\right)\right.\right. \\
& \left.\left. \sqrt{e^{-idx} \left((1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]\right)}\right) / \left(d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]}\right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left( 2 i e^{\frac{i dx}{2}} \operatorname{ArcTanh} \left[ \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \sqrt{(1 + e^{2 i dx}) \cos [c] + i (-1 + e^{2 i dx}) \sin [c]} \right] \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \right. \\
& \quad \left. \sqrt{e^{-i dx} \left( (1 + e^{2 i dx}) \cos [c] + i (-1 + e^{2 i dx}) \sin [c] \right)} \right] / \left( d \sqrt{2 (1 + e^{2 i dx}) \cos [c] + 2 i (-1 + e^{2 i dx}) \sin [c]} \right) \right) + \\
& \frac{1}{2} \cos \left[ \frac{c}{2} \right] \left( - \left( 2 i e^{\frac{i dx}{2}} \operatorname{Log} \left[ 2 \left( e^{i dx} \cos \left[ \frac{c}{2} \right] + i e^{i dx} \sin \left[ \frac{c}{2} \right] + \sqrt{(1 + e^{2 i dx}) \cos [c] + i (-1 + e^{2 i dx}) \sin [c]} \right) \right] \left( \cos \left[ \frac{c}{2} \right] - i \sin \left[ \frac{c}{2} \right] \right) \right. \right. \\
& \quad \left. \left. \sqrt{e^{-i dx} \left( (1 + e^{2 i dx}) \cos [c] + i (-1 + e^{2 i dx}) \sin [c] \right)} \right] / \left( d \sqrt{2 (1 + e^{2 i dx}) \cos [c] + 2 i (-1 + e^{2 i dx}) \sin [c]} \right) \right) + \\
& \left( 2 i e^{\frac{i dx}{2}} \operatorname{ArcTanh} \left[ \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \sqrt{(1 + e^{2 i dx}) \cos [c] + i (-1 + e^{2 i dx}) \sin [c]} \right] \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \right. \\
& \quad \left. \sqrt{e^{-i dx} \left( (1 + e^{2 i dx}) \cos [c] + i (-1 + e^{2 i dx}) \sin [c] \right)} \right] / \left( d \sqrt{2 (1 + e^{2 i dx}) \cos [c] + 2 i (-1 + e^{2 i dx}) \sin [c]} \right) \right) \Big) + \\
& \sqrt{\cos [c + d x]} (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \left( \frac{(155 A + 125 B + 112 C) \cos \left[ \frac{d x}{2} \right] \sin \left[ \frac{c}{2} \right]}{240 d} + \right. \\
& \quad \frac{5 (16 A + 18 B + 17 C) \cos \left[ \frac{3 d x}{2} \right] \sin \left[ \frac{3 c}{2} \right]}{512 d} + \\
& \quad \frac{(20 A + 50 B + 61 C) \cos \left[ \frac{5 d x}{2} \right] \sin \left[ \frac{5 c}{2} \right]}{960 d} + \\
& \quad \frac{(2 B + 5 C) \cos \left[ \frac{7 d x}{2} \right] \sin \left[ \frac{7 c}{2} \right]}{256 d} + \\
& \quad \frac{C \cos \left[ \frac{9 d x}{2} \right] \sin \left[ \frac{9 c}{2} \right]}{320 d} + \\
& \quad \frac{(155 A + 125 B + 112 C) \cos \left[ \frac{c}{2} \right] \sin \left[ \frac{d x}{2} \right]}{240 d} + \\
& \quad \frac{5 (16 A + 18 B + 17 C) \cos \left[ \frac{3 c}{2} \right] \sin \left[ \frac{3 d x}{2} \right]}{512 d} + \\
& \quad \left. \frac{(20 A + 50 B + 61 C) \cos \left[ \frac{5 c}{2} \right] \sin \left[ \frac{5 d x}{2} \right]}{960 d} + \right. \\
& \quad \left. \frac{(2 B + 5 C) \cos \left[ \frac{7 c}{2} \right] \sin \left[ \frac{7 d x}{2} \right]}{256 d} + \frac{C \cos \left[ \frac{9 c}{2} \right] \sin \left[ \frac{9 d x}{2} \right]}{320 d} \right)
\end{aligned}$$

■ **Problem 495: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x] + C \cos [c + d x]^2)}{\sqrt{\cos [c + d x]}} dx$$

Optimal (type 3, 233 leaves, 6 steps) :

$$\frac{a^{5/2} (304 A + 200 B + 163 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{64 d} + \frac{a^3 (432 A + 392 B + 299 C) \sqrt{\cos[c+dx]} \sin[c+dx]}{192 d \sqrt{a+a \cos[c+dx]}} +$$

$$\frac{a^2 (16 A + 24 B + 17 C) \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{32 d} +$$

$$\frac{a (8 B + 5 C) \sqrt{\cos[c+dx]} (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{24 d} + \frac{C \sqrt{\cos[c+dx]} (a+a \cos[c+dx])^{5/2} \sin[c+dx]}{4 d}$$

Result (type 3, 364 leaves) :

$$-\frac{1}{1536 d \sqrt{2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c]}} (a (1 + \cos[c+dx]))^{5/2} \operatorname{Sec}\left[\frac{1}{2} (c+dx)\right]^5$$

$$\left(-3 i (304 A + 200 B + 163 C) e^{\frac{i dx}{2}} \left(\operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right)\right] \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c]}\right) -\right.$$

$$\left.\operatorname{Log}\left[2 \left(e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c]}\right)\right]\right)$$

$$\sqrt{e^{-i dx} \left((1 + e^{2 i dx}) \cos[c] + i (-1 + e^{2 i dx}) \sin[c]\right)} - 4 \sqrt{\cos[c+dx]} (528 A + 632 B + 581 C + (96 A + 272 B + 362 C) \cos[c+dx] +$$

$$4 (8 B + 23 C) \cos[2 (c+dx)] + 12 C \cos[3 (c+dx)]) \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2} (c+dx)\right] \Bigg)$$

■ **Problem 496: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \cos[c+dx])^{5/2} (A + B \cos[c+dx] + C \cos[c+dx]^2)}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 3, 231 leaves, 6 steps) :

$$\frac{a^{5/2} (40 A + 38 B + 25 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{8 d} -$$

$$\frac{a^3 (24 A - 54 B - 49 C) \sqrt{\cos[c+dx]} \sin[c+dx]}{24 d \sqrt{a+a \cos[c+dx]}} - \frac{a^2 (8 A - 2 B - 3 C) \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{4 d} -$$

$$\frac{a (6 A - C) \sqrt{\cos[c+dx]} (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{3 d} + \frac{2 A (a+a \cos[c+dx])^{5/2} \sin[c+dx]}{d \sqrt{\cos[c+dx]}}$$

Result (type 3, 364 leaves) :



$$\begin{aligned}
& - \frac{1}{192 d \sqrt{2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c]}} (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^5 \\
& \left( -3 i (40 A + 38 B + 25 C) e^{\frac{i d x}{2}} \left( \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right)\right] \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) - \right. \\
& \quad \left. \operatorname{Log}\left[2 \left(e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] \right) \\
& \frac{\sqrt{e^{-i d x} ((1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c])}}{\sqrt{\cos [c + d x]}} - \frac{1}{\sqrt{\cos [c + d x]}} 4 (48 A + 6 B + 17 C + 3 (8 A + 22 B + 27 C) \cos [c + d x] + \\
& \quad (6 B + 17 C) \cos [2 (c + d x)] + 2 C \cos [3 (c + d x)]) \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin\left[\frac{1}{2} (c + d x)\right] \Big)
\end{aligned}$$

■ **Problem 497: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x] + C \cos [c + d x]^2)}{\cos [c + d x]^{5/2}} dx$$

Optimal (type 3, 233 leaves, 6 steps):

$$\begin{aligned}
& \frac{a^{5/2} (8 A + 20 B + 19 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}}\right]}{4 d} - \\
& \frac{a^3 (56 A + 12 B - 27 C) \sqrt{\cos [c + d x]} \sin [c + d x]}{12 d \sqrt{a + a \cos [c + d x]}} - \frac{a^2 (8 A + 4 B - C) \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{2 d} + \\
& \frac{2 a (5 A + 3 B) (a + a \cos [c + d x])^{3/2} \sin [c + d x]}{3 d \sqrt{\cos [c + d x]}} + \frac{2 A (a + a \cos [c + d x])^{5/2} \sin [c + d x]}{3 d \cos [c + d x]^{3/2}}
\end{aligned}$$

Result (type 3, 364 leaves):

$$\begin{aligned}
& - \frac{1}{96 d \sqrt{2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c]}} (a (1 + \cos [c + d x]))^{5/2} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^5 \\
& \left( -3 i (8 A + 20 B + 19 C) e^{\frac{i d x}{2}} \left( \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right)\right] \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]} \right) - \right. \\
& \quad \left. \operatorname{Log}\left[2 \left(e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] \right) \\
& \frac{\sqrt{e^{-i d x} ((1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c])}}{\cos [c + d x]^{3/2}} - \frac{1}{\cos [c + d x]^{3/2}} 2 (16 A + 12 B + 33 C + (128 A + 48 B + 9 C) \cos [c + d x] + \\
& \quad 3 (4 B + 11 C) \cos [2 (c + d x)] + 3 C \cos [3 (c + d x)]) \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin\left[\frac{1}{2} (c + d x)\right] \Big)
\end{aligned}$$

- **Problem 498: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x] + C \cos [c + d x]^2)}{\cos [c + d x]^{7/2}} dx$$

Optimal (type 3, 223 leaves, 6 steps) :

$$\frac{a^{5/2} (2 B + 5 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{d} - \frac{a^3 (64 A + 70 B + 15 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{15 d \sqrt{a+a \cos [c+d x]}} +$$

$$\frac{2 a^2 (8 A + 10 B + 5 C) \sqrt{a+a \cos [c+d x]} \sin [c+d x]}{5 d \sqrt{\cos [c+d x]}} + \frac{2 a (A+B) (a+a \cos [c+d x])^{3/2} \sin [c+d x]}{3 d \cos [c+d x]^{3/2}} + \frac{2 A (a+a \cos [c+d x])^{5/2} \sin [c+d x]}{5 d \cos [c+d x]^{5/2}}$$

Result (type 3, 984 leaves) :

$$\frac{1}{8} (2B + 5C) (a (1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5$$

$$\left(\frac{1}{2} i \sin\left[\frac{c}{2}\right] \left(-\left(2 i e^{\frac{idx}{2}} \log\left[2 \left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]}\right]\right)\right) \left(\cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right]\right)\right.\right.$$

$$\left.\frac{\sqrt{e^{-idx} \left((1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]\right)}}{\left(d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]}\right)} - \right.$$

$$\left.\left(2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]}\right]\right) \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right)\right.$$

$$\left.\frac{\sqrt{e^{-idx} \left((1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]\right)}}{\left(d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]}\right)}\right) +$$

$$\frac{1}{2} \cos\left[\frac{c}{2}\right] \left(-\left(2 i e^{\frac{idx}{2}} \log\left[2 \left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]}\right]\right)\right) \left(\cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right]\right)\right.$$

$$\left.\frac{\sqrt{e^{-idx} \left((1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]\right)}}{\left(d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]}\right)} + \right.$$

$$\left.\left(2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]}\right]\right) \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right)\right.$$

$$\left.\frac{\sqrt{e^{-idx} \left((1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \sin[c]\right)}}{\left(d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c]}\right)}\right) +$$

$$\sqrt{\cos[c + dx]} (a (1 + \cos[c + dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left(\frac{C \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{4d} + \frac{C \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{4d} + \right.$$

$$\frac{A \sec[c + dx]^3 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{10d} +$$

$$\frac{\sec[c + dx]^2 (14A \sin\left[\frac{c}{2} + \frac{dx}{2}\right] + 5B \sin\left[\frac{c}{2} + \frac{dx}{2}\right])}{30d} +$$

$$\left.\frac{\sec[c + dx] (43A \sin\left[\frac{c}{2} + \frac{dx}{2}\right] + 40B \sin\left[\frac{c}{2} + \frac{dx}{2}\right] + 15C \sin\left[\frac{c}{2} + \frac{dx}{2}\right])}{30d}\right)$$

■ **Problem 499: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx])^{5/2} (A + B \cos[c + dx] + C \cos[c + dx]^2)}{\cos[c + dx]^{9/2}} dx$$

Optimal (type 3, 222 leaves, 6 steps):

$$\frac{2a^{5/2} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right]}{d} + \frac{2a^3 (160A + 224B + 245C) \sin[c + dx]}{105d \sqrt{\cos[c + dx]} \sqrt{a + a \cos[c + dx]}} + \frac{2a^2 (40A + 56B + 35C) \sqrt{a + a \cos[c + dx]} \sin[c + dx]}{105d \cos[c + dx]^{3/2}} +$$

$$\frac{2a (5A + 7B) (a + a \cos[c + dx])^{3/2} \sin[c + dx]}{35d \cos[c + dx]^{5/2}} + \frac{2A (a + a \cos[c + dx])^{5/2} \sin[c + dx]}{7d \cos[c + dx]^{7/2}}$$

Result (type 3, 1000 leaves) :

$$\begin{aligned} & \frac{1}{4} C (a (1 + \cos [c + dx]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \\ & \left( \frac{1}{2} i \sin \left[ \frac{c}{2} \right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log} \left[ 2 \left( e^{idx} \cos \left[ \frac{c}{2} \right] + i e^{idx} \sin \left[ \frac{c}{2} \right] + \sqrt{(1 + e^{2idx}) \cos [c] + i (-1 + e^{2idx}) \sin [c]} \right) \right] \right) \left( \cos \left[ \frac{c}{2} \right] - i \sin \left[ \frac{c}{2} \right] \right) \right. \\ & \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos [c] + i (-1 + e^{2idx}) \sin [c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos [c] + 2 i (-1 + e^{2idx}) \sin [c]} \right) - \\ & \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh} \left[ \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \sqrt{(1 + e^{2idx}) \cos [c] + i (-1 + e^{2idx}) \sin [c]} \right] \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \right. \\ & \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos [c] + i (-1 + e^{2idx}) \sin [c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos [c] + 2 i (-1 + e^{2idx}) \sin [c]} \right) \right) + \\ & \frac{1}{2} \cos \left[ \frac{c}{2} \right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log} \left[ 2 \left( e^{idx} \cos \left[ \frac{c}{2} \right] + i e^{idx} \sin \left[ \frac{c}{2} \right] + \sqrt{(1 + e^{2idx}) \cos [c] + i (-1 + e^{2idx}) \sin [c]} \right) \right] \right) \left( \cos \left[ \frac{c}{2} \right] - i \sin \left[ \frac{c}{2} \right] \right) \right. \\ & \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos [c] + i (-1 + e^{2idx}) \sin [c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos [c] + 2 i (-1 + e^{2idx}) \sin [c]} \right) + \\ & \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh} \left[ \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \sqrt{(1 + e^{2idx}) \cos [c] + i (-1 + e^{2idx}) \sin [c]} \right] \left( \cos \left[ \frac{c}{2} \right] + i \sin \left[ \frac{c}{2} \right] \right) \right. \\ & \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos [c] + i (-1 + e^{2idx}) \sin [c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos [c] + 2 i (-1 + e^{2idx}) \sin [c]} \right) \right) \right) + \\ & \sqrt{\cos [c + dx]} (a (1 + \cos [c + dx]))^{5/2} \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \left( \frac{A \operatorname{Sec} [c + dx]^4 \sin \left[ \frac{c}{2} + \frac{dx}{2} \right]}{14 d} + \right. \\ & \quad \frac{\operatorname{Sec} [c + dx]^3 (20 A \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] + 7 B \sin \left[ \frac{c}{2} + \frac{dx}{2} \right])}{70 d} + \\ & \quad \frac{\operatorname{Sec} [c + dx]^2 (115 A \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] + 98 B \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] + 35 C \sin \left[ \frac{c}{2} + \frac{dx}{2} \right])}{210 d} + \\ & \quad \left. \frac{\operatorname{Sec} [c + dx] (230 A \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] + 301 B \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] + 280 C \sin \left[ \frac{c}{2} + \frac{dx}{2} \right])}{210 d} \right) \end{aligned}$$

■ **Problem 503: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c + dx]^{3/2} (A + B \cos [c + dx] + C \cos [c + dx]^2)}{\sqrt{a + a \cos [c + dx]}} dx$$

Optimal (type 3, 241 leaves, 8 steps) :

$$\begin{aligned}
& - \frac{(8A - 14B + 9C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] + \sqrt{2} (A - B + C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{8\sqrt{a} d} + \\
& \frac{(8A - 2B + 7C) \sqrt{\cos[c+dx]} \sin[c+dx]}{8d \sqrt{a+a \cos[c+dx]}} + \frac{(6B - C) \cos[c+dx]^{3/2} \sin[c+dx]}{12d \sqrt{a+a \cos[c+dx]}} + \frac{C \cos[c+dx]^{5/2} \sin[c+dx]}{3d \sqrt{a+a \cos[c+dx]}}
\end{aligned}$$

Result (type 3, 449 leaves):

$$\begin{aligned}
& \frac{1}{48d \sqrt{a} (1 + \cos[c+dx])} \\
& \cos\left[\frac{1}{2}(c+dx)\right] \left( \frac{1}{\sqrt{1+e^{2i(c+dx)}}} 3\sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} \left( -8Adx + 14Bdx - 9Cdx + i(8A - 14B + 9C) \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] - \right. \right. \\
& 16i\sqrt{2} (A - B + C) \operatorname{Log}\left[1 + e^{i(c+dx)}\right] - 8iA \operatorname{Log}\left[1 + \sqrt{1+e^{2i(c+dx)}}\right] + 14iB \operatorname{Log}\left[1 + \sqrt{1+e^{2i(c+dx)}}\right] - \\
& 9iC \operatorname{Log}\left[1 + \sqrt{1+e^{2i(c+dx)}}\right] + 16i\sqrt{2} A \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}\right] - \\
& \left. 16i\sqrt{2} B \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}\right] + 16i\sqrt{2} C \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}\right] \right) + \\
& 4\sqrt{\cos[c+dx]} (24A - 6B + 25C + 2(6B - C) \cos[c+dx] + 4C \cos[2(c+dx)]) \sin\left[\frac{1}{2}(c+dx)\right]
\end{aligned}$$

■ **Problem 504: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]} (A + B \cos[c+dx] + C \cos[c+dx]^2)}{\sqrt{a+a \cos[c+dx]}} dx$$

Optimal (type 3, 195 leaves, 7 steps):

$$\begin{aligned}
& \frac{(8A - 4B + 7C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] - \sqrt{2} (A - B + C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{4\sqrt{a} d} + \\
& \frac{(4B - C) \sqrt{\cos[c+dx]} \sin[c+dx]}{4d \sqrt{a+a \cos[c+dx]}} + \frac{C \cos[c+dx]^{3/2} \sin[c+dx]}{2d \sqrt{a+a \cos[c+dx]}}
\end{aligned}$$

Result (type 3, 431 leaves):

$$\frac{1}{8\sqrt{a(1+\cos[c+dx])}} \cos\left[\frac{1}{2}(c+dx)\right] \left( \frac{1}{d\sqrt{1+e^{2i(c+dx)}}} \sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})} \left( 8Adx - 4Bdx + 7Cdx - i(8A-4B+7C) \operatorname{ArcSinh}[e^{i(c+dx)}] \right) + \right. \\ \left. 8i\sqrt{2}(A-B+C) \operatorname{Log}[1+e^{i(c+dx)}] + 8iA \operatorname{Log}[1+\sqrt{1+e^{2i(c+dx)}}] - 4iB \operatorname{Log}[1+\sqrt{1+e^{2i(c+dx)}}] + \right. \\ \left. 7iC \operatorname{Log}[1+\sqrt{1+e^{2i(c+dx)}}] - 8i\sqrt{2}A \operatorname{Log}[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}] + 8i\sqrt{2}B \operatorname{Log}[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}] - \right. \\ \left. 8i\sqrt{2}C \operatorname{Log}[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}] \right) + \frac{4\sqrt{\cos[c+dx]}(4B-C+2C\cos[c+dx])\sin\left[\frac{1}{2}(c+dx)\right]}{d}$$

- **Problem 505: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A+B\cos[c+dx]+C\cos[c+dx]^2}{\sqrt{\cos[c+dx]}\sqrt{a+a\cos[c+dx]}} dx$$

Optimal (type 3, 141 leaves, 6 steps):

$$\frac{(2B-C) \operatorname{ArcSin}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{\sqrt{a}d} + \frac{\sqrt{2}(A-B+C) \operatorname{ArcTan}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{2}\sqrt{\cos[c+dx]}\sqrt{a+a\cos[c+dx]}}\right]}{\sqrt{a}d} + \frac{C\sqrt{\cos[c+dx]}\sin[c+dx]}{d\sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 383 leaves):

$$\frac{1}{2\sqrt{a(1+\cos[c+dx])}} \cos\left[\frac{1}{2}(c+dx)\right] \left( \frac{1}{d\sqrt{1+e^{2i(c+dx)}}} \sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})} \left( 2Bdx - Cdx - i(2B-C) \operatorname{ArcSinh}[e^{i(c+dx)}] - 2i\sqrt{2}(A-B+C) \operatorname{Log}[1+e^{i(c+dx)}] \right) + \right. \\ \left. 2iB \operatorname{Log}[1+\sqrt{1+e^{2i(c+dx)}}] - iC \operatorname{Log}[1+\sqrt{1+e^{2i(c+dx)}}] + 2i\sqrt{2}A \operatorname{Log}[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}] - \right. \\ \left. 2i\sqrt{2}B \operatorname{Log}[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}] + 2i\sqrt{2}C \operatorname{Log}[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}] \right) + \frac{4C\sqrt{\cos[c+dx]}\sin\left[\frac{1}{2}(c+dx)\right]}{d}$$

- **Problem 506: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A+B\cos[c+dx]+C\cos[c+dx]^2}{\cos[c+dx]^{3/2}\sqrt{a+a\cos[c+dx]}} dx$$

Optimal (type 3, 138 leaves, 6 steps):

$$\frac{2 C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{\sqrt{a} d}-\frac{\sqrt{2}(A-B+C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right]}{\sqrt{a} d}+\frac{2 A \sin [c+d x]}{d \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}$$

Result (type 3, 340 leaves):

$$\frac{1}{d \sqrt{a}(1+\cos [c+d x])} \cos \left[\frac{1}{2}(c+d x)\right] \left(1 / \left(\sqrt{1+e^{2 i(c+d x)}}\right) \sqrt{2} e^{\frac{1}{2} i(c+d x)} \sqrt{e^{-i(c+d x)}\left(1+e^{2 i(c+d x)}\right)}\right. \\ \left.(C d x-i C \operatorname{ArcSinh}\left[e^{i(c+d x)}\right]+i \sqrt{2}(A-B+C) \operatorname{Log}\left[1+e^{i(c+d x)}\right]+i C \operatorname{Log}\left[1+\sqrt{1+e^{2 i(c+d x)}}\right]-i \sqrt{2} A \operatorname{Log}\left[1-e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]+i \sqrt{2} B \operatorname{Log}\left[1-e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]-i \sqrt{2} C \operatorname{Log}\left[1-e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]\right) +\frac{4 A \sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}$$

■ **Problem 507: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B \cos [c+d x]+C \cos [c+d x]^2}{\cos [c+d x]^{5 / 2} \sqrt{a+a \cos [c+d x]}} d x$$

Optimal (type 3, 143 leaves, 5 steps):

$$\frac{\sqrt{2}(A-B+C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right]}{\sqrt{a} d}+\frac{2 A \sin [c+d x]}{3 d \cos [c+d x]^{3 / 2} \sqrt{a+a \cos [c+d x]}}-\frac{2(A-3 B) \sin [c+d x]}{3 d \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}$$

Result (type 3, 207 leaves):

$$\frac{1}{3 d \sqrt{a}(1+\cos [c+d x])} 2 \cos \left[\frac{1}{2}(c+d x)\right] \\ \left(-1 / \left(\sqrt{1+e^{2 i(c+d x)}}\right) 3 i(A-B+C) e^{\frac{1}{2} i(c+d x)} \sqrt{e^{-i(c+d x)}\left(1+e^{2 i(c+d x)}\right)}\left(\operatorname{Log}\left[1+e^{i(c+d x)}\right]-\operatorname{Log}\left[1-e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]\right)\right) +\frac{2 A \sin \left[\frac{1}{2}(c+d x)\right]}{\cos [c+d x]^{3 / 2}}-\frac{2(A-3 B) \sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}$$

■ **Problem 508: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B \cos [c+d x]+C \cos [c+d x]^2}{\cos [c+d x]^{7 / 2} \sqrt{a+a \cos [c+d x]}} d x$$

Optimal (type 3, 191 leaves, 6 steps):

$$\begin{aligned}
& - \frac{\sqrt{2} (A - B + C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} + \frac{2 A \sin[c+dx]}{5 d \cos[c+dx]^{5/2} \sqrt{a+a \cos[c+dx]}} - \\
& \frac{2 (A - 5 B) \sin[c+dx]}{15 d \cos[c+dx]^{3/2} \sqrt{a+a \cos[c+dx]}} + \frac{2 (13 A - 5 B + 15 C) \sin[c+dx]}{15 d \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}
\end{aligned}$$

Result (type 3, 239 leaves):

$$\begin{aligned}
& \left( 2 \operatorname{Cos}\left[\frac{1}{2} (c+dx)\right] \right. \\
& \left. \left( 1 / \left( \sqrt{1+e^{2i(c+dx)}} \right) 15 i (A - B + C) e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} \left( \operatorname{Log}[1+e^{i(c+dx)}] - \operatorname{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) \right. \\
& \left. \left. \frac{6 A \sin\left[\frac{1}{2} (c+dx)\right]}{\cos[c+dx]^{5/2}} - \frac{2 (A - 5 B) \sin\left[\frac{1}{2} (c+dx)\right]}{\cos[c+dx]^{3/2}} + \frac{2 (13 A - 5 B + 15 C) \sin\left[\frac{1}{2} (c+dx)\right]}{\sqrt{\cos[c+dx]}} \right) \right) / \left( 15 d \sqrt{a (1 + \cos[c+dx])} \right)
\end{aligned}$$

■ **Problem 509: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos[c+dx] + C \cos^2[c+dx]}{\cos^9[c+dx] \sqrt{a+a \cos[c+dx]}} dx$$

Optimal (type 3, 237 leaves, 7 steps):

$$\begin{aligned}
& \frac{\sqrt{2} (A - B + C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{\sqrt{a} d} + \frac{2 A \sin[c+dx]}{7 d \cos[c+dx]^{7/2} \sqrt{a+a \cos[c+dx]}} - \\
& \frac{2 (A - 7 B) \sin[c+dx]}{35 d \cos[c+dx]^{5/2} \sqrt{a+a \cos[c+dx]}} + \frac{2 (31 A - 7 B + 35 C) \sin[c+dx]}{105 d \cos[c+dx]^{3/2} \sqrt{a+a \cos[c+dx]}} - \frac{2 (43 A - 91 B + 35 C) \sin[c+dx]}{105 d \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}
\end{aligned}$$

Result (type 3, 262 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{a (1 + \cos[c+dx])}} \operatorname{Cos}\left[\frac{1}{2} (c+dx)\right] \\
& \left( -1 / \left( d \sqrt{1+e^{2i(c+dx)}} \right) 2 i (A - B + C) e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} \left( \operatorname{Log}[1+e^{i(c+dx)}] - \operatorname{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) - \\
& 1 / \left( 105 d \cos[c+dx]^{7/2} \right) (-122 A + 14 B - 70 C + 3 (47 A - 119 B + 35 C) \cos[c+dx] - \\
& 2 (31 A - 7 B + 35 C) \cos[2 (c+dx)] + 43 A \cos[3 (c+dx)] - 91 B \cos[3 (c+dx)] + 35 C \cos[3 (c+dx)]) \operatorname{Sin}\left[\frac{1}{2} (c+dx)\right]
\end{aligned}$$



- **Problem 510: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]} (aA + (Ab + aB) \cos[c+dx] + bB \cos[c+dx]^2)}{\sqrt{a+a\cos[c+dx]}} dx$$

Optimal (type 3, 213 leaves, 7 steps):

$$\frac{(8aA - 4Ab - 4aB + 7bB) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{4\sqrt{a}d} - \frac{\sqrt{2}(a-b)(A-B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2}\sqrt{\cos[c+dx]}\sqrt{a+a\cos[c+dx]}}\right]}{\sqrt{a}d} +$$

$$\frac{(4Ab + 4aB - bB) \sqrt{\cos[c+dx]} \sin[c+dx]}{4d\sqrt{a+a\cos[c+dx]}} + \frac{bB \cos[c+dx]^{3/2} \sin[c+dx]}{2d\sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 540 leaves):

$$\frac{1}{8\sqrt{a}(1+\cos[c+dx])}$$

$$\cos\left[\frac{1}{2}(c+dx)\right] \left( \frac{1}{d\sqrt{1+e^{2i(c+dx)}}} \sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})} \left( 8aAdx - 4Abdx - 4aBdx + 7bBdx - i(8aA - 4Ab - 4aB + 7bB) \right. \right.$$

$$\operatorname{ArcSinh}\left[e^{i(c+dx)}\right] + 8i\sqrt{2}(a-b)(A-B) \operatorname{Log}\left[1+e^{i(c+dx)}\right] + 8iaA \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] - 4iAb \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] -$$

$$4iaB \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] + 7ibB \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] - 8i\sqrt{2}aA \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] +$$

$$8i\sqrt{2}Ab \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] + 8i\sqrt{2}aB \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] -$$

$$\left. \left. 8i\sqrt{2}bB \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right]\right) + \frac{4\sqrt{\cos[c+dx]}(4Ab + 4aB - bB + 2bB \cos[c+dx]) \sin\left[\frac{1}{2}(c+dx)\right]}{d} \right)$$

- **Problem 511: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^{3/2} (A + B \cos[c+dx] + C \cos[c+dx]^2)}{(a+a\cos[c+dx])^{3/2}} dx$$

Optimal (type 3, 260 leaves, 8 steps):

$$\frac{(8A - 12B + 19C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right]}{4a^{3/2}d} - \frac{(5A - 9B + 13C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2}\sqrt{\cos[c+dx]}\sqrt{a+a\cos[c+dx]}}\right]}{2\sqrt{2}a^{3/2}d} -$$

$$\frac{(A-B+C) \cos[c+dx]^{5/2} \sin[c+dx]}{2d(a+a\cos[c+dx])^{3/2}} - \frac{(2A-6B+7C) \sqrt{\cos[c+dx]} \sin[c+dx]}{4ad\sqrt{a+a\cos[c+dx]}} + \frac{(A-B+2C) \cos[c+dx]^{3/2} \sin[c+dx]}{2ad\sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 462 leaves):

$$\frac{1}{4 d (a (1 + \operatorname{Cos}[c + d x]))^{3/2}} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^3 \left( \frac{1}{\sqrt{1 + e^{2i(c+dx)}}} \sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1 + e^{2i(c+dx)})} (8 A d x - 12 B d x + 19 C d x - i(8 A - 12 B + 19 C) \operatorname{ArcSinh}[e^{i(c+dx)}]) + \right. \\ \left. 2 i \sqrt{2} (5 A - 9 B + 13 C) \operatorname{Log}[1 + e^{i(c+dx)}] + 8 i A \operatorname{Log}[1 + \sqrt{1 + e^{2i(c+dx)}}] - 12 i B \operatorname{Log}[1 + \sqrt{1 + e^{2i(c+dx)}}] + \right. \\ \left. 19 i C \operatorname{Log}[1 + \sqrt{1 + e^{2i(c+dx)}}] - 10 i \sqrt{2} A \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] + \right. \\ \left. 18 i \sqrt{2} B \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] - 26 i \sqrt{2} C \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) + \\ \left. 2 \sqrt{\operatorname{Cos}[c + d x]} (-2 A + 6 B - 6 C + (4 B - 3 C) \operatorname{Cos}[c + d x] + C \operatorname{Cos}[2(c + d x)]) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] \right)$$

■ **Problem 512: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Cos}[c + d x]} (A + B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2)}{(a + a \operatorname{Cos}[c + d x])^{3/2}} dx$$

Optimal (type 3, 202 leaves, 7 steps):

$$\frac{(2 B - 3 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{a^{3/2} d} + \frac{(A - 5 B + 9 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \\ \frac{(A - B + C) \operatorname{Cos}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{2 d (a + a \operatorname{Cos}[c + d x])^{3/2}} + \frac{(A - B + 3 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{2 a d \sqrt{a + a \operatorname{Cos}[c + d x]}}$$

Result (type 3, 413 leaves):

$$\frac{1}{2 (a (1 + \operatorname{Cos}[c + d x]))^{3/2}} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^3 \left( \frac{1}{d \sqrt{1 + e^{2i(c+dx)}}} \sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1 + e^{2i(c+dx)})} (4 B d x - 6 C d x - 2 i(2 B - 3 C) \operatorname{ArcSinh}[e^{i(c+dx)}] - i \sqrt{2} (A - 5 B + 9 C) \right. \\ \left. \operatorname{Log}[1 + e^{i(c+dx)}] + 4 i B \operatorname{Log}[1 + \sqrt{1 + e^{2i(c+dx)}}] - 6 i C \operatorname{Log}[1 + \sqrt{1 + e^{2i(c+dx)}}] + i \sqrt{2} A \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] - \right. \\ \left. 5 i \sqrt{2} B \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] + 9 i \sqrt{2} C \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}] \right) + \\ \left. \frac{2 \sqrt{\operatorname{Cos}[c + d x]} (A - B + 3 C + 2 C \operatorname{Cos}[c + d x]) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{d} \right)$$

- **Problem 513: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx] + C \cos[c + dx]^2}{\sqrt{\cos[c + dx]} (a + a \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 149 leaves, 6 steps):

$$\frac{2 C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right]}{a^{3/2} d} + \frac{(3 A + B - 5 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{(A - B + C) \sqrt{\cos[c + dx]} \sin[c + dx]}{2 d (a + a \cos[c + dx])^{3/2}}$$

Result (type 3, 366 leaves):

$$\frac{1}{2 (a (1 + \cos[c + dx]))^{3/2}} \cos\left[\frac{1}{2} (c + dx)\right]^3$$

$$\left( 1 / \left( d \sqrt{1 + e^{2i(c+dx)}} \right) \sqrt{2} e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( 4 C dx - 4 i C \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] - i \sqrt{2} (3 A + B - 5 C) \operatorname{Log}\left[1 + e^{i(c+dx)}\right] + \right. \right.$$

$$4 i C \operatorname{Log}\left[1 + \sqrt{1 + e^{2i(c+dx)}}\right] + 3 i \sqrt{2} A \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] + i \sqrt{2} B \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] -$$

$$\left. \left. 5 i \sqrt{2} C \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] \right) - \frac{2 (A - B + C) \sqrt{\cos[c + dx]} \operatorname{Sec}\left[\frac{1}{2} (c + dx)\right] \operatorname{Tan}\left[\frac{1}{2} (c + dx)\right]}{d} \right)$$

- **Problem 514: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos[c + dx] + C \cos[c + dx]^2}{\cos[c + dx]^{3/2} (a + a \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 161 leaves, 5 steps):

$$- \frac{(7 A - 3 B - C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{(A - B + C) \sin[c + dx]}{2 d \sqrt{\cos[c + dx]} (a + a \cos[c + dx])^{3/2}} + \frac{(5 A - B + C) \sin[c + dx]}{2 a d \sqrt{\cos[c + dx]} \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 213 leaves):

$$\frac{1}{(a (1 + \cos[c + dx]))^{3/2}} \cos\left[\frac{1}{2} (c + dx)\right]^3$$

$$\left( 1 / \left( d \sqrt{1 + e^{2i(c+dx)}} \right) i (7 A - 3 B - C) e^{\frac{1}{2} i (c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( \operatorname{Log}\left[1 + e^{i(c+dx)}\right] - \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] \right) + \right.$$

$$\left. \frac{(4 A + (5 A - B + C) \cos[c + dx]) \operatorname{Sec}\left[\frac{1}{2} (c + dx)\right] \operatorname{Tan}\left[\frac{1}{2} (c + dx)\right]}{d \sqrt{\cos[c + dx]}} \right)$$

■ **Problem 515: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos[c + dx] + C \cos[c + dx]^2}{\cos[c + dx]^{5/2} (a + a \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 213 leaves, 6 steps):

$$\frac{(11A - 7B + 3C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{2\sqrt{2} a^{3/2} d} - \frac{(A - B + C) \sin[c + dx]}{2d \cos[c + dx]^{3/2} (a + a \cos[c + dx])^{3/2}} +$$

$$\frac{(7A - 3B + 3C) \sin[c + dx]}{6ad \cos[c + dx]^{3/2} \sqrt{a + a \cos[c + dx]}} - \frac{(19A - 15B + 3C) \sin[c + dx]}{6ad \sqrt{\cos[c + dx]} \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 239 leaves):

$$\frac{1}{(a(1 + \cos[c + dx]))^{3/2}} \cos\left[\frac{1}{2}(c + dx)\right]^3$$

$$\left(-1/\left(d\sqrt{1 + e^{2i(c+dx)}}\right) i (11A - 7B + 3C) e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1 + e^{2i(c+dx)})} \left(\operatorname{Log}[1 + e^{i(c+dx)}] - \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}]\right) -$$

$$1/(6d \cos[c + dx]^{3/2}) (11A - 15B + 3C + 24(A - B) \cos[c + dx] + (19A - 15B + 3C) \cos[2(c + dx)]) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right)$$

■ **Problem 516: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos[c + dx] + C \cos[c + dx]^2}{\cos[c + dx]^{7/2} (a + a \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 263 leaves, 7 steps):

$$-\frac{(15A - 11B + 7C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right]}{2\sqrt{2} a^{3/2} d} - \frac{(A - B + C) \sin[c + dx]}{2d \cos[c + dx]^{5/2} (a + a \cos[c + dx])^{3/2}} +$$

$$\frac{(9A - 5B + 5C) \sin[c + dx]}{10ad \cos[c + dx]^{5/2} \sqrt{a + a \cos[c + dx]}} - \frac{(39A - 35B + 15C) \sin[c + dx]}{30ad \cos[c + dx]^{3/2} \sqrt{a + a \cos[c + dx]}} + \frac{(147A - 95B + 75C) \sin[c + dx]}{30ad \sqrt{\cos[c + dx]} \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 277 leaves):

$$\frac{1}{(a(1 + \cos[c + dx]))^{3/2}} \cos\left[\frac{1}{2}(c + dx)\right]^3$$

$$\left(1/\left(d\sqrt{1 + e^{2i(c+dx)}}\right) i (15A - 11B + 7C) e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1 + e^{2i(c+dx)})} \left(\operatorname{Log}[1 + e^{i(c+dx)}] - \operatorname{Log}[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}]\right) +$$

$$1/(60d \cos[c + dx]^{5/2}) (264A - 120B + 120C + (393A - 205B + 225C) \cos[c + dx] + 24(9A - 5B + 5C) \cos[2(c + dx)] +$$

$$147A \cos[3(c + dx)] - 95B \cos[3(c + dx)] + 75C \cos[3(c + dx)]) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right)$$

■ **Problem 517: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c+d x]^{3 / 2} (A+B \cos [c+d x]+C \cos [c+d x]^2)}{(a+a \cos [c+d x])^{5 / 2}} d x$$

Optimal (type 3, 254 leaves, 8 steps):

$$\frac{(2 B-5 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{a^{5 / 2} d}+\frac{(3 A-43 B+115 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right]}{16 \sqrt{2} a^{5 / 2} d}$$

$$\frac{(A-B+C) \cos [c+d x]^{5 / 2} \sin [c+d x]}{4 d(a+a \cos [c+d x])^{5 / 2}}+\frac{(A+7 B-15 C) \cos [c+d x]^{3 / 2} \sin [c+d x]}{16 a d(a+a \cos [c+d x])^{3 / 2}}+\frac{(3 A-11 B+35 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{16 a^2 d \sqrt{a+a \cos [c+d x]}}$$

Result (type 3, 434 leaves):

$$\frac{1}{8 d(a(1+\cos [c+d x]))^{5 / 2}} \cos \left[\frac{1}{2}(c+d x)\right]^5$$

$$\left(\frac{1}{\sqrt{1+e^{2 i(c+d x)}}} \sqrt{2} e^{\frac{1}{2} i(c+d x)} \sqrt{e^{-i(c+d x)}\left(1+e^{2 i(c+d x)}\right)}\left(32 B d x-80 C d x-16 i(2 B-5 C) \operatorname{ArcSinh}\left[e^{i(c+d x)}\right]-i \sqrt{2}(3 A-43 B+115 C)\right.\right.$$

$$\left.\left.\operatorname{Log}\left[1+e^{i(c+d x)}\right]+32 i B \operatorname{Log}\left[1+\sqrt{1+e^{2 i(c+d x)}}\right]-80 i C \operatorname{Log}\left[1+\sqrt{1+e^{2 i(c+d x)}}\right]+3 i \sqrt{2} A \operatorname{Log}\left[1-e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]-\right.$$

$$\left.43 i \sqrt{2} B \operatorname{Log}\left[1-e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]+115 i \sqrt{2} C \operatorname{Log}\left[1-e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]\right)+$$

$$\left.\sqrt{\cos [c+d x]}(3 A-11 B+43 C+(7 A-15 B+55 C) \cos [c+d x]+8 C \cos [2(c+d x)]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)$$

■ **Problem 518: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{\cos [c+d x]}(A+B \cos [c+d x]+C \cos [c+d x]^2)}{(a+a \cos [c+d x])^{5 / 2}} d x$$

Optimal (type 3, 201 leaves, 7 steps):

$$\frac{2 C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right]}{a^{5 / 2} d}+\frac{(5 A+3 B-43 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{2} \sqrt{\cos [c+d x]} \sqrt{a+a \cos [c+d x]}}\right]}{16 \sqrt{2} a^{5 / 2} d}$$

$$\frac{(A-B+C) \cos [c+d x]^{3 / 2} \sin [c+d x]}{4 d(a+a \cos [c+d x])^{5 / 2}}+\frac{(5 A+3 B-11 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{16 a d(a+a \cos [c+d x])^{3 / 2}}$$

Result (type 3, 385 leaves):

$$\frac{1}{8 d (a (1 + \operatorname{Cos}[c + d x]))^{5/2}} \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]^5$$

$$\left( \frac{1}{\sqrt{1 + e^{2i(c+dx)}}} \sqrt{2} e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( 32 C d x - 32 i C \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] - i \sqrt{2} (5 A + 3 B - 43 C) \operatorname{Log}\left[1 + e^{i(c+dx)}\right] + \right. \right.$$

$$32 i C \operatorname{Log}\left[1 + \sqrt{1 + e^{2i(c+dx)}}\right] + 5 i \sqrt{2} A \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] +$$

$$\left. 3 i \sqrt{2} B \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] - 43 i \sqrt{2} C \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] \right) +$$

$$\sqrt{\operatorname{Cos}[c + d x]} (5 A + 3 B - 11 C + (A + 7 B - 15 C) \operatorname{Cos}[c + d x]) \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^3 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]$$

■ **Problem 519: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2}{\sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Cos}[c + d x])^{5/2}} dx$$

Optimal (type 3, 163 leaves, 5 steps):

$$\frac{(19 A + 5 B + 3 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A - B + C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{4 d (a + a \operatorname{Cos}[c + d x])^{5/2}} - \frac{(9 A - B - 7 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{16 a d (a + a \operatorname{Cos}[c + d x])^{3/2}}$$

Result (type 3, 226 leaves):

$$\left( \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]^5 \right.$$

$$\left. \left( -1 / \left( \sqrt{1 + e^{2i(c+dx)}} \right) i (19 A + 5 B + 3 C) e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( \operatorname{Log}\left[1 + e^{i(c+dx)}\right] - \operatorname{Log}\left[1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}}\right] \right) - \right.$$

$$\left. \left. \frac{1}{2} \sqrt{\operatorname{Cos}[c + d x]} (13 A - 5 B - 3 C + (9 A - B - 7 C) \operatorname{Cos}[c + d x]) \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^3 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \right) \right) / (4 d (a (1 + \operatorname{Cos}[c + d x]))^{5/2})$$

■ **Problem 520: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2}{\operatorname{Cos}[c + d x]^{3/2} (a + a \operatorname{Cos}[c + d x])^{5/2}} dx$$

Optimal (type 3, 211 leaves, 6 steps):

$$- \frac{(75 A - 19 B - 5 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Cos}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A - B + C) \operatorname{Sin}[c + d x]}{4 d \sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Cos}[c + d x])^{5/2}} -$$

$$\frac{(13 A - 5 B - 3 C) \operatorname{Sin}[c + d x]}{16 a d \sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Cos}[c + d x])^{3/2}} + \frac{(49 A - 9 B + C) \operatorname{Sin}[c + d x]}{16 a^2 d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + a \operatorname{Cos}[c + d x]}}$$

Result (type 3, 242 leaves) :

$$\left( \cos \left[ \frac{1}{2} (c + dx) \right] \right)^5$$

$$\left( \frac{1}{\sqrt{1 + e^{2i(c+dx)}}} \right)^3 \frac{i (75A - 19B - 5C) e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( \log[1 + e^{i(c+dx)}] - \log[1 - e^{i(c+dx)}] + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}} \right)}{1 / \left( 4 \sqrt{\cos[c+dx]} \right) \left( 113A - 9B + C + 2 (85A - 13B + 5C) \cos[c+dx] + (49A - 9B + C) \cos[2(c+dx)] \right)}$$

$$\left. \left. \left. \sec \left[ \frac{1}{2} (c + dx) \right]^3 \tan \left[ \frac{1}{2} (c + dx) \right] \right) \right) \right) / \left( 4d (a (1 + \cos[c + dx]))^{5/2} \right)$$

■ **Problem 521: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos[c + dx] + C \cos[c + dx]^2}{\cos[c + dx]^{5/2} (a + a \cos[c + dx])^{5/2}} dx$$

Optimal (type 3, 261 leaves, 7 steps) :

$$\frac{(163A - 75B + 19C) \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}} \right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A - B + C) \sin[c + dx]}{4d \cos[c + dx]^{3/2} (a + a \cos[c + dx])^{5/2}}$$

$$+ \frac{(17A - 9B + C) \sin[c + dx]}{16ad \cos[c + dx]^{3/2} (a + a \cos[c + dx])^{3/2}} + \frac{(95A - 39B + 15C) \sin[c + dx]}{48a^2 d \cos[c + dx]^{3/2} \sqrt{a + a \cos[c + dx]}}$$

$$- \frac{(299A - 147B + 27C) \sin[c + dx]}{48a^2 d \sqrt{\cos[c + dx]} \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 279 leaves) :

$$- \left( \cos \left[ \frac{1}{2} (c + dx) \right] \right)^5$$

$$\left( \frac{1}{\sqrt{1 + e^{2i(c+dx)}}} \right)^3 \frac{i (163A - 75B + 19C) e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left( \log[1 + e^{i(c+dx)}] - \log[1 - e^{i(c+dx)}] + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}} \right)}{1 / \left( 8 \cos[c + dx] \right)^{3/2} \left( 878A - 510B + 78C + (1537A - 825B + 81C) \cos[c + dx] + 2 (503A - 255B + 39C) \cos[2(c + dx)] + 299A \right)}$$

$$\left. \left. \left. \cos[3(c + dx)] - 147B \cos[3(c + dx)] + 27C \cos[3(c + dx)] \right) \sec \left[ \frac{1}{2} (c + dx) \right]^3 \tan \left[ \frac{1}{2} (c + dx) \right] \right) \right) / \left( 12d (a (1 + \cos[c + dx]))^{5/2} \right)$$

■ **Problem 525: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + dx]) (A + C \cos[c + dx]^2) \sec[c + dx] dx$$

Optimal (type 3, 58 leaves, 4 steps) :

$$\frac{1}{2} b (2A + C) x + \frac{a A \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{a C \sin[c + dx]}{d} + \frac{b C \cos[c + dx] \sin[c + dx]}{2d}$$

Result (type 3, 131 leaves) :

$$A b x + \frac{b C (c + d x)}{2 d} - \frac{a A \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{d x}{2}\right] - \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} +$$

$$\frac{a A \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{d x}{2}\right] + \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a C \cos[d x] \sin[c]}{d} + \frac{a C \cos[c] \sin[d x]}{d} + \frac{b C \sin[2(c + d x)]}{4 d}$$

■ **Problem 526: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + d x]) (A + C \cos[c + d x])^2 \sec[c + d x]^2 dx$$

Optimal (type 3, 42 leaves, 4 steps):

$$a C x + \frac{A b \operatorname{ArcTanh}[\sin[c + d x]]}{d} + \frac{b C \sin[c + d x]}{d} + \frac{a A \tan[c + d x]}{d}$$

Result (type 3, 112 leaves):

$$a C x - \frac{A b \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{d x}{2}\right] - \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{A b \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{d x}{2}\right] + \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{b C \cos[d x] \sin[c]}{d} + \frac{b C \cos[c] \sin[d x]}{d} + \frac{a A \tan[c + d x]}{d}$$

■ **Problem 527: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + d x]) (A + C \cos[c + d x])^2 \sec[c + d x]^3 dx$$

Optimal (type 3, 58 leaves, 4 steps):

$$b C x + \frac{a (A + 2 C) \operatorname{ArcTanh}[\sin[c + d x]]}{2 d} + \frac{A b \tan[c + d x]}{d} + \frac{a A \sec[c + d x] \tan[c + d x]}{2 d}$$

Result (type 3, 218 leaves):

$$b C x - \frac{a C \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{d x}{2}\right] - \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a C \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{d x}{2}\right] + \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} -$$

$$\frac{a A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} + \frac{a A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} +$$

$$\frac{a A}{4 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{a A}{4 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{A b \tan[c + d x]}{d}$$

■ **Problem 529: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + d x]) (A + C \cos[c + d x])^2 \sec[c + d x]^5 dx$$

Optimal (type 3, 117 leaves, 7 steps):



$$\frac{a(3A+4C)\operatorname{ArcTanh}[\sin[c+dx]]}{8d} + \frac{b(2A+3C)\tan[c+dx]}{3d} +$$

$$\frac{a(3A+4C)\sec[c+dx]\tan[c+dx]}{8d} + \frac{Ab\sec[c+dx]^2\tan[c+dx]}{3d} + \frac{aA\sec[c+dx]^3\tan[c+dx]}{4d}$$

Result (type 3, 377 leaves):

$$-\frac{3aA\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} - \frac{aC\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \frac{3aA\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} +$$

$$\frac{aC\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \frac{aA}{16d\left(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{3aA}{16d\left(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right)^2} +$$

$$\frac{aC}{4d\left(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{aA}{16d\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{3aA}{16d\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)^2} -$$

$$\frac{aC}{4d\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{2Ab\tan[c+dx]}{3d} + \frac{bC\tan[c+dx]}{d} + \frac{Ab\sec[c+dx]^2\tan[c+dx]}{3d}$$

■ **Problem 530: Result more than twice size of optimal antiderivative.**

$$\int (a+b\cos[c+dx])(A+C\cos[c+dx]^2)\sec[c+dx]^6 dx$$

Optimal (type 3, 140 leaves, 7 steps):

$$\frac{b(3A+4C)\operatorname{ArcTanh}[\sin[c+dx]]}{8d} + \frac{a(4A+5C)\tan[c+dx]}{5d} + \frac{b(3A+4C)\sec[c+dx]\tan[c+dx]}{8d} +$$

$$\frac{Ab\sec[c+dx]^3\tan[c+dx]}{4d} + \frac{aA\sec[c+dx]^4\tan[c+dx]}{5d} + \frac{a(4A+5C)\tan[c+dx]^3}{15d}$$

Result (type 3, 426 leaves):

$$\begin{aligned}
& - \frac{3 A b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right]}{8 d}-\frac{b C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right]}{2 d}+ \\
& \frac{3 A b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right]}{8 d}+\frac{b C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right]}{2 d}+\frac{A b}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^4}+ \\
& \frac{3 A b}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}+\frac{b C}{4 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}-\frac{A b}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^4}- \\
& \frac{3 A b}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}-\frac{b C}{4 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}+\frac{8 a A \operatorname{Tan}[c+d x]}{15 d}+ \\
& \frac{2 a C \operatorname{Tan}[c+d x]}{3 d}+\frac{4 a A \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{15 d}+\frac{a C \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{3 d}+\frac{a A \operatorname{Sec}[c+d x]^4 \operatorname{Tan}[c+d x]}{5 d}
\end{aligned}$$

■ **Problem 536: Result more than twice size of optimal antiderivative.**

$$\int (a+b \cos [c+d x])^2 (A+C \cos [c+d x]^2) \operatorname{Sec}[c+d x]^3 dx$$

Optimal (type 3, 103 leaves, 5 steps):

$$\begin{aligned}
& 2 a b C x + \frac{(2 A b^2 + a^2 (A + 2 C)) \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{2 d} - \\
& \frac{b^2 (A - 2 C) \operatorname{Sin}[c+d x]}{2 d} + \frac{a A b \operatorname{Tan}[c+d x]}{d} + \frac{A (a+b \cos [c+d x])^2 \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{2 d}
\end{aligned}$$

Result (type 3, 249 leaves):

$$\begin{aligned}
& \frac{1}{4 d} \left( 8 a b C (c+d x) - 2 (2 A b^2 + a^2 (A + 2 C)) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right] \right) + \\
& 2 (2 A b^2 + a^2 (A + 2 C)) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right] + \frac{a^2 A}{\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^2} + \\
& \frac{8 a A b \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]} - \frac{a^2 A}{\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{8 a A b \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]} + 4 b^2 C \operatorname{Sin}[c+d x] \right)
\end{aligned}$$

■ **Problem 537: Result more than twice size of optimal antiderivative.**

$$\int (a+b \cos [c+d x])^2 (A+C \cos [c+d x]^2) \operatorname{Sec}[c+d x]^4 dx$$

Optimal (type 3, 112 leaves, 5 steps):

$$b^2 C x + \frac{a b (A + 2 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} + \frac{(2 A b^2 + a^2 (2 A + 3 C)) \operatorname{Tan}[c + d x]}{3 d} +$$

$$\frac{a A b \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{3 d} + \frac{A (a + b \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d}$$

Result (type 3, 424 leaves):

$$\frac{b^2 C (c + d x)}{d} + \frac{(-a A b - 2 a b C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right]}{d} +$$

$$\frac{(a A b + 2 a b C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right]}{d} + \frac{a^2 A + 6 a A b}{12 d \left(\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right)^2} +$$

$$\frac{a^2 A \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]}{6 d \left(\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right)^3} + \frac{a^2 A \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]}{6 d \left(\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right)^3} + \frac{-a^2 A - 6 a A b}{12 d \left(\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right)^2} +$$

$$\frac{2 a^2 A \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] + 3 A b^2 \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] + 3 a^2 C \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]}{3 d \left(\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right)} + \frac{2 a^2 A \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] + 3 A b^2 \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] + 3 a^2 C \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]}{3 d \left(\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right)}$$

■ **Problem 538: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Cos}[c + d x])^2 (A + C \operatorname{Cos}[c + d x]^2) \operatorname{Sec}[c + d x]^5 dx$$

Optimal (type 3, 154 leaves, 7 steps):

$$\frac{(4 b^2 (A + 2 C) + a^2 (3 A + 4 C)) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{2 a b (2 A + 3 C) \operatorname{Tan}[c + d x]}{3 d} +$$

$$\frac{(2 A b^2 + a^2 (3 A + 4 C)) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{a A b \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{6 d} + \frac{A (a + b \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d}$$

Result (type 3, 503 leaves):

$$\frac{(-3a^2A - 4Ab^2 - 4a^2C - 8b^2C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{(3a^2A + 4Ab^2 + 4a^2C + 8b^2C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} +$$

$$\frac{a^2A}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{9a^2A + 8aAb + 12Ab^2 + 12a^2C}{48d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{aAb \sin\left[\frac{1}{2}(c+dx)\right]}{3d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} -$$

$$\frac{a^2A}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{aAb \sin\left[\frac{1}{2}(c+dx)\right]}{3d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{-9a^2A - 8aAb - 12Ab^2 - 12a^2C}{48d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} +$$

$$\frac{2(2aAb \sin\left[\frac{1}{2}(c+dx)\right] + 3aAbC \sin\left[\frac{1}{2}(c+dx)\right])}{3d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)} + \frac{2(2aAb \sin\left[\frac{1}{2}(c+dx)\right] + 3aAbC \sin\left[\frac{1}{2}(c+dx)\right])}{3d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}$$

■ **Problem 545: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + dx])^3 (A + C \cos[c + dx])^2 \sec[c + dx]^4 dx$$

Optimal (type 3, 163 leaves, 6 steps):

$$3ab^2Cx + \frac{b(2Ab^2 + 3a^2(A + 2C)) \operatorname{ArcTanh}[\sin[c + dx]]}{2d} - \frac{b^3(5A - 6C) \sin[c + dx]}{6d} + \frac{a(3Ab^2 + a^2(2A + 3C)) \tan[c + dx]}{3d} +$$

$$\frac{Ab(a + b \cos[c + dx])^2 \sec[c + dx] \tan[c + dx]}{2d} + \frac{A(a + b \cos[c + dx])^3 \sec[c + dx]^2 \tan[c + dx]}{3d}$$

Result (type 3, 473 leaves):

$$\frac{3ab^2C(c + dx)}{d} + \frac{(-3a^2Ab - 2Ab^3 - 6a^2bC) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{2d} +$$

$$\frac{(3a^2Ab + 2Ab^3 + 6a^2bC) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \frac{a^3A + 9a^2Ab}{12d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} +$$

$$\frac{a^3A \sin\left[\frac{1}{2}(c+dx)\right]}{6d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{a^3A \sin\left[\frac{1}{2}(c+dx)\right]}{6d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} +$$

$$\frac{-a^3A - 9a^2Ab}{12d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{2a^3A \sin\left[\frac{1}{2}(c+dx)\right] + 9aAb^2 \sin\left[\frac{1}{2}(c+dx)\right] + 3a^3C \sin\left[\frac{1}{2}(c+dx)\right]}{3d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)} +$$

$$\frac{2a^3A \sin\left[\frac{1}{2}(c+dx)\right] + 9aAb^2 \sin\left[\frac{1}{2}(c+dx)\right] + 3a^3C \sin\left[\frac{1}{2}(c+dx)\right]}{3d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)} + \frac{b^3C \sin[c + dx]}{d}$$

■ **Problem 546: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^3 (A + C \cos [c + d x]^2) \sec [c + d x]^5 dx$$

Optimal (type 3, 182 leaves, 6 steps):

$$b^3 C x + \frac{a (12 b^2 (A + 2 C) + a^2 (3 A + 4 C)) \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} +$$

$$\frac{b (A b^2 + a^2 (4 A + 6 C)) \tan [c + d x]}{2 d} + \frac{a (2 A b^2 + a^2 (3 A + 4 C)) \sec [c + d x] \tan [c + d x]}{8 d} +$$

$$\frac{A b (a + b \cos [c + d x])^2 \sec [c + d x]^2 \tan [c + d x]}{4 d} + \frac{A (a + b \cos [c + d x])^3 \sec [c + d x]^3 \tan [c + d x]}{4 d}$$

Result (type 3, 562 leaves):

$$\frac{b^3 C (c + d x)}{d} + \frac{(-3 a^3 A - 12 a A b^2 - 4 a^3 C - 24 a b^2 C) \operatorname{Log}[\cos [\frac{1}{2} (c + d x)] - \sin [\frac{1}{2} (c + d x)]]}{8 d} +$$

$$\frac{(3 a^3 A + 12 a A b^2 + 4 a^3 C + 24 a b^2 C) \operatorname{Log}[\cos [\frac{1}{2} (c + d x)] + \sin [\frac{1}{2} (c + d x)]]}{8 d} + \frac{a^3 A}{16 d (\cos [\frac{1}{2} (c + d x)] - \sin [\frac{1}{2} (c + d x)])^4} +$$

$$\frac{3 a^3 A + 4 a^2 A b + 12 a A b^2 + 4 a^3 C}{16 d (\cos [\frac{1}{2} (c + d x)] - \sin [\frac{1}{2} (c + d x)])^2} + \frac{a^2 A b \sin [\frac{1}{2} (c + d x)]}{2 d (\cos [\frac{1}{2} (c + d x)] - \sin [\frac{1}{2} (c + d x)])^3} -$$

$$\frac{a^3 A}{16 d (\cos [\frac{1}{2} (c + d x)] + \sin [\frac{1}{2} (c + d x)])^4} + \frac{a^2 A b \sin [\frac{1}{2} (c + d x)]}{2 d (\cos [\frac{1}{2} (c + d x)] + \sin [\frac{1}{2} (c + d x)])^3} +$$

$$\frac{-3 a^3 A - 4 a^2 A b - 12 a A b^2 - 4 a^3 C}{16 d (\cos [\frac{1}{2} (c + d x)] + \sin [\frac{1}{2} (c + d x)])^2} + \frac{2 a^2 A b \sin [\frac{1}{2} (c + d x)] + A b^3 \sin [\frac{1}{2} (c + d x)] + 3 a^2 b C \sin [\frac{1}{2} (c + d x)]}{d (\cos [\frac{1}{2} (c + d x)] - \sin [\frac{1}{2} (c + d x)])} +$$

$$\frac{2 a^2 A b \sin [\frac{1}{2} (c + d x)] + A b^3 \sin [\frac{1}{2} (c + d x)] + 3 a^2 b C \sin [\frac{1}{2} (c + d x)]}{d (\cos [\frac{1}{2} (c + d x)] + \sin [\frac{1}{2} (c + d x)])}$$

■ **Problem 554: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^4 (A + C \cos [c + d x]^2) \sec [c + d x]^4 dx$$

Optimal (type 3, 251 leaves, 7 steps):

$$\frac{1}{2} b^2 (2 A b^2 + (12 a^2 + b^2) C) x + \frac{2 a b (2 A b^2 + a^2 (A + 2 C)) \operatorname{ArcTanh}[\sin[c + d x]]}{d} - \frac{2 a b (b^2 (11 A - 6 C) + a^2 (2 A + 3 C)) \sin[c + d x]}{3 d} -$$

$$\frac{b^2 (3 b^2 (6 A - C) + a^2 (4 A + 6 C)) \cos[c + d x] \sin[c + d x]}{6 d} + \frac{(6 A b^2 + a^2 (2 A + 3 C)) (a + b \cos[c + d x])^2 \tan[c + d x]}{3 d} +$$

$$\frac{2 A b (a + b \cos[c + d x])^3 \sec[c + d x] \tan[c + d x]}{3 d} + \frac{A (a + b \cos[c + d x])^4 \sec[c + d x]^2 \tan[c + d x]}{3 d}$$

Result (type 3, 864 leaves):

$$\frac{b^2 (2 A b^2 + 12 a^2 C + b^2 C) (c + d x) \cos[c + d x]^4 (b + a \sec[c + d x])^4}{2 d (a + b \cos[c + d x])^4} -$$

$$\frac{2 (a^3 A b + 2 a A b^3 + 2 a^3 b C) \cos[c + d x]^4 \log\left[\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right] (b + a \sec[c + d x])^4}{d (a + b \cos[c + d x])^4} +$$

$$\frac{2 (a^3 A b + 2 a A b^3 + 2 a^3 b C) \cos[c + d x]^4 \log\left[\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right] (b + a \sec[c + d x])^4}{d (a + b \cos[c + d x])^4} +$$

$$\frac{(a^4 A + 12 a^3 A b) \cos[c + d x]^4 (b + a \sec[c + d x])^4}{12 d (a + b \cos[c + d x])^4 \left(\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right)^2} + \frac{a^4 A \cos[c + d x]^4 (b + a \sec[c + d x])^4 \sin\left[\frac{1}{2} (c + d x)\right]}{6 d (a + b \cos[c + d x])^4 \left(\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right)^3} +$$

$$\frac{a^4 A \cos[c + d x]^4 (b + a \sec[c + d x])^4 \sin\left[\frac{1}{2} (c + d x)\right]}{6 d (a + b \cos[c + d x])^4 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right)^3} + \frac{(-a^4 A - 12 a^3 A b) \cos[c + d x]^4 (b + a \sec[c + d x])^4}{12 d (a + b \cos[c + d x])^4 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right)^2} +$$

$$\left(\cos[c + d x]^4 (b + a \sec[c + d x])^4 \left(2 a^4 A \sin\left[\frac{1}{2} (c + d x)\right] + 18 a^2 A b^2 \sin\left[\frac{1}{2} (c + d x)\right] + 3 a^4 C \sin\left[\frac{1}{2} (c + d x)\right]\right)\right) /$$

$$\left(3 d (a + b \cos[c + d x])^4 \left(\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right)\right) +$$

$$\left(\cos[c + d x]^4 (b + a \sec[c + d x])^4 \left(2 a^4 A \sin\left[\frac{1}{2} (c + d x)\right] + 18 a^2 A b^2 \sin\left[\frac{1}{2} (c + d x)\right] + 3 a^4 C \sin\left[\frac{1}{2} (c + d x)\right]\right)\right) /$$

$$\left(3 d (a + b \cos[c + d x])^4 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right)\right) +$$

$$\frac{4 a b^3 C \cos[c + d x]^4 (b + a \sec[c + d x])^4 \sin[c + d x]}{d (a + b \cos[c + d x])^4} + \frac{b^4 C \cos[c + d x]^4 (b + a \sec[c + d x])^4 \sin[2 (c + d x)]}{4 d (a + b \cos[c + d x])^4}$$

■ **Problem 555: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + d x])^4 (A + C \cos[c + d x]^2) \sec[c + d x]^5 dx$$

Optimal (type 3, 246 leaves, 7 steps):

$$4 a b^3 C x + \frac{(8 A b^4 + 24 a^2 b^2 (A + 2 C) + a^4 (3 A + 4 C)) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} - \frac{b^2 (2 b^2 (13 A - 12 C) + 3 a^2 (3 A + 4 C)) \operatorname{Sin}[c + d x]}{24 d} +$$

$$\frac{a b (12 A b^2 + a^2 (23 A + 36 C)) \operatorname{Tan}[c + d x]}{12 d} + \frac{(4 A b^2 + a^2 (3 A + 4 C)) (a + b \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} +$$

$$\frac{A b (a + b \operatorname{Cos}[c + d x])^3 \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d} + \frac{A (a + b \operatorname{Cos}[c + d x])^4 \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d}$$

Result (type 3, 612 leaves):

$$\frac{4 a b^3 C (c + d x)}{d} + \frac{(-3 a^4 A - 24 a^2 A b^2 - 8 A b^4 - 4 a^4 C - 48 a^2 b^2 C) \operatorname{Log}[\operatorname{Cos}[\frac{1}{2} (c + d x)] - \operatorname{Sin}[\frac{1}{2} (c + d x)]]}{8 d} +$$

$$\frac{(3 a^4 A + 24 a^2 A b^2 + 8 A b^4 + 4 a^4 C + 48 a^2 b^2 C) \operatorname{Log}[\operatorname{Cos}[\frac{1}{2} (c + d x)] + \operatorname{Sin}[\frac{1}{2} (c + d x)]]}{8 d} +$$

$$\frac{a^4 A}{16 d (\operatorname{Cos}[\frac{1}{2} (c + d x)] - \operatorname{Sin}[\frac{1}{2} (c + d x)])^4} + \frac{9 a^4 A + 16 a^3 A b + 72 a^2 A b^2 + 12 a^4 C}{48 d (\operatorname{Cos}[\frac{1}{2} (c + d x)] - \operatorname{Sin}[\frac{1}{2} (c + d x)])^2} +$$

$$\frac{2 a^3 A b \operatorname{Sin}[\frac{1}{2} (c + d x)]}{3 d (\operatorname{Cos}[\frac{1}{2} (c + d x)] - \operatorname{Sin}[\frac{1}{2} (c + d x)])^3} - \frac{a^4 A}{16 d (\operatorname{Cos}[\frac{1}{2} (c + d x)] + \operatorname{Sin}[\frac{1}{2} (c + d x)])^4} + \frac{2 a^3 A b \operatorname{Sin}[\frac{1}{2} (c + d x)]}{3 d (\operatorname{Cos}[\frac{1}{2} (c + d x)] + \operatorname{Sin}[\frac{1}{2} (c + d x)])^3} +$$

$$\frac{-9 a^4 A - 16 a^3 A b - 72 a^2 A b^2 - 12 a^4 C}{48 d (\operatorname{Cos}[\frac{1}{2} (c + d x)] + \operatorname{Sin}[\frac{1}{2} (c + d x)])^2} + \frac{4 (2 a^3 A b \operatorname{Sin}[\frac{1}{2} (c + d x)] + 3 a A b^3 \operatorname{Sin}[\frac{1}{2} (c + d x)] + 3 a^3 b C \operatorname{Sin}[\frac{1}{2} (c + d x)])}{3 d (\operatorname{Cos}[\frac{1}{2} (c + d x)] - \operatorname{Sin}[\frac{1}{2} (c + d x)])} +$$

$$\frac{4 (2 a^3 A b \operatorname{Sin}[\frac{1}{2} (c + d x)] + 3 a A b^3 \operatorname{Sin}[\frac{1}{2} (c + d x)] + 3 a^3 b C \operatorname{Sin}[\frac{1}{2} (c + d x)])}{3 d (\operatorname{Cos}[\frac{1}{2} (c + d x)] + \operatorname{Sin}[\frac{1}{2} (c + d x)])} + \frac{b^4 C \operatorname{Sin}[c + d x]}{d}$$

■ **Problem 566: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \operatorname{Cos}[c + d x]^2) \operatorname{Sec}[c + d x]}{a + b \operatorname{Cos}[c + d x]} dx$$

Optimal (type 3, 88 leaves, 4 steps):

$$\frac{C x}{b} - \frac{2 (A b^2 + a^2 C) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a+b}}\right]}{a \sqrt{a-b} b \sqrt{a+b} d} + \frac{A \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{a d}$$

Result (type 3, 234 leaves):

$$\left( 2 (A + C \cos [c + d x]) \right. \\ \left. \left( \left( a C d x - A b \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] + A b \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] \right) \sqrt{-(a^2 - b^2) (\cos [c] - i \sin [c])^2} + \right. \\ \left. 2 (A b^2 + a^2 C) \operatorname{ArcTan} \left[ \frac{(i \cos [c] + \sin [c]) (b \sin [c] + (-a + b \cos [c]) \tan \left[ \frac{d x}{2} \right])}{\sqrt{-(a^2 - b^2) (\cos [c] - i \sin [c])^2}} \right] (i \cos [c] + \sin [c]) \right) \right) / \\ \left( a b d (2 A + C + C \cos [2 (c + d x)]) \sqrt{(-a^2 + b^2) (\cos [2 c] - i \sin [2 c])} \right)$$

■ **Problem 567: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos [c + d x])^2 \operatorname{Sec} [c + d x]^2}{a + b \cos [c + d x]} dx$$

Optimal (type 3, 95 leaves, 5 steps):

$$\frac{2 (A b^2 + a^2 C) \operatorname{ArcTan} \left[ \frac{\sqrt{a-b} \tan \left[ \frac{1}{2} (c + d x) \right]}{\sqrt{a+b}} \right]}{a^2 \sqrt{a-b} \sqrt{a+b} d} - \frac{A b \operatorname{ArcTanh} [\sin [c + d x]]}{a^2 d} + \frac{A \tan [c + d x]}{a d}$$

Result (type 3, 306 leaves):

$$\left( 2 \cos [c + d x]^2 (C + A \operatorname{Sec} [c + d x])^2 \left( A b \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] - \right. \right. \\ \left. \left. A b \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] - \frac{2 i (A b^2 + a^2 C) \operatorname{ArcTan} \left[ \frac{(i \cos [c] + \sin [c]) (b \sin [c] + (-a + b \cos [c]) \tan \left[ \frac{d x}{2} \right])}{\sqrt{-(a^2 - b^2) (\cos [c] - i \sin [c])^2}} \right] (\cos [c] - i \sin [c])}{\sqrt{(-a^2 + b^2) (\cos [c] - i \sin [c])^2}} + \right. \right. \\ \left. \left. \frac{a A \sin \left[ \frac{d x}{2} \right]}{(\cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right])} + \right. \right. \\ \left. \left. \frac{a A \sin \left[ \frac{d x}{2} \right]}{(\cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right]) (\cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right])} \right) \right) / (a^2 d (2 A + C + C \cos [2 (c + d x)]))$$



- **Problem 568: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos[c + dx])^2 \sec[c + dx]^3}{a + b \cos[c + dx]} dx$$

Optimal (type 3, 137 leaves, 6 steps):

$$-\frac{2b(Ab^2 + a^2C) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^3 \sqrt{a-b} \sqrt{a+b} d} + \frac{(2Ab^2 + a^2(A+2C)) \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{2a^3 d} - \frac{Ab \operatorname{Tan}[c+dx]}{a^2 d} + \frac{A \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{2ad}$$

Result (type 3, 399 leaves):

$$\frac{1}{2a^3 d (2A + C + C \cos[2(c+dx)])} \cos[c+dx]^2 (C + A \sec[c+dx])^2$$

$$\left( -2(2Ab^2 + a^2(A+2C)) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + 2(2Ab^2 + a^2(A+2C)) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) +$$

$$\frac{8b(Ab^2 + a^2C) \operatorname{ArcTan}\left[\frac{(i \cos[c] + \sin[c]) (b \sin[c] + (-a+b \cos[c]) \operatorname{Tan}\left[\frac{dx}{2}\right])}{\sqrt{-(a^2-b^2)} (\cos[c] - i \sin[c])^2}\right] (i \cos[c] + \sin[c])}{\sqrt{(-a^2+b^2)} (\cos[c] - i \sin[c])^2} +$$

$$\frac{a^2 A}{(\cos[\frac{1}{2}(c+dx)] - \sin[\frac{1}{2}(c+dx)])^2} - \frac{4aAb \operatorname{Sin}\left[\frac{dx}{2}\right]}{(\cos[\frac{c}{2}] - \sin[\frac{c}{2}]) (\cos[\frac{1}{2}(c+dx)] - \sin[\frac{1}{2}(c+dx)])}$$

$$\left. \frac{a^2 A}{(\cos[\frac{1}{2}(c+dx)] + \sin[\frac{1}{2}(c+dx)])^2} - \frac{4aAb \operatorname{Sin}\left[\frac{dx}{2}\right]}{(\cos[\frac{c}{2}] + \sin[\frac{c}{2}]) (\cos[\frac{1}{2}(c+dx)] + \sin[\frac{1}{2}(c+dx)])} \right)$$

- **Problem 569: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos[c + dx])^2 \sec[c + dx]^4}{a + b \cos[c + dx]} dx$$

Optimal (type 3, 184 leaves, 7 steps):

$$\frac{2 b^2 (A b^2 + a^2 C) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a+b}}\right]}{a^4 \sqrt{a-b} \sqrt{a+b} d} - \frac{b (2 A b^2 + a^2 (A + 2 C)) \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{2 a^4 d} +$$

$$\frac{(3 A b^2 + a^2 (2 A + 3 C)) \operatorname{Tan}[c+d x]}{3 a^3 d} - \frac{A b \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{2 a^2 d} + \frac{A \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{3 a d}$$

Result (type 3, 413 leaves):

$$\frac{1}{12 a^4 d} \left( - \frac{24 b^2 (A b^2 + a^2 C) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}} + 6 b (2 A b^2 + a^2 (A + 2 C)) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - \right.$$

$$6 b (2 A b^2 + a^2 (A + 2 C)) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + \frac{a^2 A (a - 3 b)}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} +$$

$$\frac{2 a^3 A \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3} + \frac{4 a (3 A b^2 + a^2 (2 A + 3 C)) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]} +$$

$$\left. \frac{2 a^3 A \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3} - \frac{a^2 A (a - 3 b)}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{4 a (3 A b^2 + a^2 (2 A + 3 C)) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]} \right)$$

■ **Problem 574: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \operatorname{Cos}[c+d x]^2) \operatorname{Sec}[c+d x]}{(a + b \operatorname{Cos}[c+d x])^2} dx$$

Optimal (type 3, 134 leaves, 5 steps):

$$- \frac{2 b (2 a^2 A - A b^2 + a^2 C) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a+b}}\right]}{a^2 (a-b)^{3/2} (a+b)^{3/2} d} + \frac{A \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{a^2 d} + \frac{(A b^2 + a^2 C) \operatorname{Sin}[c+d x]}{a (a^2 - b^2) d (a + b \operatorname{Cos}[c+d x])}$$

Result (type 3, 306 leaves):

$$\left( 2 \cos [c + d x] (C \cos [c + d x] + A \sec [c + d x]) \left( -A \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] + A \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) + \right.$$

$$\frac{2 b (-A b^2 + a^2 (2 A + C)) \operatorname{ArcTan} \left[ \frac{(i \cos [c] + \sin [c]) (b \sin [c] + (-a + b \cos [c]) \tan \left[ \frac{d x}{2} \right])}{\sqrt{-(a^2 - b^2)} (\cos [c] - i \sin [c])^2} \right] (i \cos [c] + \sin [c])}{(a^2 - b^2) \sqrt{(-a^2 + b^2)} (\cos [c] - i \sin [c])^2} +$$

$$\left. \frac{a (A b^2 + a^2 C) (-a \sin [c] + b \sin [d x])}{(a - b) b (a + b) (a + b \cos [c + d x]) \left( \cos \left[ \frac{c}{2} \right] - \sin \left[ \frac{c}{2} \right] \right) \left( \cos \left[ \frac{c}{2} \right] + \sin \left[ \frac{c}{2} \right] \right)} \right) / (a^2 d (2 A + C + C \cos [2 (c + d x)]))$$

■ **Problem 576: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos [c + d x])^2 \sec [c + d x]^3}{(a + b \cos [c + d x])^2} dx$$

Optimal (type 3, 265 leaves, 7 steps):

$$-\frac{2 b (4 a^2 A b^2 - 3 A b^4 + 2 a^4 C - a^2 b^2 C) \operatorname{ArcTan} \left[ \frac{\sqrt{a-b} \tan \left[ \frac{1}{2} (c + d x) \right]}{\sqrt{a+b}} \right]}{a^4 (a-b)^{3/2} (a+b)^{3/2} d} + \frac{(6 A b^2 + a^2 (A + 2 C)) \operatorname{ArcTanh} [\sin [c + d x]]}{2 a^4 d} +$$

$$\frac{b (3 A b^2 - a^2 (2 A - C)) \tan [c + d x]}{a^3 (a^2 - b^2) d} - \frac{(3 A b^2 - a^2 (A - 2 C)) \sec [c + d x] \tan [c + d x]}{2 a^2 (a^2 - b^2) d} + \frac{(A b^2 + a^2 C) \sec [c + d x] \tan [c + d x]}{a (a^2 - b^2) d (a + b \cos [c + d x])}$$

Result (type 3, 712 leaves):

$$\begin{aligned}
& \frac{4 b \left( 4 a^2 A b^2 - 3 A b^4 + 2 a^4 C - a^2 b^2 C \right) \operatorname{ArcTanh} \left[ \frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]}{\sqrt{-a^2+b^2}} \right] \operatorname{Cos} [c+dx]^2 \left( C+A \operatorname{Sec} [c+dx]^2 \right)}{a^4 \left( a^2 - b^2 \right) \sqrt{-a^2+b^2} d \left( 2 A + C + C \operatorname{Cos} [2 c + 2 d x] \right)} + \\
& \frac{\left( -a^2 A - 6 A b^2 - 2 a^2 C \right) \operatorname{Cos} [c+dx]^2 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right] \right] \left( C+A \operatorname{Sec} [c+dx]^2 \right)}{a^4 d \left( 2 A + C + C \operatorname{Cos} [2 c + 2 d x] \right)} + \\
& \frac{\left( a^2 A + 6 A b^2 + 2 a^2 C \right) \operatorname{Cos} [c+dx]^2 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right] \right] \left( C+A \operatorname{Sec} [c+dx]^2 \right)}{a^4 d \left( 2 A + C + C \operatorname{Cos} [2 c + 2 d x] \right)} + \\
& \frac{A \operatorname{Cos} [c+dx]^2 \left( C+A \operatorname{Sec} [c+dx]^2 \right)}{2 a^2 d \left( 2 A + C + C \operatorname{Cos} [2 c + 2 d x] \right) \left( \operatorname{Cos} \left[ \frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right] \right)^2} - \frac{4 A b \operatorname{Cos} [c+dx]^2 \left( C+A \operatorname{Sec} [c+dx]^2 \right) \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right]}{a^3 d \left( 2 A + C + C \operatorname{Cos} [2 c + 2 d x] \right) \left( \operatorname{Cos} \left[ \frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right] \right)} - \\
& \frac{A \operatorname{Cos} [c+dx]^2 \left( C+A \operatorname{Sec} [c+dx]^2 \right)}{2 a^2 d \left( 2 A + C + C \operatorname{Cos} [2 c + 2 d x] \right) \left( \operatorname{Cos} \left[ \frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right] \right)^2} - \frac{4 A b \operatorname{Cos} [c+dx]^2 \left( C+A \operatorname{Sec} [c+dx]^2 \right) \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right]}{a^3 d \left( 2 A + C + C \operatorname{Cos} [2 c + 2 d x] \right) \left( \operatorname{Cos} \left[ \frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right] \right)} + \\
& \frac{2 \operatorname{Cos} [c+dx]^2 \left( C+A \operatorname{Sec} [c+dx]^2 \right) \left( A b^4 \operatorname{Sin} [c+dx] + a^2 b^2 C \operatorname{Sin} [c+dx] \right)}{a^3 (a-b) (a+b) d (a+b \operatorname{Cos} [c+dx]) \left( 2 A + C + C \operatorname{Cos} [2 c + 2 d x] \right)}
\end{aligned}$$

■ **Problem 582: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+C \operatorname{Cos} [c+dx]^2) \operatorname{Sec} [c+dx]}{(a+b \operatorname{Cos} [c+dx])^3} dx$$

Optimal (type 3, 211 leaves, 6 steps):

$$\begin{aligned}
& \frac{b \left( 5 a^2 A b^2 - 2 A b^4 - 3 a^4 (2 A + C) \right) \operatorname{ArcTan} \left[ \frac{\sqrt{a-b} \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]}{\sqrt{a+b}} \right]}{a^3 (a-b)^{5/2} (a+b)^{5/2} d} + \\
& \frac{A \operatorname{ArcTanh} [\operatorname{Sin} [c+dx]]}{a^3 d} + \frac{(A b^2 + a^2 C) \operatorname{Sin} [c+dx]}{2 a (a^2 - b^2) d (a+b \operatorname{Cos} [c+dx])^2} - \frac{(2 A b^4 - a^4 C - a^2 b^2 (5 A + 2 C)) \operatorname{Sin} [c+dx]}{2 a^2 (a^2 - b^2)^2 d (a+b \operatorname{Cos} [c+dx])}
\end{aligned}$$

Result (type 3, 409 leaves):

$$\frac{1}{2 a^3 d (2 A + C + C \cos [2 (c + d x)])}$$

$$\cos [c + d x] (C \cos [c + d x] + A \sec [c + d x]) \left( -4 A \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] + 4 A \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] \right) +$$

$$\left( 4 b (-5 a^2 A b^2 + 2 A b^4 + 3 a^4 (2 A + C)) \operatorname{ArcTan} \left[ \frac{(i \cos [c] + \sin [c]) (b \sin [c] + (-a + b \cos [c]) \tan \left[ \frac{dx}{2} \right])}{\sqrt{-(a^2 - b^2) (\cos [c] - i \sin [c])^2}} \right] (i \cos [c] + \sin [c]) \right) /$$

$$\left( (a^2 - b^2)^2 \sqrt{(-a^2 + b^2) (\cos [c] - i \sin [c])^2} \right) -$$

$$(a \sec [c] ((2 a^2 + b^2) (-2 A b^4 + a^4 C + a^2 b^2 (5 A + 2 C)) \sin [c] + b (-a (-7 A b^4 + 4 a^4 C + a^2 b^2 (16 A + 5 C)) \sin [d x] +$$

$$b (a b (-A b^2 + a^2 (4 A + 3 C)) \sin [2 c + d x] - (-2 A b^4 + a^4 C + a^2 b^2 (5 A + 2 C)) \sin [c + 2 d x])))) / (b (a^2 - b^2)^2 (a + b \cos [c + d x])^2)$$

■ **Problem 583: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos [c + d x])^2 \sec [c + d x]^2}{(a + b \cos [c + d x])^3} dx$$

Optimal (type 3, 275 leaves, 7 steps):

$$-\frac{(15 a^2 A b^4 - 6 A b^6 - 2 a^6 C - a^4 b^2 (12 A + C)) \operatorname{ArcTan} \left[ \frac{\sqrt{a-b} \tan \left[ \frac{1}{2} (c + d x) \right]}{\sqrt{a+b}} \right]}{a^4 (a-b)^{5/2} (a+b)^{5/2} d} - \frac{3 A b \operatorname{ArcTanh} [\sin [c + d x]]}{a^4 d} -$$

$$\frac{(11 a^2 A b^2 - 6 A b^4 - a^4 (2 A - 3 C)) \tan [c + d x]}{2 a^3 (a^2 - b^2)^2 d} + \frac{(A b^2 + a^2 C) \tan [c + d x]}{2 a (a^2 - b^2) d (a + b \cos [c + d x])^2} - \frac{(3 A b^4 - 2 a^4 C - a^2 b^2 (6 A + C)) \tan [c + d x]}{2 a^2 (a^2 - b^2)^2 d (a + b \cos [c + d x])}$$

Result (type 3, 649 leaves):

$$\begin{aligned}
& - \left( 2 \left( 12 a^4 A b^2 - 15 a^2 A b^4 + 6 A b^6 + 2 a^6 C + a^4 b^2 C \right) \operatorname{ArcTanh} \left[ \frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]}{\sqrt{-a^2+b^2}} \right] \operatorname{Cos}[c+dx]^2 (C+A \operatorname{Sec}[c+dx]^2) \right) / \\
& \left( a^4 (a^2-b^2)^2 \sqrt{-a^2+b^2} d (2A+C+C \operatorname{Cos}[2c+2dx]) \right) + \frac{6Ab \operatorname{Cos}[c+dx]^2 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right] \right] (C+A \operatorname{Sec}[c+dx]^2)}{a^4 d (2A+C+C \operatorname{Cos}[2c+2dx])} - \\
& \frac{6Ab \operatorname{Cos}[c+dx]^2 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right] \right] (C+A \operatorname{Sec}[c+dx]^2)}{a^4 d (2A+C+C \operatorname{Cos}[2c+2dx])} + \\
& \frac{2A \operatorname{Cos}[c+dx]^2 (C+A \operatorname{Sec}[c+dx]^2) \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right]}{a^3 d (2A+C+C \operatorname{Cos}[2c+2dx]) \left( \operatorname{Cos} \left[ \frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right] \right)} + \frac{2A \operatorname{Cos}[c+dx]^2 (C+A \operatorname{Sec}[c+dx]^2) \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right]}{a^3 d (2A+C+C \operatorname{Cos}[2c+2dx]) \left( \operatorname{Cos} \left[ \frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right] \right)} + \\
& \frac{\operatorname{Cos}[c+dx]^2 (C+A \operatorname{Sec}[c+dx]^2) (-Ab^3 \operatorname{Sin}[c+dx] - a^2 b C \operatorname{Sin}[c+dx])}{a^2 (a-b)(a+b) d (a+b \operatorname{Cos}[c+dx])^2 (2A+C+C \operatorname{Cos}[2c+2dx])} + \\
& \frac{\operatorname{Cos}[c+dx]^2 (C+A \operatorname{Sec}[c+dx]^2) (-7a^2 A b^3 \operatorname{Sin}[c+dx] + 4A b^5 \operatorname{Sin}[c+dx] - 3a^4 b C \operatorname{Sin}[c+dx])}{a^3 (a-b)^2 (a+b)^2 d (a+b \operatorname{Cos}[c+dx]) (2A+C+C \operatorname{Cos}[2c+2dx])}
\end{aligned}$$

■ **Problem 584: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+C \operatorname{Cos}[c+dx]^2) \operatorname{Sec}[c+dx]^3}{(a+b \operatorname{Cos}[c+dx])^3} dx$$

Optimal (type 3, 378 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b \left( 12 A b^6 - a^2 b^4 (29 A - 2 C) + 5 a^4 b^2 (4 A - C) + 6 a^6 C \right) \operatorname{ArcTan} \left[ \frac{\sqrt{a-b} \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]}{\sqrt{a+b}} \right]}{a^5 (a-b)^{5/2} (a+b)^{5/2} d} + \frac{(12 A b^2 + a^2 (A + 2 C)) \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{2 a^5 d} - \\
& \frac{b \left( 12 A b^4 + a^4 (6 A - 5 C) - a^2 b^2 (21 A - 2 C) \right) \operatorname{Tan}[c+dx]}{2 a^4 (a^2 - b^2)^2 d} + \frac{(6 A b^4 + a^4 (A - 4 C) - a^2 b^2 (10 A - C)) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{2 a^3 (a^2 - b^2)^2 d} + \\
& \frac{(A b^2 + a^2 C) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{2 a (a^2 - b^2) d (a+b \operatorname{Cos}[c+dx])^2} + \frac{(7 a^2 A b^2 - 4 A b^4 + 3 a^4 C) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{2 a^2 (a^2 - b^2)^2 d (a+b \operatorname{Cos}[c+dx])}
\end{aligned}$$

Result (type 3, 856 leaves):

$$\begin{aligned}
& \left( 2b \left( 20a^4Ab^2 - 29a^2Ab^4 + 12Ab^6 + 6a^6C - 5a^4b^2C + 2a^2b^4C \right) \operatorname{ArcTanh} \left[ \frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2}(c+dx) \right]}{\sqrt{-a^2+b^2}} \right] \operatorname{Cos}[c+dx]^2 (C+A \operatorname{Sec}[c+dx]^2) \right) / \\
& \left( a^5 (a^2-b^2)^2 \sqrt{-a^2+b^2} d (2A+C+C \operatorname{Cos}[2c+2dx]) \right) + \\
& \frac{(-a^2A - 12Ab^2 - 2a^2C) \operatorname{Cos}[c+dx]^2 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2}(c+dx) \right] - \operatorname{Sin} \left[ \frac{1}{2}(c+dx) \right] \right]}{a^5 d (2A+C+C \operatorname{Cos}[2c+2dx])} + \\
& \frac{(a^2A + 12Ab^2 + 2a^2C) \operatorname{Cos}[c+dx]^2 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2}(c+dx) \right] + \operatorname{Sin} \left[ \frac{1}{2}(c+dx) \right] \right]}{a^5 d (2A+C+C \operatorname{Cos}[2c+2dx])} + \\
& \frac{A \operatorname{Cos}[c+dx]^2 (C+A \operatorname{Sec}[c+dx]^2)}{2a^3 d (2A+C+C \operatorname{Cos}[2c+2dx]) (\operatorname{Cos} \left[ \frac{1}{2}(c+dx) \right] - \operatorname{Sin} \left[ \frac{1}{2}(c+dx) \right])^2} - \frac{6Ab \operatorname{Cos}[c+dx]^2 (C+A \operatorname{Sec}[c+dx]^2) \operatorname{Sin} \left[ \frac{1}{2}(c+dx) \right]}{a^4 d (2A+C+C \operatorname{Cos}[2c+2dx]) (\operatorname{Cos} \left[ \frac{1}{2}(c+dx) \right] - \operatorname{Sin} \left[ \frac{1}{2}(c+dx) \right])} - \\
& \frac{A \operatorname{Cos}[c+dx]^2 (C+A \operatorname{Sec}[c+dx]^2)}{2a^3 d (2A+C+C \operatorname{Cos}[2c+2dx]) (\operatorname{Cos} \left[ \frac{1}{2}(c+dx) \right] + \operatorname{Sin} \left[ \frac{1}{2}(c+dx) \right])^2} - \frac{6Ab \operatorname{Cos}[c+dx]^2 (C+A \operatorname{Sec}[c+dx]^2) \operatorname{Sin} \left[ \frac{1}{2}(c+dx) \right]}{a^4 d (2A+C+C \operatorname{Cos}[2c+2dx]) (\operatorname{Cos} \left[ \frac{1}{2}(c+dx) \right] + \operatorname{Sin} \left[ \frac{1}{2}(c+dx) \right])} + \\
& \frac{\operatorname{Cos}[c+dx]^2 (C+A \operatorname{Sec}[c+dx]^2) (Ab^4 \operatorname{Sin}[c+dx] + a^2b^2C \operatorname{Sin}[c+dx])}{a^3 (a-b)(a+b) d (a+b \operatorname{Cos}[c+dx])^2 (2A+C+C \operatorname{Cos}[2c+2dx])} + \\
& \frac{(\operatorname{Cos}[c+dx]^2 (C+A \operatorname{Sec}[c+dx]^2) (9a^2Ab^4 \operatorname{Sin}[c+dx] - 6Ab^6 \operatorname{Sin}[c+dx] + 5a^4b^2C \operatorname{Sin}[c+dx] - 2a^2b^4C \operatorname{Sin}[c+dx]))}{(a^4 (a-b)^2 (a+b)^2 d (a+b \operatorname{Cos}[c+dx]) (2A+C+C \operatorname{Cos}[2c+2dx]))} /
\end{aligned}$$

■ **Problem 585: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^4 (A+C \operatorname{Cos}[c+dx]^2)}{(a+b \operatorname{Cos}[c+dx])^4} dx$$

Optimal (type 3, 514 leaves, 8 steps):

$$\begin{aligned}
& \frac{(2Ab^2 + (20a^2 + b^2)C)x}{2b^6} + \frac{(8aAb^8 - a^7b^2(2A - 69C) + 7a^5b^4(A - 12C) - 8a^3b^6(A - 5C) - 20a^9C) \operatorname{ArcTan} \left[ \frac{\sqrt{a-b} \operatorname{Tan} \left[ \frac{1}{2}(c+dx) \right]}{\sqrt{a+b}} \right]}{\sqrt{a-b} b^6 \sqrt{a+b} (a^2 - b^2)^3 d} - \\
& \frac{a (a^4 b^2 (6A - 167C) - a^2 b^4 (17A - 146C) + 2b^6 (13A - 12C) + 60a^6 C) \operatorname{Sin}[c+dx]}{6b^5 (a^2 - b^2)^3 d} + \\
& \frac{(a^4 b^2 (A - 27C) - a^2 b^4 (2A - 23C) + b^6 (6A - C) + 10a^6 C) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{2b^4 (a^2 - b^2)^3 d} - \frac{(Ab^2 + a^2 C) \operatorname{Cos}[c+dx]^4 \operatorname{Sin}[c+dx]}{3b (a^2 - b^2) d (a+b \operatorname{Cos}[c+dx])^3} + \\
& \frac{(4Ab^4 - 5a^4 C + a^2 b^2 (A + 10C)) \operatorname{Cos}[c+dx]^3 \operatorname{Sin}[c+dx]}{6b^2 (a^2 - b^2)^2 d (a+b \operatorname{Cos}[c+dx])^2} - \frac{(12Ab^6 + a^4 b^2 (2A - 53C) + 20a^6 C + a^2 b^4 (A + 48C)) \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{6b^3 (a^2 - b^2)^3 d (a+b \operatorname{Cos}[c+dx])}
\end{aligned}$$

Result (type 3, 1452 leaves) :

$$\frac{1}{b^6 (a^2 - b^2)^3 \sqrt{-a^2 + b^2} d} a (2 a^6 A b^2 - 7 a^4 A b^4 + 8 a^2 A b^6 - 8 A b^8 + 20 a^8 C - 69 a^6 b^2 C + 84 a^4 b^4 C - 40 a^2 b^6 C) \operatorname{ArcTanh}\left[\frac{(a - b) \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{-a^2 + b^2}}\right] -$$

$$\frac{1}{96 b^6 (-a^2 + b^2)^3 d (a + b \operatorname{Cos}[c + d x])^3}$$

$$\begin{aligned} & (96 a^9 A b^2 (c + d x) - 144 a^7 A b^4 (c + d x) - 144 a^5 A b^6 (c + d x) + 336 a^3 A b^8 (c + d x) - 144 a A b^{10} (c + d x) + 960 a^{11} C (c + d x) - \\ & 1392 a^9 b^2 C (c + d x) - 1512 a^7 b^4 C (c + d x) + 3288 a^5 b^6 C (c + d x) - 1272 a^3 b^8 C (c + d x) - 72 a b^{10} C (c + d x) + 288 a^8 A b^3 (c + d x) \operatorname{Cos}[c + d x] - \\ & 792 a^6 A b^5 (c + d x) \operatorname{Cos}[c + d x] + 648 a^4 A b^7 (c + d x) \operatorname{Cos}[c + d x] - 72 a^2 A b^9 (c + d x) \operatorname{Cos}[c + d x] - 72 A b^{11} (c + d x) \operatorname{Cos}[c + d x] + \\ & 2880 a^{10} b C (c + d x) \operatorname{Cos}[c + d x] - 7776 a^8 b^3 C (c + d x) \operatorname{Cos}[c + d x] + 6084 a^6 b^5 C (c + d x) \operatorname{Cos}[c + d x] - \\ & 396 a^4 b^7 C (c + d x) \operatorname{Cos}[c + d x] - 756 a^2 b^9 C (c + d x) \operatorname{Cos}[c + d x] - 36 b^{11} C (c + d x) \operatorname{Cos}[c + d x] + 144 a^7 A b^4 (c + d x) \operatorname{Cos}[2 (c + d x)] - \\ & 432 a^5 A b^6 (c + d x) \operatorname{Cos}[2 (c + d x)] + 432 a^3 A b^8 (c + d x) \operatorname{Cos}[2 (c + d x)] - 144 a A b^{10} (c + d x) \operatorname{Cos}[2 (c + d x)] + \\ & 1440 a^9 b^2 C (c + d x) \operatorname{Cos}[2 (c + d x)] - 4248 a^7 b^4 C (c + d x) \operatorname{Cos}[2 (c + d x)] + 4104 a^5 b^6 C (c + d x) \operatorname{Cos}[2 (c + d x)] - \\ & 1224 a^3 b^8 C (c + d x) \operatorname{Cos}[2 (c + d x)] - 72 a b^{10} C (c + d x) \operatorname{Cos}[2 (c + d x)] + 24 a^6 A b^5 (c + d x) \operatorname{Cos}[3 (c + d x)] - \\ & 72 a^4 A b^7 (c + d x) \operatorname{Cos}[3 (c + d x)] + 72 a^2 A b^9 (c + d x) \operatorname{Cos}[3 (c + d x)] - 24 A b^{11} (c + d x) \operatorname{Cos}[3 (c + d x)] + \\ & 240 a^8 b^3 C (c + d x) \operatorname{Cos}[3 (c + d x)] - 708 a^6 b^5 C (c + d x) \operatorname{Cos}[3 (c + d x)] + 684 a^4 b^7 C (c + d x) \operatorname{Cos}[3 (c + d x)] - \\ & 204 a^2 b^9 C (c + d x) \operatorname{Cos}[3 (c + d x)] - 12 b^{11} C (c + d x) \operatorname{Cos}[3 (c + d x)] - 96 a^8 A b^3 \operatorname{Sin}[c + d x] + 228 a^6 A b^5 \operatorname{Sin}[c + d x] - \\ & 288 a^4 A b^7 \operatorname{Sin}[c + d x] - 144 a^2 A b^9 \operatorname{Sin}[c + d x] - 960 a^{10} b C \operatorname{Sin}[c + d x] + 2232 a^8 b^3 C \operatorname{Sin}[c + d x] - 1086 a^6 b^5 C \operatorname{Sin}[c + d x] - \\ & 750 a^4 b^7 C \operatorname{Sin}[c + d x] + 270 a^2 b^9 C \operatorname{Sin}[c + d x] - 6 b^{11} C \operatorname{Sin}[c + d x] - 120 a^7 A b^4 \operatorname{Sin}[2 (c + d x)] + 360 a^5 A b^6 \operatorname{Sin}[2 (c + d x)] - \\ & 480 a^3 A b^8 \operatorname{Sin}[2 (c + d x)] - 1200 a^9 b^2 C \operatorname{Sin}[2 (c + d x)] + 3300 a^7 b^4 C \operatorname{Sin}[2 (c + d x)] - 2772 a^5 b^6 C \operatorname{Sin}[2 (c + d x)] + \\ & 372 a^3 b^8 C \operatorname{Sin}[2 (c + d x)] + 60 a b^{10} C \operatorname{Sin}[2 (c + d x)] - 44 a^6 A b^5 \operatorname{Sin}[3 (c + d x)] + 128 a^4 A b^7 \operatorname{Sin}[3 (c + d x)] - \\ & 144 a^2 A b^9 \operatorname{Sin}[3 (c + d x)] - 440 a^8 b^3 C \operatorname{Sin}[3 (c + d x)] + 1253 a^6 b^5 C \operatorname{Sin}[3 (c + d x)] - 1143 a^4 b^7 C \operatorname{Sin}[3 (c + d x)] + \\ & 279 a^2 b^9 C \operatorname{Sin}[3 (c + d x)] - 9 b^{11} C \operatorname{Sin}[3 (c + d x)] - 30 a^7 b^4 C \operatorname{Sin}[4 (c + d x)] + 90 a^5 b^6 C \operatorname{Sin}[4 (c + d x)] - 90 a^3 b^8 C \operatorname{Sin}[4 (c + d x)] + \\ & 30 a b^{10} C \operatorname{Sin}[4 (c + d x)] + 3 a^6 b^5 C \operatorname{Sin}[5 (c + d x)] - 9 a^4 b^7 C \operatorname{Sin}[5 (c + d x)] + 9 a^2 b^9 C \operatorname{Sin}[5 (c + d x)] - 3 b^{11} C \operatorname{Sin}[5 (c + d x)] \end{aligned}$$

■ **Problem 586: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^3 (A + C \operatorname{Cos}[c + d x]^2)}{(a + b \operatorname{Cos}[c + d x])^4} dx$$

Optimal (type 3, 369 leaves, 7 steps) :

$$\frac{4 a C x}{b^5} - \frac{(2 A b^8 - 8 a^8 C + 28 a^6 b^2 C - 35 a^4 b^4 C + a^2 b^6 (3 A + 20 C)) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a+b}}\right]}{(a-b)^{7/2} b^5 (a+b)^{7/2} d} -$$

$$\frac{(5 A b^4 - (12 a^4 - 23 a^2 b^2 + 6 b^4) C) \operatorname{Sin}[c + d x]}{6 b^4 (a^2 - b^2)^2 d} - \frac{(A b^2 + a^2 C) \operatorname{Cos}[c + d x]^3 \operatorname{Sin}[c + d x]}{3 b (a^2 - b^2) d (a + b \operatorname{Cos}[c + d x])^3} +$$

$$\frac{(3 A b^4 - 4 a^4 C + a^2 b^2 (2 A + 9 C)) \operatorname{Cos}[c + d x]^2 \operatorname{Sin}[c + d x]}{6 b^2 (a^2 - b^2)^2 d (a + b \operatorname{Cos}[c + d x])^2} + \frac{a (2 A b^6 + 4 a^6 C - 11 a^4 b^2 C + 3 a^2 b^4 (A + 4 C)) \operatorname{Sin}[c + d x]}{2 b^4 (a^2 - b^2)^3 d (a + b \operatorname{Cos}[c + d x])}$$

Result (type 3, 893 leaves) :



$$\frac{(-3 a^2 A b^6 - 2 A b^8 + 8 a^8 C - 28 a^6 b^2 C + 35 a^4 b^4 C - 20 a^2 b^6 C) \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a^2+b^2}}\right]}{b^5 (a^2 - b^2)^3 \sqrt{-a^2 + b^2} d} -$$

$$\frac{1}{24 b^5 (-a^2 + b^2)^3 d (a + b \operatorname{Cos}[c + dx])^3} (-96 a^{10} C (c + dx) + 144 a^8 b^2 C (c + dx) + 144 a^6 b^4 C (c + dx) - 336 a^4 b^6 C (c + dx) +$$

$$144 a^2 b^8 C (c + dx) - 288 a^9 b C (c + dx) \operatorname{Cos}[c + dx] + 792 a^7 b^3 C (c + dx) \operatorname{Cos}[c + dx] - 648 a^5 b^5 C (c + dx) \operatorname{Cos}[c + dx] +$$

$$72 a^3 b^7 C (c + dx) \operatorname{Cos}[c + dx] + 72 a b^9 C (c + dx) \operatorname{Cos}[c + dx] - 144 a^8 b^2 C (c + dx) \operatorname{Cos}[2(c + dx)] + 432 a^6 b^4 C (c + dx) \operatorname{Cos}[2(c + dx)] -$$

$$432 a^4 b^6 C (c + dx) \operatorname{Cos}[2(c + dx)] + 144 a^2 b^8 C (c + dx) \operatorname{Cos}[2(c + dx)] - 24 a^7 b^3 C (c + dx) \operatorname{Cos}[3(c + dx)] +$$

$$72 a^5 b^5 C (c + dx) \operatorname{Cos}[3(c + dx)] - 72 a^3 b^7 C (c + dx) \operatorname{Cos}[3(c + dx)] + 24 a b^9 C (c + dx) \operatorname{Cos}[3(c + dx)] + 18 a^5 A b^5 \operatorname{Sin}[c + dx] +$$

$$39 a^3 A b^7 \operatorname{Sin}[c + dx] + 18 a A b^9 \operatorname{Sin}[c + dx] + 96 a^9 b C \operatorname{Sin}[c + dx] - 228 a^7 b^3 C \operatorname{Sin}[c + dx] + 135 a^5 b^5 C \operatorname{Sin}[c + dx] +$$

$$90 a^3 b^7 C \operatorname{Sin}[c + dx] - 18 a b^9 C \operatorname{Sin}[c + dx] + 6 a^4 A b^6 \operatorname{Sin}[2(c + dx)] + 54 a^2 A b^8 \operatorname{Sin}[2(c + dx)] + 120 a^8 b^2 C \operatorname{Sin}[2(c + dx)] -$$

$$336 a^6 b^4 C \operatorname{Sin}[2(c + dx)] + 300 a^4 b^6 C \operatorname{Sin}[2(c + dx)] - 18 a^2 b^8 C \operatorname{Sin}[2(c + dx)] - 6 b^{10} C \operatorname{Sin}[2(c + dx)] + 2 a^5 A b^5 \operatorname{Sin}[3(c + dx)] -$$

$$5 a^3 A b^7 \operatorname{Sin}[3(c + dx)] + 18 a A b^9 \operatorname{Sin}[3(c + dx)] + 44 a^7 b^3 C \operatorname{Sin}[3(c + dx)] - 125 a^5 b^5 C \operatorname{Sin}[3(c + dx)] + 114 a^3 b^7 C \operatorname{Sin}[3(c + dx)] -$$

$$18 a b^9 C \operatorname{Sin}[3(c + dx)] + 3 a^6 b^4 C \operatorname{Sin}[4(c + dx)] - 9 a^4 b^6 C \operatorname{Sin}[4(c + dx)] + 9 a^2 b^8 C \operatorname{Sin}[4(c + dx)] - 3 b^{10} C \operatorname{Sin}[4(c + dx)])$$

■ **Problem 587: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + dx]^2 (A + C \operatorname{Cos}[c + dx])^2}{(a + b \operatorname{Cos}[c + dx])^4} dx$$

Optimal (type 3, 304 leaves, 6 steps):

$$\frac{Cx}{b^4} + \frac{a (a^2 b^4 (A - 8 C) - 2 a^6 C + 7 a^4 b^2 C + 4 b^6 (A + 2 C)) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{(a-b)^{7/2} b^4 (a+b)^{7/2} d} - \frac{(A b^2 + a^2 C) \operatorname{Cos}[c + dx]^2 \operatorname{Sin}[c + dx]}{3 b (a^2 - b^2) d (a + b \operatorname{Cos}[c + dx])^3} -$$

$$\frac{a (2 A b^4 - 3 a^4 C + a^2 b^2 (3 A + 8 C)) \operatorname{Sin}[c + dx]}{6 b^3 (a^2 - b^2)^2 d (a + b \operatorname{Cos}[c + dx])^2} - \frac{(4 A b^6 + 9 a^6 C + 2 a^2 b^4 (7 A + 17 C) - a^4 b^2 (3 A + 28 C)) \operatorname{Sin}[c + dx]}{6 b^3 (a^2 - b^2)^3 d (a + b \operatorname{Cos}[c + dx])}$$

Result (type 3, 773 leaves):

$$\frac{a \left( -a^2 A b^4 - 4 A b^6 + 2 a^6 C - 7 a^4 b^2 C + 8 a^2 b^4 C - 8 b^6 C \right) \operatorname{ArcTanh} \left[ \frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]}{\sqrt{-a^2+b^2}} \right]}{b^4 (a^2 - b^2)^3 \sqrt{-a^2+b^2} d} +$$

$$\frac{1}{24 b^4 (-a^2 + b^2)^3 d (a + b \operatorname{Cos}[c + dx])^3} \left( -24 a^9 C (c + dx) + 36 a^7 b^2 C (c + dx) + 36 a^5 b^4 C (c + dx) - 84 a^3 b^6 C (c + dx) + 36 a b^8 C (c + dx) - \right.$$

$$72 a^8 b C (c + dx) \operatorname{Cos}[c + dx] + 198 a^6 b^3 C (c + dx) \operatorname{Cos}[c + dx] - 162 a^4 b^5 C (c + dx) \operatorname{Cos}[c + dx] + 18 a^2 b^7 C (c + dx) \operatorname{Cos}[c + dx] +$$

$$18 b^9 C (c + dx) \operatorname{Cos}[c + dx] - 36 a^7 b^2 C (c + dx) \operatorname{Cos}[2 (c + dx)] + 108 a^5 b^4 C (c + dx) \operatorname{Cos}[2 (c + dx)] - 108 a^3 b^6 C (c + dx) \operatorname{Cos}[2 (c + dx)] +$$

$$36 a b^8 C (c + dx) \operatorname{Cos}[2 (c + dx)] - 6 a^6 b^3 C (c + dx) \operatorname{Cos}[3 (c + dx)] + 18 a^4 b^5 C (c + dx) \operatorname{Cos}[3 (c + dx)] - 18 a^2 b^7 C (c + dx) \operatorname{Cos}[3 (c + dx)] +$$

$$6 b^9 C (c + dx) \operatorname{Cos}[3 (c + dx)] + 51 a^4 A b^5 \operatorname{Sin}[c + dx] + 18 a^2 A b^7 \operatorname{Sin}[c + dx] + 6 A b^9 \operatorname{Sin}[c + dx] + 24 a^8 b C \operatorname{Sin}[c + dx] -$$

$$57 a^6 b^3 C \operatorname{Sin}[c + dx] + 72 a^4 b^5 C \operatorname{Sin}[c + dx] + 36 a^2 b^7 C \operatorname{Sin}[c + dx] - 6 a^5 A b^4 \operatorname{Sin}[2 (c + dx)] + 54 a^3 A b^6 \operatorname{Sin}[2 (c + dx)] +$$

$$12 a A b^8 \operatorname{Sin}[2 (c + dx)] + 30 a^7 b^2 C \operatorname{Sin}[2 (c + dx)] - 90 a^5 b^4 C \operatorname{Sin}[2 (c + dx)] + 120 a^3 b^6 C \operatorname{Sin}[2 (c + dx)] - a^4 A b^5 \operatorname{Sin}[3 (c + dx)] +$$

$$10 a^2 A b^7 \operatorname{Sin}[3 (c + dx)] + 6 A b^9 \operatorname{Sin}[3 (c + dx)] + 11 a^6 b^3 C \operatorname{Sin}[3 (c + dx)] - 32 a^4 b^5 C \operatorname{Sin}[3 (c + dx)] + 36 a^2 b^7 C \operatorname{Sin}[3 (c + dx)] \left. \right)$$

■ **Problem 590: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \operatorname{Cos}[c + dx])^2 \operatorname{Sec}[c + dx]}{(a + b \operatorname{Cos}[c + dx])^4} dx$$

Optimal (type 3, 301 leaves, 7 steps):

$$- \frac{b \left( 7 a^2 A b^4 - 2 A b^6 - a^4 b^2 (8 A - C) + 4 a^6 (2 A + C) \right) \operatorname{ArcTan} \left[ \frac{\sqrt{a-b} \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]}{\sqrt{a+b}} \right]}{a^4 (a-b)^{7/2} (a+b)^{7/2} d} + \frac{A \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{a^4 d} +$$

$$\frac{(A b^2 + a^2 C) \operatorname{Sin}[c + dx]}{3 a (a^2 - b^2) d (a + b \operatorname{Cos}[c + dx])^3} - \frac{(3 A b^4 - 2 a^4 C - a^2 b^2 (8 A + 3 C)) \operatorname{Sin}[c + dx]}{6 a^2 (a^2 - b^2)^2 d (a + b \operatorname{Cos}[c + dx])^2} - \frac{(17 a^2 A b^4 - 6 A b^6 - 2 a^6 C - 13 a^4 b^2 (2 A + C)) \operatorname{Sin}[c + dx]}{6 a^3 (a^2 - b^2)^3 d (a + b \operatorname{Cos}[c + dx])}$$

Result (type 3, 1088 leaves):

$$\begin{aligned}
& - \frac{2 A \cos [c+d x] \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right](C \cos [c+d x]+A \sec [c+d x])}{a^4 d(2 A+C+C \cos [2 c+2 d x])} + \\
& \frac{2 A \cos [c+d x] \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right](C \cos [c+d x]+A \sec [c+d x])}{a^4 d(2 A+C+C \cos [2 c+2 d x])} + \\
& \frac{1}{\left(a^2-b^2\right)^3(2 A+C+C \cos [2 c+2 d x])} \left(8 a^6 A-8 a^4 A b^2+7 a^2 A b^4-2 A b^6+4 a^6 C+a^4 b^2 C\right) \cos [c+d x] \\
& (C \cos [c+d x]+A \sec [c+d x]) \left( \left( 2 i b \operatorname{ArcTan}\left[\sec \left[\frac{d x}{2}\right]\right] \left( \frac{\cos [c]}{\sqrt{-a^2 \cos [2 c]+b^2 \cos [2 c]+i a^2 \sin [2 c]-i b^2 \sin [2 c]}} - \right. \right. \right. \\
& \left. \left. \left. \frac{i \sin [c]}{\sqrt{-a^2 \cos [2 c]+b^2 \cos [2 c]+i a^2 \sin [2 c]-i b^2 \sin [2 c]}} \right) \left(-i a \sin \left[\frac{d x}{2}\right]+i b \sin \left[c+\frac{d x}{2}\right]\right)\right) \cos [c] \right) / \\
& \left(a^4 d \sqrt{-a^2 \cos [2 c]+b^2 \cos [2 c]+i a^2 \sin [2 c]-i b^2 \sin [2 c]}\right) + \left(2 b \operatorname{ArcTan}\left[\sec \left[\frac{d x}{2}\right]\right] \right. \\
& \left. \left( \frac{\cos [c]}{\sqrt{-a^2 \cos [2 c]+b^2 \cos [2 c]+i a^2 \sin [2 c]-i b^2 \sin [2 c]}} - \frac{i \sin [c]}{\sqrt{-a^2 \cos [2 c]+b^2 \cos [2 c]+i a^2 \sin [2 c]-i b^2 \sin [2 c]}} \right) \right. \\
& \left. \left(-i a \sin \left[\frac{d x}{2}\right]+i b \sin \left[c+\frac{d x}{2}\right]\right)\right) \sin [c] \right) / \left(a^4 d \sqrt{-a^2 \cos [2 c]+b^2 \cos [2 c]+i a^2 \sin [2 c]-i b^2 \sin [2 c]}\right) - \\
& \left(2 \cos [c+d x] \sec [c](C \cos [c+d x]+A \sec [c+d x])\left(a A b^2 \sin [c]+a^3 C \sin [c]-A b^3 \sin [d x]-a^2 b C \sin [d x]\right)\right) / \\
& \left(3 a b\left(a^2-b^2\right) d(a+b \cos [c+d x])^3(2 A+C+C \cos [2 c+2 d x])\right) + \\
& \left(\cos [c+d x] \sec [c](C \cos [c+d x]+A \sec [c+d x])\right. \\
& \left.(-6 a^3 A b \sin [c]+a A b^3 \sin [c]-5 a^3 b C \sin [c]+8 a^2 A b^2 \sin [d x]-3 A b^4 \sin [d x]+2 a^4 C \sin [d x]+3 a^2 b^2 C \sin [d x])\right) / \\
& \left(3 a^2\left(a^2-b^2\right)^2 d(a+b \cos [c+d x])^2(2 A+C+C \cos [2 c+2 d x])\right) + \\
& \left(\cos [c+d x] \sec [c](C \cos [c+d x]+A \sec [c+d x])\left(-18 a^5 A b \sin [c]+6 a^3 A b^3 \sin [c]-3 a A b^5 \sin [c]-12 a^5 b C \sin [c]-\right.\right. \\
& \left.3 a^3 b^3 C \sin [c]+26 a^4 A b^2 \sin [d x]-17 a^2 A b^4 \sin [d x]+6 A b^6 \sin [d x]+2 a^6 C \sin [d x]+13 a^4 b^2 C \sin [d x]\right) / \\
& \left.3 a^3\left(a^2-b^2\right)^3 d(a+b \cos [c+d x])(2 A+C+C \cos [2 c+2 d x])\right)
\end{aligned}$$

■ **Problem 592: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+C \cos [c+d x])^2 \sec [c+d x]^3}{(a+b \cos [c+d x])^4} d x$$

Optimal (type 3, 522 leaves, 9 steps):

$$\begin{aligned}
& \frac{(20 A b^9 - a^2 b^7 (69 A - 2 C) - 8 a^6 b^3 (5 A - C) + 7 a^4 b^5 (12 A - C) - 8 a^8 b C) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^6 \sqrt{a-b} \sqrt{a+b} (a^2 - b^2)^3 d} + \\
& \frac{(20 A b^2 + a^2 (A + 2 C)) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2 a^6 d} + \frac{b (60 A b^6 - a^6 (24 A - 26 C) + a^4 b^2 (146 A - 17 C) - a^2 b^4 (167 A - 6 C)) \operatorname{Tan}[c + dx]}{6 a^5 (a^2 - b^2)^3 d} - \\
& \frac{(10 A b^6 - a^6 (A - 6 C) + a^4 b^2 (23 A - 2 C) - a^2 b^4 (27 A - C)) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2 a^4 (a^2 - b^2)^3 d} + \frac{(A b^2 + a^2 C) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{3 a (a^2 - b^2) d (a + b \operatorname{Cos}[c + dx])^3} - \\
& \frac{(5 A b^4 - 4 a^4 C - a^2 b^2 (10 A + C)) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{6 a^2 (a^2 - b^2)^2 d (a + b \operatorname{Cos}[c + dx])^2} + \frac{(20 A b^6 - a^2 b^4 (53 A - 2 C) + 12 a^6 C + a^4 b^2 (48 A + C)) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{6 a^3 (a^2 - b^2)^3 d (a + b \operatorname{Cos}[c + dx])}
\end{aligned}$$

Result (type 3, 1065 leaves):

$$\begin{aligned}
& \left( 2 b (40 a^6 A b^2 - 84 a^4 A b^4 + 69 a^2 A b^6 - 20 A b^8 + 8 a^8 C - 8 a^6 b^2 C + 7 a^4 b^4 C - 2 a^2 b^6 C) \right. \\
& \left. \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a^2+b^2}}\right] \operatorname{Cos}[c+dx]^2 (C+A \operatorname{Sec}[c+dx]^2) \right) / \left( a^6 (a^2 - b^2)^3 \sqrt{-a^2+b^2} d (2A+C+C \operatorname{Cos}[2c+2dx]) \right) + \\
& \frac{(-a^2 A - 20 A b^2 - 2 a^2 C) \operatorname{Cos}[c+dx]^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] (C+A \operatorname{Sec}[c+dx]^2)}{a^6 d (2A+C+C \operatorname{Cos}[2c+2dx])} + \\
& \frac{(a^2 A + 20 A b^2 + 2 a^2 C) \operatorname{Cos}[c+dx]^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] (C+A \operatorname{Sec}[c+dx]^2)}{a^6 d (2A+C+C \operatorname{Cos}[2c+2dx])} + \\
& \frac{1}{48 a^5 (a^2 - b^2)^3 d (a + b \operatorname{Cos}[c + dx])^3 (2A + C + C \operatorname{Cos}[2c + 2dx])} \\
& (C + A \operatorname{Sec}[c + dx]^2) (48 a^{10} A \operatorname{Sin}[c + dx] - 396 a^8 A b^2 \operatorname{Sin}[c + dx] + 1212 a^6 A b^4 \operatorname{Sin}[c + dx] - 1000 a^4 A b^6 \operatorname{Sin}[c + dx] + 106 a^2 A b^8 \operatorname{Sin}[c + dx] + \\
& 120 A b^{10} \operatorname{Sin}[c + dx] + 144 a^8 b^2 C \operatorname{Sin}[c + dx] - 76 a^6 b^4 C \operatorname{Sin}[c + dx] + 10 a^4 b^6 C \operatorname{Sin}[c + dx] + 12 a^2 b^8 C \operatorname{Sin}[c + dx] - \\
& 120 a^9 A b \operatorname{Sin}[2(c + dx)] + 84 a^7 A b^3 \operatorname{Sin}[2(c + dx)] + 1116 a^5 A b^5 \operatorname{Sin}[2(c + dx)] - 1560 a^3 A b^7 \operatorname{Sin}[2(c + dx)] + \\
& 600 a A b^9 \operatorname{Sin}[2(c + dx)] + 240 a^7 b^3 C \operatorname{Sin}[2(c + dx)] - 180 a^5 b^5 C \operatorname{Sin}[2(c + dx)] + 60 a^3 b^7 C \operatorname{Sin}[2(c + dx)] - \\
& 252 a^8 A b^2 \operatorname{Sin}[3(c + dx)] + 1044 a^6 A b^4 \operatorname{Sin}[3(c + dx)] - 806 a^4 A b^6 \operatorname{Sin}[3(c + dx)] - 61 a^2 A b^8 \operatorname{Sin}[3(c + dx)] + \\
& 180 A b^{10} \operatorname{Sin}[3(c + dx)] + 144 a^8 b^2 C \operatorname{Sin}[3(c + dx)] - 50 a^6 b^4 C \operatorname{Sin}[3(c + dx)] - 7 a^4 b^6 C \operatorname{Sin}[3(c + dx)] + 18 a^2 b^8 C \operatorname{Sin}[3(c + dx)] - \\
& 138 a^7 A b^3 \operatorname{Sin}[4(c + dx)] + 738 a^5 A b^5 \operatorname{Sin}[4(c + dx)] - 840 a^3 A b^7 \operatorname{Sin}[4(c + dx)] + 300 a A b^9 \operatorname{Sin}[4(c + dx)] + \\
& 120 a^7 b^3 C \operatorname{Sin}[4(c + dx)] - 90 a^5 b^5 C \operatorname{Sin}[4(c + dx)] + 30 a^3 b^7 C \operatorname{Sin}[4(c + dx)] - 24 a^6 A b^4 \operatorname{Sin}[5(c + dx)] + 146 a^4 A b^6 \operatorname{Sin}[5(c + dx)] - \\
& 167 a^2 A b^8 \operatorname{Sin}[5(c + dx)] + 60 A b^{10} \operatorname{Sin}[5(c + dx)] + 26 a^6 b^4 C \operatorname{Sin}[5(c + dx)] - 17 a^4 b^6 C \operatorname{Sin}[5(c + dx)] + 6 a^2 b^8 C \operatorname{Sin}[5(c + dx)])
\end{aligned}$$

■ **Problem 599: Result more than twice size of optimal antiderivative.**

$$\int \frac{(1 - \cos[c + dx])^2 \sec[c + dx]^3}{a + b \cos[c + dx]} dx$$

Optimal (type 3, 117 leaves, 6 steps):

$$\frac{2\sqrt{a-b} b \sqrt{a+b} \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^3 d} - \frac{(a^2 - 2b^2) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2a^3 d} - \frac{b \operatorname{Tan}[c + dx]}{a^2 d} + \frac{\sec[c + dx] \operatorname{Tan}[c + dx]}{2a d}$$

Result (type 3, 236 leaves):

$$\frac{1}{4a^3 d} \left( 8b \sqrt{-a^2 + b^2} \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a^2 + b^2}}\right] + 2a^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - \right. \\ \left. 4b^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - 2a^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] + 4b^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] + \right. \\ \left. \frac{a^2}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{a^2}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - 4ab \operatorname{Tan}[c + dx] \right)$$

■ **Problem 609: Result more than twice size of optimal antiderivative.**

$$\int \frac{(1 - \cos[c + dx])^2 \sec[c + dx]^4}{(a + b \cos[c + dx])^2} dx$$

Optimal (type 3, 195 leaves, 8 steps):

$$- \frac{2b^2 (3a^2 - 4b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^5 \sqrt{a-b} \sqrt{a+b} d} + \frac{b (a^2 - 4b^2) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{a^5 d} - \\ \frac{(a^2 - 12b^2) \operatorname{Tan}[c + dx]}{3a^4 d} - \frac{2b \sec[c + dx] \operatorname{Tan}[c + dx]}{a^3 d} + \frac{4 \sec[c + dx]^2 \operatorname{Tan}[c + dx]}{3a^2 d} - \frac{\sec[c + dx]^2 \operatorname{Tan}[c + dx]}{a d (a + b \cos[c + dx])}$$

Result (type 3, 475 leaves):

$$\begin{aligned}
& \frac{2 b^2 (3 a^2 - 4 b^2) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{-a^2+b^2}}\right]}{a^5 \sqrt{-a^2+b^2} d} + \frac{(-a^2 b + 4 b^3) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{a^5 d} + \\
& \frac{(a^2 b - 4 b^3) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{a^5 d} + \frac{a - 6 b}{12 a^3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} + \\
& \frac{\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{6 a^2 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3} + \frac{\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{6 a^2 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3} + \frac{-a + 6 b}{12 a^3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} + \\
& \frac{-a^2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] + 9 b^2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{3 a^4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)} + \frac{-a^2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] + 9 b^2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{3 a^4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)} + \frac{b^3 \operatorname{Sin}[c+d x]}{a^4 d (a+b \operatorname{Cos}[c+d x])}
\end{aligned}$$

■ **Problem 626: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+b \operatorname{Cos}[c+d x]} (A+C \operatorname{Cos}[c+d x]^2) \operatorname{Sec}[c+d x] dx$$

Optimal (type 4, 231 leaves, 9 steps):

$$\begin{aligned}
& \frac{2 a C \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{3 b d \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{a+b}}} - \frac{2 \left(a^2 C - b^2 (3 A + C)\right) \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{3 b d \sqrt{a+b \operatorname{Cos}[c+d x]}} + \\
& \frac{2 a A \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{d \sqrt{a+b \operatorname{Cos}[c+d x]}} + \frac{2 C \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 d}
\end{aligned}$$

Result (type 4, 371 leaves):

$$\frac{1}{6d} \left( \frac{4b(3A+C) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \right.$$

$$\frac{2a(6A+C) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \frac{1}{b^2 \sqrt{-\frac{1}{a+b}}} 2iC \sqrt{-\frac{b(-1+\cos[c+dx])}{a+b}} \sqrt{\frac{b(1+\cos[c+dx])}{-a+b}} \operatorname{Csc}[c+dx]$$

$$\left. \left( -2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( -2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] \right) \right) + 4C \sqrt{a+b\cos[c+dx]} \operatorname{Sin}[c+dx] \right)$$

■ **Problem 627: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+b\cos[c+dx]} (A+C\cos[c+dx])^2 \operatorname{Sec}[c+dx]^2 dx$$

Optimal (type 4, 205 leaves, 9 steps):

$$-\frac{(A-2C) \sqrt{a+b\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{d \sqrt{\frac{a+b\cos[c+dx]}{a+b}}} + \frac{aA \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{d \sqrt{a+b\cos[c+dx]}}$$

$$+\frac{Ab \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{d \sqrt{a+b\cos[c+dx]}} + \frac{A \sqrt{a+b\cos[c+dx]} \operatorname{Tan}[c+dx]}{d}$$

Result (type 4, 374 leaves):

$$\frac{1}{4d} \left( \frac{8aC \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b \cos[c+dx]}} + \frac{2b(A+2C) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b \cos[c+dx]}} - \right.$$

$$\left. \frac{1}{ab \sqrt{-\frac{1}{a+b}}} 2i(A-2C) \sqrt{-\frac{b(-1+\cos[c+dx])}{a+b}} \sqrt{\frac{b(1+\cos[c+dx])}{-a+b}} \operatorname{Csc}[c+dx] \right.$$

$$\left. \left( -2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( -2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] \right) \right) + 4A \sqrt{a+b \cos[c+dx]} \operatorname{Tan}[c+dx] \right)$$

■ **Problem 628: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+b \cos[c+dx]} (A+C \cos[c+dx])^2 \operatorname{Sec}[c+dx]^3 dx$$

Optimal (type 4, 277 leaves, 10 steps):

$$\frac{Ab \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{4ad \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} +$$

$$\frac{b(3A+8C) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right] - (Ab^2 - 4a^2(A+2C)) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{4d \sqrt{a+b \cos[c+dx]}} +$$

$$\frac{Ab \sqrt{a+b \cos[c+dx]} \operatorname{Tan}[c+dx]}{4ad} + \frac{A \sqrt{a+b \cos[c+dx]} \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{2d}$$

Result (type 4, 535 leaves):



$$\begin{aligned}
& \frac{1}{16 a d} \\
& \left( \frac{2 (4 a A b + 16 a b C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{2 (8 a^2 A - 3 A b^2 + 16 a^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} \right) + \\
& \left( 2 i A b^2 \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \Bigg/ \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \right. \\
& \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) + \\
& \frac{\sqrt{a+b \cos [c+d x]} \left( \frac{A b \tan [c+d x]}{4 a} + \frac{1}{2} A \sec [c+d x] \tan [c+d x] \right)}{d}
\end{aligned}$$

- **Problem 629: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+b \cos [c+d x]} (A+C \cos [c+d x]^2) \sec [c+d x]^4 dx$$

Optimal (type 4, 365 leaves, 11 steps):

$$\begin{aligned}
& \frac{(3 A b^2 - 8 a^2 (2 A + 3 C)) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right] - (A b^2 - 8 a^2 (2 A + 3 C)) \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{24 a^2 d \sqrt{\frac{a + b \cos[c + d x]}{a + b}}} - \frac{24 a d \sqrt{a + b \cos[c + d x]}}{24 a^2 d} + \\
& \frac{b (A b^2 + 4 a^2 (A + 2 C)) \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2 b}{a + b}\right] - (3 A b^2 - 8 a^2 (2 A + 3 C)) \sqrt{a + b \cos[c + d x]} \operatorname{Tan}[c + d x]}{8 a^2 d \sqrt{a + b \cos[c + d x]}} - \frac{24 a^2 d}{24 a^2 d} + \\
& \frac{A b \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{12 a d} + \frac{A \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d}
\end{aligned}$$

Result (type 4, 601 leaves):

$$\begin{aligned}
& -\frac{1}{96 a^2 d} b \left( -\frac{8 a A b \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{2(-8 a^2 A-9 A b^2-24 a^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} \right. \\
& \left( 2 i(16 a^2 A-3 A b^2+24 a^2 C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left( 2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \Big/ \\
& \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} \right. \\
& \left. \left. \left. \left( 2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2 \right) \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{\sec [c+d x] \left( 16 a^2 A \sin [c+d x]-3 A b^2 \sin [c+d x]+24 a^2 C \sin [c+d x] \right)}{24 a^2} + \right. \\
& \left. \frac{A b \sec [c+d x] \tan [c+d x]}{12 a} + \right. \\
& \left. \frac{1}{3} A \sec [c+d x]^2 \tan [c+d x] \right)
\end{aligned}$$

■ **Problem 633: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos [c+d x])^{3/2} (A+C \cos [c+d x])^2 \sec [c+d x] dx$$

Optimal (type 4, 281 leaves, 10 steps):

$$\frac{2(a^2 C + b^2(5A + 3C))\sqrt{a + b\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] + \frac{2a(5Ab^2 - (a^2 - b^2)c)\sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{5bd\sqrt{\frac{a+b\cos[c+dx]}{a+b}}} + \frac{2a^2 A \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{d\sqrt{a + b\cos[c + dx]}} + \frac{2aC\sqrt{a + b\cos[c + dx]} \sin[c + dx]}{5d} + \frac{2C(a + b\cos[c + dx])^{3/2} \sin[c + dx]}{5d}$$

Result (type 4, 421 leaves):

$$\frac{1}{10d} \left( \frac{8ab(5A + 2C)\sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b\cos[c + dx]}} + \frac{2(a^2(10A + C) + b^2(5A + 3C))\sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b\cos[c + dx]}} \right) + \frac{1}{ab^2\sqrt{-\frac{1}{a+b}}} 2i(5Ab^2 + (a^2 + 3b^2)c) \sqrt{-\frac{b(-1 + \cos[c + dx])}{a+b}} \sqrt{\frac{b(1 + \cos[c + dx])}{-a+b}} \operatorname{Csc}[c + dx] \left( -2a(a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b\cos[c + dx]}\right], \frac{a+b}{a-b}\right] + b \left( -2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b\cos[c + dx]}\right], \frac{a+b}{a-b}\right] + b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b\cos[c + dx]}\right], \frac{a+b}{a-b}\right] \right) \right) + 4C\sqrt{a + b\cos[c + dx]}(2a + b\cos[c + dx])\sin[c + dx]$$

■ **Problem 634: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + b\cos[c + dx])^{3/2} (A + C\cos[c + dx])^2 \operatorname{Sec}[c + dx]^2 dx$$

Optimal (type 4, 270 leaves, 10 steps):

$$\begin{aligned}
& - \frac{a(3A-8C)\sqrt{a+b\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right] + (a^2(3A-2C)+2b^2(3A+C))\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{3d\sqrt{\frac{a+b\cos[c+dx]}{a+b}}} + \frac{(a^2(3A-2C)+2b^2(3A+C))\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{3d\sqrt{a+b\cos[c+dx]}} + \\
& \frac{3aAb\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{d\sqrt{a+b\cos[c+dx]}} - \frac{b(3A-2C)\sqrt{a+b\cos[c+dx]}\sin[c+dx]}{3d} + \frac{A(a+b\cos[c+dx])^{3/2}\tan[c+dx]}{d}
\end{aligned}$$

Result (type 4, 528 leaves):

$$\begin{aligned}
& \frac{1}{12d} \\
& \left( \frac{2(12Ab^2+12a^2C+4b^2C)\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right] + 2(15aAb+8abC)\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \frac{2(15aAb+8abC)\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} \right) - \\
& \left( 2i(-3aAb+8abC)\sqrt{\frac{b-b\cos[c+dx]}{a+b}}\sqrt{\frac{b+b\cos[c+dx]}{a-b}}\cos[2(c+dx)] \right. \\
& \left. \left( 2a(a-b)\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b\left( 2a\operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right) - b\operatorname{EllipticPi}\left[\frac{a+b}{a}, i\operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] \right) \right) \sin[c+dx] \Big/ \left( a\sqrt{-\frac{1}{a+b}}\sqrt{1-\cos[c+dx]^2} \right. \\
& \left. \left. \sqrt{-\frac{a^2-b^2-2a(a+b\cos[c+dx])+(a+b\cos[c+dx])^2}{b^2}}(2a^2-b^2-4a(a+b\cos[c+dx])+2(a+b\cos[c+dx])^2) \right) \right) + \\
& \frac{\sqrt{a+b\cos[c+dx]}\left(\frac{2}{3}bC\sin[c+dx]+aA\tan[c+dx]\right)}{d}
\end{aligned}$$

■ **Problem 635: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b\cos[c+dx])^{3/2}(A+C\cos[c+dx])^2\sec[c+dx]^3dx$$

Optimal (type 4, 276 leaves, 10 steps):

$$\begin{aligned}
 & - \frac{b (5 A - 8 C) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{4 d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} + \\
 & \frac{a b (7 A + 8 C) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right] + (3 A b^2 + 4 a^2 (A + 2 C)) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{4 d \sqrt{a + b \cos [c + d x]} + 4 d \sqrt{a + b \cos [c + d x]}} + \\
 & \frac{3 A b \sqrt{a + b \cos [c + d x]} \tan [c + d x]}{4 d} + \frac{A (a + b \cos [c + d x])^{3/2} \sec [c + d x] \tan [c + d x]}{2 d}
 \end{aligned}$$

Result (type 4, 544 leaves):

$$\begin{aligned}
& \frac{1}{16d} \left( \frac{2(4Ab + 32abc) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \right. \\
& \frac{2(8a^2A + Ab^2 + 16a^2C + 8b^2C) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} - \\
& \left. \left( 2i(-5Ab^2 + 8b^2C) \sqrt{\frac{b-b\cos[c+dx]}{a+b}} \sqrt{-\frac{b+b\cos[c+dx]}{a-b}} \cos[2(c+dx)] \right. \right. \\
& \left. \left. \left( 2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \right. \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin[c+dx] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]^2} \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2a(a+b\cos[c+dx])+(a+b\cos[c+dx])^2}{b^2}} (2a^2-b^2-4a(a+b\cos[c+dx])+2(a+b\cos[c+dx])^2) \right) \right) \right) + \\
& \frac{\sqrt{a+b\cos[c+dx]} \left( \frac{5}{4}Ab \tan[c+dx] + \frac{1}{2}aA \sec[c+dx] \tan[c+dx] \right)}{d}
\end{aligned}$$

■ **Problem 636: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b\cos[c+dx])^{3/2} (A+C\cos[c+dx]^2) \sec[c+dx]^4 dx$$

Optimal (type 4, 365 leaves, 11 steps):

$$\begin{aligned}
& - \frac{(3 A b^2 + 8 a^2 (2 A + 3 C)) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{24 a d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} + \\
& \frac{(8 a^2 (2 A + 3 C) + b^2 (17 A + 48 C)) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{24 d \sqrt{a + b \cos [c + d x]}} - \\
& \frac{b (A b^2 - 12 a^2 (A + 2 C)) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{8 a d \sqrt{a + b \cos [c + d x]}} + \frac{(3 A b^2 + 8 a^2 (2 A + 3 C)) \sqrt{a + b \cos [c + d x]} \tan [c + d x]}{24 a d} + \\
& \frac{A b \sqrt{a + b \cos [c + d x]} \sec [c + d x] \tan [c + d x]}{4 d} + \frac{A (a + b \cos [c + d x])^{3/2} \sec [c + d x]^2 \tan [c + d x]}{3 d}
\end{aligned}$$

Result (type 4, 607 leaves):



$$\begin{aligned}
& -\frac{1}{96ad}b \left( \frac{2(-28Ab - 96abc) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \right. \\
& \frac{2(-56a^2A + 9Ab^2 - 120a^2C) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} - \\
& \left( 2i(16a^2A + 3Ab^2 + 24a^2C) \sqrt{\frac{b-b\cos[c+dx]}{a+b}} \sqrt{-\frac{b+b\cos[c+dx]}{a-b}} \cos[2(c+dx)] \right. \\
& \left. \left( 2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin[c+dx] \Big/ \\
& \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]} \sqrt{-\frac{a^2-b^2-2a(a+b\cos[c+dx])+(a+b\cos[c+dx])^2}{b^2}} \right. \\
& \left. \left. \left. \left( 2a^2-b^2-4a(a+b\cos[c+dx]) + 2(a+b\cos[c+dx])^2 \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b\cos[c+dx]} \left( \frac{\sec[c+dx] (16a^2A \sin[c+dx] + 3Ab^2 \sin[c+dx] + 24a^2C \sin[c+dx])}{24a} + \right. \\
& \left. \frac{7}{12} Ab \sec[c+dx] \tan[c+dx] + \right. \\
& \left. \frac{1}{3} aA \sec[c+dx]^2 \tan[c+dx] \right)
\end{aligned}$$

■ **Problem 637: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b\cos[c+dx])^{3/2} (A+C\cos[c+dx]^2) \sec[c+dx]^5 dx$$

Optimal (type 4, 436 leaves, 12 steps):

$$\begin{aligned}
 & \frac{b \left( 3 A b^2 - 4 a^2 (13 A + 20 C) \right) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), \frac{2 b}{a + b} \right]}{64 a^2 d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} - \\
 & \frac{b \left( A b^2 - 4 a^2 (19 A + 28 C) \right) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), \frac{2 b}{a + b} \right]}{64 a d \sqrt{a + b \cos [c + d x]}} + \\
 & \frac{\left( 3 A b^4 + 24 a^2 b^2 (A + 2 C) + 16 a^4 (3 A + 4 C) \right) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi} \left[ 2, \frac{1}{2} (c + d x), \frac{2 b}{a + b} \right]}{64 a^2 d \sqrt{a + b \cos [c + d x]}} - \\
 & \frac{b \left( 3 A b^2 - 4 a^2 (13 A + 20 C) \right) \sqrt{a + b \cos [c + d x]} \tan [c + d x]}{64 a^2 d} + \frac{\left( A b^2 + 4 a^2 (3 A + 4 C) \right) \sqrt{a + b \cos [c + d x]} \sec [c + d x] \tan [c + d x]}{32 a d} + \\
 & \frac{A b \sqrt{a + b \cos [c + d x]} \sec [c + d x]^2 \tan [c + d x]}{8 d} + \frac{A (a + b \cos [c + d x])^{3/2} \sec [c + d x]^3 \tan [c + d x]}{4 d}
 \end{aligned}$$

Result (type 4, 696 leaves):

$$\begin{aligned}
& \frac{1}{256 a^2 d} \left( \frac{2 (48 a^3 A b + 4 a A b^3 + 64 a^3 b C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2 (96 a^4 A - 4 a^2 A b^2 + 9 A b^4 + 128 a^4 C + 16 a^2 b^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\
& \left( 2 i (-52 a^2 A b^2 + 3 A b^4 - 80 a^2 b^2 C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{\sec [c+d x]^2 (12 a^2 A \sin [c+d x]+A b^2 \sin [c+d x]+16 a^2 C \sin [c+d x])}{32 a} + \right. \\
& \frac{\sec [c+d x] (52 a^2 A b \sin [c+d x]-3 A b^3 \sin [c+d x]+80 a^2 b C \sin [c+d x])}{64 a^2} + \\
& \left. \left. \frac{3}{8} A b \sec [c+d x]^2 \tan [c+d x] + \frac{1}{4} a A \sec [c+d x]^3 \tan [c+d x] \right) \right)
\end{aligned}$$

■ **Problem 641: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos [c+d x])^{5/2} (A+C \cos [c+d x]^2) \sec [c+d x] dx$$

Optimal (type 4, 342 leaves, 11 steps):

$$\begin{aligned}
& \frac{2 a (49 A b^2 + 3 a^2 C + 29 b^2 C) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right] + 21 b d \sqrt{\frac{a + b \cos[c + d x]}{a + b}}}{21 b d \sqrt{a + b \cos[c + d x]}} \\
& + \frac{2 (2 a^2 b^2 (7 A - C) - 3 a^4 C + b^4 (7 A + 5 C)) \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right] + 2 a^3 A \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{21 b d \sqrt{a + b \cos[c + d x]} + d \sqrt{a + b \cos[c + d x]}} \\
& + \frac{2 (3 a^2 C + b^2 (7 A + 5 C)) \sqrt{a + b \cos[c + d x]} \sin[c + d x]}{21 d} + \frac{2 a C (a + b \cos[c + d x])^{3/2} \sin[c + d x]}{7 d} + \frac{2 C (a + b \cos[c + d x])^{5/2} \sin[c + d x]}{7 d}
\end{aligned}$$

Result (type 4, 468 leaves):

$$\begin{aligned}
& \frac{1}{42 d} \left( \frac{4 b (9 a^2 (7 A + 3 C) + b^2 (7 A + 5 C)) \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{\sqrt{a + b \cos[c + d x]}} + \right. \\
& \frac{2 a (3 a^2 (14 A + C) + b^2 (49 A + 29 C)) \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{\sqrt{a + b \cos[c + d x]}} + \frac{1}{b^2 \sqrt{-\frac{1}{a + b}}} \\
& 2 i (49 A b^2 + 3 a^2 C + 29 b^2 C) \sqrt{-\frac{b (-1 + \cos[c + d x])}{a + b}} \sqrt{\frac{b (1 + \cos[c + d x])}{-a + b}} \operatorname{Csc}[c + d x] \\
& \left. \left( -2 a (a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos[c + d x]}\right], \frac{a + b}{a - b}\right] + b \left( -2 a \operatorname{EllipticF}\left[ \right. \right. \right. \\
& \left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos[c + d x]}\right], \frac{a + b}{a - b}\right] + b \operatorname{EllipticPi}\left[\frac{a + b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos[c + d x]}\right], \frac{a + b}{a - b}\right]\right) \right) + \right. \\
& \left. 2 \sqrt{a + b \cos[c + d x]} (14 A b^2 + 18 a^2 C + 13 b^2 C + 18 a b C \cos[c + d x] + 3 b^2 C \cos[2(c + d x)]) \sin[c + d x] \right)
\end{aligned}$$

■ **Problem 642: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + b \cos [c + d x])^{5/2} (A + C \cos [c + d x]^2) \sec [c + d x]^2 dx$$

Optimal (type 4, 327 leaves, 11 steps):

$$\begin{aligned} & - \frac{(a^2 (15 A - 46 C) - 6 b^2 (5 A + 3 C)) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2b}{a+b}\right]}{15 d \sqrt{\frac{a+b \cos [c+d x]}{a+b}}} + \\ & \frac{a (a^2 (15 A - 16 C) + 4 b^2 (15 A + 4 C)) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2b}{a+b}\right] + 5 a^2 A b \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2b}{a+b}\right]}{15 d \sqrt{a + b \cos [c + d x]} + d \sqrt{a + b \cos [c + d x]}} - \\ & \frac{a b (15 A - 16 C) \sqrt{a + b \cos [c + d x]} \sin [c + d x]}{15 d} - \frac{b (5 A - 2 C) (a + b \cos [c + d x])^{3/2} \sin [c + d x]}{5 d} + \frac{A (a + b \cos [c + d x])^{5/2} \tan [c + d x]}{d} \end{aligned}$$

Result (type 4, 581 leaves):

$$\begin{aligned}
& \frac{1}{60 d} \left( \frac{2 (180 a A b^2 + 60 a^3 C + 68 a b^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2 (135 a^2 A b + 30 A b^3 + 46 a^2 b C + 18 b^3 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\
& \left. \frac{2 i (-15 a^2 A b + 30 A b^3 + 46 a^2 b C + 18 b^3 C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)]}{\sqrt{a+b \cos [c+d x]}} \right) \\
& \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \Big/ \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) + \\
& \frac{\sqrt{a+b \cos [c+d x]} \left( \frac{22}{15} a b C \sin [c+d x] + \frac{1}{5} b^2 C \sin [2(c+d x)] + a^2 A \tan [c+d x] \right)}{d}
\end{aligned}$$

■ **Problem 643: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos [c+d x])^{5/2} (A+C \cos [c+d x]^2) \sec [c+d x]^3 dx$$

Optimal (type 4, 329 leaves, 11 steps):

$$\begin{aligned}
& - \frac{a b (27 A - 56 C) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right] + b (8 b^2 (3 A + C) + a^2 (33 A + 16 C)) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{12 d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}} + \frac{b^2 (21 A - 8 C) \sqrt{a + b \cos[c + d x]} \sin[c + d x]}{12 d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{a (15 A b^2 + 4 a^2 (A + 2 C)) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{4 d \sqrt{a + b \cos[c + d x]}} - \frac{b^2 (21 A - 8 C) \sqrt{a + b \cos[c + d x]} \sin[c + d x]}{12 d} + \\
& \frac{5 A b (a + b \cos[c + d x])^{3/2} \tan[c + d x]}{4 d} + \frac{A (a + b \cos[c + d x])^{5/2} \sec[c + d x] \tan[c + d x]}{2 d}
\end{aligned}$$

Result (type 4, 582 leaves):

$$\begin{aligned}
& \frac{1}{48d} \left( \frac{2(12a^2Ab + 48Ab^3 + 144a^2bC + 16b^3C) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \right. \\
& \frac{2(24a^3A + 63aAb^2 + 48a^3C + 56ab^2C) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} - \\
& \left. \left( 2i(-27aAb^2 + 56ab^2C) \sqrt{\frac{b-b\cos[c+dx]}{a+b}} \sqrt{\frac{b+b\cos[c+dx]}{a-b}} \cos[2(c+dx)] \right. \right. \\
& \left. \left. \left( 2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \right. \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin[c+dx] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]^2} \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2a(a+b\cos[c+dx])+(a+b\cos[c+dx])^2}{b^2}} (2a^2-b^2-4a(a+b\cos[c+dx])+2(a+b\cos[c+dx])^2) \right) \right) \right) + \\
& \frac{\sqrt{a+b\cos[c+dx]} \left( \frac{2}{3}b^2C \sin[c+dx] + \frac{9}{4}aAb \tan[c+dx] + \frac{1}{2}a^2A \sec[c+dx] \tan[c+dx] \right)}{d}
\end{aligned}$$

■ **Problem 644: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b\cos[c+dx])^{5/2} (A+C\cos[c+dx]^2) \sec[c+dx]^4 dx$$

Optimal (type 4, 363 leaves, 11 steps):



$$\begin{aligned}
& - \frac{(3b^2(11A - 16C) + 8a^2(2A + 3C)) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{24d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \\
& \frac{a(8a^2(2A + 3C) + b^2(59A + 96C)) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{24d \sqrt{a + b \cos[c + dx]}} + \\
& \frac{5b(Ab^2 + 4a^2(A + 2C)) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{8d \sqrt{a + b \cos[c + dx]}} + \frac{(15Ab^2 + 8a^2(2A + 3C)) \sqrt{a + b \cos[c + dx]} \tan[c + dx]}{24d} + \\
& \frac{5Ab(a + b \cos[c + dx])^{3/2} \sec[c + dx] \tan[c + dx]}{12d} + \frac{A(a + b \cos[c + dx])^{5/2} \sec[c + dx]^2 \tan[c + dx]}{3d}
\end{aligned}$$

Result (type 4, 616 leaves):

$$\begin{aligned}
& -\frac{1}{96d}b \left( \frac{2(-52aAb - 288abC) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \right. \\
& \frac{2(-104a^2A + 3Ab^2 - 216a^2C - 48b^2C) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} - \\
& \left. \left( 2i(16a^2A + 33Ab^2 + 24a^2C - 48b^2C) \sqrt{\frac{b-b\cos[c+dx]}{a+b}} \sqrt{\frac{b+b\cos[c+dx]}{a-b}} \cos[2(c+dx)] \right. \right. \\
& \left. \left. \left( 2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin[c+dx] \right) / \\
& \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]} \sqrt{-\frac{a^2-b^2-2a(a+b\cos[c+dx])+(a+b\cos[c+dx])^2}{b^2}} \right. \\
& \left. \left. \left. \left( 2a^2-b^2-4a(a+b\cos[c+dx])+2(a+b\cos[c+dx])^2 \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b\cos[c+dx]} \left( \frac{1}{24} \sec[c+dx] (16a^2A \sin[c+dx] + 33Ab^2 \sin[c+dx] + 24a^2C \sin[c+dx]) + \right. \\
& \frac{13}{12} aAb \sec[c+dx] \tan[c+dx] + \\
& \left. \frac{1}{3} a^2A \sec[c+dx]^2 \tan[c+dx] \right)
\end{aligned}$$

■ **Problem 645: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b\cos[c+dx])^{5/2} (A+C\cos[c+dx]^2) \sec[c+dx]^5 dx$$

Optimal (type 4, 437 leaves, 12 steps):

$$\begin{aligned}
 & - \frac{b (15 A b^2 + 4 a^2 (71 A + 108 C)) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2b}{a+b}\right]}{192 a d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}} + \\
 & \frac{b (4 a^2 (89 A + 132 C) + b^2 (133 A + 384 C)) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2b}{a+b}\right]}{192 d \sqrt{a + b \cos[c + d x]}} - \\
 & \frac{(5 A b^4 - 120 a^2 b^2 (A + 2 C) - 16 a^4 (3 A + 4 C)) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2b}{a+b}\right]}{64 a d \sqrt{a + b \cos[c + d x]}} + \\
 & \frac{b (15 A b^2 + 4 a^2 (71 A + 108 C)) \sqrt{a + b \cos[c + d x]} \tan[c + d x]}{192 a d} + \frac{(5 A b^2 + 4 a^2 (3 A + 4 C)) \sqrt{a + b \cos[c + d x]} \sec[c + d x] \tan[c + d x]}{32 d} + \\
 & \frac{5 A b (a + b \cos[c + d x])^{3/2} \sec[c + d x]^2 \tan[c + d x]}{24 d} + \frac{A (a + b \cos[c + d x])^{5/2} \sec[c + d x]^3 \tan[c + d x]}{4 d}
 \end{aligned}$$

Result (type 4, 704 leaves):

$$\begin{aligned}
& \frac{1}{768 a d} \left( \frac{2 (144 a^3 A b + 236 a A b^3 + 192 a^3 b C + 768 a b^3 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{1}{\sqrt{a+b \cos [c+d x]}} \right. \\
& 2 (288 a^4 A + 436 a^2 A b^2 - 45 A b^4 + 384 a^4 C + 1008 a^2 b^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] - \\
& \left. \left( 2 i (-284 a^2 A b^2 - 15 A b^4 - 432 a^2 b^2 C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \right. \\
& \left. \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right)\right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{1}{96} \sec [c+d x]^2 (36 a^2 A \sin [c+d x] + 59 A b^2 \sin [c+d x] + 48 a^2 C \sin [c+d x]) + \right. \\
& \left. \frac{\sec [c+d x] (284 a^2 A b \sin [c+d x] + 15 A b^3 \sin [c+d x] + 432 a^2 b C \sin [c+d x])}{192 a} + \right. \\
& \left. \left. \frac{17}{24} a A b \sec [c+d x]^2 \tan [c+d x] + \frac{1}{4} a^2 A \sec [c+d x]^3 \tan [c+d x] \right) \right)
\end{aligned}$$

■ **Problem 652: Unable to integrate problem.**

$$\int \frac{(A + C \cos [c + d x]^2) \sec [c + d x]}{\sqrt{a + b \cos [c + d x]}} dx$$

Optimal (type 4, 183 leaves, 8 steps):

$$\frac{2 C \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{b d \sqrt{\frac{a+b \cos [c+d x]}{a+b}}}$$

$$\frac{2 a C \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{b d \sqrt{a+b \cos [c+d x]}} + \frac{2 A \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{d \sqrt{a+b \cos [c+d x]}}$$

Result (type 8, 35 leaves):

$$\int \frac{(A+C \cos [c+d x]^2) \sec [c+d x]}{\sqrt{a+b \cos [c+d x]}} dx$$

- **Problem 653: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A+C \cos [c+d x]^2) \sec [c+d x]^2}{\sqrt{a+b \cos [c+d x]}} dx$$

Optimal (type 4, 214 leaves, 9 steps):

$$\frac{A \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{a d \sqrt{\frac{a+b \cos [c+d x]}{a+b}}} + \frac{(A+2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{d \sqrt{a+b \cos [c+d x]}}$$

$$\frac{A b \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{a d \sqrt{a+b \cos [c+d x]}} + \frac{A \sqrt{a+b \cos [c+d x]} \tan [c+d x]}{a d}$$

Result (type 4, 559 leaves):

$$\begin{aligned}
& \frac{2 A \cos [c+d x] \sqrt{a+b \cos [c+d x]} (C+A \sec [c+d x])^2 \sin [c+d x]}{a d (2 A+C+C \cos [2 c+2 d x])} + \\
& \frac{1}{2 a d (2 A+C+C \cos [2 c+2 d x])} \cos [c+d x]^2 (C+A \sec [c+d x])^2 \left( \frac{8 a c \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \right. \\
& \frac{6 A b \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \left( 2 i A b \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \right. \\
& \left. \left. \left. \sqrt{1-\cos [c+d x]}^2 \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) \right)
\end{aligned}$$

- **Problem 654: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A+C \cos [c+d x])^2 \sec [c+d x]^3}{\sqrt{a+b \cos [c+d x]}} dx$$

Optimal (type 4, 278 leaves, 10 steps):

$$\frac{3 A b \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{4 a^2 d \sqrt{\frac{a+b \cos [c+d x]}{a+b}}}$$

$$\frac{A b \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] + (3 A b^2 + 4 a^2 (A+2 C)) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{4 a d \sqrt{a+b \cos [c+d x]} + 4 a^2 d \sqrt{a+b \cos [c+d x]}}$$

$$\frac{3 A b \sqrt{a+b \cos [c+d x]} \tan [c+d x]}{4 a^2 d} + \frac{A \sqrt{a+b \cos [c+d x]} \operatorname{Sec}[c+d x] \tan [c+d x]}{2 a d}$$

Result (type 4, 603 leaves):

$$\frac{1}{8 a^2 d (2 A+C+C \cos [2 c+2 d x])} \cos [c+d x]^2 (C+A \operatorname{Sec}[c+d x])^2$$

$$\left( \frac{8 a A b \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{2 (8 a^2 A+9 A b^2+16 a^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} \right)$$

$$\left( 6 i A b^2 \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \left( 2 a (a-b) \right. \right.$$

$$\left. \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] - \right. \right.$$

$$\left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \left/ \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \right. \right.$$

$$\left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) +$$

$$\frac{\cos [c+d x]^2 \sqrt{a+b \cos [c+d x]} (C+A \operatorname{Sec}[c+d x])^2 \left( -\frac{3 A b \tan [c+d x]}{2 a^2} + \frac{A \operatorname{Sec}[c+d x] \tan [c+d x]}{a} \right)}{d (2 A+C+C \cos [2 c+2 d x])}$$

■ **Problem 655: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + C \cos[c + dx])^2 \sec[c + dx]^4}{\sqrt{a + b \cos[c + dx]}} dx$$

Optimal (type 4, 370 leaves, 11 steps):

$$\begin{aligned}
 & - \frac{(15 A b^2 + 8 a^2 (2 A + 3 C)) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] + (5 A b^2 + 8 a^2 (2 A + 3 C)) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{24 a^3 d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \frac{(5 A b^2 + 8 a^2 (2 A + 3 C)) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{24 a^2 d \sqrt{a + b \cos[c + dx]}} \\
 & + \frac{b (5 A b^2 + 4 a^2 (A + 2 C)) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{8 a^3 d \sqrt{a + b \cos[c + dx]}} + \frac{(15 A b^2 + 8 a^2 (2 A + 3 C)) \sqrt{a + b \cos[c + dx]} \operatorname{Tan}[c + dx]}{24 a^3 d} \\
 & + \frac{5 A b \sqrt{a + b \cos[c + dx]} \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{12 a^2 d} + \frac{A \sqrt{a + b \cos[c + dx]} \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{3 a d}
 \end{aligned}$$

Result (type 4, 604 leaves):



$$\begin{aligned}
& -\frac{1}{96 a^3 d} b \left( \frac{40 a A b \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{2\left(40 a^2 A+45 A b^2+72 a^2 C\right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} \right. \\
& \left. \left( 2 i\left(16 a^2 A+15 A b^2+24 a^2 C\right) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \right. \\
& \left. \left. \left( 2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]+b\left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right]-b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right)\right) \sin [c+d x] \right) / \\
& \left. \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} \right. \right. \\
& \left. \left. \left( 2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2 \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{\sec [c+d x]\left(16 a^2 A \sin [c+d x]+15 A b^2 \sin [c+d x]+24 a^2 C \sin [c+d x]\right)}{24 a^3} \right. \\
& \left. \frac{5 A b \sec [c+d x] \tan [c+d x]}{12 a^2} + \frac{A \sec [c+d x]^2 \tan [c+d x]}{3 a} \right)
\end{aligned}$$

■ **Problem 660: Unable to integrate problem.**

$$\int \frac{(A+C \cos [c+d x]^2) \sec [c+d x]}{(a+b \cos [c+d x])^{3/2}} dx$$

Optimal (type 4, 259 leaves, 9 steps):

$$\begin{aligned}
& - \frac{2 (A b^2 + a^2 C) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right] + 2 C \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{a b (a^2 - b^2) d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} + \frac{b d \sqrt{a + b \cos [c + d x]}}{a b (a^2 - b^2) d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} \\
& + \frac{2 A \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{a d \sqrt{a + b \cos [c + d x]}} + \frac{2 (A b^2 + a^2 C) \sin [c + d x]}{a (a^2 - b^2) d \sqrt{a + b \cos [c + d x]}}
\end{aligned}$$

Result (type 8, 35 leaves):

$$\int \frac{(A + C \cos [c + d x]^2) \sec [c + d x]}{(a + b \cos [c + d x])^{3/2}} dx$$

■ **Problem 661: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos [c + d x]^2) \sec [c + d x]^2}{(a + b \cos [c + d x])^{3/2}} dx$$

Optimal (type 4, 296 leaves, 10 steps):

$$\begin{aligned}
& \frac{(3 A b^2 - a^2 (A - 2 C)) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right] + A \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{a^2 (a^2 - b^2) d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} + \frac{A d \sqrt{a + b \cos [c + d x]}}{a^2 (a^2 - b^2) d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} \\
& - \frac{3 A b \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{a^2 d \sqrt{a + b \cos [c + d x]}} - \frac{b (3 A b^2 - a^2 (A - 2 C)) \sin [c + d x]}{a^2 (a^2 - b^2) d \sqrt{a + b \cos [c + d x]}} + \frac{A \tan [c + d x]}{a d \sqrt{a + b \cos [c + d x]}}
\end{aligned}$$

Result (type 4, 677 leaves):

$$\begin{aligned}
& \frac{1}{2 a^2 (a-b)(a+b) d (2 A+C+C \cos [2 c+2 d x])} \cos [c+d x]^2 (C+A \sec [c+d x]^2) \left( \frac{2 (4 a A b^2+4 a^3 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2(-7 a^2 A b+9 A b^3+2 a^2 b C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\
& \left. \left( 2 i(-a^2 A b+3 A b^3+2 a^2 b C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \right. \\
& \left. \left. \left( 2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \\
& \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} \right. \\
& \left. \left. \left( 2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2 \right) \right) \right) + \\
& \frac{\cos [c+d x]^2 \sqrt{a+b \cos [c+d x]} (C+A \sec [c+d x]^2) \left( -\frac{4(A b^3 \sin [c+d x]+a^2 b C \sin [c+d x])}{a^2(a^2-b^2)(a+b \cos [c+d x])} + \frac{2 A \tan [c+d x]}{a^2} \right)}{d(2 A+C+C \cos [2 c+2 d x])}
\end{aligned}$$

■ **Problem 662: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+C \cos [c+d x]^2) \sec [c+d x]^3}{(a+b \cos [c+d x])^{3/2}} dx$$

Optimal (type 4, 370 leaves, 11 steps):

$$\begin{aligned}
& - \frac{b (15 A b^2 - a^2 (7 A - 8 C)) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{4 a^3 (a^2 - b^2) d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} \\
& + \frac{5 A b \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{4 a^2 d \sqrt{a + b \cos [c + d x]}} + \frac{(15 A b^2 + 4 a^2 (A + 2 C)) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{4 a^3 d \sqrt{a + b \cos [c + d x]}} + \\
& \frac{b^2 (15 A b^2 - a^2 (7 A - 8 C)) \sin [c + d x]}{4 a^3 (a^2 - b^2) d \sqrt{a + b \cos [c + d x]}} - \frac{5 A b \tan [c + d x]}{4 a^2 d \sqrt{a + b \cos [c + d x]}} + \frac{A \sec [c + d x] \tan [c + d x]}{2 a d \sqrt{a + b \cos [c + d x]}}
\end{aligned}$$

Result (type 4, 727 leaves):

$$\begin{aligned}
& - \frac{1}{8 a^3 (-a+b) (a+b) d (2 A+C+C \operatorname{Cos}[2 c+2 d x])} \\
& \operatorname{Cos}[c+d x]^2 (C+A \operatorname{Sec}[c+d x])^2 \left( \frac{2 (4 a^3 A b-20 a A b^3-16 a^3 b C) \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \operatorname{Cos}[c+d x]}} + \right. \\
& \frac{2 (8 a^4 A+29 a^2 A b^2-45 A b^4+16 a^4 C-24 a^2 b^2 C) \sqrt{\frac{a+b \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \operatorname{Cos}[c+d x]}} - \\
& \left. \left( 2 i (7 a^2 A b^2-15 A b^4-8 a^2 b^2 C) \sqrt{\frac{b-b \operatorname{Cos}[c+d x]}{a+b}} \sqrt{-\frac{b+b \operatorname{Cos}[c+d x]}{a-b}} \operatorname{Cos}[2(c+d x)] \left( 2 a(a-b) \right. \right. \right. \\
& \left. \left. \left. \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \operatorname{Cos}[c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \operatorname{Cos}[c+d x]}\right], \frac{a+b}{a-b}\right] - \right. \right. \right. \right. \\
& \left. \left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \operatorname{Cos}[c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \operatorname{Sin}[c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\operatorname{Cos}[c+d x]^2} \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \operatorname{Cos}[c+d x])+(a+b \operatorname{Cos}[c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \operatorname{Cos}[c+d x])+2(a+b \operatorname{Cos}[c+d x])^2) \right) \right) \right) + \\
& \left( \operatorname{Cos}[c+d x]^2 \sqrt{a+b \operatorname{Cos}[c+d x]} (C+A \operatorname{Sec}[c+d x])^2 \left( \frac{4 (A b^4 \operatorname{Sin}[c+d x]+a^2 b^2 C \operatorname{Sin}[c+d x])}{a^3 (a^2-b^2)(a+b \operatorname{Cos}[c+d x])} - \frac{7 A b \operatorname{Tan}[c+d x]}{2 a^3} + \right. \right. \\
& \left. \left. \frac{A \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{a^2} \right) \right) / (d (2 A+C+C \operatorname{Cos}[2 c+2 d x]))
\end{aligned}$$

■ **Problem 667: Unable to integrate problem.**

$$\int \frac{(A+C \operatorname{Cos}[c+d x])^2 \operatorname{Sec}[c+d x]}{(a+b \operatorname{Cos}[c+d x])^{5/2}} dx$$

Optimal (type 4, 375 leaves, 10 steps):

$$\frac{2(3Ab^4 - a^4C - a^2b^2(7A + 3C))\sqrt{a + b\cos[c + dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{3a^2b(a^2 - b^2)^2d\sqrt{\frac{a+b\cos[c+dx]}{a+b}}} + \frac{2(Ab^2 + a^2C)\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{3ab(a^2 - b^2)d\sqrt{a + b\cos[c + dx]}}$$

$$\frac{2A\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{a^2d\sqrt{a + b\cos[c + dx]}} + \frac{2(Ab^2 + a^2C)\sin[c + dx]}{3a(a^2 - b^2)d(a + b\cos[c + dx])^{3/2}} - \frac{2(3Ab^4 - a^4C - a^2b^2(7A + 3C))\sin[c + dx]}{3a^2(a^2 - b^2)^2d\sqrt{a + b\cos[c + dx]}}$$

Result (type 8, 35 leaves):

$$\int \frac{(A + C\cos[c + dx])^2 \operatorname{Sec}[c + dx]}{(a + b\cos[c + dx])^{5/2}} dx$$

■ **Problem 668: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + C\cos[c + dx])^2 \operatorname{Sec}[c + dx]^2}{(a + b\cos[c + dx])^{5/2}} dx$$

Optimal (type 4, 416 leaves, 11 steps):

$$\frac{(26a^2Ab^2 - 15Ab^4 - a^4(3A - 8C))\sqrt{a + b\cos[c + dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{3a^3(a^2 - b^2)^2d\sqrt{\frac{a+b\cos[c+dx]}{a+b}}}$$

$$\frac{(5Ab^2 - a^2(3A - 2C))\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] - 5Ab\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{3a^2(a^2 - b^2)d\sqrt{a + b\cos[c + dx]} - a^3d\sqrt{a + b\cos[c + dx]}}$$

$$\frac{b(5Ab^2 - a^2(3A - 2C))\sin[c + dx]}{3a^2(a^2 - b^2)d(a + b\cos[c + dx])^{3/2}} - \frac{b(26a^2Ab^2 - 15Ab^4 - a^4(3A - 8C))\sin[c + dx]}{3a^3(a^2 - b^2)^2d\sqrt{a + b\cos[c + dx]}} + \frac{A\tan[c + dx]}{ad(a + b\cos[c + dx])^{3/2}}$$

Result (type 4, 786 leaves):

1

$$6 a^3 (-a+b)^2 (a+b)^2 d (2 A+C+C \cos [2 c+2 d x])$$

$$\begin{aligned} & \cos [c+d x]^2 (C+A \sec [c+d x]^2) \left( \frac{2 (36 a^3 A b^2 - 20 a A b^4 + 12 a^5 C + 4 a^3 b^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\ & \frac{2 (-33 a^4 A b + 86 a^2 A b^3 - 45 A b^5 + 8 a^4 b C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\ & \left. \left( 2 i (-3 a^4 A b + 26 a^2 A b^3 - 15 A b^5 + 8 a^4 b C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \left( 2 a (a-b) \right. \right. \right. \\ & \left. \left. \left. \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] - \right. \right. \right. \right. \\ & \left. \left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\ & \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) \right) + \\ & \left( \cos [c+d x]^2 \sqrt{a+b \cos [c+d x]} (C+A \sec [c+d x]^2) \left( -\frac{4(A b^3 \sin [c+d x]+a^2 b C \sin [c+d x])}{3 a^2(a^2-b^2)(a+b \cos [c+d x])^2} - \right. \right. \\ & \left. \left. \frac{8(5 a^2 A b^3 \sin [c+d x]-3 A b^5 \sin [c+d x]+2 a^4 b C \sin [c+d x])}{3 a^3(a^2-b^2)^2(a+b \cos [c+d x])} + \frac{2 A \tan [c+d x]}{a^3} \right) \right) / (d(2 A+C+C \cos [2 c+2 d x])) \end{aligned}$$

■ **Problem 709: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+C \cos [c+d x]^2}{\cos [c+d x]^{3/2} (a+b \cos [c+d x])} dx$$

Optimal (type 4, 112 leaves, 6 steps):

$$-\frac{2 A \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{a d} + \frac{2 C \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{b d} - \frac{2\left(A b^2+a^2 C\right) \operatorname{EllipticPi}\left[\frac{2 b}{a+b}, \frac{1}{2}(c+d x), 2\right]}{a b(a+b) d} + \frac{2 A \sin [c+d x]}{a d \sqrt{\cos [c+d x]}}$$

Result (type 4, 242 leaves):

$$\frac{2 A \sin [c+d x]}{a d \sqrt{\cos [c+d x]}} - \frac{1}{2 a d} \left( \frac{6 A b \operatorname{EllipticPi}\left[\frac{2 b}{a+b}, \frac{1}{2}(c+d x), 2\right]}{a+b} + \frac{(2 a A-2 a C)\left(2 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] - \frac{2 a \operatorname{EllipticPi}\left[\frac{2 b}{a+b}, \frac{1}{2}(c+d x), 2\right]}{a+b}\right)}{b} \right) + \left( 2 A \cos [2(c+d x)] \left( -2 a b \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\cos [c+d x]}\right], -1\right] + 2 a(a+b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\cos [c+d x]}\right], -1\right] + (2 a^2-b^2) \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\cos [c+d x]}\right], -1\right] \sin [c+d x] \right) / \left( a b \sqrt{1-\cos [c+d x]^2} (-1+2 \cos [c+d x]^2) \right) \right)$$

- **Problem 727: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} (A+C \cos [c+d x]^2) dx$$

Optimal (type 4, 553 leaves, 8 steps):

$$\frac{1}{24 a b^2 d} (a-b) \sqrt{a+b} (3 a^2 C-8 b^2(3 A+2 C)) \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \\ \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \frac{1}{24 b^2 d} \sqrt{a+b} (3 a^2 C-2 a b C-8 b^2(3 A+2 C)) \cot [c+d x] \\ \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \\ \frac{1}{8 b^3 d} a \sqrt{a+b} (8 A b^2+(a^2+4 b^2) C) \cot [c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \\ \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \frac{(3 a^2 C-8 b^2(3 A+2 C)) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{24 b^2 d \sqrt{\cos [c+d x]}} - \\ \frac{a C \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{4 b d} + \frac{C \sqrt{\cos [c+d x]} (a+b \cos [c+d x])^{3 / 2} \sin [c+d x]}{3 b d}$$

Result (type 4, 1220 leaves):



$$\frac{1}{48 b d}$$

$$\left( - \left( \begin{aligned} & 4 a (24 A b^2 - a^2 C + 16 b^2 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \\ & \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \end{aligned} \right)$$

$$\left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a (48 a A b + 28 a b C)$$

$$\left( \left( \begin{aligned} & \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \\ & \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \end{aligned} \right)$$

$$\left( \begin{aligned} & \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \\ & \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \end{aligned} \right) +$$

$$\begin{aligned}
& 2 (24 A b^2 - 3 a^2 C + 16 b^2 C) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}}}\right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + \\
& \left. \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{a C \sin[c+dx]}{12b} + \frac{1}{6} C \sin[2(c+dx)] \right)}{d}
\end{aligned}$$

■ **Problem 728: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \cos[c+dx]} (A+C \cos[c+dx]^2)}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 455 leaves, 7 steps):

$$\begin{aligned}
& -\frac{1}{4bd} (a-b) \sqrt{a+b} C \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{4bd} \\
& \sqrt{a+b} (8Ab + (a+2b)C) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{1}{4b^2d} \sqrt{a+b} (a^2C - 4b^2(2A+C)) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \\
& \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{aC \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{4bd \sqrt{\cos[c+dx]}} + \frac{C \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2d}
\end{aligned}$$

Result (type 4, 1169 leaves):

$$\begin{aligned}
& \frac{C \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2d} + \frac{1}{8d} \\
& \left( - \left( 4a(8aA + 3aC) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a \right. \\
& \left. (8Ab + 4bC) \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& 2 a c \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /
\end{aligned}$$

$$\left( \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right)$$

■ **Problem 729: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \cos[c+dx]} (A+C \cos[c+dx]^2)}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 439 leaves, 7 steps):

$$\begin{aligned} & \frac{1}{ad} (a-b) \sqrt{a+b} (2A-C) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{ad} \\ & \sqrt{a+b} (2aA-2Ab-aC) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\ & \frac{1}{bd} a \sqrt{a+b} C \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\ & \frac{2A \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}} - \frac{(2A-C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}} \end{aligned}$$

Result (type 4, 1166 leaves):

$$\begin{aligned} & \frac{2A \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}} + \\ & \frac{1}{2d} \left( \left( 4abc \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ & \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a \right) \end{aligned}$$

$$\begin{aligned}
& (-2 a A + 2 a C) \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2(-2 A b + b C) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2 a \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\ \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\ \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \Bigg)$$

- **Problem 731: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} (A+C \operatorname{Cos}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{7/2}} dx$$

Optimal (type 4, 345 leaves, 5 steps):

$$-\frac{1}{15a^3d} 2(a-b) \sqrt{a+b} (2Ab^2 - 3a^2(3A+5C)) \operatorname{Cot}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{15a^2d}$$

$$2(a-b) \sqrt{a+b} (9aA+2Ab+15aC) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}}$$

$$\sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{2A \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{5d \operatorname{Cos}[c+dx]^{5/2}} + \frac{2Ab \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{15ad \operatorname{Cos}[c+dx]^{3/2}}$$

Result (type 4, 1288 leaves):

$$-\frac{1}{15a^2d}$$

$$\begin{aligned}
& \left( - \left( 4 a (2 a^2 A b - 2 A b^3) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a (9 a^3 A - 2 a A b^2 + 15 a^3 C) \right. \\
& \quad \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \quad \left. 2 (9 a^2 A b - 2 A b^3 + 15 a^2 b C) \frac{\left( i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+dx] \right. \right. \\
& \quad \left. \left. b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right)}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right)
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \left. \right) + \frac{1}{d} \\
& \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2 \operatorname{Sec}[c+dx] (9a^2 A \operatorname{Sin}[c+dx] - 2Ab^2 \operatorname{Sin}[c+dx] + 15a^2 C \operatorname{Sin}[c+dx])}{15a^2} + \right. \\
& \quad \frac{2Ab \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{15a} + \\
& \quad \frac{2}{5} \\
& \quad A \operatorname{Sec}[c+dx]^2 \\
& \quad \left. \operatorname{Tan}[c+dx] \right)
\end{aligned}$$

■ **Problem 732:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+b \cos [c+d x]} (A+C \cos [c+d x]^2)}{\cos [c+d x]^{9/2}} dx$$

Optimal (type 4, 415 leaves, 6 steps):

$$\begin{aligned} & \frac{1}{105 a^4 d} 2 (a-b) b \sqrt{a+b} (8 A b^2 + a^2 (19 A + 35 C)) \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \\ & \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \frac{1}{105 a^3 d} 2 (a-b) \sqrt{a+b} (6 a A b + 8 A b^2 + 5 a^2 (5 A + 7 C)) \\ & \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \\ & \frac{2 A \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{7 d \cos [c+d x]^{7/2}} + \frac{2 A b \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{35 a d \cos [c+d x]^{5/2}} - \frac{2 (4 A b^2 - 5 a^2 (5 A + 7 C)) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{105 a^2 d \cos [c+d x]^{3/2}} \end{aligned}$$

Result (type 4, 1373 leaves):

$$\begin{aligned} & \frac{1}{105 a^3 d} \left( \left( 4 a (25 a^4 A - 17 a^2 A b^2 - 8 A b^4 + 35 a^4 C - 35 a^2 b^2 C) \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\ & \left. \left. \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right) / \right. \\ & \left. \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - 4 a (-19 a^3 A b - 8 a A b^3 - 35 a^3 b C) \right. \\ & \left. \left( \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right. \\ & \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \Bigg) + \\
& 2(-19a^2Ab^2 - 8Ab^4 - 35a^2b^2c) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \Bigg) + \frac{1}{d}$$

$$\sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left( \frac{2 \sec [c+d x]^2 (25 a^2 A \sin [c+d x] - 4 A b^2 \sin [c+d x] + 35 a^2 C \sin [c+d x])}{105 a^2} + \right.$$

$$\frac{2 \sec [c+d x] (19 a^2 A b \sin [c+d x] + 8 A b^3 \sin [c+d x] + 35 a^2 b C \sin [c+d x])}{105 a^3} +$$

$$\frac{2 A b \sec [c+d x]^2 \tan [c+d x]}{35 a} +$$

$$\left. \frac{2}{7} A \sec [c+d x]^3 \tan [c+d x] \right)$$

■ **Problem 733: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\cos [c+d x]} (a+b \cos [c+d x])^{3/2} (A+C \cos [c+d x]^2) dx$$

Optimal (type 4, 638 leaves, 9 steps):

$$-\frac{1}{64 b^2 d} (a-b) \sqrt{a+b} (80 A b^2 - 3 a^2 C + 52 b^2 C) \cot [c+d x]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \frac{1}{64 b^2 d}$$

$$\sqrt{a+b} (3 a^3 C - 2 a^2 b C - 8 b^3 (4 A + 3 C) - 4 a b^2 (20 A + 13 C)) \cot [c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \frac{1}{64 b^3 d} \sqrt{a+b} (3 a^4 C + 24 a^2 b^2 (2 A + C) + 16 b^4 (4 A + 3 C)) \cot [c+d x]$$

$$\text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} +$$

$$\frac{a (80 A b^2 - 3 a^2 C + 52 b^2 C) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{64 b^2 d \sqrt{\cos [c+d x]}} - \frac{(3 a^2 C - 4 b^2 (4 A + 3 C)) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{32 b d}$$

$$\frac{a C \sqrt{\cos [c+d x]} (a+b \cos [c+d x])^{3/2} \sin [c+d x]}{8 b d} + \frac{C \sqrt{\cos [c+d x]} (a+b \cos [c+d x])^{5/2} \sin [c+d x]}{4 b d}$$

Result (type 4, 1270 leaves):

$$\begin{aligned}
& -\frac{1}{128bd} \left( - \left( 4a(-112aAb^2 + a^3C - 76ab^2C) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - 4a(-128a^2Ab - 64Ab^3 - 76a^2bC - 48b^3C) \\
& \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) + \\
& 2(-80aAb^2 + 3a^3C - 52ab^2C) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \Bigg) + \\
& \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{(16 A b^2 + a^2 C + 14 b^2 C) \operatorname{Sin}[c+dx]}{32 b} + \right. \\
& \quad \frac{3}{16} \\
& \quad a \\
& \quad C \\
& \quad \operatorname{Sin}[2(c+dx)] + \frac{1}{16} \\
& \quad b \\
& \quad C \\
& \quad \left. \operatorname{Sin}[3(c+dx)] \right)
\end{aligned}$$

**Problem 734: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \cos[c + dx])^{3/2} (A + C \cos[c + dx]^2)}{\sqrt{\cos[c + dx]}} dx$$

Optimal (type 4, 553 leaves, 8 steps):

$$\begin{aligned} & -\frac{1}{24abd} (a-b) \sqrt{a+b} (3a^2C + 8b^2(3A+2C)) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\ & \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{24bd} \sqrt{a+b} (48aAb + 24Ab^2 + 3a^2C + 14abC + 16b^2C) \\ & \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\ & \frac{1}{8b^2d} a \sqrt{a+b} (24Ab^2 - a^2C + 12b^2C) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\ & \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(3a^2C + 8b^2(3A+2C)) \sqrt{a+b\cos[c+dx]} \sin[c+dx]}{24bd\sqrt{\cos[c+dx]}} + \\ & \frac{aC\sqrt{\cos[c+dx]} \sqrt{a+b\cos[c+dx]} \sin[c+dx]}{4d} + \frac{C\sqrt{\cos[c+dx]} (a+b\cos[c+dx])^{3/2} \sin[c+dx]}{3d} \end{aligned}$$

Result (type 4, 1221 leaves):

$$\begin{aligned} & \frac{1}{48d} \left( - \left( 4a (48a^2A + 24Ab^2 + 17a^2C + 16b^2C) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ & \left. \left. \sqrt{\frac{(a+b\cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) \\ & \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b\cos[c+dx]} \right) - 4a (96aAb + 52abC) \end{aligned}$$

$$\begin{aligned}
& \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \quad \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \quad \left. \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + \\
& 2(24Ab^2 + 3a^2c + 16b^2c) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \quad \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.
\end{aligned}$$



$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\ \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \\ \left. \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) + \frac{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{7}{12} a C \operatorname{Sin}[c+dx] + \frac{1}{6} b C \operatorname{Sin}[2(c+dx)] \right)}{d}$$

■ **Problem 735: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{3/2} (A+C \operatorname{Cos}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{3/2}} dx$$

Optimal (type 4, 509 leaves, 8 steps):

$$\frac{1}{4d} (a-b) \sqrt{a+b} (8A-5C) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \\ \frac{1}{4d} \sqrt{a+b} (8aA-16Ab-5aC-2bC) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\ \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{4bd} \sqrt{a+b} (8Ab^2+3a^2C+4b^2C) \operatorname{Cot}[c+dx] \\ \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \\ \frac{a(8A-5C) \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{4d \sqrt{\operatorname{Cos}[c+dx]}} - \frac{b(4A-C) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{2d} + \frac{2A(a+b \operatorname{Cos}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{d \sqrt{\operatorname{Cos}[c+dx]}}$$

Result (type 4, 1209 leaves):

$$\begin{aligned}
& \frac{1}{8d} \left( \left( 4a(-8aAb - 7abc) \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\operatorname{Cos}[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b)\sqrt{\operatorname{Cos}[c+dx]}\sqrt{a+b\operatorname{Cos}[c+dx]} \right) + 4a(8a^2A - 8Ab^2 - 8a^2C - 4b^2C) \\
& \left( \left( \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\operatorname{Cos}[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b)\sqrt{\operatorname{Cos}[c+dx]}\sqrt{a+b\operatorname{Cos}[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\operatorname{Cos}[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b\sqrt{\operatorname{Cos}[c+dx]}\sqrt{a+b\operatorname{Cos}[c+dx]} \right) \right) - \\
& 2(8aAb - 5abc) \left( \frac{i\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\operatorname{Cos}[c+dx]}\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right]\operatorname{Sec}[c+dx]}{b\sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2}\operatorname{Sec}[c+dx]}\sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\frac{1}{b} - 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\ \left. \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + \\ \left. \left. \frac{\sqrt{a+b \cos[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right) + \frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{1}{2} b C \operatorname{Sin}[c+dx] + 2 a A \operatorname{Tan}[c+dx] \right)}{d}$$

- **Problem 736: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{3/2} (A+C \cos[c+dx]^2)}{\cos[c+dx]^{5/2}} dx$$

Optimal (type 4, 500 leaves, 8 steps) :

$$\begin{aligned}
& \frac{1}{3ad} (a-b) b \sqrt{a+b} (8A-3C) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{1}{3ad} \sqrt{a+b} (6Ab^2 + 2a^2(A+3C) - a(8Ab-3bC)) \cot[c+dx] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{d} \\
& 3a\sqrt{a+b} C \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{2Ab\sqrt{a+b}\cos[c+dx] \sin[c+dx]}{d\sqrt{\cos[c+dx]}} - \frac{b(8A-3C)\sqrt{a+b}\cos[c+dx] \sin[c+dx]}{3d\sqrt{\cos[c+dx]}} + \frac{2A(a+b\cos[c+dx])^{3/2} \sin[c+dx]}{3d\cos[c+dx]^{3/2}}
\end{aligned}$$

Result (type 4, 1219 leaves):

$$\begin{aligned}
& \frac{1}{6d} \left( - \left( 4a(2a^2A - 2Ab^2 + 6a^2C + 3b^2C) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}}{a\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \left. \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - 4a(-8aAb + 12abC) \right. \\
& \left. \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}}{a\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \Bigg) + \\
& 2(-8Ab^2 + 3b^2c) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) +
\end{aligned}$$

$$\left. \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) + \frac{\sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left( \frac{8}{3} A b \tan [c+d x] + \frac{2}{3} a A \sec [c+d x] \tan [c+d x] \right)}{d}$$

■ **Problem 737: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos [c+d x])^{3/2} (A+C \cos [c+d x]^2)}{\cos [c+d x]^{7/2}} dx$$

Optimal (type 4, 465 leaves, 7 steps):

$$\frac{1}{5 a^2 d} 2 (a-b) \sqrt{a+b} (A b^2+a^2 (3 A+5 C)) \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \frac{1}{5 a d} 2 \sqrt{a+b} (A b^2-2 a b(2 A+5 C)+a^2(3 A+5 C)) \cot [c+d x]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \frac{1}{d}$$

$$2 b \sqrt{a+b} C \cot [c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} +$$

$$\frac{2 A b \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{5 d \cos [c+d x]^{3/2}} + \frac{2 A (a+b \cos [c+d x])^{3/2} \sin [c+d x]}{5 d \cos [c+d x]^{5/2}}$$

Result (type 4, 1296 leaves):

$$-\frac{1}{5 a d}$$

$$\left( -4 a \left( -a^2 A b + A b^3 - 5 a^2 b C \right) \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos [c+d x] \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \right.$$

$$\left. \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right) /$$

$$\begin{aligned}
& \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a \left( 3a^3 A + aAb^2 + 5a^3 C - 5ab^2 C \right) \\
& \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + \\
& 2 \left( 3a^2 Ab + Ab^3 + 5a^2 bC \right) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) -
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\ \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\ \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\cos[c+dx]}} \left. \right) + \frac{1}{d} \\ \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2 \operatorname{Sec}[c+dx] (3 a^2 A \operatorname{Sin}[c+dx] + A b^2 \operatorname{Sin}[c+dx] + 5 a^2 C \operatorname{Sin}[c+dx])}{5 a} + \right. \\ \frac{4}{5} \\ A \\ b \\ \operatorname{Sec}[c+dx] \\ \operatorname{Tan}[c+dx] + \frac{2}{5} a A \\ \left. \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right)$$

■ **Problem 738: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{3/2} (A+C \cos[c+dx]^2)}{\cos[c+dx]^{9/2}} dx$$

Optimal (type 4, 418 leaves, 6 steps):



$$\begin{aligned}
& -\frac{1}{105 a^3 d} 4 (a-b) b \sqrt{a+b} (3 A b^2 - a^2 (41 A + 70 C)) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{105 a^2 d} \\
& 2(a-b) \sqrt{a+b} (25 a^2 A - 57 a A b - 6 A b^2 + 35 a^2 C - 105 a b C) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{6 A b \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{35 d \operatorname{Cos}[c+d x]^{5/2}} + \\
& \frac{2(3 A b^2 + 5 a^2 (5 A + 7 C)) \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{105 a d \operatorname{Cos}[c+d x]^{3/2}} + \frac{2 A (a+b \operatorname{Cos}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{7 d \operatorname{Cos}[c+d x]^{7/2}}
\end{aligned}$$

Result (type 4, 1371 leaves):

$$\begin{aligned}
& \frac{1}{105 a^2 d} \left( - \left( 4 a (25 a^4 A - 31 a^2 A b^2 + 6 A b^4 + 35 a^4 C - 35 a^2 b^2 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (-82 a^3 A b + 6 a A b^3 - 140 a^3 b C) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \Bigg) + \\
& 2(-82a^2Ab^2 + 6Ab^4 - 140a^2b^2c) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \Bigg) + \frac{1}{d}$$

$$\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2 \sec[c+dx]^2 (25 a^2 A \sin[c+dx] + 3 A b^2 \sin[c+dx] + 35 a^2 C \sin[c+dx])}{105 a} + \right.$$

$$\frac{4 \sec[c+dx] (41 a^2 A b \sin[c+dx] - 3 A b^3 \sin[c+dx] + 70 a^2 b C \sin[c+dx])}{105 a^2} +$$

$$\frac{16}{35} A b \sec[c+dx]^2 \tan[c+dx] +$$

$$\left. \frac{2}{7} a A \sec[c+dx]^3 \tan[c+dx] \right)$$

■ **Problem 739: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{3/2} (A+C \cos[c+dx]^2)}{\cos[c+dx]^{11/2}} dx$$

Optimal (type 4, 502 leaves, 7 steps):

$$\frac{1}{315 a^4 d} 2 (a-b) \sqrt{a+b} (8 A b^4 + 21 a^4 (7 A + 9 C) + 3 a^2 b^2 (11 A + 21 C)) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{315 a^3 d} 2 (a-b) \sqrt{a+b} (6 a A b^2 + 8 A b^3 - 21 a^3 (7 A + 9 C) + a^2 (39 A b + 63 b C))$$

$$\cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2 A b \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{21 d \cos[c+dx]^{7/2}} + \frac{2 (3 A b^2 + 7 a^2 (7 A + 9 C)) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{315 a d \cos[c+dx]^{5/2}} -$$

$$\frac{4 b (2 A b^2 - a^2 (44 A + 63 C)) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{315 a^2 d \cos[c+dx]^{3/2}} + \frac{2 A (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{9 d \cos[c+dx]^{9/2}}$$

Result (type 4, 1485 leaves):

$$\begin{aligned}
& -\frac{1}{315 a^3 d} \left( - \left( 4 a \left( -39 a^4 A b + 31 a^2 A b^3 + 8 A b^5 - 63 a^4 b c + 63 a^2 b^3 c \right) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a \left( 147 a^5 A + 33 a^3 A b^2 + 8 a A b^4 + 189 a^5 c + 63 a^3 b^2 c \right) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + 2 \left( 147 a^4 A b + 33 a^2 A b^3 + 8 A b^5 + 189 a^4 b c + 63 a^2 b^3 c \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} + \right. \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \Bigg) + \\
& \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2 \operatorname{Sec}[c+dx]^3 (49 a^2 A \sin[c+dx] + 3 A b^2 \sin[c+dx] + 63 a^2 C \sin[c+dx])}{315 a} + \right. \\
& \left. \frac{4 \operatorname{Sec}[c+dx]^2 (44 a^2 A b \sin[c+dx] - 2 A b^3 \sin[c+dx] + 63 a^2 b C \sin[c+dx])}{315 a^2} + \right. \\
& \left. \frac{1}{315 a^3} \right)
\end{aligned}$$

$$\left. \begin{aligned} & 2 \operatorname{Sec}[c+dx] \left( 147 a^4 A \operatorname{Sin}[c+dx] + 33 a^2 A b^2 \operatorname{Sin}[c+dx] + 8 A b^4 \operatorname{Sin}[c+dx] + 189 a^4 C \operatorname{Sin}[c+dx] + 63 a^2 b^2 C \operatorname{Sin}[c+dx] \right) + \\ & \frac{20}{63} A b \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx] + \\ & \frac{2}{9} a A \operatorname{Sec}[c+dx]^4 \operatorname{Tan}[c+dx] \end{aligned} \right)$$

■ **Problem 740: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\operatorname{Cos}[c+dx]} (a+b \operatorname{Cos}[c+dx])^{5/2} (A+C \operatorname{Cos}[c+dx]^2) dx$$

Optimal (type 4, 746 leaves, 10 steps):

$$\begin{aligned} & \frac{1}{1920 a b^2 d} (a-b) \sqrt{a+b} \left( 45 a^4 C - 256 b^4 (5 A + 4 C) - 12 a^2 b^2 (220 A + 141 C) \right) \\ & \operatorname{Cot}[c+dx] \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \\ & \frac{1}{1920 b^2 d} \sqrt{a+b} \left( 45 a^4 C - 30 a^3 b C - 256 b^4 (5 A + 4 C) - 12 a^2 b^2 (220 A + 141 C) - 8 a b^3 (260 A + 193 C) \right) \operatorname{Cot}[c+dx] \\ & \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{128 b^3 d} \\ & a \sqrt{a+b} \left( 3 a^4 C + 40 a^2 b^2 (2 A + C) + 80 b^4 (4 A + 3 C) \right) \operatorname{Cot}[c+dx] \operatorname{EllipticPi} \left[ \frac{a+b}{b}, \operatorname{ArcSin} \left[ \frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}} \right], -\frac{a+b}{a-b} \right] \\ & \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{(45 a^4 C - 256 b^4 (5 A + 4 C) - 12 a^2 b^2 (220 A + 141 C)) \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{1920 b^2 d \sqrt{\operatorname{Cos}[c+dx]}} + \\ & \frac{a(240 A b^2 - 15 a^2 C + 172 b^2 C) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{320 b d} - \\ & \frac{(15 a^2 C - 16 b^2 (5 A + 4 C)) \sqrt{\operatorname{Cos}[c+dx]} (a+b \operatorname{Cos}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{240 b d} - \\ & \frac{3 a C \sqrt{\operatorname{Cos}[c+dx]} (a+b \operatorname{Cos}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{40 b d} + \frac{C \sqrt{\operatorname{Cos}[c+dx]} (a+b \operatorname{Cos}[c+dx])^{7/2} \operatorname{Sin}[c+dx]}{5 b d} \end{aligned}$$

Result (type 4, 1341 leaves):

$$\begin{aligned}
& -\frac{1}{3840bd} \left( \left( 4a(-4720a^2Ab^2 - 1280Ab^4 + 15a^4c - 3236a^2b^2c - 1024b^4c) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - 4a(-3840a^3Ab - 6080aAb^3 - 2292a^3bc - 4624ab^3c) \\
& \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left. \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) + 2(-2640a^2Ab^2 - 1280Ab^4 + 45a^4c - 1692a^2b^2c - 1024b^4c)
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} + \right. \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \Bigg) + \\
& \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{a(1040Ab^2 + 15a^2c + 898b^2c) \sin[c+dx]}{960b} + \right. \\
& \frac{1}{480} \\
& (80Ab^2 + 93a^2c + 88b^2c) \\
& \left. \sin[2(c+dx)] + \frac{21}{160} \right)
\end{aligned}$$



$$\left. \begin{aligned} &a \\ &b \\ &c \\ &\sin[3(c+dx)] + \frac{1}{40} \\ &b^2 \\ &c \\ &\sin[4(c+dx)] \end{aligned} \right)$$

■ **Problem 741: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{5/2} (A+C \cos[c+dx]^2)}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 635 leaves, 9 steps):

$$\begin{aligned} &-\frac{1}{192bd} (a-b) \sqrt{a+b} (432Ab^2 + 15a^2c + 284b^2c) \cot[c+dx] \\ &\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{192bd} \\ &\sqrt{a+b} (15a^3c + 24b^3(4A+3C) + 2a^2b(192A+59C) + 4ab^2(108A+71C)) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\ &\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{64b^2d} \sqrt{a+b} (5a^4c - 120a^2b^2(2A+C) - 16b^4(4A+3C)) \cot[c+dx] \\ &\text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\ &\frac{a(432Ab^2 + 15a^2c + 284b^2c) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{192bd \sqrt{\cos[c+dx]}} + \frac{(5a^2c + 4b^2(4A+3C)) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{32d} + \\ &\frac{5aC \sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{24d} + \frac{c \sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{5/2} \sin[c+dx]}{4d} \end{aligned}$$

Result (type 4, 1275 leaves):

$$\begin{aligned}
& \frac{1}{384 d} \left( - \left( 4 a \left( 384 a^3 A + 528 a A b^2 + 133 a^3 C + 356 a b^2 C \right) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right]}, -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a \left( 1152 a^2 A b + 192 A b^3 + 644 a^2 b C + 144 b^3 C \right) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right]}, -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right]}, -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2 \left( 432 a A b^2 + 15 a^3 C + 284 a b^2 C \right) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right]}, -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\ \left. \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right) + \frac{1}{d} \\ \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{1}{96} (48 A b^2 + 59 a^2 C + 42 b^2 C) \operatorname{Sin}[c+dx] + \frac{17}{48} a b C \operatorname{Sin}[2(c+dx)] + \right. \\ \left. \frac{1}{16} b^2 C \operatorname{Sin}[3(c+dx)] \right)$$

- **Problem 742: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{5/2} (A+C \operatorname{Cos}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{3/2}} dx$$

Optimal (type 4, 609 leaves, 9 steps):

$$\frac{1}{24 a d} (a-b) \sqrt{a+b} \left( a^2 (48 A - 33 C) - 8 b^2 (3 A + 2 C) \right) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{24 d}$$

$$\sqrt{a+b} \left( a^2 (48 A - 33 C) - 8 b^2 (3 A + 2 C) - 2 a b (72 A + 13 C) \right) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{8 b d} 5 a \sqrt{a+b} \left( 8 A b^2 + (a^2 + 4 b^2) C \right) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} -$$

$$\frac{\left( a^2 (48 A - 33 C) - 8 b^2 (3 A + 2 C) \right) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{24 d \sqrt{\operatorname{Cos}[c+d x]}} - \frac{a b (8 A - 3 C) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{4 d}$$

$$\frac{b(6 A - C) \sqrt{\operatorname{Cos}[c+d x]} (a+b \operatorname{Cos}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{3 d} + \frac{2 A (a+b \operatorname{Cos}[c+d x])^{5/2} \operatorname{Sin}[c+d x]}{d \sqrt{\operatorname{Cos}[c+d x]}}$$

Result (type 4, 1262 leaves):

$$\frac{1}{48 d} \left( \left( 4 a \left( -96 a^2 A b - 24 A b^3 - 59 a^2 b C - 16 b^3 C \right) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right)$$

$$\left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) + 4 a \left( 48 a^3 A - 144 a A b^2 - 48 a^3 C - 76 a b^2 C \right)$$

$$\left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \csc [c+d x] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& 2\left(48 a^2 A b-24 A b^3-33 a^2 b C-16 b^3 C\right) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \sec [c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \sec [c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \sec [c+d x]}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \csc [c+d x] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \right) \right)
\end{aligned}$$

$$\left. \begin{aligned} & \left( \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \text{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\ & \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) + \frac{\sqrt{a+b\cos[c+dx]}\text{Sin}[c+dx]}{b\sqrt{\cos[c+dx]}} \end{aligned} \right) +$$

$$\frac{\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\left(\frac{13}{12}abC\text{Sin}[c+dx] + \frac{1}{6}b^2C\text{Sin}[2(c+dx)] + 2a^2A\text{Tan}[c+dx]\right)}{d}$$

■ **Problem 743: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b\cos[c+dx])^{5/2}(A+C\cos[c+dx]^2)}{\cos[c+dx]^{5/2}} dx$$

Optimal (type 4, 567 leaves, 9 steps):

$$\begin{aligned} & \frac{1}{12d}(a-b)b\sqrt{a+b}(56A-27C)\text{Cot}[c+dx]\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]\sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}}\sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} + \\ & \frac{1}{12d}\sqrt{a+b}(6b^2(12A+C)+8a^2(A+3C)-a(56Ab-27bC))\text{Cot}[c+dx]\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\ & \sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}}\sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} - \frac{1}{4d}\sqrt{a+b}(8Ab^2+15a^2C+4b^2C)\text{Cot}[c+dx] \\ & \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]\sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}}\sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} - \\ & \frac{ab(56A-27C)\sqrt{a+b\cos[c+dx]}\text{Sin}[c+dx]}{12d\sqrt{\cos[c+dx]}} - \frac{b^2(8A-C)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\text{Sin}[c+dx]}{2d} + \\ & \frac{10Ab(a+b\cos[c+dx])^{3/2}\text{Sin}[c+dx]}{3d\sqrt{\cos[c+dx]}} + \frac{2A(a+b\cos[c+dx])^{5/2}\text{Sin}[c+dx]}{3d\cos[c+dx]^{3/2}} \end{aligned}$$

Result (type 4, 1256 leaves):

$$\begin{aligned}
& \frac{1}{24 d} \left( - \left( 4 a (8 a^3 A + 16 a A b^2 + 24 a^3 C + 33 a b^2 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a (-56 a^2 A b + 24 A b^3 + 72 a^2 b C + 12 b^3 C) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2 (-56 a A b^2 + 27 a b^2 C) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \left. \right) + \frac{1}{d} \\
& \quad \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{1}{2} b^2 C \operatorname{Sin}[c+dx] + \frac{14}{3} a A b \operatorname{Tan}[c+dx] + \frac{2}{3} a^2 A \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx] \right)
\end{aligned}$$

■ **Problem 744: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{5/2} (A+C \operatorname{Cos}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{7/2}} dx$$

Optimal (type 4, 606 leaves, 9 steps):



$$\frac{1}{15 a d} (a-b) \sqrt{a+b} \left( b^2 (46 A - 15 C) + 6 a^2 (3 A + 5 C) \right) \cot [c+d x] \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a+b} \cos [c+d x]}{\sqrt{a+b} \sqrt{\cos [c+d x]}} \right], -\frac{a+b}{a-b} \right]$$

$$\sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \frac{1}{15 a d} \sqrt{a+b} \left( 30 A b^3 - a b^2 (46 A - 15 C) - 6 a^3 (3 A + 5 C) + a^2 (34 A b + 90 b C) \right)$$

$$\cot [c+d x] \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a+b} \cos [c+d x]}{\sqrt{a+b} \sqrt{\cos [c+d x]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \frac{1}{d}$$

$$5 a b \sqrt{a+b} C \cot [c+d x] \operatorname{EllipticPi} \left[ \frac{a+b}{b}, \operatorname{ArcSin} \left[ \frac{\sqrt{a+b} \cos [c+d x]}{\sqrt{a+b} \sqrt{\cos [c+d x]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} +$$

$$\frac{2 \left( 5 A b^2 + a^2 (3 A + 5 C) \right) \sqrt{a+b} \cos [c+d x] \sin [c+d x]}{5 d \sqrt{\cos [c+d x]}} - \frac{\left( b^2 (46 A - 15 C) + 6 a^2 (3 A + 5 C) \right) \sqrt{a+b} \cos [c+d x] \sin [c+d x]}{15 d \sqrt{\cos [c+d x]}} +$$

$$\frac{2 A b (a+b \cos [c+d x])^{3/2} \sin [c+d x]}{3 d \cos [c+d x]^{3/2}} + \frac{2 A (a+b \cos [c+d x])^{5/2} \sin [c+d x]}{5 d \cos [c+d x]^{5/2}}$$

Result (type 4, 1309 leaves):

$$\frac{1}{30 d} \left( \left( 4 a \left( -16 a^2 A b + 16 A b^3 - 60 a^2 b C - 15 b^3 C \right) \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos [c+d x] \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \operatorname{Csc} [c+d x] \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}}{a \sqrt{2}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right) \right)$$

$$\left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + 4 a \left( 18 a^3 A + 46 a A b^2 + 30 a^3 C - 90 a b^2 C \right)$$

$$\left( \left( \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos [c+d x] \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \operatorname{Csc} [c+d x] \right. \right.$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left(\sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x]\right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& 2\left(18 a^2 A b+46 A b^3+30 a^2 b C-15 b^3 C\right) \left(\frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \text{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \sec [c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \sec [c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \sec [c+d x]}{a+b}}\right) + \\
& \frac{1}{b} 2 a \left(\left(a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x]\right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \\
& \left. \left(a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}\right) - \right.
\end{aligned}$$

$$\left( \left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \text{Cos}[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \text{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right/$$

$$\left. \left( b \sqrt{\text{Cos}[c+dx]} \sqrt{a+b \text{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \text{Cos}[c+dx]} \text{Sin}[c+dx]}{b \sqrt{\text{Cos}[c+dx]}} + \frac{1}{d} \right)$$

$$\sqrt{\text{Cos}[c+dx]} \sqrt{a+b \text{Cos}[c+dx]} \left( \frac{2}{15} \text{Sec}[c+dx] (9 a^2 A \text{Sin}[c+dx] + 23 A b^2 \text{Sin}[c+dx] + 15 a^2 C \text{Sin}[c+dx]) + \right.$$

$$\left. \frac{22}{15} a A b \text{Sec}[c+dx] \text{Tan}[c+dx] + \frac{2}{5} a^2 A \text{Sec}[c+dx]^2 \text{Tan}[c+dx] \right)$$

- **Problem 745: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \text{Cos}[c+dx])^{5/2} (A+C \text{Cos}[c+dx]^2)}{\text{Cos}[c+dx]^{9/2}} dx$$

Optimal (type 4, 540 leaves, 8 steps):

$$\frac{1}{21 a^2 d} 2 (a-b) b \sqrt{a+b} (3 A b^2 + a^2 (29 A + 49 C)) \text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\text{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} - \frac{1}{21 a d} 2 \sqrt{a+b} (3 A b^3 - 9 a b^2 (3 A + 7 C) - a^3 (5 A + 7 C) + a^2 b (29 A + 49 C))$$

$$\text{Cot}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\text{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} - \frac{1}{d}$$

$$2 b^2 \sqrt{a+b} C \text{Cot}[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \text{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\text{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} +$$

$$\frac{2(3 A b^2 + a^2 (5 A + 7 C)) \sqrt{a+b \text{Cos}[c+dx]} \text{Sin}[c+dx]}{21 d \text{Cos}[c+dx]^{3/2}} + \frac{2 A b (a+b \text{Cos}[c+dx])^{3/2} \text{Sin}[c+dx]}{7 d \text{Cos}[c+dx]^{5/2}} + \frac{2 A (a+b \text{Cos}[c+dx])^{5/2} \text{Sin}[c+dx]}{7 d \text{Cos}[c+dx]^{7/2}}$$

Result (type 4, 1378 leaves):

$$\begin{aligned}
& \frac{1}{21 a d} \left( - \left( 4 a (5 a^4 A - 2 a^2 A b^2 - 3 A b^4 + 7 a^4 C + 14 a^2 b^2 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a (-29 a^3 A b - 3 a A b^3 - 49 a^3 b C + 21 a b^3 C) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2 (-29 a^2 A b^2 - 3 A b^4 - 49 a^2 b^2 C) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right) + \frac{1}{d} \\
& \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2}{21} \operatorname{Sec}[c+dx]^2 (5 a^2 A \operatorname{Sin}[c+dx] + 9 A b^2 \operatorname{Sin}[c+dx] + 7 a^2 C \operatorname{Sin}[c+dx]) + \right. \\
& \quad \left. \frac{2 \operatorname{Sec}[c+dx] (29 a^2 A b \operatorname{Sin}[c+dx] + 3 A b^3 \operatorname{Sin}[c+dx] + 49 a^2 b C \operatorname{Sin}[c+dx])}{21 a} + \right. \\
& \quad \left. \frac{6}{7} a A b \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] + \right. \\
& \quad \left. \frac{2}{7} a^2 A \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx] \right)
\end{aligned}$$

■ **Problem 746:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \cos[c + dx])^{5/2} (A + C \cos[c + dx]^2)}{\cos[c + dx]^{11/2}} dx$$

Optimal (type 4, 504 leaves, 7 steps):

$$-\frac{1}{315 a^3 d} 2 (a - b) \sqrt{a + b} (10 A b^4 - 21 a^4 (7 A + 9 C) - 3 a^2 b^2 (93 A + 161 C))$$

$$\cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} -$$

$$\frac{1}{315 a^2 d} 2 (a - b) \sqrt{a + b} (10 A b^3 + 21 a^3 (7 A + 9 C) + 15 a b^2 (11 A + 21 C) - 6 a^2 b (19 A + 28 C)) \cot[c + dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} +$$

$$\frac{2(15 A b^2 + 7 a^2 (7 A + 9 C)) \sqrt{a + b \cos[c + dx]} \sin[c + dx]}{315 d \cos[c + dx]^{5/2}} + \frac{2b(5 A b^2 + a^2 (163 A + 231 C)) \sqrt{a + b \cos[c + dx]} \sin[c + dx]}{315 a d \cos[c + dx]^{3/2}} +$$

$$\frac{10 A b (a + b \cos[c + dx])^{3/2} \sin[c + dx]}{63 d \cos[c + dx]^{7/2}} + \frac{2 A (a + b \cos[c + dx])^{5/2} \sin[c + dx]}{9 d \cos[c + dx]^{9/2}}$$

Result (type 4, 1485 leaves):

$$-\frac{1}{315 a^2 d} \left( \left( 4 a (-114 a^4 A b + 124 a^2 A b^3 - 10 A b^5 - 168 a^4 b C + 168 a^2 b^3 C) \sqrt{\frac{(a + b) \cot\left[\frac{1}{2}(c + dx)\right]^2}{-a + b}} \sqrt{\frac{(a + b) \cos[c + dx] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \operatorname{Csc}[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a + b}\right] \sin\left[\frac{1}{2}(c + dx)\right]^4 \right) / \right.$$

$$\left. \left( (a + b) \sqrt{\cos[c + dx]} \sqrt{a + b \cos[c + dx]} \right) - 4 a (147 a^5 A + 279 a^3 A b^2 - 10 a A b^4 + 189 a^5 C + 483 a^3 b^2 C) \right.$$

$$\left. \left( \left( \sqrt{\frac{(a + b) \cot\left[\frac{1}{2}(c + dx)\right]^2}{-a + b}} \sqrt{\frac{(a + b) \cos[c + dx] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \operatorname{Csc}[c + dx] \right) \right) \right)$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right) / \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \text{Csc}[c+dx]\text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right) / \right. \\
& \left. \left(b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) \right) + 2(147a^4Ab + 279a^2Ab^3 - 10Ab^5 + 189a^4bC + 483a^2b^3C) \\
& \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\text{Sec}[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\text{Sec}[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\text{Sec}[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b}2a\left(\left(a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \text{Csc}[c+dx]\right) \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right) / \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \quad \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \Bigg) + \\
& \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2}{315} \operatorname{Sec}[c+dx]^3 (49 a^2 A \operatorname{Sin}[c+dx] + 75 A b^2 \operatorname{Sin}[c+dx] + 63 a^2 C \operatorname{Sin}[c+dx]) + \right. \\
& \quad \left. \frac{2 \operatorname{Sec}[c+dx]^2 (163 a^2 A b \operatorname{Sin}[c+dx] + 5 A b^3 \operatorname{Sin}[c+dx] + 231 a^2 b C \operatorname{Sin}[c+dx])}{315 a} + \right. \\
& \quad \frac{1}{315 a^2} \\
& \quad \left. 2 \operatorname{Sec}[c+dx] (147 a^4 A \operatorname{Sin}[c+dx] + 279 a^2 A b^2 \operatorname{Sin}[c+dx] - 10 A b^4 \operatorname{Sin}[c+dx] + 189 a^4 C \operatorname{Sin}[c+dx] + 483 a^2 b^2 C \operatorname{Sin}[c+dx]) + \right. \\
& \quad \frac{38}{63} a A b \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx] + \\
& \quad \left. \frac{2}{9} a^2 A \operatorname{Sec}[c+dx]^4 \operatorname{Tan}[c+dx] \right)
\end{aligned}$$

- **Problem 747: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{5/2} (A+C \operatorname{Cos}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{13/2}} dx$$

Optimal (type 4, 587 leaves, 8 steps):



$$\frac{1}{693 a^4 d} 2 (a-b) b \sqrt{a+b} (8 A b^4 + 3 a^2 b^2 (17 A + 33 C) + a^4 (741 A + 957 C)) \operatorname{Cot}[c + d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c + d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c + d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c + d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c + d x])}{a-b}} + \frac{1}{693 a^3 d}$$

$$2 (a-b) \sqrt{a+b} (6 a A b^3 + 8 A b^4 + 15 a^4 (9 A + 11 C) + 3 a^2 b^2 (19 A + 33 C) - 6 a^3 b (101 A + 132 C)) \operatorname{Cot}[c + d x]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c + d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c + d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c + d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c + d x])}{a-b}} +$$

$$\frac{2 (5 A b^2 + 3 a^2 (9 A + 11 C)) \sqrt{a+b} \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{231 d \operatorname{Cos}[c + d x]^{7/2}} + \frac{2 b (3 A b^2 + a^2 (229 A + 297 C)) \sqrt{a+b} \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{693 a d \operatorname{Cos}[c + d x]^{5/2}} -$$

$$\frac{2 (4 A b^4 - 15 a^4 (9 A + 11 C) - a^2 b^2 (205 A + 297 C)) \sqrt{a+b} \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{693 a^2 d \operatorname{Cos}[c + d x]^{3/2}} +$$

$$\frac{10 A b (a+b \operatorname{Cos}[c + d x])^{3/2} \operatorname{Sin}[c + d x]}{99 d \operatorname{Cos}[c + d x]^{9/2}} + \frac{2 A (a+b \operatorname{Cos}[c + d x])^{5/2} \operatorname{Sin}[c + d x]}{11 d \operatorname{Cos}[c + d x]^{11/2}}$$

Result (type 4, 1591 leaves):

$$\frac{1}{693 a^3 d} \left( - \left( 4 a (135 a^6 A - 78 a^4 A b^2 - 49 a^2 A b^4 - 8 A b^6 + 165 a^6 C - 66 a^4 b^2 C - 99 a^2 b^4 C) \right. \right.$$

$$\left. \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right.$$

$$\left. \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a (-741 a^5 A b - 51 a^3 A b^3 - 8 a A b^5 - 957 a^5 b C - 99 a^3 b^3 C)$$

$$\left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + 2(-741 a^4 A b^2 - 51 a^2 A b^4 - 8 A b^6 - 957 a^4 b^2 c - 99 a^2 b^4 c) \\
& \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \text{Csc}[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
\left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
\left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \Bigg) + \\
\frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2}{693} \operatorname{Sec}[c+dx]^4 (81 a^2 A \operatorname{Sin}[c+dx] + 113 A b^2 \operatorname{Sin}[c+dx] + 99 a^2 C \operatorname{Sin}[c+dx]) + \right. \\
\frac{2 \operatorname{Sec}[c+dx]^3 (229 a^2 A b \operatorname{Sin}[c+dx] + 3 A b^3 \operatorname{Sin}[c+dx] + 297 a^2 b C \operatorname{Sin}[c+dx])}{693 a} + \frac{1}{693 a^2} \\
2 \operatorname{Sec}[c+dx]^2 (135 a^4 A \operatorname{Sin}[c+dx] + 205 a^2 A b^2 \operatorname{Sin}[c+dx] - 4 A b^4 \operatorname{Sin}[c+dx] + 165 a^4 C \operatorname{Sin}[c+dx] + 297 a^2 b^2 C \operatorname{Sin}[c+dx]) + \\
\frac{1}{693 a^3} 2 \operatorname{Sec}[c+dx] (741 a^4 A b \operatorname{Sin}[c+dx] + 51 a^2 A b^3 \operatorname{Sin}[c+dx] + 8 A b^5 \operatorname{Sin}[c+dx] + 957 a^4 b C \operatorname{Sin}[c+dx] + 99 a^2 b^3 C \operatorname{Sin}[c+dx]) + \\
\left. \frac{46}{99} a A b \operatorname{Sec}[c+dx]^4 \operatorname{Tan}[c+dx] + \frac{2}{11} a^2 A \operatorname{Sec}[c+dx]^5 \operatorname{Tan}[c+dx] \right)$$

■ **Problem 748: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^{3/2} (A + C \operatorname{Cos}[c+dx]^2)}{\sqrt{a+b \operatorname{Cos}[c+dx]}} dx$$

Optimal (type 4, 554 leaves, 8 steps):

$$\begin{aligned}
& -\frac{1}{24 a b^3 d} (a-b) \sqrt{a+b} (15 a^2 C+8 b^2 (3 A+2 C)) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{24 b^3 d} \sqrt{a+b} (15 a^2 C-10 a b C+8 b^2 (3 A+2 C)) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{8 b^4 d} \\
& a \sqrt{a+b} (8 A b^2+5 a^2 C+4 b^2 C) \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{(15 a^2 C+8 b^2 (3 A+2 C)) \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{24 b^3 d \sqrt{\operatorname{Cos}[c+d x]}} - \\
& \frac{5 a C \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{12 b^2 d} + \frac{C \operatorname{Cos}[c+d x]^{3/2} \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 b d}
\end{aligned}$$

Result (type 4, 1216 leaves):

$$\begin{aligned}
& \frac{1}{48 b^2 d} \\
& \left( \left( \left( 4 a (24 A b^2+5 a^2 C+16 b^2 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \right. \\
& \left. \left. \left. \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right],-\frac{2 a}{-a+b}\right] \right. \right. \right. \\
& \left. \left. \left. \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 16 a^2 b c \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2(24Ab^2 + 15a^2C + 16b^2C) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\ \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\ \left. \frac{\sqrt{a+b \cos[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left(-\frac{5a \operatorname{CSc}[c+dx]}{12b^2} + \frac{\operatorname{CSc}[2(c+dx)]}{6b}\right)}{d}$$

■ **Problem 749: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]} (A + C \cos[c+dx]^2)}{\sqrt{a+b \cos[c+dx]}} dx$$

Optimal (type 4, 455 leaves, 7 steps):

$$\frac{1}{4b^2d} 3(a-b) \sqrt{a+b} C \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{4b^2d} \\ (3a-2b) \sqrt{a+b} C \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\ \frac{1}{4b^3d} \sqrt{a+b} (3a^2C + 4b^2(2A+C)) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \\ \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{3aC \sqrt{a+b \cos[c+dx]} \operatorname{Sin}[c+dx]}{4b^2d \sqrt{\cos[c+dx]}} + \frac{C \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \operatorname{Sin}[c+dx]}{2bd}$$

Result (type 4, 1169 leaves):

$$\frac{C \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \operatorname{Sin}[c+dx]}{2bd} -$$

$$\begin{aligned}
& \frac{1}{8bd} \left( - \left( 4a^2 C \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \quad \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a \right. \\
& \quad (-8Ab - 4bC) \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \quad \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \quad \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \quad \left. \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + \\
& \quad 6aC \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx] \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}}} \right) +
\end{aligned}$$

$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\ \left. \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right)$$

■ **Problem 750: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + C \operatorname{Cos}[c+dx]^2}{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]}} dx$$

Optimal (type 4, 393 leaves, 6 steps):



$$\begin{aligned}
& -\frac{1}{abd} (a-b) \sqrt{a+b} C \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{abd} \\
& \sqrt{a+b} (2Ab+aC) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{1}{b^2 d} a \sqrt{a+b} C \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{C \sqrt{a+b} \cos[c+dx] \sin[c+dx]}{bd \sqrt{\cos[c+dx]}}
\end{aligned}$$

Result (type 4, 741 leaves):

$$\begin{aligned}
& \frac{1}{2d} \left( -4a(2A+C) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& 2C \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx] \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.
\end{aligned}$$

$$\left. \left( \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right) / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \right.$$

$$\left( a \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \right.$$

$$\left. \left. \text{Csc} [c+d x] \text{EllipticPi} \left[ -\frac{a}{b}, \text{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right) / \right.$$

$$\left. \left. \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) \right)$$

■ **Problem 751: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos [c+d x]^2}{\cos [c+d x]^{3/2} \sqrt{a+b \cos [c+d x]}} dx$$

Optimal (type 4, 343 leaves, 6 steps):

$$\frac{1}{a^2 d} 2 A (a-b) \sqrt{a+b} \cot [c+d x] \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} -$$

$$\frac{2 A \sqrt{a+b} \cot [c+d x] \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}}{a d} - \frac{1}{b d}$$

$$2 \sqrt{a+b} C \cot [c+d x] \text{EllipticPi} \left[ \frac{a+b}{b}, \text{ArcSin} \left[ \frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}$$

Result (type 4, 2642 leaves):

$$\begin{aligned}
& \frac{2 A \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{a d \sqrt{\cos [c+d x]}} - \left( (1+\cos [c+d x])^{3 / 2} \left( -\frac{A \sqrt{\cos [c+d x]}}{\sqrt{a+b \cos [c+d x]}} + \frac{C \sqrt{\cos [c+d x]}}{\sqrt{a+b \cos [c+d x]}} - \frac{2 A b \cos [c+d x]^{3 / 2}}{a \sqrt{a+b \cos [c+d x]}} \right) \right. \\
& \sec \left[ \frac{1}{2} (c+d x) \right]^2 \left( 2 A (a+b) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan \left[ \frac{1}{2} (c+d x) \right]\right], \frac{-a+b}{a+b}\right] - \right. \\
& 2 a (A-C) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan \left[ \frac{1}{2} (c+d x) \right]\right], \frac{-a+b}{a+b}\right] + \\
& 4 a C \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan \left[ \frac{1}{2} (c+d x) \right]\right], \frac{-a+b}{a+b}\right] + A b \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \\
& \left. \left. \sec \left[ \frac{1}{2} (c+d x) \right] \sin \left[ \frac{3}{2} (c+d x) \right] + 2 a A \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[ \frac{1}{2} (c+d x) \right] - A b \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[ \frac{1}{2} (c+d x) \right] \right) \right) / \\
& \left( 2 a d \sqrt{a+b \cos [c+d x]} \left( -\frac{1}{4 a (a+b \cos [c+d x])^{3 / 2}} b (1+\cos [c+d x])^{3 / 2} \sec \left[ \frac{1}{2} (c+d x) \right]^2 \sin [c+d x] \right. \right. \\
& \left. \left( 2 A (a+b) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan \left[ \frac{1}{2} (c+d x) \right]\right], \frac{-a+b}{a+b}\right] - \right. \right. \\
& 2 a (A-C) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan \left[ \frac{1}{2} (c+d x) \right]\right], \frac{-a+b}{a+b}\right] + 4 a C \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \\
& \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan \left[ \frac{1}{2} (c+d x) \right]\right], \frac{-a+b}{a+b}\right] + A b \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sec \left[ \frac{1}{2} (c+d x) \right] \sin \left[ \frac{3}{2} (c+d x) \right] + \\
& \left. \left. 2 a A \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[ \frac{1}{2} (c+d x) \right] - A b \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[ \frac{1}{2} (c+d x) \right] \right) + \frac{1}{4 a \sqrt{a+b \cos [c+d x]}} \right. \\
& \left. 3 \sqrt{1+\cos [c+d x]} \sec \left[ \frac{1}{2} (c+d x) \right]^2 \sin [c+d x] \left( 2 A (a+b) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan \left[ \frac{1}{2} (c+d x) \right]\right], \frac{-a+b}{a+b}\right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 a (A - C) \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] + \\
& 4 a C \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] + A b \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \\
& \left. \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \operatorname{Sin}\left[\frac{3}{2} (c + d x)\right] + 2 a A \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - A b \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right) - \\
& \frac{1}{2 a \sqrt{a + b \cos [c + d x]}} (1 + \cos [c + d x])^{3/2} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \left(2 A (a + b) \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticE}\left[\right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] - 2 a (A - C) \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] + \right. \\
& \left. 4 a C \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] + A b \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \operatorname{Sin}\left[\frac{3}{2} (c + d x)\right] + 2 a A \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - A b \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right) - \\
& \frac{1}{2 a \sqrt{a + b \cos [c + d x]}} (1 + \cos [c + d x])^{3/2} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \left(\frac{3}{2} A b \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \cos\left[\frac{3}{2} (c + d x)\right] \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] + \right. \\
& \left. a A \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 - \frac{1}{2} A b \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 + \frac{1}{\sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}}} A (a + b) \operatorname{EllipticE}\left[\right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] \left(-\frac{b \sin [c + d x]}{(a + b) (1 + \cos [c + d x])} + \frac{(a + b \cos [c + d x]) \sin [c + d x]}{(a + b) (1 + \cos [c + d x])^2}\right) - \frac{1}{\sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}}} a (A - C) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \left( -\frac{b \sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(a+b \cos[c+dx]) \sin[c+dx]}{(a+b)(1+\cos[c+dx])^2} \right) + \frac{1}{\sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} \\
& 2 a C \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \left( -\frac{b \sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(a+b \cos[c+dx]) \sin[c+dx]}{(a+b)(1+\cos[c+dx])^2} \right) + \\
& \frac{A b \text{Sec}\left[\frac{1}{2}(c+dx)\right] \left( \frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} - \frac{\sin[c+dx]}{1+\cos[c+dx]} \right) \text{Sin}\left[\frac{3}{2}(c+dx)\right] + a A \left( \frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} - \frac{\sin[c+dx]}{1+\cos[c+dx]} \right) \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{2 \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}}} + \frac{\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}}}{\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}}} \\
& \frac{A b \left( \frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} - \frac{\sin[c+dx]}{1+\cos[c+dx]} \right) \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{2 \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}}} + \frac{1}{2} A b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{Sec}\left[\frac{1}{2}(c+dx)\right] \text{Sin}\left[\frac{3}{2}(c+dx)\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right] - \\
& \frac{a(A-C) \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{1-\frac{(-a+b) \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} - \frac{2 a C \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{1-\frac{(-a+b) \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} + \\
& \frac{A(a+b) \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\frac{(-a+b) \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}{\sqrt{1-\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) \right)
\end{aligned}$$

■ **Problem 752: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c+dx]^2}{\cos[c+dx]^{5/2} \sqrt{a+b \cos[c+dx]}} dx$$

Optimal (type 4, 283 leaves, 4 steps):

$$\begin{aligned}
& -\frac{1}{3a^3d} 4A(a-b)b\sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Cos}[c+dx]}{\sqrt{a+b}\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{3a^2d} \\
& 2\sqrt{a+b} (2Ab+a(A+3C)) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Cos}[c+dx]}{\sqrt{a+b}\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \\
& \frac{2A\sqrt{a+b}\operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{3ad\operatorname{Cos}[c+dx]^{3/2}}
\end{aligned}$$

Result (type 4, 1204 leaves):

$$\begin{aligned}
& \frac{1}{3a^2d} \\
& \left( - \left( 4a(a^2A+2Ab^2+3a^2C) \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\operatorname{Cos}[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\operatorname{Cos}[c+dx]}\sqrt{a+b\operatorname{Cos}[c+dx]} \right) - \right. \\
& \left. 8a^2Ab \left( \left( \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\operatorname{Cos}[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\operatorname{Cos}[c+dx]}\sqrt{a+b\operatorname{Cos}[c+dx]} \right) - \right. \\
& \left. \left( \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\operatorname{Cos}[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4 \right/ \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) \right\} + \\
& 4 A b^2 \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Sec}[c+d x] \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Sec}[c+d x]}{a+b}}}} + \right. \\
& \left. \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right. \\
& \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4 \right) \right/ \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\
& \left. \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4 \right) \right/ \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) \right) \right\} + \\
& \left. \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) + \frac{\sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left(-\frac{4 A b \tan [c+d x]}{3 a^2} + \frac{2 A \operatorname{Sec}[c+d x] \tan [c+d x]}{3 a}\right)}{d}
\end{aligned}$$

■ **Problem 753: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos [c + d x]^2}{\cos [c + d x]^{7/2} \sqrt{a + b \cos [c + d x]}} dx$$

Optimal (type 4, 354 leaves, 5 steps):

$$\frac{1}{15 a^4 d} 2 (a - b) \sqrt{a + b} (8 A b^2 + 3 a^2 (3 A + 5 C)) \cot [c + d x]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \sec [c + d x])}{a + b}} \sqrt{\frac{a (1 + \sec [c + d x])}{a - b}} + \frac{1}{15 a^3 d}$$

$$2 \sqrt{a + b} (2 a A b - 8 A b^2 - 3 a^2 (3 A + 5 C)) \cot [c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \sec [c + d x])}{a + b}}$$

$$\sqrt{\frac{a (1 + \sec [c + d x])}{a - b}} + \frac{2 A \sqrt{a + b \cos [c + d x]} \sin [c + d x]}{5 a d \cos [c + d x]^{5/2}} - \frac{8 A b \sqrt{a + b \cos [c + d x]} \sin [c + d x]}{15 a^2 d \cos [c + d x]^{3/2}}$$

Result (type 4, 1298 leaves):

$$-\frac{1}{15 a^3 d} \left( - \left( 4 a (7 a^2 A b + 8 A b^3 + 15 a^2 b C) \sqrt{\frac{(a + b) \cot \left[ \frac{1}{2} (c + d x) \right]^2}{-a + b}} \sqrt{-\frac{(a + b) \cos [c + d x] \csc \left[ \frac{1}{2} (c + d x) \right]^2}{a}} \right. \right. \\ \left. \left. \sqrt{\frac{(a + b \cos [c + d x]) \csc \left[ \frac{1}{2} (c + d x) \right]^2}{a}} \csc [c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a + b \cos [c + d x]) \csc \left[ \frac{1}{2} (c + d x) \right]^2}{a}}}{\sqrt{2}}\right], -\frac{2 a}{-a + b}\right] \sin \left[ \frac{1}{2} (c + d x) \right]^4 \right) / \right. \\ \left. \left( (a + b) \sqrt{\cos [c + d x]} \sqrt{a + b \cos [c + d x]} \right) - 4 a (9 a^3 A + 8 a A b^2 + 15 a^3 C) \right. \\ \left. \left( \left( \sqrt{\frac{(a + b) \cot \left[ \frac{1}{2} (c + d x) \right]^2}{-a + b}} \sqrt{-\frac{(a + b) \cos [c + d x] \csc \left[ \frac{1}{2} (c + d x) \right]^2}{a}} \sqrt{\frac{(a + b \cos [c + d x]) \csc \left[ \frac{1}{2} (c + d x) \right]^2}{a}} \csc [c + d x] \right. \right. \right. \\ \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a + b \cos [c + d x]) \csc \left[ \frac{1}{2} (c + d x) \right]^2}{a}}}{\sqrt{2}}\right], -\frac{2 a}{-a + b}\right] \sin \left[ \frac{1}{2} (c + d x) \right]^4 \right) / \left( (a + b) \sqrt{\cos [c + d x]} \sqrt{a + b \cos [c + d x]} \right) - \right. \right. \right.$$



$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& 2(9a^2Ab + 8Ab^3 + 15a^2bC) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \left. \csc[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left( \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \frac{1}{d}$$

$$\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2 \sec[c+dx] (9 a^2 A \sin[c+dx] + 8 A b^2 \sin[c+dx] + 15 a^2 C \sin[c+dx])}{15 a^3} - \frac{8 A b \sec[c+dx] \tan[c+dx]}{15 a^2} + \frac{2 A \sec[c+dx]^2 \tan[c+dx]}{5 a} \right)$$

■ **Problem 754: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c+dx]^2}{\cos[c+dx]^{9/2} \sqrt{a+b \cos[c+dx]}} dx$$

Optimal (type 4, 429 leaves, 6 steps):

$$-\frac{1}{105 a^5 d} 4 (a-b) b \sqrt{a+b} (24 A b^2 + a^2 (22 A + 35 C)) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{105 a^4 d}$$

$$2 \sqrt{a+b} (12 a A b^2 - 48 A b^3 - 5 a^3 (5 A + 7 C) - a^2 (44 A b + 70 b C)) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{2 A \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{7 a d \cos[c+dx]^{7/2}} -$$

$$\frac{12 A b \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{35 a^2 d \cos[c+dx]^{5/2}} + \frac{2 (24 A b^2 + 5 a^2 (5 A + 7 C)) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{105 a^3 d \cos[c+dx]^{3/2}}$$

Result (type 4, 1376 leaves):

$$\frac{1}{105 a^4 d} \left( - \left( 4 a (25 a^4 A + 32 a^2 A b^2 + 48 A b^4 + 35 a^4 C + 70 a^2 b^2 C) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a-b}} \sqrt{\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right) \right)$$

$$\left. \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right)} \right/$$

$$\left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - 4 a \left( 44 a^3 A b + 48 a A b^3 + 70 a^3 b C \right)$$

$$\left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right) \right/ \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \right.$$

$$\left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right.$$

$$\left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right) \right/ \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) \right) +$$

$$2 \left( 44 a^2 A b^2 + 48 A b^4 + 70 a^2 b^2 C \right) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Sec}[c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Sec}[c+d x]}{a+b}} \right)$$

$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \right. \\
& \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} + \frac{1}{d} \right) \\
& \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2 \sec[c+dx]^2 (25 a^2 A \sin[c+dx] + 24 A b^2 \sin[c+dx] + 35 a^2 C \sin[c+dx])}{105 a^3} - \right. \\
& \left. \frac{4 \sec[c+dx] (22 a^2 A b \sin[c+dx] + 24 A b^3 \sin[c+dx] + 35 a^2 b C \sin[c+dx])}{105 a^4} - \right. \\
& \left. \frac{12 A b \sec[c+dx]^2 \tan[c+dx]}{35 a^2} + \right. \\
& \left. \frac{2 A \sec[c+dx]^3 \tan[c+dx]}{7 a} \right)
\end{aligned}$$

■ **Problem 755: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{3/2} (A + C \cos[c+dx]^2)}{(a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 604 leaves, 8 steps):

$$\frac{1}{4 b^3 \sqrt{a+b} d} (8 A b^2 + 15 a^2 C - 7 b^2 C) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{4 b^3 \sqrt{a+b} d}$$

$$(8 A b^2 + (15 a^2 + 5 a b - 2 b^2) C) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} -$$

$$\frac{1}{4 b^4 d} \sqrt{a+b} (8 A b^2 + 15 a^2 C + 4 b^2 C) \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{2(A b^2 + a^2 C) \operatorname{Cos}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{b(a^2 - b^2) d \sqrt{a+b} \operatorname{Cos}[c+d x]}$$

$$\frac{a(8 A b^2 + 15 a^2 C - 7 b^2 C) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{4 b^3 (a^2 - b^2) d \sqrt{\operatorname{Cos}[c+d x]}} + \frac{(4 A b^2 + 5 a^2 C - b^2 C) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{2 b^2 (a^2 - b^2) d}$$

Result (type 4, 1276 leaves):

$$\frac{\sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b} \operatorname{Cos}[c+d x] \left( \frac{C \operatorname{Sin}[c+d x]}{2 b^2} - \frac{2(a A b^2 \operatorname{Sin}[c+d x] + a^3 C \operatorname{Sin}[c+d x])}{b^2 (-a^2 + b^2) (a+b) \operatorname{Cos}[c+d x]} \right)}{d}$$

$$\frac{1}{8(a-b) b^2 (a+b) d} \left( - \left( 4 a (5 a^3 C - 5 a b^2 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b) \operatorname{Cos}[c+d x]} \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right/$$

$$\left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b} \operatorname{Cos}[c+d x] \right) - 4 a (8 A b^3 + 4 a^2 b C + 4 b^3 C)$$

$$\left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) + \\
& 2(8aAb^2 + 15a^3c - 7ab^2c) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b}2a \left( \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right. \\
& \left. \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right) \right)
\end{aligned}$$

$$\left( \left( \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right)$$

■ **Problem 756: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]} (A + C \cos[c+dx]^2)}{(a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 503 leaves, 7 steps):

$$-\frac{1}{ab^2 \sqrt{a+b} d} (2Ab^2 + 3a^2C - b^2C) \cot[c+dx] \\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{ab^2 \sqrt{a+b} d} \\ (2Ab^2 + a(3a+b)C) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\ \frac{1}{b^3 d} 3a \sqrt{a+b} C \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\ \frac{2(Ab^2 + a^2C) \sqrt{\cos[c+dx]} \sin[c+dx]}{b(a^2 - b^2)d \sqrt{a+b \cos[c+dx]}} + \frac{(2Ab^2 + 3a^2C - b^2C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b^2(a^2 - b^2)d \sqrt{\cos[c+dx]}}$$

Result (type 4, 1234 leaves):

$$\frac{2 \sqrt{\cos[c+dx]} (Ab^2 \sin[c+dx] + a^2 C \sin[c+dx])}{b(-a^2 + b^2)d \sqrt{a+b \cos[c+dx]}} + \frac{1}{2(a-b)b(a+b)d}$$

$$\begin{aligned}
& \left( - \left( 4 a (a^2 C - b^2 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left. 4 a (2 a A b + 2 a b C) \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+ \right. \right. \right. \\
& \quad \left. \left. \left. dx \right] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \quad \left. 2 (2 A b^2 + 3 a^2 C - b^2 C) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right]\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \right.
\end{aligned}$$



$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\ \left. \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right)$$

- **Problem 757: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Cos}[c+dx]^2}{\sqrt{\operatorname{Cos}[c+dx]} (a+b \operatorname{Cos}[c+dx])^{3/2}} dx$$

Optimal (type 4, 421 leaves, 6 steps):

$$\frac{2 (A b^2 + a^2 C) \cot [c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos [c+d x]}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}}{a^2 b \sqrt{a+b} d} +$$

$$\frac{2 (A b - a C) \cot [c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos [c+d x]}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}}{a b \sqrt{a+b} d} - \frac{1}{b^2 d}$$

$$\frac{2 \sqrt{a+b} C \cot [c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos [c+d x]}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}}{2 (A b^2 + a^2 C) \sin [c + d x]}$$

$$b (a^2 - b^2) d \sqrt{\cos [c + d x]} \sqrt{a + b \cos [c + d x]}$$

Result (type 4, 1225 leaves):

$$\frac{2 \sqrt{\cos [c + d x]} (A b^2 \sin [c + d x] + a^2 C \sin [c + d x])}{a (a^2 - b^2) d \sqrt{a + b \cos [c + d x]}} + \frac{1}{a (a - b) (a + b) d}$$

$$\left( \left( -4 a (a^2 A - A b^2) \sqrt{\frac{(a+b) \cot \left[\frac{1}{2} (c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2} (c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - 4 a \right.$$

$$\left. (-a A b - a b C) \left( \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2} (c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2} (c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \right.$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \Bigg) + \\
& 2(-Ab^2 - a^2c) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /
\end{aligned}$$

$$\left( \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right)$$

■ **Problem 758: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c+dx]^2}{\cos[c+dx]^{3/2} (a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 308 leaves, 4 steps):

$$\begin{aligned} & - \frac{1}{a^3 \sqrt{a+b} d} 2 (2 A b^2 - a^2 (A - C)) \cot[c+dx] \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\ & \frac{1}{a^2 \sqrt{a+b} d} 2 (2 A b + a (A - C)) \cot[c+dx] \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\ & \frac{2 (A b^2 + a^2 C) \sin[c+dx]}{a (a^2 - b^2) d \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}} \end{aligned}$$

Result (type 4, 1269 leaves):

$$\begin{aligned} & \frac{1}{a^2 (-a+b) (a+b) d} \\ & \left( - \left( 4 a (2 a^2 A b - 2 A b^3) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ & \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}{\sqrt{2}}} \right], -\frac{2a}{-a+b} \right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\ & \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4 a (a^3 A - 2 a A b^2 - a^3 C) \end{aligned}$$

$$\begin{aligned}
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left. \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2(a^2 A b - 2 A b^3 - a^2 b C) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\ \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \\ \left. \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) + \frac{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( -\frac{2(Ab^3 \operatorname{Sin}[c+dx] + a^2 b C \operatorname{Sin}[c+dx])}{a^2 (a^2 - b^2) (a+b \operatorname{Cos}[c+dx])} + \frac{2A \operatorname{Tan}[c+dx]}{a^2} \right)}{d}$$

■ **Problem 759: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Cos}[c+dx]^2}{\operatorname{Cos}[c+dx]^{5/2} (a+b \operatorname{Cos}[c+dx])^{3/2}} dx$$

Optimal (type 4, 392 leaves, 5 steps):

$$\frac{1}{3a^4 \sqrt{a+b} d} - 2b(8Ab^2 - a^2(5A - 3C)) \operatorname{Cot}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1 - \operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{3a^3 \sqrt{a+b} d}$$

$$2(6aAb + 8Ab^2 + a^2(A + 3C)) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1 - \operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{2(Ab^2 + a^2C) \operatorname{Sin}[c+dx]}{a(a^2 - b^2) d \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+b \operatorname{Cos}[c+dx]}} - \frac{2(4Ab^2 - a^2(A - 3C)) \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{3a^2(a^2 - b^2) d \operatorname{Cos}[c+dx]^{3/2}}$$

Result (type 4, 1327 leaves):

$$\frac{1}{3a^3(a-b)(a+b)d} \left( - \left( 4a(a^4A + 7a^2Ab^2 - 8Ab^4 + 3a^4C - 3a^2b^2C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\begin{aligned}
& \left. \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right\} / \\
& \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - 4 a \left( 5 a^3 A b - 8 a A b^3 - 3 a^3 b C \right) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right\} / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right\} / \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) \right) + \\
& 2 \left( 5 a^2 A b^2 - 8 A b^4 - 3 a^2 b^2 C \right) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Sec}[c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Sec}[c+d x]}{a+b}} \right. \\
& \left. \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \right. \\
& \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \\
& \frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2(Ab^4 \sin[c+dx] + a^2 b^2 C \sin[c+dx])}{a^3 (a^2 - b^2) (a+b \cos[c+dx])} - \frac{10Ab \tan[c+dx]}{3a^3} + \frac{2A \sec[c+dx] \tan[c+dx]}{3a^2} \right)}{d}
\end{aligned}$$

- **Problem 760: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c+dx]^2}{\cos[c+dx]^{7/2} (a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 494 leaves, 6 steps):



$$\begin{aligned}
& - \frac{1}{5 a^5 \sqrt{a+b} d} 2 (16 A b^4 - 2 a^2 b^2 (4 A - 5 C) - a^4 (3 A + 5 C)) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{5 a^4 \sqrt{a+b} d} \\
& 2 (12 a A b^2 + 16 A b^3 + 2 a^2 b (2 A + 5 C) + a^3 (3 A + 5 C)) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{2 (A b^2 + a^2 C) \operatorname{Sin}[c+d x]}{a (a^2 - b^2) d \operatorname{Cos}[c+d x]^{5/2} \sqrt{a+b} \operatorname{Cos}[c+d x]} - \\
& \frac{2 (6 A b^2 - a^2 (A - 5 C)) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{5 a^2 (a^2 - b^2) d \operatorname{Cos}[c+d x]^{5/2}} + \frac{2 b (8 A b^2 - a^2 (3 A - 5 C)) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{5 a^3 (a^2 - b^2) d \operatorname{Cos}[c+d x]^{3/2}}
\end{aligned}$$

Result (type 4, 1418 leaves):

$$\begin{aligned}
& \frac{1}{5 a^4 (-a+b) (a+b) d} \\
& \left( \left( - \left( 4 a (4 a^4 A b + 12 a^2 A b^3 - 16 A b^5 + 10 a^4 b C - 10 a^2 b^3 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2} (c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}} \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}}{a \sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]^4 \right) \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (3 a^5 A + 8 a^3 A b^2 - 16 a A b^4 + 5 a^5 C - 10 a^3 b^2 C) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2} (c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \\
& \left. (b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}) \right) + 2(3a^4Ab + 8a^2Ab^3 - 16Ab^5 + 5a^4bC - 10a^2b^3C) \\
& \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \text{Csc}[c+dx] \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\ \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\ \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} + \frac{1}{d} \\ \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2 \operatorname{Sec}[c+dx] (3 a^2 A \operatorname{Sin}[c+dx] + 11 A b^2 \operatorname{Sin}[c+dx] + 5 a^2 C \operatorname{Sin}[c+dx])}{5 a^4} - \right. \\ \left. \frac{2 (A b^5 \operatorname{Sin}[c+dx] + a^2 b^3 C \operatorname{Sin}[c+dx])}{a^4 (a^2 - b^2) (a+b \operatorname{Cos}[c+dx])} - \right. \\ \left. \frac{6 A b \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{5 a^3} + \right. \\ \left. \frac{2 A \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{5 a^2} \right)$$

- **Problem 761: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^{3/2} (A + C \operatorname{Cos}[c+dx]^2)}{(a+b \operatorname{Cos}[c+dx])^{5/2}} dx$$

Optimal (type 4, 650 leaves, 8 steps):

$$\begin{aligned}
& \frac{1}{3 a (a-b) b^3 (a+b)^{3/2} d} (8 A b^4 - 15 a^4 C + 26 a^2 b^2 C - 3 b^4 C) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{3 a (a-b) b^3 (a+b)^{3/2} d} (2 a A b^3 - 6 A b^4 + 15 a^4 C + 5 a^3 b C - 21 a^2 b^2 C - 3 a b^3 C) \\
& \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{b^4 d} \\
& 5 a \sqrt{a+b} C \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \\
& \frac{2(A b^2 + a^2 C) \operatorname{Cos}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{3 b (a^2 - b^2) d (a+b \operatorname{Cos}[c+d x])^{3/2}} + \frac{2(3 A b^4 - 5 a^4 C + a^2 b^2 (A + 9 C)) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 b^2 (a^2 - b^2)^2 d \sqrt{a+b \operatorname{Cos}[c+d x]}} - \\
& \frac{(8 A b^4 - (15 a^4 - 26 a^2 b^2 + 3 b^4) C) \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 b^3 (a^2 - b^2)^2 d \sqrt{\operatorname{Cos}[c+d x]}}
\end{aligned}$$

Result (type 4, 1366 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \\
& \left( -\frac{2(a A b^2 \operatorname{Sin}[c+d x] + a^3 C \operatorname{Sin}[c+d x])}{3 b^2 (-a^2 + b^2) (a+b \operatorname{Cos}[c+d x])^2} + \frac{4(2 A b^4 \operatorname{Sin}[c+d x] - 3 a^4 C \operatorname{Sin}[c+d x] + 5 a^2 b^2 C \operatorname{Sin}[c+d x])}{3 b^2 (-a^2 + b^2)^2 (a+b \operatorname{Cos}[c+d x])} \right) + \\
& \frac{1}{6 (a-b)^2 b^2 (a+b)^2 d} \left( -\left( 4 a (2 a^2 A b^2 - 2 A b^4 + 5 a^4 C - 8 a^2 b^2 C + 3 b^4 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (-8 a A b^3 + 4 a^3 b C - 12 a b^3 C)
\end{aligned}$$

$$\begin{aligned}
& \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \left. \right) + \\
& 2(-8Ab^4 + 15a^4c - 26a^2b^2c + 3b^4c) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\ \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\ \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \Bigg)$$

■ **Problem 762: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Cos}[c+dx]} (A + C \operatorname{Cos}[c+dx]^2)}{(a+b \operatorname{Cos}[c+dx])^{5/2}} dx$$

Optimal (type 4, 563 leaves, 7 steps):

$$-\frac{1}{3a^2b^2\sqrt{a+b}(a^2-b^2)d} 2(Ab^4 - 3a^4C + a^2b^2(3A+7C)) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\ \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{3a(a-b)b^2(a+b)^{3/2}d} 2(3aAb^2 - Ab^3 - 3a^3C - a^2bC + 6ab^2C) \\ \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{b^3d} \\ 2\sqrt{a+b} C \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \\ \frac{2(Ab^2+a^2C) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{3b(a^2-b^2)d(a+b \operatorname{Cos}[c+dx])^{3/2}} + \frac{2(Ab^4 - 3a^4C + a^2b^2(3A+7C)) \operatorname{Sin}[c+dx]}{3b^2(a^2-b^2)^2d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]}}$$

Result (type 4, 1388 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \\
& \left( \frac{2 (A b^2 \sin[c+dx] + a^2 C \sin[c+dx])}{3 b (-a^2 + b^2) (a + b \cos[c+dx])^2} + \frac{2 (-3 a^2 A b^2 \sin[c+dx] - A b^4 \sin[c+dx] + 3 a^4 C \sin[c+dx] - 7 a^2 b^2 C \sin[c+dx])}{3 a b (a^2 - b^2)^2 (a + b \cos[c+dx])} \right) - \\
& \frac{1}{3 a (a - b)^2 b (a + b)^2 d} \left( - \left( 4 a (a^2 A b^2 - A b^4 + a^4 C - a^2 b^2 C) \sqrt{\frac{(a + b) \cot\left[\frac{1}{2} (c + d x)\right]^2}{-a + b}} \sqrt{\frac{(a + b) \cos[c + d x] \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a + b \cos[c + d x]) \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a + b \cos[c + d x]) \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a + b}\right] \sin\left[\frac{1}{2} (c + d x)\right]^4 \right) / \right. \\
& \left. \left( (a + b) \sqrt{\cos[c + d x]} \sqrt{a + b \cos[c + d x]} \right) - 4 a (-3 a^3 A b - a A b^3 - a^3 b C - 3 a b^3 C) \right. \\
& \left( \left( \sqrt{\frac{(a + b) \cot\left[\frac{1}{2} (c + d x)\right]^2}{-a + b}} \sqrt{\frac{(a + b) \cos[c + d x] \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}} \sqrt{\frac{(a + b \cos[c + d x]) \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}} \operatorname{Csc}[c + d x] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a + b \cos[c + d x]) \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a + b}\right] \sin\left[\frac{1}{2} (c + d x)\right]^4 \right) / \left( (a + b) \sqrt{\cos[c + d x]} \sqrt{a + b \cos[c + d x]} \right) - \right. \\
& \left( \sqrt{\frac{(a + b) \cot\left[\frac{1}{2} (c + d x)\right]^2}{-a + b}} \sqrt{\frac{(a + b) \cos[c + d x] \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}} \sqrt{\frac{(a + b \cos[c + d x]) \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}} \operatorname{Csc}[c + d x] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a + b \cos[c + d x]) \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a + b}\right] \sin\left[\frac{1}{2} (c + d x)\right]^4 \right) / \left( b \sqrt{\cos[c + d x]} \sqrt{a + b \cos[c + d x]} \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 2(-3a^2Ab^2 - Ab^4 + 3a^4c - 7a^2b^2c) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \left. \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right)
\end{aligned}$$

■ **Problem 763: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c+dx]^2}{\sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{5/2}} dx$$

Optimal (type 4, 417 leaves, 5 steps):



$$\frac{1}{3 a^3 (a-b) (a+b)^{3/2} d}$$

$$4 b (3 a^2 A - A b^2 + 2 a^2 C) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} -$$

$$\frac{1}{3 a^2 \sqrt{a+b} (a^2-b^2) d} 2 (2 A b^2 + 3 a b (A+C) - a^2 (3 A+C)) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{2 (A b^2 + a^2 C) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 a (a^2-b^2) d (a+b \operatorname{Cos}[c+d x])^{3/2}} + \frac{4 b (A b^2 - a^2 (3 A+2 C)) \operatorname{Sin}[c+d x]}{3 a (a^2-b^2)^2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]}}$$

Result (type 4, 1364 leaves):

$$\frac{1}{d} \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \left( \frac{2 (A b^2 \operatorname{Sin}[c+d x] + a^2 C \operatorname{Sin}[c+d x])}{3 a (a^2-b^2) (a+b \operatorname{Cos}[c+d x])^2} + \frac{4 (3 a^2 A b^2 \operatorname{Sin}[c+d x] - A b^4 \operatorname{Sin}[c+d x] + 2 a^2 b^2 C \operatorname{Sin}[c+d x])}{3 a^2 (a^2-b^2)^2 (a+b \operatorname{Cos}[c+d x])} \right) +$$

$$\frac{1}{3 a^2 (a-b)^2 (a+b)^2 d} \left( - \left( 4 a (3 a^4 A - 5 a^2 A b^2 + 2 A b^4 + a^4 C - a^2 b^2 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2} (c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}}{a}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]^4 \right) /$$

$$\left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (-6 a^3 A b + 2 a A b^3 - 4 a^3 b C)$$

$$\left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2} (c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2}}{a}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) -$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \Bigg) + \\
& 2(-6a^2Ab^2 + 2Ab^4 - 4a^2b^2c) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}}$$

■ **Problem 764: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c+dx]^2}{\cos[c+dx]^{3/2} (a+b \cos[c+dx])^{5/2}} dx$$

Optimal (type 4, 449 leaves, 5 steps):

$$\frac{1}{3 a^4 \sqrt{a+b} (a^2-b^2) d} {}_2F_1\left(8 A b^4 + 3 a^4 (A-C) - a^2 b^2 (15 A+C), \cot[c+dx], \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right)$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{3 a^3 \sqrt{a+b} (a^2-b^2) d} {}_2F_1(6 a A b^2 + 8 A b^3 - 3 a^3 (A-C) - a^2 b (9 A+C), \cot[c+dx], \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b})$$

$$\frac{2(A b^2 + a^2 C) \sin[c+dx]}{3 a (a^2-b^2) d \sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{3/2}} - \frac{4(2 A b^4 - a^4 C - a^2 b^2 (4 A+C)) \sin[c+dx]}{3 a^2 (a^2-b^2)^2 d \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}}$$

Result (type 4, 1421 leaves):

$$-\frac{1}{3 a^3 (a-b)^2 (a+b)^2 d} \left( \left( 4 a (9 a^4 A b - 17 a^2 A b^3 + 8 A b^5 + a^4 b C - a^2 b^3 C) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right]}, -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right)$$

$$\left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4 a (3 a^5 A - 15 a^3 A b^2 + 8 a A b^4 - 3 a^5 C - a^3 b^2 C)$$

$$\begin{aligned}
& \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \quad \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + 2 \left( 3 a^4 A b - 15 a^2 A b^3 + 8 A b^5 - 3 a^4 b C - a^2 b^3 C \right) \\
& \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right] / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+d x] \operatorname{EllipticPi} \left[ -\frac{a}{b}, \text{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right] / \right. \\
& \left. \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) + \\
& \frac{1}{d} \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left( -\frac{2 (A b^3 \sin [c+d x] + a^2 b C \sin [c+d x])}{3 a^2 (a^2 - b^2) (a+b \cos [c+d x])^2} - \right. \\
& \left. \frac{2 (9 a^2 A b^3 \sin [c+d x] - 5 A b^5 \sin [c+d x] + 3 a^4 b C \sin [c+d x] + a^2 b^3 C \sin [c+d x])}{3 a^3 (a^2 - b^2)^2 (a+b \cos [c+d x])} + \right. \\
& \left. \frac{2 A \tan [c+d x]}{a^3} \right)
\end{aligned}$$

- **Problem 765: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos [c+d x]^2}{\cos [c+d x]^{5/2} (a+b \cos [c+d x])^{5/2}} dx$$

Optimal (type 4, 549 leaves, 6 steps):

$$\begin{aligned}
& - \frac{1}{3 a^5 \sqrt{a+b} (a^2-b^2) d} 4 b (8 A b^4 + a^4 (4 A - 3 C) - a^2 b^2 (14 A - C)) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{3 a^4 \sqrt{a+b} (a^2-b^2) d} \\
& 2 (12 a A b^3 + 16 A b^4 - 2 a^2 b^2 (8 A - C) - a^4 (A+3 C) - a^3 (9 A b - 3 b C)) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{2 (A b^2 + a^2 C) \operatorname{Sin}[c+d x]}{3 a (a^2-b^2) d \operatorname{Cos}[c+d x]^{3/2} (a+b \operatorname{Cos}[c+d x])^{3/2}} + \\
& \frac{4 (5 a^2 A b^2 - 3 A b^4 + 2 a^4 C) \operatorname{Sin}[c+d x]}{3 a^2 (a^2-b^2)^2 d \operatorname{Cos}[c+d x]^{3/2} \sqrt{a+b \operatorname{Cos}[c+d x]}} + \frac{2 (8 A b^4 + a^4 (A-5 C) - a^2 b^2 (13 A - C)) \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 a^3 (a^2-b^2)^2 d \operatorname{Cos}[c+d x]^{3/2}}
\end{aligned}$$

Result (type 4, 1471 leaves):

$$\begin{aligned}
& \frac{1}{3 a^4 (a-b)^2 (a+b)^2 d} \\
& \left( - \left( 4 a (a^6 A + 15 a^4 A b^2 - 32 a^2 A b^4 + 16 A b^6 + 3 a^6 C - 5 a^4 b^2 C + 2 a^2 b^4 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (8 a^5 A b - 28 a^3 A b^3 + 16 a A b^5 - 6 a^5 b C + 2 a^3 b^3 C) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \\
& \left. (b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}) \right) + 2 (8 a^4 A b^2 - 28 a^2 A b^4 + 16 A b^6 - 6 a^4 b^2 C + 2 a^2 b^4 C) \\
& \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \text{Csc}[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \\
& \left. \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) + \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2(Ab^4 \operatorname{Sin}[c+dx] + a^2 b^2 C \operatorname{Sin}[c+dx])}{3a^3(a^2-b^2)(a+b \operatorname{Cos}[c+dx])^2} + \right. \\
& \left. \frac{4(6a^2 A b^4 \operatorname{Sin}[c+dx] - 4Ab^6 \operatorname{Sin}[c+dx] + 3a^4 b^2 C \operatorname{Sin}[c+dx] - a^2 b^4 C \operatorname{Sin}[c+dx])}{3a^4(a^2-b^2)^2(a+b \operatorname{Cos}[c+dx])} - \right. \\
& \left. \frac{16Ab \operatorname{Tan}[c+dx]}{3a^4} + \frac{2A \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{3a^3} \right)
\end{aligned}$$

■ **Problem 766: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^m (a+b \operatorname{Cos}[c+dx])^2 (A+C \operatorname{Cos}[c+dx])^2 dx$$

Optimal (type 5, 318 leaves, 6 steps):

$$\begin{aligned}
& \frac{(2a^2 C + b^2(C(3+m) + A(4+m))) \operatorname{Cos}[c+dx]^{1+m} \operatorname{Sin}[c+dx]}{d(2+m)(4+m)} + \frac{2abC \operatorname{Cos}[c+dx]^{2+m} \operatorname{Sin}[c+dx]}{d(3+m)(4+m)} + \\
& \frac{C \operatorname{Cos}[c+dx]^{1+m} (a+b \operatorname{Cos}[c+dx])^2 \operatorname{Sin}[c+dx]}{d(4+m)} - \left( (a^2(4+m)(C(1+m) + A(2+m)) + b^2(1+m)(C(3+m) + A(4+m))) \right. \\
& \left. \operatorname{Cos}[c+dx]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Cos}[c+dx]^2\right] \operatorname{Sin}[c+dx] \right) / \left( d(1+m)(2+m)(4+m) \sqrt{\operatorname{Sin}[c+dx]^2} \right) - \\
& \frac{2ab(C(2+m) + A(3+m)) \operatorname{Cos}[c+dx]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \operatorname{Cos}[c+dx]^2\right] \operatorname{Sin}[c+dx]}{d(2+m)(3+m) \sqrt{\operatorname{Sin}[c+dx]^2}}
\end{aligned}$$



Result (type 5, 664 leaves) :

$$\frac{1}{8d} \operatorname{Cos}[c+dx]^{1+m} \operatorname{Csc}[c+dx] \left( -\frac{b^2 C \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{1+m} + \frac{4(Ab^2 + (a^2 + b^2)C) \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{1+m} + \frac{12abc \operatorname{Cos}[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{2+m} + \frac{6b^2 C \operatorname{Cos}[c+dx]^2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{3+m} - \frac{8a^2 A \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{1+m} - \frac{4Ab^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{1+m} - \frac{4a^2 C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{1+m} - \frac{3b^2 C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{1+m} - \frac{16ab \operatorname{Cos}[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{2+m} - \frac{12abc \operatorname{Cos}[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{2+m} - \frac{4Ab^2 \operatorname{Cos}[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{3+m} - \frac{4a^2 C \operatorname{Cos}[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{3+m} - \frac{4b^2 C \operatorname{Cos}[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{3+m} - \frac{4abc \operatorname{Cos}[c+dx]^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{4+m}{2}, \frac{6+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{4+m} - \frac{b^2 C \operatorname{Cos}[c+dx]^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5+m}{2}, \frac{7+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{5+m} \right) \sqrt{\operatorname{Sin}[c+dx]^2}$$

■ **Problem 768: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^m (A + C \operatorname{Cos}[c+dx]^2)}{a + b \operatorname{Cos}[c+dx]} dx$$

Optimal (type 6, 353 leaves, 8 steps) :

$$\frac{1}{b^2 (a^2 - b^2) d} a (A b^2 + a^2 C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1-m}{2}, 1, \frac{3}{2}, \sin[c+dx]^2, -\frac{b^2 \sin[c+dx]^2}{a^2 - b^2}\right] \cos[c+dx]^{-1+m} (\cos[c+dx]^2)^{\frac{1-m}{2}} \sin[c+dx] -$$

$$\frac{1}{b (a^2 - b^2) d} (A b^2 + a^2 C) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, \sin[c+dx]^2, -\frac{b^2 \sin[c+dx]^2}{a^2 - b^2}\right] \cos[c+dx]^m (\cos[c+dx]^2)^{-m/2} \sin[c+dx] +$$

$$\frac{a C \cos[c+dx]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]}{b^2 d (1+m) \sqrt{\sin[c+dx]^2}} -$$

$$\frac{C \cos[c+dx]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]}{b d (2+m) \sqrt{\sin[c+dx]^2}}$$

Result (type 6, 10459 leaves):

$$\left( \left( \frac{A \cos[c+dx]^m}{a+b \cos[c+dx]} + \frac{C \cos[c+dx]^m}{2(a+b \cos[c+dx])} + \frac{C \cos[c+dx]^m \cos[2(c+dx)]}{2(a+b \cos[c+dx])} \right) \tan[c+dx] \right.$$

$$\left. - \frac{a C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+\frac{m}{2}, \frac{3}{2}, -\tan[c+dx]^2\right]}{b^2} + \frac{A \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3}{2}, -\tan[c+dx]^2\right]}{b} \right.$$

$$\left. + \frac{a^2 C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3}{2}, -\tan[c+dx]^2\right]}{b^3} + \frac{C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{3}{2}, -\tan[c+dx]^2\right]}{b} \right.$$

$$\left. \left( 3 a^2 (a^2 - b^2) (A b^2 + a^2 C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2 + b^2}\right] (1 + \tan[c+dx]^2)^{\frac{1-m}{2}} \right) / \right.$$

$$\left. \left( b^3 \left( -3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] + \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \right.$$

$$\left. \left. \left. - \frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right) + (a^2 - b^2) (-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] \right) \tan[c+dx]^2 \right)$$

$$\left. (-b^2 + a^2 (1 + \tan[c+dx]^2)) \right) - \left( 3 a A (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] (1 + \tan[c+dx]^2)^{-m/2} \right) /$$

$$\left( \left( -3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] + \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \right.$$

$$\left. \left. \left. - \frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right) + (a^2 - b^2) m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] \right) \tan[c+dx]^2 \right)$$

$$\left. (-b^2 + a^2 (1 + \tan[c+dx]^2)) \right) + \left( 3 a^3 C \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2 + b^2}\right] (1 + \tan[c+dx]^2)^{-m/2} \right) /$$

$$\left( \left( -3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] + \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \right.$$

$$\begin{aligned}
& \left. -\frac{a^2 \tan^2[c+dx]}{a^2-b^2} \right] + (a^2-b^2) m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \left. \tan^2[c+dx] \right) \\
& \left. (-b^2+a^2(1+\tan^2[c+dx])) \right) - \left( 3 a^5 C \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan^2[c+dx], \frac{a^2 \tan^2[c+dx]}{-a^2+b^2}\right] (1+\tan^2[c+dx])^{-m/2} \right) / \\
& \left( b^2 \left( -3 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \right) \tan^2[c+dx] \right) (-b^2+a^2(1+\tan^2[c+dx])) \right) \Big) / \\
& \left( d \left( \operatorname{Sec}[c+dx]^2 \left( -\frac{a C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+\frac{m}{2}, \frac{3}{2}, -\tan^2[c+dx]\right]}{b^2} + \frac{A \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3}{2}, -\tan^2[c+dx]\right]}{b} \right. \right. \right. \\
& \quad \left. \left. \frac{a^2 C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3}{2}, -\tan^2[c+dx]\right]}{b^3} + \frac{C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{3}{2}, -\tan^2[c+dx]\right]}{b} \right. \right. \\
& \quad \left. \left. \left( 3 a^2 (a^2-b^2) (A b^2+a^2 C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\tan^2[c+dx], \frac{a^2 \tan^2[c+dx]}{-a^2+b^2}\right] (1+\tan^2[c+dx])^{\frac{1-m}{2}} \right) \right) / \right. \\
& \quad \left( b^3 \left( -3 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) (-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \right) \tan^2[c+dx] \right) (-b^2+a^2(1+\tan^2[c+dx])) \right) - \\
& \left( 3 a A (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] (1+\tan^2[c+dx])^{-m/2} \right) / \left( \left( -3 (a^2-b^2) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] + \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] + \right. \right. \right. \\
& \quad \left. \left. (a^2-b^2) m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] \right) \tan^2[c+dx] \right) (-b^2+a^2(1+\tan^2[c+dx])) \right) + \\
& \left( 3 a^3 C \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan^2[c+dx], \frac{a^2 \tan^2[c+dx]}{-a^2+b^2}\right] (1+\tan^2[c+dx])^{-m/2} \right) / \left( \left( -3 (a^2-b^2) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] + \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan^2[c+dx], -\frac{a^2 \tan^2[c+dx]}{a^2-b^2}\right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] \operatorname{Tan}[c+dx]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2)) \right) - \right. \\
& \left. \left( 3 a^5 C \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2 + b^2} \right] (1 + \operatorname{Tan}[c+dx]^2)^{-m/2} \right) / \left( b^2 \left( -3 (a^2 - b^2) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] \right) + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] \operatorname{Tan}[c+dx]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2)) \right) \right) \Bigg) + \\
& \operatorname{Tan}[c+dx] \left( - \left( 6 a^4 (a^2 - b^2) (A b^2 + a^2 C) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2 + b^2} \right] \operatorname{Sec}[c+dx]^2 \right. \right. \right. \\
& \left. \left. \left. \operatorname{Tan}[c+dx] (1 + \operatorname{Tan}[c+dx]^2)^{\frac{1}{2} - \frac{m}{2}} \right) / \left( b^3 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) (-1+m) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2))^2 \right) + \\
& \left( 6 a^3 A (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] (1 + \operatorname{Tan}[c+dx]^2)^{-m/2} \right) / \\
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] \operatorname{Tan}[c+dx]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2))^2 \right) - \right. \\
& \left. \left( 6 a^5 C \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2 + b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] (1 + \operatorname{Tan}[c+dx]^2)^{-m/2} \right) / \right. \\
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] \operatorname{Tan}[c+dx]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2))^2 \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \left( 6 a^7 C \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2+b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] (1+\operatorname{Tan}[c+dx]^2)^{-m/2} \right) / \\
& \left( b^2 \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2))^2 \Big) + \\
& \left( 6 a^2 (a^2-b^2) (A b^2+a^2 C) \left( \frac{1}{2}-\frac{m}{2} \right) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2+b^2} \right] \operatorname{Sec}[c+dx]^2 \right. \\
& \quad \left. \operatorname{Tan}[c+dx] (1+\operatorname{Tan}[c+dx]^2)^{-\frac{1}{2}-\frac{m}{2}} \right) / \left( b^3 \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2)) \Big) + \\
& \left( 3 a^2 (a^2-b^2) (A b^2+a^2 C) \left( -\frac{1}{3}(-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1+\frac{1}{2}(-1+m), 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2+b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] + \right. \right. \\
& \quad \left. \left. 1 / (3(-a^2+b^2)) 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2+b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right. \\
& \quad \left. (1+\operatorname{Tan}[c+dx]^2)^{\frac{1}{2}-\frac{m}{2}} \right) / \left( b^3 \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2)) \Big) + \\
& \left( 3 a A (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] (1+\operatorname{Tan}[c+dx]^2)^{-1-\frac{m}{2}} \right) / \\
& \left( \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2)) \Big) -
\end{aligned}$$

$$\begin{aligned}
& \left( 3 a^3 C m \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+d x]^2, \frac{a^2 \operatorname{Tan}[c+d x]^2}{-a^2+b^2} \right] \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] (1+\operatorname{Tan}[c+d x]^2)^{-1-\frac{m}{2}} \right) / \\
& \left( \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+d x]^2 \right) (-b^2+a^2 (1+\operatorname{Tan}[c+d x]^2)) \Big) + \\
& \left( 3 a^5 C m \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+d x]^2, \frac{a^2 \operatorname{Tan}[c+d x]^2}{-a^2+b^2} \right] \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] (1+\operatorname{Tan}[c+d x]^2)^{-1-\frac{m}{2}} \right) / \\
& \left( b^2 \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+d x]^2 \right) (-b^2+a^2 (1+\operatorname{Tan}[c+d x]^2)) \Big) - \\
& \left( 3 a A (a^2-b^2) \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+\frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2} \right] \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] - \right. \right. \\
& \quad \left. \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2} \right] \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{3 (a^2-b^2)} \right) (1+\operatorname{Tan}[c+d x]^2)^{-m/2} \right) / \\
& \left( \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, \right. \right. \right. \\
& \quad \left. \left. -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2} \right] + (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+d x]^2 \right) \\
& \quad (-b^2+a^2 (1+\operatorname{Tan}[c+d x]^2)) \Big) + \left( 3 a^3 C \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+\frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, \frac{a^2 \operatorname{Tan}[c+d x]^2}{-a^2+b^2} \right] \operatorname{Sec}[c+d x]^2 \right. \right. \\
& \quad \left. \left. \operatorname{Tan}[c+d x] + \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, \frac{a^2 \operatorname{Tan}[c+d x]^2}{-a^2+b^2} \right] \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{3 (-a^2+b^2)} \right) (1+\operatorname{Tan}[c+d x]^2)^{-m/2} \right) / \\
& \left( \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} + (a^2-b^2) m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Tan}[c+dx]^2 \Big) \\
& (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2)) \Big) - \left( 3 a^5 C \left[ -\frac{1}{3} m \operatorname{AppellF1}\left[\frac{3}{2}, 1+\frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] + \right. \right. \\
& \left. \left. \frac{2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3(-a^2+b^2)} \right] (1+\operatorname{Tan}[c+dx]^2)^{-m/2} \right) / \left( b^2 \left( -3(a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (a^2-b^2) m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Tan}[c+dx]^2 \right) (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2)) \right) \right) + \\
& \frac{1}{b} C \operatorname{Csc}[c+dx] \operatorname{Sec}[c+dx] \left( -\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2\right] + (1+\operatorname{Tan}[c+dx]^2)^{\frac{1}{2}(-3-m)} \right) + \frac{1}{b} \\
& A \operatorname{Csc}[c+dx] \operatorname{Sec}[c+dx] \left( -\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2\right] + (1+\operatorname{Tan}[c+dx]^2)^{\frac{1}{2}(-1-m)} \right) + \frac{1}{b^3} \\
& a^2 C \operatorname{Csc}[c+dx] \operatorname{Sec}[c+dx] \left( -\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2\right] + (1+\operatorname{Tan}[c+dx]^2)^{\frac{1}{2}(-1-m)} \right) - \\
& \frac{1}{b^2} a C \operatorname{Csc}[c+dx] \operatorname{Sec}[c+dx] \left( -\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+\frac{m}{2}, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2\right] + (1+\operatorname{Tan}[c+dx]^2)^{-1-\frac{m}{2}} \right) - \\
& \left( 3 a^2 (a^2-b^2) (A b^2+a^2 C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2+b^2}\right] \right. \\
& \left. (1+\operatorname{Tan}[c+dx]^2)^{\frac{1}{2}-\frac{m}{2}} \left( 2 \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (a^2-b^2) (-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \right) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \right. \right. \\
& \left. \left. 3 (a^2-b^2) \left( -\frac{1}{3} (-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 1+\frac{1}{2}(-1+m), 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \right. \right. \right. \\
& \left. \left. \left. \frac{1}{3(a^2-b^2)} 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right) + \right. \\
& \left. \operatorname{Tan}[c+dx]^2 \left( 2 a^2 \left( -\frac{3}{5} (-1+m) \operatorname{AppellF1}\left[\frac{5}{2}, 1+\frac{1}{2}(-1+m), 2, \frac{7}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \right. \right. \right. \\
& \left. \left. \left. \frac{1}{5(a^2-b^2)} 12 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}(-1+m), 3, \frac{7}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& (a^2 - b^2)^{-1+m} \left( -\frac{1}{5(a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1+m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] - \right. \\
& \left. \frac{3}{5} (1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1+m}{2}, 1, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] \right) \Bigg) \Bigg) \Bigg) / \\
& \left( b^3 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), \right. \right. \right. \right. \\
& \left. \left. \left. \frac{5}{2}, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + (a^2 - b^2)^{-1+m} \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \right. \right. \right. \\
& \left. \left. \left. \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right)^2 \tan[c+dx]^2 \right) (-b^2 + a^2 (1 + \tan[c+dx]^2)) \Bigg) + \\
& \left( 3 a A (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] (1 + \tan[c+dx]^2)^{-m/2} \left( 2 \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \frac{m}{2}, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + (a^2 - b^2)^m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right) \right) \\
& \sec[c+dx]^2 \tan[c+dx] - 3 (a^2 - b^2) \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right. \\
& \left. \sec[c+dx]^2 \tan[c+dx] - \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx]}{3 (a^2 - b^2)} \right) \Bigg) + \\
& \tan[c+dx]^2 \left( 2 a^2 \left( -\frac{3}{5} m \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \left. \left. \frac{1}{5 (a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{m}{2}, 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] \right) \right) + \\
& (a^2 - b^2)^m \left( -\frac{1}{5 (a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{2+m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] - \right. \\
& \left. \frac{3}{5} (2+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{2+m}{2}, 1, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] \right) \Bigg) \Bigg) \Bigg) / \\
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \right. \right.
\end{aligned}$$



$$\begin{aligned}
& -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} + (a^2-b^2) m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Tan}[c+dx]^2 \Big)^2 \\
& (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2)) \Big) - \left( 3 a^3 C \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2+b^2}\right] (1+\operatorname{Tan}[c+dx]^2)^{-m/2} \right. \\
& \left. \left( 2 \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (a^2-b^2) m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \right) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \right. \right. \\
& \left. \left. 3 (a^2-b^2) \left( -\frac{1}{3} m \operatorname{AppellF1}\left[\frac{3}{2}, 1+\frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \right. \right. \right. \\
& \left. \left. \left. \frac{2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right) \right) \Big) + \\
& \operatorname{Tan}[c+dx]^2 \left( 2 a^2 \left( -\frac{3}{5} m \operatorname{AppellF1}\left[\frac{5}{2}, 1+\frac{m}{2}, 2, \frac{7}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \right. \right. \\
& \left. \left. \frac{1}{5 (a^2-b^2)} 12 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{m}{2}, 3, \frac{7}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right) + \\
& (a^2-b^2) m \left( -\frac{1}{5 (a^2-b^2)} 6 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2+m}{2}, 2, \frac{7}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \right. \\
& \left. \left. \frac{3}{5} (2+m) \operatorname{AppellF1}\left[\frac{5}{2}, 1+\frac{2+m}{2}, 1, \frac{7}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right) \Big) \Big) \Big) / \\
& \left( \left( -3 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right) + (a^2-b^2) m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] \right) \operatorname{Tan}[c+dx]^2 \Big)^2 \right. \\
& \left. (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2)) \right) + \left( 3 a^5 C \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2+b^2}\right] (1+\operatorname{Tan}[c+dx]^2)^{-m/2} \right. \\
& \left. \left( 2 \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2}\right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (a^2 - b^2)^m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \\
& 3(a^2 - b^2) \left( -\frac{1}{3} m \operatorname{AppellF1}\left[\frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \\
& \left. \frac{2a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx]}{3(a^2 - b^2)} \right) + \\
& \tan[c+dx]^2 \left( 2a^2 \left( -\frac{3}{5} m \operatorname{AppellF1}\left[\frac{5}{2}, 1 + \frac{m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \left. \left. \frac{1}{5(a^2 - b^2)} 12a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{m}{2}, 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \right. \\
& (a^2 - b^2)^m \left( -\frac{1}{5(a^2 - b^2)} 6a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2+m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \\
& \left. \left. \frac{3}{5} (2+m) \operatorname{AppellF1}\left[\frac{5}{2}, 1 + \frac{2+m}{2}, 1, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( b^2 \left( -3(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] + \left( 2a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] + (a^2 - b^2)^m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] \right) \tan[c+dx]^2 \right)^2 (-b^2 + a^2(1 + \tan[c+dx]^2)) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 769: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^m (A + C \cos[c+dx]^2)}{(a + b \cos[c+dx])^2} dx$$

Optimal (type 6, 514 leaves, 9 steps):

$$\frac{1}{b^2 (a^2 - b^2)^2 d} (A b^4 m - a^4 C (1+m) + a^2 b^2 (A - A m + C (2+m))) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1-m}{2}, 1, \frac{3}{2}, \sin[c+dx]^2, -\frac{b^2 \sin[c+dx]^2}{a^2 - b^2}\right]$$

$$\cos[c+dx]^{-1+m} (\cos[c+dx]^2)^{\frac{1-m}{2}} \sin[c+dx] - \frac{1}{a b (a^2 - b^2)^2 d} (A b^4 m - a^4 C (1+m) + a^2 b^2 (A - A m + C (2+m)))$$

$$\operatorname{AppellF1}\left[\frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, \sin[c+dx]^2, -\frac{b^2 \sin[c+dx]^2}{a^2 - b^2}\right] \cos[c+dx]^m (\cos[c+dx]^2)^{-m/2} \sin[c+dx] +$$

$$\frac{(A b^2 + a^2 C) \cos[c+dx]^{1+m} \sin[c+dx] (a^2 C (1+m) - b^2 (C - A m)) \cos[c+dx]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]}{a (a^2 - b^2) d (a + b \cos[c+dx])} - \frac{b^2 (a^2 - b^2) d (1+m) \sqrt{\sin[c+dx]^2}}{b^2 (a^2 - b^2) d (1+m) \sqrt{\sin[c+dx]^2}} +$$

$$\frac{(A b^2 + a^2 C) (1+m) \cos[c+dx]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]}{a b (a^2 - b^2) d (2+m) \sqrt{\sin[c+dx]^2}}$$

Result (type 6, 17999 leaves):

$$\left( \left( \frac{A \cos[c+dx]^m}{(a+b \cos[c+dx])^2} + \frac{C \cos[c+dx]^m}{2 (a+b \cos[c+dx])^2} + \frac{C \cos[c+dx]^m \cos[2(c+dx)]}{2 (a+b \cos[c+dx])^2} \right) \right.$$

$$\left. - \frac{2 a C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} + \frac{m}{2}, \frac{3}{2}, -\tan[c+dx]^2\right] \tan[c+dx]}{b^3} + \frac{C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1 + \frac{m}{2}, \frac{3}{2}, -\tan[c+dx]^2\right] \tan[c+dx]}{b^2} \right.$$

$$\left. \left( 6 A b^2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] \tan[c+dx] (1 + \tan[c+dx]^2)^{-m/2} \right) / \right.$$

$$\left( \left( -3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] + \right. \right.$$

$$\left. \left( (a^2 - b^2) m \operatorname{AppellF1}\left[\frac{3}{2}, 1 + \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] + \right. \right.$$

$$\left. \left. 4 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] \right) \tan[c+dx]^2 (b^2 - a^2 (1 + \tan[c+dx]^2))^2 \right) -$$

$$\left( 6 a^2 (a^2 - b^2) C \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] \tan[c+dx] (1 + \tan[c+dx]^2)^{-m/2} \right) / \right.$$

$$\left( \left( -3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] + \right. \right.$$

$$\left. \left( (a^2 - b^2) m \operatorname{AppellF1}\left[\frac{3}{2}, 1 + \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] + \right. \right.$$

$$\left. \left. 4 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2}\right] \right) \tan[c+dx]^2 (b^2 - a^2 (1 + \tan[c+dx]^2))^2 \right) +$$

$$\begin{aligned}
& \left( 6 a A b (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] \operatorname{Tan}[c+dx] (1 + \operatorname{Tan}[c+dx]^2)^{\frac{1}{2} - \frac{m}{2}} \right) / \\
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) (b^2 - a^2 (1 + \operatorname{Tan}[c+dx]^2))^2 \Big) + \\
& \left( 6 a^3 (a^2 - b^2) C \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2 + b^2} \right] \operatorname{Tan}[c+dx] (1 + \operatorname{Tan}[c+dx]^2)^{\frac{1}{2} - \frac{m}{2}} \right) / \\
& \left( b \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) (b^2 - a^2 (1 + \operatorname{Tan}[c+dx]^2))^2 \Big) - \\
& \left( 6 a^3 (a^2 - b^2) C \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2 + b^2} \right] \operatorname{Tan}[c+dx] (1 + \operatorname{Tan}[c+dx]^2)^{\frac{1}{2} - \frac{m}{2}} \right) / \\
& \left( b^3 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2)) \Big) - \\
& \left( 3 A (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] \operatorname{Tan}[c+dx] (1 + \operatorname{Tan}[c+dx]^2)^{-m/2} \right) / \\
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2)) \Big) + \\
& \left( 3 a^2 (a^2 - b^2) C \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2 + b^2} \right] \operatorname{Tan}[c+dx] (1 + \operatorname{Tan}[c+dx]^2)^{-m/2} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( b^2 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \operatorname{Tan}[c + d x]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c + d x]^2)) \right) \right) \Big/ \\
& \left( d \left( -\frac{2 a C \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{2} + \frac{m}{2}, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2}{b^3} + \frac{C \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, 1 + \frac{m}{2}, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2}{b^2} \right. \right. \\
& \quad \left. \left( 24 a^2 A b^2 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]^2 (1 + \operatorname{Tan}[c + d x]^2)^{-m/2} \right) \right) \Big/ \\
& \quad \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + d x]^2 \right) (b^2 - a^2 (1 + \operatorname{Tan}[c + d x]^2))^3 \Big) - \\
& \left( 24 a^4 (a^2 - b^2) C \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]^2 (1 + \operatorname{Tan}[c + d x]^2)^{-m/2} \right) \Big/ \\
& \quad \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + d x]^2 \right) (b^2 - a^2 (1 + \operatorname{Tan}[c + d x]^2))^3 \Big) + \\
& \left( 24 a^3 A b (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 2, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]^2 (1 + \operatorname{Tan}[c + d x]^2)^{\frac{1}{2} - \frac{m}{2}} \right) \Big/ \\
& \quad \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 2, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 + m), 3, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (-1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + d x]^2 \right) (b^2 - a^2 (1 + \operatorname{Tan}[c + d x]^2))^3 \Big) +
\end{aligned}$$



$$\begin{aligned}
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) (b^2 - a^2 (1 + \tan[c + dx]^2))^2 \Big) - \\
& \left( 6 a b^2 (a^2 - b^2) \tan[c + dx] \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \quad \left. \left. \frac{4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 (a^2 - b^2)} \right) (1 + \tan[c + dx]^2)^{-m/2} \right) / \\
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) (b^2 - a^2 (1 + \tan[c + dx]^2))^2 \Big) - \\
& \left( 6 a^2 (a^2 - b^2) c \tan[c + dx] \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \quad \left. \left. \frac{4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 (a^2 - b^2)} \right) (1 + \tan[c + dx]^2)^{-m/2} \right) / \\
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) (b^2 - a^2 (1 + \tan[c + dx]^2))^2 \Big) + \\
& \left( 12 a A b (a^2 - b^2) \left( \frac{1}{2} - \frac{m}{2} \right) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 2, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]^2 \right. \\
& \quad \left. (1 + \tan[c + dx]^2)^{-\frac{1}{2} - \frac{m}{2}} \right) / \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 2, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \\
& \quad \left. (a^2-b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Tan}[c+dx]^2 \right) (b^2-a^2 (1+\operatorname{Tan}[c+dx]^2))^2 \Big) + \\
& \left( 12 a^3 (a^2-b^2) c \left( \frac{1}{2} - \frac{m}{2} \right) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2+b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]^2 \right. \\
& \quad \left. (1+\operatorname{Tan}[c+dx]^2)^{-\frac{1}{2}-\frac{m}{2}} \right) / \left( b \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Tan}[c+dx]^2 \right) (b^2-a^2 (1+\operatorname{Tan}[c+dx]^2))^2 \right) \Big) + \\
& \left( 6 a A b (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 (1+\operatorname{Tan}[c+dx]^2)^{\frac{1}{2}-\frac{m}{2}} \right) / \\
& \left( \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Tan}[c+dx]^2 \right) (b^2-a^2 (1+\operatorname{Tan}[c+dx]^2))^2 \right) \Big) + \\
& \left( 6 a^3 (a^2-b^2) c \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2+b^2} \right] \operatorname{Sec}[c+dx]^2 (1+\operatorname{Tan}[c+dx]^2)^{\frac{1}{2}-\frac{m}{2}} \right) / \\
& \left( b \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Tan}[c+dx]^2 \right) (b^2-a^2 (1+\operatorname{Tan}[c+dx]^2))^2 \right) \Big) + \\
& \left( 6 a A b (a^2-b^2) \operatorname{Tan}[c+dx] \left[ -\frac{1}{3} (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1+\frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \right. \right. \\
& \quad \left. \left. \frac{4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3 (a^2-b^2)} \right] \right) (1+\operatorname{Tan}[c+dx]^2)^{\frac{1}{2}-\frac{m}{2}} \Big) /
\end{aligned}$$



$$\begin{aligned}
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 2, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 + m), 3, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) (-1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1 + m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + d x]^2 (b^2 - a^2 (1 + \operatorname{Tan}[c + d x]^2))^2 \right) + \\
& \left( 6 a^3 (a^2 - b^2) C \operatorname{Tan}[c + d x] \left( -\frac{1}{3} (-1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{1}{2} (-1 + m), 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \frac{a^2 \operatorname{Tan}[c + d x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] + \right. \right. \\
& \quad \left. \left. \frac{4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 + m), 3, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \frac{a^2 \operatorname{Tan}[c + d x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 (-a^2 + b^2)} \right) (1 + \operatorname{Tan}[c + d x]^2)^{\frac{1}{2} - \frac{m}{2}} \right) / \\
& \left( b \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 2, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 + m), 3, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) (-1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1 + m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + d x]^2 (b^2 - a^2 (1 + \operatorname{Tan}[c + d x]^2))^2 \right) + \\
& \left( 12 a^5 (a^2 - b^2) C \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \frac{a^2 \operatorname{Tan}[c + d x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]^2 (1 + \operatorname{Tan}[c + d x]^2)^{\frac{1}{2} - \frac{m}{2}} \right) / \\
& \left( b^3 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 + m), 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) (-1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1 + m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + d x]^2 (-b^2 + a^2 (1 + \operatorname{Tan}[c + d x]^2))^2 \right) + \\
& \left( 6 a^2 A (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]^2 (1 + \operatorname{Tan}[c + d x]^2)^{-m/2} \right) / \\
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2 + m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + d x]^2 (-b^2 + a^2 (1 + \operatorname{Tan}[c + d x]^2))^2 \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( 6 a^4 (a^2 - b^2) C \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \frac{a^2 \operatorname{Tan}[c + d x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]^2 (1 + \operatorname{Tan}[c + d x]^2)^{-m/2} \right) / \\
& \left( b^2 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + d x]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c + d x]^2))^2 \Big) - \\
& \left( 12 a^3 (a^2 - b^2) C \left( \frac{1}{2} - \frac{m}{2} \right) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \frac{a^2 \operatorname{Tan}[c + d x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]^2 \right. \\
& \quad \left. (1 + \operatorname{Tan}[c + d x]^2)^{-\frac{1}{2} - \frac{m}{2}} \right) / \left( b^3 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + d x]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c + d x]^2)) \Big) - \\
& \left( 6 a^3 (a^2 - b^2) C \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \frac{a^2 \operatorname{Tan}[c + d x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[c + d x]^2 (1 + \operatorname{Tan}[c + d x]^2)^{\frac{1}{2} - \frac{m}{2}} \right) / \\
& \left( b^3 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[c + d x]^2 \right) (-b^2 + a^2 (1 + \operatorname{Tan}[c + d x]^2)) \Big) - \\
& \left( 6 a^3 (a^2 - b^2) C \operatorname{Tan}[c + d x] \left[ -\frac{1}{3} (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{1}{2} (-1+m), 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \frac{a^2 \operatorname{Tan}[c + d x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] + \right. \right. \\
& \quad \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \frac{a^2 \operatorname{Tan}[c + d x]^2}{-a^2 + b^2} \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 (-a^2 + b^2)} \right) (1 + \operatorname{Tan}[c + d x]^2)^{\frac{1}{2} - \frac{m}{2}} \Big) / \\
& \left( b^3 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, -\frac{a^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \left( 3 A (a^2 - b^2) \tan[c + dx] \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \quad \left. \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 (a^2 - b^2)} \right) (1 + \tan[c + dx]^2)^{-m/2} \right) / \\
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) (-b^2 + a^2 (1 + \tan[c + dx]^2)) \Big) + \\
& \left( 3 a^2 (a^2 - b^2) C \tan[c + dx] \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, \frac{a^2 \tan[c + dx]^2}{-a^2 + b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] + \right. \right. \\
& \quad \left. \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, \frac{a^2 \tan[c + dx]^2}{-a^2 + b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 (-a^2 + b^2)} \right) (1 + \tan[c + dx]^2)^{-m/2} \right) / \\
& \left( b^2 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \\
& \quad \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) \\
& \quad \left. (-b^2 + a^2 (1 + \tan[c + dx]^2)) \right) + \frac{C \operatorname{Sec}[c + dx]^2 \left( -\operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, 1 + \frac{m}{2}, \frac{3}{2}, -\tan[c + dx]^2 \right] + (1 + \tan[c + dx]^2)^{-1 - \frac{m}{2}} \right)}{b^2} - \\
& \frac{2 a C \operatorname{Sec}[c + dx]^2 \left( -\operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{2} + \frac{m}{2}, \frac{3}{2}, -\tan[c + dx]^2 \right] + (1 + \tan[c + dx]^2)^{-\frac{1}{2} - \frac{m}{2}} \right)}{b^3} + \\
& \left( 6 A b^2 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \tan[c + dx] \right. \\
& \quad \left. (1 + \tan[c + dx]^2)^{-m/2} \left( 2 \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right.
\end{aligned}$$

$$\begin{aligned}
& 3 (a^2 - b^2) \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \\
& \left. \frac{4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 (a^2 - b^2)} \right) + \\
& \tan[c + dx]^2 \left( (a^2 - b^2) m \left( -\frac{1}{5 (a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{m}{2}, 3, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \left. \left. \frac{6}{5} \left( 1 + \frac{m}{2} \right) \operatorname{AppellF1} \left[ \frac{5}{2}, 2 + \frac{m}{2}, 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) + \right. \\
& \left. 4 a^2 \left( -\frac{3}{5} m \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{m}{2}, 3, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \left. \left. \frac{18 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{m}{2}, 4, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{5 (a^2 - b^2)} \right) \right) \Bigg) / \\
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 2, \frac{5}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right)^2 \\
& (b^2 - a^2 (1 + \tan[c + dx]^2))^2 \Bigg) + \left( 6 a^2 (a^2 - b^2) \operatorname{C AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \tan[c + dx] \right. \\
& \left. (1 + \tan[c + dx]^2)^{-m/2} \left( 2 \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \left. \left. 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \\
& \left. 3 (a^2 - b^2) \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \left. \left. \frac{4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 (a^2 - b^2)} \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \tan[c+dx]^2 \left( (a^2-b^2) m \left( -\frac{1}{5(a^2-b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, 1+\frac{m}{2}, 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \quad \left. \frac{6}{5} \left( 1+\frac{m}{2} \right) \operatorname{AppellF1} \left[ \frac{5}{2}, 2+\frac{m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \\
& \quad 4 a^2 \left( -\frac{3}{5} m \operatorname{AppellF1} \left[ \frac{5}{2}, 1+\frac{m}{2}, 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \\
& \quad \left. \frac{18 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{m}{2}, 4, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx]}{5(a^2-b^2)} \right) \Big) \Big) \Big) \Big) / \\
& \left( \left( -3(a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 2, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] + \left( (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+\frac{m}{2}, 2, \frac{5}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] + 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 3, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \right) \tan[c+dx]^2 \right)^2 \\
& \quad (b^2 - a^2 (1 + \tan[c+dx]^2))^2 + \left( 6 a^3 (a^2 - b^2) \operatorname{CAppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2+b^2} \right] \right. \\
& \quad \left. \tan[c+dx] (1 + \tan[c+dx]^2)^{\frac{1}{2}-\frac{m}{2}} \left( 2 \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \quad \left. 3 (a^2 - b^2) \left( -\frac{1}{3} (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1+\frac{1}{2} (-1+m), 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \quad \left. \left. \frac{1}{3 (a^2 - b^2)} 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \right. \\
& \quad \left. \tan[c+dx]^2 \left( 2 a^2 \left( -\frac{3}{5} (-1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1+\frac{1}{2} (-1+m), 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{5 (a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2} (-1+m), 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (-1+m) \left( -\frac{1}{5 (a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1+m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{3}{5} (1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1+\frac{1+m}{2}, 1, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \right) \right) \Big) \Big) \Big) \Big) /
\end{aligned}$$

$$\begin{aligned}
& \left( b^3 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right) \tan[c+dx]^2 \right)^2 \\
& \quad \left( -b^2 + a^2 (1 + \tan[c+dx]^2) \right) - \left( 6 a A b (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right. \\
& \quad \tan[c+dx] (1 + \tan[c+dx]^2)^{\frac{1}{2} - \frac{m}{2}} \left( 2 \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right) \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \\
& \quad 3 (a^2 - b^2) \left( -\frac{1}{3} (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \\
& \quad \left. \frac{1}{3 (a^2 - b^2)} 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \\
& \quad \tan[c+dx]^2 \left( 4 a^2 \left( -\frac{3}{5} (-1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1}{2} (-1+m), 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \quad \left. \left. \frac{1}{5 (a^2 - b^2)} 18 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2} (-1+m), 4, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \right. \\
& \quad \left. (a^2 - b^2) (-1+m) \left( -\frac{1}{5 (a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1+m}{2}, 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \quad \left. \left. \frac{3}{5} (1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1+m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \right) \right) \Big/ \\
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right) \tan[c+dx]^2 \right)^2 \\
& \quad \left( b^2 - a^2 (1 + \tan[c+dx]^2) \right)^2 - \left( 6 a^3 (a^2 - b^2) C \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2 + b^2} \right] \right. \\
& \quad \tan[c+dx] (1 + \tan[c+dx]^2)^{\frac{1}{2} - \frac{m}{2}} \left( 2 \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right) \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right.
\end{aligned}$$

$$\begin{aligned}
& 3 (a^2 - b^2) \left( -\frac{1}{3} (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] - \right. \\
& \quad \left. \frac{1}{3 (a^2 - b^2)} 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] \right) + \\
& \tan[c+dx]^2 \left( 4 a^2 \left( -\frac{3}{5} (-1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1}{2} (-1+m), 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \quad \left. \left. \frac{1}{5 (a^2 - b^2)} 18 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2} (-1+m), 4, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] \right) + \right. \\
& \quad (a^2 - b^2) (-1+m) \left( -\frac{1}{5 (a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1+m}{2}, 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] - \right. \\
& \quad \left. \left. \frac{3}{5} (1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1+m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( b \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 2, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + \left( 4 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 3, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right) \tan[c+dx]^2 \right)^2 \\
& (b^2 - a^2 (1 + \tan[c+dx]^2))^2 \Bigg) + \left( 3 A (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \tan[c+dx] \right. \\
& (1 + \tan[c+dx]^2)^{-m/2} \left( 2 \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right) \sec[c+dx]^2 \tan[c+dx] - \right. \\
& 3 (a^2 - b^2) \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] - \right. \\
& \quad \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx]}{3 (a^2 - b^2)} \right) + \\
& \tan[c+dx]^2 \left( 2 a^2 \left( -\frac{3}{5} m \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] - \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \left. \frac{12 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{m}{2}, 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx]}{5(a^2-b^2)} \right) + \\
& (a^2-b^2) m \left( -\frac{1}{5(a^2-b^2)} 6 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2+m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \\
& \left. \frac{3}{5} (2+m) \operatorname{AppellF1}\left[\frac{5}{2}, 1+\frac{2+m}{2}, 1, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \Bigg) \Bigg) / \\
& \left( \left( -3(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + (a^2-b^2) m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) \tan[c+dx]^2 \right)^2 \\
& \left. (-b^2+a^2(1+\tan[c+dx]^2)) \right) - \left( 3 a^2 (a^2-b^2) \operatorname{CAppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2+b^2}\right] \tan[c+dx] \right. \\
& \left. (1+\tan[c+dx]^2)^{-m/2} \left( 2 \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (a^2-b^2) m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \left. 3(a^2-b^2) \left( -\frac{1}{3} m \operatorname{AppellF1}\left[\frac{3}{2}, 1+\frac{m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \left. \left. \frac{2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx]}{3(a^2-b^2)} \right) \right) + \\
& \tan[c+dx]^2 \left( 2 a^2 \left( -\frac{3}{5} m \operatorname{AppellF1}\left[\frac{5}{2}, 1+\frac{m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \left. \left. \frac{12 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{m}{2}, 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx]}{5(a^2-b^2)} \right) \right) + \\
& (a^2-b^2) m \left( -\frac{1}{5(a^2-b^2)} 6 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2+m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right.
\end{aligned}$$

$$\left. \left. \left. \left. \frac{3}{5} (2+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{2+m}{2}, 1, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \right) \right) \right) /$$

$$\left( b^2 \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, \right. \right. \right. \right.$$

$$\left. \left. \left. -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] + (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, \right. \right. \right.$$

$$\left. \left. \left. -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2} \right] \right) \tan[c+dx]^2 \right)^2 (-b^2+a^2(1+\tan[c+dx]^2)) \right) \right) \right)$$

■ **Problem 773: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + dx]) (B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^2 dx$$

Optimal (type 3, 35 leaves, 5 steps):

$$(bB + aC)x + \frac{aB \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{bC \sin[c + dx]}{d}$$

Result (type 3, 104 leaves):

$$bBx + aCx - \frac{aB \operatorname{Log} \left[ \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{aB \operatorname{Log} \left[ \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{bC \cos[dx] \sin[c]}{d} + \frac{bC \cos[c] \sin[dx]}{d}$$

■ **Problem 774: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + dx]) (B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^3 dx$$

Optimal (type 3, 35 leaves, 5 steps):

$$bCx + \frac{(bB + aC) \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{aB \tan[c + dx]}{d}$$

Result (type 3, 159 leaves):

$$bCx - \frac{bB \operatorname{Log} \left[ \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right]}{d} - \frac{aC \operatorname{Log} \left[ \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] - \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right]}{d} +$$

$$\frac{bB \operatorname{Log} \left[ \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{aC \operatorname{Log} \left[ \cos \left[ \frac{c}{2} + \frac{dx}{2} \right] + \sin \left[ \frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{aB \tan[c + dx]}{d}$$

■ **Problem 775: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + dx]) (B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^4 dx$$

Optimal (type 3, 61 leaves, 7 steps) :

$$\frac{(a B + 2 b C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{(b B + a C) \operatorname{Tan}[c + d x]}{d} + \frac{a B \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 d}$$

Result (type 3, 164 leaves) :

$$\frac{1}{4 d} \left( -2 (a B + 2 b C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \right. \\ \left. 2 a B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 4 b C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \right. \\ \left. \frac{a B}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{a B}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + 4 (b B + a C) \operatorname{Tan}[c + d x] \right)$$

■ **Problem 777: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Cos}[c + d x]) (B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2) \operatorname{Sec}[c + d x]^6 dx$$

Optimal (type 3, 114 leaves, 8 steps) :

$$\frac{(3 a B + 4 b C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{(b B + a C) \operatorname{Tan}[c + d x]}{d} + \\ \frac{(3 a B + 4 b C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{a B \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d} + \frac{(b B + a C) \operatorname{Tan}[c + d x]^3}{3 d}$$

Result (type 3, 403 leaves) :

$$- \frac{3 a B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} - \frac{b C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} + \frac{3 a B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \\ \frac{b C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} + \frac{a B}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{3 a B}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \\ \frac{b C}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{a B}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} - \frac{3 a B}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \\ \frac{b C}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{2 b B \operatorname{Tan}[c + d x]}{3 d} + \frac{2 a C \operatorname{Tan}[c + d x]}{3 d} + \frac{b B \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d} + \frac{a C \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d}$$

■ **Problem 783: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Cos}[c + d x])^2 (B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2) \operatorname{Sec}[c + d x]^4 dx$$

Optimal (type 3, 80 leaves, 5 steps) :

$$b^2 C x + \frac{(a^2 B + 2 b^2 B + 4 a b C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{a (2 b B + a C) \operatorname{Tan}[c + d x]}{d} + \frac{a^2 B \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 d}$$

Result (type 3, 225 leaves):

$$\frac{1}{4 d} \left( 4 b^2 C c + 4 b^2 C d x - 2 (a^2 B + 2 b^2 B + 4 a b C) \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] + 2 a^2 B \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] + \right. \\ \left. 4 b^2 B \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] + 8 a b C \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] + \right. \\ \left. \frac{a^2 B}{\left( \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \frac{a^2 B}{\left( \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right)^2} + 4 a (2 b B + a C) \operatorname{Tan}[c + d x] \right)$$

■ **Problem 785: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Cos}[c + d x])^2 (B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2) \operatorname{Sec}[c + d x]^6 dx$$

Optimal (type 3, 156 leaves, 8 steps):

$$\frac{(3 a^2 B + 4 b^2 B + 8 a b C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{(4 a b B + 2 a^2 C + 3 b^2 C) \operatorname{Tan}[c + d x]}{3 d} + \\ \frac{(3 a^2 B + 4 b^2 B + 8 a b C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{a (2 b B + a C) \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d} + \frac{a^2 B \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d}$$

Result (type 3, 457 leaves):

$$\frac{1}{48 d} \left( -6 (3 a^2 B + 4 b^2 B + 8 a b C) \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] + 6 (3 a^2 B + 4 b^2 B + 8 a b C) \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] + \right. \\ \left. \frac{3 a^2 B}{\left( \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right)^4} + \frac{12 b^2 B + 8 a b (B + 3 C) + a^2 (9 B + 4 C)}{\left( \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{8 a (2 b B + a C) \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right]}{\left( \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right)^3} + \right. \\ \left. \frac{16 (4 a b B + 2 a^2 C + 3 b^2 C) \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right]}{\operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right]} - \frac{3 a^2 B}{\left( \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right)^4} + \right. \\ \left. \frac{8 a (2 b B + a C) \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right]}{\left( \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right)^3} - \frac{12 b^2 B + 8 a b (B + 3 C) + a^2 (9 B + 4 C)}{\left( \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{16 (4 a b B + 2 a^2 C + 3 b^2 C) \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right]}{\operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right]} \right)$$

■ **Problem 790: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Cos}[c + d x])^3 (B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2) \operatorname{Sec}[c + d x]^4 dx$$

Optimal (type 3, 124 leaves, 6 steps):

$$b^2 (bB + 3aC) x + \frac{a (a^2 B + 6b^2 B + 6abC) \operatorname{ArcTanh}[\sin[c + dx]]}{2d} -$$

$$\frac{b^2 (aB - 2bC) \sin[c + dx]}{2d} + \frac{a^2 (2bB + aC) \tan[c + dx]}{d} + \frac{aB (a + b \cos[c + dx])^2 \operatorname{Sec}[c + dx] \tan[c + dx]}{2d}$$

Result (type 3, 277 leaves):

$$\frac{1}{4d} \left( 4b^2 (bB + 3aC) (c + dx) - 2a (a^2 B + 6b^2 B + 6abC) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right] \right) +$$

$$2a (a^2 B + 6b^2 B + 6abC) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right] + \frac{a^3 B}{\left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right)^2} +$$

$$\frac{4a^2 (3bB + aC) \sin \left[ \frac{1}{2} (c + dx) \right]}{\cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right]} - \frac{a^3 B}{\left( \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right)^2} + \frac{4a^2 (3bB + aC) \sin \left[ \frac{1}{2} (c + dx) \right]}{\cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right]} + 4b^3 C \sin[c + dx] \right)$$

■ **Problem 791: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + dx])^3 (B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^5 dx$$

Optimal (type 3, 145 leaves, 6 steps):

$$b^3 C x + \frac{(3a^2 bB + 2b^3 B + a^3 C + 6ab^2 C) \operatorname{ArcTanh}[\sin[c + dx]]}{2d} + \frac{a (2a^2 B + 8b^2 B + 9abC) \tan[c + dx]}{3d} +$$

$$\frac{a^2 (5bB + 3aC) \operatorname{Sec}[c + dx] \tan[c + dx]}{6d} + \frac{aB (a + b \cos[c + dx])^2 \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3d}$$

Result (type 3, 392 leaves):

$$\frac{1}{12d} \left( 12b^3 C (c + dx) - 6 (3a^2 bB + 2b^3 B + a^3 C + 6ab^2 C) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right] \right) +$$

$$6 (3a^2 bB + 2b^3 B + a^3 C + 6ab^2 C) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right] + \frac{a^2 (9bB + a(B + 3C))}{\left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right)^2} +$$

$$\frac{2a^3 B \sin \left[ \frac{1}{2} (c + dx) \right]}{\left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right)^3} + \frac{4a (2a^2 B + 9b^2 B + 9abC) \sin \left[ \frac{1}{2} (c + dx) \right]}{\cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right]} +$$

$$\frac{2a^3 B \sin \left[ \frac{1}{2} (c + dx) \right]}{\left( \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right)^3} - \frac{a^2 (9bB + a(B + 3C))}{\left( \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right)^2} + \frac{4a (2a^2 B + 9b^2 B + 9abC) \sin \left[ \frac{1}{2} (c + dx) \right]}{\cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right]} \right)$$

■ **Problem 792: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^3 (B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^6 dx$$

Optimal (type 3, 188 leaves, 8 steps):

$$\frac{(3 a^3 B + 12 a b^2 B + 12 a^2 b C + 8 b^3 C) \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} +$$

$$\frac{(6 a^2 b B + 3 b^3 B + 2 a^3 C + 9 a b^2 C) \tan [c + d x]}{3 d} + \frac{a (3 a^2 B + 10 b^2 B + 12 a b C) \sec [c + d x] \tan [c + d x]}{8 d} +$$

$$\frac{a^2 (3 b B + 2 a C) \sec [c + d x]^2 \tan [c + d x]}{6 d} + \frac{a B (a + b \cos [c + d x])^2 \sec [c + d x]^3 \tan [c + d x]}{4 d}$$

Result (type 3, 639 leaves):

$$\frac{(-3 a^3 B - 12 a b^2 B - 12 a^2 b C - 8 b^3 C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} +$$

$$\frac{(3 a^3 B + 12 a b^2 B + 12 a^2 b C + 8 b^3 C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{a^3 B}{16 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^4} +$$

$$\frac{9 a^3 B + 12 a^2 b B + 36 a b^2 B + 4 a^3 C + 36 a^2 b C}{48 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{a^3 B}{16 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{-9 a^3 B - 12 a^2 b B - 36 a b^2 B - 4 a^3 C - 36 a^2 b C}{48 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^2} +$$

$$\frac{3 a^2 b B \sin\left[\frac{1}{2}(c + d x)\right] + a^3 C \sin\left[\frac{1}{2}(c + d x)\right]}{6 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^3} + \frac{3 a^2 b B \sin\left[\frac{1}{2}(c + d x)\right] + a^3 C \sin\left[\frac{1}{2}(c + d x)\right]}{6 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^3} +$$

$$\frac{6 a^2 b B \sin\left[\frac{1}{2}(c + d x)\right] + 3 b^3 B \sin\left[\frac{1}{2}(c + d x)\right] + 2 a^3 C \sin\left[\frac{1}{2}(c + d x)\right] + 9 a b^2 C \sin\left[\frac{1}{2}(c + d x)\right]}{3 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)} +$$

$$\frac{6 a^2 b B \sin\left[\frac{1}{2}(c + d x)\right] + 3 b^3 B \sin\left[\frac{1}{2}(c + d x)\right] + 2 a^3 C \sin\left[\frac{1}{2}(c + d x)\right] + 9 a b^2 C \sin\left[\frac{1}{2}(c + d x)\right]}{3 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)}$$

■ **Problem 800: Result more than twice size of optimal antiderivative.**

$$\int \frac{(B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^4}{a + b \cos [c + d x]} dx$$

Optimal (type 3, 143 leaves, 7 steps):

$$-\frac{2 b^2 (b B - a C) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \tan\left[\frac{1}{2}(c + d x)\right]}{\sqrt{a+b}}\right]}{a^3 \sqrt{a-b} \sqrt{a+b} d} + \frac{(a^2 B + 2 b^2 B - 2 a b C) \operatorname{ArcTanh}[\sin [c + d x]]}{2 a^3 d} - \frac{(b B - a C) \tan [c + d x]}{a^2 d} + \frac{B \sec [c + d x] \tan [c + d x]}{2 a d}$$

Result (type 3, 300 leaves) :

$$\frac{1}{4 a^3 d} \left( \frac{8 b^2 (b B - a C) \operatorname{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}} - 2 (a^2 B + 2 b^2 B - 2 a b C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + \right.$$

$$2 (a^2 B + 2 b^2 B - 2 a b C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] + \frac{a^2 B}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} +$$

$$\left. \frac{4 a (-b B + a C) \sin\left[\frac{1}{2}(c+dx)\right]}{\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]} - \frac{a^2 B}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{4 a (-b B + a C) \sin\left[\frac{1}{2}(c+dx)\right]}{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]} \right)$$

- **Problem 818: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \cos[c + dx]} (B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^3 dx$$

Optimal (type 4, 213 leaves, 10 steps) :

$$-\frac{B \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \frac{(a B + 2 b C) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{d \sqrt{a + b \cos[c + dx]}} +$$

$$\frac{(b B + 2 a C) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{d \sqrt{a + b \cos[c + dx]}} + \frac{B \sqrt{a + b \cos[c + dx]} \tan[c + dx]}{d}$$

Result (type 4, 484 leaves) :

$$\begin{aligned}
& \frac{1}{4d} \left( \frac{8bc \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b \cos[c+dx]}} + \right. \\
& \frac{2(bB+4aC) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b \cos[c+dx]}} + \left( 2ibB \sqrt{\frac{b-b \cos[c+dx]}{a+b}} \sqrt{-\frac{b+b \cos[c+dx]}{a-b}} \cos[2(c+dx)] \right. \\
& \left. \left( 2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin[c+dx] \right) / \\
& \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]}^2 \sqrt{-\frac{a^2-b^2-2a(a+b \cos[c+dx])+(a+b \cos[c+dx])^2}{b^2}} \right. \\
& \left. \left. \left. (2a^2-b^2-4a(a+b \cos[c+dx])+2(a+b \cos[c+dx])^2) \right) \right) + \frac{B \sqrt{a+b \cos[c+dx]} \tan[c+dx]}{d}
\end{aligned}$$

■ **Problem 819: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+b \cos[c+dx]} (B \cos[c+dx] + C \cos[c+dx]^2) \sec[c+dx]^4 dx$$

Optimal (type 4, 292 leaves, 11 steps):



$$\begin{aligned}
& - \frac{(bB + 4aC) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4ad \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \\
& \frac{(3bB + 4aC) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] + (4a^2B - b^2B + 4abc) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4d \sqrt{a + b \cos[c + dx]}} + \\
& \frac{(bB + 4aC) \sqrt{a + b \cos[c + dx]} \operatorname{Tan}[c + dx]}{4ad} + \frac{B \sqrt{a + b \cos[c + dx]} \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2d}
\end{aligned}$$

Result (type 4, 552 leaves):

$$\begin{aligned}
& \frac{1}{16ad} \left( \frac{8abB \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} + \frac{2(8a^2B - 3b^2B + 4abc) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} - \right. \\
& \left. \left( 2i(-b^2B - 4abc) \sqrt{\frac{b - b \cos[c + dx]}{a + b}} \sqrt{\frac{b + b \cos[c + dx]}{a - b}} \operatorname{Cos}[2(c + dx)] \right. \right. \\
& \left. \left. \left( 2a(a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \right. \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \operatorname{Sin}[c + dx] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1 - \cos[c + dx]^2} \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2 - b^2 - 2a(a + b \cos[c + dx]) + (a + b \cos[c + dx])^2}{b^2}} (2a^2 - b^2 - 4a(a + b \cos[c + dx]) + 2(a + b \cos[c + dx])^2) \right) \right) + \\
& \frac{\sqrt{a + b \cos[c + dx]} \left( \frac{\operatorname{Sec}[c + dx] (bB \operatorname{Sin}[c + dx] + 4aC \operatorname{Sin}[c + dx])}{4a} + \frac{1}{2} B \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx] \right)}{d}
\end{aligned}$$

■ **Problem 823: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + b \cos[c + dx])^{3/2} (B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^2 dx$$

Optimal (type 4, 236 leaves, 10 steps):

$$\frac{2(3bB + 4aC)\sqrt{a+b\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{3d\sqrt{\frac{a+b\cos[c+dx]}{a+b}}} + \frac{2(3abB - a^2C + b^2C)\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{3d\sqrt{a+b\cos[c+dx]}}$$

$$\frac{2a^2B\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{d\sqrt{a+b\cos[c+dx]}} + \frac{2bC\sqrt{a+b\cos[c+dx]}\sin[c+dx]}{3d}$$

Result (type 4, 406 leaves):

$$\frac{1}{6d}$$

$$\left( \frac{4(6abB + 3a^2C + b^2C)\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \frac{2(6a^2B + 3b^2B + 4abC)\sqrt{\frac{a+b\cos[c+dx]}{a+b}}\operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} \right) +$$

$$\frac{1}{ab\sqrt{-\frac{1}{a+b}}} 2i(3bB + 4aC) \sqrt{-\frac{b(-1 + \cos[c+dx])}{a+b}} \sqrt{\frac{b(1 + \cos[c+dx])}{-a+b}} \operatorname{Csc}[c+dx]$$

$$\left( -2a(a-b)\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b\left( -2a\operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b\operatorname{EllipticPi}\left[\frac{a+b}{a}, i\operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] \right) \right) + 4bC\sqrt{a+b\cos[c+dx]}\sin[c+dx]$$

■ **Problem 824: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b\cos[c+dx])^{3/2} (B\cos[c+dx] + C\cos[c+dx]^2) \operatorname{Sec}[c+dx]^3 dx$$

Optimal (type 4, 232 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(aB - 2bC) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] + (a^2B + 2b^2B + 2abC) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \frac{(a^2B + 2b^2B + 2abC) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{d \sqrt{a + b \cos[c + dx]}} + \\
& \frac{a(3bB + 2aC) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{d \sqrt{a + b \cos[c + dx]}} + \frac{aB \sqrt{a + b \cos[c + dx]} \operatorname{Tan}[c + dx]}{d}
\end{aligned}$$

Result (type 4, 398 leaves):

$$\begin{aligned}
& \frac{1}{4d} \left( \frac{8b(bB + 2aC) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} + \right. \\
& \frac{2(5abB + 4a^2C + 2b^2C) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} + \frac{1}{ab \sqrt{-\frac{1}{a+b}}} 2i(-aB + 2bC) \sqrt{-\frac{b(-1 + \cos[c + dx])}{a+b}} \\
& \sqrt{\frac{b(1 + \cos[c + dx])}{-a+b}} \operatorname{Csc}[c + dx] \left( -2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right] + \right. \\
& b \left( -2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right] + \right. \\
& \left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right]\right) \right) + 4aB \sqrt{a + b \cos[c + dx]} \operatorname{Tan}[c + dx] \left. \right)
\end{aligned}$$

■ **Problem 825: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + b \cos[c + dx])^{3/2} (B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^4 dx$$

Optimal (type 4, 295 leaves, 11 steps):

$$\begin{aligned}
& - \frac{(5bB + 4aC) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4d \sqrt{\frac{a + b \cos[c + dx]}{a+b}}} + \\
& \frac{(7abB + 4a^2C + 8b^2C) \sqrt{\frac{a + b \cos[c + dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] + (4a^2B + 3b^2B + 12abC) \sqrt{\frac{a + b \cos[c + dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4d \sqrt{a + b \cos[c + dx]}} + \\
& \frac{(5bB + 4aC) \sqrt{a + b \cos[c + dx]} \tan[c + dx]}{4d} + \frac{aB \sqrt{a + b \cos[c + dx]} \operatorname{Sec}[c + dx] \tan[c + dx]}{2d}
\end{aligned}$$

Result (type 4, 556 leaves):

$$\begin{aligned}
& \frac{1}{16d} \left( \frac{2(4abB + 16b^2C) \sqrt{\frac{a + b \cos[c + dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} + \frac{2(8a^2B + b^2B + 20abC) \sqrt{\frac{a + b \cos[c + dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} \right) - \\
& \left( 2i(-5b^2B - 4abC) \sqrt{\frac{b - b \cos[c + dx]}{a+b}} \sqrt{\frac{b + b \cos[c + dx]}{a-b}} \cos[2(c + dx)] \right. \\
& \left. \left( 2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right] \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b} \right) - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right] \right) \right) \sin[c + dx] \Big/ \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1 - \cos[c + dx]}^2 \right. \\
& \left. \left. \sqrt{-\frac{a^2 - b^2 - 2a(a + b \cos[c + dx]) + (a + b \cos[c + dx])^2}{b^2}} (2a^2 - b^2 - 4a(a + b \cos[c + dx]) + 2(a + b \cos[c + dx])^2) \right) \right) + \\
& \frac{1}{d} \sqrt{a + b \cos[c + dx]} \left( \frac{1}{4} \operatorname{Sec}[c + dx] (5bB \sin[c + dx] + 4aC \sin[c + dx]) + \frac{1}{2} aB \operatorname{Sec}[c + dx] \tan[c + dx] \right)
\end{aligned}$$

■ **Problem 826: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + b \cos[c + dx])^{3/2} (B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^5 dx$$

Optimal (type 4, 375 leaves, 12 steps):

$$\begin{aligned}
 & - \frac{(16 a^2 B + 3 b^2 B + 30 a b C) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{24 a d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}} + \\
 & \frac{(16 a^2 B + 17 b^2 B + 42 a b C) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{24 d \sqrt{a + b \cos[c + d x]}} + \\
 & \frac{(12 a^2 b B - b^3 B + 8 a^3 C + 6 a b^2 C) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{8 a d \sqrt{a + b \cos[c + d x]}} + \frac{(16 a^2 B + 3 b^2 B + 30 a b C) \sqrt{a + b \cos[c + d x]} \operatorname{Tan}[c + d x]}{24 a d} + \\
 & \frac{(7 b B + 6 a C) \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{12 d} + \frac{a B \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d}
 \end{aligned}$$

Result (type 4, 634 leaves):

$$\begin{aligned}
& \frac{1}{96 a d} \left( \frac{2 (28 a b^2 B + 24 a^2 b C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2 (56 a^2 b B - 9 b^3 B + 48 a^3 C + 6 a b^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\
& \left( 2 i (-16 a^2 b B - 3 b^3 B - 30 a b^2 C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{1}{12} \sec [c+d x]^2 (7 b B \sin [c+d x]+6 a C \sin [c+d x]) + \right. \\
& \left. \frac{\sec [c+d x] (16 a^2 B \sin [c+d x]+3 b^2 B \sin [c+d x]+30 a b C \sin [c+d x])}{24 a} + \frac{1}{3} a B \sec [c+d x]^2 \tan [c+d x] \right)
\end{aligned}$$

■ **Problem 830: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos [c+d x])^{5/2} (B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]^2 dx$$

Optimal (type 4, 292 leaves, 11 steps):

$$\begin{aligned}
& \frac{2 (35 a b B + 23 a^2 C + 9 b^2 C) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{15 d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}} + \\
& \frac{2 (10 a^2 b B + 5 b^3 B - 8 a^3 C + 8 a b^2 C) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{15 d \sqrt{a + b \cos[c + d x]}} + \frac{2 a^3 B \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{2 b (5 b B + 8 a C) \sqrt{a + b \cos[c + d x]} \sin[c + d x]}{15 d} + \frac{2 b C (a + b \cos[c + d x])^{3/2} \sin[c + d x]}{5 d}
\end{aligned}$$

Result (type 4, 453 leaves):

$$\begin{aligned}
& \frac{1}{30 d} \left( \frac{4 (45 a^2 b B + 5 b^3 B + 15 a^3 C + 17 a b^2 C) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + d x]}} + \right. \\
& \frac{2 (30 a^3 B + 35 a b^2 B + 23 a^2 b C + 9 b^3 C) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + d x]}} + \\
& \frac{1}{a b \sqrt{-\frac{1}{a+b}}} 2 i (35 a b B + 23 a^2 C + 9 b^2 C) \sqrt{-\frac{b(-1 + \cos[c + d x])}{a+b}} \sqrt{-\frac{b(1 + \cos[c + d x])}{a-b}} \\
& \left. \operatorname{Csc}[c + d x] \left( -2 a (a - b) \operatorname{EllipticE}\left[ i \operatorname{ArcSinh}\left[ \sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + d x]} \right], \frac{a+b}{a-b} \right] + b \left( -2 a \operatorname{EllipticF}\left[ \right. \right. \right. \right. \\
& \left. \left. \left. \left. i \operatorname{ArcSinh}\left[ \sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + d x]} \right], \frac{a+b}{a-b} \right] + b \operatorname{EllipticPi}\left[ \frac{a+b}{a}, i \operatorname{ArcSinh}\left[ \sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + d x]} \right], \frac{a+b}{a-b} \right] \right) \right) \right) + \\
& \left. 4 b \sqrt{a + b \cos[c + d x]} (5 b B + 11 a C + 3 b C \cos[c + d x]) \sin[c + d x] \right)
\end{aligned}$$

■ **Problem 831: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + b \cos [c + d x])^{5/2} (B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^3 dx$$

Optimal (type 4, 296 leaves, 11 steps):

$$\begin{aligned} & - \frac{(3 a^2 B - 6 b^2 B - 14 a b C) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{3 d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} + \\ & \frac{(3 a^3 B + 12 a b^2 B + 4 a^2 b C + 2 b^3 C) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right] - a^2 (5 b B + 2 a C) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{3 d \sqrt{a + b \cos [c + d x]} + d \sqrt{a + b \cos [c + d x]}} - \\ & \frac{b (3 a B - 2 b C) \sqrt{a + b \cos [c + d x]} \sin [c + d x]}{3 d} + \frac{a B (a + b \cos [c + d x])^{3/2} \tan [c + d x]}{d} \end{aligned}$$

Result (type 4, 560 leaves):



$$\begin{aligned}
& \frac{1}{12d} \left( \frac{2(36ab^2B + 36a^2bC + 4b^3C) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \right. \\
& \frac{2(27a^2bB + 6b^3B + 12a^3C + 14ab^2C) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} - \\
& \left. \frac{2i(-3a^2bB + 6b^3B + 14ab^2C) \sqrt{\frac{b-b\cos[c+dx]}{a+b}} \sqrt{-\frac{b+b\cos[c+dx]}{a-b}} \cos[2(c+dx)]}{\sqrt{a+b\cos[c+dx]}} \right) \\
& \left( 2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin[c+dx] \Big/ \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]}^2 \right. \\
& \left. \left. \sqrt{-\frac{a^2-b^2-2a(a+b\cos[c+dx])+(a+b\cos[c+dx])^2}{b^2}} (2a^2-b^2-4a(a+b\cos[c+dx])+2(a+b\cos[c+dx])^2) \right) \right) + \\
& \frac{\sqrt{a+b\cos[c+dx]} \left( \frac{2}{3}b^2C \sin[c+dx] + a^2B \tan[c+dx] \right)}{d}
\end{aligned}$$

■ **Problem 832: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b\cos[c+dx])^{5/2} (B\cos[c+dx] + C\cos[c+dx]^2) \sec[c+dx]^4 dx$$

Optimal (type 4, 315 leaves, 11 steps):

$$\begin{aligned}
& - \frac{(9 a b B + 4 a^2 C - 8 b^2 C) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{4 d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}} + \\
& \frac{(11 a^2 b B + 8 b^3 B + 4 a^3 C + 16 a b^2 C) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{4 d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{a (4 a^2 B + 15 b^2 B + 20 a b C) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{4 d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{a (7 b B + 4 a C) \sqrt{a + b \cos[c + d x]} \tan[c + d x]}{4 d} + \frac{a B (a + b \cos[c + d x])^{3/2} \sec[c + d x] \tan[c + d x]}{2 d}
\end{aligned}$$

Result (type 4, 589 leaves):

$$\begin{aligned}
& \frac{1}{16d} \left( \frac{2(4a^2bB + 16b^3B + 48ab^2C) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \right. \\
& \frac{2(8a^3B + 21a^2bB + 36a^2bC + 8b^3C) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} - \\
& \left. \left( 2i(-9ab^2B - 4a^2bC + 8b^3C) \sqrt{\frac{b-b\cos[c+dx]}{a+b}} \sqrt{-\frac{b+b\cos[c+dx]}{a-b}} \cos[2(c+dx)] \right. \right. \\
& \left. \left. \left( 2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \right) \sin[c+dx] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]} \right)^2 \\
& \left. \left. \left. \left. \sqrt{-\frac{a^2-b^2-2a(a+b\cos[c+dx])+(a+b\cos[c+dx])^2}{b^2}} (2a^2-b^2-4a(a+b\cos[c+dx])+2(a+b\cos[c+dx])^2) \right) \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b\cos[c+dx]} \left( \frac{1}{4} \sec[c+dx] (9abB \sin[c+dx] + 4a^2C \sin[c+dx]) + \frac{1}{2} a^2B \sec[c+dx] \tan[c+dx] \right)
\end{aligned}$$

■ **Problem 833: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b\cos[c+dx])^{5/2} (B\cos[c+dx] + C\cos[c+dx]^2) \sec[c+dx]^5 dx$$

Optimal (type 4, 376 leaves, 12 steps):

$$\begin{aligned}
& - \frac{(16 a^2 B + 33 b^2 B + 54 a b C) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{24 d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} + \\
& \frac{(16 a^3 B + 59 a b^2 B + 66 a^2 b C + 48 b^3 C) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{24 d \sqrt{a + b \cos [c + d x]}} + \\
& \frac{(20 a^2 b B + 5 b^3 B + 8 a^3 C + 30 a b^2 C) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{8 d \sqrt{a + b \cos [c + d x]}} + \\
& \frac{(16 a^2 B + 33 b^2 B + 54 a b C) \sqrt{a + b \cos [c + d x]} \tan [c + d x]}{24 d} + \\
& \frac{a(3 b B + 2 a C) \sqrt{a + b \cos [c + d x]} \sec [c + d x] \tan [c + d x]}{4 d} + \frac{a B (a + b \cos [c + d x])^{3/2} \sec [c + d x]^2 \tan [c + d x]}{3 d}
\end{aligned}$$

Result (type 4, 639 leaves):

$$\begin{aligned}
& \frac{1}{96 d} \left( \frac{2 (52 a b^2 B + 24 a^2 b C + 96 b^3 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2 (104 a^2 b B - 3 b^3 B + 48 a^3 C + 126 a b^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\
& \left( 2 i (-16 a^2 b B - 33 b^3 B - 54 a b^2 C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{1}{12} \sec [c+d x]^2 (13 a b B \sin [c+d x] + 6 a^2 C \sin [c+d x]) + \right. \\
& \left. \frac{1}{24} \sec [c+d x] (16 a^2 B \sin [c+d x] + 33 b^2 B \sin [c+d x] + 54 a b C \sin [c+d x]) + \frac{1}{3} a^2 B \sec [c+d x]^2 \tan [c+d x] \right)
\end{aligned}$$

■ **Problem 834: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos [c+d x])^{5/2} (B \cos [c+d x] + C \cos [c+d x]^2) \sec [c+d x]^6 dx$$

Optimal (type 4, 465 leaves, 13 steps):

$$\begin{aligned}
& - \frac{(284 a^2 b B + 15 b^3 B + 128 a^3 C + 264 a b^2 C) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{192 a d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \\
& \frac{(356 a^2 b B + 133 b^3 B + 128 a^3 C + 472 a b^2 C) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{192 d \sqrt{a + b \cos[c + dx]}} + \\
& \frac{(48 a^4 B + 120 a^2 b^2 B - 5 b^4 B + 160 a^3 b C + 40 a b^3 C) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{64 a d \sqrt{a + b \cos[c + dx]}} + \\
& \frac{(284 a^2 b B + 15 b^3 B + 128 a^3 C + 264 a b^2 C) \sqrt{a + b \cos[c + dx]} \tan[c + dx]}{192 a d} + \\
& \frac{(36 a^2 B + 59 b^2 B + 104 a b C) \sqrt{a + b \cos[c + dx]} \sec[c + dx] \tan[c + dx]}{96 d} + \\
& \frac{a (11 b B + 8 a C) \sqrt{a + b \cos[c + dx]} \sec[c + dx]^2 \tan[c + dx]}{24 d} + \frac{a B (a + b \cos[c + dx])^{3/2} \sec[c + dx]^3 \tan[c + dx]}{4 d}
\end{aligned}$$

Result (type 4, 729 leaves):

$$\begin{aligned}
& \frac{1}{768 a d} \left( \frac{2 (144 a^3 b B + 236 a b^3 B + 416 a^2 b^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{1}{\sqrt{a+b \cos [c+d x]}} \right. \\
& 2 (288 a^4 B + 436 a^2 b^2 B - 45 b^4 B + 832 a^3 b C - 24 a b^3 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] - \\
& \left. \left( 2 i (-284 a^2 b^2 B - 15 b^4 B - 128 a^3 b C - 264 a b^3 C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \right. \\
& \left. \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{1}{24} \sec [c+d x]^3 (17 a b B \sin [c+d x]+8 a^2 C \sin [c+d x]) + \right. \\
& \frac{1}{96} \sec [c+d x]^2 (36 a^2 B \sin [c+d x]+59 b^2 B \sin [c+d x]+104 a b C \sin [c+d x]) + \\
& 1 / (192 a) \sec [c+d x] (284 a^2 b B \sin [c+d x]+15 b^3 B \sin [c+d x]+128 a^3 C \sin [c+d x]+264 a b^2 C \sin [c+d x]) + \\
& \left. \frac{1}{4} a^2 B \sec [c+d x]^3 \tan [c+d x] \right)
\end{aligned}$$

■ **Problem 839: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]^3}{\sqrt{a+b \cos [c+d x]}} dx$$

Optimal (type 4, 216 leaves, 10 steps):

$$\begin{aligned}
& - \frac{B \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{a d \sqrt{\frac{a+b \cos [c+d x]}{a+b}}} + \frac{B \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{d \sqrt{a+b \cos [c+d x]}} \\
& \frac{(b B-2 a C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{a d \sqrt{a+b \cos [c+d x]}} + \frac{B \sqrt{a+b \cos [c+d x]} \operatorname{Tan}[c+d x]}{a d}
\end{aligned}$$

Result (type 4, 320 leaves):

$$\begin{aligned}
& \frac{1}{4 a d} \left( \frac{2(-3 b B+4 a C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \right. \\
& \left. 1 / \left( a b \sqrt{-\frac{1}{a+b}} \right) 2 i B \sqrt{-\frac{b(-1+\cos [c+d x])}{a+b}} \sqrt{\frac{b(1+\cos [c+d x])}{-a+b}} \operatorname{Csc}[c+d x] \right. \\
& \left. \left( -2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( -2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] + b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) + 4 B \sqrt{a+b \cos [c+d x]} \operatorname{Tan}[c+d x] \right)
\end{aligned}$$

■ **Problem 840: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(B \cos [c+d x]+C \cos [c+d x]^2) \operatorname{Sec}[c+d x]^4}{\sqrt{a+b \cos [c+d x]}} dx$$

Optimal (type 4, 299 leaves, 11 steps):



$$\frac{(3bB - 4aC) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4a^2 d \sqrt{\frac{a + b \cos[c + dx]}{a+b}}} -$$

$$\frac{(bB - 4aC) \sqrt{\frac{a + b \cos[c + dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] + (4a^2 B + 3b^2 B - 4abC) \sqrt{\frac{a + b \cos[c + dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4ad \sqrt{a + b \cos[c + dx]} + 4a^2 d \sqrt{a + b \cos[c + dx]}} -$$

$$\frac{(3bB - 4aC) \sqrt{a + b \cos[c + dx]} \tan[c + dx]}{4a^2 d} + \frac{B \sqrt{a + b \cos[c + dx]} \sec[c + dx] \tan[c + dx]}{2ad}$$

Result (type 4, 556 leaves):

$$\frac{1}{16a^2 d} \left( \frac{8abB \sqrt{\frac{a + b \cos[c + dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} + \frac{2(8a^2 B + 9b^2 B - 12abC) \sqrt{\frac{a + b \cos[c + dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} - \right.$$

$$\left. \left( 2i(3b^2 B - 4abC) \sqrt{\frac{b - b \cos[c + dx]}{a+b}} \sqrt{\frac{b + b \cos[c + dx]}{a-b}} \cos[2(c + dx)] \right) \right.$$

$$\left. \left( 2a(a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right] \right. \right. \right.$$

$$\left. \left. \left. - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin[c + dx] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1 - \cos[c + dx]^2} \right.$$

$$\left. \left. \left. \sqrt{-\frac{a^2 - b^2 - 2a(a + b \cos[c + dx]) + (a + b \cos[c + dx])^2}{b^2}} (2a^2 - b^2 - 4a(a + b \cos[c + dx]) + 2(a + b \cos[c + dx])^2) \right) \right) +$$

$$\frac{\sqrt{a + b \cos[c + dx]} \left( \frac{\sec[c + dx] (-3bB \sin[c + dx] + 4aC \sin[c + dx])}{4a^2} + \frac{B \sec[c + dx] \tan[c + dx]}{2a} \right)}{d}$$

■ **Problem 845: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^2}{(a + b \cos[c + dx])^{3/2}} dx$$

Optimal (type 4, 190 leaves, 8 steps) :

$$\begin{aligned}
 & - \frac{2 (b B - a C) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{a (a^2 - b^2) d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} + \\
 & \frac{2 B \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{a d \sqrt{a + b \cos [c + d x]}} + \frac{2 b (b B - a C) \sin [c + d x]}{a (a^2 - b^2) d \sqrt{a + b \cos [c + d x]}}
 \end{aligned}$$

Result (type 4, 614 leaves) :

$$\begin{aligned}
 & - \frac{2 \cos [c + d x] (C + B \sec [c + d x]) (-b^2 B \sin [c + d x] + a b C \sin [c + d x])}{a (a^2 - b^2) d \sqrt{a + b \cos [c + d x]} (B + C \cos [c + d x])} - \\
 & \frac{1}{2 a (-a + b) (a + b) d (B + C \cos [c + d x])} \cos [c + d x] (C + B \sec [c + d x]) \\
 & \left( \frac{2 (-2 a b B + 2 a^2 C) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{\sqrt{a + b \cos [c + d x]}} + \frac{2 (2 a^2 B - 3 b^2 B + a b C) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{\sqrt{a + b \cos [c + d x]}} - \right. \\
 & \left. \left( 2 i (-b^2 B + a b C) \sqrt{\frac{b - b \cos [c + d x]}{a + b}} \sqrt{-\frac{b + b \cos [c + d x]}{a - b}} \cos [2 (c + d x)] \right. \right. \\
 & \left. \left( 2 a (a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos [c + d x]}\right], \frac{a + b}{a - b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos [c + d x]}\right], \right. \right. \right. \right. \\
 & \left. \left. \left. \frac{a + b}{a - b}\right] - b \operatorname{EllipticPi}\left[\frac{a + b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos [c + d x]}\right], \frac{a + b}{a - b}\right]\right) \right) \sin [c + d x] \left. \right) / \left( a \sqrt{-\frac{1}{a + b}} \right. \\
 & \left. \left. \left. \sqrt{1 - \cos [c + d x]^2} \sqrt{-\frac{a^2 - b^2 - 2 a (a + b \cos [c + d x]) + (a + b \cos [c + d x])^2}{b^2}} (2 a^2 - b^2 - 4 a (a + b \cos [c + d x]) + 2 (a + b \cos [c + d x])^2) \right) \right) \right)
 \end{aligned}$$

- **Problem 846: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^3}{(a + b \cos[c + dx])^{3/2}} dx$$

Optimal (type 4, 303 leaves, 11 steps):

$$\begin{aligned} & - \frac{(a^2 B - 3 b^2 B + 2 a b C) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] + B \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{a^2 (a^2 - b^2) d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \frac{B \sqrt{a + b \cos[c + dx]}}{a d \sqrt{a + b \cos[c + dx]}} \\ & \frac{(3 b B - 2 a C) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{a^2 d \sqrt{a + b \cos[c + dx]}} + \frac{b (a^2 B - 3 b^2 B + 2 a b C) \sin[c + dx]}{a^2 (a^2 - b^2) d \sqrt{a + b \cos[c + dx]}} + \frac{B \tan[c + dx]}{a d \sqrt{a + b \cos[c + dx]}} \end{aligned}$$

Result (type 4, 608 leaves):

$$\begin{aligned}
& \frac{1}{4 a^2 (a-b)(a+b) d} \left( \frac{2 (4 a b^2 B - 4 a^2 b C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2 (-7 a^2 b B + 9 b^3 B + 4 a^3 C - 6 a b^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\
& \left( 2 i (-a^2 b B + 3 b^3 B - 2 a b^2 C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) + \\
& \frac{\sqrt{a+b \cos [c+d x]} \left( \frac{2(-b^3 B \sin [c+d x]+a b^2 C \sin [c+d x])}{a^2(a^2-b^2)(a+b \cos [c+d x])} + \frac{B \tan [c+d x]}{a^2} \right)}{d}
\end{aligned}$$

- **Problem 851: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(B \cos [c+d x] + C \cos [c+d x]^2) \sec [c+d x]^2}{(a+b \cos [c+d x])^{5/2}} dx$$

Optimal (type 4, 349 leaves, 11 steps):

$$\begin{aligned}
& - \frac{2 (7 a^2 b B - 3 b^3 B - 4 a^3 C) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{3 a^2 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos[c + d x]}{a + b}}} + \frac{2 (b B - a C) \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{3 a (a^2 - b^2) d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{2 B \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{a^2 d \sqrt{a + b \cos[c + d x]}} + \frac{2 b (b B - a C) \sin[c + d x]}{3 a (a^2 - b^2) d (a + b \cos[c + d x])^{3/2}} + \frac{2 b (7 a^2 b B - 3 b^3 B - 4 a^3 C) \sin[c + d x]}{3 a^2 (a^2 - b^2)^2 d \sqrt{a + b \cos[c + d x]}}
\end{aligned}$$

Result (type 4, 743 leaves):

$$\begin{aligned}
& \frac{1}{6 a^2 (a-b)^2 (a+b)^2 d (B+C \cos [c+d x])} \\
& \cos [c+d x] (C+B \sec [c+d x]) \left( \frac{2 \left(-12 a^3 b B+4 a b^3 B+6 a^4 C+2 a^2 b^2 C\right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2 \left(6 a^4 B-19 a^2 b^2 B+9 b^4 B+4 a^3 b C\right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\
& \left. \left( 2 i \left(-7 a^2 b^2 B+3 b^4 B+4 a^3 b C\right) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \left( 2 a(a-b) \right. \right. \right. \\
& \left. \left. \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]+b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]- \right. \right. \right. \\
& \left. \left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right)\right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right) \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} \left( 2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2 \right) \right) \right) \right) + \\
& \frac{1}{d(B+C \cos [c+d x])} \cos [c+d x] \sqrt{a+b \cos [c+d x]} (C+B \sec [c+d x]) \\
& \left( -\frac{2\left(-b^2 B \sin [c+d x]+a b C \sin [c+d x]\right)}{3 a\left(a^2-b^2\right)(a+b \cos [c+d x])^2} - \right. \\
& \left. \frac{2\left(-7 a^2 b^2 B \sin [c+d x]+3 b^4 B \sin [c+d x]+4 a^3 b C \sin [c+d x]\right)}{3 a^2\left(a^2-b^2\right)^2(a+b \cos [c+d x])} \right)
\end{aligned}$$

■ **Problem 852: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]^3}{(a+b \cos [c+d x])^{5/2}} dx$$

Optimal (type 4, 437 leaves, 12 steps):

$$\begin{aligned}
 & - \frac{(3 a^4 B - 26 a^2 b^2 B + 15 b^4 B + 14 a^3 b C - 6 a b^3 C) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{3 a^3 (a^2 - b^2)^2 d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}} + \\
 & \frac{(3 a^2 B - 5 b^2 B + 2 a b C) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right] - (5 b B - 2 a C) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{3 a^2 (a^2 - b^2) d \sqrt{a + b \cos[c + d x]} - a^3 d \sqrt{a + b \cos[c + d x]}} + \\
 & \frac{b (3 a^2 B - 5 b^2 B + 2 a b C) \sin[c + d x]}{3 a^2 (a^2 - b^2) d (a + b \cos[c + d x])^{3/2}} + \frac{b (3 a^4 B - 26 a^2 b^2 B + 15 b^4 B + 14 a^3 b C - 6 a b^3 C) \sin[c + d x]}{3 a^3 (a^2 - b^2)^2 d \sqrt{a + b \cos[c + d x]}} + \frac{B \tan[c + d x]}{a d (a + b \cos[c + d x])^{3/2}}
 \end{aligned}$$

Result (type 4, 750 leaves):

$$\begin{aligned}
& \frac{1}{12 a^3 (-a+b)^2 (a+b)^2 d} \left( \frac{2 (36 a^3 b^2 B - 20 a b^4 B - 24 a^4 b C + 8 a^2 b^3 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{1}{\sqrt{a+b \cos [c+d x]}} \right. \\
& 2 (-33 a^4 b B + 86 a^2 b^3 B - 45 b^5 B + 12 a^5 C - 38 a^3 b^2 C + 18 a b^4 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c+d x), \frac{2 b}{a+b}\right] - \\
& \left. \left( 2 i (-3 a^4 b B + 26 a^2 b^3 B - 15 b^5 B - 14 a^3 b^2 C + 6 a b^4 C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2 (c+d x)] \right. \right. \\
& \left. \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right)\right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a (a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a (a+b \cos [c+d x])+2 (a+b \cos [c+d x])^2) \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{2 (-b^3 B \sin [c+d x]+a b^2 C \sin [c+d x])}{3 a^2 (a^2-b^2)(a+b \cos [c+d x])^2} + \right. \\
& \left. \frac{2 (-10 a^2 b^3 B \sin [c+d x]+6 b^5 B \sin [c+d x]+7 a^3 b^2 C \sin [c+d x]-3 a b^4 C \sin [c+d x])}{3 a^3 (a^2-b^2)^2 (a+b \cos [c+d x])} + \frac{B \tan [c+d x]}{a^3} \right)
\end{aligned}$$

■ **Problem 881: Result more than twice size of optimal antiderivative.**

$$\int \frac{B \cos [c+d x]+C \cos [c+d x]^2}{\cos [c+d x]^{5/2} (a+b \cos [c+d x])} dx$$

Optimal (type 4, 86 leaves, 6 steps):

$$-\frac{2 B \operatorname{EllipticE}\left[\frac{1}{2} (c+d x), 2\right]}{a d} - \frac{2 (b B-a C) \operatorname{EllipticPi}\left[\frac{2 b}{a+b}, \frac{1}{2} (c+d x), 2\right]}{a (a+b) d} + \frac{2 B \sin [c+d x]}{a d \sqrt{\cos [c+d x]}}$$

Result (type 4, 210 leaves):



$$\frac{1}{2ad} \left( \frac{2(-3bB + 2aC) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a+b} - \frac{2aB \left( 2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] - \frac{2a \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a+b} \right)}{b} + \right. \\ \left. \frac{4B \sin[c+dx]}{\sqrt{\cos[c+dx]}} - 1 \right) / \left( ab \sqrt{\sin[c+dx]^2} \right) 2B \left( -2ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\cos[c+dx]}\right], -1\right] + \right. \\ \left. 2a(a+b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\cos[c+dx]}\right], -1\right] + (2a^2 - b^2) \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\cos[c+dx]}\right], -1\right] \right) \sin[c+dx] \right)$$

- **Problem 897: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} (B \cos[c+dx] + C \cos[c+dx]^2) dx$$

Optimal (type 4, 560 leaves, 9 steps):

$$-\frac{1}{24ab^2d} (a-b) \sqrt{a+b} (6abB - 3a^2C + 16b^2C) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\ \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{24b^2d} \sqrt{a+b} (a+2b) (6bB - 3aC + 8bC) \cot[c+dx] \\ \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{8b^3d} \\ \sqrt{a+b} (2a^2bB - 8b^3B - a^3C - 4ab^2C) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\ \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(6abB - 3a^2C + 16b^2C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{24b^2d \sqrt{\cos[c+dx]}} + \\ \frac{(2bB - aC) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{4bd} + \frac{C \sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{3bd}$$

Result (type 4, 1224 leaves):

$$\begin{aligned}
& -\frac{1}{48bd} \left( - \left( 4a(-18abB + a^2C - 16b^2C) \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\operatorname{Cos}[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b)\sqrt{\operatorname{Cos}[c+dx]}\sqrt{a+b\operatorname{Cos}[c+dx]} \right) - 4a(-24b^2B - 28abC) \\
& \left( \left( \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\operatorname{Cos}[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\operatorname{Cos}[c+dx]}\sqrt{a+b\operatorname{Cos}[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\operatorname{Cos}[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b\sqrt{\operatorname{Cos}[c+dx]}\sqrt{a+b\operatorname{Cos}[c+dx]} \right) \right) + \\
& 2(-6abB + 3a^2C - 16b^2C) \left( \frac{i\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\operatorname{Cos}[c+dx]}\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right]\operatorname{Sec}[c+dx]}{b\sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2}\operatorname{Sec}[c+dx]}\sqrt{\frac{(a+b\operatorname{Cos}[c+dx])\operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \Bigg) + \\
& \quad \frac{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{(6bB+aC) \operatorname{Sin}[c+dx]}{12b} + \frac{1}{6} C \operatorname{Sin}[2(c+dx)] \right)}{d}
\end{aligned}$$

■ **Problem 898: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} (B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2)}{\sqrt{\operatorname{Cos}[c+dx]}} dx$$

Optimal (type 4, 473 leaves, 8 steps):

$$-\frac{1}{4abd}$$

$$(a-b)\sqrt{a+b}(4bB+aC)\cot[c+dx]\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]\sqrt{\frac{a(1-\sec[c+dx])}{a+b}}\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{4bd}\sqrt{a+b}(aC+2b(2B+C))\cot[c+dx]\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]\sqrt{\frac{a(1-\sec[c+dx])}{a+b}}\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{4b^2d}\sqrt{a+b}(4abB-a^2C+4b^2C)\cot[c+dx]\operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]\sqrt{\frac{a(1-\sec[c+dx])}{a+b}}$$

$$\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(4bB+aC)\sqrt{a+b}\cos[c+dx]\sin[c+dx]}{4bd\sqrt{\cos[c+dx]}} + \frac{C\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx]\sin[c+dx]}{2d}$$

Result (type 4, 1175 leaves):

$$\frac{C\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx]\sin[c+dx]}{2d} + \frac{1}{8d}$$

$$\left( - \left( 4a(4bB+3aC)\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}}\sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\csc[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}}{a\sqrt{2}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - 4a \right.$$

$$\left. (8aB+4bC) \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}}\sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\csc[c+dx] \right. \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}}{a\sqrt{2}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right.$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& 2(4bB+aC) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /
\end{aligned}$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}}$$

■ **Problem 899: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \cos[c+dx]} (B \cos[c+dx] + C \cos[c+dx]^2)}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 385 leaves, 7 steps):

$$\begin{aligned} & -\frac{1}{ad} (a-b) \sqrt{a+b} C \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{d} \\ & \sqrt{a+b} (2B+C) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{bd} \\ & \sqrt{a+b} (2bB+aC) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\ & \frac{C \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}} \end{aligned}$$

Result (type 4, 3054 leaves):

$$\begin{aligned} & \left( (1+\cos[c+dx])^{3/2} \left( \frac{B \sqrt{a+b \cos[c+dx]}}{\sqrt{\cos[c+dx]}} + C \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \right. \\ & \left. \left( 2(a+b) C \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - 4(bB+a(-B+C)) \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \right. \right. \\ & \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - 8bB \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - \right. \right. \\ & \left. \left. 4aC \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + bC \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \right) \end{aligned}$$



$$\begin{aligned}
& 2 a C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[\frac{1}{2}(c+d x)\right]-b C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[\frac{1}{2}(c+d x)\right]+\frac{1}{4 \sqrt{a+b \cos [c+d x]}}(1+\cos [c+d x])^{3 / 2} \\
& \sec \left[\frac{1}{2}(c+d x)\right]^2 \tan \left[\frac{1}{2}(c+d x)\right]\left(2(a+b) C \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]-\right. \\
& 4(b B+a(-B+C)) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]- \\
& 8 b B \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]- \\
& 4 a C \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]+b C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \\
& \sec \left[\frac{1}{2}(c+d x)\right] \sin \left[\frac{3}{2}(c+d x)\right]+2 a C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[\frac{1}{2}(c+d x)\right]-b C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[\frac{1}{2}(c+d x)\right]+ \\
& \frac{1}{4 \sqrt{a+b \cos [c+d x]}}(1+\cos [c+d x])^{3 / 2} \sec \left[\frac{1}{2}(c+d x)\right]^2\left(\frac{3}{2} b C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \cos \left[\frac{3}{2}(c+d x)\right] \sec \left[\frac{1}{2}(c+d x)\right]+ \right. \\
& a C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sec \left[\frac{1}{2}(c+d x)\right]^2-\frac{1}{2} b C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sec \left[\frac{1}{2}(c+d x)\right]^2+\frac{1}{\sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}}(a+b) C \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]\left(-\frac{b \sin [c+d x]}{(a+b)(1+\cos [c+d x])}+\frac{(a+b \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2}\right)-\frac{1}{\sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} \\
& 2(b B+a(-B+C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]\left(-\frac{b \sin [c+d x]}{(a+b)(1+\cos [c+d x])}+\frac{(a+b \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2}\right)-
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{\sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} 4 b B \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \left(-\frac{b \sin [c+d x]}{(a+b)(1+\cos [c+d x])}+\frac{(a+b \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2}\right)-\frac{1}{\sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} \\
& 2 a C \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]\left(-\frac{b \sin [c+d x]}{(a+b)(1+\cos [c+d x])}+\frac{(a+b \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2}\right)+ \\
& \frac{b C \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]\left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2}-\frac{\sin [c+d x]}{1+\cos [c+d x]}\right) \sin \left[\frac{3}{2}(c+d x)\right]+a C\left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2}-\frac{\sin [c+d x]}{1+\cos [c+d x]}\right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{2 \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}}+\frac{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}}{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}} \\
& \frac{b C\left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2}-\frac{\sin [c+d x]}{1+\cos [c+d x]}\right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{2 \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}}+\frac{1}{2} b C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \sin \left[\frac{3}{2}(c+d x)\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]- \\
& \frac{2(b B+a(-B+C)) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{1-\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}+\frac{4 b B \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{1-\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}+ \\
& \frac{2 a C \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{1-\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}+ \\
& \frac{(a+b) C \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}{\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right)
\end{aligned}$$

■ Problem 900: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+b \cos [c+d x]} (B \cos [c+d x]+C \cos [c+d x]^2)}{\cos [c+d x]^{5/2}} dx$$

Optimal (type 4, 351 leaves, 6 steps):

$$\begin{aligned} & \frac{1}{a d} 2(a-b) \sqrt{a+b} B \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \frac{1}{a d} \\ & 2 \sqrt{a+b} (b B-a(B-C)) \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \\ & \frac{1}{d} 2 \sqrt{a+b} C \cot [c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} \end{aligned}$$

Result (type 4, 1161 leaves):

$$\begin{aligned} & -\left(4 a^2 C \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \\ & \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) d \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\ & \frac{1}{d} 4 a(-a B+b C) \left(\left(\sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\ & \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& \frac{2B \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}} - \frac{1}{d} 2bB \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \left. \csc[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}}$$

■ **Problem 901: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \cos [c+d x]} (B \cos [c+d x]+C \cos [c+d x]^2)}{\cos [c+d x]^{7/2}} dx$$

Optimal (type 4, 284 leaves, 5 steps):

$$\frac{1}{3 a^2 d} 2 (a-b) \sqrt{a+b} (b B+3 a C) \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} +$$

$$\frac{1}{3 a d} 2 (a-b) \sqrt{a+b} (B-3 C) \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} +$$

$$\frac{2 B \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{3 d \cos [c+d x]^{3/2}}$$

Result (type 4, 1229 leaves):

$$\frac{1}{3 a d} \left( - \left( 4 a (a^2 B - b^2 B) \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) -$$

$$4 a (-a b B - 3 a^2 C) \left( \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\begin{aligned}
& dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right] / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left(\sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x]\right. \\
& \left.\operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right] / \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right)\right) + \\
& 2(-b^2 B-3 a b C) \left(\frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Sec}[c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Sec}[c+d x]}{a+b}}}\right) + \\
& \frac{1}{b} 2 a \left(\left(a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x]\right. \right. \\
& \left.\left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right] / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \\
& \left.\left(a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}\right)\right)
\end{aligned}$$

$$\left. \left( \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \text{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right/$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \text{Sin}[c+dx]}{b \sqrt{\cos[c+dx]}} \left. \right)$$

$$\frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2 \text{Sec}[c+dx] (bB \text{Sin}[c+dx] + 3aC \text{Sin}[c+dx])}{3a} + \frac{2}{3} B \text{Sec}[c+dx] \text{Tan}[c+dx] \right)}{d}$$

■ **Problem 902: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \cos[c+dx]} (B \cos[c+dx] + C \cos[c+dx]^2)}{\cos[c+dx]^{9/2}} dx$$

Optimal (type 4, 350 leaves, 6 steps):

$$\frac{1}{15a^3d} 2(a-b)\sqrt{a+b} (9a^2B - 2b^2B + 5abC) \text{Cot}[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} - \frac{1}{15a^2d}$$

$$2(a-b)\sqrt{a+b} (9aB + 2bB - 5aC) \text{Cot}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}}$$

$$\sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} + \frac{2B\sqrt{a+b \cos[c+dx]} \text{Sin}[c+dx]}{5d \cos[c+dx]^{5/2}} + \frac{2(bB + 5aC)\sqrt{a+b \cos[c+dx]} \text{Sin}[c+dx]}{15ad \cos[c+dx]^{3/2}}$$

Result (type 4, 1315 leaves):

$$-\frac{1}{15a^2d} \left( - \left( 4a(2a^2bB - 2b^3B - 5a^3C + 5ab^2C) \sqrt{\frac{(a+b) \text{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right) \right)$$

$$\begin{aligned}
& \left. \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right\} / \\
& \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a (9a^3 B - 2ab^2 B + 5a^2 b C) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right\} / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right\} / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + \\
& 2 (9a^2 b B - 2b^3 B + 5a b^2 C) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \right. \\
& \left. \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) + \\
& \frac{1}{d} \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left( \frac{2 \operatorname{Sec}[c+d x]^2 (b B \sin [c+d x] + 5 a C \sin [c+d x])}{15 a} + \right. \\
& \left. \frac{2 \operatorname{Sec}[c+d x] (9 a^2 B \sin [c+d x] - 2 b^2 B \sin [c+d x] + 5 a b C \sin [c+d x])}{15 a^2} + \right. \\
& \left. \frac{2}{5} B \operatorname{Sec}[c+d x]^2 \right. \\
& \left. \operatorname{Tan}[c+d x] \right)
\end{aligned}$$

■ **Problem 903: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \cos [c+d x]} (B \cos [c+d x] + C \cos [c+d x]^2)}{\cos [c+d x]^{11/2}} dx$$

Optimal (type 4, 433 leaves, 7 steps):



$$\frac{1}{105 a^4 d} 2 (a-b) \sqrt{a+b} (19 a^2 b B + 8 b^3 B + 63 a^3 C - 14 a b^2 C) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{105 a^3 d} 2 (a-b) \sqrt{a+b} (8 b^2 B + a^2 (25 B - 63 C) + 2 a b (3 B - 7 C))$$

$$\operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{2 B \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{7 d \operatorname{Cos}[c+d x]^{7/2}} + \frac{2 (b B + 7 a C) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{35 a d \operatorname{Cos}[c+d x]^{5/2}} + \frac{2 (25 a^2 B - 4 b^2 B + 7 a b C) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{105 a^2 d \operatorname{Cos}[c+d x]^{3/2}}$$

Result (type 4, 1408 leaves):

$$\frac{1}{105 a^3 d} \left( - \left( 4 a (25 a^4 B - 17 a^2 b^2 B - 8 b^4 B - 14 a^3 b C + 14 a b^3 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a \sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \right.$$

$$\left. \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (-19 a^3 b B - 8 a b^3 B - 63 a^4 C + 14 a^2 b^2 C) \right.$$

$$\left. \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a \sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - \right.$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& 2(-19a^2b^2B - 8b^4B - 63a^3bC + 14ab^3C) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) +
\end{aligned}$$

$$\left. \left. \left. \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) \right) + \frac{1}{d} \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left( \frac{2 \sec [c+d x]^3 (b B \sin [c+d x]+7 a C \sin [c+d x])}{35 a} + \right.$$

$$\frac{2 \sec [c+d x]^2 (25 a^2 B \sin [c+d x]-4 b^2 B \sin [c+d x]+7 a b C \sin [c+d x])}{105 a^2} +$$

$$\frac{2 \sec [c+d x] (19 a^2 b B \sin [c+d x]+8 b^3 B \sin [c+d x]+63 a^3 C \sin [c+d x]-14 a b^2 C \sin [c+d x])}{105 a^3} +$$

$$\left. \left. \left. \frac{2}{7} B \sec [c+d x]^3 \tan [c+d x] \right) \right) \right)$$

■ **Problem 904: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\cos [c+d x]} (a+b \cos [c+d x])^{3/2} (B \cos [c+d x]+C \cos [c+d x]^2) dx$$

Optimal (type 4, 670 leaves, 10 steps):

$$-\frac{1}{192 a b^2 d} (a-b) \sqrt{a+b} (24 a^2 b B+128 b^3 B-9 a^3 C+156 a b^2 C) \cot [c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} -$$

$$\frac{1}{192 b^2 d} \sqrt{a+b} (9 a^3 C-6 a^2 b(4 B+C)-8 b^3(16 B+9 C)-4 a b^2(28 B+39 C)) \cot [c+d x]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \frac{1}{64 b^3 d}$$

$$\sqrt{a+b} (8 a^3 b B-96 a b^3 B-3 a^4 C-24 a^2 b^2 C-48 b^4 C) \cot [c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \frac{(24 a^2 b B+128 b^3 B-9 a^3 C+156 a b^2 C) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{192 b^2 d \sqrt{\cos [c+d x]}} +$$

$$\frac{(8 a b B-3 a^2 C+12 b^2 C) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{32 b d} +$$

$$\frac{(8 b B-3 a C) \sqrt{\cos [c+d x]} (a+b \cos [c+d x])^{3/2} \sin [c+d x]}{24 b d} + \frac{C \sqrt{\cos [c+d x]} (a+b \cos [c+d x])^{5/2} \sin [c+d x]}{4 b d}$$

Result (type 4, 1284 leaves):

$$\begin{aligned}
& -\frac{1}{384 b d} \left( -4 a \left( -136 a^2 b B - 128 b^3 B + 3 a^3 C - 228 a b^2 C \right) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \\
& \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a \left( -416 a b^2 B - 228 a^2 b C - 144 b^3 C \right) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - \right. \\
& \left. \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) \right) + \\
& 2 \left( -24 a^2 b B - 128 b^3 B + 9 a^3 C - 156 a b^2 C \right) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Sec}[c+d x]} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Sec}[c+d x]}{a+b}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right) + \\
& \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{(56 a b B+3 a^2 C+42 b^2 C) \operatorname{Sin}[c+dx]}{96 b} + \right. \\
& \quad \frac{1}{48} \\
& \quad (8 b B+9 a C) \\
& \quad \operatorname{Sin}[2(c+dx)] + \frac{1}{16} \\
& \quad b \\
& \quad c \\
& \quad \left. \operatorname{Sin}[3(c+dx)] \right)
\end{aligned}$$

■ **Problem 905:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \cos[c + dx])^{3/2} (B \cos[c + dx] + C \cos[c + dx]^2)}{\sqrt{\cos[c + dx]}} dx$$

Optimal (type 4, 566 leaves, 9 steps):

$$\begin{aligned}
& -\frac{1}{24abd} (a-b) \sqrt{a+b} (30abB + 3a^2C + 16b^2C) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{24bd} \sqrt{a+b} (30abB + 12b^2B + 3a^2C + 14abC + 16b^2C) \\
& \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{8b^2d} \\
& \sqrt{a+b} (6a^2bB + 8b^3B - a^3C + 12ab^2C) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(30abB + 3a^2C + 16b^2C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{24bd \sqrt{\cos[c+dx]}} + \\
& \frac{(6bB + 7aC) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{12d} + \frac{bC \cos[c+dx]^{3/2} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{3d}
\end{aligned}$$

Result (type 4, 1227 leaves):

$$\begin{aligned}
& \frac{1}{48d} \left( - \left( 4a (42abB + 17a^2C + 16b^2C) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a \sqrt{2}}\right]}, -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a (48a^2B + 24b^2B + 52abC)
\end{aligned}$$

$$\begin{aligned}
& \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \Big) + \\
& 2(30abB + 3a^2C + 16b^2C) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\ \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \\ \left. \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) + \frac{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{1}{12} (6bB+7aC) \operatorname{Sin}[c+dx] + \frac{1}{6} bC \operatorname{Sin}[2(c+dx)] \right)}{d}$$

■ **Problem 906: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{3/2} (B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{3/2}} dx$$

Optimal (type 4, 472 leaves, 8 steps):

$$-\frac{1}{4ad} \\ (a-b) \sqrt{a+b} (4bB+5aC) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \\ \frac{1}{4d} \sqrt{a+b} (8aB+4bB+5aC+2bC) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\ \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{4bd} \sqrt{a+b} (12abB+3a^2C+4b^2C) \operatorname{Cot}[c+dx] \\ \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \\ \frac{(4bB+5aC) \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{4d \sqrt{\operatorname{Cos}[c+dx]}} + \frac{bC \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{2d}$$

Result (type 4, 1198 leaves):



$$\frac{b C \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{2 d} +$$

$$\frac{1}{8 d} \left( - \left( 4 a (8 a^2 B + 4 b^2 B + 7 a b C) \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right) \right) /$$

$$\left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - 4 a (16 a b B + 8 a^2 C + 4 b^2 C)$$

$$\left( \left( \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\left. \left. \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right) \right) / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) -$$

$$\left( \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \operatorname{Csc}[c+d x] \right.$$

$$\left. \left. \operatorname{EllipticPi} \left[ -\frac{a}{b}, \operatorname{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right) \right) / \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) +$$

$$\begin{aligned}
& 2 (4 b^2 B + 5 a b C) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \left. \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right)
\end{aligned}$$

■ **Problem 907: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{3/2} (B \cos[c+dx] + C \cos[c+dx]^2)}{\cos[c+dx]^{5/2}} dx$$

Optimal (type 4, 449 leaves, 8 steps):

$$\frac{1}{ad} (a-b) \sqrt{a+b} (2aB-bC) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{d}$$

$$\sqrt{a+b} (2a(B-C) - b(4B+C)) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{d} \sqrt{a+b} (2bB+3aC) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2aB\sqrt{a+b}\cos[c+dx] \sin[c+dx]}{d\sqrt{\cos[c+dx]}} - \frac{(2aB-bC)\sqrt{a+b}\cos[c+dx] \sin[c+dx]}{d\sqrt{\cos[c+dx]}}$$

Result (type 4, 1196 leaves):

$$\frac{2aB\sqrt{a+b}\cos[c+dx] \sin[c+dx]}{d\sqrt{\cos[c+dx]}} +$$

$$\frac{1}{2d} \left( \left( 4a(-2abB - 2a^2C - b^2C) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.$$

$$\left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx] \right) + 4a(2a^2B - 2b^2B - 4abC)$$

$$\left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx] \right) -$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& 2(2abB - b^2c) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /
\end{aligned}$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}}$$

■ **Problem 908: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{3/2} (B \cos[c+dx] + C \cos[c+dx]^2)}{\cos[c+dx]^{7/2}} dx$$

Optimal (type 4, 418 leaves, 7 steps):

$$\frac{1}{3ad} 2(a-b) \sqrt{a+b} (4bB+3aC) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{3ad} 2\sqrt{a+b} (3b^2B-ab(4B-6C)+a^2(B-3C)) \operatorname{Cot}[c+dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{d}$$

$$2b\sqrt{a+b} C \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{2aB \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{3d \cos[c+dx]^{3/2}}$$

Result (type 4, 1236 leaves):

$$\frac{1}{3d} \left( -4a(a^2B-b^2B+3abC) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right.$$

$$\left. \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/$$

$$\left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a(-4abB-3a^2C+3b^2C)$$

$$\begin{aligned}
& \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \left. \right) + \\
& 2(-4b^2B - 3abc) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\ \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\ \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} + \frac{1}{d} \\ \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2}{3} \operatorname{Sec}[c+dx] (4bB \operatorname{Sin}[c+dx] + 3aC \operatorname{Sin}[c+dx]) + \frac{2}{3} aB \operatorname{Sec}[c+dx] \right. \\ \left. \operatorname{Tan}[c+dx] \right)$$

■ **Problem 909: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{3/2} (B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{9/2}} dx$$

Optimal (type 4, 353 leaves, 6 steps):

$$\frac{1}{15 a^2 d} 2 (a-b) \sqrt{a+b} (9 a^2 B + 3 b^2 B + 20 a b C) \operatorname{Cot}[c+dx] \\ \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{15 a d} \\ 2 (a-b) \sqrt{a+b} (9 a B - 3 b B - 5 a C + 15 b C) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \\ \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{2 a B \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{5 d \operatorname{Cos}[c+dx]^{5/2}} + \frac{2 (6 b B + 5 a C) \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{15 d \operatorname{Cos}[c+dx]^{3/2}}$$

Result (type 4, 1314 leaves):

$$\begin{aligned}
& -\frac{1}{15ad} \left( - \left( 4a(-3a^2bB + 3b^3B - 5a^3C + 5ab^2C) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - 4a(9a^3B + 3ab^2B + 20a^2bC) \\
& \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) + \\
& 2(9a^2bB + 3b^3B + 20a^2b^2C) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\operatorname{Sec}[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\operatorname{Sec}[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right) + \\
& \quad \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2}{15} \operatorname{Sec}[c+dx]^2 (6 b B \operatorname{Sin}[c+dx] + 5 a C \operatorname{Sin}[c+dx]) + \right. \\
& \quad \left. \frac{2 \operatorname{Sec}[c+dx] (9 a^2 B \operatorname{Sin}[c+dx] + 3 b^2 B \operatorname{Sin}[c+dx] + 20 a b C \operatorname{Sin}[c+dx])}{15 a} + \right. \\
& \quad \left. \frac{2}{5} a B \operatorname{Sec}[c+dx]^2 \right. \\
& \quad \left. \left. \operatorname{Tan}[c+dx] \right) \right)
\end{aligned}$$

■ **Problem 910:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \cos[c + dx])^{3/2} (B \cos[c + dx] + C \cos[c + dx]^2)}{\cos[c + dx]^{11/2}} dx$$

Optimal (type 4, 433 leaves, 7 steps):

$$\frac{1}{105 a^3 d} 2 (a - b) \sqrt{a + b} (82 a^2 b B - 6 b^3 B + 63 a^3 C + 21 a b^2 C) \cot[c + dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \frac{1}{105 a^2 d}$$

$$2 (a - b) \sqrt{a + b} (6 b^2 B - a^2 (25 B - 63 C) + 3 a b (19 B - 7 C)) \cot[c + dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right]$$

$$\sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \frac{2 a B \sqrt{a + b \cos[c + dx]} \sin[c + dx]}{7 d \cos[c + dx]^{7/2}} +$$

$$\frac{2 (8 b B + 7 a C) \sqrt{a + b \cos[c + dx]} \sin[c + dx]}{35 d \cos[c + dx]^{5/2}} + \frac{2 (25 a^2 B + 3 b^2 B + 42 a b C) \sqrt{a + b \cos[c + dx]} \sin[c + dx]}{105 a d \cos[c + dx]^{3/2}}$$

Result (type 4, 1407 leaves):

$$\frac{1}{105 a^2 d} \left( - \left( 4 a (25 a^4 B - 31 a^2 b^2 B + 6 b^4 B + 21 a^3 b C - 21 a b^3 C) \sqrt{\frac{(a + b) \cot\left[\frac{1}{2}(c + dx)\right]^2}{-a + b}} \sqrt{\frac{(a + b) \cos[c + dx] \csc\left[\frac{1}{2}(c + dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a + b \cos[c + dx]) \csc\left[\frac{1}{2}(c + dx)\right]^2}{a}} \csc[c + dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(a + b \cos[c + dx]) \csc\left[\frac{1}{2}(c + dx)\right]^2}}{a \sqrt{2}}\right], -\frac{2 a}{-a + b}\right] \sin\left[\frac{1}{2}(c + dx)\right]^4 \right) \right/$$

$$\left( (a + b) \sqrt{\cos[c + dx]} \sqrt{a + b \cos[c + dx]} \right) - 4 a (-82 a^3 b B + 6 a b^3 B - 63 a^4 C - 21 a^2 b^2 C)$$

$$\left( \left( \sqrt{\frac{(a + b) \cot\left[\frac{1}{2}(c + dx)\right]^2}{-a + b}} \sqrt{\frac{(a + b) \cos[c + dx] \csc\left[\frac{1}{2}(c + dx)\right]^2}{a}} \sqrt{\frac{(a + b \cos[c + dx]) \csc\left[\frac{1}{2}(c + dx)\right]^2}{a}} \csc[c + dx] \right. \right.$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) \right) + \\
& 2 \left(-82 a^2 b^2 B+6 b^4 B-63 a^3 b C-21 a b^3 C\right) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Sec}[c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Sec}[c+d x]}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \text{Csc}[c + dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \text{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \text{Sin}[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \Bigg) + \\
& \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2}{35} \text{Sec}[c+dx]^3 (8bB \text{Sin}[c+dx] + 7aC \text{Sin}[c+dx]) + \right. \\
& \frac{2 \text{Sec}[c+dx]^2 (25a^2 B \text{Sin}[c+dx] + 3b^2 B \text{Sin}[c+dx] + 42abC \text{Sin}[c+dx])}{105a} + \\
& \left. \frac{2 \text{Sec}[c+dx] (82a^2 b B \text{Sin}[c+dx] - 6b^3 B \text{Sin}[c+dx] + 63a^3 C \text{Sin}[c+dx] + 21ab^2 C \text{Sin}[c+dx])}{105a^2} + \right. \\
& \left. \frac{2}{7} a B \text{Sec}[c+dx]^3 \text{Tan}[c+dx] \right)
\end{aligned}$$

■ **Problem 911: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{5/2} (B \cos[c+dx] + C \cos[c+dx]^2) dx$$

Optimal (type 4, 779 leaves, 11 steps):

$$\begin{aligned}
& - \frac{1}{1920 a b^2 d} (a-b) \sqrt{a+b} (150 a^3 b B + 2840 a b^3 B - 45 a^4 C + 1692 a^2 b^2 C + 1024 b^4 C) \\
& \quad \text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\
& \frac{1}{1920 b^2 d} \sqrt{a+b} (45 a^4 C - 30 a^3 b (5B+C) - 16 b^4 (45B+64C) - 8 a b^3 (355B+193C) - 4 a^2 b^2 (295B+423C)) \text{Cot}[c+dx] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{1}{128 b^3 d} \sqrt{a+b} (10 a^4 b B - 240 a^2 b^3 B - 96 b^5 B - 3 a^5 C - 40 a^3 b^2 C - 240 a b^4 C) \text{Cot}[c+dx] \\
& \quad \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{(150 a^3 b B + 2840 a b^3 B - 45 a^4 C + 1692 a^2 b^2 C + 1024 b^4 C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{1920 b^2 d \sqrt{\cos[c+dx]}} + \\
& \frac{(50 a^2 b B + 120 b^3 B - 15 a^3 C + 172 a b^2 C) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{320 b d} + \\
& \frac{(50 a b B - 15 a^2 C + 64 b^2 C) \sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{240 b d} + \\
& \frac{(10 b B - 3 a C) \sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{5/2} \sin[c+dx]}{40 b d} + \frac{C \sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{7/2} \sin[c+dx]}{5 b d}
\end{aligned}$$

Result (type 4, 1353 leaves):

$$\begin{aligned}
& - \frac{1}{3840 b d} \\
& \left( - \left( 4 a (-1330 a^3 b B - 3560 a b^3 B + 15 a^4 C - 3236 a^2 b^2 C - 1024 b^4 C) \sqrt{\frac{(a+b) \text{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \quad \left. \left. \sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \text{Csc}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right]}, -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right)
\end{aligned}$$

$$\begin{aligned}
& \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a \left( -6440 a^2 b^2 B - 1440 b^4 B - 2292 a^3 b C - 4624 a b^3 C \right) \\
& \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \quad \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \csc[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + 2 \left( -150 a^3 b B - 2840 a b^3 B + 45 a^4 C - 1692 a^2 b^2 C - 1024 b^4 C \right) \\
& \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} + \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} \right) \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right.
\end{aligned}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right] / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) -$$

$$\left(a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}\right.$$

$$\left.\operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right] /$$

$$\left.\left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}}\right) +$$

$$\frac{1}{d} \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left(\frac{(590 a^2 b B+420 b^3 B+15 a^3 C+898 a b^2 C) \sin [c+d x]}{960 b} +$$

$$\frac{1}{480}$$

$$(170 a b B+93 a^2 C+88 b^2 C) \sin [2(c+d x)] +$$

$$\frac{1}{160} b(10 b B+21 a C) \sin [3(c+d x)] + \frac{1}{40} b^2 C \sin [4(c+d x)]\right)$$

■ **Problem 912: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a+b \cos [c+d x])^{5/2} (B \cos [c+d x]+C \cos [c+d x]^2)}{\sqrt{\cos [c+d x]}} dx$$

Optimal (type 4, 664 leaves, 10 steps):

$$\begin{aligned}
& -\frac{1}{192abd} (a-b) \sqrt{a+b} (264a^2bB + 128b^3B + 15a^3C + 284ab^2C) \operatorname{Cot}[c+dx] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{1}{192bd} \sqrt{a+b} (15a^3C + 8b^3(16B+9C) + 2a^2b(132B+59C) + 4ab^2(52B+71C)) \operatorname{Cot}[c+dx] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{64b^2d} \\
& \sqrt{a+b} (40a^3bB + 160ab^3B - 5a^4C + 120a^2b^2C + 48b^4C) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(264a^2bB + 128b^3B + 15a^3C + 284ab^2C) \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{192bd\sqrt{\cos[c+dx]}} + \\
& \frac{(24abB + 5a^2C + 12b^2C) \sqrt{\cos[c+dx]} \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{32d} + \\
& \frac{(8bB + 11aC) \sqrt{\cos[c+dx]} (a+b\cos[c+dx])^{3/2} \sin[c+dx]}{24d} + \frac{bC\cos[c+dx]^{3/2} (a+b\cos[c+dx])^{3/2} \sin[c+dx]}{4d}
\end{aligned}$$

Result (type 4, 1287 leaves):

$$\begin{aligned}
& \frac{1}{384d} \left( - \left( 4a (472a^2bB + 128b^3B + 133a^3C + 356ab^2C) \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right]}, -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b\cos[c+dx]} \right) - 4a (384a^3B + 608ab^2B + 644a^2bC + 144b^3C) \\
& \left( \left( \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \csc [c+d x] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) \right) + \\
& 2 \left(264 a^2 b B+128 b^3 B+15 a^3 C+284 a b^2 C\right) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \sec [c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \sec [c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \sec [c+d x]}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \csc [c+d x] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right.
\end{aligned}$$

$$\left. \left( \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right/$$

$$\left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) + \frac{\sqrt{a+b\cos[c+dx]}\sin[c+dx]}{b\sqrt{\cos[c+dx]}} + \frac{1}{d}$$

$$\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \left( \frac{1}{96} (104abB + 59a^2C + 42b^2C) \sin[c+dx] + \frac{1}{48} b(8bB + 17aC) \right.$$

$$\left. \sin[2(c+dx)] + \frac{1}{16} b^2C \sin[3(c+dx)] \right)$$

■ **Problem 913: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b\cos[c+dx])^{5/2} (B\cos[c+dx] + C\cos[c+dx]^2)}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 563 leaves, 9 steps):

$$-\frac{1}{24ad} (a-b)\sqrt{a+b} (54abB + 33a^2C + 16b^2C) \cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{24d} \sqrt{a+b} (4b^2(3B+4C) + ab(54B+26C) + a^2(48B+33C))$$

$$\cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{8bd}$$

$$\sqrt{a+b} (30a^2bB + 8b^3B + 5a^3C + 20ab^2C) \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(54abB + 33a^2C + 16b^2C) \sqrt{a+b\cos[c+dx]}\sin[c+dx]}{24d\sqrt{\cos[c+dx]}} +$$

$$\frac{b(2bB + 3aC) \sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\sin[c+dx]}{4d} + \frac{bc\sqrt{\cos[c+dx]}(a+b\cos[c+dx])^{3/2}\sin[c+dx]}{3d}$$

Result (type 4, 1251 leaves):

$$\begin{aligned}
& \frac{1}{48 d} \left( - \left( 4 a (48 a^3 B + 66 a b^2 B + 59 a^2 b C + 16 b^3 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a (144 a^2 b B + 24 b^3 B + 48 a^3 C + 76 a b^2 C) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \Bigg) + \\
& 2 (54 a b^2 B + 33 a^2 b C + 16 b^3 C) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \Bigg) + \\
& \quad \frac{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{1}{12} b (6 b B + 13 a C) \operatorname{Sin}[c+dx] + \frac{1}{6} b^2 C \operatorname{Sin}[2(c+dx)] \right)}{d}
\end{aligned}$$

■ **Problem 914: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{5/2} (B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{5/2}} dx$$

Optimal (type 4, 547 leaves, 9 steps):

$$\begin{aligned}
& \frac{1}{4ad} (a-b) \sqrt{a+b} (8a^2B - 4b^2B - 9abC) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{4d} \sqrt{a+b} (8a^2(B-C) - 2b^2(2B+C) - 3ab(8B+3C)) \\
& \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\
& \frac{1}{4d} \sqrt{a+b} (20abB + 15a^2C + 4b^2C) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{(8a^2B - 4b^2B - 9abC) \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{4d\sqrt{\cos[c+dx]}} - \\
& \frac{b(4aB - bC) \sqrt{\cos[c+dx]} \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{2d} + \frac{2aB(a+b)\cos[c+dx]^{3/2} \sin[c+dx]}{d\sqrt{\cos[c+dx]}}
\end{aligned}$$

Result (type 4, 1241 leaves):

$$\begin{aligned}
& \frac{1}{8d} \left( \left( 4a(-16a^2bB - 4b^3B - 8a^3C - 11ab^2C) \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b}\cos[c+dx] \right) + 4a(8a^3B - 24a^2bB - 24a^2bC - 4b^3C) \\
& \left( \left( \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) - \\
& 2(8a^2bB - 4b^3B - 9ab^2C) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b}2a \left( \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right. \right. \\
& \left. \left. \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right) \right) \right) -
\end{aligned}$$

$$\left. \left. \left. \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left(b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}\right) \right) + \right. \right. \\ \left. \left. \left. \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right) + \frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left(\frac{1}{2} b^2 C \sin[c+dx] + 2 a^2 B \tan[c+dx]\right)}{d} \right)$$

- **Problem 915: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{5/2} (B \cos[c+dx] + C \cos[c+dx]^2)}{\cos[c+dx]^{7/2}} dx$$

Optimal (type 4, 536 leaves, 9 steps):

$$\frac{1}{3ad} (a-b) \sqrt{a+b} (14abB + 6a^2C - 3b^2C) \cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\ \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{3d} \sqrt{a+b} (2ab(7B-9C) - 2a^2(B-3C) - 3b^2(6B+C)) \\ \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{d} \\ b \sqrt{a+b} (2bB + 5aC) \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\ \frac{2a(2bB + aC) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}} - \\ \frac{(14abB + 6a^2C - 3b^2C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{3d \sqrt{\cos[c+dx]}} + \frac{2aB(a+b \cos[c+dx])^{3/2} \sin[c+dx]}{3d \cos[c+dx]^{3/2}}$$

Result (type 4, 1269 leaves):

$$\begin{aligned}
& \frac{1}{6d} \left( - \left( 4a \left( 2a^3 B + 4ab^2 B + 12a^2 b C + 3b^3 C \right) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right]}, -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4a \left( -14a^2 b B + 6b^3 B - 6a^3 C + 18ab^2 C \right) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right]}, -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right]}, -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2 \left( -14ab^2 B - 6a^2 b C + 3b^3 C \right) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right]}, -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$



$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\ \left. \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right) + \\ \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2}{3} \operatorname{Sec}[c+dx] (7 a b B \operatorname{Sin}[c+dx] + 3 a^2 C \operatorname{Sin}[c+dx]) + \right. \\ \left. \frac{2}{3} a^2 B \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx] \right)$$

- **Problem 916: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{5/2} (B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{9/2}} dx$$

Optimal (type 4, 493 leaves, 8 steps):

$$\frac{1}{15 a d} 2 (a-b) \sqrt{a+b} (9 a^2 B+23 b^2 B+35 a b C) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}}+\frac{1}{15 a d} 2 \sqrt{a+b} (15 b^3 B-a b^2(23 B-45 C)+a^2 b(17 B-35 C)-a^3(9 B-5 C))$$

$$\operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}}-\frac{1}{d}$$

$$2 b^2 \sqrt{a+b} C \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}}+$$

$$\frac{2 a(8 b B+5 a C) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{15 d \operatorname{Cos}[c+d x]^{3 / 2}}+\frac{2 a B(a+b) \operatorname{Cos}[c+d x]^{3 / 2} \operatorname{Sin}[c+d x]}{5 d \operatorname{Cos}[c+d x]^{5 / 2}}$$

Result (type 4, 1319 leaves):

$$\frac{1}{15 d} \left( \left( 4 a (-8 a^2 b B+8 b^3 B-5 a^3 C-10 a b^2 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}\right],-\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \right.$$

$$\left. \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b} \operatorname{Cos}[c+d x] \right) + 4 a (9 a^3 B+23 a b^2 B+35 a^2 b C-15 b^3 C) \right.$$

$$\left. \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}\right],-\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b} \operatorname{Cos}[c+d x] \right) - \right.$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& 2(9a^2bB + 23b^3B + 35ab^2C) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left( \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2}{15} \sec^2[c+dx] (11 a b B \sin[c+dx] + 5 a^2 C \sin[c+dx]) + \frac{2}{15} \sec[c+dx] (9 a^2 B \sin[c+dx] + 23 b^2 B \sin[c+dx] + 35 a b C \sin[c+dx]) + \frac{2}{5} a^2 B \sec[c+dx]^2 \tan[c+dx] \right)$$

- **Problem 917: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{5/2} (B \cos[c+dx] + C \cos^2[c+dx])}{\cos[c+dx]^{11/2}} dx$$

Optimal (type 4, 434 leaves, 7 steps):

$$\frac{1}{105 a^2 d} 2 (a-b) \sqrt{a+b} (145 a^2 b B + 15 b^3 B + 63 a^3 C + 161 a b^2 C) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{105 a d}$$

$$2 (a-b) \sqrt{a+b} (a^2 (25 B - 63 C) + 15 b^2 (B - 7 C) - 8 a b (15 B - 7 C)) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{2 a (10 b B + 7 a C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{35 d \cos[c+dx]^{5/2}} +$$

$$\frac{2 (25 a^2 B + 45 b^2 B + 77 a b C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{105 d \cos[c+dx]^{3/2}} + \frac{2 a B (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{7 d \cos[c+dx]^{7/2}}$$

Result (type 4, 1409 leaves):

$$\frac{1}{105 a d} \left( - \left( 4 a (25 a^4 B - 10 a^2 b^2 B - 15 b^4 B + 56 a^3 b C - 56 a b^3 C) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right) \right)$$

$$\left( \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a \left( -145a^3bB - 15ab^3B - 63a^4C - 161a^2b^2C \right)$$

$$\left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.$$

$$\left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right.$$

$$\left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + 2 \left( -145a^2b^2B - 15b^4B - 63a^3bC - 161ab^3C \right)$$

$$\left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) +$$

$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right) + \\
& \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2}{35} \operatorname{Sec}[c+dx]^3 (15 a b B \operatorname{Sin}[c+dx] + 7 a^2 C \operatorname{Sin}[c+dx]) + \right. \\
& \quad \frac{2}{105} \operatorname{Sec}[c+dx]^2 \\
& \quad \left. (25 a^2 B \operatorname{Sin}[c+dx] + 45 b^2 B \operatorname{Sin}[c+dx] + 77 a b C \operatorname{Sin}[c+dx]) + \right. \\
& \quad \left. 1 / (105 a)^2 \operatorname{Sec}[c+dx] (145 a^2 b B \operatorname{Sin}[c+dx] + 15 b^3 B \operatorname{Sin}[c+dx] + 63 a^3 C \operatorname{Sin}[c+dx] + 161 a b^2 C \operatorname{Sin}[c+dx]) + \right. \\
& \quad \left. \frac{2}{7} a^2 B \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx] \right)
\end{aligned}$$

■ **Problem 918: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{5/2} (B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{13/2}} dx$$

Optimal (type 4, 522 leaves, 8 steps) :

$$\frac{1}{315 a^3 d} 2 (a-b) \sqrt{a+b} (147 a^4 B + 279 a^2 b^2 B - 10 b^4 B + 435 a^3 b C + 45 a b^3 C)$$

$$\text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} -$$

$$\frac{1}{315 a^2 d} 2 (a-b) \sqrt{a+b} (10 b^3 B - 6 a^2 b (19 B - 60 C) + 3 a^3 (49 B - 25 C) + 15 a b^2 (11 B - 3 C)) \text{Cot}[c+dx]$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} +$$

$$\frac{2 a (4 b B + 3 a C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{21 d \cos[c+dx]^{7/2}} + \frac{2 (49 a^2 B + 75 b^2 B + 135 a b C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{315 d \cos[c+dx]^{5/2}} +$$

$$\frac{2 (163 a^2 b B + 5 b^3 B + 75 a^3 C + 135 a b^2 C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{315 a d \cos[c+dx]^{3/2}} + \frac{2 a B (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{9 d \cos[c+dx]^{9/2}}$$

Result (type 4, 1517 leaves) :

$$-\frac{1}{315 a^2 d}$$

$$\left( \left( -4 a (-114 a^4 b B + 124 a^2 b^3 B - 10 b^5 B - 75 a^5 C + 30 a^3 b^2 C + 45 a b^4 C) \sqrt{\frac{(a+b) \text{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \text{Csc}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4 a (147 a^5 B + 279 a^3 b^2 B - 10 a b^4 B + 435 a^4 b C + 45 a^2 b^3 C)$$

$$\left( \left( \sqrt{\frac{(a+b) \text{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \text{Csc}[c+dx] \right. \right.$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\text{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b)\text{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \text{Csc}[c+dx]\text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\text{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( \right. \\
& \left. (b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}) \right) + 2(147a^4bB + 279a^2b^3B - 10b^5B + 435a^3b^2C + 45ab^4C) \\
& \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\text{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\text{Sec}[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\text{Sec}[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\text{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b}2a \left( \left( a\sqrt{\frac{(a+b)\text{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \text{Csc}[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\text{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right.
\end{aligned}$$



$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
\left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
\left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \left. \right) + \\
\frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2}{63} \operatorname{Sec}[c+dx]^4 (19abB \operatorname{Sin}[c+dx] + 9a^2C \operatorname{Sin}[c+dx]) + \right. \\
\frac{2}{315} \operatorname{Sec}[c+dx]^3 (49a^2B \operatorname{Sin}[c+dx] + 75b^2B \operatorname{Sin}[c+dx] + 135abC \operatorname{Sin}[c+dx]) + \\
\frac{1}{315a} \\
2 \operatorname{Sec}[c+dx]^2 (163a^2bB \operatorname{Sin}[c+dx] + 5b^3B \operatorname{Sin}[c+dx] + 75a^3C \operatorname{Sin}[c+dx] + 135ab^2C \operatorname{Sin}[c+dx]) + \\
\frac{1}{315a^2} \\
2 \operatorname{Sec}[c+dx] (147a^4B \operatorname{Sin}[c+dx] + 279a^2b^2B \operatorname{Sin}[c+dx] - 10b^4B \operatorname{Sin}[c+dx] + 435a^3bC \operatorname{Sin}[c+dx] + 45ab^3C \operatorname{Sin}[c+dx]) + \\
\left. \frac{2}{9} a^2B \operatorname{Sec}[c+dx]^4 \operatorname{Tan}[c+dx] \right)$$

- **Problem 919: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{5/2} (B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{15/2}} dx$$

Optimal (type 4, 622 leaves, 9 steps):

$$\frac{1}{3465 a^4 d} 2 (a-b) \sqrt{a+b} (3705 a^4 b B + 255 a^2 b^3 B + 40 b^5 B + 1617 a^5 C + 3069 a^3 b^2 C - 110 a b^4 C)$$

$$\text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{3465 a^3 d} 2 (a-b) \sqrt{a+b} (40 b^4 B + 3 a^4 (225 B - 539 C) - 6 a^3 b (505 B - 209 C) + 15 a^2 b^2 (19 B - 121 C) + 10 a b^3 (3 B - 11 C))$$

$$\text{Cot}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2 a (14 b B + 11 a C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{99 d \cos[c+dx]^{9/2}} + \frac{2 (81 a^2 B + 113 b^2 B + 209 a b C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{693 d \cos[c+dx]^{7/2}} +$$

$$\frac{2 (1145 a^2 b B + 15 b^3 B + 539 a^3 C + 825 a b^2 C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{3465 a d \cos[c+dx]^{5/2}} +$$

$$\frac{2 (675 a^4 B + 1025 a^2 b^2 B - 20 b^4 B + 1793 a^3 b C + 55 a b^3 C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{3465 a^2 d \cos[c+dx]^{3/2}} + \frac{2 a B (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{11 d \cos[c+dx]^{11/2}}$$

Result (type 4, 1640 leaves):

$$\frac{1}{3465 a^3 d} \left( - \left( 4 a (675 a^6 B - 390 a^4 b^2 B - 245 a^2 b^4 B - 40 b^6 B + 1254 a^5 b C - 1364 a^3 b^3 C + 110 a b^5 C) \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right.$$

$$\left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}}{a \sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.$$

$$\left. 4 a (-3705 a^5 b B - 255 a^3 b^3 B - 40 a b^5 B - 1617 a^6 C - 3069 a^4 b^2 C + 110 a^2 b^4 C) \right.$$

$$\left. \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \right.$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \Bigg/ \left((a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}\right) -$$

$$\left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right.$$

$$\left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \Bigg/ \left(b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}\right) \right) +$$

$$2 \left(-3705 a^4 b^2 B - 255 a^2 b^4 B - 40 b^6 B - 1617 a^5 b C - 3069 a^3 b^3 C + 110 a b^5 C\right)$$

$$\left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} \right. +$$

$$\left. \frac{1}{b} 2a \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right.$$

$$\left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \Bigg/ \left((a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}\right) - \right.$$

$$\left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right.$$

$$\left. \text{Csc}[c + dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right)$$

$$\begin{aligned} & \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2}{99} \sec[c+dx]^5 (23abB \sin[c+dx] + 11a^2C \sin[c+dx]) + \right. \\ & \frac{2}{693} \sec[c+dx]^4 (81a^2B \sin[c+dx] + 113b^2B \sin[c+dx] + 209abc \sin[c+dx]) + \frac{1}{3465a} \\ & 2 \sec[c+dx]^3 (1145a^2bB \sin[c+dx] + 15b^3B \sin[c+dx] + 539a^3C \sin[c+dx] + 825ab^2C \sin[c+dx]) + \frac{1}{3465a^2} \\ & 2 \sec[c+dx]^2 (675a^4B \sin[c+dx] + 1025a^2b^2B \sin[c+dx] - 20b^4B \sin[c+dx] + 1793a^3bC \sin[c+dx] + 55ab^3C \sin[c+dx]) + \\ & \frac{1}{3465a^3} 2 \sec[c+dx] (3705a^4bB \sin[c+dx] + 255a^2b^3B \sin[c+dx] + 40b^5B \sin[c+dx] + \\ & \left. 1617a^5C \sin[c+dx] + 3069a^3b^2C \sin[c+dx] - 110ab^4C \sin[c+dx]) + \frac{2}{11} a^2B \sec[c+dx]^5 \tan[c+dx] \right) \end{aligned}$$

- **Problem 920: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{3/2} (B \cos[c+dx] + C \cos[c+dx]^2)}{\sqrt{a+b \cos[c+dx]}} dx$$

Optimal (type 4, 571 leaves, 9 steps):

$$\frac{1}{24 a b^3 d} (a-b) \sqrt{a+b} (18 a b B - 15 a^2 C - 16 b^2 C) \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \frac{1}{24 b^3 d} \sqrt{a+b} (18 a b B - 12 b^2 B - 15 a^2 C + 10 a b C - 16 b^2 C)$$

$$\cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} -$$

$$\frac{1}{8 b^4 d} \sqrt{a+b} (6 a^2 b B + 8 b^3 B - 5 a^3 C - 4 a b^2 C) \cot [c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \frac{(18 a b B - 15 a^2 C - 16 b^2 C) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{24 b^3 d \sqrt{\cos [c+d x]}} +$$

$$\frac{(6 b B - 5 a C) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{12 b^2 d} + \frac{C \cos [c+d x]^{3/2} \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{3 b d}$$

Result (type 4, 1229 leaves):

$$\frac{1}{48 b^2 d}$$

$$\left( \left( -4 a (-6 a b B + 5 a^2 C + 16 b^2 C) \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right.$$

$$\left. \left. \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right) \right/$$

$$\left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - 4 a (24 b^2 B + 4 a b C)$$

$$\left( \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + \\
& 2(-18abB + 15a^2C + 16b^2C) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right) \right)
\end{aligned}$$

$$\left. \left. \left. \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left(b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}\right) \right) \right) + \left. \left. \left. \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right) + \frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left(\frac{(6bB-5aC) \sin[c+dx]}{12b^2} + \frac{C \sin[2(c+dx)]}{6b}\right)}{d} \right)$$

■ **Problem 921: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]} (B \cos[c+dx] + C \cos[c+dx]^2)}{\sqrt{a+b \cos[c+dx]}} dx$$

Optimal (type 4, 479 leaves, 8 steps):

$$-\frac{1}{4ab^2d} (a-b) \sqrt{a+b} (4bB-3aC) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{4b^2d}$$

$$\sqrt{a+b} (3aC-2b(2B+C)) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{4b^3d} \sqrt{a+b} (4abB-3a^2C-4b^2C) \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}}$$

$$\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(4bB-3aC) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{4b^2d \sqrt{\cos[c+dx]}} + \frac{C \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2bd}$$

Result (type 4, 1175 leaves):

$$\frac{C \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2bd} + \frac{1}{8bd}$$

$$\begin{aligned}
& \left( - \left( 4 a (4 b B - a C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad 16 a b C \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \quad 2 (4 b B - 3 a C) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) +
\end{aligned}$$



$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\ \left. \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right)$$

- **Problem 922: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2}{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]}} dx$$

Optimal (type 4, 391 leaves, 8 steps):

$$\begin{aligned}
& -\frac{1}{abd} (a-b) \sqrt{a+b} C \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{\sqrt{a+b} C \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{bd} - \frac{1}{b^2 d} \\
& \sqrt{a+b} (2bB - aC) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{C \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{bd \sqrt{\cos[c+dx]}}
\end{aligned}$$

Result (type 4, 4017 leaves):

$$\begin{aligned}
& \left( (1 + \cos[c+dx])^{3/2} \left( \frac{B \sqrt{\cos[c+dx]}}{\sqrt{a+b \cos[c+dx]}} + \frac{C \cos[c+dx]^{3/2}}{\sqrt{a+b \cos[c+dx]}} \right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \left. \left( 2i(a-b)C \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + \right. \right. \\
& \left. \left. 4i(bB - aC) \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] - \right. \right. \\
& \left. \left. 8i b B \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + \right. \right. \\
& \left. \left. 4i a C \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + b \sqrt{\frac{a-b}{a+b}} C \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \right. \right. \\
& \left. \left. \sec\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] + 2a \sqrt{\frac{a-b}{a+b}} C \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] - b \sqrt{\frac{a-b}{a+b}} C \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) /
\end{aligned}$$

$$\left( 4 b \sqrt{\frac{a-b}{a+b}} d \sqrt{a+b \cos[c+dx]} \left( \frac{1}{8 \sqrt{\frac{a-b}{a+b}} (a+b \cos[c+dx])^{3/2}} (1 + \cos[c+dx])^{3/2} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] \right. \right.$$

$$\left. \left. + 2 i (a-b) C \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + \right. \right.$$

$$4 i (bB - aC) \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] - 8 i bB$$

$$\sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + 4 i aC \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}}$$

$$\operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + b \sqrt{\frac{a-b}{a+b}} C \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] +$$

$$2 a \sqrt{\frac{a-b}{a+b}} C \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - b \sqrt{\frac{a-b}{a+b}} C \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left. \right) -$$

$$\frac{1}{8 b \sqrt{\frac{a-b}{a+b}} \sqrt{a+b \cos[c+dx]}} 3 \sqrt{1 + \cos[c+dx]} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx]$$

$$\left( 2 i (a-b) C \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + \right.$$

$$4 i (bB - aC) \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] - 8 i bB$$

$$\sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + 4 i aC \sqrt{\frac{a+b \cos[c+dx]}{(a+b)(1+\cos[c+dx])}}$$

$$\operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] + b \sqrt{\frac{a-b}{a+b}} C \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] +$$

$$\begin{aligned}
& \left. 2 a \sqrt{\frac{a-b}{a+b}} c \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[\frac{1}{2}(c+d x)\right] - b \sqrt{\frac{a-b}{a+b}} c \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[\frac{1}{2}(c+d x)\right] \right) + \\
& \frac{1}{4 b \sqrt{\frac{a-b}{a+b}} \sqrt{a+b \cos [c+d x]}} (1+\cos [c+d x])^{3 / 2} \sec \left[\frac{1}{2}(c+d x)\right]^2 \tan \left[\frac{1}{2}(c+d x)\right] \\
& \left( 2 i (a-b) c \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] + \right. \\
& 4 i (b B-a C) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] - 8 i b B \\
& \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] + 4 i a c \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \\
& \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] + b \sqrt{\frac{a-b}{a+b}} c \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sec \left[\frac{1}{2}(c+d x)\right] \sin \left[\frac{3}{2}(c+d x)\right] + \\
& \left. 2 a \sqrt{\frac{a-b}{a+b}} c \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[\frac{1}{2}(c+d x)\right] - b \sqrt{\frac{a-b}{a+b}} c \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \tan \left[\frac{1}{2}(c+d x)\right] \right) + \\
& \frac{1}{4 b \sqrt{\frac{a-b}{a+b}} \sqrt{a+b \cos [c+d x]}} (1+\cos [c+d x])^{3 / 2} \sec \left[\frac{1}{2}(c+d x)\right]^2 \left( \frac{3}{2} b \sqrt{\frac{a-b}{a+b}} c \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \cos \left[\frac{3}{2}(c+d x)\right] \sec \left[\frac{1}{2}(c+d x)\right] + \right. \\
& a \sqrt{\frac{a-b}{a+b}} c \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sec \left[\frac{1}{2}(c+d x)\right]^2 - \frac{1}{2} b \sqrt{\frac{a-b}{a+b}} c \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sec \left[\frac{1}{2}(c+d x)\right]^2 + \frac{1}{\sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} \\
& \left. i (a-b) c \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] \left( -\frac{b \sin [c+d x]}{(a+b)(1+\cos [c+d x])} + \frac{(a+b \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2} \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} 2 i(b B-a C) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] \\
& \left(-\frac{b \sin [c+d x]}{(a+b)(1+\cos [c+d x])}+\frac{(a+b \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2}\right)-\frac{1}{\sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} 4 i b B \\
& \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right]\left(-\frac{b \sin [c+d x]}{(a+b)(1+\cos [c+d x])}+\frac{(a+b \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2}\right)+ \\
& \frac{1}{\sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} 2 i a C \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right]\left(-\frac{b \sin [c+d x]}{(a+b)(1+\cos [c+d x])}+\right. \\
& \left.\frac{(a+b \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2}\right)+\frac{b \sqrt{\frac{a-b}{a+b}} C \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]\left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2}-\frac{\sin [c+d x]}{1+\cos [c+d x]}\right) \sin \left[\frac{3}{2}(c+d x)\right]}{2 \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}}+ \\
& \frac{a \sqrt{\frac{a-b}{a+b}} C\left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2}-\frac{\sin [c+d x]}{1+\cos [c+d x]}\right) \tan \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}}-\frac{b \sqrt{\frac{a-b}{a+b}} C\left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2}-\frac{\sin [c+d x]}{1+\cos [c+d x]}\right) \tan \left[\frac{1}{2}(c+d x)\right]}{2 \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}}+\frac{1}{2} b \sqrt{\frac{a-b}{a+b}} \\
& c \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \sin \left[\frac{3}{2}(c+d x)\right] \tan \left[\frac{1}{2}(c+d x)\right]-\frac{2 \sqrt{\frac{a-b}{a+b}}(b B-a C) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{\sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{1+\frac{(a-b) \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}} \\
& \frac{(a-b) \sqrt{\frac{a-b}{a+b}} C \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}}{\sqrt{1+\frac{(a-b) \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}+
\end{aligned}$$

$$\frac{4 b \sqrt{\frac{a-b}{a+b}} B \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{\sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{1+\frac{(a-b) \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}$$

$$\frac{2 a \sqrt{\frac{a-b}{a+b}} C \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{\sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{1+\frac{(a-b) \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}$$

■ **Problem 924: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{B \cos [c+d x]+C \cos [c+d x]^2}{\cos [c+d x]^{5/2} \sqrt{a+b \cos [c+d x]}} dx$$

Optimal (type 4, 230 leaves, 4 steps):

$$\frac{1}{a^2 d} 2(a-b) \sqrt{a+b} B \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} -$$

$$\frac{1}{a d} 2 \sqrt{a+b} (B-C) \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}$$

Result (type 4, 1164 leaves):

$$\frac{2 B \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{a d \sqrt{\cos [c+d x]}}$$

$$\frac{1}{a d} \left( \left( 4 a (b B - a C) \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) -$$

$$\begin{aligned}
& 4 a^2 B \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2 b B \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\ \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\ \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \Bigg)$$

■ **Problem 925: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2}{\operatorname{Cos}[c+dx]^{7/2} \sqrt{a+b \operatorname{Cos}[c+dx]}} dx$$

Optimal (type 4, 290 leaves, 5 steps):

$$-\frac{1}{3a^3d} \\ 2(a-b)\sqrt{a+b} (2bB-3aC) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \\ \frac{1}{3a^2d} 2\sqrt{a+b} (2bB+a(B-3C)) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\ \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{2B\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{3ad \operatorname{Cos}[c+dx]^{3/2}}$$

Result (type 4, 1238 leaves):

$$\frac{1}{3a^2d}$$



$$\begin{aligned}
& \left( - \left( 4 a (a^2 B + 2 b^2 B - 3 a b C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left. 4 a (2 a b B - 3 a^2 C) \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+ \right. \right. \right. \\
& \quad \left. \left. \left. dx\right] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left. \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \quad \left. 2 (2 b^2 B - 3 a b C) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right]\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \right.
\end{aligned}$$

$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\ \left. \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right) + \\ \frac{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2 \operatorname{Sec}[c+dx] (-2 b B \operatorname{Sin}[c+dx] + 3 a C \operatorname{Sin}[c+dx])}{3 a^2} + \frac{2 B \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{3 a} \right)}{d}$$

■ **Problem 926: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2}{\operatorname{Cos}[c+dx]^{9/2} \sqrt{a+b \operatorname{Cos}[c+dx]}} dx$$

Optimal (type 4, 363 leaves, 6 steps):

$$\frac{1}{15 a^4 d} 2 (a-b) \sqrt{a+b} (9 a^2 B + 8 b^2 B - 10 a b C) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{15 a^3 d}$$

$$2 \sqrt{a+b} (8 b^2 B + a^2 (9 B - 5 C) - 2 a b (B + 5 C)) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{2 B \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{5 a d \operatorname{Cos}[c+d x]^{5/2}} - \frac{2 (4 b B - 5 a C) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{15 a^2 d \operatorname{Cos}[c+d x]^{3/2}}$$

Result (type 4, 1319 leaves):

$$-\frac{1}{15 a^3 d} \left( - \left( 4 a (7 a^2 b B + 8 b^3 B - 5 a^3 C - 10 a b^2 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\ \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right) / \\ \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (9 a^3 B + 8 a b^2 B - 10 a^2 b C) \\ \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) -$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& 2(9a^2bB + 8b^3B - 10ab^2C) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \left. \csc[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left. \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) +$$

$$\frac{1}{d} \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left( \frac{2 \sec [c+d x]^2 (-4 b B \sin [c+d x]+5 a C \sin [c+d x])}{15 a^2} + \right.$$

$$\frac{2 \sec [c+d x] (9 a^2 B \sin [c+d x]+8 b^2 B \sin [c+d x]-10 a b C \sin [c+d x])}{15 a^3} +$$

$$\left. \frac{2 B \sec [c+d x]^2 \tan [c+d x]}{5 a} \right)$$

- **Problem 927: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^{3/2} (B \cos [c+d x]+C \cos [c+d x]^2)}{(a+b \cos [c+d x])^{3/2}} dx$$

Optimal (type 4, 620 leaves, 9 steps):

$$-\frac{1}{4 a b^3 \sqrt{a+b} d} (12 a^2 b B-4 b^3 B-15 a^3 C+7 a b^2 C) \cot [c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} +$$

$$\frac{1}{4 b^3 \sqrt{a+b} d} (a b(12 B-5 C)-15 a^2 C+2 b^2(2 B+C)) \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \frac{1}{4 b^4 d} \sqrt{a+b} (12 a b B-15 a^2 C-4 b^2 C) \cot [c+d x]$$

$$\operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} +$$

$$\frac{2 a(b B-a C) \cos [c+d x]^{3/2} \sin [c+d x]}{b\left(a^2-b^2\right) d \sqrt{a+b \cos [c+d x]}} + \frac{\left(12 a^2 b B-4 b^3 B-15 a^3 C+7 a b^2 C\right) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{4 b^3\left(a^2-b^2\right) d \sqrt{\cos [c+d x]}}$$

$$\frac{\left(4 a b B-5 a^2 C+b^2 C\right) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{2 b^2\left(a^2-b^2\right) d}$$

Result (type 4, 1297 leaves) :

$$\frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{c \sin[c+dx]}{2b^2} - \frac{2(-a^2 b B \sin[c+dx] + a^3 C \sin[c+dx])}{b^2(-a^2+b^2)(a+b \cos[c+dx])} \right)}{d} -$$

$$\frac{1}{8(a-b)b^2(a+b)d} \left( - \left( 4a(-4a^2 b B + 4b^3 B + 5a^3 C - 5ab^2 C) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.$$

$$\left. \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a(-8ab^2 B + 4a^2 b C + 4b^3 C) \right.$$

$$\left. \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.$$

$$\left. \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) +$$

$$\begin{aligned}
& 2 (-12 a^2 b B + 4 b^3 B + 15 a^3 C - 7 a b^2 C) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}}} + \right. \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right)
\end{aligned}$$

■ **Problem 928: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Cos}[c+dx]} (B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2)}{(a+b \operatorname{Cos}[c+dx])^{3/2}} dx$$

Optimal (type 4, 500 leaves, 8 steps):

$$\frac{1}{a b^2 \sqrt{a+b} d} (2 a b B - 3 a^2 C + b^2 C) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{b^2 \sqrt{a+b} d}$$

$$(2 b B - 3 a C - b C) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{b^3 d}$$

$$\sqrt{a+b} (2 b B - 3 a C) \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{2 a (b B - a C) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{b (a^2 - b^2) d \sqrt{a+b} \operatorname{Cos}[c+d x]} - \frac{(2 a b B - 3 a^2 C + b^2 C) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{b^2 (a^2 - b^2) d \sqrt{\operatorname{Cos}[c+d x]}}$$

Result (type 4, 1234 leaves):

$$\frac{2 \sqrt{\operatorname{Cos}[c+d x]} (-a b B \operatorname{Sin}[c+d x] + a^2 C \operatorname{Sin}[c+d x])}{b (-a^2 + b^2) d \sqrt{a+b} \operatorname{Cos}[c+d x]} + \frac{1}{2 (a-b) b (a+b) d}$$

$$\left( - \left( 4 a (a^2 C - b^2 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b} \operatorname{Cos}[c+d x] \right) -$$

$$4 a (-2 b^2 B + 2 a b C) \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$



$$\begin{aligned}
& dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right] / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left(\sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x]\right. \\
& \left.\operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right] / \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right)\right) + \\
& 2(-2 a b B+3 a^2 C-b^2 C)\left(\frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Sec}[c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Sec}[c+d x]}{a+b}}}\right) + \\
& \frac{1}{b} 2 a\left(\left(a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x]\right. \right. \\
& \left.\left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right] / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \\
& \left.\left(a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}\right.\right)
\end{aligned}$$

$$\left( \left( \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right)$$

■ **Problem 929: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{B \cos[c+dx] + C \cos[c+dx]^2}{\sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 416 leaves, 7 steps):

$$\frac{2(bB - aC) \cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{ab \sqrt{a+b} d} + \\ \frac{2(bB - aC) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{ab \sqrt{a+b} d} - \frac{1}{b^2 d} \\ \frac{2\sqrt{a+b} C \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{b(a^2 - b^2) d \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}}$$

Result (type 4, 1012 leaves):

$$\frac{2\sqrt{\cos[c+dx]} (-bB \sin[c+dx] + aC \sin[c+dx])}{(a^2 - b^2) d \sqrt{a+b \cos[c+dx]}} - \frac{1}{(-a+b)(a+b)d} \\ \left( -4a(aB - bC) \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \text{Csc}[c+dx] \right) \right) \right)$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) \right) + \\
& 2(b B - a C) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Sec}[c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Sec}[c+d x]}{a+b}} \right. + \\
& \left. \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \right. \\
& \left. \left. \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \right.
\end{aligned}$$

$$\left( \text{Csc}[c + dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}}$$

■ **Problem 930: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{B \cos[c+dx] + C \cos[c+dx]^2}{\cos[c+dx]^{3/2} (a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 284 leaves, 5 steps):

$$\frac{2(bB - aC) \cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{a^2 \sqrt{a+b} d} +$$

$$\frac{2(B+C) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}}{a \sqrt{a+b} d} - \frac{2(bB - aC) \sin[c+dx]}{(a^2 - b^2) d \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}}$$

Result (type 4, 1223 leaves):

$$-\frac{2 \sqrt{\cos[c+dx]} (-b^2 B \sin[c+dx] + a b C \sin[c+dx])}{a (a^2 - b^2) d \sqrt{a+b \cos[c+dx]}} + \frac{1}{a (a-b) (a+b) d}$$

$$\left( - \left( 4 a (a^2 B - b^2 B) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \text{Csc}[c+dx] \right. \right.$$

$$\left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4 a$$

$$\begin{aligned}
& (-a b B + a^2 C) \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2(-b^2 B + a b C) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\ \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\ \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}}$$

■ **Problem 931: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{B \cos[c+dx] + C \cos[c+dx]^2}{\cos[c+dx]^{5/2} (a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 305 leaves, 5 steps):

$$\frac{1}{a^3 \sqrt{a+b} d} 2 (a^2 B - 2b^2 B + a b C) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\ \frac{1}{a^2 \sqrt{a+b} d} 2 (2bB + a(B-C)) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\ \frac{2b(bB - aC) \sin[c+dx]}{a(a^2 - b^2) d \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}}$$

Result (type 4, 1281 leaves):

$$\frac{1}{a^2 (-a+b) (a+b) d} \left( - \left( 4a (2a^2 b B - 2b^3 B - a^3 C + a b^2 C) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\begin{aligned}
& \left. \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right\} / \\
& \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - 4 a \left( a^3 B - 2 a b^2 B + a^2 b C \right) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right\} / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right\} / \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) \right) + \\
& 2 \left( a^2 b B - 2 b^3 B + a b^2 C \right) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Sec}[c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Sec}[c+d x]}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + \\
& \left. \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2(-b^3 B \sin[c+dx] + a b^2 C \sin[c+dx])}{a^2 (a^2 - b^2) (a+b \cos[c+dx])} + \frac{2 B \tan[c+dx]}{a^2} \right)}{d}
\end{aligned}$$

■ **Problem 932: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{B \cos[c+dx] + C \cos[c+dx]^2}{\cos[c+dx]^{7/2} (a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 393 leaves, 6 steps):

$$-\frac{1}{3 a^4 \sqrt{a+b} d} 2 (5 a^2 b B - 8 b^3 B - 3 a^3 C + 6 a b^2 C) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{3 a^3 \sqrt{a+b} d}$$

$$2(a+2b)(4bB+a(B-3C)) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2b(bB-aC) \sin[c+dx]}{a(a^2-b^2) d \cos[c+dx]^{3/2} \sqrt{a+b \cos[c+dx]}} + \frac{2(a^2 B - 4b^2 B + 3abC) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{3a^2(a^2-b^2) d \cos[c+dx]^{3/2}}$$

Result (type 4, 1357 leaves):



$$\begin{aligned}
& \frac{1}{3 a^3 (a-b)(a+b) d} \left( - \left( 4 a \left( a^4 B + 7 a^2 b^2 B - 8 b^4 B - 6 a^3 b C + 6 a b^3 C \right) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a \left( 5 a^3 b B - 8 a b^3 B - 3 a^4 C + 6 a^2 b^2 C \right) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) \right) + \\
& 2 \left( 5 a^2 b^2 B - 8 b^4 B - 3 a^3 b C + 6 a b^3 C \right) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Sec}[c+d x]} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Sec}[c+d x]}{a+b}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \Bigg) + \\
& \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2 \operatorname{Sec}[c+dx] (-5 b B \operatorname{Sin}[c+dx] + 3 a C \operatorname{Sin}[c+dx])}{3 a^3} - \right. \\
& \quad \frac{2 (-b^4 B \operatorname{Sin}[c+dx] + a b^3 C \operatorname{Sin}[c+dx])}{a^3 (a^2 - b^2) (a+b \operatorname{Cos}[c+dx])} + \\
& \quad \left. \frac{2 B \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{3 a^2} \right)
\end{aligned}$$

■ **Problem 933: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^{3/2} (B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2)}{(a+b \operatorname{Cos}[c+dx])^{5/2}} dx$$

Optimal (type 4, 674 leaves, 9 steps):

$$\begin{aligned}
& \frac{1}{3 a (a-b) b^3 (a+b)^{3/2} d} (6 a^3 b B - 14 a b^3 B - 15 a^4 C + 26 a^2 b^2 C - 3 b^4 C) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{3 b^3 \sqrt{a+b} (a^2-b^2) d} (a^2 b (6 B-5 C) - 3 b^3 (4 B-C) - 15 a^3 C + a b^2 (2 B+21 C)) \\
& \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{b^4 d} \\
& \sqrt{a+b} (2 b B-5 a C) \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \\
& \frac{2 a (b B-a C) \operatorname{Cos}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{3 b (a^2-b^2) d (a+b \operatorname{Cos}[c+d x])^{3/2}} + \frac{2 a (2 a^2 b B-6 b^3 B-5 a^3 C+9 a b^2 C) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 b^2 (a^2-b^2)^2 d \sqrt{a+b \operatorname{Cos}[c+d x]}} - \\
& \frac{(6 a^3 b B-14 a b^3 B-15 a^4 C+26 a^2 b^2 C-3 b^4 C) \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 b^3 (a^2-b^2)^2 d \sqrt{\operatorname{Cos}[c+d x]}}
\end{aligned}$$

Result (type 4, 1396 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \\
& \left( -\frac{2(-a^2 b B \operatorname{Sin}[c+d x] + a^3 C \operatorname{Sin}[c+d x])}{3 b^2 (-a^2 + b^2) (a+b \operatorname{Cos}[c+d x])^2} - \frac{2(-3 a^3 b B \operatorname{Sin}[c+d x] + 7 a b^3 B \operatorname{Sin}[c+d x] + 6 a^4 C \operatorname{Sin}[c+d x] - 10 a^2 b^2 C \operatorname{Sin}[c+d x])}{3 b^2 (-a^2 + b^2)^2 (a+b \operatorname{Cos}[c+d x])} \right) + \\
& \frac{1}{6 (a-b)^2 b^2 (a+b)^2 d} \left( -\left( 4 a (-2 a^3 b B + 2 a b^3 B + 5 a^4 C - 8 a^2 b^2 C + 3 b^4 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \right. \\
& \left. \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (2 a^2 b^2 B + 6 b^4 B + 4 a^3 b C - 12 a b^3 C) \right)
\end{aligned}$$

$$\left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.$$

$$\left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right.$$

$$\left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + 2 \left( -6 a^3 b B + 14 a b^3 B + 15 a^4 C - 26 a^2 b^2 C + 3 b^4 C \right) \right)$$

$$\left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} + \right.$$

$$\left. \frac{1}{b} 2 a \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right) \right)$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+d x] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \right. \\
& \left. \left. \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) \right)
\end{aligned}$$

- **Problem 934: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos [c+d x]} (B \cos [c+d x] + C \cos [c+d x]^2)}{(a+b \cos [c+d x])^{5/2}} dx$$

Optimal (type 4, 545 leaves, 8 steps):

$$\frac{1}{3 a (a-b) b^2 (a+b)^{3/2} d}$$

$$2 (4 b^3 B + 3 a^3 C - 7 a b^2 C) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{1}{3 a (a-b) b^2 (a+b)^{3/2} d} 2 (a b^2 B - 3 b^3 B - 3 a^3 C - a^2 b C + 6 a b^2 C) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{b^3 d}$$

$$2 \sqrt{a+b} C \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{2 a (b B - a C) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 b (a^2 - b^2) d (a+b \operatorname{Cos}[c+d x])^{3/2}} - \frac{2 a (4 b^3 B + 3 a^3 C - 7 a b^2 C) \operatorname{Sin}[c+d x]}{3 b^2 (a^2 - b^2)^2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]}}$$

Result (type 4, 1342 leaves):

$$\frac{1}{d} \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \left( \frac{2 (-a b B \operatorname{Sin}[c+d x] + a^2 C \operatorname{Sin}[c+d x])}{3 b (-a^2 + b^2) (a+b \operatorname{Cos}[c+d x])^2} + \frac{2 (4 b^3 B \operatorname{Sin}[c+d x] + 3 a^3 C \operatorname{Sin}[c+d x] - 7 a b^2 C \operatorname{Sin}[c+d x])}{3 b (-a^2 + b^2)^2 (a+b \operatorname{Cos}[c+d x])} \right) -$$

$$\frac{1}{3 (a-b)^2 b (a+b)^2 d} \left( - \left( 4 a (-a^2 b B + b^3 B + a^3 C - a b^2 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \right.$$

$$\left. \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (4 a b^2 B - a^2 b C - 3 b^3 C) \right.$$

$$\left. \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \right.$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \csc [c+d x] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) \right) + \\
& 2\left(4 b^3 B+3 a^3 C-7 a b^2 C\right) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \sec [c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \sec [c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \sec [c+d x]}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \csc [c+d x] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right.
\end{aligned}$$

$$\left( \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}}$$

■ **Problem 935: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{B \cos[c+dx] + C \cos[c+dx]^2}{\sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{5/2}} dx$$

Optimal (type 4, 391 leaves, 6 steps):

$$-\frac{1}{3a^2(a-b)(a+b)^{3/2}d} 2(3a^2B + b^2B - 4abC) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{3a(a-b)(a+b)^{3/2}d}$$

$$2(3aB - bB + aC - 3bC) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{2(bB - aC) \sqrt{\cos[c+dx]} \sin[c+dx]}{3(a^2 - b^2)d(a+b \cos[c+dx])^{3/2}} + \frac{2(3a^2B + b^2B - 4abC) \sin[c+dx]}{3(a^2 - b^2)^2d \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}}$$

Result (type 4, 1335 leaves):

$$\frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2(-bB \sin[c+dx] + aC \sin[c+dx])}{3(a^2 - b^2)(a+b \cos[c+dx])^2} - \frac{2(3a^2bB \sin[c+dx] + b^3B \sin[c+dx] - 4ab^2C \sin[c+dx])}{3a(a^2 - b^2)^2(a+b \cos[c+dx])} \right) +$$

$$\frac{1}{3a(a-b)^2(a+b)^2d} \left( - \left( 4a(-a^2bB + b^3B + a^3C - ab^2C) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right) \right)$$



$$\begin{aligned}
& \left. \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right\} / \\
& \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - 4 a \left( 3 a^3 B+a b^2 B-4 a^2 b C \right) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right\} / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4\right\} / \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) \right) + \\
& 2 \left( 3 a^2 b B+b^3 B-4 a b^2 C \right) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Sec}[c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Sec}[c+d x]}{a+b}} \right. \\
& \left. \frac{1}{b} 2 a \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right) \right)
\end{aligned}$$

$$\left. \left( \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.$$

$$\left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right.$$

$$\left. \left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.$$

$$\left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right)$$

■ **Problem 936: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{B \cos[c+dx] + C \cos[c+dx]^2}{\cos[c+dx]^{3/2} (a+b \cos[c+dx])^{5/2}} dx$$

Optimal (type 4, 429 leaves, 6 steps):

$$\frac{1}{3 a^3 (a-b) (a+b)^{3/2} d} 2 (6 a^2 b B - 2 b^3 B - 3 a^3 C - a b^2 C) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{3 a^2 \sqrt{a+b} (a^2-b^2) d}$$

$$2 (2 b^2 B - 3 a^2 (B+C) + a b (3 B+C)) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}}$$

$$\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{2 b (b B - a C) \sqrt{\cos[c+dx]} \sin[c+dx]}{3 a (a^2-b^2) d (a+b \cos[c+dx])^{3/2}} - \frac{2 (6 a^2 b B - 2 b^3 B - 3 a^3 C - a b^2 C) \sin[c+dx]}{3 a (a^2-b^2)^2 d \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}}$$

Result (type 4, 1384 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \\
& \left( -\frac{2(-b^2 B \sin[c+dx] + a b C \sin[c+dx])}{3 a (a^2 - b^2) (a + b \cos[c+dx])^2} - \frac{2(-6 a^2 b^2 B \sin[c+dx] + 2 b^4 B \sin[c+dx] + 3 a^3 b C \sin[c+dx] + a b^3 C \sin[c+dx])}{3 a^2 (a^2 - b^2)^2 (a + b \cos[c+dx])} \right) + \\
& \frac{1}{3 a^2 (a - b)^2 (a + b)^2 d} \left( - \left( 4 a (3 a^4 B - 5 a^2 b^2 B + 2 b^4 B - a^3 b C + a b^3 C) \sqrt{\frac{(a + b) \cot\left[\frac{1}{2}(c + dx)\right]^2}{-a + b}} \sqrt{\frac{(a + b) \cos[c + dx] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \operatorname{Csc}[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a + b}\right] \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]^4 \right) / \right. \\
& \left. \left( (a + b) \sqrt{\cos[c + dx]} \sqrt{a + b \cos[c + dx]} \right) - 4 a (-6 a^3 b B + 2 a b^3 B + 3 a^4 C + a^2 b^2 C) \right. \\
& \left. \left( \left( \sqrt{\frac{(a + b) \cot\left[\frac{1}{2}(c + dx)\right]^2}{-a + b}} \sqrt{\frac{(a + b) \cos[c + dx] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \operatorname{Csc}[c + dx] \right. \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a + b}\right] \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]^4 \right) / \left( (a + b) \sqrt{\cos[c + dx]} \sqrt{a + b \cos[c + dx]} \right) - \right. \\
& \left. \left( \sqrt{\frac{(a + b) \cot\left[\frac{1}{2}(c + dx)\right]^2}{-a + b}} \sqrt{\frac{(a + b) \cos[c + dx] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \operatorname{Csc}[c + dx] \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a + b}\right] \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]^4 \right) / \left( b \sqrt{\cos[c + dx]} \sqrt{a + b \cos[c + dx]} \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 2 \left( -6 a^2 b^2 B + 2 b^4 B + 3 a^3 b C + a b^3 C \right) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}}} + \right. \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right)
\end{aligned}$$

■ **Problem 937: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2}{\operatorname{Cos}[c+dx]^{5/2} (a+b \operatorname{Cos}[c+dx])^{5/2}} dx$$

Optimal (type 4, 456 leaves, 6 steps):

$$\frac{1}{3 a^4 (a-b) (a+b)^{3/2} d} \left( 3 a^4 B - 15 a^2 b^2 B + 8 b^4 B + 6 a^3 b C - 2 a b^3 C \right) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{3 a^3 \sqrt{a+b} (a^2-b^2) d} \left( 8 b^3 B - 3 a^3 (B-C) + 2 a b^2 (3 B-C) - 3 a^2 b (3 B+C) \right)$$

$$\operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{2 b (b B - a C) \operatorname{Sin}[c+d x]}{3 a (a^2-b^2) d \sqrt{\operatorname{Cos}[c+d x]} (a+b \operatorname{Cos}[c+d x])^{3/2}} + \frac{2 b (8 a^2 b B - 4 b^3 B - 5 a^3 C + a b^2 C) \operatorname{Sin}[c+d x]}{3 a^2 (a^2-b^2)^2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]}}$$

Result (type 4, 1431 leaves):

$$-\frac{1}{3 a^3 (a-b)^2 (a+b)^2 d}$$

$$\left( -4 a \left( 9 a^4 b B - 17 a^2 b^3 B + 8 b^5 B - 3 a^5 C + 5 a^3 b^2 C - 2 a b^4 C \right) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right.$$

$$\left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) /$$

$$\left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a \left( 3 a^5 B - 15 a^3 b^2 B + 8 a b^4 B + 6 a^4 b C - 2 a^2 b^3 C \right)$$

$$\left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) -$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + 2(3a^4 b B - 15a^2 b^3 B + 8b^5 B + 6a^3 b^2 C - 2ab^4 C) \\
& \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right. \\
& \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) - \\
& \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right)
\end{aligned}$$

$$\left. \begin{aligned} & \left. \left( \text{Csc}[c + dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ & \left. \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right) + \\ & \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2(-b^3 B \sin[c+dx] + a b^2 C \sin[c+dx])}{3 a^2 (a^2 - b^2) (a+b \cos[c+dx])^2} + \right. \\ & \frac{2(-9 a^2 b^3 B \sin[c+dx] + 5 b^5 B \sin[c+dx] + 6 a^3 b^2 C \sin[c+dx] - 2 a b^4 C \sin[c+dx])}{3 a^3 (a^2 - b^2)^2 (a+b \cos[c+dx])} + \\ & \left. \frac{2 B \tan[c+dx]}{a^3} \right) \end{aligned} \right)$$

■ **Problem 942: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + dx]) (A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^2 dx$$

Optimal (type 3, 52 leaves, 4 steps):

$$(b B + a C) x + \frac{(A b + a B) \text{ArcTanh}[\sin[c + dx]]}{d} + \frac{b C \sin[c + dx]}{d} + \frac{a A \tan[c + dx]}{d}$$

Result (type 3, 187 leaves):

$$b B x + a C x - \frac{A b \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} - \frac{a B \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{A b \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a B \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{b C \cos[d x] \sin[c]}{d} + \frac{b C \cos[c] \sin[d x]}{d} + \frac{a A \tan[c + dx]}{d}$$

■ **Problem 945: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + dx]) (A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^5 dx$$

Optimal (type 3, 137 leaves, 7 steps):

$$\frac{(3 a A + 4 b B + 4 a C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{(2 A b + 2 a B + 3 b C) \operatorname{Tan}[c + d x]}{3 d} + \frac{(3 a A + 4 b B + 4 a C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{(A b + a B) \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d} + \frac{a A \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d}$$

Result (type 3, 545 leaves):

$$\begin{aligned} & - \frac{3 a A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} - \frac{b B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} - \\ & \frac{a C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} + \frac{3 a A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{b B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} + \\ & \frac{a C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} + \frac{a A}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{3 a A}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \\ & \frac{b B}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{a C}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{a A}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} - \\ & \frac{3 a A}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{b B}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{a C}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \\ & \frac{2 A b \operatorname{Tan}[c + d x]}{3 d} + \frac{2 a B \operatorname{Tan}[c + d x]}{3 d} + \frac{b C \operatorname{Tan}[c + d x]}{d} + \frac{A b \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d} + \frac{a B \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d} \end{aligned}$$

■ **Problem 946: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Cos}[c + d x]) (A + B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2) \operatorname{Sec}[c + d x]^6 dx$$

Optimal (type 3, 165 leaves, 7 steps):

$$\frac{(3 A b + 3 a B + 4 b C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{(4 a A + 5 b B + 5 a C) \operatorname{Tan}[c + d x]}{5 d} + \frac{(3 A b + 3 a B + 4 b C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{(A b + a B) \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d} + \frac{a A \operatorname{Sec}[c + d x]^4 \operatorname{Tan}[c + d x]}{5 d} + \frac{(4 a A + 5 b B + 5 a C) \operatorname{Tan}[c + d x]^3}{15 d}$$

Result (type 3, 660 leaves):



$$\begin{aligned}
& - \frac{3 A b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} - \frac{3 a B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} - \frac{b C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \\
& \frac{3 A b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{3 a B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{b C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \\
& \frac{A b}{16 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} + \frac{a B}{16 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} + \frac{3 A b}{16 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} + \\
& \frac{3 a B}{16 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{b C}{4 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{A b}{16 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} - \\
& \frac{a B}{16 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} - \frac{3 A b}{16 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{3 a B}{16 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} - \\
& \frac{b C}{4 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{8 a A \operatorname{Tan}[c+d x]}{15 d} + \frac{2 b B \operatorname{Tan}[c+d x]}{3 d} + \frac{2 a C \operatorname{Tan}[c+d x]}{3 d} + \\
& \frac{4 a A \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{15 d} + \frac{b B \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{3 d} + \frac{a C \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{3 d} + \frac{a A \operatorname{Sec}[c+d x]^4 \operatorname{Tan}[c+d x]}{5 d}
\end{aligned}$$

■ **Problem 951: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Cos}[c+d x])^2 (A+B \operatorname{Cos}[c+d x]+C \operatorname{Cos}[c+d x]^2) \operatorname{Sec}[c+d x]^3 dx$$

Optimal (type 3, 118 leaves, 5 steps):

$$\begin{aligned}
& b (b B+2 a C) x + \frac{(2 A b^2+4 a b B+a^2(A+2 C)) \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{2 d} - \\
& \frac{b^2(A-2 C) \operatorname{Sin}[c+d x]}{2 d} + \frac{a(A b+a B) \operatorname{Tan}[c+d x]}{d} + \frac{A(a+b \operatorname{Cos}[c+d x])^2 \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{2 d}
\end{aligned}$$

Result (type 3, 277 leaves):

$$\begin{aligned}
& \frac{1}{4 d} \left( 4 b (b B+2 a C) (c+d x) - 2 (2 A b^2+4 a b B+a^2(A+2 C)) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + \right. \\
& \left. 2 (2 A b^2+4 a b B+a^2(A+2 C)) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + \frac{a^2 A}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} + \right. \\
& \left. \frac{4 a (2 A b+a B) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]} - \frac{a^2 A}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{4 a (2 A b+a B) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]} + 4 b^2 C \operatorname{Sin}[c+d x] \right)
\end{aligned}$$

■ **Problem 952: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^2 (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^4 dx$$

Optimal (type 3, 141 leaves, 5 steps):

$$b^2 C x + \frac{(a^2 B + 2 b^2 B + 2 a b (A + 2 C)) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{(2 A b^2 + 6 a b B + a^2 (2 A + 3 C)) \tan [c + d x]}{3 d} + \frac{a (2 A b + 3 a B) \sec [c + d x] \tan [c + d x]}{6 d} + \frac{A (a + b \cos [c + d x])^2 \sec [c + d x]^2 \tan [c + d x]}{3 d}$$

Result (type 3, 386 leaves):

$$\frac{1}{12 d} \left( 12 b^2 C (c + d x) - 6 (a^2 B + 2 b^2 B + 2 a b (A + 2 C)) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] \right) + 6 (a^2 B + 2 b^2 B + 2 a b (A + 2 C)) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \frac{a (6 A b + a (A + 3 B))}{(\cos [\frac{1}{2} (c + d x)] - \sin [\frac{1}{2} (c + d x)])^2} + \frac{2 a^2 A \sin [\frac{1}{2} (c + d x)]}{(\cos [\frac{1}{2} (c + d x)] - \sin [\frac{1}{2} (c + d x)])^3} + \frac{4 (3 A b^2 + 6 a b B + a^2 (2 A + 3 C)) \sin [\frac{1}{2} (c + d x)]}{\cos [\frac{1}{2} (c + d x)] - \sin [\frac{1}{2} (c + d x)]} + \frac{2 a^2 A \sin [\frac{1}{2} (c + d x)]}{(\cos [\frac{1}{2} (c + d x)] + \sin [\frac{1}{2} (c + d x)])^3} - \frac{a (6 A b + a (A + 3 B))}{(\cos [\frac{1}{2} (c + d x)] + \sin [\frac{1}{2} (c + d x)])^2} + \frac{4 (3 A b^2 + 6 a b B + a^2 (2 A + 3 C)) \sin [\frac{1}{2} (c + d x)]}{\cos [\frac{1}{2} (c + d x)] + \sin [\frac{1}{2} (c + d x)]}$$

■ **Problem 960: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^3 (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^4 dx$$

Optimal (type 3, 196 leaves, 6 steps):

$$b^2 (b B + 3 a C) x + \frac{(2 A b^3 + a^3 B + 6 a b^2 B + 3 a^2 b (A + 2 C)) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} - \frac{b^2 (5 A b + 3 a B - 6 b C) \sin [c + d x]}{6 d} + \frac{a (3 A b^2 + 6 a b B + a^2 (2 A + 3 C)) \tan [c + d x]}{3 d} + \frac{(A b + a B) (a + b \cos [c + d x])^2 \sec [c + d x] \tan [c + d x]}{2 d} + \frac{A (a + b \cos [c + d x])^3 \sec [c + d x]^2 \tan [c + d x]}{3 d}$$

Result (type 3, 549 leaves):

$$\begin{aligned}
& \frac{b^2 (bB + 3aC) (c + dx)}{d} + \frac{(-3a^2Ab - 2Ab^3 - a^3B - 6ab^2B - 6a^2bC) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{2d} + \\
& \frac{(3a^2Ab + 2Ab^3 + a^3B + 6ab^2B + 6a^2bC) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{2d} + \frac{a^3A + 9a^2Ab + 3a^3B}{12d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{a^3A \sin\left[\frac{1}{2}(c + dx)\right]}{6d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{a^3A \sin\left[\frac{1}{2}(c + dx)\right]}{6d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{-a^3A - 9a^2Ab - 3a^3B}{12d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{2a^3A \sin\left[\frac{1}{2}(c + dx)\right] + 9aAb^2 \sin\left[\frac{1}{2}(c + dx)\right] + 9a^2bB \sin\left[\frac{1}{2}(c + dx)\right] + 3a^3C \sin\left[\frac{1}{2}(c + dx)\right]}{3d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \\
& \frac{2a^3A \sin\left[\frac{1}{2}(c + dx)\right] + 9aAb^2 \sin\left[\frac{1}{2}(c + dx)\right] + 9a^2bB \sin\left[\frac{1}{2}(c + dx)\right] + 3a^3C \sin\left[\frac{1}{2}(c + dx)\right]}{3d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \frac{b^3C \sin[c + dx]}{d}
\end{aligned}$$

- **Problem 981: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]}{a + b \cos[c + dx]} dx$$

Optimal (type 3, 94 leaves, 4 steps):

$$\frac{cx}{b} - \frac{2(Ab^2 - a(bB - aC)) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a\sqrt{a-b} b\sqrt{a+b} d} + \frac{A \operatorname{ArcTanh}[\sin[c + dx]]}{ad}$$

Result (type 3, 256 leaves):

$$\begin{aligned}
& \left( 2(A + B \cos[c + dx] + C \cos[c + dx]^2) \right. \\
& \left. \left( \left( a C dx - A b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + A b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) \sqrt{-(a^2 - b^2) (\cos[c] - i \sin[c])^2} + \right. \right. \\
& \left. \left. 2(Ab^2 + a(-bB + aC)) \operatorname{ArcTan}\left[\frac{(i \cos[c] + \sin[c]) (b \sin[c] + (-a + b \cos[c]) \operatorname{Tan}\left[\frac{dx}{2}\right])}{\sqrt{-(a^2 - b^2) (\cos[c] - i \sin[c])^2}}\right] (i \cos[c] + \sin[c]) \right) \right) / \\
& \left( a b d (2A + C + 2B \cos[c + dx] + C \cos[2(c + dx)]) \sqrt{(-a^2 + b^2) (\cos[2c] - i \sin[2c])} \right)
\end{aligned}$$

- **Problem 982: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^2}{a + b \cos[c + dx]} dx$$

Optimal (type 3, 107 leaves, 5 steps):

$$\frac{2 (A b^2 - a (b B - a C)) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^2 \sqrt{a-b} \sqrt{a+b} d} - \frac{(A b - a B) \operatorname{ArcTanh}[\sin[c + dx]]}{a^2 d} + \frac{A \tan[c + dx]}{a d}$$

Result (type 3, 339 leaves):

$$\frac{1}{a^2 d (2 A + C + 2 B \cos[c + dx] + C \cos[2(c + dx)])} 2 \cos[c + dx]^2 (C + B \sec[c + dx] + A \sec[c + dx]^2)$$

$$\left( (A b - a B) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + (-A b + a B) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] - \right.$$

$$\frac{2 i (A b^2 + a (-b B + a C)) \operatorname{ArcTan}\left[\frac{(i \cos[c] + \sin[c]) (b \sin[c] + (-a + b \cos[c]) \tan\left[\frac{dx}{2}\right])}{\sqrt{-(a^2 - b^2)} (\cos[c] - i \sin[c])^2}\right] (\cos[c] - i \sin[c])}{\sqrt{-(a^2 + b^2)} (\cos[c] - i \sin[c])^2} +$$

$$\left. \frac{a A \sin\left[\frac{dx}{2}\right]}{(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right])} + \frac{a A \sin\left[\frac{dx}{2}\right]}{(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])} \right)$$

- **Problem 983: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^3}{a + b \cos[c + dx]} dx$$

Optimal (type 3, 154 leaves, 6 steps):

$$-\frac{2 b (A b^2 - a (b B - a C)) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^3 \sqrt{a-b} \sqrt{a+b} d} +$$

$$\frac{(2 A b^2 - 2 a b B + a^2 (A + 2 C)) \operatorname{ArcTanh}[\sin[c + dx]]}{2 a^3 d} - \frac{(A b - a B) \tan[c + dx]}{a^2 d} + \frac{A \sec[c + dx] \tan[c + dx]}{2 a d}$$

Result (type 3, 314 leaves):

$$\frac{1}{4 a^3 d} \left( \frac{8 b (A b^2 + a (-b B + a C)) \operatorname{ArcTanh} \left[ \frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (c+d x) \right]}{\sqrt{-a^2+b^2}} \right]}{\sqrt{-a^2+b^2}} - 2 (2 A b^2 - 2 a b B + a^2 (A + 2 C)) \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c+d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c+d x) \right] \right] \right) +$$

$$2 (2 A b^2 - 2 a b B + a^2 (A + 2 C)) \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c+d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c+d x) \right] \right] + \frac{a^2 A}{\left( \operatorname{Cos} \left[ \frac{1}{2} (c+d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c+d x) \right] \right)^2} +$$

$$\left. \frac{4 a (-A b + a B) \operatorname{Sin} \left[ \frac{1}{2} (c+d x) \right]}{\operatorname{Cos} \left[ \frac{1}{2} (c+d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c+d x) \right]} - \frac{a^2 A}{\left( \operatorname{Cos} \left[ \frac{1}{2} (c+d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c+d x) \right] \right)^2} + \frac{4 a (-A b + a B) \operatorname{Sin} \left[ \frac{1}{2} (c+d x) \right]}{\operatorname{Cos} \left[ \frac{1}{2} (c+d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c+d x) \right]} \right)$$

■ **Problem 984: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2) \operatorname{Sec}[c + d x]^4}{a + b \operatorname{Cos}[c + d x]} dx$$

Optimal (type 3, 214 leaves, 7 steps) :

$$\frac{2 b^2 (A b^2 - a (b B - a C)) \operatorname{ArcTan} \left[ \frac{\sqrt{a-b} \operatorname{Tan} \left[ \frac{1}{2} (c+d x) \right]}{\sqrt{a+b}} \right]}{a^4 \sqrt{a-b} \sqrt{a+b} d} - \frac{(2 A b^3 - a^3 B - 2 a b^2 B + a^2 b (A + 2 C)) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 a^4 d} +$$

$$\frac{(3 A b^2 - 3 a b B + a^2 (2 A + 3 C)) \operatorname{Tan}[c + d x]}{3 a^3 d} - \frac{(A b - a B) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 a^2 d} + \frac{A \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 a d}$$

Result (type 3, 466 leaves) :

$$\frac{1}{12 a^4 d} \left( -\frac{24 b^2 (A b^2 + a (-b B + a C)) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}} + 6 (2 A b^3 - a^3 B - 2 a b^2 B + a^2 b (A + 2 C)) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) +$$

$$6 (-2 A b^3 + a^3 B + 2 a b^2 B - a^2 b (A + 2 C)) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \frac{a^2 (-3 A b + a (A + 3 B))}{(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^2} +$$

$$\frac{2 a^3 A \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^3} + \frac{4 a (3 A b^2 - 3 a b B + a^2 (2 A + 3 C)) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]} +$$

$$\left. \frac{2 a^3 A \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^3} - \frac{a^2 (-3 A b + a (A + 3 B))}{(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^2} + \frac{4 a (3 A b^2 - 3 a b B + a^2 (2 A + 3 C)) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]} \right)$$

■ **Problem 990: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \operatorname{Cos}[c + dx] + C \operatorname{Cos}[c + dx]^2) \operatorname{Sec}[c + dx]}{(a + b \operatorname{Cos}[c + dx])^2} dx$$

Optimal (type 3, 147 leaves, 5 steps):

$$-\frac{2 (2 a^2 A b - A b^3 - a^3 B + a^2 b C) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^2 (a-b)^{3/2} (a+b)^{3/2} d} + \frac{A \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{a^2 d} + \frac{(A b^2 - a (b B - a C)) \operatorname{Sin}[c + dx]}{a (a^2 - b^2) d (a + b \operatorname{Cos}[c + dx])}$$

Result (type 3, 319 leaves):

$$\left( 2 \operatorname{Cos}[c + dx] (B + C \operatorname{Cos}[c + dx] + A \operatorname{Sec}[c + dx]) \right. \\ \left. - A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + \left( 2 i (A b^3 + a^3 B - a^2 b (2 A + C)) \right. \right. \\ \left. \left. \operatorname{ArcTan}\left[\frac{(i \operatorname{Cos}[c] + \operatorname{Sin}[c]) (b \operatorname{Sin}[c] + (-a + b \operatorname{Cos}[c]) \operatorname{Tan}\left[\frac{dx}{2}\right])}{\sqrt{-(a^2 - b^2) (\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2}}\right] (\operatorname{Cos}[c] - i \operatorname{Sin}[c])^3 \right] \right) / \left( (-a^2 + b^2) (\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2 \right)^{3/2} + \\ \left. \left. \frac{a (A b^2 + a (-b B + a C)) (-a \operatorname{Sin}[c] + b \operatorname{Sin}[dx])}{(a - b) b (a + b) (a + b \operatorname{Cos}[c + dx]) \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right)} \right) \right) / (a^2 d (2 A + C + 2 B \operatorname{Cos}[c + dx] + C \operatorname{Cos}[2(c + dx)]))$$

■ **Problem 994: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + dx]^3 (A + B \operatorname{Cos}[c + dx] + C \operatorname{Cos}[c + dx]^2)}{(a + b \operatorname{Cos}[c + dx])^3} dx$$

Optimal (type 3, 456 leaves, 7 steps):

$$\frac{(2 A b^2 - 6 a b B + 12 a^2 C + b^2 C) x}{2 b^5} - \frac{1}{(a - b)^{5/2} b^5 (a + b)^{5/2} d} \\ a (6 A b^6 - 6 a^5 b B + 15 a^3 b^3 B - 12 a b^5 B + a^4 b^2 (2 A - 29 C) - 5 a^2 b^4 (A - 4 C) + 12 a^6 C) \operatorname{ArcTan}\left[\frac{\sqrt{a - b} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a + b}}\right] + \\ \frac{(6 a^4 b B - 11 a^2 b^3 B + 2 b^5 B - a^3 b^2 (2 A - 21 C) + a b^4 (5 A - 6 C) - 12 a^5 C) \operatorname{Sin}[c + dx]}{2 b^4 (a^2 - b^2)^2 d} - \\ \frac{(3 a^3 b B - 6 a b^3 B - a^2 b^2 (A - 10 C) + b^4 (4 A - C) - 6 a^4 C) \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]}{2 b^3 (a^2 - b^2)^2 d} - \\ \frac{(A b^2 - a (b B - a C)) \operatorname{Cos}[c + dx]^3 \operatorname{Sin}[c + dx]}{2 b (a^2 - b^2) d (a + b \operatorname{Cos}[c + dx])^2} + \frac{(3 A b^4 + a (2 a^2 b B - 5 b^3 B - 4 a^3 C + 7 a b^2 C)) \operatorname{Cos}[c + dx]^2 \operatorname{Sin}[c + dx]}{2 b^2 (a^2 - b^2)^2 d (a + b \operatorname{Cos}[c + dx])}$$

Result (type 3, 1150 leaves):

$$\frac{1}{b^5 (a^2 - b^2)^2 \sqrt{-a^2 + b^2} d}$$

$$a (2 a^4 A b^2 - 5 a^2 A b^4 + 6 A b^6 - 6 a^5 b B + 15 a^3 b^3 B - 12 a b^5 B + 12 a^6 C - 29 a^4 b^2 C + 20 a^2 b^4 C) \operatorname{ArcTanh} \left[ \frac{(a - b) \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right]}{\sqrt{-a^2 + b^2}} \right] +$$

$$\frac{1}{16 b^5 (-a^2 + b^2)^2 d (a + b \operatorname{Cos}[c + d x])^2}$$

$$\begin{aligned} & (16 a^6 A b^2 (c + d x) - 24 a^4 A b^4 (c + d x) + 8 A b^8 (c + d x) - 48 a^7 b B (c + d x) + 72 a^5 b^3 B (c + d x) - 24 a b^7 B (c + d x) + \\ & 96 a^8 C (c + d x) - 136 a^6 b^2 C (c + d x) - 12 a^4 b^4 C (c + d x) + 48 a^2 b^6 C (c + d x) + 4 b^8 C (c + d x) + 32 a^5 A b^3 (c + d x) \operatorname{Cos}[c + d x] - \\ & 64 a^3 A b^5 (c + d x) \operatorname{Cos}[c + d x] + 32 a A b^7 (c + d x) \operatorname{Cos}[c + d x] - 96 a^6 b^2 B (c + d x) \operatorname{Cos}[c + d x] + 192 a^4 b^4 B (c + d x) \operatorname{Cos}[c + d x] - \\ & 96 a^2 b^6 B (c + d x) \operatorname{Cos}[c + d x] + 192 a^7 b C (c + d x) \operatorname{Cos}[c + d x] - 368 a^5 b^3 C (c + d x) \operatorname{Cos}[c + d x] + 160 a^3 b^5 C (c + d x) \operatorname{Cos}[c + d x] + \\ & 16 a b^7 C (c + d x) \operatorname{Cos}[c + d x] + 8 a^4 A b^4 (c + d x) \operatorname{Cos}[2 (c + d x)] - 16 a^2 A b^6 (c + d x) \operatorname{Cos}[2 (c + d x)] + 8 A b^8 (c + d x) \operatorname{Cos}[2 (c + d x)] - \\ & 24 a^5 b^3 B (c + d x) \operatorname{Cos}[2 (c + d x)] + 48 a^3 b^5 B (c + d x) \operatorname{Cos}[2 (c + d x)] - 24 a b^7 B (c + d x) \operatorname{Cos}[2 (c + d x)] + \\ & 48 a^6 b^2 C (c + d x) \operatorname{Cos}[2 (c + d x)] - 92 a^4 b^4 C (c + d x) \operatorname{Cos}[2 (c + d x)] + 40 a^2 b^6 C (c + d x) \operatorname{Cos}[2 (c + d x)] + \\ & 4 b^8 C (c + d x) \operatorname{Cos}[2 (c + d x)] - 16 a^5 A b^3 \operatorname{Sin}[c + d x] + 40 a^3 A b^5 \operatorname{Sin}[c + d x] + 48 a^6 b^2 B \operatorname{Sin}[c + d x] - 84 a^4 b^4 B \operatorname{Sin}[c + d x] + \\ & 8 a^2 b^6 B \operatorname{Sin}[c + d x] + 4 b^8 B \operatorname{Sin}[c + d x] - 96 a^7 b C \operatorname{Sin}[c + d x] + 160 a^5 b^3 C \operatorname{Sin}[c + d x] - 32 a^3 b^5 C \operatorname{Sin}[c + d x] - \\ & 8 a b^7 C \operatorname{Sin}[c + d x] - 12 a^4 A b^4 \operatorname{Sin}[2 (c + d x)] + 24 a^2 A b^6 \operatorname{Sin}[2 (c + d x)] + 36 a^5 b^3 B \operatorname{Sin}[2 (c + d x)] - 64 a^3 b^5 B \operatorname{Sin}[2 (c + d x)] + \\ & 16 a b^7 B \operatorname{Sin}[2 (c + d x)] - 72 a^6 b^2 C \operatorname{Sin}[2 (c + d x)] + 130 a^4 b^4 C \operatorname{Sin}[2 (c + d x)] - 48 a^2 b^6 C \operatorname{Sin}[2 (c + d x)] + \\ & 2 b^8 C \operatorname{Sin}[2 (c + d x)] + 4 a^4 b^4 B \operatorname{Sin}[3 (c + d x)] - 8 a^2 b^6 B \operatorname{Sin}[3 (c + d x)] + 4 b^8 B \operatorname{Sin}[3 (c + d x)] - 8 a^5 b^3 C \operatorname{Sin}[3 (c + d x)] + \\ & 16 a^3 b^5 C \operatorname{Sin}[3 (c + d x)] - 8 a b^7 C \operatorname{Sin}[3 (c + d x)] + a^4 b^4 C \operatorname{Sin}[4 (c + d x)] - 2 a^2 b^6 C \operatorname{Sin}[4 (c + d x)] + b^8 C \operatorname{Sin}[4 (c + d x)] \end{aligned}$$

- **Problem 998: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2) \operatorname{Sec}[c + d x]}{(a + b \operatorname{Cos}[c + d x])^3} dx$$

Optimal (type 3, 238 leaves, 6 steps):

$$\frac{(5 a^2 A b^3 - 2 A b^5 + 2 a^5 B + a^3 b^2 B - 3 a^4 b (2 A + C)) \operatorname{ArcTan} \left[ \frac{\sqrt{a-b} \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right]}{\sqrt{a+b}} \right]}{a^3 (a - b)^{5/2} (a + b)^{5/2} d} + \frac{A \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{a^3 d} + \frac{(A b^2 - a (b B - a C)) \operatorname{Sin}[c + d x]}{2 a (a^2 - b^2) d (a + b \operatorname{Cos}[c + d x])^2} - \frac{(2 A b^4 + 3 a^3 b B - a^4 C - a^2 b^2 (5 A + 2 C)) \operatorname{Sin}[c + d x]}{2 a^2 (a^2 - b^2)^2 d (a + b \operatorname{Cos}[c + d x])}$$

Result (type 3, 1065 leaves):



$$\begin{aligned}
& - \frac{2 A \operatorname{Cos}[c+d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2}+\frac{d x}{2}\right]-\operatorname{Sin}\left[\frac{c}{2}+\frac{d x}{2}\right]\right](B+C \operatorname{Cos}[c+d x]+A \operatorname{Sec}[c+d x])}{a^3 d(2 A+C+2 B \operatorname{Cos}[c+d x]+C \operatorname{Cos}[2 c+2 d x])} + \\
& \frac{2 A \operatorname{Cos}[c+d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2}+\frac{d x}{2}\right]+\operatorname{Sin}\left[\frac{c}{2}+\frac{d x}{2}\right]\right](B+C \operatorname{Cos}[c+d x]+A \operatorname{Sec}[c+d x])}{a^3 d(2 A+C+2 B \operatorname{Cos}[c+d x]+C \operatorname{Cos}[2 c+2 d x])} + \\
& \frac{1}{\left(a^2-b^2\right)^2(2 A+C+2 B \operatorname{Cos}[c+d x]+C \operatorname{Cos}[2 c+2 d x])}(-6 a^4 A b+5 a^2 A b^3-2 A b^5+2 a^5 B+a^3 b^2 B-3 a^4 b C) \operatorname{Cos}[c+d x] \\
& (B+C \operatorname{Cos}[c+d x]+A \operatorname{Sec}[c+d x])\left(-\left(2 i \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{d x}{2}\right]\right]\left(\frac{\operatorname{Cos}[c]}{\sqrt{-a^2 \operatorname{Cos}[2 c]+b^2 \operatorname{Cos}[2 c]+i a^2 \operatorname{Sin}[2 c]-i b^2 \operatorname{Sin}[2 c]}}-\right.\right.\right. \\
& \left.\left.\left.\frac{i \operatorname{Sin}[c]}{\sqrt{-a^2 \operatorname{Cos}[2 c]+b^2 \operatorname{Cos}[2 c]+i a^2 \operatorname{Sin}[2 c]-i b^2 \operatorname{Sin}[2 c]}}\right)\left(-i a \operatorname{Sin}\left[\frac{d x}{2}\right]+i b \operatorname{Sin}\left[c+\frac{d x}{2}\right]\right)\right) \operatorname{Cos}[c]\right) / \\
& \left(a^3 d \sqrt{-a^2 \operatorname{Cos}[2 c]+b^2 \operatorname{Cos}[2 c]+i a^2 \operatorname{Sin}[2 c]-i b^2 \operatorname{Sin}[2 c]}\right)-\left(2 \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{d x}{2}\right]\right]\right. \\
& \left.\left(\frac{\operatorname{Cos}[c]}{\sqrt{-a^2 \operatorname{Cos}[2 c]+b^2 \operatorname{Cos}[2 c]+i a^2 \operatorname{Sin}[2 c]-i b^2 \operatorname{Sin}[2 c]}}-\frac{i \operatorname{Sin}[c]}{\sqrt{-a^2 \operatorname{Cos}[2 c]+b^2 \operatorname{Cos}[2 c]+i a^2 \operatorname{Sin}[2 c]-i b^2 \operatorname{Sin}[2 c]}}\right)\right. \\
& \left.\left(-i a \operatorname{Sin}\left[\frac{d x}{2}\right]+i b \operatorname{Sin}\left[c+\frac{d x}{2}\right]\right)\right) \operatorname{Sin}[c]\right) / \left(a^3 d \sqrt{-a^2 \operatorname{Cos}[2 c]+b^2 \operatorname{Cos}[2 c]+i a^2 \operatorname{Sin}[2 c]-i b^2 \operatorname{Sin}[2 c]}\right) + \\
& \left(\operatorname{Cos}[c+d x] \operatorname{Sec}[c](B+C \operatorname{Cos}[c+d x]+A \operatorname{Sec}[c+d x])\left(-10 a^4 A b^2 \operatorname{Sin}[c]-a^2 A b^4 \operatorname{Sin}[c]+2 A b^6 \operatorname{Sin}[c]+6 a^5 b B \operatorname{Sin}[c]+3 a^3 b^3 B \operatorname{Sin}[c]-2 a^6 C \operatorname{Sin}[c]-5 a^4 b^2 C \operatorname{Sin}[c]-2 a^2 b^4 C \operatorname{Sin}[c]+16 a^3 A b^3 \operatorname{Sin}[d x]-7 a A b^5 \operatorname{Sin}[d x]-10 a^4 b^2 B \operatorname{Sin}[d x]+a^2 b^4 B \operatorname{Sin}[d x]+4 a^5 b C \operatorname{Sin}[d x]+5 a^3 b^3 C \operatorname{Sin}[d x]-4 a^3 A b^3 \operatorname{Sin}[2 c+d x]+a A b^5 \operatorname{Sin}[2 c+d x]+2 a^4 b^2 B \operatorname{Sin}[2 c+d x]+a^2 b^4 B \operatorname{Sin}[2 c+d x]-3 a^3 b^3 C \operatorname{Sin}[2 c+d x]+5 a^2 A b^4 \operatorname{Sin}[c+2 d x]-2 A b^6 \operatorname{Sin}[c+2 d x]-3 a^3 b^3 B \operatorname{Sin}[c+2 d x]+a^4 b^2 C \operatorname{Sin}[c+2 d x]+2 a^2 b^4 C \operatorname{Sin}[c+2 d x]\right)\right) / \\
& \left(2 a^2 b\left(a^2-b^2\right)^2 d(a+b \operatorname{Cos}[c+d x])^2(2 A+C+2 B \operatorname{Cos}[c+d x]+C \operatorname{Cos}[2 c+2 d x])\right)
\end{aligned}$$

■ **Problem 1001: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[c+d x]^4(A+B \operatorname{Cos}[c+d x]+C \operatorname{Cos}[c+d x]^2)}{(a+b \operatorname{Cos}[c+d x])^4} d x$$

Optimal (type 3, 649 leaves, 8 steps):

$$\frac{(2Ab^2 - 8abB + 20a^2C + b^2C)x}{2b^6} +$$

$$\left( a \left( 8Ab^8 + 8a^7bB - 28a^5b^3B + 35a^3b^5B - 20ab^7B - a^6b^2(2A - 69C) + 7a^4b^4(A - 12C) - 8a^2b^6(A - 5C) - 20a^8C \right) \right.$$

$$\left. \text{ArcTan}\left[\frac{\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right] \right) / \left( \sqrt{a-b} b^6 \sqrt{a+b} (a^2 - b^2)^3 d \right) + \frac{1}{6b^5(a^2 - b^2)^3 d}$$

$$(24a^6bB - 68a^4b^3B + 65a^2b^5B - 6b^7B - a^5b^2(6A - 167C) + a^3b^4(17A - 146C) - 2ab^6(13A - 12C) - 60a^7C) \sin[c+dx] -$$

$$\frac{1}{2b^4(a^2 - b^2)^3 d} (4a^5bB - 11a^3b^3B + 12ab^5B - a^4b^2(A - 27C) + a^2b^4(2A - 23C) - b^6(6A - C) - 10a^6C) \cos[c+dx] \sin[c+dx] -$$

$$\frac{(Ab^2 - a(bB - aC)) \cos[c+dx]^4 \sin[c+dx]}{3b(a^2 - b^2)d(a+b\cos[c+dx])^3} + \frac{(4Ab^4 + 2a^3bB - 7ab^3B - 5a^4C + a^2b^2(A + 10C)) \cos[c+dx]^3 \sin[c+dx]}{6b^2(a^2 - b^2)^2 d(a+b\cos[c+dx])^2} -$$

$$\left( (12Ab^6 - 8a^5bB + 20a^3b^3B - 27ab^5B + a^4b^2(2A - 53C) + 20a^6C + a^2b^4(A + 48C)) \cos[c+dx]^2 \sin[c+dx] \right) /$$

$$(6b^3(a^2 - b^2)^3 d(a+b\cos[c+dx]))$$

Result (type 3, 658 leaves) :

$$\frac{(2Ab^2 - 8abB + 20a^2C + b^2C)(c+dx)}{2b^6d} + \frac{1}{b^6(a^2 - b^2)^3 \sqrt{-a^2 + b^2} d}$$

$$a \left( 2a^6Ab^2 - 7a^4Ab^4 + 8a^2Ab^6 - 8Ab^8 - 8a^7bB + 28a^5b^3B - 35a^3b^5B + 20ab^7B + 20a^8C - 69a^6b^2C + 84a^4b^4C - 40a^2b^6C \right)$$

$$\text{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a^2 + b^2}}\right] + \frac{(-bB + 4aC) \left( -\frac{i \cos[c+dx]}{2b^5} - \frac{\sin[c+dx]}{2b^5} \right)}{d} +$$

$$\frac{(-bB + 4aC) \left( \frac{i \cos[c+dx]}{2b^5} - \frac{\sin[c+dx]}{2b^5} \right)}{d} + \frac{a^4Ab^2 \sin[c+dx] - a^5bB \sin[c+dx] + a^6C \sin[c+dx]}{3b^5(-a^2 + b^2)d(a+b\cos[c+dx])^3} +$$

$$(7a^5Ab^2 \sin[c+dx] - 12a^3Ab^4 \sin[c+dx] - 10a^6bB \sin[c+dx] + 15a^4b^3B \sin[c+dx] + 13a^7C \sin[c+dx] - 18a^5b^2C \sin[c+dx]) /$$

$$(6b^5(-a^2 + b^2)^2 d(a+b\cos[c+dx])^2) +$$

$$(11a^6Ab^2 \sin[c+dx] - 32a^4Ab^4 \sin[c+dx] + 36a^2Ab^6 \sin[c+dx] - 26a^7bB \sin[c+dx] + 71a^5b^3B \sin[c+dx] - 60a^3b^5B \sin[c+dx] +$$

$$47a^8C \sin[c+dx] - 122a^6b^2C \sin[c+dx] + 90a^4b^4C \sin[c+dx]) / (6b^5(-a^2 + b^2)^3 d(a+b\cos[c+dx])) + \frac{C \sin[2(c+dx)]}{4b^4d}$$

■ **Problem 1003: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^2 (A + B \cos[c+dx] + C \cos[c+dx]^2)}{(a+b\cos[c+dx])^4} dx$$

Optimal (type 3, 349 leaves, 6 steps) :

$$\frac{C x}{b^4} - \frac{(3 a^2 b^5 B + 2 b^7 B - a^3 b^4 (A - 8 C) + 2 a^7 C - 7 a^5 b^2 C - 4 a b^6 (A + 2 C)) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{(a-b)^{7/2} b^4 (a+b)^{7/2} d} -$$

$$\frac{(A b^2 - a (b B - a C)) \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{3 b (a^2 - b^2) d (a+b \operatorname{Cos}[c+dx])^3} - \frac{a (2 A b^4 - 5 a b^3 B - 3 a^4 C + a^2 b^2 (3 A + 8 C)) \operatorname{Sin}[c+dx]}{6 b^3 (a^2 - b^2)^2 d (a+b \operatorname{Cos}[c+dx])^2} -$$

$$\frac{(4 A b^6 + a^3 b^3 B - 16 a b^5 B + 9 a^6 C + 2 a^2 b^4 (7 A + 17 C) - a^4 b^2 (3 A + 28 C)) \operatorname{Sin}[c+dx]}{6 b^3 (a^2 - b^2)^3 d (a+b \operatorname{Cos}[c+dx])}$$

Result (type 3, 915 leaves):

$$\frac{(-a^3 A b^4 - 4 a A b^6 + 3 a^2 b^5 B + 2 b^7 B + 2 a^7 C - 7 a^5 b^2 C + 8 a^3 b^4 C - 8 a b^6 C) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a^2+b^2}}\right]}{b^4 (a^2 - b^2)^3 \sqrt{-a^2+b^2} d} +$$

$$\frac{1}{24 b^4 (-a^2 + b^2)^3 d (a+b \operatorname{Cos}[c+dx])^3} (-24 a^9 C (c+dx) + 36 a^7 b^2 C (c+dx) + 36 a^5 b^4 C (c+dx) - 84 a^3 b^6 C (c+dx) +$$

$$36 a b^8 C (c+dx) - 72 a^8 b C (c+dx) \operatorname{Cos}[c+dx] + 198 a^6 b^3 C (c+dx) \operatorname{Cos}[c+dx] - 162 a^4 b^5 C (c+dx) \operatorname{Cos}[c+dx] +$$

$$18 a^2 b^7 C (c+dx) \operatorname{Cos}[c+dx] + 18 b^9 C (c+dx) \operatorname{Cos}[c+dx] - 36 a^7 b^2 C (c+dx) \operatorname{Cos}[2(c+dx)] + 108 a^5 b^4 C (c+dx) \operatorname{Cos}[2(c+dx)] -$$

$$108 a^3 b^6 C (c+dx) \operatorname{Cos}[2(c+dx)] + 36 a b^8 C (c+dx) \operatorname{Cos}[2(c+dx)] - 6 a^6 b^3 C (c+dx) \operatorname{Cos}[3(c+dx)] +$$

$$18 a^4 b^5 C (c+dx) \operatorname{Cos}[3(c+dx)] - 18 a^2 b^7 C (c+dx) \operatorname{Cos}[3(c+dx)] + 6 b^9 C (c+dx) \operatorname{Cos}[3(c+dx)] + 51 a^4 A b^5 \operatorname{Sin}[c+dx] +$$

$$18 a^2 A b^7 \operatorname{Sin}[c+dx] + 6 A b^9 \operatorname{Sin}[c+dx] - 18 a^5 b^4 B \operatorname{Sin}[c+dx] - 39 a^3 b^6 B \operatorname{Sin}[c+dx] - 18 a b^8 B \operatorname{Sin}[c+dx] + 24 a^8 b C \operatorname{Sin}[c+dx] -$$

$$57 a^6 b^3 C \operatorname{Sin}[c+dx] + 72 a^4 b^5 C \operatorname{Sin}[c+dx] + 36 a^2 b^7 C \operatorname{Sin}[c+dx] - 6 a^5 A b^4 \operatorname{Sin}[2(c+dx)] + 54 a^3 A b^6 \operatorname{Sin}[2(c+dx)] +$$

$$12 a A b^8 \operatorname{Sin}[2(c+dx)] - 6 a^4 b^5 B \operatorname{Sin}[2(c+dx)] - 54 a^2 b^7 B \operatorname{Sin}[2(c+dx)] + 30 a^7 b^2 C \operatorname{Sin}[2(c+dx)] - 90 a^5 b^4 C \operatorname{Sin}[2(c+dx)] +$$

$$120 a^3 b^6 C \operatorname{Sin}[2(c+dx)] - a^4 A b^5 \operatorname{Sin}[3(c+dx)] + 10 a^2 A b^7 \operatorname{Sin}[3(c+dx)] + 6 A b^9 \operatorname{Sin}[3(c+dx)] - 2 a^5 b^4 B \operatorname{Sin}[3(c+dx)] +$$

$$5 a^3 b^6 B \operatorname{Sin}[3(c+dx)] - 18 a b^8 B \operatorname{Sin}[3(c+dx)] + 11 a^6 b^3 C \operatorname{Sin}[3(c+dx)] - 32 a^4 b^5 C \operatorname{Sin}[3(c+dx)] + 36 a^2 b^7 C \operatorname{Sin}[3(c+dx)] )$$

■ **Problem 1006: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2) \operatorname{Sec}[c+dx]}{(a+b \operatorname{Cos}[c+dx])^4} dx$$

Optimal (type 3, 345 leaves, 7 steps):

$$\frac{(7 a^2 A b^5 - 2 A b^7 - 2 a^7 B - 3 a^5 b^2 B - a^4 b^3 (8 A - C) + 4 a^6 b (2 A + C)) \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^4 (a-b)^{7/2} (a+b)^{7/2} d} +$$

$$\frac{A \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{a^4 d} + \frac{(A b^2 - a (b B - a C)) \operatorname{Sin}[c+dx]}{3 a (a^2 - b^2) d (a+b \operatorname{Cos}[c+dx])^3} - \frac{(3 A b^4 + 5 a^3 b B - 2 a^4 C - a^2 b^2 (8 A + 3 C)) \operatorname{Sin}[c+dx]}{6 a^2 (a^2 - b^2)^2 d (a+b \operatorname{Cos}[c+dx])^2} -$$

$$\frac{(17 a^2 A b^4 - 6 A b^6 + 11 a^5 b B + 4 a^3 b^3 B - 2 a^6 C - 13 a^4 b^2 (2 A + C)) \operatorname{Sin}[c+dx]}{6 a^3 (a^2 - b^2)^3 d (a+b \operatorname{Cos}[c+dx])}$$

Result (type 3, 1256 leaves) :

$$\begin{aligned}
& - \frac{2 A \cos [c+d x] \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right](B+C \cos [c+d x]+A \operatorname{Sec}[c+d x])}{a^4 d(2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x])} + \\
& \frac{2 A \cos [c+d x] \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right](B+C \cos [c+d x]+A \operatorname{Sec}[c+d x])}{a^4 d(2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x])} + \\
& \frac{1}{\left(a^2-b^2\right)^3(2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x])} \left(-8 a^6 A b+8 a^4 A b^3-7 a^2 A b^5+2 A b^7+2 a^7 B+3 a^5 b^2 B-4 a^6 b C-a^4 b^3 C\right) \cos [c+d x] \\
& (B+C \cos [c+d x]+A \operatorname{Sec}[c+d x]) \left(-\left(2 i \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{d x}{2}\right]\right]\left(\frac{\cos [c]}{\sqrt{-a^2 \cos [2 c]+b^2 \cos [2 c]+i a^2 \sin [2 c]-i b^2 \sin [2 c]}}-\right.\right.\right. \\
& \left.\left.\left.\frac{i \sin [c]}{\sqrt{-a^2 \cos [2 c]+b^2 \cos [2 c]+i a^2 \sin [2 c]-i b^2 \sin [2 c]}}\right)\left(-i a \sin \left[\frac{d x}{2}\right]+i b \sin \left[c+\frac{d x}{2}\right]\right)\right) \cos [c]\right) / \\
& \left(a^4 d \sqrt{-a^2 \cos [2 c]+b^2 \cos [2 c]+i a^2 \sin [2 c]-i b^2 \sin [2 c]}\right)-\left(2 \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{d x}{2}\right]\right]\right. \\
& \left.\left(\frac{\cos [c]}{\sqrt{-a^2 \cos [2 c]+b^2 \cos [2 c]+i a^2 \sin [2 c]-i b^2 \sin [2 c]}}-\frac{i \sin [c]}{\sqrt{-a^2 \cos [2 c]+b^2 \cos [2 c]+i a^2 \sin [2 c]-i b^2 \sin [2 c]}}\right)\right) \\
& \left(-i a \sin \left[\frac{d x}{2}\right]+i b \sin \left[c+\frac{d x}{2}\right]\right) \sin [c]\right) / \left(a^4 d \sqrt{-a^2 \cos [2 c]+b^2 \cos [2 c]+i a^2 \sin [2 c]-i b^2 \sin [2 c]}\right) \left. \right) - \\
& (2 \cos [c+d x] \operatorname{Sec}[c](B+C \cos [c+d x]+A \operatorname{Sec}[c+d x])\left(a A b^2 \sin [c]-a^2 b B \sin [c]+a^3 C \sin [c]-\right. \\
& \left. A b^3 \sin [d x]+a b^2 B \sin [d x]-a^2 b C \sin [d x]\right) / \\
& (3 a b\left(a^2-b^2\right) d(a+b \cos [c+d x])^3(2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]))+ \\
& (\cos [c+d x] \operatorname{Sec}[c](B+C \cos [c+d x]+A \operatorname{Sec}[c+d x])\left(-6 a^3 A b \sin [c]+a A b^3 \sin [c]+3 a^4 B \sin [c]+2 a^2 b^2 B \sin [c]-\right. \\
& \left. 5 a^3 b C \sin [c]+8 a^2 A b^2 \sin [d x]-3 A b^4 \sin [d x]-5 a^3 b B \sin [d x]+2 a^4 C \sin [d x]+3 a^2 b^2 C \sin [d x]\right) / \\
& (3 a^2\left(a^2-b^2\right)^2 d(a+b \cos [c+d x])^2(2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]))+ \\
& (\cos [c+d x] \operatorname{Sec}[c](B+C \cos [c+d x]+A \operatorname{Sec}[c+d x]) \\
& \left(-18 a^5 A b \sin [c]+6 a^3 A b^3 \sin [c]-3 a A b^5 \sin [c]+6 a^6 B \sin [c]+9 a^4 b^2 B \sin [c]-12 a^5 b C \sin [c]-3 a^3 b^3 C \sin [c]+ \right. \\
& \left. 26 a^4 A b^2 \sin [d x]-17 a^2 A b^4 \sin [d x]+6 A b^6 \sin [d x]-11 a^5 b B \sin [d x]-4 a^3 b^3 B \sin [d x]+2 a^6 C \sin [d x]+13 a^4 b^2 C \sin [d x]\right) / \\
& (3 a^3\left(a^2-b^2\right)^3 d(a+b \cos [c+d x])(2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]))
\end{aligned}$$

■ **Problem 1007: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^2}{(a + b \cos[c + dx])^4} dx$$

Optimal (type 3, 480 leaves, 8 steps):

$$\begin{aligned} & - \frac{1}{a^5 (a - b)^{7/2} (a + b)^{7/2} d} \\ & (35 a^4 A b^4 - 28 a^2 A b^6 + 8 A b^8 + 8 a^7 b B - 8 a^5 b^3 B + 7 a^3 b^5 B - 2 a b^7 B - 2 a^8 C - a^6 b^2 (20 A + 3 C)) \operatorname{ArcTan}\left[\frac{\sqrt{a - b} \operatorname{Tan}\left[\frac{1}{2} (c + dx)\right]}{\sqrt{a + b}}\right] - \\ & \frac{(4 A b - a B) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{a^5 d} + \frac{(68 a^2 A b^4 - 24 A b^6 + 26 a^5 b B - 17 a^3 b^3 B + 6 a b^5 B + a^6 (6 A - 11 C) - a^4 b^2 (65 A + 4 C)) \operatorname{Tan}[c + dx]}{6 a^4 (a^2 - b^2)^3 d} + \\ & \frac{(A b^2 - a (b B - a C)) \operatorname{Tan}[c + dx]}{3 a (a^2 - b^2) d (a + b \cos[c + dx])^3} - \frac{(4 A b^4 + 6 a^3 b B - a b^3 B - 3 a^4 C - a^2 b^2 (9 A + 2 C)) \operatorname{Tan}[c + dx]}{6 a^2 (a^2 - b^2)^2 d (a + b \cos[c + dx])^2} - \\ & \frac{(11 a^2 A b^4 - 4 A b^6 + 6 a^5 b B - 2 a^3 b^3 B + a b^5 B - 2 a^6 C - 3 a^4 b^2 (4 A + C)) \operatorname{Tan}[c + dx]}{2 a^3 (a^2 - b^2)^3 d (a + b \cos[c + dx])} \end{aligned}$$

Result (type 3, 1113 leaves):

$$\begin{aligned}
& - \left( 2 \left( 20 a^6 A b^2 - 35 a^4 A b^4 + 28 a^2 A b^6 - 8 A b^8 - 8 a^7 b B + 8 a^5 b^3 B - 7 a^3 b^5 B + 2 a b^7 B + 2 a^8 C + 3 a^6 b^2 C \right) \operatorname{ArcTanh} \left[ \frac{(a-b) \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]}{\sqrt{-a^2+b^2}} \right] \right. \\
& \quad \left. \operatorname{Cos}[c+dx]^2 (C+B \operatorname{Sec}[c+dx]+A \operatorname{Sec}[c+dx]^2) \right) / \left( a^5 (a^2-b^2)^3 \sqrt{-a^2+b^2} d (2A+C+2B \operatorname{Cos}[c+dx]+C \operatorname{Cos}[2c+2dx]) \right) - \\
& \left( 2 (-4Ab+ab) \operatorname{Cos}[c+dx]^2 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right] \right] (C+B \operatorname{Sec}[c+dx]+A \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (a^5 d (2A+C+2B \operatorname{Cos}[c+dx]+C \operatorname{Cos}[2c+2dx])) + \\
& \left( 2 (-4Ab+ab) \operatorname{Cos}[c+dx]^2 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c+dx) \right] \right] (C+B \operatorname{Sec}[c+dx]+A \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (a^5 d (2A+C+2B \operatorname{Cos}[c+dx]+C \operatorname{Cos}[2c+2dx])) + \\
& \left( \operatorname{Cos}[c+dx] (C+B \operatorname{Sec}[c+dx]+A \operatorname{Sec}[c+dx]^2) (48 a^9 A \operatorname{Sin}[c+dx] - 108 a^7 A b^2 \operatorname{Sin}[c+dx] - 174 a^5 A b^4 \operatorname{Sin}[c+dx] + \right. \\
& \quad 294 a^3 A b^6 \operatorname{Sin}[c+dx] - 120 a A b^8 \operatorname{Sin}[c+dx] + 120 a^6 b^3 B \operatorname{Sin}[c+dx] - 90 a^4 b^5 B \operatorname{Sin}[c+dx] + 30 a^2 b^7 B \operatorname{Sin}[c+dx] - \\
& \quad 54 a^7 b^2 C \operatorname{Sin}[c+dx] - 6 a^5 b^4 C \operatorname{Sin}[c+dx] + 72 a^8 A b \operatorname{Sin}[2(c+dx)] - 444 a^6 A b^3 \operatorname{Sin}[2(c+dx)] + 370 a^4 A b^5 \operatorname{Sin}[2(c+dx)] - \\
& \quad 40 a^2 A b^7 \operatorname{Sin}[2(c+dx)] - 48 A b^9 \operatorname{Sin}[2(c+dx)] + 144 a^7 b^2 B \operatorname{Sin}[2(c+dx)] - 76 a^5 b^4 B \operatorname{Sin}[2(c+dx)] + 10 a^3 b^6 B \operatorname{Sin}[2(c+dx)] + \\
& \quad 12 a b^8 B \operatorname{Sin}[2(c+dx)] - 72 a^8 b C \operatorname{Sin}[2(c+dx)] - 2 a^6 b^3 C \operatorname{Sin}[2(c+dx)] - 16 a^4 b^5 C \operatorname{Sin}[2(c+dx)] + 36 a^7 A b^2 \operatorname{Sin}[3(c+dx)] - \\
& \quad 318 a^5 A b^4 \operatorname{Sin}[3(c+dx)] + 342 a^3 A b^6 \operatorname{Sin}[3(c+dx)] - 120 a A b^8 \operatorname{Sin}[3(c+dx)] + 120 a^6 b^3 B \operatorname{Sin}[3(c+dx)] - \\
& \quad 90 a^4 b^5 B \operatorname{Sin}[3(c+dx)] + 30 a^2 b^7 B \operatorname{Sin}[3(c+dx)] - 54 a^7 b^2 C \operatorname{Sin}[3(c+dx)] - 6 a^5 b^4 C \operatorname{Sin}[3(c+dx)] + \\
& \quad 6 a^6 A b^3 \operatorname{Sin}[4(c+dx)] - 65 a^4 A b^5 \operatorname{Sin}[4(c+dx)] + 68 a^2 A b^7 \operatorname{Sin}[4(c+dx)] - 24 A b^9 \operatorname{Sin}[4(c+dx)] + 26 a^5 b^4 B \operatorname{Sin}[4(c+dx)] - \\
& \quad \left. 17 a^3 b^6 B \operatorname{Sin}[4(c+dx)] + 6 a b^8 B \operatorname{Sin}[4(c+dx)] - 11 a^6 b^3 C \operatorname{Sin}[4(c+dx)] - 4 a^4 b^5 C \operatorname{Sin}[4(c+dx)] \right) / \\
& \quad (24 a^4 (a^2-b^2)^3 d (a+b \operatorname{Cos}[c+dx])^3 (2A+C+2B \operatorname{Cos}[c+dx]+C \operatorname{Cos}[2c+2dx]))
\end{aligned}$$

■ **Problem 1017: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+b \operatorname{Cos}[c+dx]} (A+B \operatorname{Cos}[c+dx]+C \operatorname{Cos}[c+dx]^2) \operatorname{Sec}[c+dx] dx$$

Optimal (type 4, 240 leaves, 9 steps):

$$\begin{aligned}
& \frac{2(3bB+aC) \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE} \left[ \frac{1}{2} (c+dx), \frac{2b}{a+b} \right]}{3bd \sqrt{\frac{a+b \operatorname{Cos}[c+dx]}{a+b}}} + \frac{2(3Ab^2-(a^2-b^2)C) \sqrt{\frac{a+b \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF} \left[ \frac{1}{2} (c+dx), \frac{2b}{a+b} \right]}{3bd \sqrt{a+b \operatorname{Cos}[c+dx]}} + \\
& \frac{2aA \sqrt{\frac{a+b \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi} \left[ 2, \frac{1}{2} (c+dx), \frac{2b}{a+b} \right]}{d \sqrt{a+b \operatorname{Cos}[c+dx]}} + \frac{2C \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{3d}
\end{aligned}$$

Result (type 4, 508 leaves):

$$\begin{aligned}
& \frac{2 C \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{3 d} + \\
& \frac{1}{6 d} \left( \frac{2(6 A b+6 a B+2 b C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{2(6 a A+3 b B+a C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} \right) - \\
& \left( 2 i(3 b B+a C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left( 2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} \left( 2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2 \right) \right) \right)
\end{aligned}$$

■ **Problem 1018: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+b \cos [c+d x]} (A+B \cos [c+d x]+C \cos [c+d x]^2) \operatorname{Sec}[c+d x]^2 dx$$

Optimal (type 4, 217 leaves, 9 steps):

$$\begin{aligned}
& -\frac{(A-2 C) \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{d \sqrt{\frac{a+b \cos [c+d x]}{a+b}}} + \frac{(a A+2 b B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{d \sqrt{a+b \cos [c+d x]}} + \\
& \frac{(A b+2 a B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{d \sqrt{a+b \cos [c+d x]}} + \frac{A \sqrt{a+b \cos [c+d x]} \tan [c+d x]}{d}
\end{aligned}$$

Result (type 4, 385 leaves):

$$\frac{1}{4d} \left( \frac{8(bB + aC) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \right.$$

$$\frac{2(Ab + 4aB + 2bC) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} - \frac{1}{ab\sqrt{-\frac{1}{a+b}}} 2i(A-2C) \sqrt{-\frac{b(-1+\cos[c+dx])}{a+b}}$$

$$\sqrt{\frac{b(1+\cos[c+dx])}{-a+b}} \operatorname{Csc}[c+dx] \left( -2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + \right.$$

$$b \left( -2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + \right.$$

$$\left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] \right) \right) + 4A \sqrt{a+b\cos[c+dx]} \operatorname{Tan}[c+dx] \right)$$

■ **Problem 1019: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+b\cos[c+dx]} (A+B\cos[c+dx] + C\cos[c+dx]^2) \operatorname{Sec}[c+dx]^3 dx$$

Optimal (type 4, 299 leaves, 10 steps):

$$\frac{(Ab + 4aB) \sqrt{a+b\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right] + (3Ab + 4aB + 8bC) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{4ad\sqrt{\frac{a+b\cos[c+dx]}{a+b}} + 4d\sqrt{a+b\cos[c+dx]}}$$

$$+ \frac{(Ab^2 - 4abB - 4a^2(A+2C)) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{4ad\sqrt{a+b\cos[c+dx]}}$$

$$+ \frac{(Ab + 4aB) \sqrt{a+b\cos[c+dx]} \operatorname{Tan}[c+dx]}{4ad} + \frac{A\sqrt{a+b\cos[c+dx]} \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{2d}$$



Result (type 4, 566 leaves) :

$$\frac{1}{16 a d} \left( \frac{2 (4 a A b + 16 a b C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right.$$

$$\frac{2 (8 a^2 A - 3 A b^2 + 4 a b B + 16 a^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} -$$

$$\left( 2 i (-A b^2 - 4 a b B) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2 (c+d x)] \right.$$

$$\left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right.$$

$$\left. \left. \sqrt{-\frac{a^2-b^2-2 a (a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a (a+b \cos [c+d x])+2 (a+b \cos [c+d x])^2) \right) \right) +$$

$$\frac{\sqrt{a+b \cos [c+d x]} \left( \frac{\sec [c+d x] (A b \sin [c+d x]+4 a B \sin [c+d x])}{4 a} + \frac{1}{2} A \sec [c+d x] \tan [c+d x] \right)}{d}$$

■ **Problem 1020: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+b \cos [c+d x]} (A+B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]^4 dx$$

Optimal (type 4, 399 leaves, 11 steps) :

$$\begin{aligned}
& \frac{(3 A b^2 - 6 a b B - 8 a^2 (2 A + 3 C)) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{24 a^2 d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} - \\
& \frac{(A b^2 - 18 a b B - 8 a^2 (2 A + 3 C)) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{24 a d \sqrt{a + b \cos [c + d x]}} + \\
& \frac{(A b^3 + 8 a^3 B - 2 a b^2 B + 4 a^2 b (A + 2 C)) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{8 a^2 d \sqrt{a + b \cos [c + d x]}} - \\
& \frac{(3 A b^2 - 6 a b B - 8 a^2 (2 A + 3 C)) \sqrt{a + b \cos [c + d x]} \tan [c + d x]}{24 a^2 d} + \\
& \frac{(A b + 6 a B) \sqrt{a + b \cos [c + d x]} \sec [c + d x] \tan [c + d x]}{12 a d} + \frac{A \sqrt{a + b \cos [c + d x]} \sec [c + d x]^2 \tan [c + d x]}{3 d}
\end{aligned}$$

Result (type 4, 661 leaves):

$$\begin{aligned}
& \frac{1}{96 a^2 d} \left( \frac{2 (4 a A b^2 + 24 a^2 b B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2 (8 a^2 A b + 9 A b^3 + 48 a^3 B - 18 a b^2 B + 24 a^2 b C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\
& \left. \frac{2 i (-16 a^2 A b + 3 A b^3 - 6 a b^2 B - 24 a^2 b C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)]}{\sqrt{a+b \cos [c+d x]}} \right) \\
& \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \Big/ \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \right. \\
& \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{\sec [c+d x]^2 (A b \sin [c+d x]+6 a B \sin [c+d x])}{12 a} + \right. \\
& \left. \frac{\sec [c+d x] (16 a^2 A \sin [c+d x]-3 A b^2 \sin [c+d x]+6 a b B \sin [c+d x]+24 a^2 C \sin [c+d x])}{24 a^2} + \frac{1}{3} A \sec [c+d x]^2 \tan [c+d x] \right)
\end{aligned}$$

■ **Problem 1024: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos [c+d x])^{3/2} (A+B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x] dx$$

Optimal (type 4, 306 leaves, 10 steps):

$$\frac{2 (15 A b^2 + 20 a b B + 3 a^2 C + 9 b^2 C) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{15 b d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}}$$

$$\frac{2 (5 a^2 b B - 5 b^3 B + 3 a^3 C - 3 a b^2 (5 A + C)) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{15 b d \sqrt{a + b \cos [c + d x]}} +$$

$$\frac{2 a^2 A \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{d \sqrt{a + b \cos [c + d x]}} + \frac{2 (5 b B + 3 a C) \sqrt{a + b \cos [c + d x]} \sin [c + d x]}{15 d} + \frac{2 C (a + b \cos [c + d x])^{3/2} \sin [c + d x]}{5 d}$$

Result (type 4, 455 leaves):

$$\frac{1}{30 d} \left( \frac{4 (15 a^2 B + 5 b^2 B + 6 a b (5 A + 2 C)) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{\sqrt{a + b \cos [c + d x]}} + \right.$$

$$\frac{2 (20 a b B + 3 a^2 (10 A + C) + 3 b^2 (5 A + 3 C)) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{\sqrt{a + b \cos [c + d x]}} +$$

$$\frac{1}{a b^2 \sqrt{-\frac{1}{a + b}}} 2 i (15 A b^2 + 20 a b B + 3 a^2 C + 9 b^2 C) \sqrt{-\frac{b (-1 + \cos [c + d x])}{a + b}} \sqrt{\frac{b (1 + \cos [c + d x])}{-a + b}}$$

$$\operatorname{Csc}[c + d x] \left( -2 a (a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos [c + d x]}\right], \frac{a + b}{a - b}\right] + b \left( -2 a \operatorname{EllipticF}\left[\right.\right.$$

$$\left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos [c + d x]}\right], \frac{a + b}{a - b}\right] + b \operatorname{EllipticPi}\left[\frac{a + b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos [c + d x]}\right], \frac{a + b}{a - b}\right]\right) \right) +$$

$$\left. 4 \sqrt{a + b \cos [c + d x]} (5 b B + 6 a C + 3 b C \cos [c + d x]) \sin [c + d x] \right)$$

■ **Problem 1025: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + b \cos [c + d x])^{3/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^2 dx$$

Optimal (type 4, 286 leaves, 10 steps):

$$\begin{aligned} & - \frac{(3 a A - 6 b B - 8 a C) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{3 d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} + \\ & \frac{(6 a b B + a^2 (3 A - 2 C) + 2 b^2 (3 A + C)) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{3 d \sqrt{a + b \cos [c + d x]}} + \\ & \frac{a (3 A b + 2 a B) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2 b}{a + b}\right]}{d \sqrt{a + b \cos [c + d x]}} - \\ & \frac{b (3 A - 2 C) \sqrt{a + b \cos [c + d x]} \sin [c + d x]}{3 d} + \frac{A (a + b \cos [c + d x])^{3/2} \tan [c + d x]}{d} \end{aligned}$$

Result (type 4, 551 leaves):

$$\begin{aligned}
& \frac{1}{12d} \left( \frac{2(12Ab^2 + 24abB + 12a^2C + 4b^2C) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \right. \\
& \frac{2(15aAb + 12a^2B + 6b^2B + 8abC) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} - \\
& \left. \frac{2i(-3aAb + 6b^2B + 8abC) \sqrt{\frac{b-b\cos[c+dx]}{a+b}} \sqrt{\frac{b+b\cos[c+dx]}{a-b}} \cos[2(c+dx)]}{\sqrt{a+b\cos[c+dx]}} \right) \\
& \left( 2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin[c+dx] \Big/ \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]^2} \right. \\
& \left. \left. \sqrt{-\frac{a^2-b^2-2a(a+b\cos[c+dx])+(a+b\cos[c+dx])^2}{b^2}} (2a^2-b^2-4a(a+b\cos[c+dx])+2(a+b\cos[c+dx])^2) \right) \right) + \\
& \frac{\sqrt{a+b\cos[c+dx]} \left( \frac{2}{3}bC \sin[c+dx] + aA \tan[c+dx] \right)}{d}
\end{aligned}$$

■ **Problem 1026: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b\cos[c+dx])^{3/2} (A+B\cos[c+dx]+C\cos[c+dx]^2) \sec[c+dx]^3 dx$$

Optimal (type 4, 307 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(5Ab + 4aB - 8bC) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \\
& \frac{(4a^2B + 8b^2B + ab(7A + 8C)) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4d \sqrt{a + b \cos[c + dx]}} + \\
& \frac{(3Ab^2 + 12abB + 4a^2(A + 2C)) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4d \sqrt{a + b \cos[c + dx]}} + \\
& \frac{(3Ab + 4aB) \sqrt{a + b \cos[c + dx]} \tan[c + dx]}{4d} + \frac{A(a + b \cos[c + dx])^{3/2} \sec[c + dx] \tan[c + dx]}{2d}
\end{aligned}$$

Result (type 4, 579 leaves):

$$\begin{aligned}
& \frac{1}{16d} \left( \frac{2(4aAb + 16b^2B + 32abc) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \right. \\
& \frac{2(8a^2A + Ab^2 + 20abB + 16a^2C + 8b^2C) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} - \\
& \left. \frac{2i(-5Ab^2 - 4abB + 8b^2C) \sqrt{\frac{b-b\cos[c+dx]}{a+b}} \sqrt{\frac{b+b\cos[c+dx]}{a-b}} \cos[2(c+dx)]}{\sqrt{a+b\cos[c+dx]}} \right) \\
& \left( 2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] \right. \right. \\
& \left. \left. - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin[c+dx] \Big/ \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]^2} \right. \\
& \left. \left. \sqrt{-\frac{a^2-b^2-2a(a+b\cos[c+dx])+(a+b\cos[c+dx])^2}{b^2}} (2a^2-b^2-4a(a+b\cos[c+dx])+2(a+b\cos[c+dx])^2) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b\cos[c+dx]} \left( \frac{1}{4} \sec[c+dx] (5Ab\sin[c+dx] + 4aB\sin[c+dx]) + \frac{1}{2} aA \sec[c+dx] \tan[c+dx] \right)
\end{aligned}$$

■ **Problem 1027: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b\cos[c+dx])^{3/2} (A+B\cos[c+dx] + C\cos[c+dx]^2) \sec[c+dx]^4 dx$$

Optimal (type 4, 399 leaves, 11 steps):



$$\begin{aligned}
& - \frac{(3Ab^2 + 30abB + 8a^2(2A + 3C))\sqrt{a + b\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{24ad\sqrt{\frac{a+b\cos[c+dx]}{a+b}}} + \\
& \frac{(42abB + 8a^2(2A + 3C) + b^2(17A + 48C))\sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{24d\sqrt{a + b\cos[c + dx]}} - \\
& \frac{(Ab^3 - 8a^3B - 6ab^2B - 12a^2b(A + 2C))\sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{8ad\sqrt{a + b\cos[c + dx]}} + \\
& \frac{(3Ab^2 + 30abB + 8a^2(2A + 3C))\sqrt{a + b\cos[c + dx]} \operatorname{Tan}[c + dx]}{24ad} + \\
& \frac{(Ab + 2aB)\sqrt{a + b\cos[c + dx]} \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{4d} + \frac{A(a + b\cos[c + dx])^{3/2} \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{3d}
\end{aligned}$$

Result (type 4, 667 leaves):

$$\begin{aligned}
& \frac{1}{96 a d} \left( \frac{2 (28 a A b^2 + 24 a^2 b B + 96 a b^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2 (56 a^2 A b - 9 A b^3 + 48 a^3 B + 6 a b^2 B + 120 a^2 b C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\
& \left( 2 i (-16 a^2 A b - 3 A b^3 - 30 a b^2 B - 24 a^2 b C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2 (c+d x)] \right. \\
& \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a (a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a (a+b \cos [c+d x])+2 (a+b \cos [c+d x])^2) \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{1}{12} \sec [c+d x]^2 (7 A b \sin [c+d x]+6 a B \sin [c+d x]) + \right. \\
& \left. \frac{\sec [c+d x] (16 a^2 A \sin [c+d x]+3 A b^2 \sin [c+d x]+30 a b B \sin [c+d x]+24 a^2 C \sin [c+d x])}{24 a} + \frac{1}{3} a A \sec [c+d x]^2 \tan [c+d x] \right)
\end{aligned}$$

■ **Problem 1028: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos [c+d x])^{3/2} (A+B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]^5 dx$$

Optimal (type 4, 503 leaves, 12 steps):

$$\begin{aligned}
& \frac{(9 A b^3 - 128 a^3 B - 24 a b^2 B - 12 a^2 b (13 A + 20 C)) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{192 a^2 d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}} - \\
& \frac{(3 A b^3 - 128 a^3 B - 136 a b^2 B - 12 a^2 b (19 A + 28 C)) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{192 a d \sqrt{a + b \cos[c + d x]}} + \\
& \left( (3 A b^4 + 96 a^3 b B - 8 a b^3 B + 24 a^2 b^2 (A + 2 C) + 16 a^4 (3 A + 4 C)) \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right] \right) / \\
& \left( 64 a^2 d \sqrt{a + b \cos[c + d x]} \right) - \frac{(9 A b^3 - 128 a^3 B - 24 a b^2 B - 12 a^2 b (13 A + 20 C)) \sqrt{a + b \cos[c + d x]} \operatorname{Tan}[c + d x]}{192 a^2 d} + \\
& \frac{(3 A b^2 + 56 a b B + 12 a^2 (3 A + 4 C)) \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{96 a d} + \\
& \frac{(3 A b + 8 a B) \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{24 d} + \frac{A (a + b \cos[c + d x])^{3/2} \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d}
\end{aligned}$$

Result (type 4, 783 leaves):

$$\begin{aligned}
& \frac{1}{768 a^2 d} \left( \frac{2 (144 a^3 A b + 12 a A b^3 + 224 a^2 b^2 B + 192 a^3 b C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{1}{\sqrt{a+b \cos [c+d x]}} \right. \\
& 2 (288 a^4 A - 12 a^2 A b^2 + 27 A b^4 + 448 a^3 b B - 72 a b^3 B + 384 a^4 C + 48 a^2 b^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] - \\
& \left. \left( 2 i (-156 a^2 A b^2 + 9 A b^4 - 128 a^3 b B - 24 a b^3 B - 240 a^2 b^2 C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \right. \\
& \left. \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right)\right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{1}{24} \sec [c+d x]^3 (9 A b \sin [c+d x]+8 a B \sin [c+d x]) + \right. \\
& \left. \frac{\sec [c+d x]^2 (36 a^2 A \sin [c+d x]+3 A b^2 \sin [c+d x]+56 a b B \sin [c+d x]+48 a^2 C \sin [c+d x])}{96 a} + \right. \\
& \left. 1 / (192 a^2) \sec [c+d x] (156 a^2 A b \sin [c+d x]-9 A b^3 \sin [c+d x]+128 a^3 B \sin [c+d x]+24 a b^2 B \sin [c+d x]+240 a^2 b C \sin [c+d x]) + \right. \\
& \left. \frac{1}{4} a A \sec [c+d x]^3 \tan [c+d x] \right)
\end{aligned}$$

■ **Problem 1032: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos [c+d x])^{5/2} (A+B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x] dx$$

Optimal (type 4, 383 leaves, 11 steps):

$$\begin{aligned}
& \frac{2 \left( 161 a^2 b B + 63 b^3 B + 15 a^3 C + 5 a b^2 (49 A + 29 C) \right) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), \frac{2 b}{a + b} \right]}{105 b d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} \\
& \left( 2 \left( 56 a^3 b B - 56 a b^3 B - 10 a^2 b^2 (7 A - C) + 15 a^4 C - 5 b^4 (7 A + 5 C) \right) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), \frac{2 b}{a + b} \right] \right) / \\
& \left( 105 b d \sqrt{a + b \cos [c + d x]} \right) + \frac{2 a^3 A \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi} \left[ 2, \frac{1}{2} (c + d x), \frac{2 b}{a + b} \right]}{d \sqrt{a + b \cos [c + d x]}} + \\
& \frac{2 \left( 35 A b^2 + 56 a b B + 15 a^2 C + 25 b^2 C \right) \sqrt{a + b \cos [c + d x]} \sin [c + d x]}{105 d} + \\
& \frac{2 \left( 7 b B + 5 a C \right) (a + b \cos [c + d x])^{3/2} \sin [c + d x]}{35 d} + \frac{2 C (a + b \cos [c + d x])^{5/2} \sin [c + d x]}{7 d}
\end{aligned}$$

Result (type 4, 652 leaves):

$$\begin{aligned}
& \frac{1}{210 d} \left( \frac{1}{\sqrt{a+b \cos [c+d x]}} 2 \left( 630 a^2 A b + 70 A b^3 + 210 a^3 B + 238 a b^2 B + 270 a^2 b C + 50 b^3 C \right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] + \right. \\
& \frac{1}{\sqrt{a+b \cos [c+d x]}} 2 \left( 210 a^3 A + 245 a A b^2 + 161 a^2 b B + 63 b^3 B + 15 a^3 C + 145 a b^2 C \right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] - \\
& \left. \left( 2 i \left( 245 a A b^2 + 161 a^2 b B + 63 b^3 B + 15 a^3 C + 145 a b^2 C \right) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \right. \\
& \left. \left( 2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} \left( 2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2 \right) \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{1}{210} \left( 140 A b^2 + 308 a b B + 180 a^2 C + 115 b^2 C \right) \sin [c+d x] + \frac{1}{35} b \left( 7 b B + 15 a C \right) \sin [2(c+d x)] + \right. \\
& \left. \frac{1}{14} b^2 C \sin [3(c+d x)] \right)
\end{aligned}$$

■ **Problem 1033: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos [c+d x])^{5/2} (A+B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]^2 dx$$

Optimal (type 4, 357 leaves, 11 steps):

$$\begin{aligned}
& \frac{(70 a b B - a^2 (15 A - 46 C) + 6 b^2 (5 A + 3 C)) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{15 d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}} + \\
& \frac{(20 a^2 b B + 10 b^3 B + a^3 (15 A - 16 C) + 4 a b^2 (15 A + 4 C)) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{15 d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{a^2 (5 A b + 2 a B) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{d \sqrt{a + b \cos[c + d x]}} - \frac{b (15 a A - 10 b B - 16 a C) \sqrt{a + b \cos[c + d x]} \sin[c + d x]}{15 d} - \\
& \frac{b (5 A - 2 C) (a + b \cos[c + d x])^{3/2} \sin[c + d x]}{5 d} + \frac{A (a + b \cos[c + d x])^{5/2} \tan[c + d x]}{d}
\end{aligned}$$

Result (type 4, 621 leaves):

$$\begin{aligned}
& \frac{1}{60d} \left( \frac{2(180aAb^2 + 180a^2bB + 20b^3B + 60a^3C + 68ab^2C) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right]}{\sqrt{a+b\cos[c+dx]}} + \frac{1}{\sqrt{a+b\cos[c+dx]}} \right. \\
& 2(135a^2Ab + 30Ab^3 + 60a^3B + 70ab^2B + 46a^2bC + 18b^3C) \sqrt{\frac{a+b\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right] - \\
& \left. \left( 2i(-15a^2Ab + 30Ab^3 + 70ab^2B + 46a^2bC + 18b^3C) \sqrt{\frac{b-b\cos[c+dx]}{a+b}} \sqrt{\frac{b+b\cos[c+dx]}{a-b}} \cos[2(c+dx)] \right. \right. \\
& \left. \left( 2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin[c+dx] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]} \right)^2 \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2a(a+b\cos[c+dx])+(a+b\cos[c+dx])^2}{b^2}} (2a^2-b^2-4a(a+b\cos[c+dx])+2(a+b\cos[c+dx])^2) \right) \right) \right) + \\
& \frac{\sqrt{a+b\cos[c+dx]} \left( \frac{2}{15}b(5bB+11aC) \sin[c+dx] + \frac{1}{5}b^2C \sin[2(c+dx)] + a^2A \tan[c+dx] \right)}{d}
\end{aligned}$$

■ **Problem 1034: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b\cos[c+dx])^{5/2} (A+B\cos[c+dx]+C\cos[c+dx]^2) \sec[c+dx]^3 dx$$

Optimal (type 4, 372 leaves, 11 steps):



$$\begin{aligned}
& - \frac{(12 a^2 B - 24 b^2 B + a b (27 A - 56 C)) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{12 d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}} + \\
& \frac{(12 a^3 B + 48 a b^2 B + 8 b^3 (3 A + C) + a^2 b (33 A + 16 C)) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{12 d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{a (15 A b^2 + 20 a b B + 4 a^2 (A + 2 C)) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{4 d \sqrt{a + b \cos[c + d x]}} - \frac{b (21 A b + 12 a B - 8 b C) \sqrt{a + b \cos[c + d x]} \sin[c + d x]}{12 d} + \\
& \frac{(5 A b + 4 a B) (a + b \cos[c + d x])^{3/2} \tan[c + d x]}{4 d} + \frac{A (a + b \cos[c + d x])^{5/2} \sec[c + d x] \tan[c + d x]}{2 d}
\end{aligned}$$

Result (type 4, 636 leaves):

$$\begin{aligned}
& \frac{1}{48 d} \left( \frac{2 (12 a^2 A b + 48 A b^3 + 144 a b^2 B + 144 a^2 b C + 16 b^3 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{1}{\sqrt{a+b \cos [c+d x]}} \right. \\
& 2 (24 a^3 A + 63 a A b^2 + 108 a^2 b B + 24 b^3 B + 48 a^3 C + 56 a b^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] - \\
& \left. \left( 2 i (-27 a A b^2 - 12 a^2 b B + 24 b^3 B + 56 a b^2 C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \right. \\
& \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]^2} \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) \right) + \\
& \left. \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{2}{3} b^2 C \sin [c+d x] + \frac{1}{4} \sec [c+d x] (9 a A b \sin [c+d x] + 4 a^2 B \sin [c+d x]) + \frac{1}{2} a^2 A \sec [c+d x] \tan [c+d x] \right) \right)
\end{aligned}$$

■ **Problem 1035: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos [c+d x])^{5/2} (A+B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]^4 dx$$

Optimal (type 4, 407 leaves, 11 steps):

$$\begin{aligned}
& - \frac{(54 a b B + 3 b^2 (11 A - 16 C) + 8 a^2 (2 A + 3 C)) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{24 d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}} + \\
& \frac{(66 a^2 b B + 48 b^3 B + 8 a^3 (2 A + 3 C) + a b^2 (59 A + 96 C)) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{24 d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{(5 A b^3 + 8 a^3 B + 30 a b^2 B + 20 a^2 b (A + 2 C)) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{8 d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{(15 A b^2 + 42 a b B + 8 a^2 (2 A + 3 C)) \sqrt{a + b \cos[c + d x]} \operatorname{Tan}[c + d x]}{24 d} + \\
& \frac{(5 A b + 6 a B) (a + b \cos[c + d x])^{3/2} \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{12 d} + \frac{A (a + b \cos[c + d x])^{5/2} \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d}
\end{aligned}$$

Result (type 4, 684 leaves):

$$\begin{aligned}
& \frac{1}{96 d} \left( \frac{2 (52 a A b^2 + 24 a^2 b B + 96 b^3 B + 288 a b^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{1}{\sqrt{a+b \cos [c+d x]}} \right. \\
& 2 (104 a^2 A b - 3 A b^3 + 48 a^3 B + 126 a b^2 B + 216 a^2 b C + 48 b^3 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] - \\
& \left. \left( 2 i (-16 a^2 A b - 33 A b^3 - 54 a b^2 B - 24 a^2 b C + 48 b^3 C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \right. \\
& \left. \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right)\right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{1}{12} \sec [c+d x]^2 (13 a A b \sin [c+d x] + 6 a^2 B \sin [c+d x]) + \right. \\
& \left. \frac{1}{24} \sec [c+d x] (16 a^2 A \sin [c+d x] + 33 A b^2 \sin [c+d x] + 54 a b B \sin [c+d x] + 24 a^2 C \sin [c+d x]) + \frac{1}{3} a^2 A \sec [c+d x]^2 \tan [c+d x] \right)
\end{aligned}$$

■ **Problem 1036: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \cos [c+d x])^{5/2} (A+B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]^5 dx$$

Optimal (type 4, 502 leaves, 12 steps):

$$\begin{aligned}
& - \frac{(15 A b^3 + 128 a^3 B + 264 a b^2 B + 4 a^2 b (71 A + 108 C)) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{192 a d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}} + \frac{1}{192 d \sqrt{a + b \cos[c + d x]}} \\
& (128 a^3 B + 472 a b^2 B + 4 a^2 b (89 A + 132 C) + b^3 (133 A + 384 C)) \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right] - \frac{1}{64 a d \sqrt{a + b \cos[c + d x]}} \\
& (5 A b^4 - 160 a^3 b B - 40 a b^3 B - 120 a^2 b^2 (A + 2 C) - 16 a^4 (3 A + 4 C)) \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right] + \\
& \frac{(15 A b^3 + 128 a^3 B + 264 a b^2 B + 4 a^2 b (71 A + 108 C)) \sqrt{a + b \cos[c + d x]} \operatorname{Tan}[c + d x]}{192 a d} + \\
& \frac{(5 A b^2 + 24 a b B + 4 a^2 (3 A + 4 C)) \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{32 d} + \\
& \frac{(5 A b + 8 a B) (a + b \cos[c + d x])^{3/2} \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{24 d} + \frac{A (a + b \cos[c + d x])^{5/2} \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d}
\end{aligned}$$

Result (type 4, 792 leaves):

$$\begin{aligned}
& \frac{1}{768 a d} \left( \frac{1}{\sqrt{a + b \cos [c + d x]}} \right. \\
& 2 (144 a^3 A b + 236 a A b^3 + 416 a^2 b^2 B + 192 a^3 b C + 768 a b^3 C) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2 b}{a + b}\right] + \frac{1}{\sqrt{a + b \cos [c + d x]}} \\
& 2 (288 a^4 A + 436 a^2 A b^2 - 45 A b^4 + 832 a^3 b B - 24 a b^3 B + 384 a^4 C + 1008 a^2 b^2 C) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2 b}{a + b}\right] - \\
& \left( 2 i (-284 a^2 A b^2 - 15 A b^4 - 128 a^3 b B - 264 a b^3 B - 432 a^2 b^2 C) \sqrt{\frac{b - b \cos [c + d x]}{a + b}} \sqrt{-\frac{b + b \cos [c + d x]}{a - b}} \cos [2(c + d x)] \right. \\
& \left. \left( 2 a (a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos [c + d x]}\right], \frac{a + b}{a - b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos [c + d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a + b}{a - b}\right] - b \operatorname{EllipticPi}\left[\frac{a + b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a + b}} \sqrt{a + b \cos [c + d x]}\right], \frac{a + b}{a - b}\right]\right) \right) \sin [c + d x] \right) / \left( a \sqrt{-\frac{1}{a + b}} \sqrt{1 - \cos [c + d x]^2} \right. \\
& \left. \left. \sqrt{-\frac{a^2 - b^2 - 2 a (a + b \cos [c + d x]) + (a + b \cos [c + d x])^2}{b^2}} (2 a^2 - b^2 - 4 a (a + b \cos [c + d x]) + 2 (a + b \cos [c + d x])^2) \right) \right) + \\
& \frac{1}{d} \sqrt{a + b \cos [c + d x]} \left( \frac{1}{24} \sec [c + d x]^3 (17 a A b \sin [c + d x] + 8 a^2 B \sin [c + d x]) + \right. \\
& \frac{1}{96} \sec [c + d x]^2 (36 a^2 A \sin [c + d x] + 59 A b^2 \sin [c + d x] + 104 a b B \sin [c + d x] + 48 a^2 C \sin [c + d x]) + \frac{1}{192 a} \\
& \sec [c + d x] (284 a^2 A b \sin [c + d x] + 15 A b^3 \sin [c + d x] + 128 a^3 B \sin [c + d x] + 264 a b^2 B \sin [c + d x] + 432 a^2 b C \sin [c + d x]) + \\
& \left. \frac{1}{4} a^2 A \sec [c + d x]^3 \tan [c + d x] \right)
\end{aligned}$$

■ **Problem 1037: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + b \cos [c + d x])^{5/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^6 dx$$

Optimal (type 4, 624 leaves, 13 steps):

$$\begin{aligned}
& \frac{1}{1920 a^2 d \sqrt{\frac{a+b \cos [c+d x]}{a+b}}} \\
& \left( 45 A b^4 - 2840 a^3 b B - 150 a b^3 B - 256 a^4 (4 A + 5 C) - 12 a^2 b^2 (141 A + 220 C) \right) \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] - \\
& \left( \left( 15 A b^4 - 3560 a^3 b B - 1330 a b^3 B - 256 a^4 (4 A + 5 C) - 4 a^2 b^2 (809 A + 1180 C) \right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] \right) / \\
& \left( 1920 a d \sqrt{a+b \cos [c+d x]} \right) + \\
& \left( \left( 3 A b^5 + 96 a^5 B + 240 a^3 b^2 B - 10 a b^4 B + 40 a^2 b^3 (A + 2 C) + 80 a^4 b (3 A + 4 C) \right) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] \right) / \\
& \left( 128 a^2 d \sqrt{a+b \cos [c+d x]} \right) - \frac{1}{1920 a^2 d} \\
& \left( 45 A b^4 - 2840 a^3 b B - 150 a b^3 B - 256 a^4 (4 A + 5 C) - 12 a^2 b^2 (141 A + 220 C) \right) \sqrt{a+b \cos [c+d x]} \operatorname{Tan}[c+d x] + \\
& \frac{\left( 15 A b^3 + 360 a^3 B + 590 a b^2 B + 4 a^2 b (193 A + 260 C) \right) \sqrt{a+b \cos [c+d x]} \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{960 a d} + \\
& \frac{\left( 15 A b^2 + 110 a b B + 16 a^2 (4 A + 5 C) \right) \sqrt{a+b \cos [c+d x]} \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{240 d} + \\
& \frac{(A b + 2 a B) (a+b \cos [c+d x])^{3/2} \operatorname{Sec}[c+d x]^3 \operatorname{Tan}[c+d x]}{8 d} + \frac{A (a+b \cos [c+d x])^{5/2} \operatorname{Sec}[c+d x]^4 \operatorname{Tan}[c+d x]}{5 d}
\end{aligned}$$

Result (type 4, 930 leaves):

$$\begin{aligned}
& \frac{1}{7680 a^2 d} \left( \frac{1}{\sqrt{a+b \cos[c+dx]}} 2 (3088 a^3 A b^2 + 60 a A b^4 + 1440 a^4 b B + 2360 a^2 b^3 B + 4160 a^3 b^2 C) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right] + \right. \\
& \frac{1}{\sqrt{a+b \cos[c+dx]}} 2 (6176 a^4 A b - 492 a^2 A b^3 + 135 A b^5 + 2880 a^5 B + 4360 a^3 b^2 B - 450 a b^4 B + 8320 a^4 b C - 240 a^2 b^3 C) \\
& \left. \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right] - \right. \\
& \left. \left( 2 i (-1024 a^4 A b - 1692 a^2 A b^3 + 45 A b^5 - 2840 a^3 b^2 B - 150 a b^4 B - 1280 a^4 b C - 2640 a^2 b^3 C) \sqrt{\frac{b-b \cos[c+dx]}{a+b}} \right. \right. \\
& \left. \sqrt{-\frac{b+b \cos[c+dx]}{a-b}} \cos[2(c+dx)] \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[ \right. \right. \right. \\
& \left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] \right) \right) \right) \\
& \left. \sin[c+dx] \right) \left/ \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]}^2 \sqrt{-\frac{a^2-b^2-2a(a+b \cos[c+dx])+(a+b \cos[c+dx])^2}{b^2}} \right. \right. \\
& \left. \left. (2 a^2 - b^2 - 4 a (a+b \cos[c+dx]) + 2 (a+b \cos[c+dx])^2) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos[c+dx]} \left( \frac{1}{40} \sec[c+dx]^4 (21 a A b \sin[c+dx] + 10 a^2 B \sin[c+dx]) + \right. \\
& \frac{1}{240} \sec[c+dx]^3 (64 a^2 A \sin[c+dx] + 93 A b^2 \sin[c+dx] + 170 a b B \sin[c+dx] + 80 a^2 C \sin[c+dx]) + \frac{1}{960 a} \\
& \sec[c+dx]^2 (772 a^2 A b \sin[c+dx] + 15 A b^3 \sin[c+dx] + 360 a^3 B \sin[c+dx] + 590 a b^2 B \sin[c+dx] + 1040 a^2 b C \sin[c+dx]) + \\
& \frac{1}{1920 a^2} \sec[c+dx] (1024 a^4 A \sin[c+dx] + 1692 a^2 A b^2 \sin[c+dx] - 45 A b^4 \sin[c+dx] + 2840 a^3 b B \sin[c+dx] + \\
& \left. 150 a b^3 B \sin[c+dx] + 1280 a^4 C \sin[c+dx] + 2640 a^2 b^2 C \sin[c+dx]) + \frac{1}{5} a^2 A \sec[c+dx]^4 \tan[c+dx] \right)
\end{aligned}$$



■ **Problem 1043: Unable to integrate problem.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]}{\sqrt{a + b \cos[c + dx]}} dx$$

Optimal (type 4, 189 leaves, 8 steps):

$$\frac{2C \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{bd \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \frac{2(bB - aC) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] + 2A \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{bd \sqrt{a + b \cos[c + dx]} + d \sqrt{a + b \cos[c + dx]}}$$

Result (type 8, 43 leaves):

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]}{\sqrt{a + b \cos[c + dx]}} dx$$

■ **Problem 1044: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^2}{\sqrt{a + b \cos[c + dx]}} dx$$

Optimal (type 4, 220 leaves, 9 steps):

$$-\frac{A \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{ad \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \frac{(A + 2C) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{d \sqrt{a + b \cos[c + dx]}} - \frac{(Ab - 2aB) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{ad \sqrt{a + b \cos[c + dx]}} + \frac{A \sqrt{a + b \cos[c + dx]} \tan[c + dx]}{ad}$$

Result (type 4, 600 leaves):

$$\begin{aligned}
& \frac{2 A \cos [c+d x] \sqrt{a+b \cos [c+d x]} (C+B \sec [c+d x]+A \sec [c+d x]^2) \sin [c+d x]}{a d (2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x])} + \\
& \frac{1}{2 a d (2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x])} \cos [c+d x]^2 (C+B \sec [c+d x]+A \sec [c+d x]^2) \\
& \left( \frac{8 a C \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{2(-3 A b+4 a B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \left. \left( 2 i A b \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \left( 2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + \right. \right. \right. \\
& \left. \left. b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] - \right. \right. \right. \\
& \left. \left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \right. \\
& \left. \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) \right)
\end{aligned}$$

■ **Problem 1045: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]^3}{\sqrt{a+b \cos [c+d x]}} dx$$

Optimal (type 4, 303 leaves, 10 steps):

$$\frac{(3Ab - 4aB) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right] - (Ab - 4aB) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4a^2 d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \frac{4ad \sqrt{a + b \cos[c + dx]}}{4a^2 d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} +$$

$$\frac{(3Ab^2 - 4abB + 4a^2(A + 2C)) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{4a^2 d \sqrt{a + b \cos[c + dx]}} -$$

$$\frac{(3Ab - 4aB) \sqrt{a + b \cos[c + dx]} \tan[c + dx]}{4a^2 d} + \frac{A \sqrt{a + b \cos[c + dx]} \operatorname{Sec}[c + dx] \tan[c + dx]}{2ad}$$

Result (type 4, 562 leaves):

$$\frac{1}{16a^2 d} \left( \frac{8aAb \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} + \frac{2(8a^2A + 9Ab^2 - 12abB + 16a^2C) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right]}{\sqrt{a + b \cos[c + dx]}} - \right.$$

$$\left. \left( 2i(3Ab^2 - 4abB) \sqrt{\frac{b - b \cos[c + dx]}{a + b}} \sqrt{-\frac{b + b \cos[c + dx]}{a - b}} \cos[2(c + dx)] \right) \right.$$

$$\left. \left( 2a(a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos[c + dx]}\right], \frac{a+b}{a-b}\right] \right) \right) \sin[c + dx] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1 - \cos[c + dx]^2} \right.$$

$$\left. \left. \sqrt{-\frac{a^2 - b^2 - 2a(a + b \cos[c + dx]) + (a + b \cos[c + dx])^2}{b^2}} (2a^2 - b^2 - 4a(a + b \cos[c + dx]) + 2(a + b \cos[c + dx])^2) \right) \right) +$$

$$\frac{\sqrt{a + b \cos[c + dx]} \left( \frac{\operatorname{Sec}[c + dx] (-3Ab \sin[c + dx] + 4aB \sin[c + dx])}{4a^2} + \frac{A \operatorname{Sec}[c + dx] \tan[c + dx]}{2a} \right)}{d}$$

■ **Problem 1046: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^4}{\sqrt{a + b \cos[c + dx]}} dx$$

Optimal (type 4, 405 leaves, 11 steps):

$$\begin{aligned} & - \frac{(15 A b^2 - 18 a b B + 8 a^2 (2 A + 3 C)) \sqrt{a + b \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2} (c + dx), \frac{2b}{a+b}\right]}{24 a^3 d \sqrt{\frac{a+b \cos[c+dx]}{a+b}}} + \\ & - \frac{(5 A b^2 - 6 a b B + 8 a^2 (2 A + 3 C)) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), \frac{2b}{a+b}\right]}{24 a^2 d \sqrt{a + b \cos[c + dx]}} - \\ & + \frac{(5 A b^3 - 8 a^3 B - 6 a b^2 B + 4 a^2 b (A + 2 C)) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + dx), \frac{2b}{a+b}\right]}{8 a^3 d \sqrt{a + b \cos[c + dx]}} + \\ & - \frac{(15 A b^2 - 18 a b B + 8 a^2 (2 A + 3 C)) \sqrt{a + b \cos[c + dx]} \operatorname{Tan}[c + dx]}{24 a^3 d} - \\ & + \frac{(5 A b - 6 a B) \sqrt{a + b \cos[c + dx]} \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{12 a^2 d} + \frac{A \sqrt{a + b \cos[c + dx]} \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{3 a d} \end{aligned}$$

Result (type 4, 665 leaves):

$$\begin{aligned}
& \frac{1}{96 a^3 d} \left( \frac{2 (-20 a A b^2 + 24 a^2 b B) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \right. \\
& \frac{2 (-40 a^2 A b - 45 A b^3 + 48 a^3 B + 54 a b^2 B - 72 a^2 b C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} - \\
& \left( 2 i (-16 a^2 A b - 15 A b^3 + 18 a b^2 B - 24 a^2 b C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right)\right) \sin [c+d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \right. \\
& \left. \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2) \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \cos [c+d x]} \left( \frac{\sec [c+d x]^2 (-5 A b \sin [c+d x]+6 a B \sin [c+d x])}{12 a^2} + \right. \\
& \left. \frac{\sec [c+d x] (16 a^2 A \sin [c+d x]+15 A b^2 \sin [c+d x]-18 a b B \sin [c+d x]+24 a^2 C \sin [c+d x])}{24 a^3} + \frac{A \sec [c+d x]^2 \tan [c+d x]}{3 a} \right)
\end{aligned}$$

■ **Problem 1050: Unable to integrate problem.**

$$\int \frac{(A+B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]}{(a+b \cos [c+d x])^{3/2}} dx$$

Optimal (type 4, 271 leaves, 9 steps):

$$\begin{aligned}
& - \frac{2 (A b^2 - a (b B - a C)) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right] + 2 C \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{a b (a^2 - b^2) d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} + \frac{b d \sqrt{a + b \cos [c + d x]}}{a b (a^2 - b^2) d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} \\
& + \frac{2 A \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{a d \sqrt{a + b \cos [c + d x]}} + \frac{2 (A b^2 - a (b B - a C)) \sin [c + d x]}{a (a^2 - b^2) d \sqrt{a + b \cos [c + d x]}}
\end{aligned}$$

Result (type 8, 43 leaves):

$$\int \frac{(A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]}{(a + b \cos [c + d x])^{3/2}} dx$$

■ **Problem 1051: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^2}{(a + b \cos [c + d x])^{3/2}} dx$$

Optimal (type 4, 313 leaves, 10 steps):

$$\begin{aligned}
& \frac{(3 A b^2 - 2 a b B - a^2 (A - 2 C)) \sqrt{a + b \cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right] + A \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{a^2 (a^2 - b^2) d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} + \frac{A d \sqrt{a + b \cos [c + d x]}}{a^2 (a^2 - b^2) d \sqrt{\frac{a + b \cos [c + d x]}{a + b}}} \\
& - \frac{(3 A b - 2 a B) \sqrt{\frac{a + b \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 b}{a + b}\right]}{a^2 d \sqrt{a + b \cos [c + d x]}} - \frac{b (3 A b^2 - 2 a b B - a^2 (A - 2 C)) \sin [c + d x]}{a^2 (a^2 - b^2) d \sqrt{a + b \cos [c + d x]}} + \frac{A \tan [c + d x]}{a d \sqrt{a + b \cos [c + d x]}}
\end{aligned}$$

Result (type 4, 751 leaves):

1

$$2 a^2 (a - b) (a + b) d (2 A + C + 2 B \cos [c + d x] + C \cos [2 c + 2 d x])$$

$$\cos [c + d x]^2 (C + B \sec [c + d x] + A \sec [c + d x]^2) \left( \frac{2 (4 a A b^2 - 4 a^2 b B + 4 a^3 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 b}{a+b}\right]}{\sqrt{a + b \cos [c + d x]}} + \right.$$

$$\frac{2 (-7 a^2 A b + 9 A b^3 + 4 a^3 B - 6 a b^2 B + 2 a^2 b C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 b}{a+b}\right]}{\sqrt{a + b \cos [c + d x]}} -$$

$$\left. \left( 2 i (-a^2 A b + 3 A b^3 - 2 a b^2 B + 2 a^2 b C) \sqrt{\frac{b - b \cos [c + d x]}{a + b}} \sqrt{-\frac{b + b \cos [c + d x]}{a - b}} \cos [2 (c + d x)] \left( 2 a (a - b) \right. \right. \right.$$

$$\left. \left. \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos [c + d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos [c + d x]}\right], \frac{a+b}{a-b}\right] - \right. \right.$$

$$\left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \cos [c + d x]}\right], \frac{a+b}{a-b}\right] \right) \right) \sin [c + d x] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1 - \cos [c + d x]^2} \right.$$

$$\left. \left. \sqrt{-\frac{a^2 - b^2 - 2 a (a + b \cos [c + d x]) + (a + b \cos [c + d x])^2}{b^2}} (2 a^2 - b^2 - 4 a (a + b \cos [c + d x]) + 2 (a + b \cos [c + d x])^2) \right) \right) +$$

$$\left( \cos [c + d x]^2 \sqrt{a + b \cos [c + d x]} (C + B \sec [c + d x] + A \sec [c + d x]^2) \right.$$

$$\left. \left( -\frac{4 (A b^3 \sin [c + d x] - a b^2 B \sin [c + d x] + a^2 b C \sin [c + d x])}{a^2 (a^2 - b^2) (a + b \cos [c + d x])} + \frac{2 A \tan [c + d x]}{a^2} \right) \right) /$$

$$(d (2 A + C + 2 B \cos [c + d x] + C \cos [2 c + 2 d x]))$$

■ **Problem 1052: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^3}{(a + b \cos [c + d x])^{3/2}} dx$$

Optimal (type 4, 416 leaves, 11 steps):

$$\begin{aligned}
& - \frac{(15 A b^3 + 4 a^3 B - 12 a b^2 B - a^2 (7 A b - 8 b C)) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2b}{a+b}\right]}{4 a^3 (a^2 - b^2) d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}} \\
& + \frac{(5 A b - 4 a B) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2b}{a+b}\right]}{4 a^2 d \sqrt{a + b \cos[c + d x]}} + \frac{(15 A b^2 - 12 a b B + 4 a^2 (A + 2 C)) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2b}{a+b}\right]}{4 a^3 d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{b (15 A b^3 + 4 a^3 B - 12 a b^2 B - a^2 (7 A b - 8 b C)) \sin[c + d x]}{4 a^3 (a^2 - b^2) d \sqrt{a + b \cos[c + d x]}} - \frac{(5 A b - 4 a B) \tan[c + d x]}{4 a^2 d \sqrt{a + b \cos[c + d x]}} + \frac{A \sec[c + d x] \tan[c + d x]}{2 a d \sqrt{a + b \cos[c + d x]}}
\end{aligned}$$

Result (type 4, 723 leaves):



$$\begin{aligned}
& - \frac{1}{16 a^3 (-a+b)(a+b) d} \left( \frac{2 (4 a^3 A b - 20 a A b^3 + 16 a^2 b^2 B - 16 a^3 b C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]}{\sqrt{a+b \cos [c+d x]}} + \frac{1}{\sqrt{a+b \cos [c+d x]}} \right. \\
& 2 (8 a^4 A + 29 a^2 A b^2 - 45 A b^4 - 28 a^3 b B + 36 a b^3 B + 16 a^4 C - 24 a^2 b^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] - \\
& \left. \left( 2 i (7 a^2 A b^2 - 15 A b^4 - 4 a^3 b B + 12 a b^3 B - 8 a^2 b^2 C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \sqrt{\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \right. \\
& \left. \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{1}{a+b}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sin [c+d x] \Bigg) / \\
& \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} \right. \\
& \left. \left. \left( 2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2 \right) \right) \right) + \frac{1}{d} \\
& \sqrt{a+b \cos [c+d x]} \left( \frac{\sec [c+d x] (-7 A b \sin [c+d x]+4 a B \sin [c+d x])}{4 a^3} + \frac{2(A b^4 \sin [c+d x]-a b^3 B \sin [c+d x]+a^2 b^2 C \sin [c+d x])}{a^3(a^2-b^2)(a+b \cos [c+d x])} + \right. \\
& \left. \frac{A \sec [c+d x] \tan [c+d x]}{2 a^2} \right)
\end{aligned}$$

■ **Problem 1057: Unable to integrate problem.**

$$\int \frac{(A+B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]}{(a+b \cos [c+d x])^{5/2}} dx$$

Optimal (type 4, 401 leaves, 10 steps):

$$\frac{2 \left( 3 A b^4 + 4 a^3 b B - a^4 C - a^2 b^2 (7 A + 3 C) \right) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2b}{a+b}\right] + 3 a^2 b (a^2 - b^2)^2 d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}}{2 \left( A b^2 - a (b B - a C) \right) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2b}{a+b}\right] + \frac{2 A \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2b}{a+b}\right]}{a^2 d \sqrt{a + b \cos[c + d x]}}} + \frac{2 \left( A b^2 - a (b B - a C) \right) \sin[c + d x]}{3 a (a^2 - b^2) d (a + b \cos[c + d x])^{3/2}} - \frac{2 \left( 3 A b^4 + 4 a^3 b B - a^4 C - a^2 b^2 (7 A + 3 C) \right) \sin[c + d x]}{3 a^2 (a^2 - b^2)^2 d \sqrt{a + b \cos[c + d x]}}$$

Result (type 8, 43 leaves):

$$\int \frac{(A + B \cos[c + d x] + C \cos[c + d x]^2) \sec[c + d x]}{(a + b \cos[c + d x])^{5/2}} dx$$

■ **Problem 1058: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \cos[c + d x] + C \cos[c + d x]^2) \sec[c + d x]^2}{(a + b \cos[c + d x])^{5/2}} dx$$

Optimal (type 4, 461 leaves, 11 steps):

$$\frac{(26 a^2 A b^2 - 15 A b^4 - 14 a^3 b B + 6 a b^3 B - a^4 (3 A - 8 C)) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2b}{a+b}\right] - 3 a^3 (a^2 - b^2)^2 d \sqrt{\frac{a+b \cos[c+d x]}{a+b}}}{(5 A b^2 - 2 a b B - a^2 (3 A - 2 C)) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2b}{a+b}\right] - (5 A b - 2 a B) \sqrt{\frac{a+b \cos[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2b}{a+b}\right]} - \frac{b (5 A b^2 - 2 a b B - a^2 (3 A - 2 C)) \sin[c + d x]}{3 a^2 (a^2 - b^2) d (a + b \cos[c + d x])^{3/2}} - \frac{b (26 a^2 A b^2 - 15 A b^4 - 14 a^3 b B + 6 a b^3 B - a^4 (3 A - 8 C)) \sin[c + d x]}{3 a^3 (a^2 - b^2)^2 d \sqrt{a + b \cos[c + d x]}} + \frac{A \tan[c + d x]}{a d (a + b \cos[c + d x])^{3/2}}$$

Result (type 4, 915 leaves):

$$\begin{aligned}
& \frac{1}{6 a^3 (-a+b)^2 (a+b)^2 d (2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x])} \cos [c+d x]^2 (C+B \sec [c+d x]+A \sec [c+d x]^2) \\
& \left( \frac{1}{\sqrt{a+b \cos [c+d x]}} 2 (36 a^3 A b^2-20 a A b^4-24 a^4 b B+8 a^2 b^3 B+12 a^5 C+4 a^3 b^2 C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 b}{a+b}\right] + \right. \\
& \frac{1}{\sqrt{a+b \cos [c+d x]}} 2 (-33 a^4 A b+86 a^2 A b^3-45 A b^5+12 a^5 B-38 a^3 b^2 B+18 a b^4 B+8 a^4 b C) \sqrt{\frac{a+b \cos [c+d x]}{a+b}} \\
& \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 b}{a+b}\right]-\left(2 i(-3 a^4 A b+26 a^2 A b^3-15 A b^5-14 a^3 b^2 B+6 a b^4 B+8 a^4 b C) \sqrt{\frac{b-b \cos [c+d x]}{a+b}} \right. \\
& \left. \sqrt{-\frac{b+b \cos [c+d x]}{a-b}} \cos [2(c+d x)]\left(2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]+b\left(2 a \operatorname{EllipticF}\left[i \right.\right.\right. \\
& \left. \left. \left. \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]-b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos [c+d x]}\right], \frac{a+b}{a-b}\right]\right)\right) \\
& \left. \sin [c+d x]\right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos [c+d x]}^2 \sqrt{-\frac{a^2-b^2-2 a(a+b \cos [c+d x])+(a+b \cos [c+d x])^2}{b^2}} \right. \\
& \left. (2 a^2-b^2-4 a(a+b \cos [c+d x])+2(a+b \cos [c+d x])^2)\right) \left. \right) + \\
& \left( \cos [c+d x]^2 \sqrt{a+b \cos [c+d x]} (C+B \sec [c+d x]+A \sec [c+d x]^2) \left(-\frac{4(A b^3 \sin [c+d x]-a b^2 B \sin [c+d x]+a^2 b C \sin [c+d x])}{3 a^2\left(a^2-b^2\right)(a+b \cos [c+d x])^2} - \right. \right. \\
& \left. \left. (4(10 a^2 A b^3 \sin [c+d x]-6 A b^5 \sin [c+d x]-7 a^3 b^2 B \sin [c+d x]+3 a b^4 B \sin [c+d x]+4 a^4 b C \sin [c+d x])) / \right. \right. \\
& \left. \left. (3 a^3\left(a^2-b^2\right)^2(a+b \cos [c+d x]))+\frac{2 A \tan [c+d x]}{a^3}\right) \right) / (d(2 A+C+2 B \cos [c+d x]+C \cos [2 c+2 d x]))
\end{aligned}$$

■ **Problem 1059: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]^3}{(a+b \cos [c+d x])^{5/2}} dx$$

Optimal (type 4, 572 leaves, 12 steps):

$$\begin{aligned}
& \left( (105 A b^5 - 12 a^5 B + 104 a^3 b^2 B - 60 a b^4 B + a^4 b (33 A - 56 C) - 2 a^2 b^3 (85 A - 12 C)) \sqrt{a + b \cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right] \right) / \\
& \left( 12 a^4 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \right) + \frac{(35 A b^3 + 12 a^3 B - 20 a b^2 B - a^2 (27 A b - 8 b C)) \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{12 a^3 (a^2 - b^2) d \sqrt{a + b \cos[c + d x]}} + \\
& \frac{(35 A b^2 - 20 a b B + 4 a^2 (A + 2 C)) \sqrt{\frac{a + b \cos[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2b}{a+b}\right]}{4 a^4 d \sqrt{a + b \cos[c + d x]}} + \frac{b (35 A b^3 + 12 a^3 B - 20 a b^2 B - a^2 (27 A b - 8 b C)) \sin[c + d x]}{12 a^3 (a^2 - b^2) d (a + b \cos[c + d x])^{3/2}} - \\
& \frac{b (105 A b^5 - 12 a^5 B + 104 a^3 b^2 B - 60 a b^4 B + a^4 b (33 A - 56 C) - 2 a^2 b^3 (85 A - 12 C)) \sin[c + d x]}{12 a^4 (a^2 - b^2)^2 d \sqrt{a + b \cos[c + d x]}} - \\
& \frac{(7 A b - 4 a B) \tan[c + d x]}{4 a^2 d (a + b \cos[c + d x])^{3/2}} + \frac{A \sec[c + d x] \tan[c + d x]}{2 a d (a + b \cos[c + d x])^{3/2}}
\end{aligned}$$

Result (type 4, 922 leaves):

$$\begin{aligned}
& \frac{1}{48 a^4 (a-b)^2 (a+b)^2 d} \left( \frac{1}{\sqrt{a+b \cos[c+dx]}} \right. \\
& 2 (12 a^5 A b - 216 a^3 A b^3 + 140 a A b^5 + 144 a^4 b^2 B - 80 a^2 b^4 B - 96 a^5 b C + 32 a^3 b^3 C) \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2b}{a+b}\right] + \\
& \frac{1}{\sqrt{a+b \cos[c+dx]}} 2 (24 a^6 A + 195 a^4 A b^2 - 566 a^2 A b^4 + 315 A b^6 - 132 a^5 b B + 344 a^3 b^3 B - 180 a b^5 B + 48 a^6 C - 152 a^4 b^2 C + 72 a^2 b^4 C) \\
& \sqrt{\frac{a+b \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right] - \\
& \left. 2 i (33 a^4 A b^2 - 170 a^2 A b^4 + 105 A b^6 - 12 a^5 b B + 104 a^3 b^3 B - 60 a b^5 B - 56 a^4 b^2 C + 24 a^2 b^4 C) \sqrt{\frac{b-b \cos[c+dx]}{a+b}} \right. \\
& \sqrt{-\frac{b+b \cos[c+dx]}{a-b}} \cos[2(c+dx)] \left( 2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left( 2 a \operatorname{EllipticF}\left[ \right. \right. \right. \\
& \left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \cos[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \\
& \left. \sin[c+dx] \right) / \left( a \sqrt{-\frac{1}{a+b}} \sqrt{1-\cos[c+dx]}^2 \sqrt{-\frac{a^2-b^2-2a(a+b \cos[c+dx])+(a+b \cos[c+dx])^2}{b^2}} \right. \\
& \left. \left. (2 a^2 - b^2 - 4 a (a+b \cos[c+dx]) + 2 (a+b \cos[c+dx])^2) \right) \right) + \frac{1}{d} \\
& \sqrt{a+b \cos[c+dx]} \left( \frac{\sec[c+dx] (-11 A b \sin[c+dx] + 4 a B \sin[c+dx])}{4 a^4} + \frac{2 (A b^4 \sin[c+dx] - a b^3 B \sin[c+dx] + a^2 b^2 C \sin[c+dx])}{3 a^3 (a^2 - b^2) (a+b \cos[c+dx])^2} + \right. \\
& \left. (2 (13 a^2 A b^4 \sin[c+dx] - 9 A b^6 \sin[c+dx] - 10 a^3 b^3 B \sin[c+dx] + 6 a b^5 B \sin[c+dx] + 7 a^4 b^2 C \sin[c+dx] - 3 a^2 b^4 C \sin[c+dx])) \right) / \\
& \left. (3 a^4 (a^2 - b^2)^2 (a+b \cos[c+dx])) + \frac{A \sec[c+dx] \tan[c+dx]}{2 a^3} \right)
\end{aligned}$$

■ **Problem 1116: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} (A+B \cos[c+dx] + C \cos[c+dx]^2) dx$$

Optimal (type 4, 586 leaves, 8 steps) :

$$\begin{aligned}
 & -\frac{1}{24 a b^2 d} (a-b) \sqrt{a+b} \left(8 b^2 (3 A+2 C)+3 a (2 b B-a C)\right) \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \\
 & \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}+\frac{1}{24 b^2 d} \sqrt{a+b}\left(24 A b^2+(a+2 b)(6 b B-3 a C+8 b C)\right) \\
 & \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}+\frac{1}{8 b^3 d} \\
 & \sqrt{a+b}\left(2 a^2 b B-8 b^3 B-a^3 C-4 a b^2(2 A+C)\right) \cot [c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \\
 & \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}+\frac{\left(8 b^2(3 A+2 C)+3 a(2 b B-a C)\right) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{24 b^2 d \sqrt{\cos [c+d x]}}+ \\
 & \frac{(2 b B-a C) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{4 b d}+\frac{C \sqrt{\cos [c+d x]}(a+b \cos [c+d x])^{3 / 2} \sin [c+d x]}{3 b d}
 \end{aligned}$$

Result (type 4, 1242 leaves) :

$$\begin{aligned}
 & \frac{1}{48 b d} \left( -\left( 4 a \left( 24 A b^2 + 18 a b B - a^2 C + 16 b^2 C \right) \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \right. \right. \\
 & \left. \left. \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \operatorname{Csc} [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}}{a}\right],-\frac{2 a}{-a+b}\right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right) \right) / \\
 & \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - 4 a \left( 48 a A b + 24 b^2 B + 28 a b C \right) \\
 & \left( \left( \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \operatorname{Csc} [c+d x] \right. \right.
 \end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) \right) + \\
& 2(24 A b^2+6 a b B-3 a^2 C+16 b^2 C) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Sec}[c+d x]}{a+b}}}\right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right) \right)
\end{aligned}$$

$$\left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left(b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}\right) \right) +$$

$$\left. \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left(\frac{(6bB+aC) \sin[c+dx]}{12b} + \frac{1}{6} C \sin[2(c+dx)]\right)}{d}$$

■ **Problem 1117: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \cos[c+dx]} (A+B \cos[c+dx] + C \cos[c+dx]^2)}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 483 leaves, 7 steps):

$$-\frac{1}{4abd} (a-b) \sqrt{a+b} (4bB+aC) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{4bd}$$

$$\sqrt{a+b} (8Ab+aC+2b(2B+C)) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{4b^2d} \sqrt{a+b} (8Ab^2+4abB-a^2C+4b^2C) \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}}$$

$$\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(4bB+aC) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{4bd \sqrt{\cos[c+dx]}} + \frac{C \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2d}$$

Result (type 4, 1183 leaves):

$$\frac{C \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2d} +$$



$$\begin{aligned}
& \frac{1}{8d} \left( - \left( 4a(8aA + 4bB + 3aC) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4a(8Ab + 8aB + 4bC) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \left. \frac{2(4bB + aC)}{\left( i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx] \right. \right. \\
& \left. \left. + b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) \right)
\end{aligned}$$

$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\ \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\ \left. \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\ \left. \left. \csc[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\ \left. \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right)$$

■ **Problem 1118: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \cos[c+dx]} (A+B \cos[c+dx]+C \cos[c+dx]^2)}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 449 leaves, 7 steps):

$$\frac{1}{ad} (a-b) \sqrt{a+b} (2A-C) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{ad}$$

$$\sqrt{a+b} (2Ab-a(2A-2B-C)) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{bd} \sqrt{a+b} (2bB+aC) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2A \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}} - \frac{(2A-C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}}$$

Result (type 4, 1176 leaves):

$$\frac{2A \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}} +$$

$$\frac{1}{2d} \left( - \left( 4a(2aB+bC) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.$$

$$\left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a(-2aA+2bB+2aC)$$

$$\left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) -$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \quad \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \Bigg) + \\
& 2(-2Ab+bc) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \quad \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \csc[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}}$$

■ **Problem 1119: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \cos[c+dx]} (A+B \cos[c+dx]+C \cos[c+dx]^2)}{\cos[c+dx]^{5/2}} dx$$

Optimal (type 4, 407 leaves, 6 steps):

$$\begin{aligned} & \frac{1}{3 a^2 d} 2 (a-b) \sqrt{a+b} (A b+3 a B) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \\ & \frac{1}{3 a d} 2 \sqrt{a+b} (b(A-3 B)-a(A-3 B+3 C)) \operatorname{Cot}[c+d x] \\ & \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{d} \\ & 2 \sqrt{a+b} C \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \\ & \frac{2 A \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{3 d \cos [c+d x]^{3 / 2}} \end{aligned}$$

Result (type 4, 1240 leaves):

$$\begin{aligned} & \frac{1}{3 a d} \\ & \left( - \left( 4 a (a^2 A - A b^2 + 3 a^2 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right],-\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a \left( -aAb - 3a^2B + 3abc \right) \\
& \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + \\
& 2(-Ab^2 - 3abB) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\ \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\ \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \left. \right) + \\ \frac{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2 \operatorname{Sec}[c+dx] (A b \operatorname{Sin}[c+dx] + 3 a B \operatorname{Sin}[c+dx])}{3 a} + \frac{2}{3} A \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx] \right)}{d}$$

■ **Problem 1120: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} (A+B \operatorname{Cos}[c+dx]+C \operatorname{Cos}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{7/2}} dx$$

Optimal (type 4, 360 leaves, 5 steps):

$$-\frac{1}{15 a^3 d} 2(a-b) \sqrt{a+b} (2 A b^2 - 5 a b B - 3 a^2 (3 A + 5 C)) \operatorname{Cot}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{15 a^2 d}$$

$$2(a-b) \sqrt{a+b} (2 A b + a (9 A - 5 B + 15 C)) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}}$$

$$\sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{2 A \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{5 d \operatorname{Cos}[c+dx]^{5/2}} + \frac{2 (A b + 5 a B) \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{15 a d \operatorname{Cos}[c+dx]^{3/2}}$$

Result (type 4, 1340 leaves):

$$\begin{aligned}
& -\frac{1}{15 a^2 d} \left( \left( 4 a \left( 2 a^2 A b - 2 A b^3 - 5 a^3 B + 5 a b^2 B \right) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a \left( 9 a^3 A - 2 a A b^2 + 5 a^2 b B + 15 a^3 C \right) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \\
& \left. 2 \left( 9 a^2 A b - 2 A b^3 + 5 a b^2 B + 15 a^2 b C \right) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+dx] \right. \right. \\
& \left. \left. b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) \right) +
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \Bigg) + \\
& \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2 \operatorname{Sec}[c+dx]^2 (A b \operatorname{Sin}[c+dx] + 5 a B \operatorname{Sin}[c+dx])}{15 a} + \right. \\
& \quad \left. \frac{2 \operatorname{Sec}[c+dx] (9 a^2 A \operatorname{Sin}[c+dx] - 2 A b^2 \operatorname{Sin}[c+dx] + 5 a b B \operatorname{Sin}[c+dx] + 15 a^2 C \operatorname{Sin}[c+dx])}{15 a^2} + \right. \\
& \quad \left. \frac{2}{5} A \operatorname{Sec}[c+dx]^2 \right. \\
& \quad \left. \operatorname{Tan}[c+dx] \right)
\end{aligned}$$

■ **Problem 1121:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+b \cos [c+d x]} (A+B \cos [c+d x]+C \cos [c+d x]^2)}{\cos [c+d x]^{9/2}} dx$$

Optimal (type 4, 447 leaves, 6 steps):

$$\frac{1}{105 a^4 d} 2 (a-b) \sqrt{a+b} (8 A b^3+63 a^3 B-14 a b^2 B+a^2 b (19 A+35 C)) \cot [c+d x]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \frac{1}{105 a^3 d}$$

$$2(a-b) \sqrt{a+b} (8 A b^2+2 a b(3 A-7 B)+a^2(25 A-63 B+35 C)) \cot [c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \frac{2 A \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{7 d \cos [c+d x]^{7/2}} +$$

$$\frac{2(A b+7 a B) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{35 a d \cos [c+d x]^{5/2}} - \frac{2(4 A b^2-7 a b B-5 a^2(5 A+7 C)) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{105 a^2 d \cos [c+d x]^{3/2}}$$

Result (type 4, 1464 leaves):

$$\frac{1}{105 a^3 d} \left( - \left( 4 a (25 a^4 A - 17 a^2 A b^2 - 8 A b^4 - 14 a^3 b B + 14 a b^3 B + 35 a^4 C - 35 a^2 b^2 C) \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right.$$

$$\left. \left. \text{Csc}[c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right) / \right.$$

$$\left. \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - 4 a (-19 a^3 A b - 8 a A b^3 - 63 a^4 B + 14 a^2 b^2 B - 35 a^3 b C) \right.$$

$$\left. \left( \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \text{Csc}[c+d x] \right. \right. \right.$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left. (b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}) \right) + 2(-19a^2Ab^2 - 8Ab^4 - 63a^3bB + 14ab^3B - 35a^2b^2C) \\
& \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\text{Sec}[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\text{Sec}[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\text{Sec}[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b}2a \left( \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \text{Csc}[c+dx] \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \\
& \left. \left. \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) + \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2 \operatorname{Sec}[c+dx]^3 (A b \operatorname{Sin}[c+dx] + 7 a B \operatorname{Sin}[c+dx])}{35 a} + \right. \right. \\
& \left. \frac{2 \operatorname{Sec}[c+dx]^2 (25 a^2 A \operatorname{Sin}[c+dx] - 4 A b^2 \operatorname{Sin}[c+dx] + 7 a b B \operatorname{Sin}[c+dx] + 35 a^2 C \operatorname{Sin}[c+dx])}{105 a^2} + \right. \\
& \left. \frac{1}{(105 a^3) 2 \operatorname{Sec}[c+dx]} \right. \\
& \left. \left. (19 a^2 A b \operatorname{Sin}[c+dx] + 8 A b^3 \operatorname{Sin}[c+dx] + 63 a^3 B \operatorname{Sin}[c+dx] - 14 a b^2 B \operatorname{Sin}[c+dx] + 35 a^2 b C \operatorname{Sin}[c+dx]) + \frac{2}{7} \right. \right. \\
& \left. \left. A \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx] \right) \right)
\end{aligned}$$

■ **Problem 1122: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\operatorname{Cos}[c+dx]} (a+b \operatorname{Cos}[c+dx])^{3/2} (A+B \operatorname{Cos}[c+dx]+C \operatorname{Cos}[c+dx]^2) dx$$

Optimal (type 4, 704 leaves, 9 steps):

$$\begin{aligned}
& -\frac{1}{192 a b^2 d} (a-b) \sqrt{a+b} (24 a^2 b B + 128 b^3 B - 9 a^3 C + 12 a b^2 (20 A + 13 C)) \\
& \quad \text{Cot}[c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+d x]}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} - \\
& \frac{1}{192 b^2 d} \sqrt{a+b} (9 a^3 C - 6 a^2 b (4 B + C) - 8 b^3 (12 A + 16 B + 9 C) - 4 a b^2 (60 A + 28 B + 39 C)) \text{Cot}[c+d x] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+d x]}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} + \\
& \frac{1}{64 b^3 d} \sqrt{a+b} (8 a^3 b B - 96 a b^3 B - 3 a^4 C - 24 a^2 b^2 (2 A + C) - 16 b^4 (4 A + 3 C)) \text{Cot}[c+d x] \\
& \quad \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+d x]}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} + \\
& \frac{(24 a^2 b B + 128 b^3 B - 9 a^3 C + 12 a b^2 (20 A + 13 C)) \sqrt{a+b} \cos[c+d x] \sin[c+d x]}{192 b^2 d \sqrt{\cos[c+d x]}} + \\
& \frac{(4 b^2 (4 A + 3 C) + a (8 b B - 3 a C)) \sqrt{\cos[c+d x]} \sqrt{a+b} \cos[c+d x] \sin[c+d x]}{32 b d} + \\
& \frac{(8 b B - 3 a C) \sqrt{\cos[c+d x]} (a+b \cos[c+d x])^{3/2} \sin[c+d x]}{24 b d} + \frac{C \sqrt{\cos[c+d x]} (a+b \cos[c+d x])^{5/2} \sin[c+d x]}{4 b d}
\end{aligned}$$

Result (type 4, 1317 leaves):

$$\begin{aligned}
& -\frac{1}{384 b d} \left( -\left( 4 a (-336 a A b^2 - 136 a^2 b B - 128 b^3 B + 3 a^3 C - 228 a b^2 C) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+d x] \csc\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\
& \quad \left. \left. \sqrt{\frac{(a+b \cos[c+d x]) \csc\left[\frac{1}{2}(c+d x)\right]^2}{a}} \csc[c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+d x]) \csc\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right]}, -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4 \right) \right) / \\
& \quad \left( (a+b) \sqrt{\cos[c+d x]} \sqrt{a+b \cos[c+d x]} \right) - 4 a (-384 a^2 A b - 192 A b^3 - 416 a b^2 B - 228 a^2 b C - 144 b^3 C)
\end{aligned}$$

$$\left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) -$$

$$\left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right.$$

$$\left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) /$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + 2(-240 a A b^2 - 24 a^2 b B - 128 b^3 B + 9 a^3 C - 156 a b^2 C)$$

$$\left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) +$$

$$\frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \right. \\
& \left. \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) + \\
& \frac{1}{d} \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left( \frac{(48 A b^2+56 a b B+3 a^2 C+42 b^2 C) \sin [c+d x]}{96 b} + \right. \\
& \frac{1}{48} \\
& (8 b B+9 a C) \\
& \sin [2(c+d x)] + \frac{1}{16} \\
& b \\
& c \\
& \left. \sin [3(c+d x)] \right)
\end{aligned}$$

- **Problem 1123: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos [c+d x])^{3/2} (A+B \cos [c+d x]+C \cos [c+d x]^2)}{\sqrt{\cos [c+d x]}} dx$$

Optimal (type 4, 587 leaves, 8 steps):

$$\begin{aligned}
& -\frac{1}{24abd} (a-b) \sqrt{a+b} (24Ab^2 + 30abB + 3a^2C + 16b^2C) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{24bd} \sqrt{a+b} (3a^2C + 4b^2(6A+3B+4C) + 2ab(24A+15B+7C)) \\
& \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{8b^2d} \\
& \sqrt{a+b} (6a^2bB + 8b^3B - a^3C + 12ab^2(2A+C)) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(24Ab^2 + 30abB + 3a^2C + 16b^2C) \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{24bd\sqrt{\cos[c+dx]}} + \\
& \frac{(2bB+aC) \sqrt{\cos[c+dx]} \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{4d} + \frac{C\sqrt{\cos[c+dx]} (a+b\cos[c+dx])^{3/2} \sin[c+dx]}{3d}
\end{aligned}$$

Result (type 4, 1250 leaves):

$$\begin{aligned}
& \frac{1}{48d} \left( - \left( 4a (48a^2A + 24Ab^2 + 42abB + 17a^2C + 16b^2C) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b\cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right. \\
& \left. \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b\cos[c+dx]} \right) - 4a (96aAb + 48a^2B + 24b^2B + 52abC) \right. \\
& \left. \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right) \right) \right)
\end{aligned}$$



$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) + \\
& 2(24Ab^2 + 30abB + 3a^2C + 16b^2C) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b}2a \left( \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right. \\
& \left. \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right) \right)
\end{aligned}$$

$$\left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left(b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}\right) \right) + \left. \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left(\frac{1}{12}(6bB+7aC) \sin[c+dx] + \frac{1}{6}bC \sin[2(c+dx)]\right)}{d}$$

■ **Problem 1124: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{3/2} (A+B \cos[c+dx] + C \cos[c+dx]^2)}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 535 leaves, 8 steps):

$$\frac{1}{4ad} (a-b) \sqrt{a+b} (8aA-4bB-5aC) \cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{4d} \sqrt{a+b} (a(8A-8B-5C) - 2b(8A+2B+C)) \cot[c+dx]$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{4bd}$$

$$\sqrt{a+b} (8Ab^2 + 12abB + 3a^2C + 4b^2C) \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{(8aA-4bB-5aC) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{4d \sqrt{\cos[c+dx]}}$$

$$\frac{b(4A-C) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2d} + \frac{2A(a+b \cos[c+dx])^{3/2} \sin[c+dx]}{d \sqrt{\cos[c+dx]}}$$

Result (type 4, 1232 leaves):

$$\begin{aligned}
& \frac{1}{8d} \left( \left( 4a(-8aAb - 8a^2B - 4b^2B - 7abc) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right]}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) + 4a(8a^2A - 8Ab^2 - 16abB - 8a^2C - 4b^2C) \\
& \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right]}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right]}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& 2(8aAb - 4b^2B - 5abc) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\operatorname{Sec}[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\operatorname{Sec}[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\operatorname{Sec}[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& \quad \left. \left. \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right) + \frac{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{1}{2} b C \operatorname{Sin}[c+dx] + 2 a A \operatorname{Tan}[c+dx] \right)}{d}
\end{aligned}$$

■ **Problem 1125: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{3/2} (A+B \operatorname{Cos}[c+dx]+C \operatorname{Cos}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{5/2}} dx$$

Optimal (type 4, 528 leaves, 8 steps):

$$\frac{1}{3ad} (a-b) \sqrt{a+b} (8Ab+6aB-3bC) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{3ad} \sqrt{a+b} (6Ab^2+2a^2(A-3B+3C)-ab(8A-3(4B+C)))$$

$$\cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{d}$$

$$\sqrt{a+b} (2bB+3aC) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2(Ab+aB)\sqrt{a+b}\cos[c+dx]\sin[c+dx]}{d\sqrt{\cos[c+dx]}} - \frac{(8Ab+6aB-3bC)\sqrt{a+b}\cos[c+dx]\sin[c+dx]}{3d\sqrt{\cos[c+dx]}} + \frac{2A(a+b)\cos[c+dx]^{3/2}\sin[c+dx]}{3d\cos[c+dx]^{3/2}}$$

Result (type 4, 1260 leaves):

$$\frac{1}{6d} \left( - \left( 4a(2a^2A-2Ab^2+6abB+6a^2C+3b^2C) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.$$

$$\left. \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - 4a(-8aAb-6a^2B+6b^2B+12abC) \right.$$

$$\left. \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right.$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& 2(-8Ab^2 - 6abB + 3b^2C) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /
\end{aligned}$$

$$\left( \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \frac{1}{d}$$

$$\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2}{3} \sec[c+dx] (4Ab \sin[c+dx] + 3aB \sin[c+dx]) + \frac{2}{3} aA \sec[c+dx] \right. \\ \left. \tan[c+dx] \right)$$

■ **Problem 1126: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{3/2} (A+B \cos[c+dx] + C \cos[c+dx]^2)}{\cos[c+dx]^{7/2}} dx$$

Optimal (type 4, 490 leaves, 7 steps):

$$\frac{1}{15 a^2 d} 2 (a-b) \sqrt{a+b} (3 A b^2 + 20 a b B + 3 a^2 (3 A + 5 C)) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{15 a d} 2 \sqrt{a+b} (3 b^2 (A-5 B) - 2 a b (6 A-10 B+15 C) + a^2 (9 A-5 B+15 C))$$

$$\operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{d}$$

$$2 b \sqrt{a+b} C \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2(3 A b + 5 a B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{15 d \cos[c+dx]^{3/2}} + \frac{2 A (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{5 d \cos[c+dx]^{5/2}}$$

Result (type 4, 1353 leaves):

$$-\frac{1}{15 a d} \left( \left( 4 a (-3 a^2 A b + 3 A b^3 - 5 a^3 B + 5 a b^2 B - 15 a^2 b C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\begin{aligned}
& \left. \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right. \\
& \left. \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a \left( 9a^3 A + 3aAb^2 + 20a^2bB + 15a^3C - 15ab^2C \right) \right. \\
& \left. \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right. \\
& \left. \left. \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \right. \\
& \left. \left. \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right. \\
& \left. \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right. \right. \\
& \left. \left. \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) \right. \right. \\
& \left. \left. \left. \left. 2 \left( 9a^2Ab + 3Ab^3 + 20a^2b^2B + 15a^2bC \right) \right. \right. \right. \\
& \left. \left. \left. \left. \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{Arcsinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx] \right. \right. \right. \\
& \left. \left. \left. \left. \left. \frac{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right. \right. \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right] / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \right. \\
& \left. \text{Csc}[c+d x] \text{EllipticPi} \left[ -\frac{a}{b}, \text{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right] / \right. \\
& \left. \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) + \\
& \frac{1}{d} \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left( \frac{2}{15} \sec [c+d x]^2 (6 A b \sin [c+d x] + 5 a B \sin [c+d x]) + \right. \\
& \left. \frac{2 \sec [c+d x] (9 a^2 A \sin [c+d x] + 3 A b^2 \sin [c+d x] + 20 a b B \sin [c+d x] + 15 a^2 C \sin [c+d x])}{15 a} + \right. \\
& \left. \frac{2}{5} \frac{a}{A} \sec [c+d x]^2 \right. \\
& \left. \tan [c+d x] \right)
\end{aligned}$$

- **Problem 1127: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos [c+d x])^{3/2} (A+B \cos [c+d x]+C \cos [c+d x]^2)}{\cos [c+d x]^{9/2}} dx$$

Optimal (type 4, 450 leaves, 6 steps):

$$\begin{aligned}
& -\frac{1}{105 a^3 d} 2 (a-b) \sqrt{a+b} (6 A b^3 - 63 a^3 B - 21 a b^2 B - 2 a^2 b (41 A + 70 C)) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{105 a^2 d} \\
& 2 (a-b) \sqrt{a+b} (6 A b^2 - a^2 (25 A - 63 B + 35 C) + 3 a b (19 A - 7 B + 35 C)) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{2(3 A b + 7 a B) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{35 d \operatorname{Cos}[c+d x]^{5/2}} + \\
& \frac{2(3 A b^2 + 42 a b B + 5 a^2 (5 A + 7 C)) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{105 a d \operatorname{Cos}[c+d x]^{3/2}} + \frac{2 A (a+b \operatorname{Cos}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{7 d \operatorname{Cos}[c+d x]^{7/2}}
\end{aligned}$$

Result (type 4, 1463 leaves):

$$\begin{aligned}
& \frac{1}{105 a^2 d} \left( - \left( 4 a (25 a^4 A - 31 a^2 A b^2 + 6 A b^4 + 21 a^3 b B - 21 a b^3 B + 35 a^4 C - 35 a^2 b^2 C) \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\
& \left. \left. \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \right. \\
& \left. \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - 4 a (-82 a^3 A b + 6 a A b^3 - 63 a^4 B - 21 a^2 b^2 B - 140 a^3 b C) \right. \\
& \left. \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \\
& \left. \text{Csc}[c+d x] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \right. \\
& \left. \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) + 2\left(-82 a^2 A b^2+6 A b^4-63 a^3 b B-21 a b^3 B-140 a^2 b^2 C\right) \right. \\
& \left. \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \sec [c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \sec [c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \sec [c+d x]}{a+b}} + \right. \\
& \left. \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \text{Csc}[c+d x] \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
\left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
\left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \left. \right) + \\
\frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2}{35} \operatorname{Sec}[c+dx]^3 (8Ab \operatorname{Sin}[c+dx] + 7aB \operatorname{Sin}[c+dx]) + \right. \\
\left. \frac{2 \operatorname{Sec}[c+dx]^2 (25a^2 A \operatorname{Sin}[c+dx] + 3Ab^2 \operatorname{Sin}[c+dx] + 42abB \operatorname{Sin}[c+dx] + 35a^2 C \operatorname{Sin}[c+dx])}{105a} + \right. \\
\left. 1 / (105a^2) 2 \operatorname{Sec}[c+dx] \right. \\
\left. (82a^2 Ab \operatorname{Sin}[c+dx] - 6Ab^3 \operatorname{Sin}[c+dx] + 63a^3 B \operatorname{Sin}[c+dx] + 21ab^2 B \operatorname{Sin}[c+dx] + 140a^2 bC \operatorname{Sin}[c+dx]) + \frac{2}{7} \right. \\
\left. aA \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx] \right)$$

■ **Problem 1128: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{3/2} (A+B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{11/2}} dx$$

Optimal (type 4, 550 leaves, 7 steps):

$$\frac{1}{315 a^4 d} 2 (a-b) \sqrt{a+b} (8 A b^4 + 246 a^3 b B - 18 a b^3 B + 21 a^4 (7 A + 9 C) + 3 a^2 b^2 (11 A + 21 C))$$

$$\text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{315 a^3 d} 2 (a-b) \sqrt{a+b} (8 A b^3 + 6 a b^2 (A-3 B) + 3 a^2 b (13 A - 57 B + 21 C) - 3 a^3 (49 A - 25 B + 63 C)) \text{Cot}[c+dx]$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2 (A b + 3 a B) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{21 d \cos[c+dx]^{7/2}} + \frac{2 (3 A b^2 + 72 a b B + 7 a^2 (7 A + 9 C)) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{315 a d \cos[c+dx]^{5/2}} -$$

$$\frac{2 (4 A b^3 - 75 a^3 B - 9 a b^2 B - 2 a^2 b (44 A + 63 C)) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{315 a^2 d \cos[c+dx]^{3/2}} + \frac{2 A (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{9 d \cos[c+dx]^{9/2}}$$

Result (type 4, 1614 leaves):

$$-\frac{1}{315 a^3 d} \left( \left( 4 a (-39 a^4 A b + 31 a^2 A b^3 + 8 A b^5 - 75 a^5 B + 93 a^3 b^2 B - 18 a b^4 B - 63 a^4 b C + 63 a^2 b^3 C) \right. \right.$$

$$\left. \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) -$$

$$4 a (147 a^5 A + 33 a^3 A b^2 + 8 a A b^4 + 246 a^4 b B - 18 a^2 b^3 B + 189 a^5 C + 63 a^3 b^2 C)$$

$$\left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right] / \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) -$$

$$\left(\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}}\sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\csc[c+dx]\right.$$

$$\left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right] / \left(b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) +$$

$$2(147a^4Ab + 33a^2Ab^3 + 8Ab^5 + 246a^3b^2B - 18ab^4B + 189a^4bC + 63a^2b^3C)$$

$$\left(\frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}}\right) +$$

$$\frac{1}{b}2a\left(\left(a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}}\sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\csc[c+dx]\right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right] / \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) -$$

$$\left(a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}}\sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\right.$$

$$\begin{aligned}
 & \left. \text{Csc}[c + d x] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \text{Cos}[c+d x]) \text{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \text{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right/ \\
 & \left. \left( b \sqrt{\text{Cos}[c+d x]} \sqrt{a+b \text{Cos}[c+d x]} \right) + \frac{\sqrt{a+b \text{Cos}[c+d x]} \text{Sin}[c+d x]}{b \sqrt{\text{Cos}[c+d x]}} \right) \\
 & \frac{1}{d} \sqrt{\text{Cos}[c+d x]} \sqrt{a+b \text{Cos}[c+d x]} \left( \frac{2}{63} \text{Sec}[c+d x]^4 (10 A b \text{Sin}[c+d x] + 9 a B \text{Sin}[c+d x]) + \right. \\
 & \left. \frac{2 \text{Sec}[c+d x]^3 (49 a^2 A \text{Sin}[c+d x] + 3 A b^2 \text{Sin}[c+d x] + 72 a b B \text{Sin}[c+d x] + 63 a^2 C \text{Sin}[c+d x])}{315 a} + \right. \\
 & \left. \frac{1}{315 a^2} \right. \\
 & \left. 2 \text{Sec}[c+d x]^2 (88 a^2 A b \text{Sin}[c+d x] - 4 A b^3 \text{Sin}[c+d x] + 75 a^3 B \text{Sin}[c+d x] + 9 a b^2 B \text{Sin}[c+d x] + 126 a^2 b C \text{Sin}[c+d x]) + \right. \\
 & \left. \frac{1}{315 a^3} \right. \\
 & \left. 2 \text{Sec}[c+d x] (147 a^4 A \text{Sin}[c+d x] + 33 a^2 A b^2 \text{Sin}[c+d x] + 8 A b^4 \text{Sin}[c+d x] + 246 a^3 b B \text{Sin}[c+d x] - \right. \\
 & \left. 18 a b^3 B \text{Sin}[c+d x] + 189 a^4 C \text{Sin}[c+d x] + 63 a^2 b^2 C \text{Sin}[c+d x]) + \frac{2}{9} a A \text{Sec}[c+d x]^4 \text{Tan}[c+d x] \right)
 \end{aligned}$$

■ **Problem 1129: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\text{Cos}[c+d x]} (a+b \text{Cos}[c+d x])^{5/2} (A+B \text{Cos}[c+d x]+C \text{Cos}[c+d x]^2) dx$$

Optimal (type 4, 834 leaves, 10 steps):

$$\begin{aligned}
& - \frac{1}{1920 a b^2 d} (a-b) \sqrt{a+b} (150 a^3 b B + 2840 a b^3 B - 45 a^4 C + 256 b^4 (5 A + 4 C) + 12 a^2 b^2 (220 A + 141 C)) \\
& \quad \text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{1920 b^2 d} \\
& \sqrt{a+b} (45 a^4 C - 30 a^3 b (5 B + C) - 16 b^4 (80 A + 45 B + 64 C) - 8 a b^3 (260 A + 355 B + 193 C) - 4 a^2 b^2 (660 A + 295 B + 423 C)) \\
& \quad \text{Cot}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{1}{128 b^3 d} \sqrt{a+b} (10 a^4 b B - 240 a^2 b^3 B - 96 b^5 B - 3 a^5 C - 40 a^3 b^2 (2 A + C) - 80 a b^4 (4 A + 3 C)) \text{Cot}[c+dx] \\
& \quad \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{1}{1920 b^2 d \sqrt{\cos[c+dx]}} (150 a^3 b B + 2840 a b^3 B - 45 a^4 C + 256 b^4 (5 A + 4 C) + 12 a^2 b^2 (220 A + 141 C)) \sqrt{a+b} \cos[c+dx] \sin[c+dx] + \\
& \frac{(50 a^2 b B + 120 b^3 B - 15 a^3 C + 4 a b^2 (60 A + 43 C)) \sqrt{\cos[c+dx]} \sqrt{a+b} \cos[c+dx] \sin[c+dx]}{320 b d} + \\
& \frac{(80 A b^2 + 50 a b B - 15 a^2 C + 64 b^2 C) \sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{240 b d} + \\
& \frac{(10 b B - 3 a C) \sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{5/2} \sin[c+dx]}{40 b d} + \frac{C \sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{7/2} \sin[c+dx]}{5 b d}
\end{aligned}$$

Result (type 4, 1410 leaves):

$$\begin{aligned}
& - \frac{1}{3840 b d} \left( - \left( 4 a (-4720 a^2 A b^2 - 1280 A b^4 - 1330 a^3 b B - 3560 a b^3 B + 15 a^4 C - 3236 a^2 b^2 C - 1024 b^4 C) \right. \right. \\
& \quad \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \\
& \quad \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \quad \left. \left. \right) \right)
\end{aligned}$$



$$4 a (-3840 a^3 A b - 6080 a A b^3 - 6440 a^2 b^2 B - 1440 b^4 B - 2292 a^3 b C - 4624 a b^3 C)$$

$$\left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\ \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\ \left. \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\ \left. \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) +$$

$$2 (-2640 a^2 A b^2 - 1280 A b^4 - 150 a^3 b B - 2840 a b^3 B + 45 a^4 C - 1692 a^2 b^2 C - 1024 b^4 C)$$

$$\left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}}} \right. \\ \left. \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \right. \\ \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \right.$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
\left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
\left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \left. \right) + \frac{1}{d} \\
\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{(1040 a A b^2 + 590 a^2 b B + 420 b^3 B + 15 a^3 C + 898 a b^2 C) \operatorname{Sin}[c+dx]}{960 b} + \right. \\
\frac{1}{480} \\
(80 A b^2 + 170 a b B + 93 a^2 C + 88 b^2 C) \\
\operatorname{Sin}[2(c+dx)] + \frac{1}{160} \\
b(10 b B + 21 a C) \\
\operatorname{Sin}[3(c+dx)] + \frac{1}{40} \\
\left. b^2 C \operatorname{Sin}[4(c+dx)] \right)$$

■ **Problem 1130: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{5/2} (A+B \operatorname{Cos}[c+dx]+C \operatorname{Cos}[c+dx]^2)}{\sqrt{\operatorname{Cos}[c+dx]}} dx$$

Optimal (type 4, 700 leaves, 9 steps):

$$\begin{aligned}
& -\frac{1}{192abd} (a-b) \sqrt{a+b} (264a^2bB + 128b^3B + 15a^3C + 4ab^2(108A + 71C)) \\
& \quad \text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{1}{192bd} \sqrt{a+b} (15a^3C + 8b^3(12A + 16B + 9C) + 2a^2b(192A + 132B + 59C) + 4ab^2(108A + 52B + 71C)) \text{Cot}[c+dx] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\
& \frac{1}{64b^2d} \sqrt{a+b} (40a^3bB + 160ab^3B - 5a^4C + 120a^2b^2(2A + C) + 16b^4(4A + 3C)) \text{Cot}[c+dx] \\
& \quad \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{(264a^2bB + 128b^3B + 15a^3C + 4ab^2(108A + 71C)) \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{192bd\sqrt{\cos[c+dx]}} + \\
& \frac{(16Ab^2 + 24abB + 5a^2C + 12b^2C) \sqrt{\cos[c+dx]} \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{32d} + \\
& \frac{(8bB + 5aC) \sqrt{\cos[c+dx]} (a+b\cos[c+dx])^{3/2} \sin[c+dx]}{24d} + \frac{C \sqrt{\cos[c+dx]} (a+b\cos[c+dx])^{5/2} \sin[c+dx]}{4d}
\end{aligned}$$

Result (type 4, 1326 leaves):

$$\begin{aligned}
& \frac{1}{384d} \\
& \left( \left( -4a(384a^3A + 528aAb^2 + 472a^2bB + 128b^3B + 133a^3C + 356ab^2C) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \quad \left. \left. \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(a+b)\cos[c+dx]}\csc\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{2}a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx] \right) - 4a(1152a^2Ab + 192Ab^3 + 384a^3B + 608ab^2B + 644a^2bC + 144b^3C)
\end{aligned}$$

$$\left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.$$

$$\left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right.$$

$$\left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + 2 (432 a A b^2 + 264 a^2 b B + 128 b^3 B + 15 a^3 C + 284 a b^2 C)$$

$$\left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right.$$

$$\left. \frac{1}{b} 2 a \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \right. \\
& \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \frac{1}{d} \\
& \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{1}{96} (48 A b^2 + 104 a b B + 59 a^2 C + 42 b^2 C) \sin[c+dx] + \frac{1}{48} b (8 b B + 17 a C) \right. \\
& \left. \sin[2(c+dx)] + \frac{1}{16} b^2 C \sin[3(c+dx)] \right)
\end{aligned}$$

- **Problem 1131: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{5/2} (A+B \cos[c+dx] + C \cos[c+dx]^2)}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 647 leaves, 9 steps):

$$\begin{aligned}
& -\frac{1}{24ad} (a-b) \sqrt{a+b} (54abB - a^2(48A - 33C) + 8b^2(3A + 2C)) \operatorname{Cot}[c+dx] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\
& \frac{1}{24d} \sqrt{a+b} (a^2(48A - 48B - 33C) - 4b^2(6A + 3B + 4C) - 2ab(72A + 27B + 13C)) \operatorname{Cot}[c+dx] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{8bd} \\
& \sqrt{a+b} (30a^2bB + 8b^3B + 5a^3C + 20ab^2(2A + C)) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(54abB - a^2(48A - 33C) + 8b^2(3A + 2C)) \sqrt{a+b}\cos[c+dx] \operatorname{Sin}[c+dx]}{24d\sqrt{\cos[c+dx]}} - \\
& \frac{b(8aA - 2bB - 3aC) \sqrt{\cos[c+dx]} \sqrt{a+b}\cos[c+dx] \operatorname{Sin}[c+dx]}{4d} - \\
& \frac{b(6A - C) \sqrt{\cos[c+dx]} (a+b\cos[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{3d} + \frac{2A(a+b\cos[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{d\sqrt{\cos[c+dx]}}
\end{aligned}$$

Result (type 4, 1302 leaves):

$$\begin{aligned}
& \frac{1}{48d} \left( \left( 4a(-96a^2Ab - 24Ab^3 - 48a^3B - 66ab^2B - 59a^2bC - 16b^3C) \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right]}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b\cos[c+dx]} \right) + 4a(48a^3A - 144aAb^2 - 144a^2bB - 24b^3B - 48a^3C - 76ab^2C)
\end{aligned}$$

$$\left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
\left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
\left. \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
\left. \left. \csc[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
\left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) - 2 (48 a^2 A b - 24 A b^3 - 54 a b^2 B - 33 a^2 b C - 16 b^3 C) \\
\left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} \right. \\
\left. \frac{1}{b} 2 a \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right) \right)$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \right. \\
& \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \\
& \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{1}{12} b (6bB + 13aC) \sin[c+dx] + \frac{1}{6} b^2 C \sin[2(c+dx)] + \right. \\
& \left. 2a^2 A \tan[c+dx] \right)
\end{aligned}$$

■ **Problem 1132: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{5/2} (A+B \cos[c+dx] + C \cos[c+dx]^2)}{\cos[c+dx]^{5/2}} dx$$

Optimal (type 4, 622 leaves, 9 steps):



$$\frac{1}{12ad} (a-b) \sqrt{a+b} (24a^2B - 12b^2B + ab(56A - 27C)) \operatorname{Cot}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{12d}$$

$$\sqrt{a+b} (ab(56A - 72B - 27C) - 6b^2(12A + 2B + C) - 8a^2(A - 3B + 3C)) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{4d} \sqrt{a+b} (8Ab^2 + 20abB + 15a^2C + 4b^2C) \operatorname{Cot}[c+dx]$$

$$\operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{(24a^2B - 12b^2B + ab(56A - 27C)) \sqrt{a+b}\cos[c+dx] \operatorname{Sin}[c+dx]}{12d\sqrt{\cos[c+dx]}} - \frac{b(8Ab + 4aB - bC) \sqrt{\cos[c+dx]} \sqrt{a+b}\cos[c+dx] \operatorname{Sin}[c+dx]}{2d} +$$

$$\frac{2(5Ab + 3aB) (a+b)\cos[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{3d\sqrt{\cos[c+dx]}} + \frac{2A(a+b)\cos[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{3d\cos[c+dx]^{3/2}}$$

Result (type 4, 1316 leaves):

$$\frac{1}{24d} \left( \left( 4a(8a^3A + 16abA^2 + 48a^2bB + 12b^3B + 24a^3C + 33ab^2C) \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right/$$

$$\left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx] \right) - 4a(-56a^2Ab + 24Ab^3 - 24a^3B + 72ab^2B + 72a^2bC + 12b^3C)$$

$$\left( \left( \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b)\cos[c+dx]\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \\
& \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left(b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) \right) + \\
& 2(-56 a A b^2 - 24 a^2 b B + 12 b^3 B + 27 a b^2 C) \left( \frac{i \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{a+b \cos [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin \left[\frac{1}{2}(c+d x)\right]}{\sqrt{\cos [c+d x]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+d x]}{b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2} \operatorname{Sec}[c+d x]} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Sec}[c+d x]}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4\right) / \left((a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]}\right) - \right. \\
& \left. \left( a \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right) \right)
\end{aligned}$$

$$\left. \left( \left( \left( \left( \left( \left( \text{Csc}[c + dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \text{Cos}[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \text{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \right. \right. \right. \right. \right. \right. \right. \right. \left. \left. \left. \left. \left. \left. \left. \left. \left. \left( b \sqrt{\text{Cos}[c+dx]} \sqrt{a+b \text{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \text{Cos}[c+dx]} \text{Sin}[c+dx]}{b \sqrt{\text{Cos}[c+dx]}} + \frac{1}{d} \right) \right) \right) \right) \right) \right) \right) \right) \sqrt{\text{Cos}[c+dx]} \sqrt{a+b \text{Cos}[c+dx]} \left( \frac{1}{2} b^2 C \text{Sin}[c+dx] + \frac{2}{3} \text{Sec}[c+dx] (7 a A b \text{Sin}[c+dx] + 3 a^2 B \text{Sin}[c+dx]) + \frac{2}{3} a^2 A \text{Sec}[c+dx] \text{Tan}[c+dx] \right)$$

■ **Problem 1133: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \text{Cos}[c + dx])^{5/2} (A + B \text{Cos}[c + dx] + C \text{Cos}[c + dx]^2)}{\text{Cos}[c + dx]^{7/2}} dx$$

Optimal (type 4, 643 leaves, 9 steps):

$$\frac{1}{15 a d} (a - b) \sqrt{a + b} (70 a b B + b^2 (46 A - 15 C) + 6 a^2 (3 A + 5 C)) \text{Cot}[c + dx] \\ + \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Cos}[c + dx]}}{\sqrt{a + b} \sqrt{\text{Cos}[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \text{Sec}[c + dx])}{a + b}} \sqrt{\frac{a (1 + \text{Sec}[c + dx])}{a - b}} + \\ \frac{1}{15 a d} \sqrt{a + b} (30 A b^3 - 2 a^3 (9 A - 5 B + 15 C) + 2 a^2 b (17 A - 35 B + 45 C) - a b^2 (46 A - 15 (6 B + C))) \text{Cot}[c + dx] \\ + \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Cos}[c + dx]}}{\sqrt{a + b} \sqrt{\text{Cos}[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \text{Sec}[c + dx])}{a + b}} \sqrt{\frac{a (1 + \text{Sec}[c + dx])}{a - b}} - \frac{1}{d} \\ + b \sqrt{a + b} (2 b B + 5 a C) \text{Cot}[c + dx] \text{EllipticPi}\left[\frac{a + b}{b}, \text{ArcSin}\left[\frac{\sqrt{a + b \text{Cos}[c + dx]}}{\sqrt{a + b} \sqrt{\text{Cos}[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \text{Sec}[c + dx])}{a + b}} \sqrt{\frac{a (1 + \text{Sec}[c + dx])}{a - b}} + \\ \frac{2 (5 A b^2 + 10 a b B + a^2 (3 A + 5 C)) \sqrt{a + b \text{Cos}[c + dx]} \text{Sin}[c + dx]}{5 d \sqrt{\text{Cos}[c + dx]}} - \frac{(70 a b B + b^2 (46 A - 15 C) + 6 a^2 (3 A + 5 C)) \sqrt{a + b \text{Cos}[c + dx]} \text{Sin}[c + dx]}{15 d \sqrt{\text{Cos}[c + dx]}} + \\ \frac{2 (A b + a B) (a + b \text{Cos}[c + dx])^{3/2} \text{Sin}[c + dx]}{3 d \text{Cos}[c + dx]^{3/2}} + \frac{2 A (a + b \text{Cos}[c + dx])^{5/2} \text{Sin}[c + dx]}{5 d \text{Cos}[c + dx]^{5/2}}$$

Result (type 4, 1370 leaves) :

$$\begin{aligned}
& \frac{1}{30 d} \left( \left( 4 a \left( -16 a^2 A b + 16 A b^3 - 10 a^3 B - 20 a b^2 B - 60 a^2 b C - 15 b^3 C \right) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) + 4 a \left( 18 a^3 A + 46 a A b^2 + 70 a^2 b B - 30 b^3 B + 30 a^3 C - 90 a b^2 C \right) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) - \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \\
& \left. \left( b \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Cos}[c+d x]} \right) \right) - 2 \left( 18 a^2 A b + 46 A b^3 + 70 a b^2 B + 30 a^2 b C - 15 b^3 C \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}}} + \right. \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right) + \\
& \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2}{15} \operatorname{Sec}[c+dx]^2 (11 a A b \operatorname{Sin}[c+dx] + 5 a^2 B \operatorname{Sin}[c+dx]) + \right. \\
& \frac{2}{15} \operatorname{Sec}[c+dx] \\
& \left. (9 a^2 A \operatorname{Sin}[c+dx] + 23 A b^2 \operatorname{Sin}[c+dx] + 35 a b B \operatorname{Sin}[c+dx] + 15 a^2 C \operatorname{Sin}[c+dx]) + \frac{2}{5} \right)
\end{aligned}$$

$$a^2 A \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]$$

- **Problem 1134: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Cos}[c + d x])^{5/2} (A + B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2)}{\operatorname{Cos}[c + d x]^{9/2}} dx$$

Optimal (type 4, 580 leaves, 8 steps):

$$\begin{aligned} & \frac{1}{105 a^2 d} 2 (a - b) \sqrt{a + b} (15 A b^3 + 63 a^3 B + 161 a b^2 B + 5 a^2 b (29 A + 49 C)) \operatorname{Cot}[c + d x] \\ & \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Cos}[c + d x]}}{\sqrt{a + b} \sqrt{\operatorname{Cos}[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} - \frac{1}{105 a d} \\ & 2 \sqrt{a + b} (15 b^3 (A - 7 B) - a^3 (25 A - 63 B + 35 C) + a^2 b (145 A - 119 B + 245 C) - a b^2 (135 A - 161 B + 315 C)) \operatorname{Cot}[c + d x] \\ & \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Cos}[c + d x]}}{\sqrt{a + b} \sqrt{\operatorname{Cos}[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} - \frac{1}{d} \\ & 2 b^2 \sqrt{a + b} C \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a + b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Cos}[c + d x]}}{\sqrt{a + b} \sqrt{\operatorname{Cos}[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} + \\ & \frac{2 (15 A b^2 + 56 a b B + 5 a^2 (5 A + 7 C)) \sqrt{a + b \operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{105 d \operatorname{Cos}[c + d x]^{3/2}} + \\ & \frac{2 (5 A b + 7 a B) (a + b \operatorname{Cos}[c + d x])^{3/2} \operatorname{Sin}[c + d x]}{35 d \operatorname{Cos}[c + d x]^{5/2}} + \frac{2 A (a + b \operatorname{Cos}[c + d x])^{5/2} \operatorname{Sin}[c + d x]}{7 d \operatorname{Cos}[c + d x]^{7/2}} \end{aligned}$$

Result (type 4, 1472 leaves):

$$\begin{aligned} & \frac{1}{105 a d} \left( - \left( 4 a (25 a^4 A - 10 a^2 A b^2 - 15 A b^4 + 56 a^3 b B - 56 a b^3 B + 35 a^4 C + 70 a^2 b^2 C) \right. \right. \\ & \left. \left. \sqrt{\frac{(a + b) \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]^2}{-a + b}} \sqrt{-\frac{(a + b) \operatorname{Cos}[c + d x] \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}} \sqrt{\frac{(a + b \operatorname{Cos}[c + d x]) \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}} \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left. \left( \text{Csc}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \text{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \left. \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a \left( -145 a^3 A b - 15 a A b^3 - 63 a^4 B - 161 a^2 b^2 B - 245 a^3 b C + 105 a b^3 C \right) \right. \\
& \left. \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \text{Csc}[c+dx] \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \text{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \right. \\
& \left. \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \text{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \right. \\
& \left. \left. \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + 2 \left( -145 a^2 A b^2 - 15 A b^4 - 63 a^3 b B - 161 a b^3 B - 245 a^2 b^2 C \right) \right. \\
& \left. \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\text{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \text{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \text{Sec}[c+dx] \sqrt{\frac{(a+b \cos[c+dx]) \text{Sec}[c+dx]}{a+b}}} \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \Bigg) + \\
& \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2}{35} \operatorname{Sec}[c+dx]^3 (15 a A b \operatorname{Sin}[c+dx] + 7 a^2 B \operatorname{Sin}[c+dx]) + \right. \\
& \quad \frac{2}{105} \operatorname{Sec}[c+dx]^2 (25 a^2 A \operatorname{Sin}[c+dx] + 45 A b^2 \operatorname{Sin}[c+dx] + 77 a b B \operatorname{Sin}[c+dx] + 35 a^2 C \operatorname{Sin}[c+dx]) + \frac{1}{105 a} \\
& \quad 2 \operatorname{Sec}[c+dx] (145 a^2 A b \operatorname{Sin}[c+dx] + 15 A b^3 \operatorname{Sin}[c+dx] + 63 a^3 B \operatorname{Sin}[c+dx] + 161 a b^2 B \operatorname{Sin}[c+dx] + 245 a^2 b C \operatorname{Sin}[c+dx]) + \\
& \quad \left. \frac{2}{7} a^2 A \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx] \right)
\end{aligned}$$

- **Problem 1135: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Cos}[c+dx])^{5/2} (A+B \operatorname{Cos}[c+dx]+C \operatorname{Cos}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{11/2}} dx$$

Optimal (type 4, 552 leaves, 7 steps):



$$\begin{aligned}
& -\frac{1}{315 a^3 d} 2 (a-b) \sqrt{a+b} \left( 10 A b^4 - 435 a^3 b B - 45 a b^3 B - 21 a^4 (7 A + 9 C) - 3 a^2 b^2 (93 A + 161 C) \right) \\
& \quad \text{Cot}[c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \\
& \frac{1}{315 a^2 d} 2 (a-b) \sqrt{a+b} \left( 10 A b^3 + 15 a b^2 (11 A - 3 B + 21 C) - 6 a^2 b (19 A - 60 B + 28 C) + 3 a^3 (49 A - 25 B + 63 C) \right) \\
& \quad \text{Cot}[c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \\
& \frac{2 \left( 15 A b^2 + 90 a b B + 7 a^2 (7 A + 9 C) \right) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{315 d \cos [c+d x]^{5/2}} + \\
& \frac{2 \left( 5 A b^3 + 75 a^3 B + 135 a b^2 B + a^2 b (163 A + 231 C) \right) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{315 a d \cos [c+d x]^{3/2}} + \\
& \frac{2 \left( 5 A b + 9 a B \right) (a+b \cos [c+d x])^{3/2} \sin [c+d x]}{63 d \cos [c+d x]^{7/2}} + \frac{2 A (a+b \cos [c+d x])^{5/2} \sin [c+d x]}{9 d \cos [c+d x]^{9/2}}
\end{aligned}$$

Result (type 4, 1616 leaves):

$$\begin{aligned}
& -\frac{1}{315 a^2 d} \left( -\left( 4 a \left( -114 a^4 A b + 124 a^2 A b^3 - 10 A b^5 - 75 a^5 B + 30 a^3 b^2 B + 45 a b^4 B - 168 a^4 b C + 168 a^2 b^3 C \right) \right. \right. \\
& \quad \left. \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \csc [c+d x] \right. \\
& \quad \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right) / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \right. \\
& \quad \left. 4 a \left( 147 a^5 A + 279 a^3 A b^2 - 10 a A b^4 + 435 a^4 b B + 45 a^2 b^3 B + 189 a^5 C + 483 a^3 b^2 C \right) \right. \\
& \quad \left. \left( \left( \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \csc [c+d x] \right) \right. \right.
\end{aligned}$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right] / \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) -$$

$$\left(\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}}\sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\csc[c+dx]\right.$$

$$\left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right] / \left(b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) +$$

$$2(147a^4Ab + 279a^2Ab^3 - 10Ab^5 + 435a^3b^2B + 45ab^4B + 189a^4bC + 483a^2b^3C)$$

$$\left(\frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}}\right) +$$

$$\frac{1}{b}2a\left(\left(a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}}\sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\csc[c+dx]\right.\right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right] / \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) -$$

$$\left(a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}}\sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\right.$$

$$\left. \text{Csc}[c + dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \text{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left. \left. \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \text{Sin}[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right) +$$

$$\begin{aligned} & \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2}{63} \text{Sec}[c+dx]^4 (19 a A b \text{Sin}[c+dx] + 9 a^2 B \text{Sin}[c+dx]) + \right. \\ & \frac{2}{315} \text{Sec}[c+dx]^3 (49 a^2 A \text{Sin}[c+dx] + 75 A b^2 \text{Sin}[c+dx] + 135 a b B \text{Sin}[c+dx] + 63 a^2 C \text{Sin}[c+dx]) + \\ & \frac{1}{315 a} \\ & 2 \text{Sec}[c+dx]^2 (163 a^2 A b \text{Sin}[c+dx] + 5 A b^3 \text{Sin}[c+dx] + 75 a^3 B \text{Sin}[c+dx] + 135 a b^2 B \text{Sin}[c+dx] + 231 a^2 b C \text{Sin}[c+dx]) + \\ & \frac{1}{315 a^2} \\ & \left. 2 \text{Sec}[c+dx] (147 a^4 A \text{Sin}[c+dx] + 279 a^2 A b^2 \text{Sin}[c+dx] - 10 A b^4 \text{Sin}[c+dx] + 435 a^3 b B \text{Sin}[c+dx] + \right. \\ & \left. 45 a b^3 B \text{Sin}[c+dx] + 189 a^4 C \text{Sin}[c+dx] + 483 a^2 b^2 C \text{Sin}[c+dx]) + \frac{2}{9} a^2 A \text{Sec}[c+dx]^4 \text{Tan}[c+dx] \right) \end{aligned}$$

- **Problem 1136: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{3/2} (A + B \cos[c+dx] + C \cos[c+dx]^2)}{\sqrt{a+b \cos[c+dx]}} dx$$

Optimal (type 4, 593 leaves, 8 steps):

$$\begin{aligned}
& -\frac{1}{24ab^3d} (a-b)\sqrt{a+b} (24Ab^2 - 18abB + 15a^2C + 16b^2C) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{24b^3d} \sqrt{a+b} (24Ab^2 - 18abB + 12b^2B + 15a^2C - 10abC + 16b^2C) \\
& \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{8b^4d} \\
& \sqrt{a+b} (6a^2bB + 8b^3B - 5a^3C - 4ab^2(2A+C)) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(24Ab^2 - 18abB + 15a^2C + 16b^2C) \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{24b^3d\sqrt{\cos[c+dx]}} + \\
& \frac{(6bB - 5aC) \sqrt{\cos[c+dx]} \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{12b^2d} + \frac{C\cos[c+dx]^{3/2} \sqrt{a+b}\cos[c+dx] \sin[c+dx]}{3bd}
\end{aligned}$$

Result (type 4, 1241 leaves):

$$\begin{aligned}
& \frac{1}{48b^2d} \left( - \left( 4a(24Ab^2 - 6abB + 5a^2C + 16b^2C) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b)\cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b)\cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) \\
& \left( (a+b)\sqrt{\cos[c+dx]} \sqrt{a+b}\cos[c+dx] \right) - 4a(24b^2B + 4abC) \\
& \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b)\cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) + \\
& 2(24Ab^2 - 18abB + 15a^2C + 16b^2C) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}} \right) + \\
& \frac{1}{b}2a \left( \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right. \\
& \left. \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right) \right)
\end{aligned}$$

$$\left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left(b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}\right) \right) +$$

$$\left. \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left(\frac{(6bB-5aC) \sin[c+dx]}{12b^2} + \frac{C \sin[2(c+dx)]}{6b}\right)}{d}$$

■ **Problem 1137: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]} (A+B \cos[c+dx] + C \cos[c+dx]^2)}{\sqrt{a+b \cos[c+dx]}} dx$$

Optimal (type 4, 485 leaves, 7 steps):

$$-\frac{1}{4ab^2d} (a-b) \sqrt{a+b} (4bB-3aC) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{4b^2d}$$

$$\sqrt{a+b} (3aC-2b(2B+C)) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{4b^3d} \sqrt{a+b} (8Ab^2-4abB+3a^2C+4b^2C) \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}}$$

$$\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(4bB-3aC) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{4b^2d \sqrt{\cos[c+dx]}} + \frac{C \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2bd}$$

Result (type 4, 1182 leaves):

$$\frac{C \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2bd} + \frac{1}{8bd}$$

$$\begin{aligned}
& \left( - \left( 4 a (4 b B - a C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a \right. \right. \\
& (8 A b + 4 b C) \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \right. \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2 (4 b B - 3 a C) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right)
\end{aligned}$$

■ **Problem 1138: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2}{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]}} dx$$

Optimal (type 4, 401 leaves, 6 steps):



$$\begin{aligned}
& -\frac{1}{abd} (a-b) \sqrt{a+b} C \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{abd} \\
& \sqrt{a+b} (2Ab+aC) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{b^2 d} \\
& \sqrt{a+b} (2bB-aC) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{C \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{bd \sqrt{\cos[c+dx]}}
\end{aligned}$$

Result (type 4, 1117 leaves):

$$\begin{aligned}
& \frac{1}{2d} \left( - \left( 4a(2A+C) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left. 8aB \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left. \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right) \right)
\end{aligned}$$

$$\left( \text{EllipticPi} \left[ -\frac{a}{b}, \text{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right) / \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) +$$

$$2 C \left( \frac{i \cos \left[ \frac{1}{2} (c+d x) \right] \sqrt{a+b \cos [c+d x]} \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sin \left[ \frac{1}{2} (c+d x) \right]}{\sqrt{\cos [c+d x]}} \right], -\frac{2 a}{-a-b} \right] \sec [c+d x]}{b \sqrt{\cos \left[ \frac{1}{2} (c+d x) \right]^2} \sec [c+d x] \sqrt{\frac{(a+b \cos [c+d x]) \sec [c+d x]}{a+b}}} \right) +$$

$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \csc [c+d x] \right. \right.$$

$$\left. \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right) / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) -$$

$$\left( a \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \right.$$

$$\left. \left. \csc [c+d x] \text{EllipticPi} \left[ -\frac{a}{b}, \text{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right) /$$

$$\left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right)$$

■ **Problem 1139: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx] + C \cos[c + dx]^2}{\cos[c + dx]^{3/2} \sqrt{a + b \cos[c + dx]}} dx$$

Optimal (type 4, 347 leaves, 5 steps):

$$\begin{aligned} & \frac{1}{a^2 d} 2A (a - b) \sqrt{a + b} \cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \frac{1}{ad} \\ & 2\sqrt{a + b} (A - B) \cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \\ & \frac{1}{bd} 2\sqrt{a + b} C \cot[c + dx] \operatorname{EllipticPi}\left[\frac{a + b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} \end{aligned}$$

Result (type 4, 1169 leaves):

$$\begin{aligned} & \frac{2A \sqrt{a + b \cos[c + dx]} \sin[c + dx]}{ad \sqrt{\cos[c + dx]}} - \\ & \frac{1}{ad} \left( \left( 4a(Ab - aB) \sqrt{\frac{(a + b) \cot\left[\frac{1}{2}(c + dx)\right]^2}{-a + b}} \sqrt{-\frac{(a + b) \cos[c + dx] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \operatorname{Csc}[c + dx] \right. \right. \\ & \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a + b}\right] \sin\left[\frac{1}{2}(c + dx)\right]^4 \right) / \left( (a + b) \sqrt{\cos[c + dx]} \sqrt{a + b \cos[c + dx]} \right) - \right. \\ & \left. 4a(aA - aC) \left( \left( \sqrt{\frac{(a + b) \cot\left[\frac{1}{2}(c + dx)\right]^2}{-a + b}} \sqrt{-\frac{(a + b) \cos[c + dx] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}} \operatorname{Csc}[c + dx] \right. \right. \right. \\ & \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a + b}\right] \sin\left[\frac{1}{2}(c + dx)\right]^4 \right) / \left( (a + b) \sqrt{\cos[c + dx]} \sqrt{a + b \cos[c + dx]} \right) - \right. \end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& 2Ab \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /
\end{aligned}$$

$$\left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}}$$

■ **Problem 1140: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A+B \cos [c+d x]+C \cos [c+d x]^2}{\cos [c+d x]^{5/2} \sqrt{a+b \cos [c+d x]}} dx$$

Optimal (type 4, 293 leaves, 4 steps):

$$\begin{aligned} & -\frac{1}{3 a^3 d} \\ & 2(a-b) \sqrt{a+b} (2 A b-3 a B) \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \\ & \frac{1}{3 a^2 d} 2 \sqrt{a+b} (2 A b+a(A-3 B+3 C)) \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \\ & \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \frac{2 A \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{3 a d \cos [c+d x]^{3/2}} \end{aligned}$$

Result (type 4, 1244 leaves):

$$\begin{aligned} & \frac{1}{3 a^2 d} \left( \left( 4 a \left( a^2 A+2 A b^2-3 a b B+3 a^2 C \right) \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2}(c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc}\left[ \frac{1}{2}(c+d x) \right]^2}{a}} \right. \right. \\ & \left. \left. \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc}\left[ \frac{1}{2}(c+d x) \right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos [c+d x]) \operatorname{Csc}\left[ \frac{1}{2}(c+d x) \right]^2}}{a}\right],-\frac{2 a}{-a+b}\right] \sin \left[ \frac{1}{2}(c+d x) \right]^4 \right) \right) / \\ & \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - 4 a \left( 2 a A b-3 a^2 B \right) \end{aligned}$$

$$\begin{aligned}
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \left. \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) \right) + \\
& 2(2Ab^2 - 3abB) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\ \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\ \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \left. \right) + \\ \frac{\sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2 \operatorname{Sec}[c+dx] (-2Ab \operatorname{Sin}[c+dx] + 3aB \operatorname{Sin}[c+dx])}{3a^2} + \frac{2A \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{3a} \right)}{d}$$

■ **Problem 1141: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2}{\operatorname{Cos}[c+dx]^{7/2} \sqrt{a+b \operatorname{Cos}[c+dx]}} dx$$

Optimal (type 4, 372 leaves, 5 steps):

$$\frac{1}{15a^4d} 2(a-b) \sqrt{a+b} (8Ab^2 - 10abB + 3a^2(3A+5C)) \operatorname{Cot}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{15a^3d}$$

$$2\sqrt{a+b} (8Ab^2 - 2ab(A+5B) + a^2(9A-5B+15C)) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{2A\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{5ad \operatorname{Cos}[c+dx]^{5/2}} - \frac{2(4Ab-5aB)\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{15a^2d \operatorname{Cos}[c+dx]^{3/2}}$$

Result (type 4, 1351 leaves):

$$\begin{aligned}
& -\frac{1}{15 a^3 d} \left( \left( 4 a (7 a^2 A b + 8 A b^3 - 5 a^3 B - 10 a b^2 B + 15 a^2 b C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - 4 a (9 a^3 A + 8 a A b^2 - 10 a^2 b B + 15 a^3 C) \\
& \left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \\
& \left. 2 (9 a^2 A b + 8 A b^3 - 10 a b^2 B + 15 a^2 b C) \left( \frac{i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Cos}[c+dx]}}\right], -\frac{2 a}{-a-b}\right] \operatorname{Sec}[c+dx] \right. \right. \\
& \left. \left. b \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) \right) +
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right) + \\
& \quad \frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2 \operatorname{Sec}[c+dx]^2 (-4 A b \operatorname{Sin}[c+dx] + 5 a B \operatorname{Sin}[c+dx])}{15 a^2} + \right. \\
& \quad \left. \frac{2 \operatorname{Sec}[c+dx] (9 a^2 A \operatorname{Sin}[c+dx] + 8 A b^2 \operatorname{Sin}[c+dx] - 10 a b B \operatorname{Sin}[c+dx] + 15 a^2 C \operatorname{Sin}[c+dx])}{15 a^3} + \right. \\
& \quad \left. \left. \frac{2 A \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{5 a} \right) \right)
\end{aligned}$$

■ **Problem 1142: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2}{\operatorname{Cos}[c+dx]^{9/2} \sqrt{a+b \operatorname{Cos}[c+dx]}} dx$$

Optimal (type 4, 466 leaves, 6 steps):

$$\begin{aligned}
& -\frac{1}{105 a^5 d} 2 (a-b) \sqrt{a+b} (48 A b^3 - 63 a^3 B - 56 a b^2 B + a^2 (44 A b + 70 b C)) \\
& \quad \text{Cot}[c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \\
& \frac{1}{105 a^4 d} 2 \sqrt{a+b} (48 A b^3 - 4 a b^2 (3 A + 14 B) + a^3 (25 A - 63 B + 35 C) + 2 a^2 b (22 A + 7 (B + 5 C))) \text{Cot}[c+d x] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \frac{2 A \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{7 a d \cos [c+d x]^{7/2}} - \\
& \frac{2 (6 A b - 7 a B) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{35 a^2 d \cos [c+d x]^{5/2}} + \frac{2 (24 A b^2 - 28 a b B + 5 a^2 (5 A + 7 C)) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{105 a^3 d \cos [c+d x]^{3/2}}
\end{aligned}$$

Result (type 4, 1468 leaves):

$$\begin{aligned}
& \frac{1}{105 a^4 d} \left( \left( 4 a (25 a^4 A + 32 a^2 A b^2 + 48 A b^4 - 49 a^3 b B - 56 a b^3 B + 35 a^4 C + 70 a^2 b^2 C) \right. \right. \\
& \quad \left. \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \\
& \quad \left. \left. \text{Csc}[c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right]^4 \right) / \right. \\
& \quad \left. \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - 4 a (44 a^3 A b + 48 a A b^3 - 63 a^4 B - 56 a^2 b^2 B + 70 a^3 b C) \right. \\
& \quad \left. \left( \left( \sqrt{\frac{(a+b) \cot \left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2}(c+d x)\right]^2}{a}} \text{Csc}[c+d x] \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left. \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) + 2(44a^2Ab^2 + 48Ab^4 - 63a^3bB - 56ab^3B + 70a^2b^2C) \\
& \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\text{Sec}[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\text{Sec}[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\text{Sec}[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b}2a \left( \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \text{Csc}[c+dx] \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right.
\end{aligned}$$

$$\left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
\left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
\left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \Bigg) + \\
\frac{1}{d} \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \left( \frac{2 \operatorname{Sec}[c+dx]^3 (-6Ab \operatorname{Sin}[c+dx] + 7aB \operatorname{Sin}[c+dx])}{35a^2} + \right. \\
\left. \frac{2 \operatorname{Sec}[c+dx]^2 (25a^2 A \operatorname{Sin}[c+dx] + 24Ab^2 \operatorname{Sin}[c+dx] - 28abB \operatorname{Sin}[c+dx] + 35a^2 C \operatorname{Sin}[c+dx])}{105a^3} + \right. \\
\left. \frac{1}{105a^4} (2 \operatorname{Sec}[c+dx] (-44a^2 Ab \operatorname{Sin}[c+dx] - 48Ab^3 \operatorname{Sin}[c+dx] + 63a^3 B \operatorname{Sin}[c+dx] + 56ab^2 B \operatorname{Sin}[c+dx] - 70a^2 bC \operatorname{Sin}[c+dx]) + \right. \\
\left. \frac{2A \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx]}{7a} \right)$$

- **Problem 1143: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Cos}[c+dx]} (aA + (Ab+aB) \operatorname{Cos}[c+dx] + bB \operatorname{Cos}[c+dx]^2)}{\sqrt{a+b \operatorname{Cos}[c+dx]}} dx$$

Optimal (type 4, 473 leaves, 8 steps):

$$-\frac{1}{4abd}$$

$$\begin{aligned} & (a-b)\sqrt{a+b} (4Ab+aB) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\ & \frac{1}{4bd}\sqrt{a+b} (4Ab+(a+2b)B) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\ & \frac{1}{4b^2d}\sqrt{a+b} (4aAb-a^2B+4b^2B) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\cos[c+dx]}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \\ & \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{(4Ab+aB)\sqrt{a+b}\cos[c+dx]\sin[c+dx]}{4bd\sqrt{\cos[c+dx]}} + \frac{B\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx]\sin[c+dx]}{2d} \end{aligned}$$

Result (type 4, 1175 leaves):

$$\begin{aligned} & \frac{B\sqrt{\cos[c+dx]}\sqrt{a+b}\cos[c+dx]\sin[c+dx]}{2d} + \frac{1}{8d} \\ & \left( - \left( 4a(4Ab+3aB) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\ & \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}}{a\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - 4a \right. \\ & \left. (8aA+4bB) \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \right. \right. \\ & \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}}{a\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
& 2(4Ab + aB) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.
\end{aligned}$$

$$\left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}}$$

■ **Problem 1145: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{3/2} (A+B \cos[c+dx]+C \cos[c+dx]^2)}{(a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 660 leaves, 8 steps):

$$\begin{aligned} & -\frac{1}{4ab^3 \sqrt{a+b} d} (12a^2bB - 4b^3B - ab^2(8A-7C) - 15a^3C) \operatorname{Cot}[c+dx] \\ & \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{4b^3 \sqrt{a+b} d} \\ & (8Ab^2 - ab(12B-5C) + 15a^2C - 2b^2(2B+C)) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\ & \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{4b^4 d} \sqrt{a+b} (8Ab^2 - 12abB + 15a^2C + 4b^2C) \operatorname{Cot}[c+dx] \\ & \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\ & \frac{2(Ab^2 - a(bB - aC)) \cos[c+dx]^{3/2} \sin[c+dx]}{b(a^2 - b^2) d \sqrt{a+b \cos[c+dx]}} + \frac{(12a^2bB - 4b^3B - ab^2(8A-7C) - 15a^3C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{4b^3(a^2 - b^2) d \sqrt{\cos[c+dx]}} + \\ & \frac{(4Ab^2 - 4abB + 5a^2C - b^2C) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{2b^2(a^2 - b^2) d} \end{aligned}$$

Result (type 4, 1322 leaves):

$$\frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{c \sin[c+dx]}{2b^2} - \frac{2(aAb^2 \sin[c+dx] - a^2bB \sin[c+dx] + a^3C \sin[c+dx])}{b^2(-a^2+b^2)(a+b \cos[c+dx])} \right)}{d}$$

$$\begin{aligned}
& \frac{1}{8(a-b)b^2(a+b)d} \left( - \left( 4a(-4a^2bB + 4b^3B + 5a^3C - 5ab^2C) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - 4a(8Ab^3 - 8ab^2B + 4a^2bC + 4b^3C) \\
& \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \csc[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left. \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) + 2(8aAb^2 - 12a^2bB + 4b^3B + 15a^3C - 7ab^2C)
\end{aligned}$$



$$\left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}}}} + \right.$$

$$\left. \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx]} \right. \right. \right.$$

$$\left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right. \right.$$

$$\left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right.$$

$$\left. \left. \left. \csc[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \right.$$

$$\left. \left. \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right) \right)$$

■ **Problem 1146: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]} (A + B \cos[c+dx] + C \cos[c+dx]^2)}{(a + b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 535 leaves, 7 steps):

$$-\frac{1}{a b^2 \sqrt{a+b} d}$$

$$(2 A b^2 - 2 a b B + 3 a^2 C - b^2 C) \cot[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+d x]}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} +$$

$$\frac{1}{a b^2 \sqrt{a+b} d} (2 A b^2 - a(b(2 B - C) - 3 a C)) \cot[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+d x]}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} - \frac{1}{b^3 d}$$

$$\sqrt{a+b} (2 b B - 3 a C) \cot[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+d x]}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} -$$

$$\frac{2(A b^2 - a(b B - a C)) \sqrt{\cos[c+d x]} \sin[c+d x]}{b(a^2 - b^2) d \sqrt{a+b} \cos[c+d x]} + \frac{(2 A b^2 - 2 a b B + 3 a^2 C - b^2 C) \sqrt{a+b} \cos[c+d x] \sin[c+d x]}{b^2(a^2 - b^2) d \sqrt{\cos[c+d x]}}$$

Result (type 4, 1256 leaves):

$$\frac{2 \sqrt{\cos[c+d x]} (A b^2 \sin[c+d x] - a b B \sin[c+d x] + a^2 C \sin[c+d x])}{b(-a^2 + b^2) d \sqrt{a+b} \cos[c+d x]} +$$

$$\frac{1}{2(a-b) b(a+b) d} \left( - \left( 4 a(a^2 C - b^2 C) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b) \cos[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b) \cos[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+d x)\right]^4 \right) \right/$$

$$\left( (a+b) \sqrt{\cos[c+d x]} \sqrt{a+b} \cos[c+d x] \right) - 4 a(2 a A b - 2 b^2 B + 2 a b C)$$

$$\left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b) \cos[c+d x] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \operatorname{Csc}[c+d x] \right. \right.$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) + \\
& 2(2Ab^2 - 2abB + 3a^2C - b^2C) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}} \right. \\
& \left. \frac{1}{b}2a \left( \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right. \right. \\
& \left. \left. \left( a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \right.
\end{aligned}$$

$$\left( \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \text{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) + \frac{\sqrt{a+b\cos[c+dx]}\text{Sin}[c+dx]}{b\sqrt{\cos[c+dx]}}$$

■ **Problem 1147: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c+dx] + C \cos[c+dx]^2}{\sqrt{\cos[c+dx]} (a + b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 436 leaves, 6 steps):

$$\frac{1}{a^2 b \sqrt{a+b} d}$$

$$2 (Ab^2 - a(bB - aC)) \text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} +$$

$$\frac{1}{ab\sqrt{a+b}d} 2 (Ab + bB - aC) \text{Cot}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} -$$

$$\frac{1}{b^2 d} 2 \sqrt{a+b} C \text{Cot}[c+dx] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} -$$

$$\frac{2 (Ab^2 - a(bB - aC)) \text{Sin}[c+dx]}{b (a^2 - b^2) d \sqrt{\cos[c+dx]} \sqrt{a+b\cos[c+dx]}}$$

Result (type 4, 1245 leaves):

$$\frac{2 \sqrt{\cos[c+dx]} (Ab^2 \text{Sin}[c+dx] - a b B \text{Sin}[c+dx] + a^2 C \text{Sin}[c+dx])}{a (a^2 - b^2) d \sqrt{a+b\cos[c+dx]}} +$$

$$\begin{aligned}
& \frac{1}{a(a-b)(a+b)d} \left( - \left( 4a(a^2A - Ab^2) \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \\
& \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - 4a(-aAb + a^2B - abC) \\
& \left( \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) - \right. \\
& \left( \sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]} \right) \right) + \\
& 2(-Ab^2 + abB - a^2C) \left( \frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) - \right. \\
& \quad \left( a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \operatorname{Cos}[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right. \\
& \quad \left. \left. \left( b \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Cos}[c+dx]} \right) + \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{b \sqrt{\operatorname{Cos}[c+dx]}} \right) \right)
\end{aligned}$$

■ **Problem 1148: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2}{\operatorname{Cos}[c+dx]^{3/2} (a+b \operatorname{Cos}[c+dx])^{3/2}} dx$$

Optimal (type 4, 322 leaves, 4 steps):

$$-\frac{1}{a^3 \sqrt{a+b} d} 2 (2 A b^2 - a b B - a^2 (A - C)) \operatorname{Cot}[c + d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c + d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c + d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c + d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c + d x])}{a-b}} - \frac{1}{a^2 \sqrt{a+b} d}$$

$$2 (2 A b + a (A - B - C)) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c + d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c + d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c + d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c + d x])}{a-b}} +$$

$$\frac{2 (A b^2 - a (b B - a C)) \operatorname{Sin}[c + d x]}{a (a^2 - b^2) d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + b \operatorname{Cos}[c + d x]}}$$

Result (type 4, 1306 leaves):

$$\frac{1}{a^2 (-a+b) (a+b) d} \left( - \left( 4 a (2 a^2 A b - 2 A b^3 - a^3 B + a b^2 B) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c + d x] \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c + d x]) \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c + d x]) \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}}{a}\right]}, -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]^4 \right) /$$

$$\left( (a+b) \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + b \operatorname{Cos}[c + d x]} \right) - 4 a (a^3 A - 2 a A b^2 + a^2 b B - a^3 C)$$

$$\left( \left( \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c + d x] \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}} \sqrt{\frac{(a+b \operatorname{Cos}[c + d x]) \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{a}} \operatorname{Csc}[c + d x] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c + d x]) \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2}}{a}\right]}, -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]^4 \right) / \left( (a+b) \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + b \operatorname{Cos}[c + d x]} \right) -$$

$$\begin{aligned}
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \Bigg) + \\
& 2(a^2 A b - 2 A b^3 + a b^2 B - a^2 b C) \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /
\end{aligned}$$



$$\left( \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \frac{\sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( -\frac{2(Ab^3 \sin[c+dx] - ab^2 B \sin[c+dx] + a^2 b C \sin[c+dx])}{a^2(a^2 - b^2)(a+b \cos[c+dx])} + \frac{2A \tan[c+dx]}{a^2} \right)}{d}$$

■ **Problem 1149: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c+dx] + C \cos[c+dx]^2}{\cos[c+dx]^{5/2} (a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 424 leaves, 5 steps):

$$\frac{1}{3a^4 \sqrt{a+b}} 2(8Ab^3 + 3a^3B - 6ab^2B - a^2(5Ab - 3bC)) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1 - \sec[c+dx])}{a+b}} \sqrt{\frac{a(1 + \sec[c+dx])}{a-b}} + \frac{1}{3a^3 \sqrt{a+b} d} 2(8Ab^2 + 6ab(A-B) + a^2(A - 3B + 3C))$$

$$\cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1 - \sec[c+dx])}{a+b}} \sqrt{\frac{a(1 + \sec[c+dx])}{a-b}} +$$

$$\frac{2(Ab^2 - a(bB - aC)) \sin[c+dx]}{a(a^2 - b^2) d \cos[c+dx]^{3/2} \sqrt{a+b \cos[c+dx]}} - \frac{2(4Ab^2 - 3abB - a^2(A - 3C)) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{3a^2(a^2 - b^2) d \cos[c+dx]^{3/2}}$$

Result (type 4, 1402 leaves):

$$\frac{1}{3a^3(a-b)(a+b)d}$$

$$\left( - \left( 4a(a^4A + 7a^2Ab^2 - 8Ab^4 - 6a^3bB + 6ab^3B + 3a^4C - 3a^2b^2C) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) /$$

$$\begin{aligned}
& \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a \left( 5a^3 A b - 8a A b^3 - 3a^4 B + 6a^2 b^2 B - 3a^3 b C \right) \\
& \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \quad \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + 2 \left( 5a^2 A b^2 - 8A b^4 - 3a^3 b B + 6a b^3 B - 3a^2 b^2 C \right) \\
& \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.
\end{aligned}$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) -$$

$$\left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right.$$

$$\left. \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) +$$

$$\left. \left. \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) + \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right)$$

$$\left( \frac{2 \sec[c+dx] (-5Ab \sin[c+dx] + 3aB \sin[c+dx])}{3a^3} + \frac{2(Ab^4 \sin[c+dx] - ab^3B \sin[c+dx] + a^2b^2C \sin[c+dx])}{a^3(a^2 - b^2)(a+b \cos[c+dx])} + \frac{2A \sec[c+dx] \tan[c+dx]}{3a^2} \right)$$

■ **Problem 1150: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c+dx] + C \cos[c+dx]^2}{\cos[c+dx]^{7/2} (a+b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 545 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{1}{15 a^5 \sqrt{a+b} d} 2 \left( 48 A b^4 + 25 a^3 b B - 40 a b^3 B - 6 a^2 b^2 (4 A - 5 C) - 3 a^4 (3 A + 5 C) \right) \\
 & \quad \text{Cot}[c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\text{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+d x])}{a-b}} - \\
 & \frac{1}{15 a^4 \sqrt{a+b} d} 2 \left( 48 A b^3 + 4 a b^2 (9 A - 10 B) + 6 a^2 b (2 A - 5 B + 5 C) + a^3 (9 A - 5 B + 15 C) \right) \text{Cot}[c+d x] \\
 & \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\text{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+d x])}{a-b}} + \\
 & \frac{2(A b^2 - a(b B - a C)) \text{Sin}[c+d x]}{a(a^2 - b^2) d \text{Cos}[c+d x]^{5/2} \sqrt{a+b \text{Cos}[c+d x]}} - \frac{2(6 A b^2 - 5 a b B - a^2(A - 5 C)) \sqrt{a+b \text{Cos}[c+d x]} \text{Sin}[c+d x]}{5 a^2(a^2 - b^2) d \text{Cos}[c+d x]^{5/2}} + \\
 & \frac{2(24 A b^3 + 5 a^3 B - 20 a b^2 B - a^2(9 A b - 15 b C)) \sqrt{a+b \text{Cos}[c+d x]} \text{Sin}[c+d x]}{15 a^3(a^2 - b^2) d \text{Cos}[c+d x]^{3/2}}
 \end{aligned}$$

Result (type 4, 1511 leaves):

$$\begin{aligned}
 & \frac{1}{15 a^4 (-a+b)(a+b) d} \left( - \left( 4 a \left( 12 a^4 A b + 36 a^2 A b^3 - 48 A b^5 - 5 a^5 B - 35 a^3 b^2 B + 40 a b^4 B + 30 a^4 b C - 30 a^2 b^3 C \right) \right. \right. \\
 & \quad \left. \left. \sqrt{\frac{(a+b) \text{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \text{Cos}[c+d x] \text{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \text{Cos}[c+d x]) \text{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \text{Csc}[c+d x] \right. \right. \\
 & \quad \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \text{Cos}[c+d x]) \text{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \text{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\text{Cos}[c+d x]} \sqrt{a+b \text{Cos}[c+d x]} \right) - \right. \\
 & \quad \left. 4 a \left( 9 a^5 A + 24 a^3 A b^2 - 48 a A b^4 - 25 a^4 b B + 40 a^2 b^3 B + 15 a^5 C - 30 a^3 b^2 C \right) \right. \\
 & \quad \left. \left( \left( \sqrt{\frac{(a+b) \text{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \text{Cos}[c+d x] \text{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \text{Cos}[c+d x]) \text{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \text{Csc}[c+d x] \right. \right. \right.
 \end{aligned}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right] / \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) -$$

$$\left(\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}}\sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\csc[c+dx]\right.$$

$$\left.\text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right] / \left(b\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right)\right] +$$

$$2(9a^4Ab + 24a^2Ab^3 - 48Ab^5 - 25a^3b^2B + 40ab^4B + 15a^4bC - 30a^2b^3C)$$

$$\left(\frac{i\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{a+b\cos[c+dx]}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right]\sec[c+dx]}{b\sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2}\sec[c+dx]}\sqrt{\frac{(a+b\cos[c+dx])\sec[c+dx]}{a+b}}\right) +$$

$$\frac{1}{b}2a\left(\left(a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}}\sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\csc[c+dx]\right.\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right]\sin\left[\frac{1}{2}(c+dx)\right]^4\right] / \left((a+b)\sqrt{\cos[c+dx]}\sqrt{a+b\cos[c+dx]}\right) -$$

$$\left(a\sqrt{\frac{(a+b)\cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}}\sqrt{-\frac{(a+b)\cos[c+dx]\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\sqrt{\frac{(a+b\cos[c+dx])\csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}\right)$$

$$\begin{aligned}
& \left. \text{Csc}[c + dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \text{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \text{Sin}[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \\
& \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2 \text{Sec}[c+dx]^2 (-9Ab \text{Sin}[c+dx] + 5aB \text{Sin}[c+dx])}{15a^3} + \right. \\
& \frac{2 \text{Sec}[c+dx] (9a^2A \text{Sin}[c+dx] + 33Ab^2 \text{Sin}[c+dx] - 25abB \text{Sin}[c+dx] + 15a^2C \text{Sin}[c+dx])}{15a^4} - \\
& \frac{2 (Ab^5 \text{Sin}[c+dx] - ab^4B \text{Sin}[c+dx] + a^2b^3C \text{Sin}[c+dx])}{a^4 (a^2 - b^2) (a + b \cos[c+dx])} + \\
& \left. \frac{2A \text{Sec}[c+dx]^2 \text{Tan}[c+dx]}{5a^2} \right)
\end{aligned}$$

- **Problem 1151: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{3/2} (A + B \cos[c+dx] + C \cos[c+dx]^2)}{(a + b \cos[c+dx])^{5/2}} dx$$

Optimal (type 4, 723 leaves, 8 steps):

$$\frac{1}{3 a (a-b) b^3 (a+b)^{3/2} d} (8 A b^4 + 6 a^3 b B - 14 a b^3 B - 15 a^4 C + 26 a^2 b^2 C - 3 b^4 C)$$

$$\text{Cot}[c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} -$$

$$\frac{1}{3 a b^3 \sqrt{a+b} (a^2-b^2) d} (6 A b^4 - a b^3 (2 A + 3 (4 B - C)) + a^3 b (6 B - 5 C) - 15 a^4 C + a^2 b^2 (2 B + 21 C)) \text{Cot}[c+d x]$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \frac{1}{b^4 d}$$

$$\sqrt{a+b} (2 b B - 5 a C) \text{Cot}[c+d x] \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} -$$

$$\frac{2 (A b^2 - a (b B - a C)) \cos [c+d x]^{3/2} \sin [c+d x]}{3 b (a^2 - b^2) d (a+b \cos [c+d x])^{3/2}} + \frac{2 (3 A b^4 + 2 a^3 b B - 6 a b^3 B - 5 a^4 C + a^2 b^2 (A + 9 C)) \sqrt{\cos [c+d x]} \sin [c+d x]}{3 b^2 (a^2 - b^2)^2 d \sqrt{a+b \cos [c+d x]}} -$$

$$\frac{(8 A b^4 + 6 a^3 b B - 14 a b^3 B - 15 a^4 C + 26 a^2 b^2 C - 3 b^4 C) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{3 b^3 (a^2 - b^2)^2 d \sqrt{\cos [c+d x]}}$$

Result (type 4, 1448 leaves):

$$\frac{1}{d} \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left( -\frac{2 (a A b^2 \sin [c+d x] - a^2 b B \sin [c+d x] + a^3 C \sin [c+d x])}{3 b^2 (-a^2 + b^2) (a+b \cos [c+d x])^2} + \right.$$

$$\left. \frac{(2 (4 A b^4 \sin [c+d x] + 3 a^3 b B \sin [c+d x] - 7 a b^3 B \sin [c+d x] - 6 a^4 C \sin [c+d x] + 10 a^2 b^2 C \sin [c+d x]))}{(3 b^2 (-a^2 + b^2)^2 (a+b \cos [c+d x]))} + \frac{1}{6 (a-b)^2 b^2 (a+b)^2 d} \right)$$

$$\left( -\left( \left( 4 a (2 a^2 A b^2 - 2 A b^4 - 2 a^3 b B + 2 a b^3 B + 5 a^4 C - 8 a^2 b^2 C + 3 b^4 C) \sqrt{\frac{(a+b) \cot \left[\frac{1}{2} (c+d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos [c+d x] \csc \left[\frac{1}{2} (c+d x)\right]^2}{a}} \right. \right. \right.$$

$$\left. \left. \left. \sqrt{\frac{(a+b \cos [c+d x]) \csc \left[\frac{1}{2} (c+d x)\right]^2}{a}} \csc [c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(a+b \cos [c+d x]) \csc \left[\frac{1}{2} (c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \sin \left[\frac{1}{2} (c+d x)\right]^4 \right) \right) \right) /$$

$$\begin{aligned}
& \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a \left( -8aAb^3 + 2a^2b^2B + 6b^4B + 4a^3bC - 12ab^3C \right) \\
& \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right. \\
& \quad \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \csc[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + 2 \left( -8Ab^4 - 6a^3bB + 14ab^3B + 15a^4C - 26a^2b^2C + 3b^4C \right) \\
& \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} \right. \\
& \quad \left. + \frac{1}{b} 2a \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right.
\end{aligned}$$



$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) -$$

$$\left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}$$

$$\text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/$$

$$\left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right)$$

- **Problem 1152: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]} (A + B \cos[c+dx] + C \cos[c+dx]^2)}{(a + b \cos[c+dx])^{5/2}} dx$$

Optimal (type 4, 589 leaves, 7 steps):

$$\begin{aligned}
& - \frac{1}{3 a^2 b^2 \sqrt{a+b} (a^2 - b^2) d} 2 (A b^4 - 4 a b^3 B - 3 a^4 C + a^2 b^2 (3 A + 7 C)) \operatorname{Cot}[c + d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c + d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c + d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a+b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a-b}} - \\
& \frac{1}{3 a b^2 \sqrt{a+b} (a^2 - b^2) d} 2 (b^3 (A + 3 B) + 3 a^3 C + a^2 b C - a b^2 (3 A + B + 6 C)) \operatorname{Cot}[c + d x] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c + d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c + d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a+b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a-b}} - \frac{1}{b^3 d} \\
& 2 \sqrt{a+b} C \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c + d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c + d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a+b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a-b}} - \\
& \frac{2 (A b^2 - a (b B - a C)) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{3 b (a^2 - b^2) d (a + b \operatorname{Cos}[c + d x])^{3/2}} + \frac{2 (A b^4 - 4 a b^3 B - 3 a^4 C + a^2 b^2 (3 A + 7 C)) \operatorname{Sin}[c + d x]}{3 b^2 (a^2 - b^2)^2 d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + b \operatorname{Cos}[c + d x]}}
\end{aligned}$$

Result (type 4, 1441 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + b \operatorname{Cos}[c + d x]} \left( \frac{2 (A b^2 \operatorname{Sin}[c + d x] - a b B \operatorname{Sin}[c + d x] + a^2 C \operatorname{Sin}[c + d x])}{3 b (-a^2 + b^2) (a + b \operatorname{Cos}[c + d x])^2} + \right. \\
& \left. (2 (-3 a^2 A b^2 \operatorname{Sin}[c + d x] - A b^4 \operatorname{Sin}[c + d x] + 4 a b^3 B \operatorname{Sin}[c + d x] + 3 a^4 C \operatorname{Sin}[c + d x] - 7 a^2 b^2 C \operatorname{Sin}[c + d x])) / \right. \\
& \left. (3 a b (a^2 - b^2)^2 (a + b \operatorname{Cos}[c + d x])) \right) - \frac{1}{3 a (a - b)^2 b (a + b)^2 d} \\
& \left( \left( \left( 4 a (a^2 A b^2 - A b^4 - a^3 b B + a b^3 B + a^4 C - a^2 b^2 C) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \operatorname{Cos}[c + d x] \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{a}} \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{(a+b \operatorname{Cos}[c + d x]) \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{a}} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \operatorname{Cos}[c + d x]) \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^2}}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]^4 \right) \right) \right) / \\
& \left( (a + b) \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + b \operatorname{Cos}[c + d x]} \right) - 4 a (-3 a^3 A b - a A b^3 + 4 a^2 b^2 B - a^3 b C - 3 a b^3 C)
\end{aligned}$$

$$\left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \right.$$

$$\left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.$$

$$\left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right.$$

$$\left. \left. \csc[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.$$

$$\left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + 2(-3a^2Ab^2 - Ab^4 + 4ab^3B + 3a^4C - 7a^2b^2C) \right)$$

$$\left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \sec[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \sec[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \sec[c+dx]}{a+b}} + \right.$$

$$\left. \frac{1}{b} 2a \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right) \right)$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right/ \left( \right. \\
& \left. \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right)
\end{aligned}$$

■ **Problem 1153: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c+dx] + C \cos[c+dx]^2}{\sqrt{\cos[c+dx]} (a+b \cos[c+dx])^{5/2}} dx$$

Optimal (type 4, 457 leaves, 5 steps):

$$\begin{aligned}
& \frac{1}{3a^3(a-b)(a+b)^{3/2}d} {}_2F_1\left(6a^2Ab - 2Ab^3 - 3a^3B - ab^2B + 4a^2bC, \cot[c+dx], \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]\right) \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{3a^2 \sqrt{a+b} (a^2-b^2)d} {}_2F_1\left(2Ab^2 - a^2(3A+3B+C) + ab(3A+B+3C), \cot[c+dx], \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}}\right) + \\
& \frac{2(Ab^2 - a(bB - aC)) \sqrt{\cos[c+dx]} \sin[c+dx]}{3a(a^2-b^2)d(a+b \cos[c+dx])^{3/2}} + \frac{2(2Ab^3 + 3a^3B + ab^2B - 2a^2b(3A+2C)) \sin[c+dx]}{3a(a^2-b^2)^2d \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}}
\end{aligned}$$

Result (type 4, 1440 leaves) :

$$\frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \left( \frac{2 (A b^2 \sin[c+dx] - a b B \sin[c+dx] + a^2 C \sin[c+dx])}{3 a (a^2 - b^2) (a + b \cos[c+dx])^2} - \right.$$

$$\left. (2 (-6 a^2 A b^2 \sin[c+dx] + 2 A b^4 \sin[c+dx] + 3 a^3 b B \sin[c+dx] + a b^3 B \sin[c+dx] - 4 a^2 b^2 C \sin[c+dx])) / \right.$$

$$\left. (3 a^2 (a^2 - b^2)^2 (a + b \cos[c+dx])) \right) + \frac{1}{3 a^2 (a - b)^2 (a + b)^2 d}$$

$$\left( - \left( \left( 4 a (3 a^4 A - 5 a^2 A b^2 + 2 A b^4 - a^3 b B + a b^3 B + a^4 C - a^2 b^2 C) \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \right. \right.$$

$$\left. \left. \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \right.$$

$$\left. ((a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}) - 4 a (-6 a^3 A b + 2 a A b^3 + 3 a^4 B + a^2 b^2 B - 4 a^3 b C) \right.$$

$$\left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / ((a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]}) - \right.$$

$$\left. \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right) \right)$$

$$\left( \left( \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right.$$

$$\left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + 2(-6a^2 A b^2 + 2A b^4 + 3a^3 b B + a b^3 B - 4a^2 b^2 C)$$

$$\left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \text{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \text{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \text{Sec}[c+dx]}{a+b}}}\right) +$$

$$\frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \text{Csc}[c+dx] \right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) -$$

$$\left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right.$$

$$\left. \text{Csc}[c+dx] \text{EllipticPi}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) /$$

$$\left( \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right)$$

■ **Problem 1154: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c+dx] + C \cos[c+dx]^2}{\cos[c+dx]^{3/2} (a + b \cos[c+dx])^{5/2}} dx$$

Optimal (type 4, 495 leaves, 5 steps):

$$\frac{1}{3 a^4 \sqrt{a+b} (a^2 - b^2) d} - 2 (8 A b^4 + 6 a^3 b B - 2 a b^3 B + 3 a^4 (A - C) - a^2 b^2 (15 A + C))$$

$$\cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{3 a^3 \sqrt{a+b} (a^2 - b^2) d} - 2 (8 A b^3 + 2 a b^2 (3 A - B) - 3 a^3 (A - B - C) - a^2 b (9 A + 3 B + C)) \cot[c+dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2 (A b^2 - a (b B - a C)) \sin[c+dx]}{3 a (a^2 - b^2) d \sqrt{\cos[c+dx]} (a + b \cos[c+dx])^{3/2}} - \frac{2 (4 A b^4 + 5 a^3 b B - a b^3 B - 2 a^4 C - 2 a^2 b^2 (4 A + C)) \sin[c+dx]}{3 a^2 (a^2 - b^2)^2 d \sqrt{\cos[c+dx]} \sqrt{a + b \cos[c+dx]}}$$

Result (type 4, 1516 leaves):

$$-\frac{1}{3 a^3 (a-b)^2 (a+b)^2 d} \left( \left( 4 a (9 a^4 A b - 17 a^2 A b^3 + 8 A b^5 - 3 a^5 B + 5 a^3 b^2 B - 2 a b^4 B + a^4 b C - a^2 b^3 C) \right. \right.$$

$$\sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}$$

$$\left. \left. \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \right)$$

$$\begin{aligned}
& \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - 4a \left( 3a^5 A - 15a^3 A b^2 + 8a A b^4 + 6a^4 b B - 2a^2 b^3 B - 3a^5 C - a^3 b^2 C \right) \\
& \left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right. \\
& \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \\
& \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \right. \\
& \quad \left. \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \\
& \left. \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) + 2 \left( 3a^4 A b - 15a^2 A b^3 + 8A b^5 + 6a^3 b^2 B - 2a b^4 B - 3a^4 b C - a^2 b^3 C \right) \\
& \left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}}} \right) + \\
& \frac{1}{b} 2a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right] / \left( (a+b) \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) - \\
& \left( a \sqrt{\frac{(a+b) \cot \left[ \frac{1}{2} (c+d x) \right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos [c+d x] \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}} \right. \\
& \left. \operatorname{Csc} [c+d x] \operatorname{EllipticPi} \left[ -\frac{a}{b}, \text{ArcSin} \left[ \frac{\sqrt{\frac{(a+b \cos [c+d x]) \operatorname{Csc} \left[ \frac{1}{2} (c+d x) \right]^2}{a}}}{\sqrt{2}} \right], -\frac{2 a}{-a+b} \right] \sin \left[ \frac{1}{2} (c+d x) \right]^4 \right] / \right. \\
& \left. \left( b \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \right) + \frac{\sqrt{a+b \cos [c+d x]} \sin [c+d x]}{b \sqrt{\cos [c+d x]}} \right) \left. \right) + \frac{1}{d} \\
& \sqrt{\cos [c+d x]} \sqrt{a+b \cos [c+d x]} \left( -\frac{2 (A b^3 \sin [c+d x] - a b^2 B \sin [c+d x] + a^2 b C \sin [c+d x])}{3 a^2 (a^2 - b^2) (a+b \cos [c+d x])^2} - \right. \\
& \left. (2 (9 a^2 A b^3 \sin [c+d x] - 5 A b^5 \sin [c+d x] - 6 a^3 b^2 B \sin [c+d x] + 2 a b^4 B \sin [c+d x] + 3 a^4 b C \sin [c+d x] + a^2 b^3 C \sin [c+d x])) / \right. \\
& \left. (3 a^3 (a^2 - b^2)^2 (a+b \cos [c+d x])) + \frac{2 A \tan [c+d x]}{a^3} \right)
\end{aligned}$$

- **Problem 1155: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos [c+d x] + C \cos [c+d x]^2}{\cos [c+d x]^{5/2} (a+b \cos [c+d x])^{5/2}} dx$$

Optimal (type 4, 620 leaves, 6 steps):

$$\begin{aligned}
& - \frac{1}{3 a^5 \sqrt{a+b} (a^2-b^2) d} 2 (16 A b^5 - 3 a^5 B + 15 a^3 b^2 B - 8 a b^4 B - 2 a^2 b^3 (14 A - C) + a^4 (8 A b - 6 b C)) \\
& \quad \text{Cot}[c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\text{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+d x])}{a-b}} - \\
& \frac{1}{3 a^4 \sqrt{a+b} (a^2-b^2) d} 2 (16 A b^4 + 4 a b^3 (3 A - 2 B) - 3 a^3 b (3 A - 3 B - C) - 2 a^2 b^2 (8 A + 3 B - C) - a^4 (A - 3 B + 3 C)) \\
& \quad \text{Cot}[c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\text{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+d x])}{a-b}} + \\
& \frac{2 (A b^2 - a (b B - a C)) \text{Sin}[c+d x]}{3 a (a^2-b^2) d \text{Cos}[c+d x]^{3/2} (a+b \text{Cos}[c+d x])^{3/2}} + \frac{2 (10 a^2 A b^2 - 6 A b^4 - 7 a^3 b B + 3 a b^3 B + 4 a^4 C) \text{Sin}[c+d x]}{3 a^2 (a^2-b^2)^2 d \text{Cos}[c+d x]^{3/2} \sqrt{a+b \text{Cos}[c+d x]}} + \\
& \frac{2 (8 A b^4 + 8 a^3 b B - 4 a b^3 B + a^4 (A - 5 C) - a^2 b^2 (13 A - C)) \sqrt{a+b \text{Cos}[c+d x]} \text{Sin}[c+d x]}{3 a^3 (a^2-b^2)^2 d \text{Cos}[c+d x]^{3/2}}
\end{aligned}$$

Result (type 4, 1601 leaves):

$$\begin{aligned}
& \frac{1}{3 a^4 (a-b)^2 (a+b)^2 d} \\
& \left( - \left( 4 a (a^6 A + 15 a^4 A b^2 - 32 a^2 A b^4 + 16 A b^6 - 9 a^5 b B + 17 a^3 b^3 B - 8 a b^5 B + 3 a^6 C - 5 a^4 b^2 C + 2 a^2 b^4 C) \sqrt{\frac{(a+b) \text{Cot}\left[\frac{1}{2}(c+d x)\right]^2}{-a+b}} \right. \right. \\
& \quad \left. \sqrt{\frac{(a+b) \text{Cos}[c+d x] \text{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \sqrt{\frac{(a+b \text{Cos}[c+d x]) \text{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{a}} \text{Csc}[c+d x] \right. \\
& \quad \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(a+b \text{Cos}[c+d x]) \text{Csc}\left[\frac{1}{2}(c+d x)\right]^2}}{a}\right], -\frac{2 a}{-a+b}\right] \text{Sin}\left[\frac{1}{2}(c+d x)\right]^4 \right) / \left( (a+b) \sqrt{\text{Cos}[c+d x]} \sqrt{a+b \text{Cos}[c+d x]} \right) - \right. \\
& \quad \left. 4 a (8 a^5 A b - 28 a^3 A b^3 + 16 a A b^5 - 3 a^6 B + 15 a^4 b^2 B - 8 a^2 b^4 B - 6 a^5 b C + 2 a^3 b^3 C) \right)
\end{aligned}$$

$$\left( \left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.$$

$$\left( \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right.$$

$$\left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) \right) +$$

$$2 (8 a^4 A b^2 - 28 a^2 A b^4 + 16 A b^6 - 3 a^5 b B + 15 a^3 b^3 B - 8 a b^5 B - 6 a^4 b^2 C + 2 a^2 b^4 C)$$

$$\left( \frac{i \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b \cos[c+dx]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\cos[c+dx]}}\right], -\frac{2a}{-a-b}\right] \operatorname{Sec}[c+dx]}{b \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx]} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Sec}[c+dx]}{a+b}} \right) +$$

$$\frac{1}{b} 2 a \left( \left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}} \operatorname{Csc}[c+dx] \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( (a+b) \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) - \right.$$

$$\left( a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}(c+dx)\right]^2}{-a+b}} \sqrt{-\frac{(a+b) \cos[c+dx] \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}} \csc[c+dx] \right. \\
\left. \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b \cos[c+dx]) \csc\left[\frac{1}{2}(c+dx)\right]^2}{a}}}{\sqrt{2}}\right], -\frac{2a}{-a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right]^4 \right) / \left( b \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \right) + \\
\left. \left. \frac{\sqrt{a+b \cos[c+dx]} \sin[c+dx]}{b \sqrt{\cos[c+dx]}} \right) \right) + \frac{1}{d} \sqrt{\cos[c+dx]} \sqrt{a+b \cos[c+dx]} \\
\left( \frac{2 \sec[c+dx] (-8Ab \sin[c+dx] + 3aB \sin[c+dx])}{3a^4} + \frac{2(Ab^4 \sin[c+dx] - ab^3B \sin[c+dx] + a^2b^2C \sin[c+dx])}{3a^3(a^2-b^2)(a+b \cos[c+dx])^2} + \right. \\
\left. (2(12a^2Ab^4 \sin[c+dx] - 8Ab^5 \sin[c+dx] - 9a^3b^3B \sin[c+dx] + 5ab^5B \sin[c+dx] + 6a^4b^2C \sin[c+dx] - 2a^2b^4C \sin[c+dx])) / \right. \\
\left. (3a^4(a^2-b^2)^2(a+b \cos[c+dx])) + \frac{2A \sec[c+dx] \tan[c+dx]}{3a^3} \right)$$

■ **Problem 1156: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^m (a+b \cos[c+dx])^2 (A+B \cos[c+dx] + C \cos[c+dx]^2) dx$$

Optimal (type 5, 367 leaves, 6 steps):

$$\begin{aligned}
& \frac{(2a^2C + b^2C(3+m) + Ab^2(4+m) + 2abB(4+m)) \cos[c+dx]^{1+m} \sin[c+dx]}{d(2+m)(4+m)} + \\
& \frac{b(2aC + bB(4+m)) \cos[c+dx]^{2+m} \sin[c+dx]}{d(3+m)(4+m)} + \frac{C \cos[c+dx]^{1+m} (a+b \cos[c+dx])^2 \sin[c+dx]}{d(4+m)} - \\
& \left( (2abB(4+5m+m^2) + a^2(4+m)(C(1+m) + A(2+m)) + b^2(1+m)(C(3+m) + A(4+m))) \cos[c+dx]^{1+m} \right. \\
& \quad \left. \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[c+dx]^2\right] \sin[c+dx] \right) / \left( d(1+m)(2+m)(4+m) \sqrt{\sin[c+dx]^2} \right) - \\
& \left( (b^2B(2+m) + a^2B(3+m) + 2ab(C(2+m) + A(3+m))) \cos[c+dx]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos[c+dx]^2\right] \sin[c+dx] \right) / \\
& \left( d(2+m)(3+m) \sqrt{\sin[c+dx]^2} \right)
\end{aligned}$$

Result (type 5, 1677 leaves):

$$\begin{aligned}
& - \frac{b^2C \cos[c+dx]^{1+m} \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]^5}{8d(1+m)(\sin[c+dx]^2)^{5/2}} + \\
& \frac{Ab^2 \cos[c+dx]^{1+m} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]^3}{2d(1+m)(\sin[c+dx]^2)^{3/2}} + \\
& \frac{abB \cos[c+dx]^{1+m} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]^3}{d(1+m)(\sin[c+dx]^2)^{3/2}} + \\
& \frac{a^2C \cos[c+dx]^{1+m} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]^3}{2d(1+m)(\sin[c+dx]^2)^{3/2}} + \\
& \frac{b^2C \cos[c+dx]^{1+m} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]^3}{2d(1+m)(\sin[c+dx]^2)^{3/2}} + \\
& \frac{3b^2B \cos[c+dx]^{2+m} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]^3}{4d(2+m)(\sin[c+dx]^2)^{3/2}} + \\
& \frac{3abC \cos[c+dx]^{2+m} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]^3}{2d(2+m)(\sin[c+dx]^2)^{3/2}} + \\
& - \frac{3b^2C \cos[c+dx]^{3+m} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \cos[c+dx]^2\right] \sin[c+dx]^3}{4d(3+m)(\sin[c+dx]^2)^{3/2}}
\end{aligned}$$

$$\frac{a^2 A \cos [c+d x]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{d(1+m) \sqrt{\sin [c+d x]^2}}$$

$$\frac{A b^2 \cos [c+d x]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{2 d(1+m) \sqrt{\sin [c+d x]^2}}$$

$$\frac{a b B \cos [c+d x]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{d(1+m) \sqrt{\sin [c+d x]^2}}$$

$$\frac{a^2 C \cos [c+d x]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{2 d(1+m) \sqrt{\sin [c+d x]^2}}$$

$$\frac{3 b^2 C \cos [c+d x]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{8 d(1+m) \sqrt{\sin [c+d x]^2}}$$

$$\frac{2 a A b \cos [c+d x]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{d(2+m) \sqrt{\sin [c+d x]^2}}$$

$$\frac{a^2 B \cos [c+d x]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{d(2+m) \sqrt{\sin [c+d x]^2}}$$

$$\frac{3 b^2 B \cos [c+d x]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{4 d(2+m) \sqrt{\sin [c+d x]^2}}$$

$$\frac{3 a b C \cos [c+d x]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{2 d(2+m) \sqrt{\sin [c+d x]^2}}$$

$$\frac{A b^2 \cos [c+d x]^{3+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{2 d(3+m) \sqrt{\sin [c+d x]^2}}$$

$$\frac{a b B \cos [c+d x]^{3+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{d(3+m) \sqrt{\sin [c+d x]^2}}$$

$$\frac{a^2 C \cos [c+d x]^{3+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{2 d(3+m) \sqrt{\sin [c+d x]^2}}$$

$$\frac{b^2 C \cos [c+d x]^{3+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{2 d(3+m) \sqrt{\sin [c+d x]^2}} -$$

$$\frac{b^2 B \cos [c+d x]^{4+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{4+m}{2}, \frac{6+m}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{4 d(4+m) \sqrt{\sin [c+d x]^2}} -$$

$$\frac{a b C \cos [c+d x]^{4+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{4+m}{2}, \frac{6+m}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{2 d(4+m) \sqrt{\sin [c+d x]^2}} -$$

$$\frac{b^2 C \cos [c+d x]^{5+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5+m}{2}, \frac{7+m}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{8 d(5+m) \sqrt{\sin [c+d x]^2}}$$

■ **Problem 1157: Result more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^m (a+b \cos [c+d x]) (A+B \cos [c+d x]+C \cos [c+d x]^2) dx$$

Optimal (type 5, 235 leaves, 5 steps):

$$\frac{(b B+a C) \cos [c+d x]^{1+m} \sin [c+d x]}{d(2+m)} + \frac{b C \cos [c+d x]^{2+m} \sin [c+d x]}{d(3+m)} -$$

$$\left( ((b B+a C)(1+m)+a A(2+m)) \cos [c+d x]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos [c+d x]^2\right] \sin [c+d x] \right) /$$

$$\left( d(1+m)(2+m) \sqrt{\sin [c+d x]^2} \right) -$$

$$\left( (b C(2+m)+A b(3+m)+a B(3+m)) \cos [c+d x]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos [c+d x]^2\right] \sin [c+d x] \right) /$$

$$\left( d(2+m)(3+m) \sqrt{\sin [c+d x]^2} \right)$$

Result (type 5, 474 leaves):

$$\frac{1}{4d} \operatorname{Cos}[c+dx]^{1+m} \operatorname{Csc}[c+dx] \left( \frac{2(bB+aC) \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{1+m} + \frac{3bC \operatorname{Cos}[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{2+m} - \frac{4aA \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{1+m} - \frac{2bB \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{1+m} - \frac{2aC \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{1+m} - \frac{4Ab \operatorname{Cos}[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{2+m} - \frac{4aB \operatorname{Cos}[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{2+m} - \frac{3bC \operatorname{Cos}[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{2+m} - \frac{2bB \operatorname{Cos}[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{3+m} - \frac{2aC \operatorname{Cos}[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{3+m} - \frac{bC \operatorname{Cos}[c+dx]^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{4+m}{2}, \frac{6+m}{2}, \operatorname{Cos}[c+dx]^2\right]}{4+m} \right) \sqrt{\operatorname{Sin}[c+dx]^2}$$

■ **Problem 1158: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^m (A+B \operatorname{Cos}[c+dx]+C \operatorname{Cos}[c+dx]^2)}{a+b \operatorname{Cos}[c+dx]} dx$$

Optimal (type 6, 372 leaves, 8 steps):

$$\frac{1}{b^2(a^2-b^2)d} a (A b^2 - a(bB - aC)) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1-m}{2}, 1, \frac{3}{2}, \operatorname{Sin}[c+dx]^2, -\frac{b^2 \operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] \operatorname{Cos}[c+dx]^{-1+m} (\operatorname{Cos}[c+dx]^2)^{\frac{1-m}{2}} \operatorname{Sin}[c+dx] - \frac{1}{b(a^2-b^2)d} (A b^2 - a(bB - aC)) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, \operatorname{Sin}[c+dx]^2, -\frac{b^2 \operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] \operatorname{Cos}[c+dx]^m (\operatorname{Cos}[c+dx]^2)^{-m/2} \operatorname{Sin}[c+dx] - \frac{(bB - aC) \operatorname{Cos}[c+dx]^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Cos}[c+dx]^2\right] \operatorname{Sin}[c+dx]}{b^2 d (1+m) \sqrt{\operatorname{Sin}[c+dx]^2}} - \frac{C \operatorname{Cos}[c+dx]^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \operatorname{Cos}[c+dx]^2\right] \operatorname{Sin}[c+dx]}{b d (2+m) \sqrt{\operatorname{Sin}[c+dx]^2}}$$

Result (type 6, 16153 leaves):

$$\left( \frac{A \operatorname{Cos}[c+dx]^m}{a+b \operatorname{Cos}[c+dx]} + \frac{C \operatorname{Cos}[c+dx]^m}{2(a+b \operatorname{Cos}[c+dx])} + \frac{B \operatorname{Cos}[c+dx]^{1+m}}{a+b \operatorname{Cos}[c+dx]} + \frac{C \operatorname{Cos}[c+dx]^m \operatorname{Cos}[2(c+dx)]}{2(a+b \operatorname{Cos}[c+dx])} \right)$$



$$\begin{aligned}
& \left( \frac{A \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} + \frac{m}{2}, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2\right] \operatorname{Tan}[c+dx]}{b} - \frac{a B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} + \frac{m}{2}, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2\right] \operatorname{Tan}[c+dx]}{b^2} + \right. \\
& \frac{a^2 C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} + \frac{m}{2}, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2\right] \operatorname{Tan}[c+dx]}{b^3} + \frac{B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1 + \frac{m}{2}, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2\right] \operatorname{Tan}[c+dx]}{b} - \\
& \left. \frac{a C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1 + \frac{m}{2}, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2\right] \operatorname{Tan}[c+dx]}{b^2} + \frac{C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{2} + \frac{m}{2}, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2\right] \operatorname{Tan}[c+dx]}{b} + \right. \\
& \left. \left( 3 a^2 A (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2 + b^2}\right] \operatorname{Tan}[c+dx] (1 + \operatorname{Tan}[c+dx]^2)^{\frac{1}{2} - \frac{m}{2}} \right) / \right. \\
& \left( b \left( -3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2}\right] + \right. \right. \\
& \left. \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2}\right] + \right. \right. \\
& \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2}\right] \right) \operatorname{Tan}[c+dx]^2 (1 + \operatorname{Tan}[c+dx]^2) \right) \right) - \\
& \left( 3 a^3 (a^2 - b^2) B \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2 + b^2}\right] \operatorname{Tan}[c+dx] (1 + \operatorname{Tan}[c+dx]^2)^{\frac{1}{2} - \frac{m}{2}} \right) / \right. \\
& \left( b^2 \left( -3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2}\right] + \right. \right. \\
& \left. \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2}\right] + \right. \right. \\
& \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2}\right] \right) \operatorname{Tan}[c+dx]^2 (1 + \operatorname{Tan}[c+dx]^2) \right) \right) + \\
& \left( 3 a^4 (a^2 - b^2) C \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2 + b^2}\right] \operatorname{Tan}[c+dx] (1 + \operatorname{Tan}[c+dx]^2)^{\frac{1}{2} - \frac{m}{2}} \right) / \right. \\
& \left( b^3 \left( -3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2}\right] + \right. \right. \\
& \left. \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2}\right] + \right. \right. \\
& \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2}\right] \right) \operatorname{Tan}[c+dx]^2 (1 + \operatorname{Tan}[c+dx]^2) \right) \right) - \\
& \left( 3 a A (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2}\right] \operatorname{Tan}[c+dx] (1 + \operatorname{Tan}[c+dx]^2)^{-m/2} \right) / \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2)^m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \tan[c + dx]^2 \right) (-b^2 + a^2 (1 + \tan[c + dx]^2)) \right) + \\
& \left( 3 a^2 (a^2 - b^2) \operatorname{B AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, \frac{a^2 \tan[c + dx]^2}{-a^2 + b^2} \right] \tan[c + dx] (1 + \tan[c + dx]^2)^{-m/2} \right) / \\
& \left( b \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2)^m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \tan[c + dx]^2 \right) (-b^2 + a^2 (1 + \tan[c + dx]^2)) \right) - \\
& \left( 3 a^3 (a^2 - b^2) \operatorname{C AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, \frac{a^2 \tan[c + dx]^2}{-a^2 + b^2} \right] \tan[c + dx] (1 + \tan[c + dx]^2)^{-m/2} \right) / \\
& \left( b^2 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2)^m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \tan[c + dx]^2 \right) (-b^2 + a^2 (1 + \tan[c + dx]^2)) \right) \right) / \\
& \left( d \left( \frac{a \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{2} + \frac{m}{2}, \frac{3}{2}, -\tan[c + dx]^2 \right] \operatorname{Sec}[c + dx]^2}{b} - \frac{a \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{2} + \frac{m}{2}, \frac{3}{2}, -\tan[c + dx]^2 \right] \operatorname{Sec}[c + dx]^2}{b^2} + \right. \right. \\
& \quad \frac{a^2 \operatorname{C Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{2} + \frac{m}{2}, \frac{3}{2}, -\tan[c + dx]^2 \right] \operatorname{Sec}[c + dx]^2}{b^3} + \\
& \quad \frac{\operatorname{B Hypergeometric2F1} \left[ \frac{1}{2}, 1 + \frac{m}{2}, \frac{3}{2}, -\tan[c + dx]^2 \right] \operatorname{Sec}[c + dx]^2}{b} - \\
& \quad \left. \left. \frac{a \operatorname{C Hypergeometric2F1} \left[ \frac{1}{2}, 1 + \frac{m}{2}, \frac{3}{2}, -\tan[c + dx]^2 \right] \operatorname{Sec}[c + dx]^2}{b^2} + \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \frac{\text{CHypergeometric2F1}\left[\frac{1}{2}, \frac{3}{2} + \frac{m}{2}, \frac{3}{2}, -\text{Tan}[c + dx]^2\right] \text{Sec}[c + dx]^2}{b} - \\
& \left( 6 a^4 A (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1 + m), 1, \frac{3}{2}, -\text{Tan}[c + dx]^2, \frac{a^2 \text{Tan}[c + dx]^2}{-a^2 + b^2}\right] \text{Sec}[c + dx]^2 \text{Tan}[c + dx]^2 (1 + \text{Tan}[c + dx]^2)^{\frac{1}{2} - \frac{m}{2}} \right) / \\
& \left( b \left( -3 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1 + m), 1, \frac{3}{2}, -\text{Tan}[c + dx]^2, -\frac{a^2 \text{Tan}[c + dx]^2}{a^2 - b^2}\right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2} (-1 + m), 2, \frac{5}{2}, -\text{Tan}[c + dx]^2, -\frac{a^2 \text{Tan}[c + dx]^2}{a^2 - b^2}\right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (-1 + m) \text{AppellF1}\left[\frac{3}{2}, \frac{1 + m}{2}, 1, \frac{5}{2}, -\text{Tan}[c + dx]^2, -\frac{a^2 \text{Tan}[c + dx]^2}{a^2 - b^2}\right] \right) \text{Tan}[c + dx]^2 \right) (-b^2 + a^2 (1 + \text{Tan}[c + dx]^2))^2 \Big) + \\
& \left( 6 a^5 (a^2 - b^2) B \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1 + m), 1, \frac{3}{2}, -\text{Tan}[c + dx]^2, \frac{a^2 \text{Tan}[c + dx]^2}{-a^2 + b^2}\right] \text{Sec}[c + dx]^2 \text{Tan}[c + dx]^2 (1 + \text{Tan}[c + dx]^2)^{\frac{1}{2} - \frac{m}{2}} \right) / \\
& \left( b^2 \left( -3 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1 + m), 1, \frac{3}{2}, -\text{Tan}[c + dx]^2, -\frac{a^2 \text{Tan}[c + dx]^2}{a^2 - b^2}\right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2} (-1 + m), 2, \frac{5}{2}, -\text{Tan}[c + dx]^2, -\frac{a^2 \text{Tan}[c + dx]^2}{a^2 - b^2}\right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (-1 + m) \text{AppellF1}\left[\frac{3}{2}, \frac{1 + m}{2}, 1, \frac{5}{2}, -\text{Tan}[c + dx]^2, -\frac{a^2 \text{Tan}[c + dx]^2}{a^2 - b^2}\right] \right) \text{Tan}[c + dx]^2 \right) (-b^2 + a^2 (1 + \text{Tan}[c + dx]^2))^2 \Big) - \\
& \left( 6 a^6 (a^2 - b^2) C \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1 + m), 1, \frac{3}{2}, -\text{Tan}[c + dx]^2, \frac{a^2 \text{Tan}[c + dx]^2}{-a^2 + b^2}\right] \text{Sec}[c + dx]^2 \text{Tan}[c + dx]^2 (1 + \text{Tan}[c + dx]^2)^{\frac{1}{2} - \frac{m}{2}} \right) / \\
& \left( b^3 \left( -3 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1 + m), 1, \frac{3}{2}, -\text{Tan}[c + dx]^2, -\frac{a^2 \text{Tan}[c + dx]^2}{a^2 - b^2}\right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2} (-1 + m), 2, \frac{5}{2}, -\text{Tan}[c + dx]^2, -\frac{a^2 \text{Tan}[c + dx]^2}{a^2 - b^2}\right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (-1 + m) \text{AppellF1}\left[\frac{3}{2}, \frac{1 + m}{2}, 1, \frac{5}{2}, -\text{Tan}[c + dx]^2, -\frac{a^2 \text{Tan}[c + dx]^2}{a^2 - b^2}\right] \right) \text{Tan}[c + dx]^2 \right) (-b^2 + a^2 (1 + \text{Tan}[c + dx]^2))^2 \Big) + \\
& \left( 6 a^3 A (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\text{Tan}[c + dx]^2, -\frac{a^2 \text{Tan}[c + dx]^2}{a^2 - b^2}\right] \text{Sec}[c + dx]^2 \text{Tan}[c + dx]^2 (1 + \text{Tan}[c + dx]^2)^{-m/2} \right) / \\
& \left( \left( -3 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\text{Tan}[c + dx]^2, -\frac{a^2 \text{Tan}[c + dx]^2}{a^2 - b^2}\right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \text{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\text{Tan}[c + dx]^2, -\frac{a^2 \text{Tan}[c + dx]^2}{a^2 - b^2}\right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) m \text{AppellF1}\left[\frac{3}{2}, \frac{2 + m}{2}, 1, \frac{5}{2}, -\text{Tan}[c + dx]^2, -\frac{a^2 \text{Tan}[c + dx]^2}{a^2 - b^2}\right] \right) \text{Tan}[c + dx]^2 \right) (-b^2 + a^2 (1 + \text{Tan}[c + dx]^2))^2 \Big) -
\end{aligned}$$

$$\begin{aligned}
& \left( 6 a^4 (a^2 - b^2) \text{B AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, \frac{a^2 \text{Tan}[c + d x]^2}{-a^2 + b^2} \right] \text{Sec}[c + d x]^2 \text{Tan}[c + d x]^2 (1 + \text{Tan}[c + d x]^2)^{-m/2} \right) / \\
& \left( b \left( -3 (a^2 - b^2) \text{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, -\frac{a^2 \text{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \text{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\text{Tan}[c + d x]^2, -\frac{a^2 \text{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) m \text{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\text{Tan}[c + d x]^2, -\frac{a^2 \text{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \text{Tan}[c + d x]^2 \right) (-b^2 + a^2 (1 + \text{Tan}[c + d x]^2))^2 \Big) + \\
& \left( 6 a^5 (a^2 - b^2) \text{C AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, \frac{a^2 \text{Tan}[c + d x]^2}{-a^2 + b^2} \right] \text{Sec}[c + d x]^2 \text{Tan}[c + d x]^2 (1 + \text{Tan}[c + d x]^2)^{-m/2} \right) / \\
& \left( b^2 \left( -3 (a^2 - b^2) \text{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, -\frac{a^2 \text{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \text{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\text{Tan}[c + d x]^2, -\frac{a^2 \text{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) m \text{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\text{Tan}[c + d x]^2, -\frac{a^2 \text{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \text{Tan}[c + d x]^2 \right) (-b^2 + a^2 (1 + \text{Tan}[c + d x]^2))^2 \Big) + \\
& \left( 6 a^2 \text{A} (a^2 - b^2) \left( \frac{1}{2} - \frac{m}{2} \right) \text{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, \frac{a^2 \text{Tan}[c + d x]^2}{-a^2 + b^2} \right] \text{Sec}[c + d x]^2 \text{Tan}[c + d x]^2 \right. \\
& \quad \left. (1 + \text{Tan}[c + d x]^2)^{-\frac{1}{2} - \frac{m}{2}} \right) / \left( b \left( -3 (a^2 - b^2) \text{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, -\frac{a^2 \text{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \text{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 + m), 2, \frac{5}{2}, -\text{Tan}[c + d x]^2, -\frac{a^2 \text{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (-1 + m) \text{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\text{Tan}[c + d x]^2, -\frac{a^2 \text{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \text{Tan}[c + d x]^2 \right) (-b^2 + a^2 (1 + \text{Tan}[c + d x]^2)) \Big) - \\
& \left( 6 a^3 (a^2 - b^2) \text{B} \left( \frac{1}{2} - \frac{m}{2} \right) \text{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, \frac{a^2 \text{Tan}[c + d x]^2}{-a^2 + b^2} \right] \text{Sec}[c + d x]^2 \text{Tan}[c + d x]^2 \right. \\
& \quad \left. (1 + \text{Tan}[c + d x]^2)^{-\frac{1}{2} - \frac{m}{2}} \right) / \left( b^2 \left( -3 (a^2 - b^2) \text{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, -\frac{a^2 \text{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \text{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 + m), 2, \frac{5}{2}, -\text{Tan}[c + d x]^2, -\frac{a^2 \text{Tan}[c + d x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (-1 + m) \text{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\text{Tan}[c + d x]^2, -\frac{a^2 \text{Tan}[c + d x]^2}{a^2 - b^2} \right] \right) \text{Tan}[c + d x]^2 \right) (-b^2 + a^2 (1 + \text{Tan}[c + d x]^2)) \Big) + \\
& \left( 6 a^4 (a^2 - b^2) \text{C} \left( \frac{1}{2} - \frac{m}{2} \right) \text{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, \frac{a^2 \text{Tan}[c + d x]^2}{-a^2 + b^2} \right] \text{Sec}[c + d x]^2 \text{Tan}[c + d x]^2 \right.
\end{aligned}$$

$$\begin{aligned}
& \left(1 + \tan[c + dx]^2\right)^{-\frac{1-m}{2}} \Big/ \left( b^3 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) (-b^2 + a^2 (1 + \tan[c + dx]^2)) \Big) + \\
& \left( 3 a^2 A (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\tan[c + dx]^2, \frac{a^2 \tan[c + dx]^2}{-a^2 + b^2} \right] \operatorname{Sec}[c + dx]^2 (1 + \tan[c + dx]^2)^{\frac{1-m}{2}} \right) \Big/ \\
& \left( b \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) (-b^2 + a^2 (1 + \tan[c + dx]^2)) \Big) - \\
& \left( 3 a^3 (a^2 - b^2) B \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\tan[c + dx]^2, \frac{a^2 \tan[c + dx]^2}{-a^2 + b^2} \right] \operatorname{Sec}[c + dx]^2 (1 + \tan[c + dx]^2)^{\frac{1-m}{2}} \right) \Big/ \\
& \left( b^2 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) (-b^2 + a^2 (1 + \tan[c + dx]^2)) \Big) + \\
& \left( 3 a^4 (a^2 - b^2) C \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\tan[c + dx]^2, \frac{a^2 \tan[c + dx]^2}{-a^2 + b^2} \right] \operatorname{Sec}[c + dx]^2 (1 + \tan[c + dx]^2)^{\frac{1-m}{2}} \right) \Big/ \\
& \left( b^3 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) (-b^2 + a^2 (1 + \tan[c + dx]^2)) \Big) + \\
& \left( 3 a^2 A (a^2 - b^2) \tan[c + dx] \left[ -\frac{1}{3} (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{1}{2} (-1+m), 1, \frac{5}{2}, -\tan[c + dx]^2, \frac{a^2 \tan[c + dx]^2}{-a^2 + b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx]}{3(-a^2+b^2)} \right) (1+\tan[c+dx]^2)^{\frac{1}{2}-\frac{m}{2}} \Big/ \\
& \left( b \left( -3(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2)(-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) \tan[c+dx]^2 \right) (-b^2+a^2(1+\tan[c+dx]^2)) \Big) - \\
& \left( 3 a^3 (a^2-b^2) B \tan[c+dx] \left( -\frac{1}{3}(-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 1+\frac{1}{2}(-1+m), 1, \frac{5}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] + \right. \right. \\
& \quad \left. \left. \frac{2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx]}{3(-a^2+b^2)} \right) (1+\tan[c+dx]^2)^{\frac{1}{2}-\frac{m}{2}} \right) \Big/ \\
& \left( b^2 \left( -3(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2)(-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) \tan[c+dx]^2 \right) (-b^2+a^2(1+\tan[c+dx]^2)) \Big) + \\
& \left( 3 a^4 (a^2-b^2) C \tan[c+dx] \left( -\frac{1}{3}(-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 1+\frac{1}{2}(-1+m), 1, \frac{5}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] + \right. \right. \\
& \quad \left. \left. \frac{2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx]}{3(-a^2+b^2)} \right) (1+\tan[c+dx]^2)^{\frac{1}{2}-\frac{m}{2}} \right) \Big/ \\
& \left( b^3 \left( -3(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1+m), 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}(-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2)(-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \right) \tan[c+dx]^2 \right) (-b^2+a^2(1+\tan[c+dx]^2)) \Big) + \\
& \left( 3 a A (a^2-b^2) m \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2-b^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx]^2 (1+\tan[c+dx]^2)^{-1-\frac{m}{2}} \right) \Big/
\end{aligned}$$



$$\begin{aligned}
& \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \\
& \quad \left. (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Tan}[c+dx]^2 \right) (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2)) \Big) - \\
& \left( 3 a^3 (a^2-b^2) C \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2+b^2} \right] \operatorname{Sec}[c+dx]^2 (1+\operatorname{Tan}[c+dx]^2)^{-m/2} \right) / \\
& \left( b^2 \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Tan}[c+dx]^2 \right) \right) (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2)) \Big) - \\
& \left( 3 a A (a^2-b^2) \operatorname{Tan}[c+dx] \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+\frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \right. \right. \\
& \quad \left. \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3 (a^2-b^2)} \right) \right) (1+\operatorname{Tan}[c+dx]^2)^{-m/2} \Big) / \\
& \left( \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Tan}[c+dx]^2 \right) \right) (-b^2+a^2(1+\operatorname{Tan}[c+dx]^2)) \Big) + \\
& \left( 3 a^2 (a^2-b^2) B \operatorname{Tan}[c+dx] \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1+\frac{m}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2+b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] + \right. \right. \\
& \quad \left. \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{a^2 \operatorname{Tan}[c+dx]^2}{-a^2+b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3 (-a^2+b^2)} \right) \right) (1+\operatorname{Tan}[c+dx]^2)^{-m/2} \Big) / \\
& \left( b \left( -3 (a^2-b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, -\frac{a^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \left. \left( (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \tan[c+dx]^2 \right) (-b^2 + a^2 (1 + \tan[c+dx]^2)) \right) - \\
& \left( 3 a^3 (a^2 - b^2) C \tan[c+dx] \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2 + b^2} \right] \sec[c+dx]^2 \tan[c+dx] + \right. \right. \\
& \left. \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2 + b^2} \right] \sec[c+dx]^2 \tan[c+dx]}{3 (-a^2 + b^2)} \right) (1 + \tan[c+dx]^2)^{-m/2} \right) / \\
& \left( b^2 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right) \tan[c+dx]^2 \right) \\
& \left. (-b^2 + a^2 (1 + \tan[c+dx]^2)) \right) + \frac{C \sec[c+dx]^2 \left( -\operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3}{2} + \frac{m}{2}, \frac{3}{2}, -\tan[c+dx]^2 \right] + (1 + \tan[c+dx]^2)^{-\frac{3}{2} - \frac{m}{2}} \right)}{b} + \\
& \frac{B \sec[c+dx]^2 \left( -\operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, 1 + \frac{m}{2}, \frac{3}{2}, -\tan[c+dx]^2 \right] + (1 + \tan[c+dx]^2)^{-1 - \frac{m}{2}} \right)}{b} - \\
& \frac{a C \sec[c+dx]^2 \left( -\operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, 1 + \frac{m}{2}, \frac{3}{2}, -\tan[c+dx]^2 \right] + (1 + \tan[c+dx]^2)^{-1 - \frac{m}{2}} \right)}{b^2} + \\
& \frac{A \sec[c+dx]^2 \left( -\operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{2} + \frac{m}{2}, \frac{3}{2}, -\tan[c+dx]^2 \right] + (1 + \tan[c+dx]^2)^{-\frac{1}{2} - \frac{m}{2}} \right)}{b} - \\
& \frac{a B \sec[c+dx]^2 \left( -\operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{2} + \frac{m}{2}, \frac{3}{2}, -\tan[c+dx]^2 \right] + (1 + \tan[c+dx]^2)^{-\frac{1}{2} - \frac{m}{2}} \right)}{b^2} + \\
& \frac{a^2 C \sec[c+dx]^2 \left( -\operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{2} + \frac{m}{2}, \frac{3}{2}, -\tan[c+dx]^2 \right] + (1 + \tan[c+dx]^2)^{-\frac{1}{2} - \frac{m}{2}} \right)}{b^3} - \\
& \left( 3 a^2 A (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2 + b^2} \right] \tan[c+dx] \right. \\
& \left. (1 + \tan[c+dx]^2)^{\frac{1}{2} - \frac{m}{2}} \left( 2 \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right) \sec[c+dx]^2 \tan[c+dx] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 3 (a^2 - b^2) \left( -\frac{1}{3} (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{1}{2} (-1+m), 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \\
& \quad \left. \frac{1}{3 (a^2 - b^2)} 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \\
& \tan[c+dx]^2 \left( 2 a^2 \left( -\frac{3}{5} (-1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1}{2} (-1+m), 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \quad \left. \left. \frac{1}{5 (a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2} (-1+m), 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \right. \\
& \quad (a^2 - b^2) (-1+m) \left( -\frac{1}{5 (a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1+m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \\
& \quad \left. \frac{3}{5} (1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1+m}{2}, 1, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \Big) \Big) \Big) \Big) \Big) / \\
& \left( b \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right) \tan[c+dx]^2 \right)^2 \\
& \left( -b^2 + a^2 (1 + \tan[c+dx]^2) \right) \Big) + \left( 3 a^3 (a^2 - b^2) \operatorname{B} \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), 1, \frac{3}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2 + b^2} \right] \right) \\
& \tan[c+dx] (1 + \tan[c+dx]^2)^{\frac{1}{2} - \frac{m}{2}} \left( 2 \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right) \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \\
& \quad 3 (a^2 - b^2) \left( -\frac{1}{3} (-1+m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{1}{2} (-1+m), 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \\
& \quad \left. \frac{1}{3 (a^2 - b^2)} 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1+m), 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \\
& \tan[c+dx]^2 \left( 2 a^2 \left( -\frac{3}{5} (-1+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1}{2} (-1+m), 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \quad \left. \left. \frac{1}{5 (a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2} (-1+m), 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& (a^2 - b^2) (-1 + m) \left( -\frac{1}{5(a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1+m}{2}, 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \\
& \left. \frac{3}{5} (1 + m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1+m}{2}, 1, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( b^2 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 + m), 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) (-1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right)^2 \\
& (-b^2 + a^2 (1 + \tan[c + dx]^2)) \Bigg) - \left( 3 a^4 (a^2 - b^2) \operatorname{CAppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 1, \frac{3}{2}, -\tan[c + dx]^2, \frac{a^2 \tan[c + dx]^2}{-a^2 + b^2} \right] \right. \\
& \tan[c + dx] (1 + \tan[c + dx]^2)^{\frac{1}{2} - \frac{m}{2}} \left( 2 \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 + m), 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) (-1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \\
& \left. 3 (a^2 - b^2) \left( -\frac{1}{3} (-1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{1}{2} (-1 + m), 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \left. \left. \frac{1}{3 (a^2 - b^2)} 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 + m), 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \right) + \\
& \tan[c + dx]^2 \left( 2 a^2 \left( -\frac{3}{5} (-1 + m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1}{2} (-1 + m), 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \left. \left. \frac{1}{5 (a^2 - b^2)} 12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2} (-1 + m), 3, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \right) + \\
& (a^2 - b^2) (-1 + m) \left( -\frac{1}{5(a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1+m}{2}, 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] - \right. \\
& \left. \frac{3}{5} (1 + m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{1+m}{2}, 1, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( b^3 \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2} (-1 + m), 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2} (-1 + m), 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) (-1 + m) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right)^2
\end{aligned}$$

$$\begin{aligned}
& \left. (-b^2 + a^2 (1 + \tan[c + dx]^2)) \right) + \left( 3 a A (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \tan[c + dx] \right. \\
& (1 + \tan[c + dx]^2)^{-m/2} \left( 2 \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \sec[c + dx]^2 \tan[c + dx] - \right. \\
& 3 (a^2 - b^2) \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sec[c + dx]^2 \tan[c + dx] - \right. \\
& \left. \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sec[c + dx]^2 \tan[c + dx]}{3 (a^2 - b^2)} \right) \right) + \\
& \tan[c + dx]^2 \left( 2 a^2 \left( -\frac{3}{5} m \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{m}{2}, 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sec[c + dx]^2 \tan[c + dx] - \right. \right. \\
& \left. \left. \frac{12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{m}{2}, 3, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sec[c + dx]^2 \tan[c + dx]}{5 (a^2 - b^2)} \right) \right) + \\
& (a^2 - b^2) m \left( -\frac{1}{5 (a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{2+m}{2}, 2, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sec[c + dx]^2 \tan[c + dx] - \right. \\
& \left. \left. \frac{3}{5} (2+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{2+m}{2}, 1, \frac{7}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sec[c + dx]^2 \tan[c + dx] \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left( \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right)^2 \\
& (-b^2 + a^2 (1 + \tan[c + dx]^2)) \Bigg) - \left( 3 a^2 (a^2 - b^2) B \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, \frac{a^2 \tan[c + dx]^2}{-a^2 + b^2} \right] \tan[c + dx] \right. \\
& (1 + \tan[c + dx]^2)^{-m/2} \left( 2 \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, -\frac{a^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right) \sec[c+dx]^2 \tan[c+dx] - \\
& 3 (a^2 - b^2) \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] - \right. \\
& \left. \frac{2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx]}{3 (a^2 - b^2)} \right) + \\
& \tan[c+dx]^2 \left( 2 a^2 \left( -\frac{3}{5} m \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \left. \left. \frac{12 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{m}{2}, 3, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx]}{5 (a^2 - b^2)} \right) \right) + \\
& (a^2 - b^2) m \left( -\frac{1}{5 (a^2 - b^2)} 6 a^2 \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{2+m}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] - \right. \\
& \left. \frac{3}{5} (2+m) \operatorname{AppellF1} \left[ \frac{5}{2}, 1 + \frac{2+m}{2}, 1, \frac{7}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] \right) \Big) \Big) \Big) / \\
& \left( b \left( -3 (a^2 - b^2) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right) \tan[c+dx]^2 \right)^2 \\
& (-b^2 + a^2 (1 + \tan[c+dx]^2)) \Big) + \left( 3 a^3 (a^2 - b^2) \operatorname{C AppellF1} \left[ \frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, \frac{a^2 \tan[c+dx]^2}{-a^2 + b^2} \right] \tan[c+dx] \right. \\
& \left. (1 + \tan[c+dx]^2)^{-m/2} \left( 2 \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right) \sec[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \left. \left. 3 (a^2 - b^2) \left( -\frac{1}{3} m \operatorname{AppellF1} \left[ \frac{3}{2}, 1 + \frac{m}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, -\frac{a^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sec[c+dx]^2 \tan[c+dx] - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
 & \frac{2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2}\right] \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{3\left(a^2-b^2\right)} \Bigg) + \\
 & \operatorname{Tan}[c+d x]^2 \left( 2 a^2 \left( -\frac{3}{5} m \operatorname{AppellF1}\left[\frac{5}{2}, 1+\frac{m}{2}, 2, \frac{7}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2}\right] \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] - \right. \right. \\
 & \left. \left. \frac{12 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{m}{2}, 3, \frac{7}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2}\right] \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{5\left(a^2-b^2\right)} \right) + \right. \\
 & \left. \left. \left(a^2-b^2\right) m \left( -\frac{1}{5\left(a^2-b^2\right)} 6 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2+m}{2}, 2, \frac{7}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2}\right] \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] - \right. \right. \right. \\
 & \left. \left. \left. \frac{3}{5}(2+m) \operatorname{AppellF1}\left[\frac{5}{2}, 1+\frac{2+m}{2}, 1, \frac{7}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2}\right] \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] \right) \right) \right) \Bigg) / \\
 & \left( b^2 \left( -3\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{m}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2}\right] + \left( 2 a^2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{m}{2}, 2, \frac{5}{2}, \right. \right. \right. \right. \\
 & \left. \left. \left. -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2}\right] + \left(a^2-b^2\right) m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+m}{2}, 1, \frac{5}{2}, \right. \right. \right. \right. \\
 & \left. \left. \left. -\operatorname{Tan}[c+d x]^2, -\frac{a^2 \operatorname{Tan}[c+d x]^2}{a^2-b^2}\right] \right) \operatorname{Tan}[c+d x]^2 \right)^2 \left( -b^2+a^2\left(1+\operatorname{Tan}[c+d x]^2\right) \right) \Bigg) \Bigg)
 \end{aligned}$$

■ **Problem 1159: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+d x]^m \left( A+B \operatorname{Cos}[c+d x]+C \operatorname{Cos}[c+d x]^2 \right)}{\left( a+b \operatorname{Cos}[c+d x] \right)^2} d x$$

Optimal (type 6, 564 leaves, 9 steps):

$$\frac{1}{b^2 (a^2 - b^2)^2 d} (A b^4 m + a^3 b B m - a b^3 B (1+m) - a^4 C (1+m) + a^2 b^2 (A - A m + C (2+m)))$$

$$\text{AppellF1}\left[\frac{1}{2}, \frac{1-m}{2}, 1, \frac{3}{2}, \text{Sin}[c+dx]^2, -\frac{b^2 \text{Sin}[c+dx]^2}{a^2 - b^2}\right] \text{Cos}[c+dx]^{-1+m} (\text{Cos}[c+dx]^2)^{\frac{1-m}{2}} \text{Sin}[c+dx] - \frac{1}{a b (a^2 - b^2)^2 d}$$

$$(A b^4 m + a^3 b B m - a b^3 B (1+m) - a^4 C (1+m) + a^2 b^2 (A - A m + C (2+m))) \text{AppellF1}\left[\frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, \text{Sin}[c+dx]^2, -\frac{b^2 \text{Sin}[c+dx]^2}{a^2 - b^2}\right]$$

$$\text{Cos}[c+dx]^m (\text{Cos}[c+dx]^2)^{-m/2} \text{Sin}[c+dx] + \frac{(A b^2 - a (b B - a C)) \text{Cos}[c+dx]^{1+m} \text{Sin}[c+dx]}{a (a^2 - b^2) d (a + b \text{Cos}[c+dx])} +$$

$$\left( (a b B m - a^2 C (1+m) + b^2 (C - A m)) \text{Cos}[c+dx]^{1+m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \text{Cos}[c+dx]^2\right] \text{Sin}[c+dx] \right) /$$

$$\left( b^2 (a^2 - b^2) d (1+m) \sqrt{\text{Sin}[c+dx]^2} \right) + \frac{(A b^2 - a (b B - a C)) (1+m) \text{Cos}[c+dx]^{2+m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \text{Cos}[c+dx]^2\right] \text{Sin}[c+dx]}{a b (a^2 - b^2) d (2+m) \sqrt{\text{Sin}[c+dx]^2}}$$

Result (type 6, 25777 leaves): Display of huge result suppressed!

■ **Problem 1160: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \text{Cos}[c+dx]) (A + C \text{Cos}[c+dx]^2) \text{Sec}[c+dx]^{9/2} dx$$

Optimal (type 4, 205 leaves, 8 steps):

$$\frac{2 a (3 A + 5 C) \sqrt{\text{Cos}[c+dx]} \text{EllipticE}\left[\frac{1}{2} (c+dx), 2\right] \sqrt{\text{Sec}[c+dx]}}{5 d} +$$

$$\frac{2 a (5 A + 7 C) \sqrt{\text{Cos}[c+dx]} \text{EllipticF}\left[\frac{1}{2} (c+dx), 2\right] \sqrt{\text{Sec}[c+dx]}}{21 d} + \frac{2 a (3 A + 5 C) \sqrt{\text{Sec}[c+dx]} \text{Sin}[c+dx]}{5 d} +$$

$$\frac{2 a (5 A + 7 C) \text{Sec}[c+dx]^{3/2} \text{Sin}[c+dx]}{21 d} + \frac{2 a A \text{Sec}[c+dx]^{5/2} \text{Sin}[c+dx]}{5 d} + \frac{2 a A \text{Sec}[c+dx]^{7/2} \text{Sin}[c+dx]}{7 d}$$

Result (type 5, 573 leaves):

$$\begin{aligned}
& a \left( -\frac{1}{5\sqrt{2}d} 3A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (1+\cos[c+dx]) \operatorname{Csc}[c] \right. \\
& \quad \left. \left( 1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 - \frac{1}{\sqrt{2}d} \right. \\
& \quad \left. C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (1+\cos[c+dx]) \operatorname{Csc}[c] \left( 1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right) \\
& \quad \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 + \frac{5A\sqrt{\cos[c+dx]} (1+\cos[c+dx]) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\operatorname{Sec}[c+dx]}}{21d} + \\
& \quad \frac{C\sqrt{\cos[c+dx]} (1+\cos[c+dx]) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\operatorname{Sec}[c+dx]}}{3d} + \\
& \quad (1+\cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\operatorname{Sec}[c+dx]} \left( \frac{(3A+5C)\cos[dx] \operatorname{Csc}[c]}{5d} + \frac{A \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^3 \sin[dx]}{7d} + \right. \\
& \quad \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (5A\sin[c] + 7A\sin[dx])}{35d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (21A\sin[c] + 25A\sin[dx] + 35C\sin[dx])}{105d} + \frac{(5A+7C)\tan[c]}{21d} \right) \Big)
\end{aligned}$$

■ **Problem 1161: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos[c+dx]) (A + C \cos[c+dx]^2) \operatorname{Sec}[c+dx]^{7/2} dx$$

Optimal (type 4, 172 leaves, 7 steps):

$$\begin{aligned}
& -\frac{2a(3A+5C)\sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{5d} + \frac{2a(A+3C)\sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{3d} + \\
& \frac{2a(3A+5C)\sqrt{\operatorname{Sec}[c+dx]} \sin[c+dx]}{5d} + \frac{2aA \operatorname{Sec}[c+dx]^{3/2} \sin[c+dx]}{3d} + \frac{2aA \operatorname{Sec}[c+dx]^{5/2} \sin[c+dx]}{5d}
\end{aligned}$$

Result (type 5, 273 leaves):



$$\frac{1}{30 d (1 + e^{2i(c+dx)})^2}$$

$$a e^{-i(2c+dx)} (-1 + e^{2ic}) (1 + \cos[c+dx]) \operatorname{Csc}[c] \left( 9A + 15C + 5A e^{i(c+dx)} + 24A e^{2i(c+dx)} + 30C e^{2i(c+dx)} + 3A e^{4i(c+dx)} + 15C e^{4i(c+dx)} - \right.$$

$$5A e^{5i(c+dx)} - 5i(A+3C) e^{i(c+dx)} (1 + e^{2i(c+dx)})^2 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] -$$

$$\left. 3(3A+5C) (1 + e^{2i(c+dx)})^{5/2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\operatorname{Sec}[c+dx]}$$

■ **Problem 1162: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos[c+dx]) (A + C \cos[c+dx])^2 \operatorname{Sec}[c+dx]^{5/2} dx$$

Optimal (type 4, 135 leaves, 6 steps):

$$-\frac{2a(A-C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{d} +$$

$$\frac{2a(A+3C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{3d} + \frac{2aA \sqrt{\operatorname{Sec}[c+dx]} \sin[c+dx]}{d} + \frac{2aA \operatorname{Sec}[c+dx]^{3/2} \sin[c+dx]}{3d}$$

Result (type 5, 193 leaves):

$$\frac{1}{3d} a e^{-i(2c+dx)} \operatorname{Sec}[c+dx]^{3/2}$$

$$\left( -3A + 3C - 3A \cos[2(c+dx)] + 3C \cos[2(c+dx)] + 2i(A+3C) \cos[c+dx]^{3/2} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] + 3(A-C) e^{-2i(c+dx)} \right.$$

$$\left. (1 + e^{2i(c+dx)})^{3/2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 2iA \sin[c+dx] + 3iA \sin[2(c+dx)] \right) (-i \cos[2c+dx] + \sin[2c+dx])$$

■ **Problem 1163: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos[c+dx]) (A + C \cos[c+dx])^2 \operatorname{Sec}[c+dx]^{3/2} dx$$

Optimal (type 4, 135 leaves, 6 steps):

$$-\frac{2a(A-C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{d} +$$

$$\frac{2a(3A+C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{3d} + \frac{2aC \sin[c+dx]}{3d \sqrt{\operatorname{Sec}[c+dx]}} + \frac{2aA \sqrt{\operatorname{Sec}[c+dx]} \sin[c+dx]}{d}$$

Result (type 5, 184 leaves):

$$-\frac{1}{3d} a e^{-i(2c+dx)} \sqrt{\sec[c+dx]} \left( 6iA \cos[c+dx] - 6iC \cos[c+dx] + 2(3A+C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] - 6i(A-C) e^{-i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 6A \sin[c+dx] + C \sin[2(c+dx)] \right) (-\cos[2c+dx] - i \sin[2c+dx])$$

■ **Problem 1164: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos[c+dx]) (A + C \cos[c+dx])^2 \sqrt{\sec[c+dx]} dx$$

Optimal (type 4, 141 leaves, 6 steps):

$$\frac{2a(5A+3C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{5d} + \frac{2a(3A+C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3d} + \frac{2aC \sin[c+dx]}{5d \sec[c+dx]^{3/2}} + \frac{2aC \sin[c+dx]}{3d \sqrt{\sec[c+dx]}}$$

Result (type 5, 183 leaves):

$$\frac{1}{30d} a e^{-i(2c+dx)} \sqrt{\sec[c+dx]} \left( 20(3A+C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] + 12i(5A+3C) e^{-i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 2 \cos[c+dx] (-6i(5A+3C) + 10C \sin[c+dx] + 3C \sin[2(c+dx)]) \right) (\cos[2c+dx] + i \sin[2c+dx])$$

■ **Problem 1165: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \cos[c+dx]) (A + C \cos[c+dx])^2}{\sqrt{\sec[c+dx]}} dx$$

Optimal (type 4, 174 leaves, 7 steps):

$$\frac{2a(5A+3C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{5d} + \frac{2a(7A+5C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{21d} + \frac{2aC \sin[c+dx]}{7d \sec[c+dx]^{5/2}} + \frac{2aC \sin[c+dx]}{5d \sec[c+dx]^{3/2}} + \frac{2a(7A+5C) \sin[c+dx]}{21d \sqrt{\sec[c+dx]}}$$

Result (type 5, 202 leaves):

$$\frac{1}{420 d} a e^{-i(2c+dx)} \sqrt{\sec[c+dx]} \left( 40(7A+5C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] + 168i(5A+3C) e^{-i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 2\cos[c+dx](-84i(5A+3C) + 5(28A+23C)\sin[c+dx] + 42C\sin[2(c+dx)] + 15C\sin[3(c+dx)]) \right) (\cos[2c+dx] + i\sin[2c+dx])$$

■ **Problem 1166: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a\cos[c+dx])(A+C\cos[c+dx]^2)}{\sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 205 leaves, 8 steps):

$$\frac{2a(9A+7C)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{15d} + \frac{2a(7A+5C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{21d} + \frac{2aC\sin[c+dx]}{9d\sec[c+dx]^{7/2}} + \frac{2aC\sin[c+dx]}{7d\sec[c+dx]^{5/2}} + \frac{2a(9A+7C)\sin[c+dx]}{45d\sec[c+dx]^{3/2}} + \frac{2a(7A+5C)\sin[c+dx]}{21d\sqrt{\sec[c+dx]}}$$

Result (type 5, 184 leaves):

$$\frac{1}{2520 d} a \sqrt{\sec[c+dx]} \left( 240(7A+5C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] + 336i(9A+7C) e^{-i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 2\cos[c+dx] (-1512iA - 1176iC + 30(28A+23C)\sin[c+dx] + 14(18A+19C)\sin[2(c+dx)] + 90C\sin[3(c+dx)] + 35C\sin[4(c+dx)]) \right)$$

■ **Problem 1167: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+a\cos[c+dx])^2 (A+C\cos[c+dx]^2) \sec[c+dx]^{11/2} dx$$

Optimal (type 4, 270 leaves, 10 steps):

$$-\frac{16a^2(2A+3C)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{15d} + \frac{4a^2(5A+7C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{21d} + \frac{16a^2(2A+3C)\sqrt{\sec[c+dx]}\sin[c+dx]}{15d} + \frac{4a^2(5A+7C)\sec[c+dx]^{3/2}\sin[c+dx]}{21d} + \frac{2a^2(19A+21C)\sec[c+dx]^{5/2}\sin[c+dx]}{105d} + \frac{8A(a^2+a^2\cos[c+dx])\sec[c+dx]^{7/2}\sin[c+dx]}{63d} + \frac{2A(a+a\cos[c+dx])^2\sec[c+dx]^{9/2}\sin[c+dx]}{9d}$$

Result (type 5, 635 leaves) :

$$\begin{aligned}
 & -\frac{1}{15d} 4\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (a+a\cos[c+dx])^2 \operatorname{Csc}[c] \\
 & \left( 1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 - \frac{1}{5d} 2\sqrt{2} c e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \\
 & (a+a\cos[c+dx])^2 \operatorname{Csc}[c] \left( 1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 + \\
 & \frac{5A\sqrt{\cos[c+dx]} (a+a\cos[c+dx])^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c+dx]}}{21d} + \\
 & \frac{C\sqrt{\cos[c+dx]} (a+a\cos[c+dx])^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c+dx]}}{3d} + \\
 & (a+a\cos[c+dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c+dx]} \left( \frac{4(2A+3C)\cos[dx]\operatorname{Csc}[c]}{15d} + \frac{A\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^4\sin[dx]}{18d} + \right. \\
 & \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^3(7A\sin[c]+18A\sin[dx])}{126d} + \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^2(90A\sin[c]+112A\sin[dx]+63C\sin[dx])}{630d} + \\
 & \left. \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx](112A\sin[c]+63C\sin[c]+150A\sin[dx]+210C\sin[dx])}{630d} + \frac{(5A+7C)\tan[c]}{21d} \right)
 \end{aligned}$$

■ **Problem 1168: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+a\cos[c+dx])^2 (A+C\cos[c+dx])^2 \operatorname{Sec}[c+dx]^{9/2} dx$$

Optimal (type 4, 237 leaves, 9 steps) :

$$\begin{aligned}
 & -\frac{4a^2(3A+5C)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{5d} + \frac{8a^2(3A+7C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{21d} + \\
 & \frac{4a^2(3A+5C)\sqrt{\operatorname{Sec}[c+dx]}\sin[c+dx]}{5d} + \frac{2a^2(33A+35C)\operatorname{Sec}[c+dx]^{3/2}\sin[c+dx]}{105d} + \\
 & \frac{8A(a^2+a^2\cos[c+dx])\operatorname{Sec}[c+dx]^{5/2}\sin[c+dx]}{35d} + \frac{2A(a+a\cos[c+dx])^2\operatorname{Sec}[c+dx]^{7/2}\sin[c+dx]}{7d}
 \end{aligned}$$

Result (type 5, 591 leaves) :

$$\begin{aligned}
& - \frac{1}{5\sqrt{2}d} 3A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (a+a\cos[c+dx])^2 \operatorname{Csc}[c] \\
& \left( 1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 - \frac{1}{\sqrt{2}d} \\
& C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (a+a\cos[c+dx])^2 \operatorname{Csc}[c] \left( 1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 + \frac{2A\sqrt{\cos[c+dx]}(a+a\cos[c+dx])^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c+dx]}}{7d} + \\
& \frac{2C\sqrt{\cos[c+dx]}(a+a\cos[c+dx])^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c+dx]}}{3d} + \\
& (a+a\cos[c+dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c+dx]} \left( \frac{(3A+5C)\cos[dx]\operatorname{Csc}[c]}{5d} + \frac{A\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^3\sin[dx]}{14d} + \right. \\
& \left. \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^2(5A\sin[c]+14A\sin[dx])}{70d} + \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx](42A\sin[c]+60A\sin[dx]+35C\sin[dx])}{210d} + \frac{(12A+7C)\tan[c]}{42d} \right)
\end{aligned}$$

■ **Problem 1169: Result unnecessarily involves higher level functions.**

$$\int (a+a\cos[c+dx])^2 (A+C\cos[c+dx])^2 \operatorname{Sec}[c+dx]^{7/2} dx$$

Optimal (type 4, 196 leaves, 8 steps):

$$\begin{aligned}
& - \frac{16a^2A\sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{5d} + \\
& \frac{4a^2(A+3C)\sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{3d} + \frac{2a^2(17A+15C)\sqrt{\operatorname{Sec}[c+dx]}\sin[c+dx]}{15d} + \\
& \frac{8A(a^2+a^2\cos[c+dx])\operatorname{Sec}[c+dx]^{3/2}\sin[c+dx]}{15d} + \frac{2A(a+a\cos[c+dx])^2\operatorname{Sec}[c+dx]^{5/2}\sin[c+dx]}{5d}
\end{aligned}$$

Result (type 5, 293 leaves):

$$\frac{1}{60 d} a^2 (1 + \cos[c + dx])^2 \sec\left[\frac{1}{2}(c + dx)\right]^4$$

$$\left( -\frac{1}{-1 + e^{2ic}} 4i\sqrt{2} e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \left( 12A(1 + e^{2i(c+dx)}) + 12A(-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + \right. \right.$$

$$\left. 5(A + 3C) e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) +$$

$$\left. \sqrt{\sec[c + dx]} (3(16A + 5C - 5C \cos[2c]) \cos[dx] \operatorname{Csc}[c] + 30C \cos[c] \sin[dx] + 2A(10 + 3 \sec[c + dx]) \tan[c + dx]) \right)$$

■ **Problem 1170: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos[c + dx])^2 (A + C \cos[c + dx])^2 \sec[c + dx]^{5/2} dx$$

Optimal (type 4, 196 leaves, 8 steps):

$$-\frac{4a^2(A - C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{d} + \frac{8a^2(A + C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{3d}$$

$$-\frac{2a^2(5A - C) \sin[c + dx]}{3d \sqrt{\sec[c + dx]}} + \frac{8A(a^2 + a^2 \cos[c + dx]) \sqrt{\sec[c + dx]} \sin[c + dx]}{3d} + \frac{2A(a + a \cos[c + dx])^2 \sec[c + dx]^{3/2} \sin[c + dx]}{3d}$$

Result (type 5, 181 leaves):

$$\frac{1}{6d} a^2 \sec[c + dx]^{3/2}$$

$$\left( 12iA - 12iC + 12iA \cos[2(c + dx)] - 12iC \cos[2(c + dx)] + 16(A + C) \cos[c + dx]^{3/2} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] - 12i(A - C) e^{-2i(c+dx)} \right.$$

$$\left. (1 + e^{2i(c+dx)})^{3/2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 4A \sin[c + dx] + C \sin[c + dx] + 12A \sin[2(c + dx)] + C \sin[3(c + dx)] \right)$$

■ **Problem 1171: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos[c + dx])^2 (A + C \cos[c + dx])^2 \sec[c + dx]^{3/2} dx$$

Optimal (type 4, 200 leaves, 8 steps):

$$\frac{16a^2C \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{5d} + \frac{4a^2(3A + C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{3d}$$

$$-\frac{2a^2(15A - 7C) \sin[c + dx]}{15d \sqrt{\sec[c + dx]}} - \frac{2(5A - C)(a^2 + a^2 \cos[c + dx]) \sin[c + dx]}{5d \sqrt{\sec[c + dx]}} + \frac{2A(a + a \cos[c + dx])^2 \sqrt{\sec[c + dx]} \sin[c + dx]}{d}$$

Result (type 5, 175 leaves) :

$$\frac{1}{30 d \sqrt{\sec [c+d x]}} a^2 \left( -96 i C + \frac{192 i C \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c+d x)}\right]}{\sqrt{1+e^{2 i (c+d x)}}} - \frac{80 i (3 A+C) e^{i (c+d x)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c+d x)}\right]}{\sqrt{1+e^{2 i (c+d x)}}} + 40 C \sin [c+d x] + 3 C \sec [c+d x] \sin [3 (c+d x)] + 60 A \tan [c+d x] + 3 C \tan [c+d x] \right)$$

■ **Problem 1172: Result unnecessarily involves higher level functions.**

$$\int (a+a \cos [c+d x])^2 (A+C \cos [c+d x])^2 \sqrt{\sec [c+d x]} dx$$

Optimal (type 4, 204 leaves, 8 steps) :

$$\frac{4 a^2 (5 A+3 C) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} + \frac{8 a^2 (7 A+3 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{21 d} + \frac{2 a^2 (35 A+33 C) \sin [c+d x]}{105 d \sqrt{\sec [c+d x]}} + \frac{2 C (a+a \cos [c+d x])^2 \sin [c+d x]}{7 d \sqrt{\sec [c+d x]}} + \frac{8 C (a^2+a^2 \cos [c+d x]) \sin [c+d x]}{35 d \sqrt{\sec [c+d x]}}$$

Result (type 5, 203 leaves) :

$$\frac{1}{420 d} a^2 e^{-i (2 c+d x)} \sqrt{\sec [c+d x]} \left( 160 (7 A+3 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] + 336 i (5 A+3 C) e^{-i (c+d x)} \sqrt{1+e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c+d x)}\right] + 2 \cos [c+d x] (-840 i A-504 i C+5 (28 A+51 C) \sin [c+d x]+84 C \sin [2 (c+d x)]+15 C \sin [3 (c+d x)]) \right) (\cos [2 c+d x]+i \sin [2 c+d x])$$

■ **Problem 1173: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \cos [c+d x])^2 (A+C \cos [c+d x])^2}{\sqrt{\sec [c+d x]}} dx$$

Optimal (type 4, 237 leaves, 9 steps) :

$$\frac{16 a^2 (3 A+2 C) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{15 d} + \frac{4 a^2 (7 A+5 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{21 d} + \frac{2 a^2 (21 A+19 C) \sin [c+d x]}{105 d \sec [c+d x]^{3/2}} + \frac{2 C (a+a \cos [c+d x])^2 \sin [c+d x]}{9 d \sec [c+d x]^{3/2}} + \frac{8 C (a^2+a^2 \cos [c+d x]) \sin [c+d x]}{63 d \sec [c+d x]^{3/2}} + \frac{4 a^2 (7 A+5 C) \sin [c+d x]}{21 d \sqrt{\sec [c+d x]}}$$

Result (type 5, 220 leaves) :

$$\frac{1}{2520 d} a^2 e^{-i(2c+dx)} \sqrt{\sec[c+dx]} \left( 480 (7A+5C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] + \right. \\ \left. 2688 i (3A+2C) e^{-i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + \right. \\ \left. 2 \cos[c+dx] (-4032 i A - 2688 i C + 60 (28A+23C) \sin[c+dx] + 14 (18A+37C) \sin[2(c+dx)] + \right. \\ \left. 180 C \sin[3(c+dx)] + 35 C \sin[4(c+dx)] \right) (\cos[2c+dx] + i \sin[2c+dx])$$

■ **Problem 1174: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c+dx])^2 (A + C \cos[c+dx])^2}{\sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 270 leaves, 10 steps) :

$$\frac{4 a^2 (9 A + 7 C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{15 d} + \\ \frac{8 a^2 (33 A + 25 C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{231 d} + \frac{2 a^2 (99 A + 89 C) \sin[c+dx]}{693 d \sec[c+dx]^{5/2}} + \\ \frac{2 C (a + a \cos[c+dx])^2 \sin[c+dx]}{11 d \sec[c+dx]^{5/2}} + \frac{8 C (a^2 + a^2 \cos[c+dx]) \sin[c+dx]}{99 d \sec[c+dx]^{5/2}} + \frac{4 a^2 (9 A + 7 C) \sin[c+dx]}{45 d \sec[c+dx]^{3/2}} + \frac{8 a^2 (33 A + 25 C) \sin[c+dx]}{231 d \sqrt{\sec[c+dx]}}$$

Result (type 5, 730 leaves) :



$$\frac{1}{5\sqrt{2}d} 3A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (a+a\cos[c+dx])^2 \operatorname{Csc}[c]$$

$$\left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 + \frac{1}{15\sqrt{2}d} 7C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}$$

$$(a+a\cos[c+dx])^2 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 +$$

$$\frac{2A\sqrt{\cos[c+dx]}(a+a\cos[c+dx])^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c+dx]}}{7d} +$$

$$\frac{50C\sqrt{\cos[c+dx]}(a+a\cos[c+dx])^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c+dx]}}{231d} +$$

$$(a+a\cos[c+dx])^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c+dx]} \left(-\frac{(198A+149C+234A\cos[2c]+187C\cos[2c])\cos[dx]\operatorname{Csc}[c]}{720d} +\right.$$

$$\frac{(2376A+2185C)\cos[2dx]\sin[2c]}{14784d} + \frac{(36A+43C)\cos[3dx]\sin[3c]}{720d} + \frac{(11A+27C)\cos[4dx]\sin[4c]}{1232d} +$$

$$\frac{C\cos[5dx]\sin[5c]}{144d} + \frac{C\cos[6dx]\sin[6c]}{704d} + \frac{(234A+187C)\cos[c]\sin[dx]}{360d} + \frac{(2376A+2185C)\cos[2c]\sin[2dx]}{14784d} +$$

$$\left.\frac{(36A+43C)\cos[3c]\sin[3dx]}{720d} + \frac{(11A+27C)\cos[4c]\sin[4dx]}{1232d} + \frac{C\cos[5c]\sin[5dx]}{144d} + \frac{C\cos[6c]\sin[6dx]}{704d}\right)$$

■ **Problem 1175: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+a\cos[c+dx])^3 (A+C\cos[c+dx]^2) \operatorname{Sec}[c+dx]^{13/2} dx$$

Optimal (type 4, 319 leaves, 11 steps):

$$\frac{4a^3(5A+7C)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{5d} +$$

$$\frac{4a^3(105A+143C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{231d} +$$

$$\frac{4a^3(5A+7C)\sqrt{\operatorname{Sec}[c+dx]}\sin[c+dx]}{5d} + \frac{4a^3(105A+143C)\operatorname{Sec}[c+dx]^{3/2}\sin[c+dx]}{231d} +$$

$$\frac{8a^3(35A+44C)\operatorname{Sec}[c+dx]^{5/2}\sin[c+dx]}{385d} + \frac{2(35A+33C)(a^3+a^3\cos[c+dx])\operatorname{Sec}[c+dx]^{7/2}\sin[c+dx]}{231d} +$$

$$\frac{4A(a^2+a^2\cos[c+dx])^2\operatorname{Sec}[c+dx]^{9/2}\sin[c+dx]}{33ad} + \frac{2A(a+a\cos[c+dx])^3\operatorname{Sec}[c+dx]^{11/2}\sin[c+dx]}{11d}$$

Result (type 5, 677 leaves) :

$$\begin{aligned}
 & -\frac{1}{2\sqrt{2}d} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \\
 & \left( 1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 - \frac{1}{10\sqrt{2}d} 7C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \\
 & (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \left( 1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 + \\
 & \frac{5A\sqrt{\cos[c+dx]} (a+a\cos[c+dx])^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]}}{22d} + \\
 & \frac{13C\sqrt{\cos[c+dx]} (a+a\cos[c+dx])^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]}}{42d} + \\
 & (a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]} \left( \frac{(5A+7C)\cos[dx]\operatorname{Csc}[c]}{10d} + \frac{A\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^5\sin[dx]}{44d} + \right. \\
 & \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^4(3A\sin[c]+11A\sin[dx])}{132d} + \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^3(77A\sin[c]+126A\sin[dx]+33C\sin[dx])}{924d} + \\
 & \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^2(630A\sin[c]+165C\sin[c]+770A\sin[dx]+693C\sin[dx])}{4620d} + \\
 & \left. \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx](770A\sin[c]+693C\sin[c]+1050A\sin[dx]+1430C\sin[dx])}{4620d} + \frac{(105A+143C)\tan[c]}{462d} \right)
 \end{aligned}$$

■ **Problem 1176: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+a\cos[c+dx])^3 (A+C\cos[c+dx]^2) \operatorname{Sec}[c+dx]^{11/2} dx$$

Optimal (type 4, 286 leaves, 10 steps) :

$$\begin{aligned}
 & \frac{4a^3(17A+27C)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{15d} + \\
 & \frac{4a^3(11A+21C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{21d} + \frac{4a^3(17A+27C)\sqrt{\operatorname{Sec}[c+dx]}\sin[c+dx]}{15d} + \\
 & \frac{8a^3(16A+21C)\operatorname{Sec}[c+dx]^{3/2}\sin[c+dx]}{105d} + \frac{2(73A+63C)(a^3+a^3\cos[c+dx])\operatorname{Sec}[c+dx]^{5/2}\sin[c+dx]}{315d} + \\
 & \frac{4A(a^2+a^2\cos[c+dx])^2\operatorname{Sec}[c+dx]^{7/2}\sin[c+dx]}{21ad} + \frac{2A(a+a\cos[c+dx])^3\operatorname{Sec}[c+dx]^{9/2}\sin[c+dx]}{9d}
 \end{aligned}$$

Result (type 5, 635 leaves) :

$$\begin{aligned}
& -\frac{1}{30\sqrt{2}d} 17A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \\
& \left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 - \frac{1}{10\sqrt{2}d} 9C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \\
& (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 + \\
& \frac{11A\sqrt{\cos[c+dx]}(a+a\cos[c+dx])^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]}}{42d} + \\
& \frac{C\sqrt{\cos[c+dx]}(a+a\cos[c+dx])^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]}}{2d} + \\
& (a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]} \left( \frac{(17A+27C)\cos[dx]\operatorname{Csc}[c]}{30d} + \frac{A\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^4\sin[dx]}{36d} + \right. \\
& \left. \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^3(7A\sin[c]+27A\sin[dx])}{252d} + \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^2(135A\sin[c]+238A\sin[dx]+63C\sin[dx])}{1260d} + \right. \\
& \left. \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx](238A\sin[c]+63C\sin[c]+330A\sin[dx]+315C\sin[dx])}{1260d} + \frac{(22A+21C)\tan[c]}{84d} \right)
\end{aligned}$$

■ **Problem 1177: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+a\cos[c+dx])^3 (A+C\cos[c+dx]^2) \operatorname{Sec}[c+dx]^{9/2} dx$$

Optimal (type 4, 253 leaves, 9 steps):

$$\begin{aligned}
& -\frac{4a^3(7A+5C)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{5d} + \\
& \frac{4a^3(13A+35C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{21d} + \\
& \frac{8a^3(53A+70C)\sqrt{\operatorname{Sec}[c+dx]}\sin[c+dx]}{105d} + \frac{2(7A+5C)(a^3+a^3\cos[c+dx])\operatorname{Sec}[c+dx]^{3/2}\sin[c+dx]}{15d} + \\
& \frac{12A(a^2+a^2\cos[c+dx])^2\operatorname{Sec}[c+dx]^{5/2}\sin[c+dx]}{35ad} + \frac{2A(a+a\cos[c+dx])^3\operatorname{Sec}[c+dx]^{7/2}\sin[c+dx]}{7d}
\end{aligned}$$

Result (type 5, 614 leaves):

$$\begin{aligned}
& -\frac{1}{10\sqrt{2}d} 7A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \\
& \left( 1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 - \frac{1}{2\sqrt{2}d} \\
& C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \left( 1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 + \frac{13A\sqrt{\cos[c+dx]}(a+a\cos[c+dx])^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]}}{42d} + \\
& \frac{5C\sqrt{\cos[c+dx]}(a+a\cos[c+dx])^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]}}{6d} + (a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \\
& \sqrt{\operatorname{Sec}[c+dx]} \left( -\frac{(-28A-25C+5C\cos[2c])\cos[dx]\operatorname{Csc}[c]}{40d} + \frac{C\cos[c]\sin[dx]}{4d} + \frac{A\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^3\sin[dx]}{28d} + \right. \\
& \left. \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^2(5A\sin[c]+21A\sin[dx])}{140d} + \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx](63A\sin[c]+130A\sin[dx]+35C\sin[dx])}{420d} + \frac{(26A+7C)\tan[c]}{84d} \right)
\end{aligned}$$

■ **Problem 1178: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+a\cos[c+dx])^3 (A+C\cos[c+dx]^2) \operatorname{Sec}[c+dx]^{7/2} dx$$

Optimal (type 4, 253 leaves, 9 steps):

$$\begin{aligned}
& -\frac{4a^3(9A-5C)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{5d} + \frac{4a^3(3A+5C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{3d} \\
& \frac{4a^3(21A+5C)\sin[c+dx]}{15d\sqrt{\operatorname{Sec}[c+dx]}} + \frac{2(11A+5C)(a^3+a^3\cos[c+dx])\sqrt{\operatorname{Sec}[c+dx]}\sin[c+dx]}{5d} + \\
& \frac{4A(a^2+a^2\cos[c+dx])^2\operatorname{Sec}[c+dx]^{3/2}\sin[c+dx]}{5ad} + \frac{2A(a+a\cos[c+dx])^3\operatorname{Sec}[c+dx]^{5/2}\sin[c+dx]}{5d}
\end{aligned}$$

Result (type 5, 604 leaves):

$$\begin{aligned}
& -\frac{1}{10\sqrt{2}d} 9A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \\
& \left( 1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 + \frac{1}{2\sqrt{2}d} \\
& C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (a+a\cos[c+dx])^3 \operatorname{Csc}[c] \left( 1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 + \frac{A\sqrt{\cos[c+dx]} (a+a\cos[c+dx])^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]}}{2d} + \\
& \frac{5C\sqrt{\cos[c+dx]} (a+a\cos[c+dx])^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]}}{6d} + \\
& (a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]} \left( -\frac{(-36A+5C+15C\cos[2c])\cos[dx]\operatorname{Csc}[c]}{40d} + \frac{C\cos[2dx]\sin[2c]}{24d} + \frac{3C\cos[c]\sin[dx]}{4d} + \right. \\
& \left. \frac{A\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^2\sin[dx]}{20d} + \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx](A\sin[c]+5A\sin[dx])}{20d} + \frac{C\cos[2c]\sin[2dx]}{24d} + \frac{A\tan[c]}{4d} \right)
\end{aligned}$$

■ **Problem 1179: Result unnecessarily involves higher level functions.**

$$\int (a+a\cos[c+dx])^3 (A+C\cos[c+dx]^2) \operatorname{Sec}[c+dx]^{5/2} dx$$

Optimal (type 4, 251 leaves, 9 steps):

$$\begin{aligned}
& -\frac{4a^3(5A-9C)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{5d} + \frac{4a^3(5A+3C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{3d} \\
& \frac{8a^3(10A-3C)\sin[c+dx]}{15d\sqrt{\operatorname{Sec}[c+dx]}} - \frac{2(35A-3C)(a^3+a^3\cos[c+dx])\sin[c+dx]}{15d\sqrt{\operatorname{Sec}[c+dx]}} + \\
& \frac{4A(a^2+a^2\cos[c+dx])^2\sqrt{\operatorname{Sec}[c+dx]}\sin[c+dx]}{ad} + \frac{2A(a+a\cos[c+dx])^3\operatorname{Sec}[c+dx]^{3/2}\sin[c+dx]}{3d}
\end{aligned}$$

Result (type 5, 245 leaves):

$$\frac{1}{60 d} a^3 e^{-i(2c+dx)} \operatorname{Sec}[c+dx]^{3/2} \left( 120 i A - 216 i C + 120 i A \operatorname{Cos}[2(c+dx)] - 216 i C \operatorname{Cos}[2(c+dx)] + 80(5A+3C) \operatorname{Cos}[c+dx]^{3/2} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] - 24 i(5A-9C) e^{-2i(c+dx)} (1+e^{2i(c+dx)})^{3/2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 40 A \operatorname{Sin}[c+dx] + 30 C \operatorname{Sin}[c+dx] + 180 A \operatorname{Sin}[2(c+dx)] + 6 C \operatorname{Sin}[2(c+dx)] + 30 C \operatorname{Sin}[3(c+dx)] + 3 C \operatorname{Sin}[4(c+dx)] \right) (\operatorname{Cos}[2c+dx] + i \operatorname{Sin}[2c+dx])$$

■ **Problem 1180: Result unnecessarily involves higher level functions.**

$$\int (a + a \operatorname{Cos}[c+dx])^3 (A + C \operatorname{Cos}[c+dx]^2) \operatorname{Sec}[c+dx]^{3/2} dx$$

Optimal (type 4, 257 leaves, 9 steps):

$$\frac{4 a^3 (5 A + 7 C) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{5 d} + \frac{4 a^3 (35 A + 13 C) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{21 d} - \frac{4 a^3 (35 A - 41 C) \operatorname{Sin}[c+dx]}{105 d \sqrt{\operatorname{Sec}[c+dx]}} - \frac{2 (7 A - C) (a^2 + a^2 \operatorname{Cos}[c+dx])^2 \operatorname{Sin}[c+dx]}{7 a d \sqrt{\operatorname{Sec}[c+dx]}} - \frac{2 (35 A - 11 C) (a^3 + a^3 \operatorname{Cos}[c+dx]) \operatorname{Sin}[c+dx]}{35 d \sqrt{\operatorname{Sec}[c+dx]}} + \frac{2 A (a + a \operatorname{Cos}[c+dx])^3 \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{d}$$

Result (type 5, 231 leaves):

$$\frac{1}{420 d} a^3 e^{-i(2c+dx)} \sqrt{\operatorname{Sec}[c+dx]} \left( -1680 i A \operatorname{Cos}[c+dx] - 2352 i C \operatorname{Cos}[c+dx] + 80(35A+13C) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] + 336 i(5A+7C) e^{-i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 840 A \operatorname{Sin}[c+dx] + 126 C \operatorname{Sin}[c+dx] + 140 A \operatorname{Sin}[2(c+dx)] + 550 C \operatorname{Sin}[2(c+dx)] + 126 C \operatorname{Sin}[3(c+dx)] + 15 C \operatorname{Sin}[4(c+dx)] \right) (\operatorname{Cos}[2c+dx] + i \operatorname{Sin}[2c+dx])$$

■ **Problem 1181: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Cos}[c+dx])^3 (A + C \operatorname{Cos}[c+dx]^2) \sqrt{\operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 253 leaves, 9 steps):

$$\frac{4 a^3 (27 A + 17 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{15 d} +$$

$$\frac{4 a^3 (21 A + 11 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{21 d} + \frac{8 a^3 (21 A + 16 C) \sin [c + d x]}{105 d \sqrt{\sec [c + d x]}} +$$

$$\frac{2 C (a + a \cos [c + d x])^3 \sin [c + d x]}{9 d \sqrt{\sec [c + d x]}} + \frac{4 C (a^2 + a^2 \cos [c + d x])^2 \sin [c + d x]}{21 a d \sqrt{\sec [c + d x]}} + \frac{2 (63 A + 73 C) (a^3 + a^3 \cos [c + d x]) \sin [c + d x]}{315 d \sqrt{\sec [c + d x]}}$$

Result (type 5, 684 leaves):

$$\frac{1}{10 \sqrt{2} d} 9 A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (a + a \cos [c + d x])^3 \operatorname{Csc}[c]$$

$$\left( (1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 + \frac{1}{30 \sqrt{2} d} 17 C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \right.$$

$$\left. (a + a \cos [c + d x])^3 \operatorname{Csc}[c] \left( (1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 + \right.$$

$$\left. \frac{A \sqrt{\cos [c + d x]} (a + a \cos [c + d x])^3 \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\sec [c + d x]}}{2 d} + \right.$$

$$\left. \frac{11 C \sqrt{\cos [c + d x]} (a + a \cos [c + d x])^3 \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\sec [c + d x]}}{42 d} + \right.$$

$$\left. (a + a \cos [c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\sec [c + d x]} \right.$$

$$\left( -\frac{(1278 A + 743 C + 1314 A \cos [2 c] + 889 C \cos [2 c]) \cos [d x] \operatorname{Csc}[c]}{2880 d} + \frac{(42 A + 53 C) \cos [2 d x] \sin [2 c]}{336 d} + \frac{(36 A + 151 C) \cos [3 d x] \sin [3 c]}{2880 d} + \right.$$

$$\frac{3 C \cos [4 d x] \sin [4 c]}{224 d} + \frac{C \cos [5 d x] \sin [5 c]}{576 d} + \frac{(1314 A + 889 C) \cos [c] \sin [d x]}{1440 d} + \frac{(42 A + 53 C) \cos [2 c] \sin [2 d x]}{336 d} +$$

$$\left. \frac{(36 A + 151 C) \cos [3 c] \sin [3 d x]}{2880 d} + \frac{3 C \cos [4 c] \sin [4 d x]}{224 d} + \frac{C \cos [5 c] \sin [5 d x]}{576 d} \right)$$

■ **Problem 1182: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \cos [c + d x])^3 (A + C \cos [c + d x])^2}{\sqrt{\sec [c + d x]}} dx$$

Optimal (type 4, 286 leaves, 10 steps):

$$\frac{4 a^3 (7 A + 5 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} +$$

$$\frac{4 a^3 (143 A + 105 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{231 d} + \frac{8 a^3 (44 A + 35 C) \sin [c + d x]}{385 d \sec [c + d x]^{3/2}} +$$

$$\frac{2 C (a + a \cos [c + d x])^3 \sin [c + d x]}{11 d \sec [c + d x]^{3/2}} + \frac{4 C (a^2 + a^2 \cos [c + d x])^2 \sin [c + d x]}{33 a d \sec [c + d x]^{3/2}} +$$

$$\frac{2 (33 A + 35 C) (a^3 + a^3 \cos [c + d x]) \sin [c + d x]}{231 d \sec [c + d x]^{3/2}} + \frac{4 a^3 (143 A + 105 C) \sin [c + d x]}{231 d \sqrt{\sec [c + d x]}}$$

Result (type 5, 208 leaves):

$$\frac{1}{18480 d} a^3 \sqrt{\sec [c + d x]} \left( 320 (143 A + 105 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + \right.$$

$$14784 i (7 A + 5 C) e^{-i(c+d x)} \sqrt{1 + e^{2i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+d x)}\right] +$$

$$2 \cos [c + d x] (-51744 i A - 36960 i C + 10(2354 A + 1953 C) \sin [c + d x] + 308(18 A + 25 C) \sin [2(c + d x)] +$$

$$660 A \sin [3(c + d x)] + 2835 C \sin [3(c + d x)] + 770 C \sin [4(c + d x)] + 105 C \sin [5(c + d x)]) \left. \right)$$

■ **Problem 1183: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos [c + d x])^3 (A + C \cos [c + d x])^2}{\sec [c + d x]^{3/2}} dx$$

Optimal (type 4, 319 leaves, 11 steps):

$$\frac{4 a^3 (221 A + 175 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{195 d} +$$

$$\frac{4 a^3 (121 A + 95 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{231 d} + \frac{40 a^3 (143 A + 118 C) \sin [c + d x]}{9009 d \sec [c + d x]^{5/2}} +$$

$$\frac{2 C (a + a \cos [c + d x])^3 \sin [c + d x]}{13 d \sec [c + d x]^{5/2}} + \frac{12 C (a^2 + a^2 \cos [c + d x])^2 \sin [c + d x]}{143 a d \sec [c + d x]^{5/2}} +$$

$$\frac{2 (143 A + 145 C) (a^3 + a^3 \cos [c + d x]) \sin [c + d x]}{1287 d \sec [c + d x]^{5/2}} + \frac{4 a^3 (221 A + 175 C) \sin [c + d x]}{585 d \sec [c + d x]^{3/2}} + \frac{4 a^3 (121 A + 95 C) \sin [c + d x]}{231 d \sqrt{\sec [c + d x]}}$$

Result (type 5, 776 leaves):



$$\frac{1}{30\sqrt{2}d} 17A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (a+a\cos[c+dx])^3 \operatorname{Csc}[c]$$

$$\left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 + \frac{1}{78\sqrt{2}d} 35C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}$$

$$(a+a\cos[c+dx])^3 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 +$$

$$\frac{11A\sqrt{\cos[c+dx]}(a+a\cos[c+dx])^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]}}{42d} +$$

$$\frac{95C\sqrt{\cos[c+dx]}(a+a\cos[c+dx])^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c+dx]}}{462d} + (a+a\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^6$$

$$\sqrt{\operatorname{Sec}[c+dx]} \left(-\frac{(77272A+59375C+92456A\cos[2c]+75025C\cos[2c])\cos[dx]\operatorname{Csc}[c]}{299520d} + \frac{(4664A+4267C)\cos[2dx]\sin[2c]}{29568d} +$$

$$\frac{(7852A+9005C)\cos[3dx]\sin[3c]}{149760d} + \frac{(33A+59C)\cos[4dx]\sin[4c]}{2464d} + \frac{(52A+245C)\cos[5dx]\sin[5c]}{29952d} + \frac{3C\cos[6dx]\sin[6c]}{1408d} +$$

$$\frac{C\cos[7dx]\sin[7c]}{3328d} + \frac{(92456A+75025C)\cos[c]\sin[dx]}{149760d} + \frac{(4664A+4267C)\cos[2c]\sin[2dx]}{29568d} + \frac{(7852A+9005C)\cos[3c]\sin[3dx]}{149760d} +$$

$$\left.\frac{(33A+59C)\cos[4c]\sin[4dx]}{2464d} + \frac{(52A+245C)\cos[5c]\sin[5dx]}{29952d} + \frac{3C\cos[6c]\sin[6dx]}{1408d} + \frac{C\cos[7c]\sin[7dx]}{3328d}\right)$$

■ **Problem 1184: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A+C\cos[c+dx])^2 \operatorname{Sec}[c+dx]^{7/2}}{a+a\cos[c+dx]} dx$$

Optimal (type 4, 232 leaves, 8 steps):

$$-\frac{3(7A+5C)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{5ad} -$$

$$\frac{(5A+3C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{3ad} + \frac{3(7A+5C)\sqrt{\operatorname{Sec}[c+dx]}\sin[c+dx]}{5ad} -$$

$$\frac{(5A+3C)\operatorname{Sec}[c+dx]^{3/2}\sin[c+dx]}{3ad} + \frac{(7A+5C)\operatorname{Sec}[c+dx]^{5/2}\sin[c+dx]}{5ad} - \frac{(A+C)\operatorname{Sec}[c+dx]^{5/2}\sin[c+dx]}{d(a+a\cos[c+dx])}$$

Result (type 5, 666 leaves):

$$\begin{aligned}
& - \left( 21 A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \right) / \left( 5\sqrt{2} d (a + a \cos[c + dx]) \right) - \\
& \quad \frac{1}{\sqrt{2} d (a + a \cos[c + dx])} 3 C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \\
& \quad \left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] - \\
& \quad \frac{5 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} \sin[c]}{3 d (a + a \cos[c + dx])} - \\
& \quad \frac{C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} \sin[c]}{d (a + a \cos[c + dx])} + \frac{1}{a + a \cos[c + dx]} \\
& \quad \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\operatorname{Sec}[c + dx]} \left( \frac{3(7A + 5C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} \right. \\
& \quad \left. \frac{4 A \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \sin[dx]}{5 d} + \frac{4 \operatorname{Sec}[c] \operatorname{Sec}[c + dx] (3 A \sin[c] - 5 A \sin[dx])}{15 d} - \frac{2 (2 A + 5 A \cos[c] + 3 C \cos[c]) \operatorname{Sec}[c] \tan\left[\frac{c}{2}\right]}{3 d} \right)
\end{aligned}$$

- **Problem 1185: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos[c + dx])^2 \operatorname{Sec}[c + dx]^{5/2}}{a + a \cos[c + dx]} dx$$

Optimal (type 4, 190 leaves, 7 steps):

$$\begin{aligned}
& \frac{(3A + C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{a d} + \frac{(5A + 3C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{3 a d} \\
& \frac{(3A + C) \sqrt{\operatorname{Sec}[c + dx]} \sin[c + dx]}{a d} + \frac{(5A + 3C) \operatorname{Sec}[c + dx]^{3/2} \sin[c + dx]}{3 a d} - \frac{(A + C) \operatorname{Sec}[c + dx]^{3/2} \sin[c + dx]}{d (a + a \cos[c + dx])}
\end{aligned}$$

Result (type 5, 628 leaves):

$$\frac{1}{\sqrt{2} d (a + a \cos [c + d x])} - 3 A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Csc} \left[ \frac{c}{2} \right]$$

$$\left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] + \frac{1}{\sqrt{2} d (a + a \cos [c + d x])}$$

$$C e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Csc} \left[ \frac{c}{2} \right] \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right)$$

$$\operatorname{Sec} \left[ \frac{c}{2} \right] + \frac{5 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])} +$$

$$\frac{C \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{d (a + a \cos [c + d x])} + \frac{1}{a + a \cos [c + d x]}$$

$$\cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \sqrt{\sec [c + d x]} \left( -\frac{(3 A + C) \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{d} + \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (A \sin \left[ \frac{d x}{2} \right] + C \sin \left[ \frac{d x}{2} \right])}{d} \right) +$$

$$\frac{4 A \operatorname{Sec} [c] \operatorname{Sec} [c + d x] \sin [d x]}{3 d} + \frac{2 (2 A + 5 A \cos [c] + 3 C \cos [c]) \operatorname{Sec} [c] \tan \left[ \frac{c}{2} \right]}{3 d}$$

- **Problem 1186: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos [c + d x])^2 \operatorname{Sec} [c + d x]^{3/2}}{a + a \cos [c + d x]} dx$$

Optimal (type 4, 153 leaves, 6 steps):

$$-\frac{(3 A + C) \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{a d} -$$

$$\frac{(A - C) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{a d} + \frac{(3 A + C) \sqrt{\sec [c + d x]} \sin [c + d x]}{a d} - \frac{(A + C) \sqrt{\sec [c + d x]} \sin [c + d x]}{d (a + a \cos [c + d x])}$$

Result (type 5, 376 leaves):

$$\frac{1}{2 a d (1 + \operatorname{Cos}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^2$$

$$\left( -6 \sqrt{2} A e^{-i(2 c + d x)} \sqrt{\frac{e^{i(c + d x)}}{1 + e^{2 i(c + d x)}}} \operatorname{Csc}[c] \left( 1 + e^{2 i(c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right] \right) - \right.$$

$$2 \sqrt{2} C e^{-i(2 c + d x)} \sqrt{\frac{e^{i(c + d x)}}{1 + e^{2 i(c + d x)}}} \operatorname{Csc}[c] \left( 1 + e^{2 i(c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right] \right) -$$

$$4 A \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} + 4 C \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} +$$

$$\left. 2 \sqrt{\operatorname{Sec}[c + d x]} \left( 2 (3 A + C) \operatorname{Cos}[d x] \operatorname{Csc}[c] - 2 (A + C) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] \right) \right)$$

- **Problem 1187: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \operatorname{Cos}[c + d x])^2 \sqrt{\operatorname{Sec}[c + d x]}}{a + a \operatorname{Cos}[c + d x]} dx$$

Optimal (type 4, 123 leaves, 5 steps):

$$\frac{(A + 3 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{a d} +$$

$$\frac{(A - C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{a d} - \frac{(A + C) \operatorname{Sin}[c + d x]}{d (a + a \operatorname{Cos}[c + d x]) \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 5, 401 leaves):

$$\frac{1}{2 a d (1 + \operatorname{Cos}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^2$$

$$\left( 2 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Csc}[c] \left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) + \right.$$

$$6 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Csc}[c] \left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) -$$

$$\frac{2 \left( (A + 2 C) \operatorname{Cos}\left[\frac{1}{2}(c - d x)\right] + C \operatorname{Cos}\left[\frac{1}{2}(3c + d x)\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{\operatorname{Sec}[c + d x]}} +$$

$$\left. 4 A \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} - 4 C \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} \right)$$

- **Problem 1188: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Cos}[c + d x]^2}{(a + a \operatorname{Cos}[c + d x]) \sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 162 leaves, 6 steps):

$$\frac{(A + 3 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{a d} +$$

$$\frac{(3 A + 5 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 a d} - \frac{(A + C) \operatorname{Sin}[c + d x]}{d (a + a \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x]^{3/2}} + \frac{(3 A + 5 C) \operatorname{Sin}[c + d x]}{3 a d \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 5, 645 leaves):

$$\begin{aligned}
& - \frac{1}{\sqrt{2} d (a + a \cos[c + dx])} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \\
& \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] - \frac{1}{\sqrt{2} d (a + a \cos[c + dx])} \\
& 3 C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \\
& \operatorname{Sec}\left[\frac{c}{2}\right] + \frac{A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} \sin[c]}{d (a + a \cos[c + dx])} + \\
& \frac{5 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} \sin[c]}{3 d (a + a \cos[c + dx])} + \frac{1}{a + a \cos[c + dx]} \\
& \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\operatorname{Sec}[c + dx]} \left( \frac{(A + 2C + C \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \frac{2 C \cos[2dx] \sin[2c]}{3 d} - \right. \\
& \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} - \frac{4 C \cos[c] \sin[dx]}{d} + \frac{2 C \cos[2c] \sin[2dx]}{3 d} - \frac{2 (A + C) \tan\left[\frac{c}{2}\right]}{d} \right)
\end{aligned}$$

- **Problem 1189: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c + dx]^2}{(a + a \cos[c + dx]) \operatorname{Sec}[c + dx]^{3/2}} dx$$

Optimal (type 4, 199 leaves, 7 steps):

$$\frac{3 (5 A + 7 C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{5 a d} - \frac{(3 A + 5 C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{3 a d} + \frac{(A + C) \sin[c + dx]}{d (a + a \cos[c + dx]) \operatorname{Sec}[c + dx]^{5/2}} + \frac{(5 A + 7 C) \sin[c + dx]}{5 a d \operatorname{Sec}[c + dx]^{3/2}} - \frac{(3 A + 5 C) \sin[c + dx]}{3 a d \sqrt{\operatorname{Sec}[c + dx]}}$$

Result (type 5, 438 leaves):

$$\frac{1}{60 a d (1 + \operatorname{Cos}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^2$$

$$\left( 180 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Csc}[c] \left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) + \right.$$

$$252 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Csc}[c] \left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) -$$

$$120 A \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} -$$

$$200 C \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} - \frac{1}{\sqrt{\operatorname{Sec}[c + d x]}} 2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]$$

$$\left. \left( (60 A + 83 C) \operatorname{Cos}\left[\frac{1}{2}(c - d x)\right] + (30 A + 43 C) \operatorname{Cos}\left[\frac{1}{2}(3c + d x)\right] + C \operatorname{Sin}[c] \left( 7 \operatorname{Sin}\left[\frac{3}{2}(c + d x)\right] - 3 \operatorname{Sin}\left[\frac{5}{2}(c + d x)\right] \right) \right) \right)$$

■ **Problem 1190: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Cos}[c + d x]^2}{(a + a \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x]^{5/2}} dx$$

Optimal (type 4, 232 leaves, 8 steps):

$$-\frac{3(5A+7C)\sqrt{\operatorname{Cos}[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]} + 5(7A+9C)\sqrt{\operatorname{Cos}[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{5ad} -$$

$$\frac{(A+C)\operatorname{Sin}[c+dx]}{d(a+a\operatorname{Cos}[c+dx])\operatorname{Sec}[c+dx]^{7/2}} + \frac{(7A+9C)\operatorname{Sin}[c+dx]}{7ad\operatorname{Sec}[c+dx]^{5/2}} - \frac{(5A+7C)\operatorname{Sin}[c+dx]}{5ad\operatorname{Sec}[c+dx]^{3/2}} + \frac{5(7A+9C)\operatorname{Sin}[c+dx]}{21ad\sqrt{\operatorname{Sec}[c+dx]}}$$

Result (type 5, 751 leaves):

$$\begin{aligned}
& - \frac{1}{\sqrt{2} d (a + a \cos[c + dx])} 3 A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \quad \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] - \\
& \left(21 C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right)\right) \\
& \operatorname{Sec}\left[\frac{c}{2}\right] \Bigg/ \left(5 \sqrt{2} d (a + a \cos[c + dx])\right) + \frac{5 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\sec[c + dx]} \sin[c]}{3 d (a + a \cos[c + dx])} + \\
& \frac{15 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\sec[c + dx]} \sin[c]}{7 d (a + a \cos[c + dx])} + \frac{1}{a + a \cos[c + dx]} \\
& \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\sec[c + dx]} \left( \frac{(40 A + 51 C + 20 A \cos[2c] + 33 C \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{20 d} + \right. \\
& \quad \frac{(14 A + 27 C) \cos[2dx] \sin[2c]}{21 d} - \frac{C \cos[3dx] \sin[3c]}{5 d} + \frac{C \cos[4dx] \sin[4c]}{14 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} \\
& \quad \left. \frac{(20 A + 33 C) \cos[c] \sin[dx]}{5 d} + \frac{(14 A + 27 C) \cos[2c] \sin[2dx]}{21 d} - \frac{C \cos[3c] \sin[3dx]}{5 d} + \frac{C \cos[4c] \sin[4dx]}{14 d} - \frac{2(A + C) \tan\left[\frac{c}{2}\right]}{d} \right)
\end{aligned}$$

- **Problem 1191: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos[c + dx])^2 \sec[c + dx]^{5/2}}{(a + a \cos[c + dx])^2} dx$$

Optimal (type 4, 229 leaves, 8 steps):

$$\begin{aligned}
& \frac{(7A + C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{a^2 d} + \\
& \frac{2(5A + C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{3 a^2 d} - \frac{(7A + C) \sqrt{\sec[c + dx]} \sin[c + dx]}{a^2 d} + \\
& \frac{2(5A + C) \sec[c + dx]^{3/2} \sin[c + dx]}{3 a^2 d} - \frac{(7A + C) \sec[c + dx]^{3/2} \sin[c + dx]}{3 a^2 d (1 + \cos[c + dx])} - \frac{(A + C) \sec[c + dx]^{3/2} \sin[c + dx]}{3 d (a + a \cos[c + dx])^2}
\end{aligned}$$

Result (type 5, 709 leaves):



$$\frac{1}{d (a + a \cos [c + d x])^2} 7 \sqrt{2} A e^{-i (2c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2i (c+dx)}}} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right]$$

$$\left( 1 + e^{2i (c+dx)} + (-1 + e^{2i c}) \sqrt{1 + e^{2i (c+dx)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i (c+dx)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] + \frac{1}{d (a + a \cos [c + d x])^2}$$

$$\sqrt{2} C e^{-i (2c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2i (c+dx)}}} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \left( 1 + e^{2i (c+dx)} + (-1 + e^{2i c}) \sqrt{1 + e^{2i (c+dx)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i (c+dx)} \right] \right)$$

$$\operatorname{Sec} \left[ \frac{c}{2} \right] + \frac{20 A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\operatorname{Sec} [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^2} +$$

$$\frac{4 C \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\operatorname{Sec} [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^2} +$$

$$\frac{1}{(a + a \cos [c + d x])^2} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \sqrt{\operatorname{Sec} [c + d x]}$$

$$\left( -\frac{2 (7 A + C) \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{d} + \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 (A \sin \left[ \frac{dx}{2} \right] + C \sin \left[ \frac{dx}{2} \right])}{3 d} + \frac{8 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right] (4 A \sin \left[ \frac{dx}{2} \right] + C \sin \left[ \frac{dx}{2} \right])}{3 d} + \right.$$

$$\left. \frac{8 A \operatorname{Sec} [c] \operatorname{Sec} [c + d x] \sin [d x]}{3 d} + \frac{8 (A + 5 A \cos [c] + C \cos [c]) \operatorname{Sec} [c] \tan \left[ \frac{c}{2} \right]}{3 d} + \frac{2 (A + C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \tan \left[ \frac{c}{2} \right]}{3 d} \right)$$

■ **Problem 1192: Result unnecessarily involves higher level functions.**

$$\int \frac{(A + C \cos [c + d x])^2 \operatorname{Sec} [c + d x]^{3/2}}{(a + a \cos [c + d x])^2} dx$$

Optimal (type 4, 195 leaves, 7 steps):

$$\frac{4 A \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]} - (5 A - C) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{a^2 d} +$$

$$\frac{4 A \sqrt{\operatorname{Sec} [c + d x]} \sin [c + d x]}{a^2 d} - \frac{(5 A - C) \sqrt{\operatorname{Sec} [c + d x]} \sin [c + d x]}{3 a^2 d (1 + \cos [c + d x])} - \frac{(A + C) \sqrt{\operatorname{Sec} [c + d x]} \sin [c + d x]}{3 d (a + a \cos [c + d x])^2}$$

Result (type 5, 286 leaves):

$$-\frac{1}{12 a^2 d (1 + \cos [c + d x])^2} e^{-3 i (c+d x)} \left(1 + e^{i (c+d x)}\right) \left( (5 A - C) e^{i (c+d x)} \left(1 + e^{i (c+d x)}\right)^3 \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] - \right. \\ \left. i \left(12 A + 31 A e^{i (c+d x)} + C e^{i (c+d x)} + 29 A e^{2 i (c+d x)} - C e^{2 i (c+d x)} + 19 A e^{3 i (c+d x)} + C e^{3 i (c+d x)} + 5 A e^{4 i (c+d x)} - \right. \right. \\ \left. \left. C e^{4 i (c+d x)} - 12 A \left(1 + e^{i (c+d x)}\right)^3 \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c+d x)}\right]\right) \right) \sqrt{\sec [c + d x]}$$

■ **Problem 1193: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos [c + d x]^2) \sqrt{\sec [c + d x]}}{(a + a \cos [c + d x])^2} dx$$

Optimal (type 4, 165 leaves, 6 steps):

$$\frac{(A - C) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\sec [c + d x]}}{a^2 d} + \frac{2 (A + C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\sec [c + d x]}}{3 a^2 d} - \\ \frac{(A - C) \sin [c + d x]}{a^2 d (1 + \cos [c + d x]) \sqrt{\sec [c + d x]}} - \frac{(A + C) \sin [c + d x]}{3 d (a + a \cos [c + d x])^2 \sqrt{\sec [c + d x]}}$$

Result (type 5, 681 leaves):

$$\frac{1}{d (a + a \cos [c + d x])^2} \sqrt{2} A e^{-i (2 c+d x)} \sqrt{\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}}} \cos \left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \\ \left(1 + e^{2 i (c+d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c+d x)}\right]\right) \sec \left[\frac{c}{2}\right] - \frac{1}{d (a + a \cos [c + d x])^2} \\ \sqrt{2} C e^{-i (2 c+d x)} \sqrt{\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}}} \cos \left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2 i (c+d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c+d x)}\right]\right) \\ \sec \left[\frac{c}{2}\right] + \frac{4 A \cos \left[\frac{c}{2} + \frac{d x}{2}\right]^4 \sqrt{\cos [c + d x]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sec \left[\frac{c}{2}\right] \sqrt{\sec [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^2} + \\ \frac{4 C \cos \left[\frac{c}{2} + \frac{d x}{2}\right]^4 \sqrt{\cos [c + d x]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sec \left[\frac{c}{2}\right] \sqrt{\sec [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^2} + \frac{1}{(a + a \cos [c + d x])^2} \\ \cos \left[\frac{c}{2} + \frac{d x}{2}\right]^4 \sqrt{\sec [c + d x]} \left(-\frac{2 (A - C) \cos [d x] \operatorname{Csc}\left[\frac{c}{2}\right] \sec \left[\frac{c}{2}\right]}{d} + \frac{8 \sec \left[\frac{c}{2}\right] \sec \left[\frac{c}{2} + \frac{d x}{2}\right] (A \sin \left[\frac{d x}{2}\right] - 2 C \sin \left[\frac{d x}{2}\right])}{3 d} + \right. \\ \left. \frac{2 \sec \left[\frac{c}{2}\right] \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^3 (A \sin \left[\frac{d x}{2}\right] + C \sin \left[\frac{d x}{2}\right])}{3 d} + \frac{8 (A - 2 C) \tan \left[\frac{c}{2}\right]}{3 d} + \frac{2 (A + C) \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^2 \tan \left[\frac{c}{2}\right]}{3 d}\right)$$

- **Problem 1194: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos [c + d x]^2}{(a + a \cos [c + d x])^2 \sqrt{\sec [c + d x]}} dx$$

Optimal (type 4, 166 leaves, 6 steps):

$$\frac{4 C \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]} + (A - 5 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{a^2 d} + \frac{(A + C) \sin [c + d x]}{3 d (a + a \cos [c + d x])^2 \sec [c + d x]^{3/2}} + \frac{(A - 5 C) \sin [c + d x]}{3 a^2 d (1 + \cos [c + d x]) \sqrt{\sec [c + d x]}}$$

Result (type 5, 539 leaves):

$$\frac{1}{d (a + a \cos [c + d x])^2} 4 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right]$$

$$\left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)} \right] \right) \sec \left[ \frac{c}{2} \right] +$$

$$\frac{2 A \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2}(c + d x), 2 \right] \sec \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^2} -$$

$$\frac{10 C \cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2}(c + d x), 2 \right] \sec \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^2} + \frac{1}{(a + a \cos [c + d x])^2}$$

$$\cos \left[ \frac{c}{2} + \frac{dx}{2} \right]^4 \sqrt{\sec [c + d x]} \left( -\frac{2 C (3 + \cos [2c]) \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} \right]}{d} - \frac{2 \sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^3 (A \sin \left[ \frac{dx}{2} \right] + C \sin \left[ \frac{dx}{2} \right])}{3 d} + \right.$$

$$\left. \frac{4 \sec \left[ \frac{c}{2} \right] \sec \left[ \frac{c}{2} + \frac{dx}{2} \right] (A \sin \left[ \frac{dx}{2} \right] + 7 C \sin \left[ \frac{dx}{2} \right])}{3 d} + \frac{8 C \cos [c] \sin [d x]}{d} + \frac{4 (A + 7 C) \tan \left[ \frac{c}{2} \right]}{3 d} - \frac{2 (A + C) \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^2 \tan \left[ \frac{c}{2} \right]}{3 d} \right)$$

- **Problem 1195: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos [c + d x]^2}{(a + a \cos [c + d x])^2 \sec [c + d x]^{3/2}} dx$$

Optimal (type 4, 201 leaves, 7 steps):

$$\frac{(A + 7 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]} + 2 (A + 5 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{a^2 d} + \frac{(A + C) \sin [c + d x]}{3 d (a + a \cos [c + d x])^2 \sec [c + d x]^{5/2}} - \frac{(A + 7 C) \sin [c + d x]}{3 a^2 d (1 + \cos [c + d x]) \sec [c + d x]^{3/2}} + \frac{2 (A + 5 C) \sin [c + d x]}{3 a^2 d \sqrt{\sec [c + d x]}}$$

Result (type 5, 738 leaves) :

$$\begin{aligned}
 & - \frac{1}{d (a + a \cos [c + d x])^2} \sqrt{2} A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \\
 & \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] - \frac{1}{d (a + a \cos [c + d x])^2} \\
 & 7 \sqrt{2} C e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \\
 & \operatorname{Sec} \left[ \frac{c}{2} \right] + \frac{4 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\operatorname{Sec} [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^2} + \\
 & \frac{20 C \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\operatorname{Sec} [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^2} + \frac{1}{(a + a \cos [c + d x])^2} \\
 & \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\operatorname{Sec} [c + d x]} \left( \frac{2 (A + 5 C + 2 C \cos [2 c]) \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{d} + \frac{4 C \cos [2 d x] \sin [2 c]}{3 d} + \right. \\
 & \left. \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (A \sin \left[ \frac{d x}{2} \right] + C \sin \left[ \frac{d x}{2} \right])}{3 d} - \frac{8 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (2 A \sin \left[ \frac{d x}{2} \right] + 5 C \sin \left[ \frac{d x}{2} \right])}{3 d} - \right. \\
 & \left. \frac{16 C \cos [c] \sin [d x]}{d} + \frac{4 C \cos [2 c] \sin [2 d x]}{3 d} - \frac{8 (2 A + 5 C) \tan \left[ \frac{c}{2} \right]}{3 d} + \frac{2 (A + C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \tan \left[ \frac{c}{2} \right]}{3 d} \right)
 \end{aligned}$$

■ **Problem 1196: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos [c + d x]^2}{(a + a \cos [c + d x])^2 \operatorname{Sec} [c + d x]^{5/2}} dx$$

Optimal (type 4, 236 leaves, 8 steps) :

$$\begin{aligned}
 & \frac{4 (5 A + 14 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{5 a^2 d} - \frac{5 (A + 3 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{3 a^2 d} - \\
 & \frac{(A + C) \sin [c + d x]}{3 d (a + a \cos [c + d x])^2 \operatorname{Sec} [c + d x]^{7/2}} - \frac{(A + 3 C) \sin [c + d x]}{a^2 d (1 + \cos [c + d x]) \operatorname{Sec} [c + d x]^{5/2}} + \frac{4 (5 A + 14 C) \sin [c + d x]}{15 a^2 d \operatorname{Sec} [c + d x]^{3/2}} - \frac{5 (A + 3 C) \sin [c + d x]}{3 a^2 d \sqrt{\operatorname{Sec} [c + d x]}}
 \end{aligned}$$

Result (type 5, 791 leaves) :

$$\begin{aligned}
& \frac{1}{d (a + a \cos [c + d x])^2} 4 \sqrt{2} A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \\
& \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] + \\
& \frac{1}{5 d (a + a \cos [c + d x])^2} 56 \sqrt{2} C e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[ \frac{c}{2} \right] \\
& \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] - \\
& \frac{10 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^2} - \\
& \frac{10 C \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{d (a + a \cos [c + d x])^2} + \frac{1}{(a + a \cos [c + d x])^2} \\
& \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\sec [c + d x]} \left( -\frac{(60 A + 151 C + 20 A \cos [2 c] + 73 C \cos [2 c]) \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{10 d} - \frac{8 C \cos [2 d x] \sin [2 c]}{3 d} + \right. \\
& \left. \frac{2 C \cos [3 d x] \sin [3 c]}{5 d} - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (A \sin \left[ \frac{d x}{2} \right] + C \sin \left[ \frac{d x}{2} \right])}{3 d} + \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (7 A \sin \left[ \frac{d x}{2} \right] + 13 C \sin \left[ \frac{d x}{2} \right])}{3 d} + \right. \\
& \left. \frac{2 (20 A + 73 C) \cos [c] \sin [d x]}{5 d} - \frac{8 C \cos [2 c] \sin [2 d x]}{3 d} + \frac{2 C \cos [3 c] \sin [3 d x]}{5 d} + \frac{4 (7 A + 13 C) \tan \left[ \frac{c}{2} \right]}{3 d} - \frac{2 (A + C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \tan \left[ \frac{c}{2} \right]}{3 d} \right)
\end{aligned}$$

■ **Problem 1197: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos [c + d x])^2 \operatorname{Sec} [c + d x]^{5/2}}{(a + a \cos [c + d x])^3} dx$$

Optimal (type 4, 282 leaves, 9 steps):

$$\begin{aligned}
& \frac{(119 A + 9 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{10 a^3 d} + \frac{(11 A + C) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{2 a^3 d} - \\
& \frac{(119 A + 9 C) \sqrt{\sec [c + d x]} \sin [c + d x]}{10 a^3 d} + \frac{(11 A + C) \operatorname{Sec} [c + d x]^{3/2} \sin [c + d x]}{2 a^3 d} - \\
& \frac{(A + C) \operatorname{Sec} [c + d x]^{3/2} \sin [c + d x]}{5 d (a + a \cos [c + d x])^3} - \frac{2 A \operatorname{Sec} [c + d x]^{3/2} \sin [c + d x]}{3 a d (a + a \cos [c + d x])^2} - \frac{(119 A + 9 C) \operatorname{Sec} [c + d x]^{3/2} \sin [c + d x]}{30 d (a^3 + a^3 \cos [c + d x])}
\end{aligned}$$

Result (type 5, 802 leaves):

$$\begin{aligned}
& \frac{1}{5 d (a + a \cos [c + d x])^3} 119 \sqrt{2} A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right]} \\
& \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] + \frac{1}{5 d (a + a \cos [c + d x])^3} \\
& 9 \sqrt{2} C e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right]} \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \\
& \operatorname{Sec} \left[ \frac{c}{2} \right] + \frac{22 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\operatorname{Sec} [c + d x]} \sin [c]}{d (a + a \cos [c + d x])^3} + \\
& \frac{2 C \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\operatorname{Sec} [c + d x]} \sin [c]}{d (a + a \cos [c + d x])^3} + \\
& \frac{1}{(a + a \cos [c + d x])^3} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\operatorname{Sec} [c + d x]} \left( -\frac{2 (119 A + 9 C) \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{5 d} + \right. \\
& \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 (A \sin \left[ \frac{d x}{2} \right] + C \sin \left[ \frac{d x}{2} \right])}{5 d} + \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (13 A \sin \left[ \frac{d x}{2} \right] + 3 C \sin \left[ \frac{d x}{2} \right])}{15 d} + \\
& \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (29 A \sin \left[ \frac{d x}{2} \right] + 3 C \sin \left[ \frac{d x}{2} \right])}{3 d} + \frac{16 A \operatorname{Sec} [c] \operatorname{Sec} [c + d x] \sin [d x]}{3 d} + \\
& \left. \frac{4 (4 A + 33 A \cos [c] + 3 C \cos [c]) \operatorname{Sec} [c] \tan \left[ \frac{c}{2} \right]}{3 d} + \frac{4 (13 A + 3 C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \tan \left[ \frac{c}{2} \right]}{15 d} + \frac{2 (A + C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \tan \left[ \frac{c}{2} \right]}{5 d} \right)
\end{aligned}$$

- **Problem 1198: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos [c + d x])^2 \operatorname{Sec} [c + d x]^{3/2}}{(a + a \cos [c + d x])^3} dx$$

Optimal (type 4, 259 leaves, 8 steps):

$$\begin{aligned}
& \frac{(49 A - C) \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{10 a^3 d} - \\
& \frac{(13 A - C) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{6 a^3 d} + \frac{(49 A - C) \sqrt{\operatorname{Sec} [c + d x]} \sin [c + d x]}{10 a^3 d} - \\
& \frac{(A + C) \sqrt{\operatorname{Sec} [c + d x]} \sin [c + d x]}{5 d (a + a \cos [c + d x])^3} - \frac{2 (4 A - C) \sqrt{\operatorname{Sec} [c + d x]} \sin [c + d x]}{15 a d (a + a \cos [c + d x])^2} - \frac{(13 A - C) \sqrt{\operatorname{Sec} [c + d x]} \sin [c + d x]}{6 d (a^3 + a^3 \cos [c + d x])}
\end{aligned}$$

Result (type 5, 777 leaves):

$$\begin{aligned}
& - \frac{1}{5 d (a + a \cos [c + d x])^3} 49 \sqrt{2} A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \\
& \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] + \frac{1}{5 d (a + a \cos [c + d x])^3} \\
& \sqrt{2} C e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \\
& \operatorname{Sec} \left[ \frac{c}{2} \right] - \frac{26 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\operatorname{Sec} [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^3} + \\
& \frac{2 C \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\operatorname{Sec} [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^3} + \\
& \frac{1}{(a + a \cos [c + d x])^3} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\operatorname{Sec} [c + d x]} \\
& \left( \frac{2 (49 A - C) \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{5 d} - \frac{8 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (4 A \sin \left[ \frac{d x}{2} \right] - C \sin \left[ \frac{d x}{2} \right])}{15 d} - \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (13 A \sin \left[ \frac{d x}{2} \right] - C \sin \left[ \frac{d x}{2} \right])}{3 d} \right. \\
& \left. - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 (A \sin \left[ \frac{d x}{2} \right] + C \sin \left[ \frac{d x}{2} \right])}{5 d} - \frac{4 (13 A - C) \tan \left[ \frac{c}{2} \right]}{3 d} - \frac{8 (4 A - C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \tan \left[ \frac{c}{2} \right]}{15 d} - \frac{2 (A + C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \tan \left[ \frac{c}{2} \right]}{5 d} \right)
\end{aligned}$$

- **Problem 1199: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos [c + d x])^2 \sqrt{\operatorname{Sec} [c + d x]}}{(a + a \cos [c + d x])^3} dx$$

Optimal (type 4, 224 leaves, 7 steps):

$$\frac{(9 A - C) \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{10 a^3 d} + \frac{(3 A + C) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{6 a^3 d} - \\
\frac{(A + C) \sin [c + d x]}{5 d (a + a \cos [c + d x])^3 \sqrt{\operatorname{Sec} [c + d x]}} - \frac{2 (3 A - 2 C) \sin [c + d x]}{15 a d (a + a \cos [c + d x])^2 \sqrt{\operatorname{Sec} [c + d x]}} - \frac{(9 A - C) \sin [c + d x]}{10 d (a^3 + a^3 \cos [c + d x]) \sqrt{\operatorname{Sec} [c + d x]}}$$

Result (type 5, 772 leaves):

$$\begin{aligned}
& \frac{1}{5 d (a + a \operatorname{Cos}[c + d x])^3} 9 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \\
& \left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] - \frac{1}{5 d (a + a \operatorname{Cos}[c + d x])^3} \\
& \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \left( 1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \\
& \operatorname{Sec}\left[\frac{c}{2}\right] + \frac{2 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c]}{d (a + a \operatorname{Cos}[c + d x])^3} + \\
& \frac{2 C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c]}{3 d (a + a \operatorname{Cos}[c + d x])^3} + \\
& \frac{1}{(a + a \operatorname{Cos}[c + d x])^3} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c + d x]} \\
& \left( -\frac{2(9A - C) \operatorname{Cos}[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5 d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (3A \operatorname{Sin}\left[\frac{dx}{2}\right] - 7C \operatorname{Sin}\left[\frac{dx}{2}\right])}{15 d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right])}{5 d} + \right. \\
& \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (3A \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right])}{3 d} + \frac{4(3A + C) \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} + \frac{4(3A - 7C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} + \frac{2(A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right)
\end{aligned}$$

- **Problem 1200: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Cos}[c + d x]^2}{(a + a \operatorname{Cos}[c + d x])^3 \sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 220 leaves, 7 steps):

$$\begin{aligned}
& \frac{(A - 9C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{10 a^3 d} + \frac{(A + 3C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{6 a^3 d} - \\
& \frac{(A + C) \operatorname{Sin}[c + d x]}{5 d (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Sec}[c + d x]^{3/2}} + \frac{2(2A - 3C) \operatorname{Sin}[c + d x]}{15 a d (a + a \operatorname{Cos}[c + d x])^2 \sqrt{\operatorname{Sec}[c + d x]}} - \frac{(A - 9C) \operatorname{Sin}[c + d x]}{10 d (a^3 + a^3 \operatorname{Cos}[c + d x]) \sqrt{\operatorname{Sec}[c + d x]}}
\end{aligned}$$

Result (type 5, 767 leaves):



$$\frac{1}{5 d (a + a \cos [c + d x])^3} \sqrt{2} A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right]$$

$$\left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] - \frac{1}{5 d (a + a \cos [c + d x])^3}$$

$$9 \sqrt{2} C e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right)$$

$$\operatorname{Sec} \left[ \frac{c}{2} \right] + \frac{2 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\operatorname{Sec} [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^3} +$$

$$\frac{2 C \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\operatorname{Sec} [c + d x]} \sin [c]}{d (a + a \cos [c + d x])^3} +$$

$$\frac{1}{(a + a \cos [c + d x])^3} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\operatorname{Sec} [c + d x]}$$

$$\left( -\frac{2 (A - 9 C) \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{5 d} + \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (A \sin \left[ \frac{d x}{2} \right] - 9 C \sin \left[ \frac{d x}{2} \right])}{3 d} - \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 (A \sin \left[ \frac{d x}{2} \right] + C \sin \left[ \frac{d x}{2} \right])}{5 d} + \right.$$

$$\left. \frac{8 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (A \sin \left[ \frac{d x}{2} \right] + 6 C \sin \left[ \frac{d x}{2} \right])}{15 d} + \frac{4 (A - 9 C) \tan \left[ \frac{c}{2} \right]}{3 d} + \frac{8 (A + 6 C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \tan \left[ \frac{c}{2} \right]}{15 d} - \frac{2 (A + C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \tan \left[ \frac{c}{2} \right]}{5 d} \right)$$

- **Problem 1201: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos [c + d x]^2}{(a + a \cos [c + d x])^3 \operatorname{Sec} [c + d x]^{3/2}} dx$$

Optimal (type 4, 218 leaves, 7 steps):

$$-\frac{(A - 49 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{10 a^3 d} + \frac{(A - 13 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{6 a^3 d}$$

$$\frac{(A + C) \sin [c + d x]}{5 d (a + a \cos [c + d x])^3 \operatorname{Sec} [c + d x]^{5/2}} + \frac{2 (A - 4 C) \sin [c + d x]}{15 a d (a + a \cos [c + d x])^2 \operatorname{Sec} [c + d x]^{3/2}} + \frac{(A - 13 C) \sin [c + d x]}{6 d (a^3 + a^3 \cos [c + d x]) \sqrt{\operatorname{Sec} [c + d x]}}$$

Result (type 5, 793 leaves):

$$\begin{aligned}
& - \frac{1}{5 d (a + a \cos [c + d x])^3} \sqrt{2} A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \\
& \quad \operatorname{Csc} \left[ \frac{c}{2} \right] \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] + \\
& \frac{1}{5 d (a + a \cos [c + d x])^3} 49 \sqrt{2} C e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[ \frac{c}{2} \right] \\
& \quad \left( 1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) \operatorname{Sec} \left[ \frac{c}{2} \right] + \\
& \frac{2 A \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^3} - \\
& \frac{26 C \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\cos [c + d x]} \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \operatorname{Sec} \left[ \frac{c}{2} \right] \sqrt{\sec [c + d x]} \sin [c]}{3 d (a + a \cos [c + d x])^3} + \frac{1}{(a + a \cos [c + d x])^3} \\
& \cos \left[ \frac{c}{2} + \frac{d x}{2} \right]^6 \sqrt{\sec [c + d x]} \left( - \frac{2 (-A + 39 C + 10 C \cos [2 c]) \cos [d x] \operatorname{Csc} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} \right]}{5 d} + \frac{2 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 (A \sin \left[ \frac{d x}{2} \right] + C \sin \left[ \frac{d x}{2} \right])}{5 d} - \right. \\
& \quad \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 (7 A \sin \left[ \frac{d x}{2} \right] + 17 C \sin \left[ \frac{d x}{2} \right])}{15 d} + \frac{4 \operatorname{Sec} \left[ \frac{c}{2} \right] \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right] (A \sin \left[ \frac{d x}{2} \right] + 23 C \sin \left[ \frac{d x}{2} \right])}{3 d} + \\
& \quad \left. \frac{16 C \cos [c] \sin [d x]}{d} + \frac{4 (A + 23 C) \tan \left[ \frac{c}{2} \right]}{3 d} - \frac{4 (7 A + 17 C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^2 \tan \left[ \frac{c}{2} \right]}{15 d} + \frac{2 (A + C) \operatorname{Sec} \left[ \frac{c}{2} + \frac{d x}{2} \right]^4 \tan \left[ \frac{c}{2} \right]}{5 d} \right)
\end{aligned}$$

■ **Problem 1202: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos [c + d x]^2}{(a + a \cos [c + d x])^3 \sec [c + d x]^{5/2}} dx$$

Optimal (type 4, 249 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(9 A + 119 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{10 a^3 d} + \\
& \frac{(A + 11 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{2 a^3 d} - \frac{(A + C) \sin [c + d x]}{5 d (a + a \cos [c + d x])^3 \sec [c + d x]^{7/2}} - \\
& \frac{2 C \sin [c + d x]}{3 a d (a + a \cos [c + d x])^2 \sec [c + d x]^{5/2}} - \frac{(9 A + 119 C) \sin [c + d x]}{30 d (a^3 + a^3 \cos [c + d x]) \sec [c + d x]^{3/2}} + \frac{(A + 11 C) \sin [c + d x]}{2 a^3 d \sqrt{\sec [c + d x]}}
\end{aligned}$$

Result (type 5, 826 leaves):

$$\begin{aligned}
& - \frac{1}{5 d (a + a \operatorname{Cos}[c + d x])^3} 9 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \\
& \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] - \\
& \frac{1}{5 d (a + a \operatorname{Cos}[c + d x])^3} 119 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \\
& \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] + \\
& \frac{2 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c]}{d (a + a \operatorname{Cos}[c + d x])^3} + \\
& \frac{22 C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c]}{d (a + a \operatorname{Cos}[c + d x])^3} + \\
& \frac{1}{(a + a \operatorname{Cos}[c + d x])^3} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Sec}[c + d x]} \\
& \left( \frac{2 (9 A + 89 C + 30 C \operatorname{Cos}[2 c]) \operatorname{Cos}[d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5 d} + \frac{8 C \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{3 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right])}{5 d} + \right. \\
& \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (6 A \operatorname{Sin}\left[\frac{dx}{2}\right] + 11 C \operatorname{Sin}\left[\frac{dx}{2}\right])}{15 d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (9 A \operatorname{Sin}\left[\frac{dx}{2}\right] + 43 C \operatorname{Sin}\left[\frac{dx}{2}\right])}{3 d} - \frac{48 C \operatorname{Cos}[c] \operatorname{Sin}[d x]}{d} + \\
& \left. \frac{8 C \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x]}{3 d} - \frac{4 (9 A + 43 C) \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} + \frac{8 (6 A + 11 C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} - \frac{2 (A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right)
\end{aligned}$$

- **Problem 1203: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Cos}[c + d x]^2}{(a + a \operatorname{Cos}[c + d x])^3 \operatorname{Sec}[c + d x]^{7/2}} dx$$

Optimal (type 4, 290 leaves, 9 steps):

$$\frac{7(7A+33C)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{10a^3d} - \frac{(13A+63C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{6a^3d} - \frac{(A+C)\sin[c+dx]}{5d(a+a\cos[c+dx])^3\sec[c+dx]^{9/2}} - \frac{2(A+6C)\sin[c+dx]}{15ad(a+a\cos[c+dx])^2\sec[c+dx]^{7/2}} - \frac{(13A+63C)\sin[c+dx]}{10d(a^3+a^3\cos[c+dx])\sec[c+dx]^{5/2}} + \frac{7(7A+33C)\sin[c+dx]}{30a^3d\sec[c+dx]^{3/2}} - \frac{(13A+63C)\sin[c+dx]}{6a^3d\sqrt{\sec[c+dx]}}$$

Result (type 5, 870 leaves):

$$\frac{1}{5d(a+a\cos[c+dx])^3} 49\sqrt{2}Ae^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^6$$

$$\operatorname{Csc}\left[\frac{c}{2}\right]\left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right)\sec\left[\frac{c}{2}\right]+$$

$$\frac{1}{5d(a+a\cos[c+dx])^3} 231\sqrt{2}Ce^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^6\operatorname{Csc}\left[\frac{c}{2}\right]$$

$$\left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right)\sec\left[\frac{c}{2}\right]-$$

$$\frac{26A\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^6\sqrt{\cos[c+dx]}\operatorname{Csc}\left[\frac{c}{2}\right]\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sec\left[\frac{c}{2}\right]\sqrt{\sec[c+dx]}\sin[c]}{3d(a+a\cos[c+dx])^3} -$$

$$\frac{42C\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^6\sqrt{\cos[c+dx]}\operatorname{Csc}\left[\frac{c}{2}\right]\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sec\left[\frac{c}{2}\right]\sqrt{\sec[c+dx]}\sin[c]}{d(a+a\cos[c+dx])^3} + \frac{1}{(a+a\cos[c+dx])^3}$$

$$\operatorname{Cos}\left[\frac{c}{2}+\frac{dx}{2}\right]^6\sqrt{\sec[c+dx]}\left(-\frac{(78A+329C+20A\cos[2c]+133C\cos[2c])\cos[dx]\operatorname{Csc}\left[\frac{c}{2}\right]\sec\left[\frac{c}{2}\right]}{5d} - \frac{8C\cos[2dx]\sin[2c]}{d} + \frac{4C\cos[3dx]\sin[3c]}{5d} + \frac{2\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2}+\frac{dx}{2}\right]^5(A\sin\left[\frac{dx}{2}\right]+C\sin\left[\frac{dx}{2}\right])}{5d} + \frac{92\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2}+\frac{dx}{2}\right](A\sin\left[\frac{dx}{2}\right]+3C\sin\left[\frac{dx}{2}\right])}{3d} + \frac{4\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2}+\frac{dx}{2}\right]^3(17A\sin\left[\frac{dx}{2}\right]+27C\sin\left[\frac{dx}{2}\right])}{15d} + \frac{4(20A+133C)\cos[c]\sin[dx]}{5d} - \frac{8C\cos[2c]\sin[2dx]}{d} + \frac{4C\cos[3c]\sin[3dx]}{5d} + \frac{92(A+3C)\tan\left[\frac{c}{2}\right]}{3d} - \frac{4(17A+27C)\sec\left[\frac{c}{2}+\frac{dx}{2}\right]^2\tan\left[\frac{c}{2}\right]}{15d} + \frac{2(A+C)\sec\left[\frac{c}{2}+\frac{dx}{2}\right]^4\tan\left[\frac{c}{2}\right]}{5d}\right)$$

- **Problem 1207: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos[c + dx]} (A + C \cos[c + dx]^2) \sec[c + dx]^{5/2} dx$$

Optimal (type 3, 136 leaves, 5 steps):

$$\frac{2\sqrt{a} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} + \frac{2aA \sqrt{\sec[c+dx]} \sin[c+dx]}{3d \sqrt{a+a \cos[c+dx]}} + \frac{2A \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{3d}$$

Result (type 3, 473 leaves):

$$\frac{1}{6d \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}} i \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{1}{2}(c+dx)\right] \sec[c+dx]^{3/2} \\ \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) \left(6C \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right] \cos[c+dx]^2 - \right. \\ \left. 3C \cos[2(c+dx)] \operatorname{Log}\left[2\left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right)\right]\right) - \\ \left(\cos\left[\frac{dx}{2}\right] - i \sin\left[\frac{dx}{2}\right]\right) \left(3C \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2\left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right)\right]\right) + \\ 3iC \operatorname{Log}\left[2\left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] + \\ \left. 4i \sqrt{2} A \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right]\right)$$

- **Problem 1208: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos[c + dx]} (A + C \cos[c + dx]^2) \sec[c + dx]^{3/2} dx$$

Optimal (type 3, 137 leaves, 5 steps):

$$\frac{\sqrt{a} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} - \\ \frac{a(2A-C) \sin[c+dx]}{d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}} + \frac{2A \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]} \sin[c+dx]}{d}$$

Result (type 3, 620 leaves):

$$\begin{aligned}
& \frac{1}{4 d \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]}} \sqrt{a (1 + \cos[c + d x])} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \sqrt{\operatorname{Sec}[c + d x]} \\
& \left( -i C \cos\left[c + \frac{d x}{2}\right] \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \right) \right] - \right. \\
& \quad \left. i C \cos\left[c + \frac{3 d x}{2}\right] \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \right) \right] \right) + \\
& 2 i C \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]}\right] \cos[c + d x] \left(\cos\left[\frac{d x}{2}\right] + i \sin\left[\frac{d x}{2}\right]\right) - \\
& C \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \right) \right] \sin\left[c + \frac{d x}{2}\right] + \\
& 8 \sqrt{2} A \sqrt{\cos[c + d x] (\cos[d x] + i \sin[d x])} \sin\left[\frac{1}{2} (c + d x)\right] - \\
& 2 \sqrt{2} C \sqrt{\cos[c + d x] (\cos[d x] + i \sin[d x])} \sin\left[\frac{1}{2} (c + d x)\right] + 2 \sqrt{2} C \sqrt{\cos[c + d x] (\cos[d x] + i \sin[d x])} \sin\left[\frac{3}{2} (c + d x)\right] + \\
& C \operatorname{Log}\left[2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \right) \right] \sin\left[c + \frac{3 d x}{2}\right]
\end{aligned}$$

- **Problem 1209: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos[c + d x]} (A + C \cos[c + d x]^2) \sqrt{\operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 144 leaves, 5 steps):

$$\frac{\sqrt{a} (8 A + 3 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c + d x]}{\sqrt{a + a \cos[c + d x]}}\right] \sqrt{\cos[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}{4 d} + \frac{a C \sin[c + d x]}{4 d \sqrt{a + a \cos[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}} + \frac{C \sqrt{a + a \cos[c + d x]} \sin[c + d x]}{2 d \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 3, 489 leaves):

$$\frac{1}{8 \sqrt{2} d \sqrt{\sec[c+dx]} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx])} \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{1}{2}(c+dx)\right] \\
\left( -i(8A+3C) \cos\left[\frac{dx}{2}\right] \log\left[2\left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right)\right] \right) + \\
i(8A+3C) \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) + \\
8A \log\left[2\left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] + \\
3C \log\left[2\left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right)\right] \sin\left[\frac{dx}{2}\right] + \\
4\sqrt{2} C \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + 2\sqrt{2} C \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] \Big)$$

- **Problem 1210: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+a\cos[c+dx]} (A+C\cos[c+dx])^2}{\sqrt{\sec[c+dx]}} dx$$

Optimal (type 3, 189 leaves, 6 steps):

$$\frac{\sqrt{a} (8A+5C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{8d} + \\
\frac{aC \sin[c+dx]}{12d \sqrt{a+a\cos[c+dx]} \sec[c+dx]^{3/2}} + \frac{C \sqrt{a+a\cos[c+dx]} \sin[c+dx]}{3d \sec[c+dx]^{3/2}} + \frac{a(8A+5C) \sin[c+dx]}{8d \sqrt{a+a\cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 573 leaves):

$$\begin{aligned}
& \frac{1}{48 \sqrt{2} d \sqrt{\sec[c+dx]} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx])} \sqrt{a(1+\cos[c+dx])} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \\
& \left( -3 i (8 A + 5 C) \cos\left[\frac{dx}{2}\right] \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2 i dx}) \cos[c] + i (-1+e^{2 i dx}) \sin[c]} \right) \right] \right) + \\
& 3 i (8 A + 5 C) \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i dx}) \cos[c] + i (-1+e^{2 i dx}) \sin[c]} \right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) + \\
& 24 A \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2 i dx}) \cos[c] + i (-1+e^{2 i dx}) \sin[c]} \right) \right] \sin\left[\frac{dx}{2}\right] + \\
& 15 C \operatorname{Log}\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2 i dx}) \cos[c] + i (-1+e^{2 i dx}) \sin[c]} \right) \right] \sin\left[\frac{dx}{2}\right] + \\
& 48 \sqrt{2} A \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + 28 \sqrt{2} C \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + \\
& 6 \sqrt{2} C \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] + 4 \sqrt{2} C \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{5}{2}(c+dx)\right]
\end{aligned}$$

■ **Problem 1211: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+a \cos[c+dx]} (A+C \cos[c+dx])^2}{\sec[c+dx]^{3/2}} dx$$

Optimal (type 3, 234 leaves, 7 steps):

$$\begin{aligned}
& \frac{\sqrt{a} (48 A + 35 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{64 d} + \frac{a C \sin[c+dx]}{24 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{5/2}} + \\
& \frac{C \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{4 d \sec[c+dx]^{5/2}} + \frac{a (48 A + 35 C) \sin[c+dx]}{96 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2}} + \frac{a (48 A + 35 C) \sin[c+dx]}{64 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}
\end{aligned}$$

Result (type 3, 1065 leaves):



$$\begin{aligned}
& \frac{1}{128} (48A + 35C) \sqrt{\cos[c + dx]} \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} \\
& \left( \frac{1}{2} i \operatorname{Sin}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) - \right. \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right) + \\
& \frac{1}{2} \operatorname{Cos}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) + \right. \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right) \right) + \\
& \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} \left( - \frac{(48A + 41C) \operatorname{Cos}\left[\frac{dx}{2}\right] \operatorname{Sin}\left[\frac{c}{2}\right]}{384d} + \frac{(12A + 11C) \operatorname{Cos}\left[\frac{3dx}{2}\right] \operatorname{Sin}\left[\frac{3c}{2}\right]}{48d} + \right. \\
& \quad \frac{(16A + 15C) \operatorname{Cos}\left[\frac{5dx}{2}\right] \operatorname{Sin}\left[\frac{5c}{2}\right]}{128d} + \frac{C \operatorname{Cos}\left[\frac{7dx}{2}\right] \operatorname{Sin}\left[\frac{7c}{2}\right]}{48d} + \\
& \quad \frac{C \operatorname{Cos}\left[\frac{9dx}{2}\right] \operatorname{Sin}\left[\frac{9c}{2}\right]}{64d} - \frac{(48A + 41C) \operatorname{Cos}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{384d} + \\
& \quad \frac{(12A + 11C) \operatorname{Cos}\left[\frac{3c}{2}\right] \operatorname{Sin}\left[\frac{3dx}{2}\right]}{48d} + \frac{(16A + 15C) \operatorname{Cos}\left[\frac{5c}{2}\right] \operatorname{Sin}\left[\frac{5dx}{2}\right]}{128d} + \\
& \quad \left. \frac{C \operatorname{Cos}\left[\frac{7c}{2}\right] \operatorname{Sin}\left[\frac{7dx}{2}\right]}{48d} + \frac{C \operatorname{Cos}\left[\frac{9c}{2}\right] \operatorname{Sin}\left[\frac{9dx}{2}\right]}{64d} \right)
\end{aligned}$$

■ **Problem 1215: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{3/2} (A + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^{7/2} dx$$

Optimal (type 3, 183 leaves, 6 steps):

$$\frac{2 a^{3/2} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} + \frac{2 a^2 (4 A+5 C) \sqrt{\sec[c+dx]} \sin[c+dx]}{5 d \sqrt{a+a \cos[c+dx]}} +$$

$$\frac{2 a A \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{5 d} + \frac{2 A (a+a \cos[c+dx])^{3/2} \sec[c+dx]^{5/2} \sin[c+dx]}{5 d}$$

Result (type 3, 824 leaves):

$$\frac{1}{20 d \sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}} (a(1+\cos[c+dx]))^{3/2} \sec\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\sec[c+dx]}$$

$$\left(5 C e^{-\frac{1}{2} i d x} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right]+i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}\right] \cos\left[\frac{c}{2}\right]^2 (i(1+e^{2 i d x}) \cos[c]-(-1+e^{2 i d x}) \sin[c])\right) +$$

$$5 C e^{-\frac{1}{2} i d x} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right]+i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}\right]$$

$$\sin\left[\frac{c}{2}\right]^2 (i(1+e^{2 i d x}) \cos[c]-(-1+e^{2 i d x}) \sin[c]) - 5 i C e^{-\frac{1}{2} i d x} \cos\left[\frac{c}{2}\right]^2$$

$$\log\left[2\left(e^{i d x} \cos\left[\frac{c}{2}\right]+i e^{i d x} \sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}\right)\right] ((1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]) -$$

$$5 i C e^{-\frac{1}{2} i d x} \log\left[2\left(e^{i d x} \cos\left[\frac{c}{2}\right]+i e^{i d x} \sin\left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]}\right)\right] \sin\left[\frac{c}{2}\right]^2$$

$$\left((1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]\right) + 24 A \cos\left[\frac{c}{2}\right] \sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]} \sin\left[\frac{dx}{2}\right] +$$

$$20 C \cos\left[\frac{c}{2}\right] \sqrt{(1+e^{2 i d x}) \cos[c]+i(-1+e^{2 i d x}) \sin[c]} \sin\left[\frac{dx}{2}\right] + 24 \sqrt{2} A \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right] \sqrt{\cos[c+dx] (\cos[dx]+i \sin[dx])} +$$

$$20 \sqrt{2} C \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right] \sqrt{\cos[c+dx] (\cos[dx]+i \sin[dx])} + 12 \sqrt{2} A \sec[c+dx] \sqrt{\cos[c+dx] (\cos[dx]+i \sin[dx])}$$

$$\sin\left[\frac{1}{2}(c+dx)\right] + 4 \sqrt{2} A \sec[c+dx]^2 \sqrt{\cos[c+dx] (\cos[dx]+i \sin[dx])} \sin\left[\frac{1}{2}(c+dx)\right]$$

■ **Problem 1216: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a+a \cos[c+dx])^{3/2} (A+C \cos[c+dx]^2) \sec[c+dx]^{5/2} dx$$

Optimal (type 3, 181 leaves, 6 steps):

$$\frac{3 a^{3/2} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} - \frac{a^2 (8 A-3 C) \sin[c+dx]}{3 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}} +$$

$$\frac{2 a A \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]} \sin[c+dx]}{d} + \frac{2 A (a+a \cos[c+dx])^{3/2} \sec[c+dx]^{3/2} \sin[c+dx]}{3 d}$$

Result (type 3, 869 leaves):

$$\frac{1}{24 d \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]}} (a (1 + \cos[c + d x]))^{3/2} \sec\left[\frac{1}{2} (c + d x)\right]^3 \sqrt{\sec[c + d x]}$$

$$\left(9 C e^{-\frac{1}{2} i d x} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]}\right] \cos\left[\frac{c}{2}\right]^2 (i (1 + e^{2 i d x}) \cos[c] - (-1 + e^{2 i d x}) \sin[c]) +\right.$$

$$9 C e^{-\frac{1}{2} i d x} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]}\right]$$

$$\sin\left[\frac{c}{2}\right]^2 (i (1 + e^{2 i d x}) \cos[c] - (-1 + e^{2 i d x}) \sin[c]) - 9 i C e^{-\frac{1}{2} i d x} \cos\left[\frac{c}{2}\right]^2$$

$$\log\left[2 \left(e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]}\right)\right] \left((1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]\right) -$$

$$9 i C e^{-\frac{1}{2} i d x} \log\left[2 \left(e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]}\right)\right] \sin\left[\frac{c}{2}\right]^2$$

$$\left((1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]\right) + 40 A \cos\left[\frac{c}{2}\right] \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \sin\left[\frac{d x}{2}\right] -$$

$$6 C \cos\left[\frac{c}{2}\right] \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \sin\left[\frac{d x}{2}\right] + 40 \sqrt{2} A \cos\left[\frac{d x}{2}\right] \sin\left[\frac{c}{2}\right] \sqrt{\cos[c + d x] (\cos[d x] + i \sin[d x])} -$$

$$6 \sqrt{2} C \cos\left[\frac{d x}{2}\right] \sin\left[\frac{c}{2}\right] \sqrt{\cos[c + d x] (\cos[d x] + i \sin[d x])} + 6 \sqrt{2} C \cos\left[\frac{3 d x}{2}\right] \sin\left[\frac{3 c}{2}\right] \sqrt{\cos[c + d x] (\cos[d x] + i \sin[d x])} +$$

$$6 C \cos\left[\frac{3 c}{2}\right] \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \sin\left[\frac{3 d x}{2}\right] +$$

$$8 \sqrt{2} A \sec[c + d x] \sqrt{\cos[c + d x] (\cos[d x] + i \sin[d x])} \sin\left[\frac{1}{2} (c + d x)\right]$$

- **Problem 1217: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + d x])^{3/2} (A + C \cos[c + d x])^2 \sec[c + d x]^{3/2} dx$$

Optimal (type 3, 195 leaves, 6 steps):

$$\frac{a^{3/2} (8 A + 7 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c + d x]}{\sqrt{a + a \cos[c + d x]}}\right] \sqrt{\cos[c + d x]} \sqrt{\sec[c + d x]}}{4 d} - \frac{a^2 (8 A - 5 C) \sin[c + d x]}{4 d \sqrt{a + a \cos[c + d x]} \sqrt{\sec[c + d x]}}$$

$$\frac{a (4 A - C) \sqrt{a + a \cos[c + d x]} \sin[c + d x]}{2 d \sqrt{\sec[c + d x]}} + \frac{2 A (a + a \cos[c + d x])^{3/2} \sqrt{\sec[c + d x]} \sin[c + d x]}{d}$$

Result (type 3, 927 leaves):

$$\begin{aligned}
& \frac{1}{16 d \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}} a \sqrt{a (1 + \cos [c + d x])} \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] \sqrt{\operatorname{Sec}[c + d x]} \\
& \left(-i (8 A + 7 C) \cos \left[c + \frac{d x}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right] + i e^{i d x} \sin \left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] - \right. \\
& 8 i A \cos \left[c + \frac{3 d x}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right] + i e^{i d x} \sin \left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] - \\
& \left. 7 i C \cos \left[c + \frac{3 d x}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right] + i e^{i d x} \sin \left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] + \right. \\
& 2 i (8 A + 7 C) \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right] + i \sin \left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right] \cos [c + d x] \left(\cos \left[\frac{d x}{2}\right] + i \sin \left[\frac{d x}{2}\right]\right) - \\
& 8 A \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right] + i e^{i d x} \sin \left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] \sin \left[c + \frac{d x}{2}\right] - \\
& 7 C \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right] + i e^{i d x} \sin \left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] \sin \left[c + \frac{d x}{2}\right] + \\
& 32 \sqrt{2} A \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin \left[\frac{1}{2}(c + d x)\right] - 10 \sqrt{2} C \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin \left[\frac{1}{2}(c + d x)\right] + \\
& 12 \sqrt{2} C \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin \left[\frac{3}{2}(c + d x)\right] + 2 \sqrt{2} C \sqrt{\cos [c + d x] (\cos [d x] + i \sin [d x])} \sin \left[\frac{5}{2}(c + d x)\right] + \\
& 8 A \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right] + i e^{i d x} \sin \left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] \sin \left[c + \frac{3 d x}{2}\right] + \\
& \left. 7 C \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right] + i e^{i d x} \sin \left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos [c] + i (-1 + e^{2 i d x}) \sin [c]}\right)\right] \sin \left[c + \frac{3 d x}{2}\right]\right)
\end{aligned}$$

■ **Problem 1218: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{3/2} (A + C \cos [c + d x])^2 \sqrt{\operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 191 leaves, 6 steps):

$$\begin{aligned}
& \frac{a^{3/2} (24 A + 11 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}}\right] \sqrt{\cos [c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}{8 d} + \\
& \frac{a^2 (24 A + 19 C) \sin [c + d x]}{24 d \sqrt{a + a \cos [c + d x]} \sqrt{\operatorname{Sec}[c + d x]}} + \frac{a C \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{4 d \sqrt{\operatorname{Sec}[c + d x]}} + \frac{C (a + a \cos [c + d x])^{3/2} \sin [c + d x]}{3 d \sqrt{\operatorname{Sec}[c + d x]}}
\end{aligned}$$

Result (type 3, 574 leaves):

$$\frac{1}{48 \sqrt{2} d \sqrt{\sec[c+dx]} \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx])} a \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{1}{2}(c+dx)\right] \\
\left( -3 i (24 A + 11 C) \cos\left[\frac{dx}{2}\right] \log\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right) \right] \right) + \\
3 i (24 A + 11 C) \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]}\right] \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) + \\
72 A \log\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right) \right] \sin\left[\frac{dx}{2}\right] + \\
33 C \log\left[2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2i dx}) \cos[c] + i(-1+e^{2i dx}) \sin[c]} \right) \right] \sin\left[\frac{dx}{2}\right] + \\
48 \sqrt{2} A \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + 52 \sqrt{2} C \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{1}{2}(c+dx)\right] + \\
18 \sqrt{2} C \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{3}{2}(c+dx)\right] + 4 \sqrt{2} C \sqrt{\cos[c+dx]} (\cos[dx] + i \sin[dx]) \sin\left[\frac{5}{2}(c+dx)\right] \Big)$$

■ **Problem 1219: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c+dx])^{3/2} (A + C \cos[c+dx]^2)}{\sqrt{\sec[c+dx]}} dx$$

Optimal (type 3, 238 leaves, 7 steps):

$$\frac{a^{3/2} (112 A + 75 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{64 d} + \frac{a^2 (16 A + 13 C) \sin[c+dx]}{32 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2}} + \\
\frac{a C \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{8 d \sec[c+dx]^{3/2}} + \frac{C (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{4 d \sec[c+dx]^{3/2}} + \frac{a^2 (112 A + 75 C) \sin[c+dx]}{64 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 1069 leaves):

$$\begin{aligned}
& \frac{1}{256} (112 A + 75 C) \sqrt{\cos[c + dx]} (a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sqrt{\operatorname{Sec}[c + dx]} \\
& \left( \frac{1}{2} i \operatorname{Sin}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right] \right) \left( \cos\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) - \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \Big) + \\
& \frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right] \right) \left( \cos\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) + \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \Big) \Big) + \\
& (a (1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sqrt{\operatorname{Sec}[c + dx]} \left( - \frac{(80 A + 43 C) \cos\left[\frac{dx}{2}\right] \operatorname{Sin}\left[\frac{c}{2}\right]}{256 d} + \frac{3 (4 A + 3 C) \cos\left[\frac{3dx}{2}\right] \operatorname{Sin}\left[\frac{3c}{2}\right]}{32 d} + \right. \\
& \quad \frac{(16 A + 23 C) \cos\left[\frac{5dx}{2}\right] \operatorname{Sin}\left[\frac{5c}{2}\right]}{256 d} + \frac{C \cos\left[\frac{7dx}{2}\right] \operatorname{Sin}\left[\frac{7c}{2}\right]}{32 d} + \\
& \quad \frac{C \cos\left[\frac{9dx}{2}\right] \operatorname{Sin}\left[\frac{9c}{2}\right]}{128 d} - \frac{(80 A + 43 C) \cos\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{256 d} + \\
& \quad \frac{3 (4 A + 3 C) \cos\left[\frac{3c}{2}\right] \operatorname{Sin}\left[\frac{3dx}{2}\right]}{32 d} + \frac{(16 A + 23 C) \cos\left[\frac{5c}{2}\right] \operatorname{Sin}\left[\frac{5dx}{2}\right]}{256 d} + \\
& \quad \left. \frac{C \cos\left[\frac{7c}{2}\right] \operatorname{Sin}\left[\frac{7dx}{2}\right]}{32 d} + \frac{C \cos\left[\frac{9c}{2}\right] \operatorname{Sin}\left[\frac{9dx}{2}\right]}{128 d} \right)
\end{aligned}$$

■ **Problem 1220: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx])^{3/2} (A + C \cos[c + dx]^2)}{\operatorname{Sec}[c + dx]^{3/2}} dx$$

Optimal (type 3, 285 leaves, 8 steps):

$$\begin{aligned}
& \frac{a^{3/2} (176 A + 133 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{128 d} + \\
& \frac{a^2 (80 A + 67 C) \sin[c+dx]}{240 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{5/2}} + \frac{3 a C \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{40 d \sec[c+dx]^{5/2}} + \frac{C (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{5 d \sec[c+dx]^{5/2}} + \\
& \frac{a^2 (176 A + 133 C) \sin[c+dx]}{192 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2}} + \frac{a^2 (176 A + 133 C) \sin[c+dx]}{128 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}
\end{aligned}$$

Result (type 3, 1123 leaves):

$$\begin{aligned}
& \frac{1}{512} (176 A + 133 C) \sqrt{\cos[c + dx]} (a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sqrt{\operatorname{Sec}[c + dx]} \\
& \left( \frac{1}{2} i \operatorname{Sin}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) - \right. \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right) + \\
& \frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) + \right. \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \cos[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right) \right) + \\
& (a(1 + \cos[c + dx]))^{3/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sqrt{\operatorname{Sec}[c + dx]} \left( - \frac{(1360 A + 1019 C) \cos\left[\frac{dx}{2}\right] \operatorname{Sin}\left[\frac{c}{2}\right]}{7680 d} + \right. \\
& \quad \frac{(280 A + 239 C) \cos\left[\frac{3dx}{2}\right] \operatorname{Sin}\left[\frac{3c}{2}\right]}{960 d} + \\
& \quad \frac{(48 A + 49 C) \cos\left[\frac{5dx}{2}\right] \operatorname{Sin}\left[\frac{5c}{2}\right]}{512 d} + \frac{(10 A + 17 C) \cos\left[\frac{7dx}{2}\right] \operatorname{Sin}\left[\frac{7c}{2}\right]}{480 d} + \\
& \quad \frac{3 C \cos\left[\frac{9dx}{2}\right] \operatorname{Sin}\left[\frac{9c}{2}\right]}{256 d} + \frac{C \cos\left[\frac{11dx}{2}\right] \operatorname{Sin}\left[\frac{11c}{2}\right]}{320 d} - \\
& \quad \frac{(1360 A + 1019 C) \cos\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{7680 d} + \frac{(280 A + 239 C) \cos\left[\frac{3c}{2}\right] \operatorname{Sin}\left[\frac{3dx}{2}\right]}{960 d} + \\
& \quad \frac{(48 A + 49 C) \cos\left[\frac{5c}{2}\right] \operatorname{Sin}\left[\frac{5dx}{2}\right]}{512 d} + \frac{(10 A + 17 C) \cos\left[\frac{7c}{2}\right] \operatorname{Sin}\left[\frac{7dx}{2}\right]}{480 d} + \\
& \quad \left. \frac{3 C \cos\left[\frac{9c}{2}\right] \operatorname{Sin}\left[\frac{9dx}{2}\right]}{256 d} + \frac{C \cos\left[\frac{11c}{2}\right] \operatorname{Sin}\left[\frac{11dx}{2}\right]}{320 d} \right)
\end{aligned}$$

■ **Problem 1224: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{5/2} (A + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^{9/2} dx$$



Optimal (type 3, 230 leaves, 7 steps) :

$$\frac{2 a^{5/2} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{d} +$$

$$\frac{2 a^3 (32 A+49 C) \sqrt{\sec [c+d x]} \sin [c+d x]}{21 d \sqrt{a+a \cos [c+d x]}} + \frac{2 a^2 (8 A+7 C) \sqrt{a+a \cos [c+d x]} \sec [c+d x]^{3/2} \sin [c+d x]}{21 d} +$$

$$\frac{2 a A (a+a \cos [c+d x])^{3/2} \sec [c+d x]^{5/2} \sin [c+d x]}{7 d} + \frac{2 A (a+a \cos [c+d x])^{5/2} \sec [c+d x]^{7/2} \sin [c+d x]}{7 d}$$

Result (type 3, 975 leaves) :

$$\frac{1}{4} C \sqrt{\cos [c+d x]} (a (1+\cos [c+d x]))^{5/2} \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^5 \sqrt{\sec [c+d x]}$$

$$\left(\frac{1}{2} i \sin \left[\frac{c}{2}\right] \left(-\left(2 i e^{\frac{i d x}{2}} \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right]\right)\left(\cos \left[\frac{c}{2}\right]-i \sin \left[\frac{c}{2}\right]\right)\right.\right.\right.$$

$$\left.\left.\left.\sqrt{e^{-i d x}\left(\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]\right)}\right)\right) / \left(d \sqrt{2\left(1+e^{2 i d x}\right) \cos [c]+2 i\left(-1+e^{2 i d x}\right) \sin [c]}\right)-\right.$$

$$\left.\left(2 i e^{\frac{i d x}{2}} \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right]\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right)\right.\right.$$

$$\left.\left.\left.\sqrt{e^{-i d x}\left(\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]\right)}\right)\right) / \left(d \sqrt{2\left(1+e^{2 i d x}\right) \cos [c]+2 i\left(-1+e^{2 i d x}\right) \sin [c]}\right)\right) +$$

$$\frac{1}{2} \cos \left[\frac{c}{2}\right] \left(-\left(2 i e^{\frac{i d x}{2}} \operatorname{Log}\left[2\left(e^{i d x} \cos \left[\frac{c}{2}\right]+i e^{i d x} \sin \left[\frac{c}{2}\right]+\sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right]\right)\left(\cos \left[\frac{c}{2}\right]-i \sin \left[\frac{c}{2}\right]\right)\right.\right.$$

$$\left.\left.\left.\sqrt{e^{-i d x}\left(\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]\right)}\right)\right) / \left(d \sqrt{2\left(1+e^{2 i d x}\right) \cos [c]+2 i\left(-1+e^{2 i d x}\right) \sin [c]}\right)+\right.$$

$$\left.\left(2 i e^{\frac{i d x}{2}} \operatorname{ArcTanh}\left[\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right) \sqrt{\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]}\right]\left(\cos \left[\frac{c}{2}\right]+i \sin \left[\frac{c}{2}\right]\right)\right.\right.$$

$$\left.\left.\left.\sqrt{e^{-i d x}\left(\left(1+e^{2 i d x}\right) \cos [c]+i\left(-1+e^{2 i d x}\right) \sin [c]\right)}\right)\right) / \left(d \sqrt{2\left(1+e^{2 i d x}\right) \cos [c]+2 i\left(-1+e^{2 i d x}\right) \sin [c]}\right)\right) +$$

$$(a (1+\cos [c+d x]))^{5/2} \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^5 \sqrt{\sec [c+d x]} \left(\frac{(23 A+28 C) \cos \left[\frac{d x}{2}\right] \sin \left[\frac{c}{2}\right]}{21 d} + \frac{(23 A+28 C) \cos \left[\frac{c}{2}\right] \sin \left[\frac{d x}{2}\right]}{21 d} +\right.$$

$$\frac{2 A \sec [c+d x]^2 \sin \left[\frac{c}{2} + \frac{d x}{2}\right]}{7 d} + \frac{A \sec [c+d x]^3 \sin \left[\frac{c}{2} + \frac{d x}{2}\right]}{14 d} +$$

$$\left.\frac{\sec [c+d x] \left(23 A \sin \left[\frac{c}{2} + \frac{d x}{2}\right] + 7 C \sin \left[\frac{c}{2} + \frac{d x}{2}\right]\right)}{42 d}\right)$$

■ **Problem 1225: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a+a \cos [c+d x])^{5/2} (A+C \cos [c+d x]^2) \sec [c+d x]^{7/2} dx$$

Optimal (type 3, 230 leaves, 7 steps) :

$$\frac{5 a^{5/2} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} -$$

$$\frac{a^3 (64 A + 15 C) \sin[c+dx]}{15 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}} + \frac{2 a^2 (8 A + 5 C) \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]} \sin[c+dx]}{5 d} +$$

$$\frac{2 a A (a+a \cos[c+dx])^{3/2} \sec[c+dx]^{3/2} \sin[c+dx]}{3 d} + \frac{2 A (a+a \cos[c+dx])^{5/2} \sec[c+dx]^{5/2} \sin[c+dx]}{5 d}$$

Result (type 3, 969 leaves) :

$$\frac{5}{8} C \sqrt{\cos[c+dx]} (a(1+\cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c+dx]}$$

$$\left(\frac{1}{2} i \sin\left[\frac{c}{2}\right] \left(-\left(2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2\left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right]\right)\right) \left(\cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right]\right)\right.\right.$$

$$\left.\left.\sqrt{e^{-idx} \left((1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]\right)}\right) / \left(d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]}\right) -\right.$$

$$\left(2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right] \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right)\right.$$

$$\left.\left.\sqrt{e^{-idx} \left((1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]\right)}\right) / \left(d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]}\right)\right) +$$

$$\frac{1}{2} \cos\left[\frac{c}{2}\right] \left(-\left(2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2\left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right]\right)\right) \left(\cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right]\right)\right.$$

$$\left.\left.\sqrt{e^{-idx} \left((1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]\right)}\right) / \left(d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]}\right) +\right.$$

$$\left(2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right] \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right)\right.$$

$$\left.\left.\sqrt{e^{-idx} \left((1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]\right)}\right) / \left(d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]}\right)\right) +$$

$$(a(1+\cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c+dx]} \left(\frac{(172 A + 45 C) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{120 d} + \frac{C \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{8 d} +\right.$$

$$\frac{(172 A + 45 C) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{120 d} + \frac{C \cos\left[\frac{3c}{2}\right] \sin\left[\frac{3dx}{2}\right]}{8 d} +$$

$$\left.\frac{7 A \sec[c+dx] \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{15 d} + \frac{A \sec[c+dx]^2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{10 d}\right)$$

■ **Problem 1226: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a+a \cos[c+dx])^{5/2} (A+C \cos[c+dx])^2 \sec[c+dx]^{5/2} dx$$

Optimal (type 3, 238 leaves, 7 steps) :

$$\frac{a^{5/2} (8A + 19C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{4d} -$$

$$\frac{a^3 (56A - 27C) \sin[c+dx]}{12d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}} - \frac{a^2 (8A - C) \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{2d \sqrt{\sec[c+dx]}} +$$

$$\frac{10aA (a+a \cos[c+dx])^{3/2} \sqrt{\sec[c+dx]} \sin[c+dx]}{3d} + \frac{2A (a+a \cos[c+dx])^{5/2} \sec[c+dx]^{3/2} \sin[c+dx]}{3d}$$

Result (type 3, 988 leaves) :

$$\frac{1}{32} (8A + 19C) \sqrt{\cos[c+dx]} (a (1 + \cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c+dx]}$$

$$\left( \frac{1}{2} i \sin\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right.$$

$$\left. \left. \frac{\sqrt{e^{-idx} ((1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c])}}{d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]}} \right) - \right.$$

$$\left. \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \right.$$

$$\left. \left. \frac{\sqrt{e^{-idx} ((1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c])}}{d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]}} \right) \right) +$$

$$\frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right.$$

$$\left. \left. \frac{\sqrt{e^{-idx} ((1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c])}}{d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]}} \right) + \right.$$

$$\left. \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \right.$$

$$\left. \left. \frac{\sqrt{e^{-idx} ((1 + e^{2idx}) \cos[c] + i (-1 + e^{2idx}) \sin[c])}}{d \sqrt{2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c]}} \right) \right) \right) +$$

$$(a (1 + \cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c+dx]} \left( \frac{(128A - 27C) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{96d} + \frac{5C \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{16d} + \right.$$

$$\frac{C \cos\left[\frac{5dx}{2}\right] \sin\left[\frac{5c}{2}\right]}{32d} + \frac{(128A - 27C) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{96d} +$$

$$\left. \frac{5C \cos\left[\frac{3c}{2}\right] \sin\left[\frac{3dx}{2}\right]}{16d} + \frac{C \cos\left[\frac{5c}{2}\right] \sin\left[\frac{5dx}{2}\right]}{32d} + \frac{A \sec[c+dx] \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{6d} \right)$$

■ **Problem 1227: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c+dx])^{5/2} (A + C \cos[c+dx])^2 \sec[c+dx]^{3/2} dx$$

Optimal (type 3, 242 leaves, 7 steps) :

$$\frac{5 a^{5/2} (8 A + 5 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{8 d} -$$

$$\frac{a^3 (24 A - 49 C) \sin[c+dx]}{24 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}} - \frac{a^2 (8 A - 3 C) \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{4 d \sqrt{\sec[c+dx]}} -$$

$$\frac{a (6 A - C) (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{3 d \sqrt{\sec[c+dx]}} + \frac{2 A (a+a \cos[c+dx])^{5/2} \sqrt{\sec[c+dx]} \sin[c+dx]}{d}$$

Result (type 3, 1015 leaves) :

$$\frac{5}{64} (8 A + 5 C) \sqrt{\cos[c+dx]} (a (1 + \cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c+dx]}$$

$$\left( \frac{1}{2} i \sin\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{id x}{2}} \operatorname{Log}\left[ 2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right. \right.$$

$$\left. \left. \left. \sqrt{e^{-i d x} \left( (1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c]} \right) - \right. \right.$$

$$\left. \left( 2 i e^{\frac{id x}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \right.$$

$$\left. \left. \left. \sqrt{e^{-i d x} \left( (1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c]} \right) \right) \right) +$$

$$\frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{id x}{2}} \operatorname{Log}\left[ 2 \left( e^{i d x} \cos\left[\frac{c}{2}\right] + i e^{i d x} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \right) \right] \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \right.$$

$$\left. \left. \left. \sqrt{e^{-i d x} \left( (1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c]} \right) + \right. \right.$$

$$\left. \left( 2 i e^{\frac{id x}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c]} \right] \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \right.$$

$$\left. \left. \left. \sqrt{e^{-i d x} \left( (1 + e^{2 i d x}) \cos[c] + i (-1 + e^{2 i d x}) \sin[c] \right)} \right) / \left( d \sqrt{2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c]} \right) \right) \right) \right) +$$

$$(a (1 + \cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c+dx]} \left( \frac{(72 A - 47 C) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{192 d} + \frac{(3 A + 8 C) \cos\left[\frac{3 dx}{2}\right] \sin\left[\frac{3 c}{2}\right]}{24 d} + \right.$$

$$\frac{5 C \cos\left[\frac{5 dx}{2}\right] \sin\left[\frac{5 c}{2}\right]}{64 d} + \frac{C \cos\left[\frac{7 dx}{2}\right] \sin\left[\frac{7 c}{2}\right]}{96 d} +$$

$$\frac{(72 A - 47 C) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{192 d} + \frac{(3 A + 8 C) \cos\left[\frac{3 c}{2}\right] \sin\left[\frac{3 dx}{2}\right]}{24 d} +$$

$$\left. \frac{5 C \cos\left[\frac{5 c}{2}\right] \sin\left[\frac{5 dx}{2}\right]}{64 d} + \frac{C \cos\left[\frac{7 c}{2}\right] \sin\left[\frac{7 dx}{2}\right]}{96 d} \right)$$

- **Problem 1228: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{5/2} (A + C \cos [c + d x]^2) \sqrt{\sec [c + d x]} dx$$

Optimal (type 3, 238 leaves, 7 steps):

$$\frac{a^{5/2} (304 A + 163 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin [c+d x]}{\sqrt{a+a \cos [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{64 d} + \frac{a^3 (432 A + 299 C) \sin [c+d x]}{192 d \sqrt{a+a \cos [c+d x]} \sqrt{\sec [c+d x]}} +$$

$$\frac{a^2 (16 A + 17 C) \sqrt{a+a \cos [c+d x]} \sin [c+d x]}{32 d \sqrt{\sec [c+d x]}} + \frac{5 a C (a+a \cos [c+d x])^{3/2} \sin [c+d x]}{24 d \sqrt{\sec [c+d x]}} + \frac{C (a+a \cos [c+d x])^{5/2} \sin [c+d x]}{4 d \sqrt{\sec [c+d x]}}$$

Result (type 3, 1069 leaves):

$$\begin{aligned}
& \frac{1}{512} (304 A + 163 C) \sqrt{\cos[c + dx]} (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\operatorname{Sec}[c + dx]} \\
& \left( \frac{1}{2} i \operatorname{Sin}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) - \right. \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right) + \\
& \frac{1}{2} \operatorname{Cos}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) + \\
& \quad \left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \\
& \quad \left. \left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i(-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) \right) / \left( d \sqrt{2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right) \right) + \\
& (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\operatorname{Sec}[c + dx]} \left( - \frac{(432 A + 265 C) \operatorname{Cos}\left[\frac{dx}{2}\right] \operatorname{Sin}\left[\frac{c}{2}\right]}{1536 d} + \frac{5(12 A + 11 C) \operatorname{Cos}\left[\frac{3dx}{2}\right] \operatorname{Sin}\left[\frac{3c}{2}\right]}{192 d} + \right. \\
& \quad \frac{(16 A + 47 C) \operatorname{Cos}\left[\frac{5dx}{2}\right] \operatorname{Sin}\left[\frac{5c}{2}\right]}{512 d} + \frac{5 C \operatorname{Cos}\left[\frac{7dx}{2}\right] \operatorname{Sin}\left[\frac{7c}{2}\right]}{192 d} + \\
& \quad \frac{C \operatorname{Cos}\left[\frac{9dx}{2}\right] \operatorname{Sin}\left[\frac{9c}{2}\right]}{256 d} - \frac{(432 A + 265 C) \operatorname{Cos}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{1536 d} + \\
& \quad \frac{5(12 A + 11 C) \operatorname{Cos}\left[\frac{3c}{2}\right] \operatorname{Sin}\left[\frac{3dx}{2}\right]}{192 d} + \frac{(16 A + 47 C) \operatorname{Cos}\left[\frac{5c}{2}\right] \operatorname{Sin}\left[\frac{5dx}{2}\right]}{512 d} + \\
& \quad \left. \frac{5 C \operatorname{Cos}\left[\frac{7c}{2}\right] \operatorname{Sin}\left[\frac{7dx}{2}\right]}{192 d} + \frac{C \operatorname{Cos}\left[\frac{9c}{2}\right] \operatorname{Sin}\left[\frac{9dx}{2}\right]}{256 d} \right)
\end{aligned}$$

- **Problem 1229: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \cos[c + dx])^{5/2} (A + C \cos[c + dx])^2}{\sqrt{\operatorname{Sec}[c + dx]}} dx$$

Optimal (type 3, 285 leaves, 8 steps):

$$\frac{a^{5/2} (400 A + 283 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{128 d} +$$

$$\frac{a^3 (1040 A + 787 C) \sin[c+dx]}{960 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2}} + \frac{a^2 (80 A + 79 C) \sqrt{a+a \cos[c+dx]} \sin[c+dx]}{240 d \sec[c+dx]^{3/2}} +$$

$$\frac{a C (a+a \cos[c+dx])^{3/2} \sin[c+dx]}{8 d \sec[c+dx]^{3/2}} + \frac{C (a+a \cos[c+dx])^{5/2} \sin[c+dx]}{5 d \sec[c+dx]^{3/2}} + \frac{a^3 (400 A + 283 C) \sin[c+dx]}{128 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 1123 leaves):

$$\frac{1}{1024} (400 A + 283 C) \sqrt{\cos[c+dx]} (a (1 + \cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c+dx]}$$

$$\left(\frac{1}{2} i \sin\left[\frac{c}{2}\right] \left(-\left(2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right]\right) \left(\cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right]\right)\right.\right.\right.$$

$$\left.\left.\left.\sqrt{e^{-idx} \left((1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]\right)}\right) / \left(d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]}\right) - \right.\right.$$

$$\left.\left.\left(2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right] \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right)\right.\right.\right.$$

$$\left.\left.\left.\sqrt{e^{-idx} \left((1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]\right)}\right) / \left(d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]}\right)\right)\right) +$$

$$\frac{1}{2} \cos\left[\frac{c}{2}\right] \left(-\left(2 i e^{\frac{idx}{2}} \operatorname{Log}\left[2 \left(e^{idx} \cos\left[\frac{c}{2}\right] + i e^{idx} \sin\left[\frac{c}{2}\right] + \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right]\right) \left(\cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right]\right)\right.\right.$$

$$\left.\left.\left.\sqrt{e^{-idx} \left((1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]\right)}\right) / \left(d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]}\right) + \right.\right.$$

$$\left.\left.\left(2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]}\right] \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right)\right.\right.\right.$$

$$\left.\left.\left.\sqrt{e^{-idx} \left((1+e^{2idx}) \cos[c] + i(-1+e^{2idx}) \sin[c]\right)}\right) / \left(d \sqrt{2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c]}\right)\right)\right) +$$

$$(a (1 + \cos[c+dx]))^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c+dx]} \left(-\frac{(3760 A + 2309 C) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{15360 d} + \right.$$

$$\frac{(640 A + 509 C) \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{1920 d} +$$

$$\frac{5 (16 A + 19 C) \cos\left[\frac{5dx}{2}\right] \sin\left[\frac{5c}{2}\right]}{1024 d} +$$

$$\frac{(5 A + 16 C) \cos\left[\frac{7dx}{2}\right] \sin\left[\frac{7c}{2}\right]}{480 d} +$$

$$\frac{5 C \cos\left[\frac{9dx}{2}\right] \sin\left[\frac{9c}{2}\right]}{512 d} +$$

$$\frac{C \operatorname{Cos}\left[\frac{11 dx}{2}\right] \operatorname{Sin}\left[\frac{11 c}{2}\right]}{640 d} -$$

$$\frac{(3760 A + 2309 C) \operatorname{Cos}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{15360 d} +$$

$$\frac{(640 A + 509 C) \operatorname{Cos}\left[\frac{3c}{2}\right] \operatorname{Sin}\left[\frac{3 dx}{2}\right]}{1920 d} +$$

$$\frac{5 (16 A + 19 C) \operatorname{Cos}\left[\frac{5c}{2}\right] \operatorname{Sin}\left[\frac{5 dx}{2}\right]}{1024 d} +$$

$$\frac{(5 A + 16 C) \operatorname{Cos}\left[\frac{7c}{2}\right] \operatorname{Sin}\left[\frac{7 dx}{2}\right]}{480 d} +$$

$$\left( \frac{5 C \operatorname{Cos}\left[\frac{9c}{2}\right] \operatorname{Sin}\left[\frac{9 dx}{2}\right]}{512 d} + \frac{C \operatorname{Cos}\left[\frac{11c}{2}\right] \operatorname{Sin}\left[\frac{11 dx}{2}\right]}{640 d} \right)$$

- **Problem 1230: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Cos}[c + dx])^{5/2} (A + C \operatorname{Cos}[c + dx]^2)}{\operatorname{Sec}[c + dx]^{3/2}} dx$$

Optimal (type 3, 332 leaves, 9 steps):

$$\frac{a^{5/2} (1304 A + 1015 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+dx]}{\sqrt{a+a \operatorname{Cos}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{512 d} + \frac{a^3 (136 A + 109 C) \operatorname{Sin}[c+dx]}{192 d \sqrt{a+a \operatorname{Cos}[c+dx]} \operatorname{Sec}[c+dx]^{5/2}} +$$

$$\frac{a^2 (24 A + 23 C) \sqrt{a+a \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{96 d \operatorname{Sec}[c+dx]^{5/2}} + \frac{a C (a+a \operatorname{Cos}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{12 d \operatorname{Sec}[c+dx]^{5/2}} +$$

$$\frac{C (a+a \operatorname{Cos}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{6 d \operatorname{Sec}[c+dx]^{5/2}} + \frac{a^3 (1304 A + 1015 C) \operatorname{Sin}[c+dx]}{768 d \sqrt{a+a \operatorname{Cos}[c+dx]} \operatorname{Sec}[c+dx]^{3/2}} + \frac{a^3 (1304 A + 1015 C) \operatorname{Sin}[c+dx]}{512 d \sqrt{a+a \operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}$$

Result (type 3, 1177 leaves):

$$\frac{1}{4096} (1304 A + 1015 C) \sqrt{\operatorname{Cos}[c+dx]} (a (1 + \operatorname{Cos}[c+dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\operatorname{Sec}[c+dx]}$$

$$\left( \frac{1}{2} i \operatorname{Sin}\left[\frac{c}{2}\right] \left( - \left( 2 i e^{\frac{idx}{2}} \operatorname{Log}\left[ 2 \left( e^{idx} \operatorname{Cos}\left[\frac{c}{2}\right] + i e^{idx} \operatorname{Sin}\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) \right] \left( \operatorname{Cos}\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \right. \right.$$

$$\left. \left. \sqrt{e^{-idx} \left( (1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c] \right)} \right) \right) / \left( d \sqrt{2 (1 + e^{2idx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right) -$$

$$\left( 2 i e^{\frac{idx}{2}} \operatorname{ArcTanh}\left[ \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2idx}) \operatorname{Cos}[c] + i (-1 + e^{2idx}) \operatorname{Sin}[c]} \right] \right) \left( \operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right)$$



$$\begin{aligned}
& \left. \frac{\sqrt{e^{-i dx} \left( (1 + e^{2i dx}) \cos[c] + i(-1 + e^{2i dx}) \sin[c] \right)}}{\left( d \sqrt{2(1 + e^{2i dx}) \cos[c] + 2i(-1 + e^{2i dx}) \sin[c]} \right)} \right) + \\
& \frac{1}{2} \cos\left[\frac{c}{2}\right] \left( - \left( 2i e^{\frac{id x}{2}} \operatorname{Log}\left[ 2 \left( e^{i dx} \cos\left[\frac{c}{2}\right] + i e^{i dx} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2i dx}) \cos[c] + i(-1 + e^{2i dx}) \sin[c]} \right) \right] \right) \left( \cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \right. \\
& \left. \frac{\sqrt{e^{-i dx} \left( (1 + e^{2i dx}) \cos[c] + i(-1 + e^{2i dx}) \sin[c] \right)}}{\left( d \sqrt{2(1 + e^{2i dx}) \cos[c] + 2i(-1 + e^{2i dx}) \sin[c]} \right)} \right) + \\
& \left( 2i e^{\frac{id x}{2}} \operatorname{ArcTanh}\left[ \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \sqrt{(1 + e^{2i dx}) \cos[c] + i(-1 + e^{2i dx}) \sin[c]} \right] \right) \left( \cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \right. \\
& \left. \frac{\sqrt{e^{-i dx} \left( (1 + e^{2i dx}) \cos[c] + i(-1 + e^{2i dx}) \sin[c] \right)}}{\left( d \sqrt{2(1 + e^{2i dx}) \cos[c] + 2i(-1 + e^{2i dx}) \sin[c]} \right)} \right) \left. \right) + \\
& (a(1 + \cos[c + dx]))^{5/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\operatorname{Sec}[c + dx]} \left( - \frac{(2120A + 1589C) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{12288d} + \right. \\
& \frac{11(20A + 17C) \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{768d} + \\
& \frac{(1128A + 1145C) \cos\left[\frac{5dx}{2}\right] \sin\left[\frac{5c}{2}\right]}{12288d} + \\
& \frac{(20A + 29C) \cos\left[\frac{7dx}{2}\right] \sin\left[\frac{7c}{2}\right]}{768d} + \\
& \frac{(24A + 83C) \cos\left[\frac{9dx}{2}\right] \sin\left[\frac{9c}{2}\right]}{6144d} + \\
& \frac{C \cos\left[\frac{11dx}{2}\right] \sin\left[\frac{11c}{2}\right]}{256d} + \frac{C \cos\left[\frac{13dx}{2}\right] \sin\left[\frac{13c}{2}\right]}{1536d} - \\
& \frac{(2120A + 1589C) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{12288d} + \\
& \frac{11(20A + 17C) \cos\left[\frac{3c}{2}\right] \sin\left[\frac{3dx}{2}\right]}{768d} + \\
& \frac{(1128A + 1145C) \cos\left[\frac{5c}{2}\right] \sin\left[\frac{5dx}{2}\right]}{12288d} + \\
& \frac{(20A + 29C) \cos\left[\frac{7c}{2}\right] \sin\left[\frac{7dx}{2}\right]}{768d} + \frac{(24A + 83C) \cos\left[\frac{9c}{2}\right] \sin\left[\frac{9dx}{2}\right]}{6144d} + \\
& \left. \frac{C \cos\left[\frac{11c}{2}\right] \sin\left[\frac{11dx}{2}\right]}{256d} + \frac{C \cos\left[\frac{13c}{2}\right] \sin\left[\frac{13dx}{2}\right]}{1536d} \right)
\end{aligned}$$

■ **Problem 1231: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + C \cos[c + dx])^2 \sec[c + dx]^{11/2}}{\sqrt{a + a \cos[c + dx]}} dx$$

Optimal (type 3, 289 leaves, 9 steps):

$$\frac{\sqrt{2} (A + C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c + dx]} \sqrt{\sec[c + dx]}}{\sqrt{a} d} +$$

$$\frac{2 (257 A + 273 C) \sqrt{\sec[c + dx]} \sin[c + dx]}{315 d \sqrt{a + a \cos[c + dx]}} - \frac{2 (29 A + 21 C) \sec[c + dx]^{3/2} \sin[c + dx]}{315 d \sqrt{a + a \cos[c + dx]}} +$$

$$\frac{2 (19 A + 21 C) \sec[c + dx]^{5/2} \sin[c + dx]}{105 d \sqrt{a + a \cos[c + dx]}} - \frac{2 A \sec[c + dx]^{7/2} \sin[c + dx]}{63 d \sqrt{a + a \cos[c + dx]}} + \frac{2 A \sec[c + dx]^{9/2} \sin[c + dx]}{9 d \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 263 leaves):

$$\frac{1}{\sqrt{a (1 + \cos[c + dx])}}$$

$$\cos\left[\frac{1}{2} (c + dx)\right] \left[ \frac{1}{d} \operatorname{Im} (A + C) e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1 + e^{2i (c+dx)}}} \sqrt{1 + e^{2i (c+dx)}} \left( \operatorname{Log}[1 + e^{i (c+dx)}] - \operatorname{Log}[1 - e^{i (c+dx)} + \sqrt{2} \sqrt{1 + e^{2i (c+dx)}}] \right) \right] +$$

$$\frac{1}{(630 d) (1279 A + 1071 C - 2 (107 A + 63 C) \cos[c + dx] + 8 (157 A + 168 C) \cos[2 (c + dx)] - 58 A \cos[3 (c + dx)] -$$

$$42 C \cos[3 (c + dx)] + 257 A \cos[4 (c + dx)] + 273 C \cos[4 (c + dx)])} \sec[c + dx]^{9/2} \sin\left[\frac{1}{2} (c + dx)\right]$$

■ **Problem 1232: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + C \cos[c + dx])^2 \sec[c + dx]^{9/2}}{\sqrt{a + a \cos[c + dx]}} dx$$

Optimal (type 3, 244 leaves, 8 steps):

$$\frac{\sqrt{2} (A + C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c + dx]} \sqrt{\sec[c + dx]}}{\sqrt{a} d} - \frac{2 (43 A + 35 C) \sqrt{\sec[c + dx]} \sin[c + dx]}{105 d \sqrt{a + a \cos[c + dx]}} +$$

$$\frac{2 (31 A + 35 C) \sec[c + dx]^{3/2} \sin[c + dx]}{105 d \sqrt{a + a \cos[c + dx]}} - \frac{2 A \sec[c + dx]^{5/2} \sin[c + dx]}{35 d \sqrt{a + a \cos[c + dx]}} + \frac{2 A \sec[c + dx]^{7/2} \sin[c + dx]}{7 d \sqrt{a + a \cos[c + dx]}}$$

Result (type 3, 214 leaves):

$$\frac{1}{\sqrt{a(1+\cos[c+dx])}}$$

$$\cos\left[\frac{1}{2}(c+dx)\right] \left( -1/d2i(A+C) e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) + \right.$$

$$\left. \frac{4(73A+35C+24A\cos[c+dx] + (43A+35C)\cos[2(c+dx)]) \sec[c+dx]^{7/2} \sin\left[\frac{1}{2}(c+dx)\right]^3}{105d} \right)$$

■ **Problem 1233: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+C\cos[c+dx])^2 \sec[c+dx]^{7/2}}{\sqrt{a+a\cos[c+dx]}} dx$$

Optimal (type 3, 201 leaves, 7 steps):

$$\frac{\sqrt{2}(A+C) \operatorname{ArcTan}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{2}\sqrt{\cos[c+dx]}\sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a}d} +$$

$$\frac{2(13A+15C)\sqrt{\sec[c+dx]}\sin[c+dx]}{15d\sqrt{a+a\cos[c+dx]}} - \frac{2A\sec[c+dx]^{3/2}\sin[c+dx]}{15d\sqrt{a+a\cos[c+dx]}} + \frac{2A\sec[c+dx]^{5/2}\sin[c+dx]}{5d\sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 209 leaves):

$$\left( 2\cos\left[\frac{1}{2}(c+dx)\right] \left( 15i(A+C) e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) + \right.$$

$$\left. \left. \left. (19A+15C-2A\cos[c+dx] + (13A+15C)\cos[2(c+dx)]) \sec[c+dx]^{5/2} \sin\left[\frac{1}{2}(c+dx)\right]^3 \right) \right) \right) / (15d\sqrt{a(1+\cos[c+dx])})$$

■ **Problem 1234: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+C\cos[c+dx])^2 \sec[c+dx]^{5/2}}{\sqrt{a+a\cos[c+dx]}} dx$$

Optimal (type 3, 156 leaves, 6 steps):

$$\frac{\sqrt{2}(A+C) \operatorname{ArcTan}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{2}\sqrt{\cos[c+dx]}\sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a}d} - \frac{2A\sqrt{\sec[c+dx]}\sin[c+dx]}{3d\sqrt{a+a\cos[c+dx]}} + \frac{2A\sec[c+dx]^{3/2}\sin[c+dx]}{3d\sqrt{a+a\cos[c+dx]}}$$

Result (type 3, 183 leaves):

$$-\frac{1}{3d\sqrt{a(1+\cos[c+dx])}} + 2i\cos\left[\frac{1}{2}(c+dx)\right] \left( 3(A+C)e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2}\sqrt{1+e^{2i(c+dx)}}] \right) + 4iA\sec[c+dx]^{3/2} \sin\left[\frac{1}{2}(c+dx)\right]^3 \right)$$

- **Problem 1235: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+C\cos[c+dx])^2 \sec[c+dx]^{3/2}}{\sqrt{a+a\cos[c+dx]}} dx$$

Optimal (type 3, 175 leaves, 7 steps):

$$\frac{2C\operatorname{ArcSin}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a}d} + \frac{\sqrt{2}(A+C)\operatorname{ArcTan}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{2}\sqrt{\cos[c+dx]}\sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} + \frac{2A\sqrt{\sec[c+dx]}\sin[c+dx]}{d\sqrt{a+a\cos[c+dx]}}}{\sqrt{a}d}$$

Result (type 3, 290 leaves):

$$\frac{1}{d\sqrt{a(1+\cos[c+dx])}} \cos\left[\frac{1}{2}(c+dx)\right] \left( \sqrt{2}e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left( Cdx - iC\operatorname{ArcSinh}[e^{i(c+dx)}] + i\sqrt{2}(A+C)\log[1+e^{i(c+dx)}] + iC\log\left[1+\sqrt{1+e^{2i(c+dx)}}\right] - i\sqrt{2}A\log\left[1-e^{i(c+dx)} + \sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] - i\sqrt{2}C\log\left[1-e^{i(c+dx)} + \sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right] \right) + 4A\sqrt{\sec[c+dx]}\sin\left[\frac{1}{2}(c+dx)\right] \right)$$

- **Problem 1236: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+C\cos[c+dx])^2 \sqrt{\sec[c+dx]}}{\sqrt{a+a\cos[c+dx]}} dx$$

Optimal (type 3, 173 leaves, 7 steps):

$$\begin{aligned}
& \frac{C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a} d} + \\
& \frac{\sqrt{2} (A+C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a} d} + \frac{C \sin[c+dx]}{d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}
\end{aligned}$$

Result (type 3, 302 leaves):

$$\begin{aligned}
& \frac{1}{2 d \sqrt{a} (1 + \cos[c+dx])} \\
& i \cos\left[\frac{1}{2} (c+dx)\right] \left( \sqrt{2} e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left( i C dx + C \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] - 2 \sqrt{2} (A+C) \operatorname{Log}\left[1+e^{i(c+dx)}\right] - \right. \right. \\
& \quad \left. \left. C \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] + 2 \sqrt{2} A \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2} \sqrt{1+e^{2i(c+dx)}}\right] + 2 \sqrt{2} C \operatorname{Log}\left[1-e^{i(c+dx)}+\sqrt{2} \sqrt{1+e^{2i(c+dx)}}\right] \right) + \right. \\
& \quad \left. 2 i C \sqrt{\sec[c+dx]} \left( \sin\left[\frac{1}{2} (c+dx)\right] - \sin\left[\frac{3}{2} (c+dx)\right] \right) \right)
\end{aligned}$$

■ **Problem 1237: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c+dx]^2}{\sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}} dx$$

Optimal (type 3, 223 leaves, 8 steps):

$$\begin{aligned}
& \frac{(8A+7C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{4 \sqrt{a} d} - \frac{\sqrt{2} (A+C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{\sqrt{a} d} + \\
& \frac{C \sin[c+dx]}{2 d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2}} - \frac{C \sin[c+dx]}{4 d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}
\end{aligned}$$

Result (type 3, 572 leaves):

$$\frac{1}{16 d \sqrt{a} (1 + \operatorname{Cos}[c + d x])}$$

$$e^{-3 i (c+d x)} \left( 1 + e^{i (c+d x)} \right) \left( i C - 2 i C e^{i (c+d x)} + 3 i C e^{2 i (c+d x)} - 3 i C e^{3 i (c+d x)} + 2 i C e^{4 i (c+d x)} - i C e^{5 i (c+d x)} + 8 A d e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} x + \right.$$

$$7 C d e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} x - i (8 A + 7 C) e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{ArcSinh}\left[e^{i (c+d x)}\right] +$$

$$8 i \sqrt{2} (A + C) e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 + e^{i (c+d x)}\right] + 8 i A e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c+d x)}}\right] +$$

$$7 i C e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c+d x)}}\right] - 8 i \sqrt{2} A e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}\right] -$$

$$\left. 8 i \sqrt{2} C e^{2 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}\right] \right) \sqrt{\operatorname{Sec}[c + d x]}$$

■ **Problem 1238: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Cos}[c + d x]^2}{\sqrt{a + a \operatorname{Cos}[c + d x]} \operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 3, 266 leaves, 9 steps):

$$\frac{(8 A + 9 C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c+d x]}{\sqrt{a+a \operatorname{Cos}[c+d x]}}\right] \sqrt{\operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]} + \sqrt{2} (A + C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c+d x]}{\sqrt{2} \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+a \operatorname{Cos}[c+d x]}}\right] \sqrt{\operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}{8 \sqrt{a} d} + \frac{\sqrt{a} d}{\frac{C \operatorname{Sin}[c + d x]}{3 d \sqrt{a + a \operatorname{Cos}[c + d x]} \operatorname{Sec}[c + d x]^{5/2}} - \frac{C \operatorname{Sin}[c + d x]}{12 d \sqrt{a + a \operatorname{Cos}[c + d x]} \operatorname{Sec}[c + d x]^{3/2}} + \frac{(8 A + 7 C) \operatorname{Sin}[c + d x]}{8 d \sqrt{a + a \operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 3, 638 leaves):

$$\frac{1}{96 d \sqrt{a} (1 + \operatorname{Cos}[c + d x])}$$

$$i e^{-4 i (c+d x)} \left( 1 + e^{i (c+d x)} \right) \left( -2 C + 3 C e^{i (c+d x)} - 24 A e^{2 i (c+d x)} - 28 C e^{2 i (c+d x)} + 24 A e^{3 i (c+d x)} + 29 C e^{3 i (c+d x)} - 24 A e^{4 i (c+d x)} - \right.$$

$$29 C e^{4 i (c+d x)} + 24 A e^{5 i (c+d x)} + 28 C e^{5 i (c+d x)} - 3 C e^{6 i (c+d x)} + 2 C e^{7 i (c+d x)} - 24 i A d e^{3 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} x -$$

$$27 i C d e^{3 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} x - 3 (8 A + 9 C) e^{3 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{ArcSinh}\left[e^{i (c+d x)}\right] +$$

$$48 \sqrt{2} (A + C) e^{3 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 + e^{i (c+d x)}\right] + 24 A e^{3 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c+d x)}}\right] +$$

$$27 C e^{3 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c+d x)}}\right] - 48 \sqrt{2} A e^{3 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}\right] -$$

$$\left. 48 \sqrt{2} C e^{3 i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}\right] \right) \sqrt{\operatorname{Sec}[c + d x]}$$

■ **Problem 1239: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + C \operatorname{Cos}[c + d x]^2) \operatorname{Sec}[c + d x]^{9/2}}{(a + a \operatorname{Cos}[c + d x])^{3/2}} dx$$

Optimal (type 3, 315 leaves, 9 steps) :

$$\frac{(19 A + 11 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{2 \sqrt{2} a^{3/2} d} -$$

$$\frac{(1201 A + 665 C) \sqrt{\sec[c+dx]} \sin[c+dx]}{210 a d \sqrt{a+a \cos[c+dx]}} + \frac{(397 A + 245 C) \sec[c+dx]^{3/2} \sin[c+dx]}{210 a d \sqrt{a+a \cos[c+dx]}} -$$

$$\frac{(67 A + 35 C) \sec[c+dx]^{5/2} \sin[c+dx]}{70 a d \sqrt{a+a \cos[c+dx]}} - \frac{(A + C) \sec[c+dx]^{7/2} \sin[c+dx]}{2 d (a+a \cos[c+dx])^{3/2}} + \frac{(11 A + 7 C) \sec[c+dx]^{7/2} \sin[c+dx]}{14 a d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 279 leaves) :

$$\frac{1}{(a(1+\cos[c+dx]))^{3/2}}$$

$$\cos\left[\frac{1}{2}(c+dx)\right]^3 \left( -1/di (19 A + 11 C) e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left( \operatorname{Log}[1+e^{i(c+dx)}] - \operatorname{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) - \right.$$

$$\left. 1 / (840 d) (2339 A + 1435 C + 72 (71 A + 35 C) \cos[c+dx] + 60 (67 A + 35 C) \cos[2(c+dx)] + 1608 A \cos[3(c+dx)] + \right.$$

$$\left. 840 C \cos[3(c+dx)] + 1201 A \cos[4(c+dx)] + 665 C \cos[4(c+dx)] \right) \sec\left[\frac{1}{2}(c+dx)\right] \sec[c+dx]^{7/2} \tan\left[\frac{1}{2}(c+dx)\right] \Bigg)$$

■ **Problem 1240: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + C \cos[c+dx])^2 \sec[c+dx]^{7/2}}{(a+a \cos[c+dx])^{3/2}} dx$$

Optimal (type 3, 268 leaves, 8 steps) :

$$- \frac{(15 A + 7 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{2 \sqrt{2} a^{3/2} d} + \frac{(49 A + 25 C) \sqrt{\sec[c+dx]} \sin[c+dx]}{10 a d \sqrt{a+a \cos[c+dx]}} -$$

$$\frac{(13 A + 5 C) \sec[c+dx]^{3/2} \sin[c+dx]}{10 a d \sqrt{a+a \cos[c+dx]}} - \frac{(A + C) \sec[c+dx]^{5/2} \sin[c+dx]}{2 d (a+a \cos[c+dx])^{3/2}} + \frac{(9 A + 5 C) \sec[c+dx]^{5/2} \sin[c+dx]}{10 a d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 256 leaves) :

$$\frac{1}{5 d (a (1 + \operatorname{Cos}[c + d x]))^{3/2}}$$

$$\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^3 \left( 5 i (15 A + 7 C) e^{-\frac{1}{2} i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \sqrt{1 + e^{2 i (c + d x)}} \left( \operatorname{Log}[1 + e^{i (c + d x)}] - \operatorname{Log}[1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}] \right) \right) +$$

$$\frac{1}{4} (88 A + 40 C + (131 A + 75 C) \operatorname{Cos}[c + d x] + 8 (9 A + 5 C) \operatorname{Cos}[2 (c + d x)] + 49 A \operatorname{Cos}[3 (c + d x)] + 25 C \operatorname{Cos}[3 (c + d x)])$$

$$\operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}[c + d x]^{5/2} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]$$

■ **Problem 1241: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + C \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^{5/2}}{(a + a \operatorname{Cos}[c + d x])^{3/2}} dx$$

Optimal (type 3, 221 leaves, 7 steps):

$$\frac{(11 A + 3 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c + d x]}{\sqrt{2} \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + a \operatorname{Cos}[c + d x]}}\right] \sqrt{\operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}{2 \sqrt{2} a^{3/2} d}$$

$$\frac{(19 A + 3 C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{6 a d \sqrt{a + a \operatorname{Cos}[c + d x]}} - \frac{(A + C) \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{2 d (a + a \operatorname{Cos}[c + d x])^{3/2}} + \frac{(7 A + 3 C) \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{6 a d \sqrt{a + a \operatorname{Cos}[c + d x]}}$$

Result (type 3, 228 leaves):

$$\frac{1}{(a (1 + \operatorname{Cos}[c + d x]))^{3/2}}$$

$$\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^3 \left( -1 / d i (11 A + 3 C) e^{-\frac{1}{2} i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \sqrt{1 + e^{2 i (c + d x)}} \left( \operatorname{Log}[1 + e^{i (c + d x)}] - \operatorname{Log}[1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}] \right) \right) -$$

$$1 / (6 d) (11 A + 3 C + 24 A \operatorname{Cos}[c + d x] + (19 A + 3 C) \operatorname{Cos}[2 (c + d x)]) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}[c + d x]^{3/2} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]$$

■ **Problem 1242: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + C \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^{3/2}}{(a + a \operatorname{Cos}[c + d x])^{3/2}} dx$$

Optimal (type 3, 172 leaves, 6 steps):



$$\frac{(7A - C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{2 \sqrt{2} a^{3/2} d} - \frac{(A+C) \sqrt{\sec[c+dx]} \sin[c+dx]}{2d(a+a \cos[c+dx])^{3/2}} + \frac{(5A+C) \sqrt{\sec[c+dx]} \sin[c+dx]}{2ad \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 206 leaves):

$$\frac{1}{d(a(1+\cos[c+dx]))^{3/2}} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^3 \left( i(7A-C) e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left( \operatorname{Log}[1+e^{i(c+dx)}] - \operatorname{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) + (4A+(5A+C) \cos[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{\sec[c+dx]} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 1243: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+C \cos[c+dx])^2 \sqrt{\sec[c+dx]}}{(a+a \cos[c+dx])^{3/2}} dx$$

Optimal (type 3, 185 leaves, 7 steps):

$$\frac{2C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{a^{3/2} d} + \frac{(3A-5C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{2 \sqrt{2} a^{3/2} d} - \frac{(A+C) \sin[c+dx]}{2d(a+a \cos[c+dx])^{3/2} \sqrt{\sec[c+dx]}}$$

Result (type 3, 326 leaves):

$$\frac{1}{2 d (a (1 + \operatorname{Cos}[c + d x]))^{3/2}}$$

$$\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^3 \left( \sqrt{2} e^{-\frac{1}{2} i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \sqrt{1 + e^{2 i (c + d x)}} \left( 4 C d x - 4 i C \operatorname{ArcSinh}\left[e^{i (c + d x)}\right] - i \sqrt{2} (3 A - 5 C) \operatorname{Log}\left[1 + e^{i (c + d x)}\right] + \right. \right.$$

$$\left. 4 i C \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c + d x)}}\right] + 3 i \sqrt{2} A \operatorname{Log}\left[1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}\right] - 5 i \sqrt{2} C \operatorname{Log}\left[1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}\right] \right) +$$

$$\left. (A + C) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \sqrt{\operatorname{Sec}[c + d x]} \left( \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{3}{2}(c + d x)\right] \right) \right)$$

■ **Problem 1244: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + C \operatorname{Cos}[c + d x]^2}{(a + a \operatorname{Cos}[c + d x])^{3/2} \sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 3, 228 leaves, 8 steps):

$$-\frac{3 C \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c + d x]}{\sqrt{a + a \operatorname{Cos}[c + d x]}}\right] \sqrt{\operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]} + (A + 9 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c + d x]}{\sqrt{2} \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + a \operatorname{Cos}[c + d x]}}\right] \sqrt{\operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}{a^{3/2} d} + \frac{(A + 3 C) \operatorname{Sin}[c + d x]}{2 a d \sqrt{a + a \operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 3, 329 leaves):

$$\frac{1}{2 d (a (1 + \operatorname{Cos}[c + d x]))^{3/2}}$$

$$\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^3 \left( -i \sqrt{2} e^{-\frac{1}{2} i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \sqrt{1 + e^{2 i (c + d x)}} \left( -6 i C d x - 6 C \operatorname{ArcSinh}\left[e^{i (c + d x)}\right] + \sqrt{2} (A + 9 C) \operatorname{Log}\left[1 + e^{i (c + d x)}\right] + \right. \right.$$

$$\left. 6 C \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c + d x)}}\right] - \sqrt{2} A \operatorname{Log}\left[1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}\right] - 9 \sqrt{2} C \operatorname{Log}\left[1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}\right] \right) +$$

$$\left. (A + 3 C + 2 C \operatorname{Cos}[c + d x]) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \sqrt{\operatorname{Sec}[c + d x]} \left( -\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{3}{2}(c + d x)\right] \right) \right)$$

■ **Problem 1245: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Cos}[c + d x]^2}{(a + a \operatorname{Cos}[c + d x])^{3/2} \operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 3, 285 leaves, 9 steps) :

$$\frac{(8A + 19C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} - (5A + 13C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{4a^{3/2}d} - \frac{(A+C) \sin[c+dx]}{2d(a+a \cos[c+dx])^{3/2} \sec[c+dx]^{5/2}} + \frac{(A+2C) \sin[c+dx]}{2ad \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2}} - \frac{(2A+7C) \sin[c+dx]}{4ad \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}$$

Result (type 3, 1004 leaves) :

$$\frac{1}{d(a(1+\cos[c+dx]))^{3/2}} i A e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(\log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}]\right) +$$

$$\left(7i C e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(\log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}]\right)\right) /$$

$$(2d(a(1+\cos[c+dx]))^{3/2}) + \frac{1}{d(a(1+\cos[c+dx]))^{3/2}} 2\sqrt{2} A e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3$$

$$\left(dx - i \operatorname{ArcSinh}[e^{i(c+dx)}] + i\sqrt{2} \log[1+e^{i(c+dx)}] + i \log[1+\sqrt{1+e^{2i(c+dx)}}] - i\sqrt{2} \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}]\right) +$$

$$\left(19C e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3$$

$$\left(dx - i \operatorname{ArcSinh}[e^{i(c+dx)}] + i\sqrt{2} \log[1+e^{i(c+dx)}] + i \log[1+\sqrt{1+e^{2i(c+dx)}}] - i\sqrt{2} \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}]\right) /$$

$$(2\sqrt{2} d(a(1+\cos[c+dx]))^{3/2}) + \frac{1}{(a(1+\cos[c+dx]))^{3/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sqrt{\sec[c+dx]}$$

$$\left(\frac{(-4A+3C) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{2d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{c}{2}\right] + C \sin\left[\frac{c}{2}\right])}{d} - \frac{3C \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{d} + \frac{C \cos\left[\frac{5dx}{2}\right] \sin\left[\frac{5c}{2}\right]}{2d} +$$

$$\frac{(-4A+3C) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{2d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} - \frac{3C \cos\left[\frac{3c}{2}\right] \sin\left[\frac{3dx}{2}\right]}{d} + \frac{C \cos\left[\frac{5c}{2}\right] \sin\left[\frac{5dx}{2}\right]}{2d}\right)$$

■ **Problem 1246: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+C \cos[c+dx])^2 \sec[c+dx]^{7/2}}{(a+a \cos[c+dx])^{5/2}} dx$$

Optimal (type 3, 315 leaves, 9 steps) :

$$\frac{(283 A + 75 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{16 \sqrt{2} a^{5/2} d} +$$

$$\frac{(2671 A + 735 C) \sqrt{\sec[c+dx]} \sin[c+dx]}{240 a^2 d \sqrt{a+a \cos[c+dx]}} - \frac{(787 A + 195 C) \sec[c+dx]^{3/2} \sin[c+dx]}{240 a^2 d \sqrt{a+a \cos[c+dx]}} -$$

$$\frac{(A+C) \sec[c+dx]^{5/2} \sin[c+dx]}{4 d (a+a \cos[c+dx])^{5/2}} - \frac{(21 A + 5 C) \sec[c+dx]^{5/2} \sin[c+dx]}{16 a d (a+a \cos[c+dx])^{3/2}} + \frac{(157 A + 45 C) \sec[c+dx]^{5/2} \sin[c+dx]}{80 a^2 d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 476 leaves) :

$$\left( i (283 A + 75 C) e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \operatorname{Log}[1+e^{i(c+dx)}] - \operatorname{Log}[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) /$$

$$(4 d (a (1 + \cos[c+dx]))^{5/2}) + \frac{1}{(a (1 + \cos[c+dx]))^{5/2}}$$

$$\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c+dx]} \left( \frac{(2671 A + 735 C) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{30 d} + \frac{(2671 A + 735 C) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{30 d} + \right.$$

$$\frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (-29 A \sin\left[\frac{dx}{2}\right] - 13 C \sin\left[\frac{dx}{2}\right])}{4 d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (-A \sin\left[\frac{dx}{2}\right] - C \sin\left[\frac{dx}{2}\right])}{2 d} -$$

$$\left. \frac{176 A \sec[c+dx] \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{15 d} + \frac{16 A \sec[c+dx]^2 \sin\left[\frac{c}{2} + \frac{dx}{2}\right]}{5 d} - \frac{(29 A + 13 C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{4 d} - \frac{(A+C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{2 d} \right)$$

■ **Problem 1247: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + C \cos[c+dx])^2 \sec[c+dx]^{5/2}}{(a + a \cos[c+dx])^{5/2}} dx$$

Optimal (type 3, 266 leaves, 8 steps) :

$$\frac{(163 A + 19 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{16 \sqrt{2} a^{5/2} d} - \frac{(299 A + 27 C) \sqrt{\sec[c+dx]} \sin[c+dx]}{48 a^2 d \sqrt{a+a \cos[c+dx]}} -$$

$$\frac{(A+C) \sec[c+dx]^{3/2} \sin[c+dx]}{4 d (a+a \cos[c+dx])^{5/2}} - \frac{(17 A + C) \sec[c+dx]^{3/2} \sin[c+dx]}{16 a d (a+a \cos[c+dx])^{3/2}} + \frac{5 (19 A + 3 C) \sec[c+dx]^{3/2} \sin[c+dx]}{48 a^2 d \sqrt{a+a \cos[c+dx]}}$$

Result (type 3, 261 leaves) :

$$\frac{1}{4 (a (1 + \cos [c + d x]))^{5/2}}$$

$$\cos \left[ \frac{1}{2} (c + d x) \right]^5 \left( -1 / d i (163 A + 19 C) e^{-\frac{1}{2} i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \sqrt{1 + e^{2 i (c + d x)}} \left( \log [1 + e^{i (c + d x)}] - \log [1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}] \right) \right) -$$

$$1 / (24 d) (878 A + 78 C + (1537 A + 81 C) \cos [c + d x] + 2 (503 A + 39 C) \cos [2 (c + d x)] + 299 A \cos [3 (c + d x)] + 27 C \cos [3 (c + d x)])$$

$$\sec \left[ \frac{1}{2} (c + d x) \right]^3 \sec [c + d x]^{3/2} \tan \left[ \frac{1}{2} (c + d x) \right]$$

- **Problem 1248: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + C \cos [c + d x]^2) \sec [c + d x]^{3/2}}{(a + a \cos [c + d x])^{5/2}} dx$$

Optimal (type 3, 219 leaves, 7 steps):

$$\frac{5 (15 A - C) \operatorname{ArcTan} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{2} \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{16 \sqrt{2} a^{5/2} d} -$$

$$\frac{(A + C) \sqrt{\sec [c + d x]} \sin [c + d x]}{4 d (a + a \cos [c + d x])^{5/2}} - \frac{(13 A - 3 C) \sqrt{\sec [c + d x]} \sin [c + d x]}{16 a d (a + a \cos [c + d x])^{3/2}} + \frac{(49 A + C) \sqrt{\sec [c + d x]} \sin [c + d x]}{16 a^2 d \sqrt{a + a \cos [c + d x]}}$$

Result (type 3, 230 leaves):

$$\left( \cos \left[ \frac{1}{2} (c + d x) \right]^5 \left( 5 i (15 A - C) e^{-\frac{1}{2} i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \sqrt{1 + e^{2 i (c + d x)}} \left( \log [1 + e^{i (c + d x)}] - \log [1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}] \right) \right) \right) +$$

$$\frac{1}{4} (113 A + C + 10 (17 A + C) \cos [c + d x] + (49 A + C) \cos [2 (c + d x)])$$

$$\sec \left[ \frac{1}{2} (c + d x) \right]^3 \sqrt{\sec [c + d x]} \tan \left[ \frac{1}{2} (c + d x) \right] \Bigg) / (4 d (a (1 + \cos [c + d x]))^{5/2})$$

- **Problem 1249: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + C \cos [c + d x]^2) \sqrt{\sec [c + d x]}}{(a + a \cos [c + d x])^{5/2}} dx$$

Optimal (type 3, 174 leaves, 6 steps):

$$\frac{(19A + 3C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{16\sqrt{2} a^{5/2} d} - \frac{(A+C) \sin[c+dx]}{4d (a+a\cos[c+dx])^{5/2} \sqrt{\sec[c+dx]}} - \frac{(9A-7C) \sin[c+dx]}{16ad (a+a\cos[c+dx])^{3/2} \sqrt{\sec[c+dx]}}$$

Result (type 3, 233 leaves):

$$- \left( i \cos\left[\frac{1}{2}(c+dx)\right] \right)^5 \left( (19A + 3C) e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) + \frac{1}{4} i (13A - 3C + (9A - 7C) \cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^4 \sqrt{\sec[c+dx]} \left( \sin\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{3}{2}(c+dx)\right] \right) \right) / (4d (a(1+\cos[c+dx]))^{5/2})$$

■ **Problem 1250: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + C \cos[c+dx]^2}{(a + a \cos[c+dx])^{5/2} \sqrt{\sec[c+dx]}} dx$$

Optimal (type 3, 232 leaves, 8 steps):

$$\frac{2C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{a^{5/2} d} + \frac{(5A - 43C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{16\sqrt{2} a^{5/2} d} - \frac{(A+C) \sin[c+dx]}{4d (a+a\cos[c+dx])^{5/2} \sec[c+dx]^{3/2}} + \frac{(5A-11C) \sin[c+dx]}{16ad (a+a\cos[c+dx])^{3/2} \sqrt{\sec[c+dx]}}$$

Result (type 3, 345 leaves):

$$\frac{1}{8 d (a (1 + \operatorname{Cos}[c + d x]))^{5/2}}$$

$$\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^5 \left( \sqrt{2} e^{-\frac{1}{2} i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \sqrt{1 + e^{2 i (c + d x)}} \left( 32 C d x - 32 i C \operatorname{ArcSinh}\left[e^{i (c + d x)}\right] - i \sqrt{2} (5 A - 43 C) \operatorname{Log}\left[1 + e^{i (c + d x)}\right] + \right. \right.$$

$$\left. \left. 32 i C \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c + d x)}}\right] + 5 i \sqrt{2} A \operatorname{Log}\left[1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}\right] - 43 i \sqrt{2} C \operatorname{Log}\left[1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}\right] \right) + \right.$$

$$\left. \frac{1}{2} (5 A - 11 C + (A - 15 C) \operatorname{Cos}[c + d x]) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \sqrt{\operatorname{Sec}[c + d x]} \left( -\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{3}{2}(c + d x)\right] \right) \right)$$

■ **Problem 1251: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + C \operatorname{Cos}[c + d x]^2}{(a + a \operatorname{Cos}[c + d x])^{5/2} \operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 3, 277 leaves, 9 steps):

$$\frac{5 C \operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[c + d x]}{\sqrt{a + a \operatorname{Cos}[c + d x]}}\right] \sqrt{\operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]} + (3 A + 115 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sin}[c + d x]}{\sqrt{2} \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + a \operatorname{Cos}[c + d x]}}\right] \sqrt{\operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}{a^{5/2} d} + \frac{16 \sqrt{2} a^{5/2} d}{16 \sqrt{2} a^{5/2} d}$$

$$\frac{(A + C) \operatorname{Sin}[c + d x]}{4 d (a + a \operatorname{Cos}[c + d x])^{5/2} \operatorname{Sec}[c + d x]^{5/2}} + \frac{(A - 15 C) \operatorname{Sin}[c + d x]}{16 a d (a + a \operatorname{Cos}[c + d x])^{3/2} \operatorname{Sec}[c + d x]^{3/2}} + \frac{(3 A + 35 C) \operatorname{Sin}[c + d x]}{16 a^2 d \sqrt{a + a \operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 3, 352 leaves):

$$\frac{1}{8 d (a (1 + \operatorname{Cos}[c + d x]))^{5/2}}$$

$$\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^5 \left( -i \sqrt{2} e^{-\frac{1}{2} i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \sqrt{1 + e^{2 i (c + d x)}} \left( -80 i C d x - 80 C \operatorname{ArcSinh}\left[e^{i (c + d x)}\right] + \sqrt{2} (3 A + 115 C) \operatorname{Log}\left[1 + e^{i (c + d x)}\right] + \right. \right.$$

$$\left. \left. 80 C \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c + d x)}}\right] - 3 \sqrt{2} A \operatorname{Log}\left[1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}\right] - 115 \sqrt{2} C \operatorname{Log}\left[1 - e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}\right] \right) + \right.$$

$$\left. \frac{1}{2} (3 A + 43 C + (7 A + 55 C) \operatorname{Cos}[c + d x] + 8 C \operatorname{Cos}[2(c + d x)]) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \sqrt{\operatorname{Sec}[c + d x]} \left( -\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{3}{2}(c + d x)\right] \right) \right)$$

■ **Problem 1252: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Cos}[c + d x]^2}{(a + a \operatorname{Cos}[c + d x])^{5/2} \operatorname{Sec}[c + d x]^{5/2}} dx$$

Optimal (type 3, 334 leaves, 10 steps):

$$\begin{aligned}
& \frac{(8A + 39C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{4a^{5/2}d} - \\
& \frac{(43A + 219C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{2} \sqrt{\cos[c+dx]} \sqrt{a+a \cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{16\sqrt{2}a^{5/2}d} - \frac{(A+C) \sin[c+dx]}{4d(a+a \cos[c+dx])^{5/2} \sec[c+dx]^{7/2}} - \\
& \frac{(3A + 19C) \sin[c+dx]}{16ad(a+a \cos[c+dx])^{3/2} \sec[c+dx]^{5/2}} + \frac{(7A + 31C) \sin[c+dx]}{16a^2d \sqrt{a+a \cos[c+dx]} \sec[c+dx]^{3/2}} - \frac{(11A + 63C) \sin[c+dx]}{16a^2d \sqrt{a+a \cos[c+dx]} \sqrt{\sec[c+dx]}}
\end{aligned}$$

Result (type 3, 1076 leaves):



$$\begin{aligned}
& \left( 11 i A e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\
& (4 d (a (1 + \cos[c + dx]))^{5/2}) + \\
& \left( 63 i C e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left( \log[1+e^{i(c+dx)}] - \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\
& (4 d (a (1 + \cos[c + dx]))^{5/2}) + \frac{1}{d (a (1 + \cos[c + dx]))^{5/2}} 4 \sqrt{2} A e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \\
& \left( dx - i \operatorname{ArcSinh}[e^{i(c+dx)}] + i \sqrt{2} \log[1+e^{i(c+dx)}] + i \log[1+\sqrt{1+e^{2i(c+dx)}}] - i \sqrt{2} \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) + \\
& \left( 39 C e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \right. \\
& \left. \left( dx - i \operatorname{ArcSinh}[e^{i(c+dx)}] + i \sqrt{2} \log[1+e^{i(c+dx)}] + i \log[1+\sqrt{1+e^{2i(c+dx)}}] - i \sqrt{2} \log[1-e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right) \right) / \\
& (\sqrt{2} d (a (1 + \cos[c + dx]))^{5/2}) + \frac{1}{(a (1 + \cos[c + dx]))^{5/2}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sqrt{\sec[c + dx]} \\
& \left( -\frac{3 (5 A + 3 C) \cos\left[\frac{dx}{2}\right] \sin\left[\frac{c}{2}\right]}{2 d} - \frac{10 C \cos\left[\frac{3dx}{2}\right] \sin\left[\frac{3c}{2}\right]}{d} + \frac{C \cos\left[\frac{5dx}{2}\right] \sin\left[\frac{5c}{2}\right]}{d} - \frac{3 (5 A + 3 C) \cos\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right]}{2 d} + \right. \\
& \left. \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (-A \sin\left[\frac{dx}{2}\right] - C \sin\left[\frac{dx}{2}\right])}{2 d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (19 A \sin\left[\frac{dx}{2}\right] + 35 C \sin\left[\frac{dx}{2}\right])}{4 d} - \right. \\
& \left. \frac{10 C \cos\left[\frac{3c}{2}\right] \sin\left[\frac{3dx}{2}\right]}{d} + \frac{C \cos\left[\frac{5c}{2}\right] \sin\left[\frac{5dx}{2}\right]}{d} + \frac{(19 A + 35 C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \tan\left[\frac{c}{2}\right]}{4 d} - \frac{(A + C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \tan\left[\frac{c}{2}\right]}{2 d} \right)
\end{aligned}$$

- **Problem 1313: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos[c + dx]} (A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^{5/2} dx$$

Optimal (type 3, 140 leaves, 5 steps):

$$\frac{2\sqrt{a} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} +$$

$$\frac{2a(A+3B) \sqrt{\sec[c+dx]} \sin[c+dx]}{3d\sqrt{a+a\cos[c+dx]}} + \frac{2A\sqrt{a+a\cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{3d}$$

Result (type 4, 424 leaves):

$$\frac{1}{d} \sqrt{a(1+\cos[c+dx])} \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \sqrt{\sec[c+dx]} \left( \frac{2}{3} (2A+3B) \sin\left[\frac{1}{2}(c+dx)\right] + \frac{2}{3} A \sec[c+dx] \sin\left[\frac{1}{2}(c+dx)\right] \right) -$$

$$\frac{1}{d} 8(-3-2\sqrt{2}) C \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2} + (10-7\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}}$$

$$\sqrt{\frac{-1+\sqrt{2} - (-2+\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2} + (-2+\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]\right) \sqrt{a(1+\cos[c+dx])}$$

$$\left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right)$$

$$\sec\left[\frac{c}{2} + \frac{dx}{2}\right] \sqrt{\left(-1-\sqrt{2} + (2+\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{4}(c+dx)\right]^2 \sec[c+dx]^{3/2} \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2}}$$

- **Problem 1314: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+a\cos[c+dx]} (A+B\cos[c+dx]+C\cos[c+dx]^2) \sec[c+dx]^{3/2} dx$$

Optimal (type 3, 141 leaves, 5 steps):

$$\frac{\sqrt{a} (2B+C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} -$$

$$\frac{a(2A-C) \sin[c+dx]}{d\sqrt{a+a\cos[c+dx]} \sqrt{\sec[c+dx]}} + \frac{2A\sqrt{a+a\cos[c+dx]} \sqrt{\sec[c+dx]} \sin[c+dx]}{d}$$

Result (type 4, 422 leaves):

$$\frac{\sqrt{a(1+\cos[c+dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sqrt{\operatorname{Sec}[c+dx]} \left(\frac{1}{2}(4A-C) \sin\left[\frac{1}{2}(c+dx)\right] + \frac{1}{2}C \sin\left[\frac{3}{2}(c+dx)\right]\right)}{d}$$

$$\frac{1}{d} 4(-3-2\sqrt{2})(2B+C) \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}}$$

$$\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \sqrt{a(1+\cos[c+dx])}$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2\operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right)$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Sec}[c+dx]^{3/2} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}}$$

- **Problem 1315: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+a\cos[c+dx]} (A+B\cos[c+dx]+C\cos[c+dx]^2) \sqrt{\operatorname{Sec}[c+dx]} dx$$

Optimal (type 3, 151 leaves, 5 steps):

$$\frac{\sqrt{a}(8A+4B+3C) \operatorname{ArcSin}\left[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{4d} +$$

$$\frac{a(4B+C) \sin[c+dx]}{4d\sqrt{a+a\cos[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}} + \frac{C\sqrt{a+a\cos[c+dx]} \sin[c+dx]}{2d\sqrt{\operatorname{Sec}[c+dx]}}$$

Result (type 4, 441 leaves):

$$\frac{1}{d} \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} \left( -\frac{1}{8} (4B + C) \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{4} (2B + C) \sin\left[\frac{3}{2}(c + dx)\right] + \frac{1}{8} C \sin\left[\frac{5}{2}(c + dx)\right] \right) +$$

$$\frac{1}{d} \left( 2 + \frac{3}{\sqrt{2}} \right) (8A + 4B + 3C) \cos\left[\frac{1}{4}(c + dx)\right]^4$$

$$\sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left( 1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right] \right) \sqrt{a(1 + \cos[c + dx])}$$

$$\left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sqrt{\left( -1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right] \right) \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2}$$

$$\sqrt{\left( -1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right] \right) \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2} \operatorname{Sec}[c + dx]^{3/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}$$

- **Problem 1316: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \cos[c + dx]} (A + B \cos[c + dx] + C \cos[c + dx]^2)}{\sqrt{\operatorname{Sec}[c + dx]}} dx$$

Optimal (type 3, 199 leaves, 6 steps):

$$\frac{\sqrt{a} (8A + 6B + 5C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right] \sqrt{\cos[c + dx]} \sqrt{\operatorname{Sec}[c + dx]}}{8d} +$$

$$\frac{a(6B + C) \sin[c + dx]}{12d \sqrt{a + a \cos[c + dx]} \operatorname{Sec}[c + dx]^{3/2}} + \frac{C \sqrt{a + a \cos[c + dx]} \sin[c + dx]}{3d \operatorname{Sec}[c + dx]^{3/2}} + \frac{a(8A + 6B + 5C) \sin[c + dx]}{8d \sqrt{a + a \cos[c + dx]} \sqrt{\operatorname{Sec}[c + dx]}}$$

Result (type 4, 472 leaves):

$$\frac{1}{d} \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} \left( -\frac{1}{48} (24A + 6B + 11C) \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{12} (6A + 3B + 4C) \sin\left[\frac{3}{2}(c + dx)\right] + \frac{1}{16} (2B + C) \sin\left[\frac{5}{2}(c + dx)\right] + \frac{1}{24} C \sin\left[\frac{7}{2}(c + dx)\right] \right) + \frac{1}{d} \left( 1 + \frac{3}{2\sqrt{2}} \right) (8A + 6B + 5C) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left( 1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right] \right) \sqrt{a(1 + \cos[c + dx])} \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2} \operatorname{Sec}[c + dx]^{3/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}$$

- **Problem 1317: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \cos[c + dx]} (A + B \cos[c + dx] + C \cos[c + dx]^2)}{\operatorname{Sec}[c + dx]^{3/2}} dx$$

Optimal (type 3, 247 leaves, 7 steps):

$$\frac{\sqrt{a} (48A + 40B + 35C) \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right] \sqrt{\cos[c + dx]} \sqrt{\operatorname{Sec}[c + dx]}}{64d} + \frac{a(8B + C) \sin[c + dx]}{24d \sqrt{a + a \cos[c + dx]} \operatorname{Sec}[c + dx]^{5/2}} + \frac{C \sqrt{a + a \cos[c + dx]} \sin[c + dx]}{4d \operatorname{Sec}[c + dx]^{5/2}} + \frac{a(48A + 40B + 35C) \sin[c + dx]}{96d \sqrt{a + a \cos[c + dx]} \operatorname{Sec}[c + dx]^{3/2}} + \frac{a(48A + 40B + 35C) \sin[c + dx]}{64d \sqrt{a + a \cos[c + dx]} \sqrt{\operatorname{Sec}[c + dx]}}$$

Result (type 4, 496 leaves):

$$\frac{1}{d} \sqrt{a(1 + \cos[c + dx])} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} \left( -\frac{1}{384} (48A + 88B + 41C) \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{48} (12A + 16B + 11C) \sin\left[\frac{3}{2}(c + dx)\right] + \frac{1}{128} (16A + 8B + 15C) \sin\left[\frac{5}{2}(c + dx)\right] + \frac{1}{48} (2B + C) \sin\left[\frac{7}{2}(c + dx)\right] + \frac{1}{64} C \sin\left[\frac{9}{2}(c + dx)\right] \right) + \frac{1}{(-64 + 48\sqrt{2})d}$$

$$(48A + 40B + 35C) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right)} \sqrt{a(1 + \cos[c + dx])}$$

$$\left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2}$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2} \operatorname{Sec}[c + dx]^{3/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}$$

■ **Problem 1321: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[c + dx])^{3/2} (A + B \cos[c + dx] + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^{7/2} dx$$

Optimal (type 3, 192 leaves, 6 steps):

$$\frac{2a^{3/2} C \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}\right] \sqrt{\cos[c + dx]} \sqrt{\operatorname{Sec}[c + dx]}}{d} + \frac{2a^2 (12A + 20B + 15C) \sqrt{\operatorname{Sec}[c + dx]} \sin[c + dx]}{15d \sqrt{a + a \cos[c + dx]}}$$

$$+ \frac{2a (3A + 5B) \sqrt{a + a \cos[c + dx]} \operatorname{Sec}[c + dx]^{3/2} \sin[c + dx]}{15d} + \frac{2A (a + a \cos[c + dx])^{3/2} \operatorname{Sec}[c + dx]^{5/2} \sin[c + dx]}{5d}$$

Result (type 4, 470 leaves):

$$\frac{1}{d} (a (1 + \cos [c + d x]))^{3/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \sqrt{\sec [c + d x]} \left( \frac{1}{15} (18 A + 25 B + 15 C) \sin \left[ \frac{1}{2} (c + d x) \right] + \frac{1}{5} A \sec [c + d x]^2 \sin \left[ \frac{1}{2} (c + d x) \right] + \frac{1}{15} \sec [c + d x] \left( 9 A \sin \left[ \frac{1}{2} (c + d x) \right] + 5 B \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) -$$

$$\frac{1}{d} 4 (-3 - 2\sqrt{2}) C \cos \left[ \frac{1}{4} (c + d x) \right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right]}{1 + \cos \left[ \frac{1}{2} (c + d x) \right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right]}{1 + \cos \left[ \frac{1}{2} (c + d x) \right]}} \left( 1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) (a (1 + \cos [c + d x]))^{3/2}$$

$$\left( \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2\sqrt{2}}} \right], 17 - 12\sqrt{2} \right] + 2 \text{EllipticPi} \left[ -3 + 2\sqrt{2}, -\text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2\sqrt{2}}} \right], 17 - 12\sqrt{2} \right] \right)$$

$$\sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \sqrt{\left( -1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2 \sec [c + d x]^{3/2} \sqrt{3 - 2\sqrt{2} - \tan \left[ \frac{1}{4} (c + d x) \right]^2}}$$

■ **Problem 1322: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{3/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^{5/2} dx$$

Optimal (type 3, 191 leaves, 6 steps):

$$\frac{a^{3/2} (2 B + 3 C) \text{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{d} - \frac{a^2 (8 A + 6 B - 3 C) \sin [c + d x]}{3 d \sqrt{a + a \cos [c + d x]} \sqrt{\sec [c + d x]}} +$$

$$\frac{2 a (A + B) \sqrt{a + a \cos [c + d x]} \sqrt{\sec [c + d x]} \sin [c + d x]}{d} + \frac{2 A (a + a \cos [c + d x])^{3/2} \sec [c + d x]^{3/2} \sin [c + d x]}{3 d}$$

Result (type 4, 452 leaves):

$$\frac{1}{d} (a (1 + \cos [c + d x]))^{3/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \sqrt{\sec [c + d x]}$$

$$\left( \frac{1}{12} (20 A + 12 B - 3 C) \sin \left[ \frac{1}{2} (c + d x) \right] + \frac{1}{3} A \sec [c + d x] \sin \left[ \frac{1}{2} (c + d x) \right] + \frac{1}{4} C \sin \left[ \frac{3}{2} (c + d x) \right] \right) -$$

$$\frac{1}{d} 2 (-3 - 2 \sqrt{2}) (2 B + 3 C) \cos \left[ \frac{1}{4} (c + d x) \right]^4 \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right]}{1 + \cos \left[ \frac{1}{2} (c + d x) \right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right]}{1 + \cos \left[ \frac{1}{2} (c + d x) \right]}} \left( 1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) (a (1 + \cos [c + d x]))^{3/2}$$

$$\left( \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \text{EllipticPi} \left[ -3 + 2 \sqrt{2}, -\text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right)$$

$$\sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \sqrt{\left( -1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2 \sec [c + d x]^{3/2} \sqrt{3 - 2 \sqrt{2} - \tan \left[ \frac{1}{4} (c + d x) \right]^2}}$$

■ **Problem 1323: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{3/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^{3/2} dx$$

Optimal (type 3, 201 leaves, 6 steps):

$$\frac{a^{3/2} (8 A + 12 B + 7 C) \text{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{4 d} - \frac{a^2 (8 A - 4 B - 5 C) \sin [c + d x]}{4 d \sqrt{a + a \cos [c + d x]} \sqrt{\sec [c + d x]}} -$$

$$\frac{a (4 A - C) \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{2 d \sqrt{\sec [c + d x]}} + \frac{2 A (a + a \cos [c + d x])^{3/2} \sqrt{\sec [c + d x]} \sin [c + d x]}{d}$$

Result (type 4, 454 leaves):



$$\frac{1}{d} (a (1 + \cos [c + d x]))^{3/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \sqrt{\sec [c + d x]}$$

$$\left( \frac{1}{16} (16 A - 4 B - 5 C) \sin \left[ \frac{1}{2} (c + d x) \right] + \frac{1}{8} (2 B + 3 C) \sin \left[ \frac{3}{2} (c + d x) \right] + \frac{1}{16} C \sin \left[ \frac{5}{2} (c + d x) \right] \right) + \frac{1}{d} \left( 1 + \frac{3}{2 \sqrt{2}} \right) (8 A + 12 B + 7 C)$$

$$\cos \left[ \frac{1}{4} (c + d x) \right]^4 \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right]}{1 + \cos \left[ \frac{1}{2} (c + d x) \right]}} \left( 1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) (a (1 + \cos [c + d x]))^{3/2}$$

$$\left( \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \text{EllipticPi} \left[ -3 + 2 \sqrt{2}, -\text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right)$$

$$\sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \sqrt{\left( -1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2}$$

$$\sqrt{\left( -1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2 \sec [c + d x]^{3/2} \sqrt{3 - 2 \sqrt{2} - \tan \left[ \frac{1}{4} (c + d x) \right]^2}}$$

- **Problem 1324: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{3/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) \sqrt{\sec [c + d x]} dx$$

Optimal (type 3, 201 leaves, 6 steps):

$$\frac{a^{3/2} (24 A + 14 B + 11 C) \text{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{8 d} +$$

$$\frac{a^2 (24 A + 30 B + 19 C) \sin [c + d x]}{24 d \sqrt{a + a \cos [c + d x]} \sqrt{\sec [c + d x]}} + \frac{a (2 B + C) \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{4 d \sqrt{\sec [c + d x]}} + \frac{C (a + a \cos [c + d x])^{3/2} \sin [c + d x]}{3 d \sqrt{\sec [c + d x]}}$$

Result (type 4, 479 leaves):

$$\frac{1}{d} (a (1 + \cos [c + d x]))^{3/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \sqrt{\sec [c + d x]} \left( -\frac{1}{96} (24 A + 30 B + 17 C) \sin \left[ \frac{1}{2} (c + d x) \right] + \frac{1}{24} (6 A + 9 B + 7 C) \sin \left[ \frac{3}{2} (c + d x) \right] + \frac{1}{32} (2 B + 3 C) \sin \left[ \frac{5}{2} (c + d x) \right] + \frac{1}{48} C \sin \left[ \frac{7}{2} (c + d x) \right] \right) + \frac{1}{8 d} (4 + 3 \sqrt{2}) (24 A + 14 B + 11 C) \cos \left[ \frac{1}{4} (c + d x) \right]^4 \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right]}{1 + \cos \left[ \frac{1}{2} (c + d x) \right]}} \left( 1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) (a (1 + \cos [c + d x]))^{3/2} \left( \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \text{EllipticPi} \left[ -3 + 2 \sqrt{2}, -\text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \sqrt{\left( -1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2} \sqrt{\left( -1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2} \sec [c + d x]^{3/2} \sqrt{3 - 2 \sqrt{2} - \tan \left[ \frac{1}{4} (c + d x) \right]^2}$$

■ **Problem 1325: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \cos [c + d x])^{3/2} (A + B \cos [c + d x] + C \cos [c + d x]^2)}{\sqrt{\sec [c + d x]}} dx$$

Optimal (type 3, 253 leaves, 7 steps):

$$\frac{a^{3/2} (112 A + 88 B + 75 C) \text{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{64 d} + \frac{a^2 (48 A + 56 B + 39 C) \sin [c + d x]}{96 d \sqrt{a + a \cos [c + d x]} \sec [c + d x]^{3/2}} + \frac{a (8 B + 3 C) \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{24 d \sec [c + d x]^{3/2}} + \frac{C (a + a \cos [c + d x])^{3/2} \sin [c + d x]}{4 d \sec [c + d x]^{3/2}} + \frac{a^2 (112 A + 88 B + 75 C) \sin [c + d x]}{64 d \sqrt{a + a \cos [c + d x]} \sqrt{\sec [c + d x]}}$$

Result (type 4, 503 leaves):

$$\frac{1}{d} (a (1 + \cos [c + d x]))^{3/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \sqrt{\sec [c + d x]} \left( -\frac{1}{768} (240 A + 136 B + 129 C) \sin \left[ \frac{1}{2} (c + d x) \right] + \frac{1}{96} (36 A + 28 B + 27 C) \sin \left[ \frac{3}{2} (c + d x) \right] + \frac{1}{256} (16 A + 24 B + 23 C) \sin \left[ \frac{5}{2} (c + d x) \right] + \frac{1}{96} (2 B + 3 C) \sin \left[ \frac{7}{2} (c + d x) \right] + \frac{1}{128} C \sin \left[ \frac{9}{2} (c + d x) \right] \right) + \frac{1}{64 d} (4 + 3 \sqrt{2}) (112 A + 88 B + 75 C) \cos \left[ \frac{1}{4} (c + d x) \right]^4 \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right]}{1 + \cos \left[ \frac{1}{2} (c + d x) \right]}} \left( 1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) (a (1 + \cos [c + d x]))^{3/2} \left( \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \text{EllipticPi} \left[ -3 + 2 \sqrt{2}, -\text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \sqrt{\left( -1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2} \sqrt{\left( -1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2} \sec [c + d x]^{3/2} \sqrt{3 - 2 \sqrt{2} - \tan \left[ \frac{1}{4} (c + d x) \right]^2}$$

■ **Problem 1326: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \cos [c + d x])^{3/2} (A + B \cos [c + d x] + C \cos [c + d x]^2)}{\sec [c + d x]^{3/2}} dx$$

Optimal (type 3, 303 leaves, 8 steps):

$$\frac{a^{3/2} (176 A + 150 B + 133 C) \text{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{128 d} + \frac{a^2 (80 A + 90 B + 67 C) \sin [c + d x]}{240 d \sqrt{a + a \cos [c + d x]} \sec [c + d x]^{5/2}} + \frac{a (10 B + 3 C) \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{40 d \sec [c + d x]^{5/2}} + \frac{C (a + a \cos [c + d x])^{3/2} \sin [c + d x]}{5 d \sec [c + d x]^{5/2}} + \frac{a^2 (176 A + 150 B + 133 C) \sin [c + d x]}{192 d \sqrt{a + a \cos [c + d x]} \sec [c + d x]^{3/2}} + \frac{a^2 (176 A + 150 B + 133 C) \sin [c + d x]}{128 d \sqrt{a + a \cos [c + d x]} \sqrt{\sec [c + d x]}}$$

Result (type 4, 527 leaves):

$$\frac{1}{d} (a (1 + \cos [c + d x]))^{3/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \sqrt{\sec [c + d x]}$$

$$\left( -\frac{(1360 A + 1290 B + 1019 C) \sin \left[ \frac{1}{2} (c + d x) \right]}{7680} + \frac{1}{960} (280 A + 270 B + 239 C) \sin \left[ \frac{3}{2} (c + d x) \right] + \frac{1}{512} (48 A + 46 B + 49 C) \sin \left[ \frac{5}{2} (c + d x) \right] + \right.$$

$$\left. \frac{1}{480} (10 A + 15 B + 17 C) \sin \left[ \frac{7}{2} (c + d x) \right] + \frac{1}{256} (2 B + 3 C) \sin \left[ \frac{9}{2} (c + d x) \right] + \frac{1}{320} C \sin \left[ \frac{11}{2} (c + d x) \right] \right) +$$

$$\frac{1}{64 d} \left( 2 + \frac{3}{\sqrt{2}} \right) (176 A + 150 B + 133 C) \cos \left[ \frac{1}{4} (c + d x) \right]^4 \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right]}{1 + \cos \left[ \frac{1}{2} (c + d x) \right]}}$$

$$\left( 1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) (a (1 + \cos [c + d x]))^{3/2}$$

$$\left( \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \text{EllipticPi} \left[ -3 + 2 \sqrt{2}, -\text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right)$$

$$\sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^3 \sqrt{\left( -1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2}$$

$$\sqrt{\left( -1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2 \sec [c + d x]^{3/2} \sqrt{3 - 2 \sqrt{2} - \tan \left[ \frac{1}{4} (c + d x) \right]^2}}$$

■ **Problem 1330: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^{9/2} dx$$

Optimal (type 3, 242 leaves, 7 steps):

$$\frac{2 a^{5/2} C \text{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{d} +$$

$$\frac{2 a^3 (160 A + 224 B + 245 C) \sqrt{\sec [c + d x]} \sin [c + d x]}{105 d \sqrt{a + a \cos [c + d x]}} + \frac{2 a^2 (40 A + 56 B + 35 C) \sqrt{a + a \cos [c + d x]} \sec [c + d x]^{3/2} \sin [c + d x]}{105 d} +$$

$$\frac{2 a (5 A + 7 B) (a + a \cos [c + d x])^{3/2} \sec [c + d x]^{5/2} \sin [c + d x]}{35 d} + \frac{2 A (a + a \cos [c + d x])^{5/2} \sec [c + d x]^{7/2} \sin [c + d x]}{7 d}$$

Result (type 4, 522 leaves):

$$\frac{1}{d} (a (1 + \cos [c + d x]))^{5/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sqrt{\sec [c + d x]}$$

$$\left( \frac{1}{210} (230 A + 301 B + 280 C) \sin \left[ \frac{1}{2} (c + d x) \right] + \frac{1}{14} A \sec [c + d x]^3 \sin \left[ \frac{1}{2} (c + d x) \right] + \frac{1}{70} \sec [c + d x]^2 \right.$$

$$\left. \left( 20 A \sin \left[ \frac{1}{2} (c + d x) \right] + 7 B \sin \left[ \frac{1}{2} (c + d x) \right] \right) + \frac{1}{210} \sec [c + d x] \left( 115 A \sin \left[ \frac{1}{2} (c + d x) \right] + 98 B \sin \left[ \frac{1}{2} (c + d x) \right] + 35 C \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right) -$$

$$\frac{1}{d} 2 (-3 - 2\sqrt{2}) C \cos \left[ \frac{1}{4} (c + d x) \right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right]}{1 + \cos \left[ \frac{1}{2} (c + d x) \right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right]}{1 + \cos \left[ \frac{1}{2} (c + d x) \right]}}$$

$$\left( 1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) (a (1 + \cos [c + d x]))^{5/2}$$

$$\left( \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2\sqrt{2}}} \right], 17 - 12\sqrt{2} \right] + 2 \text{EllipticPi} \left[ -3 + 2\sqrt{2}, -\text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2\sqrt{2}}} \right], 17 - 12\sqrt{2} \right] \right)$$

$$\sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sqrt{\left( -1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right)} \sec \left[ \frac{1}{4} (c + d x) \right]^2 \sec [c + d x]^{3/2} \sqrt{3 - 2\sqrt{2} - \tan \left[ \frac{1}{4} (c + d x) \right]^2}$$

- **Problem 1331: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^{7/2} dx$$

Optimal (type 3, 243 leaves, 7 steps):

$$\frac{a^{5/2} (2 B + 5 C) \text{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{d} -$$

$$\frac{a^3 (64 A + 70 B + 15 C) \sin [c + d x]}{15 d \sqrt{a + a \cos [c + d x]} \sqrt{\sec [c + d x]}} + \frac{2 a^2 (8 A + 10 B + 5 C) \sqrt{a + a \cos [c + d x]} \sqrt{\sec [c + d x]} \sin [c + d x]}{5 d} +$$

$$\frac{2 a (A + B) (a + a \cos [c + d x])^{3/2} \sec [c + d x]^{3/2} \sin [c + d x]}{3 d} + \frac{2 A (a + a \cos [c + d x])^{5/2} \sec [c + d x]^{5/2} \sin [c + d x]}{5 d}$$

Result (type 4, 488 leaves):

$$\begin{aligned} & \frac{1}{d} (a (1 + \cos [c + d x]))^{5/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sqrt{\sec [c + d x]} \left( \frac{1}{120} (172 A + 160 B + 45 C) \sin \left[ \frac{1}{2} (c + d x) \right] + \right. \\ & \quad \left. \frac{1}{10} A \sec [c + d x]^2 \sin \left[ \frac{1}{2} (c + d x) \right] + \frac{1}{30} \sec [c + d x] \left( 14 A \sin \left[ \frac{1}{2} (c + d x) \right] + 5 B \sin \left[ \frac{1}{2} (c + d x) \right] \right) + \frac{1}{8} C \sin \left[ \frac{3}{2} (c + d x) \right] \right) + \\ & \frac{1}{d} \left( 2 + \frac{3}{\sqrt{2}} \right) (2 B + 5 C) \cos \left[ \frac{1}{4} (c + d x) \right]^4 \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right]}{1 + \cos \left[ \frac{1}{2} (c + d x) \right]}} \\ & \left( 1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) (a (1 + \cos [c + d x]))^{5/2} \\ & \left( \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \text{EllipticPi} \left[ -3 + 2 \sqrt{2}, -\text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \\ & \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sqrt{\left( -1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2} \\ & \sqrt{\left( -1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2} \sec [c + d x]^{3/2} \sqrt{3 - 2 \sqrt{2} - \tan \left[ \frac{1}{4} (c + d x) \right]^2} \end{aligned}$$

■ **Problem 1332: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^{5/2} dx$$

Optimal (type 3, 253 leaves, 7 steps):

$$\begin{aligned} & \frac{a^{5/2} (8 A + 20 B + 19 C) \text{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{4 d} - \\ & \frac{a^3 (56 A + 12 B - 27 C) \sin [c + d x]}{12 d \sqrt{a + a \cos [c + d x]} \sqrt{\sec [c + d x]}} - \frac{a^2 (8 A + 4 B - C) \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{2 d \sqrt{\sec [c + d x]}} + \\ & \frac{2 a (5 A + 3 B) (a + a \cos [c + d x])^{3/2} \sqrt{\sec [c + d x]} \sin [c + d x]}{3 d} + \frac{2 A (a + a \cos [c + d x])^{5/2} \sec [c + d x]^{3/2} \sin [c + d x]}{3 d} \end{aligned}$$

Result (type 4, 476 leaves):

$$\frac{1}{d} (a (1 + \cos [c + d x]))^{5/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sqrt{\sec [c + d x]} \left( \frac{1}{96} (128 A + 36 B - 27 C) \sin \left[ \frac{1}{2} (c + d x) \right] + \frac{1}{6} A \sec [c + d x] \sin \left[ \frac{1}{2} (c + d x) \right] + \frac{1}{16} (2 B + 5 C) \sin \left[ \frac{3}{2} (c + d x) \right] + \frac{1}{32} C \sin \left[ \frac{5}{2} (c + d x) \right] \right) + \frac{1}{8 d} (4 + 3 \sqrt{2}) (8 A + 20 B + 19 C) \cos \left[ \frac{1}{4} (c + d x) \right]^4 \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right]}{1 + \cos \left[ \frac{1}{2} (c + d x) \right]}} \left( 1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) (a (1 + \cos [c + d x]))^{5/2} \left( \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \text{EllipticPi} \left[ -3 + 2 \sqrt{2}, -\text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sqrt{\left( -1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2} \sqrt{\left( -1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2} \sec [c + d x]^{3/2} \sqrt{3 - 2 \sqrt{2} - \tan \left[ \frac{1}{4} (c + d x) \right]^2}$$

■ **Problem 1333: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) \sec [c + d x]^{3/2} dx$$

Optimal (type 3, 251 leaves, 7 steps):

$$\frac{a^{5/2} (40 A + 38 B + 25 C) \text{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{8 d} - \frac{a^3 (24 A - 54 B - 49 C) \sin [c + d x]}{24 d \sqrt{a + a \cos [c + d x]} \sqrt{\sec [c + d x]}} - \frac{a^2 (8 A - 2 B - 3 C) \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{4 d \sqrt{\sec [c + d x]}} - \frac{a (6 A - C) (a + a \cos [c + d x])^{3/2} \sin [c + d x]}{3 d \sqrt{\sec [c + d x]}} + \frac{2 A (a + a \cos [c + d x])^{5/2} \sqrt{\sec [c + d x]} \sin [c + d x]}{d}$$

Result (type 4, 479 leaves):

$$\frac{1}{d} (a (1 + \cos [c + d x]))^{5/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sqrt{\sec [c + d x]}$$

$$\left( \frac{1}{192} (72 A - 54 B - 47 C) \sin \left[ \frac{1}{2} (c + d x) \right] + \frac{1}{48} (6 A + 15 B + 16 C) \sin \left[ \frac{3}{2} (c + d x) \right] + \frac{1}{64} (2 B + 5 C) \sin \left[ \frac{5}{2} (c + d x) \right] + \frac{1}{96} C \sin \left[ \frac{7}{2} (c + d x) \right] \right) +$$

$$\frac{1}{8 d} \left( 2 + \frac{3}{\sqrt{2}} \right) (40 A + 38 B + 25 C) \cos \left[ \frac{1}{4} (c + d x) \right]^4 \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right]}{1 + \cos \left[ \frac{1}{2} (c + d x) \right]}}$$

$$\left( 1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) (a (1 + \cos [c + d x]))^{5/2}$$

$$\left( \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \text{EllipticPi} \left[ -3 + 2 \sqrt{2}, -\text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right)$$

$$\sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sqrt{\left( -1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2}$$

$$\sqrt{\left( -1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2 \sec [c + d x]^{3/2} \sqrt{3 - 2 \sqrt{2} - \tan \left[ \frac{1}{4} (c + d x) \right]^2}}$$

■ **Problem 1334: Result unnecessarily involves higher level functions.**

$$\int (a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x] + C \cos [c + d x]^2) \sqrt{\sec [c + d x]} dx$$

Optimal (type 3, 253 leaves, 7 steps):

$$\frac{a^{5/2} (304 A + 200 B + 163 C) \text{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{64 d} + \frac{a^3 (432 A + 392 B + 299 C) \sin [c + d x]}{192 d \sqrt{a + a \cos [c + d x]} \sqrt{\sec [c + d x]}}$$

$$\frac{a^2 (16 A + 24 B + 17 C) \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{32 d \sqrt{\sec [c + d x]}} + \frac{a (8 B + 5 C) (a + a \cos [c + d x])^{3/2} \sin [c + d x]}{24 d \sqrt{\sec [c + d x]}} + \frac{C (a + a \cos [c + d x])^{5/2} \sin [c + d x]}{4 d \sqrt{\sec [c + d x]}}$$

Result (type 4, 503 leaves):



$$\frac{1}{d} (a (1 + \cos [c + dx]))^{5/2} \sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sqrt{\sec [c + dx]} \left( -\frac{(432 A + 376 B + 265 C) \sin \left[ \frac{1}{2} (c + dx) \right]}{1536} + \frac{1}{192} (60 A + 64 B + 55 C) \sin \left[ \frac{3}{2} (c + dx) \right] + \right.$$

$$\left. \frac{1}{512} (16 A + 40 B + 47 C) \sin \left[ \frac{5}{2} (c + dx) \right] + \frac{1}{192} (2 B + 5 C) \sin \left[ \frac{7}{2} (c + dx) \right] + \frac{1}{256} C \sin \left[ \frac{9}{2} (c + dx) \right] \right) +$$

$$\frac{1}{64 d} \left( 2 + \frac{3}{\sqrt{2}} \right) (304 A + 200 B + 163 C) \cos \left[ \frac{1}{4} (c + dx) \right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos \left[ \frac{1}{2} (c + dx) \right]}{1 + \cos \left[ \frac{1}{2} (c + dx) \right]}}$$

$$\left( 1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + dx) \right] \right) (a (1 + \cos [c + dx]))^{5/2}$$

$$\left( \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + dx) \right]}{\sqrt{3 - 2\sqrt{2}}} \right], 17 - 12\sqrt{2} \right] + 2 \text{EllipticPi} \left[ -3 + 2\sqrt{2}, -\text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + dx) \right]}{\sqrt{3 - 2\sqrt{2}}} \right], 17 - 12\sqrt{2} \right] \right)$$

$$\sec \left[ \frac{c}{2} + \frac{dx}{2} \right]^5 \sqrt{\left( -1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + dx) \right] \right) \sec \left[ \frac{1}{4} (c + dx) \right]^2}$$

$$\sqrt{\left( -1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + dx) \right] \right) \sec \left[ \frac{1}{4} (c + dx) \right]^2 \sec [c + dx]^{3/2} \sqrt{3 - 2\sqrt{2} - \tan \left[ \frac{1}{4} (c + dx) \right]^2}}$$

■ **Problem 1335: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \cos [c + dx])^{5/2} (A + B \cos [c + dx] + C \cos [c + dx]^2)}{\sqrt{\sec [c + dx]}} dx$$

Optimal (type 3, 301 leaves, 8 steps):

$$\frac{a^{5/2} (400 A + 326 B + 283 C) \text{ArcSin} \left[ \frac{\sqrt{a} \sin [c + dx]}{\sqrt{a + a \cos [c + dx]}} \right] \sqrt{\cos [c + dx]} \sqrt{\sec [c + dx]}}{128 d} +$$

$$\frac{a^3 (1040 A + 950 B + 787 C) \sin [c + dx]}{960 d \sqrt{a + a \cos [c + dx]} \sec [c + dx]^{3/2}} + \frac{a^2 (80 A + 110 B + 79 C) \sqrt{a + a \cos [c + dx]} \sin [c + dx]}{240 d \sec [c + dx]^{3/2}} +$$

$$\frac{a (2 B + C) (a + a \cos [c + dx])^{3/2} \sin [c + dx]}{8 d \sec [c + dx]^{3/2}} + \frac{C (a + a \cos [c + dx])^{5/2} \sin [c + dx]}{5 d \sec [c + dx]^{3/2}} + \frac{a^3 (400 A + 326 B + 283 C) \sin [c + dx]}{128 d \sqrt{a + a \cos [c + dx]} \sqrt{\sec [c + dx]}}$$

Result (type 4, 527 leaves):

$$\frac{1}{d} (a (1 + \cos [c + d x]))^{5/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sqrt{\sec [c + d x]}$$

$$\left( -\frac{(3760 A + 2650 B + 2309 C) \sin \left[ \frac{1}{2} (c + d x) \right]}{15360} + \frac{(640 A + 550 B + 509 C) \sin \left[ \frac{3}{2} (c + d x) \right]}{1920} + \frac{(80 A + 94 B + 95 C) \sin \left[ \frac{5}{2} (c + d x) \right]}{1024} + \right.$$

$$\left. \frac{1}{960} (10 A + 25 B + 32 C) \sin \left[ \frac{7}{2} (c + d x) \right] + \frac{1}{512} (2 B + 5 C) \sin \left[ \frac{9}{2} (c + d x) \right] + \frac{1}{640} C \sin \left[ \frac{11}{2} (c + d x) \right] \right) +$$

$$\frac{1}{256 d} (4 + 3 \sqrt{2}) (400 A + 326 B + 283 C) \cos \left[ \frac{1}{4} (c + d x) \right]^4 \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right]}{1 + \cos \left[ \frac{1}{2} (c + d x) \right]}}$$

$$\left( (1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right]) (a (1 + \cos [c + d x]))^{5/2} \right.$$

$$\left. \left[ \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \text{EllipticPi} \left[ -3 + 2 \sqrt{2}, -\text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right] \right)$$

$$\sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sqrt{\left( -1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2}$$

$$\sqrt{\left( -1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2 \sec [c + d x]^{3/2} \sqrt{3 - 2 \sqrt{2} - \tan \left[ \frac{1}{4} (c + d x) \right]^2}}$$

■ **Problem 1336: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \cos [c + d x])^{5/2} (A + B \cos [c + d x] + C \cos [c + d x]^2)}{\sec [c + d x]^{3/2}} dx$$

Optimal (type 3, 353 leaves, 9 steps):

$$\frac{a^{5/2} (1304 A + 1132 B + 1015 C) \text{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{512 d} + \frac{a^3 (680 A + 628 B + 545 C) \sin [c + d x]}{960 d \sqrt{a + a \cos [c + d x]} \sec [c + d x]^{5/2}} +$$

$$\frac{a^2 (120 A + 156 B + 115 C) \sqrt{a + a \cos [c + d x]} \sin [c + d x]}{480 d \sec [c + d x]^{5/2}} + \frac{a (12 B + 5 C) (a + a \cos [c + d x])^{3/2} \sin [c + d x]}{60 d \sec [c + d x]^{5/2}} +$$

$$\frac{C (a + a \cos [c + d x])^{5/2} \sin [c + d x]}{6 d \sec [c + d x]^{5/2}} + \frac{a^3 (1304 A + 1132 B + 1015 C) \sin [c + d x]}{768 d \sqrt{a + a \cos [c + d x]} \sec [c + d x]^{3/2}} + \frac{a^3 (1304 A + 1132 B + 1015 C) \sin [c + d x]}{512 d \sqrt{a + a \cos [c + d x]} \sqrt{\sec [c + d x]}}$$

Result (type 4, 551 leaves):

$$\frac{1}{d} (a (1 + \cos [c + d x]))^{5/2} \sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sqrt{\sec [c + d x]}$$

$$\left( -\frac{(10600 A + 9236 B + 7945 C) \sin \left[ \frac{1}{2} (c + d x) \right]}{61440} + \frac{(1100 A + 1018 B + 935 C) \sin \left[ \frac{3}{2} (c + d x) \right]}{3840} + \frac{(1128 A + 1140 B + 1145 C) \sin \left[ \frac{5}{2} (c + d x) \right]}{12288} + \right.$$

$$\left. \frac{(100 A + 128 B + 145 C) \sin \left[ \frac{7}{2} (c + d x) \right]}{3840} + \frac{(24 A + 60 B + 83 C) \sin \left[ \frac{9}{2} (c + d x) \right]}{6144} + \frac{(2 B + 5 C) \sin \left[ \frac{11}{2} (c + d x) \right]}{1280} + \frac{C \sin \left[ \frac{13}{2} (c + d x) \right]}{1536} \right) +$$

$$\frac{1}{512 d} \left( 2 + \frac{3}{\sqrt{2}} \right) (1304 A + 1132 B + 1015 C) \cos \left[ \frac{1}{4} (c + d x) \right]^4 \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right]}{1 + \cos \left[ \frac{1}{2} (c + d x) \right]}}$$

$$\left( \left( 1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) (a (1 + \cos [c + d x]))^{5/2} \right.$$

$$\left. \left( \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \text{EllipticPi} \left[ -3 + 2 \sqrt{2}, -\text{ArcSin} \left[ \frac{\tan \left[ \frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \right)$$

$$\sec \left[ \frac{c}{2} + \frac{d x}{2} \right]^5 \sqrt{\left( -1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2}$$

$$\sqrt{\left( -1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[ \frac{1}{2} (c + d x) \right] \right) \sec \left[ \frac{1}{4} (c + d x) \right]^2 \sec [c + d x]^{3/2} \sqrt{3 - 2 \sqrt{2} - \tan \left[ \frac{1}{4} (c + d x) \right]^2}}$$

■ **Problem 1345: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a A + (A b + a B) \cos [c + d x] + b B \cos [c + d x]^2) \sqrt{\sec [c + d x]}}{\sqrt{a + a \cos [c + d x]}} dx$$

Optimal (type 3, 192 leaves, 7 steps):

$$\frac{(2 A b + 2 a B - b B) \text{ArcSin} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{\sqrt{a} d} +$$

$$\frac{\sqrt{2} (a - b) (A - B) \text{ArcTan} \left[ \frac{\sqrt{a} \sin [c + d x]}{\sqrt{2} \sqrt{\cos [c + d x]} \sqrt{a + a \cos [c + d x]}} \right] \sqrt{\cos [c + d x]} \sqrt{\sec [c + d x]}}{\sqrt{a} d} + \frac{b B \sin [c + d x]}{d \sqrt{a + a \cos [c + d x]} \sqrt{\sec [c + d x]}}$$

Result (type 3, 807 leaves):

$$\frac{1}{4 d \sqrt{a (1 + \operatorname{Cos}[c + d x])}}$$

$$e^{-2 i (c+d x)} \left( 1 + e^{i (c+d x)} \right) \left( i b B - i b B e^{i (c+d x)} + i b B e^{2 i (c+d x)} - i b B e^{3 i (c+d x)} + 2 A b d e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} x + 2 a B d e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} x - \right.$$

$$b B d e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} x - i (2 A b + 2 a B - b B) e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{ArcSinh}\left[e^{i (c+d x)}\right] -$$

$$2 i \sqrt{2} (a - b) (A - B) e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 + e^{i (c+d x)}\right] + 2 i A b e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c+d x)}}\right] +$$

$$2 i a B e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c+d x)}}\right] - i b B e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c+d x)}}\right] +$$

$$2 i \sqrt{2} a A e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}\right] - 2 i \sqrt{2} A b e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}}$$

$$\operatorname{Log}\left[1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}\right] - 2 i \sqrt{2} a B e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}\right] +$$

$$\left. 2 i \sqrt{2} b B e^{i (c+d x)} \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Log}\left[1 - e^{i (c+d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c+d x)}}\right] \right) \sqrt{\operatorname{Sec}[c + d x]}$$

■ **Problem 1391: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \operatorname{Cos}[c + d x]^2) \operatorname{Sec}[c + d x]^{7/2}}{a + b \operatorname{Cos}[c + d x]} dx$$

Optimal (type 4, 266 leaves, 9 steps):

$$\frac{2 (5 A b^2 + a^2 (3 A + 5 C)) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 a^3 d} -$$

$$\frac{2 A b \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 a^2 d} - \frac{2 b (A b^2 + a^2 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticPi}\left[\frac{2 b}{a + b}, \frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{a^3 (a + b) d} +$$

$$\frac{2 (5 A b^2 + a^2 (3 A + 5 C)) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{5 a^3 d} - \frac{2 A b \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{3 a^2 d} + \frac{2 A \operatorname{Sec}[c + d x]^{5/2} \operatorname{Sin}[c + d x]}{5 a d}$$

Result (type 4, 648 leaves):

$$\begin{aligned}
& -\frac{1}{30 a^3 d} \left( -\left( 2 \left( 18 a^3 A + 40 a A b^2 + 30 a^3 C \right) \cos [c+d x]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. (b+a \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) \right) / \left( b(a+b \cos [c+d x]) (1-\cos [c+d x]^2) \right) + \\
& \quad \left( 2 \left( 19 a^2 A b + 45 A b^3 + 45 a^2 b C \right) \cos [c+d x]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right) \right. \\
& \quad \left. (b+a \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / \left( a(a+b \cos [c+d x]) (1-\cos [c+d x]^2) \right) + \\
& \quad \left( \left( 9 a^2 A b + 15 A b^3 + 15 a^2 b C \right) \cos [2(c+d x)] (b+a \sec [c+d x]) \left( -4 a b + 4 a b \sec [c+d x]^2 - \right. \right. \\
& \quad \left. \left. 4 a b \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + 2(2 a-b) b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + 4 a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} - \right. \right. \\
& \quad \left. \left. 2 b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} \right) \sin [c+d x] \right) / \\
& \quad \left( a b^2 (a+b \cos [c+d x]) (1-\cos [c+d x]^2) \sqrt{\sec [c+d x]} (2-\sec [c+d x]^2) \right) \Big) + \\
& \quad \frac{\sqrt{\sec [c+d x]} \left( \frac{2(3 a^2 A+5 A b^2+5 a^2 C) \sin [c+d x]}{5 a^3} - \frac{2 A b \tan [c+d x]}{3 a^2} + \frac{2 A \sec [c+d x] \tan [c+d x]}{5 a} \right)}{d}
\end{aligned}$$

■ **Problem 1395: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+C \cos [c+d x]^2}{(a+b \cos [c+d x]) \sqrt{\sec [c+d x]}} dx$$

Optimal (type 4, 190 leaves, 7 steps):

$$\begin{aligned}
& -\frac{2 a C \sqrt{\cos [c+d x]} \operatorname{EllipticE} \left[ \frac{1}{2}(c+d x), 2 \right] \sqrt{\sec [c+d x]}}{b^2 d} + \frac{2(3 a^2 C+b^2(3 A+C)) \sqrt{\cos [c+d x]} \operatorname{EllipticF} \left[ \frac{1}{2}(c+d x), 2 \right] \sqrt{\sec [c+d x]}}{3 b^3 d} - \\
& \frac{2 a(A b^2+a^2 C) \sqrt{\cos [c+d x]} \operatorname{EllipticPi} \left[ \frac{2 b}{a+b}, \frac{1}{2}(c+d x), 2 \right] \sqrt{\sec [c+d x]}}{b^3(a+b) d} + \frac{2 C \sin [c+d x]}{3 b d \sqrt{\sec [c+d x]}}
\end{aligned}$$

Result (type 4, 539 leaves):

$$\begin{aligned}
& \frac{1}{6bd} \left( - \left( 2(6Ab + 2bC) \cos[c+dx]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] (b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) \right) / \\
& \left( b(a+b \cos[c+dx]) (1-\cos[c+dx]^2) \right) - \\
& \left( 2C \cos[c+dx]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \right) \right) \\
& \left( b+a \sec[c+dx] \right) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \Big/ \left( (a+b \cos[c+dx]) (1-\cos[c+dx]^2) \right) - \\
& \left( 3C \cos[2(c+dx)] (b+a \sec[c+dx]) \left( -4ab + 4ab \sec[c+dx]^2 - 4ab \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \right. \right. \\
& \left. \left. \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + 2(2a-b)b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + \right. \right. \\
& \left. \left. 4a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} - \right. \right. \\
& \left. \left. 2b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \right) \sin[c+dx] \right) \Big/ \\
& \left( b^2 (a+b \cos[c+dx]) (1-\cos[c+dx]^2) \sqrt{\sec[c+dx]} (2-\sec[c+dx]^2) \right) + \frac{C \sqrt{\sec[c+dx]} \sin[2(c+dx)]}{3bd}
\end{aligned}$$

■ **Problem 1396: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c+dx]^2}{(a+b \cos[c+dx]) \sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 241 leaves, 8 steps):

$$\begin{aligned}
& \frac{2(5a^2C + b^2(5A + 3C)) \sqrt{\cos[c+dx]} \operatorname{EllipticE} \left[ \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{5b^3d} - \\
& \frac{2a(3Ab^2 + (3a^2 + b^2)C) \sqrt{\cos[c+dx]} \operatorname{EllipticF} \left[ \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{3b^4d} + \\
& \frac{2a^2(Ab^2 + a^2C) \sqrt{\cos[c+dx]} \operatorname{EllipticPi} \left[ \frac{2b}{a+b}, \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{b^4(a+b)d} + \frac{2C \sin[c+dx]}{5bd \sec[c+dx]^{3/2}} - \frac{2aC \sin[c+dx]}{3b^2d \sqrt{\sec[c+dx]}}
\end{aligned}$$

Result (type 4, 603 leaves):

$$\frac{1}{30 b^2 d} \left( - \left( 16 a C \cos [c+d x]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] (b+a \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / \right. \\ \left. \left( (a+b \cos [c+d x]) (1-\cos [c+d x]^2) \right) + \right. \\ \left. \left( 2 (15 A b^2+5 a^2 C+9 b^2 C) \cos [c+d x]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right) \right. \right. \\ \left. \left. (b+a \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / \left( a (a+b \cos [c+d x]) (1-\cos [c+d x]^2) \right) + \right. \\ \left. \left( (15 A b^2+15 a^2 C+9 b^2 C) \cos [2(c+d x)] (b+a \sec [c+d x]) \left( -4 a b+4 a b \sec [c+d x]^2-4 a b \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right. \right. \right. \\ \left. \left. \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2}+2(2 a-b) b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2}+ \right. \right. \\ \left. \left. 4 a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2}- \right. \right. \\ \left. \left. 2 b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} \right) \sin [c+d x] \right) / \\ \left. \left( a b^2 (a+b \cos [c+d x]) (1-\cos [c+d x]^2) \sqrt{\sec [c+d x]} (2-\sec [c+d x]^2) \right) \right) + \\ \frac{\sqrt{\sec [c+d x]} \left( \frac{C \sin [c+d x]}{10 b}-\frac{a C \sin [2(c+d x)]}{3 b^2}+\frac{C \sin [3(c+d x)]}{10 b} \right)}{d}$$

■ **Problem 1397: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+C \cos [c+d x]^2}{(a+b \cos [c+d x]) \sec [c+d x]^{5/2}} dx$$

Optimal (type 4, 299 leaves, 9 steps):

$$\frac{2 a (5 A b^2+5 a^2 C+3 b^2 C) \sqrt{\cos [c+d x]} \operatorname{EllipticE} \left[ \frac{1}{2}(c+d x), 2 \right] \sqrt{\sec [c+d x]}}{5 b^4 d} + \\ \frac{2 (21 a^4 C+7 a^2 b^2 (3 A+C)+b^4 (7 A+5 C)) \sqrt{\cos [c+d x]} \operatorname{EllipticF} \left[ \frac{1}{2}(c+d x), 2 \right] \sqrt{\sec [c+d x]}}{21 b^5 d} - \\ \frac{2 a^3 (A b^2+a^2 C) \sqrt{\cos [c+d x]} \operatorname{EllipticPi} \left[ \frac{2 b}{a+b}, \frac{1}{2}(c+d x), 2 \right] \sqrt{\sec [c+d x]}}{b^5 (a+b) d} + \\ \frac{2 C \sin [c+d x]}{7 b d \sec [c+d x]^{5/2}} - \frac{2 a C \sin [c+d x]}{5 b^2 d \sec [c+d x]^{3/2}} + \frac{2 (7 a^2 C+b^2 (7 A+5 C)) \sin [c+d x]}{21 b^3 d \sqrt{\sec [c+d x]}}$$

Result (type 4, 663 leaves):

$$\begin{aligned}
& -\frac{1}{210 b^3 d} \left( -\left( 2 \left( -70 A b^3 + 56 a^2 b C - 50 b^3 C \right) \cos [c+d x]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. (b+a \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) \right) / \left( (b+(a+b \cos [c+d x]) (1-\cos [c+d x]^2)) \right) + \\
& \quad \left( 2 \left( 35 a A b^2 + 35 a^3 C + 13 a b^2 C \right) \cos [c+d x]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right) \right. \\
& \quad \left. (b+a \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / \left( a (a+b \cos [c+d x]) (1-\cos [c+d x]^2) \right) + \\
& \quad \left( \left( 105 a A b^2 + 105 a^3 C + 63 a b^2 C \right) \cos [2(c+d x)] (b+a \sec [c+d x]) \left( -4 a b + 4 a b \sec [c+d x]^2 - \right. \right. \\
& \quad \left. \left. 4 a b \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + 2 (2 a-b) b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + 4 a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} - \right. \right. \\
& \quad \left. \left. 2 b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} \right) \sin [c+d x] \right) / \\
& \quad \left( a b^2 (a+b \cos [c+d x]) (1-\cos [c+d x]^2) \sqrt{\sec [c+d x]} (2-\sec [c+d x]^2) \right) \Big) + \\
& \quad \frac{\sqrt{\sec [c+d x]} \left( -\frac{a C \sin [c+d x]}{10 b^2} + \frac{(14 A b^2+14 a^2 C+13 b^2 C) \sin [2(c+d x)]}{42 b^3} - \frac{a C \sin [3(c+d x)]}{10 b^2} + \frac{C \sin [4(c+d x)]}{28 b} \right)}{d}
\end{aligned}$$

■ **Problem 1399: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+C \cos [c+d x])^2 \sec [c+d x]^{3/2}}{(a+b \cos [c+d x])^2} dx$$

Optimal (type 4, 330 leaves, 8 steps):

$$\begin{aligned}
& \frac{(3 A b^2 - a^2 (2 A - C)) \sqrt{\cos [c+d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{a^2 (a^2 - b^2) d} + \\
& \frac{(A b^2 + a^2 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{a b (a^2 - b^2) d} + \\
& \frac{(3 A b^4 - a^4 C - a^2 b^2 (5 A + C)) \sqrt{\cos [c+d x]} \operatorname{EllipticPi} \left[ \frac{2 b}{a+b}, \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{a^2 (a-b) b (a+b)^2 d} - \\
& \frac{(3 A b^2 - a^2 (2 A - C)) \sqrt{\sec [c+d x]} \sin [c+d x]}{a^2 (a^2 - b^2) d} + \frac{(A b^2 + a^2 C) \sqrt{\sec [c+d x]} \sin [c+d x]}{a (a^2 - b^2) d (a+b \cos [c+d x])}
\end{aligned}$$

Result (type 4, 682 leaves):



$$\begin{aligned}
& - \frac{1}{4 a^2 (a-b)(a+b) d} \\
& \left( - \left( 2 \left( 4 a^3 A - 8 a A b^2 - 4 a^3 C \right) \operatorname{Cos}[c+d x]^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] (b+a \operatorname{Sec}[c+d x]) \sqrt{1-\operatorname{Sec}[c+d x]^2} \right. \right. \\
& \quad \left. \left. \operatorname{Sin}[c+d x] \right) \right) / \left( b (a+b \operatorname{Cos}[c+d x]) (1-\operatorname{Cos}[c+d x]^2) \right) + \\
& \left( 2 \left( 10 a^2 A b - 9 A b^3 + a^2 b C \right) \operatorname{Cos}[c+d x]^2 \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \right) \right. \\
& \quad \left. (b+a \operatorname{Sec}[c+d x]) \sqrt{1-\operatorname{Sec}[c+d x]^2} \operatorname{Sin}[c+d x] \right) / \left( a (a+b \operatorname{Cos}[c+d x]) (1-\operatorname{Cos}[c+d x]^2) \right) + \\
& \left( \left( 2 a^2 A b - 3 A b^3 - a^2 b C \right) \operatorname{Cos}[2(c+d x)] (b+a \operatorname{Sec}[c+d x]) \left( -4 a b + 4 a b \operatorname{Sec}[c+d x]^2 - 4 a b \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \right. \right. \\
& \quad \left. \left. \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} + 2(2 a-b) b \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} + \right. \right. \\
& \quad \left. \left. 4 a^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} - \right. \right. \\
& \quad \left. \left. 2 b^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} \right) \operatorname{Sin}[c+d x] \right) / \\
& \quad \left( a b^2 (a+b \operatorname{Cos}[c+d x]) (1-\operatorname{Cos}[c+d x]^2) \sqrt{\operatorname{Sec}[c+d x]} (2-\operatorname{Sec}[c+d x]^2) \right) + \\
& \frac{\sqrt{\operatorname{Sec}[c+d x]} \left( \frac{(2 a^2 A - 3 A b^2 - a^2 C) \operatorname{Sin}[c+d x]}{a^2 (a^2 - b^2)} + \frac{A b^2 \operatorname{Sin}[c+d x] + a^2 C \operatorname{Sin}[c+d x]}{a (a^2 - b^2) (a+b \operatorname{Cos}[c+d x])} \right)}{d}
\end{aligned}$$

■ **Problem 1400: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \operatorname{Cos}[c+d x]^2) \sqrt{\operatorname{Sec}[c+d x]}}{(a+b \operatorname{Cos}[c+d x])^2} dx$$

Optimal (type 4, 274 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(A b^2 + a^2 C) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\operatorname{Sec}[c+d x]}}{a b (a^2 - b^2) d} \\
& - \frac{(A b^2 - a^2 C + 2 b^2 C) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\operatorname{Sec}[c+d x]}}{b^2 (a^2 - b^2) d} \\
& + \frac{(A b^4 + a^4 C - 3 a^2 b^2 (A + C)) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+d x), 2\right] \sqrt{\operatorname{Sec}[c+d x]}}{a (a-b) b^2 (a+b)^2 d} + \frac{(A b^2 + a^2 C) \operatorname{Sin}[c+d x]}{a (a^2 - b^2) d (a+b \operatorname{Cos}[c+d x]) \sqrt{\operatorname{Sec}[c+d x]}}
\end{aligned}$$

Result (type 4, 657 leaves):

$$\frac{1}{4 a (-a+b) (a+b) d} \left( - \left( 2 (4 a A b + 4 a b C) \cos [c+d x]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] (b+a \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / \right. \\ \left. (b (a+b \cos [c+d x]) (1-\cos [c+d x]^2)) + \right. \\ \left. \left( 2 (-4 a^2 A + 3 A b^2 - a^2 C) \cos [c+d x]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right) \right. \right. \\ \left. \left. (b+a \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / (a (a+b \cos [c+d x]) (1-\cos [c+d x]^2)) + \right. \\ \left. \left( (A b^2 + a^2 C) \cos [2 (c+d x)] (b+a \sec [c+d x]) \left( -4 a b + 4 a b \sec [c+d x]^2 - 4 a b \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right. \right. \right. \\ \left. \left. \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + 2 (2 a-b) b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + \right. \right. \\ \left. \left. 4 a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} - \right. \right. \\ \left. \left. 2 b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} \right) \sin [c+d x] \right) / \\ \left. \left( a b^2 (a+b \cos [c+d x]) (1-\cos [c+d x]^2) \sqrt{\sec [c+d x]} (2-\sec [c+d x]^2) \right) \right) + \\ \frac{\sqrt{\sec [c+d x]} \left( \frac{(A b^2 + a^2 C) \sin [c+d x]}{a b (a^2 - b^2)} + \frac{A b^2 \sin [c+d x] + a^2 C \sin [c+d x]}{b (-a^2 + b^2) (a+b \cos [c+d x])} \right)}{d}$$

■ **Problem 1401: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos [c+d x]^2}{(a+b \cos [c+d x])^2 \sqrt{\sec [c+d x]}} dx$$

Optimal (type 4, 277 leaves, 7 steps):

$$\frac{(A b^2 + 3 a^2 C - 2 b^2 C) \sqrt{\cos [c+d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{b^2 (a^2 - b^2) d} + \\ \frac{a (A b^2 - 3 a^2 C + 4 b^2 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{b^3 (a^2 - b^2) d} - \\ \frac{(A b^4 - 3 a^4 C + a^2 b^2 (A + 5 C)) \sqrt{\cos [c+d x]} \operatorname{EllipticPi} \left[ \frac{2b}{a+b}, \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{(a-b) b^3 (a+b)^2 d} - \frac{(A b^2 + a^2 C) \sin [c+d x]}{b (a^2 - b^2) d (a+b \cos [c+d x]) \sqrt{\sec [c+d x]}}$$

Result (type 4, 663 leaves):

$$\frac{1}{4(a-b)b(a+b)d} \left( - \left( 2(4aAb + 4abC) \cos[c+dx]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] (b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / \right. \\ \left. (b(a+b \cos[c+dx]) (1-\cos[c+dx]^2)) + \right. \\ \left. \left( 2(-Ab^2 + a^2C - 2b^2C) \cos[c+dx]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \right) \right. \right. \\ \left. \left. (b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / (a(a+b \cos[c+dx]) (1-\cos[c+dx]^2)) + \right. \\ \left. \left( (Ab^2 + 3a^2C - 2b^2C) \cos[2(c+dx)] (b+a \sec[c+dx]) \left( -4ab + 4ab \sec[c+dx]^2 - 4ab \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \right. \right. \right. \\ \left. \left. \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + 2(2a-b)b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + \right. \right. \\ \left. \left. 4a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} - \right. \right. \\ \left. \left. 2b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \right) \sin[c+dx] \right) / \\ \left. \left( ab^2(a+b \cos[c+dx]) (1-\cos[c+dx]^2) \sqrt{\sec[c+dx]} (2-\sec[c+dx]^2) \right) \right) + \\ \frac{\sqrt{\sec[c+dx]} \left( \frac{(Ab^2+a^2C) \sin[c+dx]}{b^2(-a^2+b^2)} + \frac{-aAb^2 \sin[c+dx] - a^3C \sin[c+dx]}{b^2(-a^2+b^2)(a+b \cos[c+dx])} \right)}{d}$$

■ **Problem 1406: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos[c+dx]^2) \sqrt{\sec[c+dx]}}{(a+b \cos[c+dx])^3} dx$$

Optimal (type 4, 405 leaves, 8 steps):

$$\frac{(3Ab^4 - a^4C - a^2b^2(9A + 5C)) \sqrt{\cos[c+dx]} \operatorname{EllipticE} \left[ \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{4a^2b(a^2-b^2)^2d} + \\ \frac{(Ab^4 + a^4C - 7a^2b^2(A+C)) \sqrt{\cos[c+dx]} \operatorname{EllipticF} \left[ \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{4ab^2(a^2-b^2)^2d} + \frac{1}{4a^2(a-b)^2b^2(a+b)^3d} \\ \frac{(3Ab^6 - 3a^2b^4(2A-C) - a^6C + 5a^4b^2(3A+2C)) \sqrt{\cos[c+dx]} \operatorname{EllipticPi} \left[ \frac{2b}{a+b}, \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{2a(a^2-b^2)d(a+b \cos[c+dx])^2 \sqrt{\sec[c+dx]}} - \frac{(3Ab^4 - a^4C - a^2b^2(9A + 5C)) \sin[c+dx]}{4a^2(a^2-b^2)^2d(a+b \cos[c+dx]) \sqrt{\sec[c+dx]}}$$

Result (type 4, 816 leaves):

$$\begin{aligned}
& \frac{1}{16 a^2 (a-b)^2 (a+b)^2 d} \\
& \left( - \left( 2 \left( -32 a^3 A b + 8 a A b^3 - 24 a^3 b C \right) \operatorname{Cos}[c+d x]^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] (b+a \operatorname{Sec}[c+d x]) \sqrt{1-\operatorname{Sec}[c+d x]^2} \right. \right. \\
& \quad \left. \left. \operatorname{Sin}[c+d x] \right) \right) / \left( (b+(a+b \operatorname{Cos}[c+d x]) (1-\operatorname{Cos}[c+d x]^2)) + \left( 2 \left( 16 a^4 A - 19 a^2 A b^2 + 9 A b^4 + 5 a^4 C + a^2 b^2 C \right) \right. \right. \\
& \quad \left. \left. \operatorname{Cos}[c+d x]^2 \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \right) \right) \right. \\
& \quad \left. (b+a \operatorname{Sec}[c+d x]) \sqrt{1-\operatorname{Sec}[c+d x]^2} \operatorname{Sin}[c+d x] \right) / \left( a (a+b \operatorname{Cos}[c+d x]) (1-\operatorname{Cos}[c+d x]^2) \right) + \\
& \quad \left( (-9 a^2 A b^2 + 3 A b^4 - a^4 C - 5 a^2 b^2 C) \operatorname{Cos}[2(c+d x)] (b+a \operatorname{Sec}[c+d x]) \left( -4 a b + 4 a b \operatorname{Sec}[c+d x]^2 - \right. \right. \\
& \quad \left. \left. 4 a b \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} + 2(2 a-b) b \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \right. \right. \\
& \quad \left. \left. \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} + 4 a^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} - \right. \right. \\
& \quad \left. \left. 2 b^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} \right) \operatorname{Sin}[c+d x] \right) / \\
& \quad \left( a b^2 (a+b \operatorname{Cos}[c+d x]) (1-\operatorname{Cos}[c+d x]^2) \sqrt{\operatorname{Sec}[c+d x]} (2-\operatorname{Sec}[c+d x]^2) \right) + \frac{1}{d} \\
& \sqrt{\operatorname{Sec}[c+d x]} \left( \frac{(9 a^2 A b^2 - 3 A b^4 + a^4 C + 5 a^2 b^2 C) \operatorname{Sin}[c+d x]}{4 a^2 b (a^2 - b^2)^2} + \frac{A b^2 \operatorname{Sin}[c+d x] + a^2 C \operatorname{Sin}[c+d x]}{2 b (-a^2 + b^2) (a+b \operatorname{Cos}[c+d x])^2} + \right. \\
& \quad \left. \frac{-7 a^2 A b^2 \operatorname{Sin}[c+d x] + A b^4 \operatorname{Sin}[c+d x] + a^4 C \operatorname{Sin}[c+d x] - 7 a^2 b^2 C \operatorname{Sin}[c+d x]}{4 a b (a^2 - b^2)^2 (a+b \operatorname{Cos}[c+d x])} \right)
\end{aligned}$$

■ **Problem 1407: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Cos}[c+d x]^2}{(a+b \operatorname{Cos}[c+d x])^3 \sqrt{\operatorname{Sec}[c+d x]}} dx$$

Optimal (type 4, 408 leaves, 8 steps):

$$\frac{(Ab^4 - 3a^4C + a^2b^2(5A + 9C))\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{4ab^2(a^2-b^2)^2d} +$$

$$\frac{(a^2b^2(3A-5C) + 3a^4C + b^4(3A+8C))\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{4b^3(a^2-b^2)^2d} + \frac{1}{4a(a-b)^2b^3(a+b)^3d}$$

$$\frac{(Ab^6 - 3a^4b^2(A-2C) - 3a^6C - 5a^2b^4(2A+3C))\sqrt{\cos[c+dx]}\operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]} - (Ab^2 + a^2C)\sin[c+dx]}{2b(a^2-b^2)d(a+b\cos[c+dx])^2\sqrt{\sec[c+dx]}} - \frac{(Ab^4 - 3a^4C + a^2b^2(5A+9C))\sin[c+dx]}{4ab(a^2-b^2)^2d(a+b\cos[c+dx])\sqrt{\sec[c+dx]}}$$

Result (type 4, 821 leaves):

$$\frac{1}{16a(a-b)^2b(a+b)^2d}$$

$$\left(-2(-16a^3Ab - 8aAb^3 - 8a^3bC - 16ab^3C)\cos[c+dx]^2\operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right](b+a\sec[c+dx])\sqrt{1-\sec[c+dx]^2}\sin[c+dx]\right) / \left(b(a+b\cos[c+dx])(1-\cos[c+dx]^2)\right) + \left(2(9a^2Ab^2 - 3Ab^4 + a^4C + 5a^2b^2C)\cos[c+dx]^2\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right]\right)(b+a\sec[c+dx])\sqrt{1-\sec[c+dx]^2}\sin[c+dx]\right) / \left(a(a+b\cos[c+dx])(1-\cos[c+dx]^2)\right) +$$

$$\left((-5a^2Ab^2 - Ab^4 + 3a^4C - 9a^2b^2C)\cos[2(c+dx)](b+a\sec[c+dx])\left(-4ab + 4ab\sec[c+dx]^2 - 4ab\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right]\sqrt{\sec[c+dx]}\sqrt{1-\sec[c+dx]^2} + 2(2a-b)b\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right]\sqrt{\sec[c+dx]}\sqrt{1-\sec[c+dx]^2} + 4a^2\operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right]\sqrt{\sec[c+dx]}\sqrt{1-\sec[c+dx]^2} - 2b^2\operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right]\sqrt{\sec[c+dx]}\sqrt{1-\sec[c+dx]^2}\right)\sin[c+dx]\right) /$$

$$\left(ab^2(a+b\cos[c+dx])(1-\cos[c+dx]^2)\sqrt{\sec[c+dx]}(2-\sec[c+dx]^2)\right) + \frac{1}{d}$$

$$\sqrt{\sec[c+dx]}\left(\frac{(-5a^2Ab^2 - Ab^4 + 3a^4C - 9a^2b^2C)\sin[c+dx]}{4ab^2(a^2-b^2)^2} - \frac{aAb^2\sin[c+dx] + a^3C\sin[c+dx]}{2b^2(-a^2+b^2)(a+b\cos[c+dx])^2} + \frac{3a^2Ab^2\sin[c+dx] + 3Ab^4\sin[c+dx] - 5a^4C\sin[c+dx] + 11a^2b^2C\sin[c+dx]}{4b^2(-a^2+b^2)^2(a+b\cos[c+dx])}\right)$$

■ **Problem 1408: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos [c + d x]^2}{(a + b \cos [c + d x])^3 \sec [c + d x]^{3/2}} dx$$

Optimal (type 4, 405 leaves, 8 steps):

$$\begin{aligned} & - \frac{(b^4 (5 A - 8 C) - 15 a^4 C + a^2 b^2 (A + 29 C)) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{4 b^3 (a^2 - b^2)^2 d} \\ & + \frac{a (15 a^4 C + b^4 (7 A + 24 C) - a^2 b^2 (A + 33 C)) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{4 b^4 (a^2 - b^2)^2 d} + \frac{1}{4 (a - b)^2 b^4 (a + b)^3 d} \\ & - \frac{(3 A b^6 + 15 a^6 C + 5 a^2 b^4 (2 A + 7 C) - a^4 b^2 (A + 38 C)) \sqrt{\cos [c + d x]} \operatorname{EllipticPi}\left[\frac{2 b}{a + b}, \frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{2 b (a^2 - b^2) d (a + b \cos [c + d x])^2 \sec [c + d x]^{3/2}} - \\ & + \frac{(A b^2 + a^2 C) \sin [c + d x]}{4 b^2 (a^2 - b^2)^2 d (a + b \cos [c + d x]) \sqrt{\sec [c + d x]}} \end{aligned}$$

Result (type 4, 824 leaves):

$$\begin{aligned}
& \frac{1}{16 (a-b)^2 b^2 (a+b)^2 d} \\
& \left( - \left( 2 \left( -24 a A b^3 + 8 a^3 b C - 32 a b^3 C \right) \cos [c+d x]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] (b+a \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \right. \right. \\
& \quad \left. \left. \sin [c+d x] \right) \right) / \left( (b+(a+b \cos [c+d x]) (1-\cos [c+d x]^2)) + \left( 2 \left( 5 a^2 A b^2 + A b^4 + 5 a^4 C - 7 a^2 b^2 C + 8 b^4 C \right) \right. \right. \\
& \quad \left. \left. \cos [c+d x]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right) \right. \right. \\
& \quad \left. \left. (b+a \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) \right) / \left( (a+(a+b \cos [c+d x]) (1-\cos [c+d x]^2)) + \right. \\
& \quad \left( (-a^2 A b^2 - 5 A b^4 + 15 a^4 C - 29 a^2 b^2 C + 8 b^4 C) \cos [2(c+d x)] (b+a \sec [c+d x]) \left( -4 a b + 4 a b \sec [c+d x]^2 - \right. \right. \\
& \quad \left. \left. 4 a b \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + 2(2 a-b) b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + 4 a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} - \right. \right. \\
& \quad \left. \left. 2 b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} \right) \sin [c+d x] \right) / \\
& \quad \left( a b^2 (a+b \cos [c+d x]) (1-\cos [c+d x]^2) \sqrt{\sec [c+d x]} (2-\sec [c+d x]^2) \right) + \frac{1}{d} \\
& \sqrt{\sec [c+d x]} \left( -\frac{(-a^2 A b^2 - 5 A b^4 + 7 a^4 C - 13 a^2 b^2 C) \sin [c+d x]}{4 b^3 (a^2 - b^2)^2} - \frac{-a^2 A b^2 \sin [c+d x] - a^4 C \sin [c+d x]}{2 b^3 (-a^2 + b^2) (a+b \cos [c+d x])^2} + \right. \\
& \quad \left. \frac{a^3 A b^2 \sin [c+d x] - 7 a A b^4 \sin [c+d x] + 9 a^5 C \sin [c+d x] - 15 a^3 b^2 C \sin [c+d x]}{4 b^3 (-a^2 + b^2)^2 (a+b \cos [c+d x])} \right)
\end{aligned}$$

■ **Problem 1411: Attempted integration timed out after 120 seconds.**

$$\int \sqrt{a+b \cos [c+d x]} (A+C \cos [c+d x]^2) \sec [c+d x]^{11/2} dx$$

Optimal (type 4, 544 leaves, 8 steps):

$$\begin{aligned}
& - \frac{1}{315 a^5 d \sqrt{\text{Sec}[c + d x]}} 2 (a - b) \sqrt{a + b} \left( 16 A b^4 + 6 a^2 b^2 (4 A + 7 C) - 21 a^4 (7 A + 9 C) \right) \sqrt{\text{Cos}[c + d x]} \\
& \quad \text{Csc}[c + d x] \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{a + b \text{Cos}[c + d x]}}{\sqrt{a + b} \sqrt{\text{Cos}[c + d x]}} \right], -\frac{a + b}{a - b} \right] \sqrt{\frac{a (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \text{Sec}[c + d x])}{a - b}} - \\
& \frac{1}{315 a^4 d \sqrt{\text{Sec}[c + d x]}} 2 (a - b) \sqrt{a + b} \left( 12 a A b^2 + 16 A b^3 + 6 a^2 b (6 A + 7 C) + 21 a^3 (7 A + 9 C) \right) \sqrt{\text{Cos}[c + d x]} \text{Csc}[c + d x] \\
& \quad \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{a + b \text{Cos}[c + d x]}}{\sqrt{a + b} \sqrt{\text{Cos}[c + d x]}} \right], -\frac{a + b}{a - b} \right] \sqrt{\frac{a (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \text{Sec}[c + d x])}{a - b}} + \\
& \frac{2 b \left( 8 A b^2 + a^2 (13 A + 21 C) \right) \sqrt{a + b \text{Cos}[c + d x]} \text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{315 a^3 d} - \\
& \frac{2 \left( 6 A b^2 - 7 a^2 (7 A + 9 C) \right) \sqrt{a + b \text{Cos}[c + d x]} \text{Sec}[c + d x]^{5/2} \text{Sin}[c + d x]}{315 a^2 d} + \\
& \frac{2 A b \sqrt{a + b \text{Cos}[c + d x]} \text{Sec}[c + d x]^{7/2} \text{Sin}[c + d x]}{63 a d} + \frac{2 A \sqrt{a + b \text{Cos}[c + d x]} \text{Sec}[c + d x]^{9/2} \text{Sin}[c + d x]}{9 d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1412: Unable to integrate problem.**

$$\int \sqrt{a + b \text{Cos}[c + d x]} \left( A + C \text{Cos}[c + d x]^2 \right) \text{Sec}[c + d x]^{9/2} dx$$

Optimal (type 4, 455 leaves, 7 steps):



$$\frac{1}{105 a^4 d \sqrt{\sec[c+dx]}} 2 (a-b) b \sqrt{a+b} (8 A b^2 + a^2 (19 A + 35 C)) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{105 a^3 d \sqrt{\sec[c+dx]}}$$

$$2 (a-b) \sqrt{a+b} (6 a A b + 8 A b^2 + 5 a^2 (5 A + 7 C)) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{2(4 A b^2 - 5 a^2 (5 A + 7 C)) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{105 a^2 d} +$$

$$\frac{2 A b \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{5/2} \sin[c+dx]}{35 a d} + \frac{2 A \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{7/2} \sin[c+dx]}{7 d}$$

Result (type 8, 39 leaves):

$$\int \sqrt{a+b \cos[c+dx]} (A+C \cos[c+dx])^2 \sec[c+dx]^{9/2} dx$$

■ **Problem 1413: Attempted integration timed out after 120 seconds.**

$$\int \sqrt{a+b \cos[c+dx]} (A+C \cos[c+dx])^2 \sec[c+dx]^{7/2} dx$$

Optimal (type 4, 385 leaves, 6 steps):

$$-\frac{1}{15 a^3 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (2 A b^2 - 3 a^2 (3 A + 5 C)) \sqrt{\cos[c+dx]}$$

$$\operatorname{Csc}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{15 a^2 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (9 a A + 2 A b + 15 a C) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2 A b \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{15 a d} + \frac{2 A \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{5/2} \sin[c+dx]}{5 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1416: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \cos[c + dx]} (A + C \cos[c + dx])^2 \sqrt{\sec[c + dx]} dx$$

Optimal (type 4, 515 leaves, 8 steps):

$$\begin{aligned} & -\frac{1}{4bd\sqrt{\sec[c+dx]}} \\ & (a-b)\sqrt{a+b}\sqrt{\cos[c+dx]}\operatorname{Csc}[c+dx]\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]\sqrt{\frac{a(1-\sec[c+dx])}{a+b}}\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\ & \frac{1}{4bd\sqrt{\sec[c+dx]}}\sqrt{a+b}(8Ab+(a+2b)C)\sqrt{\cos[c+dx]}\operatorname{Csc}[c+dx]\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\ & \sqrt{\frac{a(1-\sec[c+dx])}{a+b}}\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{4b^2d\sqrt{\sec[c+dx]}}\sqrt{a+b}(a^2C-4b^2(2A+C))\sqrt{\cos[c+dx]} \\ & \operatorname{Csc}[c+dx]\operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b\cos[c+dx]}}{\sqrt{a+b}\sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]\sqrt{\frac{a(1-\sec[c+dx])}{a+b}}\sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\ & \frac{C\sqrt{a+b\cos[c+dx]}\sin[c+dx]}{2d\sqrt{\sec[c+dx]}} + \frac{aC\sqrt{a+b\cos[c+dx]}\sqrt{\sec[c+dx]}\sin[c+dx]}{4bd} \end{aligned}$$

Result (type 4, 1391 leaves):

$$\begin{aligned} & \frac{C\sqrt{a+b\cos[c+dx]}\sqrt{\sec[c+dx]}\sin[2(c+dx)]}{4d} + \\ & \left( -a^2\sqrt{\frac{a-b}{a+b}}\operatorname{C Tan}\left[\frac{1}{2}(c+dx)\right] - ab\sqrt{\frac{a-b}{a+b}}\operatorname{C Tan}\left[\frac{1}{2}(c+dx)\right] + 2ab\sqrt{\frac{a-b}{a+b}}\operatorname{C Tan}\left[\frac{1}{2}(c+dx)\right]^3 + a^2\sqrt{\frac{a-b}{a+b}}\operatorname{C Tan}\left[\frac{1}{2}(c+dx)\right]^5 - \right. \\ & ab\sqrt{\frac{a-b}{a+b}}\operatorname{C Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 16iAb^2\operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i\operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right]\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\ & \sqrt{\frac{a+b+a\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2ia^2C\operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i\operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \\ & \left. \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\sqrt{\frac{a+b+a\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \right) \end{aligned}$$

$$\begin{aligned}
& 8 i b^2 C \text{EllipticPi} \left[ \frac{a+b}{a-b}, i \text{ArcSinh} \left[ \sqrt{\frac{a-b}{a+b}} \tan \left[ \frac{1}{2} (c+dx) \right] \right], -\frac{a+b}{a-b} \right] \sqrt{1 - \tan \left[ \frac{1}{2} (c+dx) \right]^2} \\
& \sqrt{\frac{a+b+a \tan \left[ \frac{1}{2} (c+dx) \right]^2 - b \tan \left[ \frac{1}{2} (c+dx) \right]^2}{a+b}} + 16 i A b^2 \text{EllipticPi} \left[ \frac{a+b}{a-b}, i \text{ArcSinh} \left[ \sqrt{\frac{a-b}{a+b}} \tan \left[ \frac{1}{2} (c+dx) \right] \right], -\frac{a+b}{a-b} \right] \\
& \tan \left[ \frac{1}{2} (c+dx) \right]^2 \sqrt{1 - \tan \left[ \frac{1}{2} (c+dx) \right]^2} \sqrt{\frac{a+b+a \tan \left[ \frac{1}{2} (c+dx) \right]^2 - b \tan \left[ \frac{1}{2} (c+dx) \right]^2}{a+b}} - \\
& 2 i a^2 C \text{EllipticPi} \left[ \frac{a+b}{a-b}, i \text{ArcSinh} \left[ \sqrt{\frac{a-b}{a+b}} \tan \left[ \frac{1}{2} (c+dx) \right] \right], -\frac{a+b}{a-b} \right] \tan \left[ \frac{1}{2} (c+dx) \right]^2 \sqrt{1 - \tan \left[ \frac{1}{2} (c+dx) \right]^2} \\
& \sqrt{\frac{a+b+a \tan \left[ \frac{1}{2} (c+dx) \right]^2 - b \tan \left[ \frac{1}{2} (c+dx) \right]^2}{a+b}} + 8 i b^2 C \text{EllipticPi} \left[ \frac{a+b}{a-b}, i \text{ArcSinh} \left[ \sqrt{\frac{a-b}{a+b}} \tan \left[ \frac{1}{2} (c+dx) \right] \right], -\frac{a+b}{a-b} \right] \\
& \tan \left[ \frac{1}{2} (c+dx) \right]^2 \sqrt{1 - \tan \left[ \frac{1}{2} (c+dx) \right]^2} \sqrt{\frac{a+b+a \tan \left[ \frac{1}{2} (c+dx) \right]^2 - b \tan \left[ \frac{1}{2} (c+dx) \right]^2}{a+b}} - \\
& i a (a-b) C \text{EllipticE} \left[ i \text{ArcSinh} \left[ \sqrt{\frac{a-b}{a+b}} \tan \left[ \frac{1}{2} (c+dx) \right] \right], -\frac{a+b}{a-b} \right] \sqrt{1 - \tan \left[ \frac{1}{2} (c+dx) \right]^2} \left( 1 + \tan \left[ \frac{1}{2} (c+dx) \right]^2 \right) \\
& \sqrt{\frac{a+b+a \tan \left[ \frac{1}{2} (c+dx) \right]^2 - b \tan \left[ \frac{1}{2} (c+dx) \right]^2}{a+b}} + 2 i (a-b) (4 A b + (a+2 b) C) \text{EllipticF} \left[ i \text{ArcSinh} \left[ \sqrt{\frac{a-b}{a+b}} \tan \left[ \frac{1}{2} (c+dx) \right] \right], -\frac{a+b}{a-b} \right] \\
& \sqrt{1 - \tan \left[ \frac{1}{2} (c+dx) \right]^2} \left( 1 + \tan \left[ \frac{1}{2} (c+dx) \right]^2 \right) \sqrt{\frac{a+b+a \tan \left[ \frac{1}{2} (c+dx) \right]^2 - b \tan \left[ \frac{1}{2} (c+dx) \right]^2}{a+b}} \Big/ \\
& \left( 4 b \sqrt{\frac{a-b}{a+b}} d \sqrt{\frac{1}{1 - \tan \left[ \frac{1}{2} (c+dx) \right]^2}} \left( -1 + \tan \left[ \frac{1}{2} (c+dx) \right]^2 \right) \left( 1 + \tan \left[ \frac{1}{2} (c+dx) \right]^2 \right)^{3/2} \sqrt{\frac{a+b+a \tan \left[ \frac{1}{2} (c+dx) \right]^2 - b \tan \left[ \frac{1}{2} (c+dx) \right]^2}{1 + \tan \left[ \frac{1}{2} (c+dx) \right]^2}} \right)
\end{aligned}$$

■ **Problem 1417: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \cos [c+dx]} (A+C \cos [c+dx]^2)}{\sqrt{\sec [c+dx]}} dx$$

Optimal (type 4, 613 leaves, 9 steps) :

$$\frac{1}{24 a b^2 d \sqrt{\sec[c+d x]}} (a-b) \sqrt{a+b} (3 a^2 C-8 b^2 (3 A+2 C)) \sqrt{\cos[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+d x]}}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} - \frac{1}{24 b^2 d \sqrt{\sec[c+d x]}} \sqrt{a+b} (3 a^2 C-2 a b C-8 b^2 (3 A+2 C)) \sqrt{\cos[c+d x]}$$

$$\operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+d x]}}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} - \frac{1}{8 b^3 d \sqrt{\sec[c+d x]}}$$

$$a \sqrt{a+b} (8 A b^2+(a^2+4 b^2) C) \sqrt{\cos[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+d x]}}{\sqrt{a+b} \sqrt{\cos[c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+d x])}{a+b}} \sqrt{\frac{a(1+\sec[c+d x])}{a-b}} - \frac{a C \sqrt{a+b \cos[c+d x]} \sin[c+d x]}{4 b d \sqrt{\sec[c+d x]}} +$$

$$\frac{C(a+b \cos[c+d x])^{3/2} \sin[c+d x]}{3 b d \sqrt{\sec[c+d x]}} - \frac{(3 a^2 C-8 b^2 (3 A+2 C)) \sqrt{a+b \cos[c+d x]} \sqrt{\sec[c+d x]} \sin[c+d x]}{24 b^2 d}$$

Result (type 4, 1317 leaves) :

$$\frac{\sqrt{a+b \cos[c+d x]} \sqrt{\sec[c+d x]} \left(\frac{1}{12} C \sin[c+d x] + \frac{a C \sin[2(c+d x)]}{24 b} + \frac{1}{12} C \sin[3(c+d x)]\right)}{d} +$$

$$\left( \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+d x)\right]^2-b \tan\left[\frac{1}{2}(c+d x)\right]^2}{1+\tan\left[\frac{1}{2}(c+d x)\right]^2}} \right.$$

$$\left( -24 a A b^2 \tan\left[\frac{1}{2}(c+d x)\right] - 24 A b^3 \tan\left[\frac{1}{2}(c+d x)\right] + 3 a^3 C \tan\left[\frac{1}{2}(c+d x)\right] + 3 a^2 b C \tan\left[\frac{1}{2}(c+d x)\right] - 16 a b^2 C \tan\left[\frac{1}{2}(c+d x)\right] - \right.$$

$$16 b^3 C \tan\left[\frac{1}{2}(c+d x)\right] + 48 A b^3 \tan\left[\frac{1}{2}(c+d x)\right]^3 - 6 a^2 b C \tan\left[\frac{1}{2}(c+d x)\right]^3 + 32 b^3 C \tan\left[\frac{1}{2}(c+d x)\right]^3 + 24 a A b^2 \tan\left[\frac{1}{2}(c+d x)\right]^5 -$$

$$24 A b^3 \tan\left[\frac{1}{2}(c+d x)\right]^5 - 3 a^3 C \tan\left[\frac{1}{2}(c+d x)\right]^5 + 3 a^2 b C \tan\left[\frac{1}{2}(c+d x)\right]^5 + 16 a b^2 C \tan\left[\frac{1}{2}(c+d x)\right]^5 - 16 b^3 C \tan\left[\frac{1}{2}(c+d x)\right]^5 +$$

$$\left. 48 a A b^2 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\tan\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+d x)\right]^2-b \tan\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) +$$

$$\begin{aligned}
& 6 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 24 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 48 a A b^2 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 6 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 24 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (-24 A b^2 + 3 a^2 C - 16 b^2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2 a b (-24 A b + (a-14 b) C) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(24 b^2 d \sqrt{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(b \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) - a \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right)
\end{aligned}$$

■ **Problem 1418: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} (A+C \operatorname{Cos}[c+dx]^2)}{\operatorname{Sec}[c+dx]^{3/2}} dx$$

Optimal (type 4, 698 leaves, 10 steps):

$$\begin{aligned}
& - \frac{1}{192 b^3 d \sqrt{\text{Sec}[c+dx]}} (a-b) \sqrt{a+b} (48 A b^2 + 15 a^2 C + 28 b^2 C) \sqrt{\text{Cos}[c+dx]} \\
& \quad \text{Csc}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\text{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} + \\
& \frac{1}{192 b^3 d \sqrt{\text{Sec}[c+dx]}} \sqrt{a+b} (15 a^3 C - 10 a^2 b C + 24 b^3 (4 A + 3 C) + 4 a b^2 (12 A + 7 C)) \sqrt{\text{Cos}[c+dx]} \text{Csc}[c+dx] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\text{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} + \\
& \frac{1}{64 b^4 d \sqrt{\text{Sec}[c+dx]}} \sqrt{a+b} (5 a^4 C + 8 a^2 b^2 (2 A + C) - 16 b^4 (4 A + 3 C)) \sqrt{\text{Cos}[c+dx]} \text{Csc}[c+dx] \\
& \quad \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \text{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\text{Cos}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\text{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Sec}[c+dx])}{a-b}} + \\
& \frac{C(a+b \text{Cos}[c+dx])^{3/2} \text{Sin}[c+dx]}{4 b d \text{Sec}[c+dx]^{3/2}} + \frac{(5 a^2 C + 4 b^2 (4 A + 3 C)) \sqrt{a+b \text{Cos}[c+dx]} \text{Sin}[c+dx]}{32 b^2 d \sqrt{\text{Sec}[c+dx]}} - \\
& \frac{5 a C (a+b \text{Cos}[c+dx])^{3/2} \text{Sin}[c+dx]}{24 b^2 d \sqrt{\text{Sec}[c+dx]}} + \frac{a (48 A b^2 + 15 a^2 C + 28 b^2 C) \sqrt{a+b \text{Cos}[c+dx]} \sqrt{\text{Sec}[c+dx]} \text{Sin}[c+dx]}{192 b^3 d}
\end{aligned}$$

Result (type 4, 3838 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{a+b \text{Cos}[c+dx]} \sqrt{\text{Sec}[c+dx]} \left( \frac{a C \text{Sin}[c+dx]}{96 b} + \frac{(48 A b^2 - 5 a^2 C + 48 b^2 C) \text{Sin}[2(c+dx)]}{192 b^2} + \frac{a C \text{Sin}[3(c+dx)]}{96 b} + \frac{1}{32} C \text{Sin}[4(c+dx)] \right) + \\
& \left( \frac{A b}{2 \sqrt{a+b \text{Cos}[c+dx]} \sqrt{\text{Sec}[c+dx]}} + \frac{a^2 C}{96 b \sqrt{a+b \text{Cos}[c+dx]} \sqrt{\text{Sec}[c+dx]}} + \right. \\
& \frac{3 b C}{8 \sqrt{a+b \text{Cos}[c+dx]} \sqrt{\text{Sec}[c+dx]}} + \frac{3 a A \sqrt{\text{Sec}[c+dx]}}{8 \sqrt{a+b \text{Cos}[c+dx]}} + \frac{25 a C \sqrt{\text{Sec}[c+dx]}}{96 \sqrt{a+b \text{Cos}[c+dx]}} + \frac{5 a^3 C \sqrt{\text{Sec}[c+dx]}}{384 b^2 \sqrt{a+b \text{Cos}[c+dx]}} + \\
& \left. \frac{a A \text{Cos}[2(c+dx)] \sqrt{\text{Sec}[c+dx]}}{8 \sqrt{a+b \text{Cos}[c+dx]}} + \frac{7 a C \text{Cos}[2(c+dx)] \sqrt{\text{Sec}[c+dx]}}{96 \sqrt{a+b \text{Cos}[c+dx]}} + \frac{5 a^3 C \text{Cos}[2(c+dx)] \sqrt{\text{Sec}[c+dx]}}{128 b^2 \sqrt{a+b \text{Cos}[c+dx]}} \right)
\end{aligned}$$

$$\left( \left( \left( a (a+b) (48 A b^2 + 15 a^2 C + 28 b^2 C) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2} (c+dx)\right]\right], \frac{-a+b}{a+b}\right] - 2b (5 a^3 C + 2 a^2 b C - 12 a b^2 (4 A + 3 C) + \right. \right. \right.$$

$$24 b^3 (4 A + 3 C) \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2} (c+dx)\right]\right], \frac{-a+b}{a+b}\right] + 6 (5 a^4 C + 8 a^2 b^2 (2 A + C) - 16 b^4 (4 A + 3 C)) \text{EllipticPi}\left[-1, \right.$$

$$\left. \left. \left. -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2} (c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2} (c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2} (c+dx)\right]^2}{1+\text{Tan}\left[\frac{1}{2} (c+dx)\right]^2}} \sqrt{1-\text{Tan}\left[\frac{1}{2} (c+dx)\right]^4} \right) /$$

$$\left( 192 b^3 (a+b) \sqrt{\frac{1}{1-\text{Tan}\left[\frac{1}{2} (c+dx)\right]^2}} \left(-1+\text{Tan}\left[\frac{1}{2} (c+dx)\right]\right)^2 \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2} (c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2} (c+dx)\right]^2}{a+b}} \right) \right) +$$

$$\frac{a (48 A b^2 + 15 a^2 C + 28 b^2 C) \text{Tan}\left[\frac{1}{2} (c+dx)\right] \sqrt{a + \frac{b-b \text{Tan}\left[\frac{1}{2} (c+dx)\right]^2}{1+\text{Tan}\left[\frac{1}{2} (c+dx)\right]^2}}}{192 b^3 \sqrt{\frac{1+\text{Tan}\left[\frac{1}{2} (c+dx)\right]^2}{1-\text{Tan}\left[\frac{1}{2} (c+dx)\right]^2}}} \right) /$$

$$d \left( \left( \left( a (a+b) (48 A b^2 + 15 a^2 C + 28 b^2 C) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2} (c+dx)\right]\right], \frac{-a+b}{a+b}\right] - \right. \right. \right.$$

$$2b (5 a^3 C + 2 a^2 b C - 12 a b^2 (4 A + 3 C) + 24 b^3 (4 A + 3 C)) \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2} (c+dx)\right]\right], \frac{-a+b}{a+b}\right] +$$

$$6 (5 a^4 C + 8 a^2 b^2 (2 A + C) - 16 b^4 (4 A + 3 C)) \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2} (c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \text{Sec}\left[\frac{1}{2} (c+dx)\right]^2$$

$$\text{Tan}\left[\frac{1}{2} (c+dx)\right]^3 \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2} (c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2} (c+dx)\right]^2}{1+\text{Tan}\left[\frac{1}{2} (c+dx)\right]^2}} \right) / \left( 192 b^3 (a+b) \sqrt{\frac{1}{1-\text{Tan}\left[\frac{1}{2} (c+dx)\right]^2}} \right)$$

$$\begin{aligned}
& \left( -1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right)^2 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \right) + \\
& \left( a(a+b)(48Ab^2 + 15a^2C + 28b^2C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]\right], \frac{-a+b}{a+b} \right) - \\
& 2b(5a^3C + 2a^2bC - 12ab^2(4A+3C) + 24b^3(4A+3C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]\right], \frac{-a+b}{a+b} \right) + \\
& 6(5a^4C + 8a^2b^2(2A+C) - 16b^4(4A+3C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]\right], \frac{-a+b}{a+b} \right) \\
& \left( a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \right) / \\
& \left( 384b^3(a+b)^2 \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left( -1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right)^2 \left( \frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right)^{3/2} \right) + \\
& \left( a(a+b)(48Ab^2 + 15a^2C + 28b^2C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]\right], \frac{-a+b}{a+b} \right) - \\
& 2b(5a^3C + 2a^2bC - 12ab^2(4A+3C) + 24b^3(4A+3C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]\right], \frac{-a+b}{a+b} \right) + \\
& 6(5a^4C + 8a^2b^2(2A+C) - 16b^4(4A+3C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]\right], \frac{-a+b}{a+b} \right) \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \right) / \\
& \left( 192b^3(a+b) \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left( -1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right)^2 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) +
\end{aligned}$$



$$\begin{aligned}
& \left( a (a+b) (48 A b^2 + 15 a^2 C + 28 b^2 C) \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] \right] \right], \frac{-a+b}{a+b} \right) - \\
& 2 b (5 a^3 C + 2 a^2 b C - 12 a b^2 (4 A + 3 C) + 24 b^3 (4 A + 3 C)) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] \right] \right], \frac{-a+b}{a+b} \right) + \\
& 6 (5 a^4 C + 8 a^2 b^2 (2 A + C) - 16 b^4 (4 A + 3 C)) \operatorname{EllipticPi} \left[ -1, -\operatorname{ArcSin} \left[ \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] \right] \right], \frac{-a+b}{a+b} \right) \operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \\
& \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] \sqrt{\frac{1}{1 - \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}} \sqrt{\frac{a+b+a \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2 - b \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}{1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}} \sqrt{1 - \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^4} \right) / \\
& \left( 384 b^3 (a+b) \left( -1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] \right)^2 \sqrt{\frac{a+b+a \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2 - b \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}{a+b}} \right) + \\
& \left( a (48 A b^2 + 15 a^2 C + 28 b^2 C) \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] \left( -\frac{b \operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2} - \right. \right. \\
& \left. \left. \frac{\operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] (b - b \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2)}{(1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2)^2} \right) \right) / \left( 384 b^3 \sqrt{\frac{1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}{1 - \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}} \sqrt{a + \frac{b - b \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}{1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}} \right) + \\
& \frac{a (48 A b^2 + 15 a^2 C + 28 b^2 C) \operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \sqrt{a + \frac{b - b \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}{1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}}}{384 b^3 \sqrt{\frac{1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}{1 - \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}}} - \left( a (48 A b^2 + 15 a^2 C + 28 b^2 C) \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] \right. \\
& \left. \left( \frac{\operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]}{1 - \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2} + \frac{\operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] (1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2)}{(1 - \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2)^2} \right) \sqrt{a + \frac{b - b \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}{1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}} \right) / \\
& \left( 384 b^3 \left( \frac{1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}{1 - \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2} \right)^{3/2} \right) - \left( a (a+b) (48 A b^2 + 15 a^2 C + 28 b^2 C) \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] \right] \right], \frac{-a+b}{a+b} \right) -
\end{aligned}$$

$$\begin{aligned}
& 2b \left( 5a^3 C + 2a^2 b C - 12ab^2 (4A + 3C) + 24b^3 (4A + 3C) \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \text{Tan} \left[ \frac{1}{2} (c + dx) \right] \right], \frac{-a+b}{a+b} \right] + \\
& 6 \left( 5a^4 C + 8a^2 b^2 (2A + C) - 16b^4 (4A + 3C) \right) \text{EllipticPi} \left[ -1, -\text{ArcSin} \left[ \text{Tan} \left[ \frac{1}{2} (c + dx) \right] \right], \frac{-a+b}{a+b} \right] \\
& \sqrt{1 - \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^4} \left( \frac{a \text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \text{Tan} \left[ \frac{1}{2} (c + dx) \right] - b \text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \text{Tan} \left[ \frac{1}{2} (c + dx) \right]}{1 + \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2} - \right. \\
& \left. \frac{\text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \text{Tan} \left[ \frac{1}{2} (c + dx) \right] \left( a + b + a \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 - b \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right)}{\left( 1 + \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right)^2} \right) \Bigg/ \\
& \left( 384b^3 (a+b) \sqrt{\frac{1}{1 - \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2}} \left( -1 + \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right) \sqrt{\frac{a + b + a \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 - b \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2}{a+b}} \right. \\
& \left. \sqrt{\frac{a + b + a \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 - b \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2}{1 + \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2}} \right) - \left( \sqrt{\frac{a + b + a \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 - b \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2}{1 + \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2}} \right. \\
& \left. \sqrt{1 - \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^4} \left( - \frac{b \left( 5a^3 C + 2a^2 b C - 12ab^2 (4A + 3C) + 24b^3 (4A + 3C) \right) \text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2}{\sqrt{1 - \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2} \sqrt{1 - \frac{(-a+b) \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2}{a+b}}} - \right. \right. \\
& \left. \frac{3 \left( 5a^4 C + 8a^2 b^2 (2A + C) - 16b^4 (4A + 3C) \right) \text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2}{\sqrt{1 - \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2} \left( 1 + \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2 \right) \sqrt{1 - \frac{(-a+b) \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2}{a+b}}} + \right. \\
& \left. \left. \frac{a(a+b) \left( 48Ab^2 + 15a^2 C + 28b^2 C \right) \text{Sec} \left[ \frac{1}{2} (c + dx) \right]^2 \sqrt{1 - \frac{(-a+b) \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2}{a+b}}}{2 \sqrt{1 - \text{Tan} \left[ \frac{1}{2} (c + dx) \right]^2}} \right) \right) \Bigg/
\end{aligned}$$

$$\left( 192 b^3 (a+b) \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a+b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right)$$

■ **Problem 1419: Attempted integration timed out after 120 seconds.**

$$\int (a + b \cos [c + d x])^{3/2} (A + C \cos [c + d x]^2) \sec [c + d x]^{11/2} dx$$

Optimal (type 4, 542 leaves, 8 steps):

$$\frac{1}{315 a^4 d \sqrt{\sec [c + d x]}} 2 (a - b) \sqrt{a + b} (8 A b^4 + 21 a^4 (7 A + 9 C) + 3 a^2 b^2 (11 A + 21 C)) \sqrt{\cos [c + d x]}$$

$$\operatorname{Csc}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \sec [c + d x])}{a + b}} \sqrt{\frac{a (1 + \sec [c + d x])}{a - b}} +$$

$$\frac{1}{315 a^3 d \sqrt{\sec [c + d x]}} 2 (a - b) \sqrt{a + b} (6 a A b^2 + 8 A b^3 - 21 a^3 (7 A + 9 C) + a^2 (39 A b + 63 b C)) \sqrt{\cos [c + d x]}$$

$$\operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \sec [c + d x])}{a + b}} \sqrt{\frac{a (1 + \sec [c + d x])}{a - b}} -$$

$$\frac{4 b (2 A b^2 - a^2 (44 A + 63 C)) \sqrt{a + b \cos [c + d x]} \sec [c + d x]^{3/2} \sin [c + d x]}{315 a^2 d} +$$

$$\frac{2 (3 A b^2 + 7 a^2 (7 A + 9 C)) \sqrt{a + b \cos [c + d x]} \sec [c + d x]^{5/2} \sin [c + d x]}{315 a d} +$$

$$\frac{2 A b \sqrt{a + b \cos [c + d x]} \sec [c + d x]^{7/2} \sin [c + d x]}{21 d} + \frac{2 A (a + b \cos [c + d x])^{3/2} \sec [c + d x]^{9/2} \sin [c + d x]}{9 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1420: Attempted integration timed out after 120 seconds.**

$$\int (a + b \cos [c + d x])^{3/2} (A + C \cos [c + d x]^2) \sec [c + d x]^{9/2} dx$$

Optimal (type 4, 458 leaves, 7 steps):

$$\begin{aligned}
& - \frac{1}{105 a^3 d \sqrt{\sec[c+dx]}} 4 (a-b) b \sqrt{a+b} (3 A b^2 - a^2 (41 A + 70 C)) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{105 a^2 d \sqrt{\sec[c+dx]}} \\
& 2 (a-b) \sqrt{a+b} (25 a^2 A - 57 a A b - 6 A b^2 + 35 a^2 C - 105 a b C) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{2(3 A b^2 + 5 a^2 (5 A + 7 C)) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{105 a d} + \\
& \frac{6 A b \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{5/2} \sin[c+dx]}{35 d} + \frac{2 A (a+b \cos[c+dx])^{3/2} \sec[c+dx]^{7/2} \sin[c+dx]}{7 d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1422: Result more than twice size of optimal antiderivative.**

$$\int (a+b \cos[c+dx])^{3/2} (A+C \cos[c+dx])^2 \sec[c+dx]^{5/2} dx$$

Optimal (type 4, 560 leaves, 9 steps):

$$\begin{aligned}
& \frac{1}{3 a d \sqrt{\sec[c+dx]}} (a-b) b \sqrt{a+b} (8 A - 3 C) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{3 a d \sqrt{\sec[c+dx]}} \sqrt{a+b} (6 A b^2 + 2 a^2 (A+3 C) - a (8 A b - 3 b C)) \\
& \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\
& \frac{1}{d \sqrt{\sec[c+dx]}} 3 a \sqrt{a+b} C \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{2 A b \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]} \sin[c+dx]}{d} - \\
& \frac{b (8 A - 3 C) \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]} \sin[c+dx]}{3 d} + \frac{2 A (a+b \cos[c+dx])^{3/2} \sec[c+dx]^{3/2} \sin[c+dx]}{3 d}
\end{aligned}$$

Result (type 4, 3392 leaves) :

$$\begin{aligned}
& \frac{\sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]} \left(\frac{8}{3} A b \sin [c+d x]+\frac{2}{3} a A \tan [c+d x]\right)}{d}+ \\
& \left(\left(-\frac{4 a A b}{3 \sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]}}+\frac{2 a b C}{\sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]}}+\right.\right. \\
& \frac{a^2 A \sqrt{\sec [c+d x]}}{3 \sqrt{a+b \cos [c+d x]}}-\frac{A b^2 \sqrt{\sec [c+d x]}}{3 \sqrt{a+b \cos [c+d x]}}+\frac{a^2 C \sqrt{\sec [c+d x]}}{\sqrt{a+b \cos [c+d x]}}+\frac{b^2 C \sqrt{\sec [c+d x]}}{2 \sqrt{a+b \cos [c+d x]}}- \\
& \left.\frac{4 A b^2 \cos [2(c+d x)] \sqrt{\sec [c+d x]}}{3 \sqrt{a+b \cos [c+d x]}}+\frac{b^2 C \cos [2(c+d x)] \sqrt{\sec [c+d x]}}{2 \sqrt{a+b \cos [c+d x]}}\right) \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2 \sec [c+d x]} \\
& \left(-2 b(a+b)(8 A-3 C) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]+ \right. \\
& 4\left(3 A b^2+a^2(A+3 C)+a(4 A b-6 b C)\right) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]- \\
& 36 a b C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]- \\
& \left. b(8 A-3 C) \cos [c+d x](a+b \cos [c+d x]) \sec \left[\frac{1}{2}(c+d x)\right]^2 \tan \left[\frac{1}{2}(c+d x)\right]\right) \Bigg) / \\
& \left(3 d \sqrt{a+b \cos [c+d x]} \sqrt{\sec \left[\frac{1}{2}(c+d x)\right]^2} \left(\frac{1}{6(a+b \cos [c+d x])^{3 / 2} \sqrt{\sec \left[\frac{1}{2}(c+d x)\right]^2}} b \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2 \sec [c+d x]} \right.\right. \\
& \left.\left.\sin [c+d x]\left(-2 b(a+b)(8 A-3 C) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]+ \right.\right.\right. \\
& \left.\left.4\left(3 A b^2+a^2(A+3 C)+a(4 A b-6 b C)\right) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right], \right.\right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{-a+b}{a+b} \right] - 36 a b C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] - \\
& b(8 A-3 C) \cos [c+d x](a+b \cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \Bigg) - \\
& \frac{1}{6 \sqrt{a+b \cos [c+d x]} \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]} \\
& \left(-2 b(a+b)(8 A-3 C) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& 4\left(3 A b^2+a^2(A+3 C)+a(4 A b-6 b C)\right) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \right. \\
& \left. \frac{-a+b}{a+b}\right] - 36 a b C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] - \\
& b(8 A-3 C) \cos [c+d x](a+b \cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \Bigg) + \frac{1}{3 \sqrt{a+b \cos [c+d x]} \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}} \\
& \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]} \left(-\frac{1}{2} b(8 A-3 C) \cos [c+d x](a+b \cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 - \frac{1}{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}} b(a+b)\right. \\
& \left.(8 A-3 C) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2} - \frac{\sin [c+d x]}{1+\cos [c+d x]}\right) + \right. \\
& \left. \frac{1}{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}} 2\left(3 A b^2+a^2(A+3 C)+a(4 A b-6 b C)\right) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} - \frac{\sin[c+dx]}{1+\cos[c+dx]} \right) - \frac{1}{\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}}} 18abc \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[-1, \right. \\
& \left. -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \left( \frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} - \frac{\sin[c+dx]}{1+\cos[c+dx]} \right) - \frac{1}{\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} b(a+b)(8A-3C) \\
& \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \left( -\frac{b\sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(a+b\cos[c+dx])\sin[c+dx]}{(a+b)(1+\cos[c+dx])^2} \right) + \\
& \frac{1}{\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} 2(3Ab^2+a^2(A+3C)+a(4Ab-6bC)) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \left( -\frac{b\sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(a+b\cos[c+dx])\sin[c+dx]}{(a+b)(1+\cos[c+dx])^2} \right) - \frac{1}{\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} 18abc \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \\
& \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \left( -\frac{b\sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(a+b\cos[c+dx])\sin[c+dx]}{(a+b)(1+\cos[c+dx])^2} \right) + \\
& b^2(8A-3C)\cos[c+dx]\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx]\tan\left[\frac{1}{2}(c+dx)\right] + b(8A-3C)(a+b\cos[c+dx])\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sin[c+dx]\tan\left[\frac{1}{2}(c+dx)\right] - b(8A-3C)\cos[c+dx](a+b\cos[c+dx])\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^2 + \\
& \frac{2(3Ab^2+a^2(A+3C)+a(4Ab-6bC)) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{1-\frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} + \\
& \frac{18abc \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{1-\frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} - \left( b(a+b)(8A-3C) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left( \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\frac{(-a+b) \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right] / \left( \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \right) \right) + \\
& \left( \left( -2 b(a+b)(8 A-3 C) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right]\right], \frac{-a+b}{a+b} \right) + \right. \\
& 4\left(3 A b^2+a^2(A+3 C)+a(4 A b-6 b C)\right) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right]\right], \\
& \left. \frac{-a+b}{a+b} \right] - 36 a b C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right]\right], \frac{-a+b}{a+b} \right] - \\
& \left. b(8 A-3 C) \cos [c+d x](a+b \cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \tan \left[\frac{1}{2}(c+d x)\right] \right) \\
& \left( -\cos \left[\frac{1}{2}(c+d x)\right] \operatorname{Sec}[c+d x] \sin \left[\frac{1}{2}(c+d x)\right] + \cos \left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x] \tan [c+d x] \right) \Bigg) / \\
& \left( 6 \sqrt{a+b \cos [c+d x]} \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]} \right) \Bigg)
\end{aligned}$$

■ **Problem 1423: Result more than twice size of optimal antiderivative.**

$$\int (a+b \cos [c+d x])^{3/2} (A+C \cos [c+d x]^2) \operatorname{Sec}[c+d x]^{3/2} dx$$

Optimal (type 4, 569 leaves, 9 steps):



$$\frac{1}{4 d \sqrt{\sec [c+d x]}} (a-b) \sqrt{a+b} (8 A-5 C) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \frac{1}{4 d \sqrt{\sec [c+d x]}} \sqrt{a+b} (8 a A-16 A b-5 a C-2 b C) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \frac{1}{4 b d \sqrt{\sec [c+d x]}}$$

$$\sqrt{a+b} (8 A b^2+3 a^2 C+4 b^2 C) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} - \frac{b(4 A-C) \sqrt{a+b \cos [c+d x]} \sin [c+d x]}{2 d \sqrt{\sec [c+d x]}}$$

$$\frac{a(8 A-5 C) \sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]} \sin [c+d x]}{4 d} + \frac{2 A(a+b \cos [c+d x])^{3 / 2} \sqrt{\sec [c+d x]} \sin [c+d x]}{d}$$

Result (type 4, 1178 leaves):

$$\frac{\sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]} (2 a A \sin [c+d x]+\frac{1}{4} b C \sin [2(c+d x)])}{d} +$$

$$\left( 8 a^2 A \tan \left[ \frac{1}{2}(c+d x) \right] + 8 a A b \tan \left[ \frac{1}{2}(c+d x) \right] - 5 a^2 C \tan \left[ \frac{1}{2}(c+d x) \right] - 5 a b C \tan \left[ \frac{1}{2}(c+d x) \right] - 16 a A b \tan \left[ \frac{1}{2}(c+d x) \right]^3 + \right.$$

$$10 a b C \tan \left[ \frac{1}{2}(c+d x) \right]^3 - 8 a^2 A \tan \left[ \frac{1}{2}(c+d x) \right]^5 + 8 a A b \tan \left[ \frac{1}{2}(c+d x) \right]^5 + 5 a^2 C \tan \left[ \frac{1}{2}(c+d x) \right]^5 - 5 a b C \tan \left[ \frac{1}{2}(c+d x) \right]^5 +$$

$$16 A b^2 \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan \left[ \frac{1}{2}(c+d x) \right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\tan \left[ \frac{1}{2}(c+d x) \right]^2} \sqrt{\frac{a+b+a \tan \left[ \frac{1}{2}(c+d x) \right]^2-b \tan \left[ \frac{1}{2}(c+d x) \right]^2}{a+b}} +$$

$$6 a^2 C \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan \left[ \frac{1}{2}(c+d x) \right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\tan \left[ \frac{1}{2}(c+d x) \right]^2} \sqrt{\frac{a+b+a \tan \left[ \frac{1}{2}(c+d x) \right]^2-b \tan \left[ \frac{1}{2}(c+d x) \right]^2}{a+b}} +$$

$$8 b^2 C \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan \left[ \frac{1}{2}(c+d x) \right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\tan \left[ \frac{1}{2}(c+d x) \right]^2} \sqrt{\frac{a+b+a \tan \left[ \frac{1}{2}(c+d x) \right]^2-b \tan \left[ \frac{1}{2}(c+d x) \right]^2}{a+b}} +$$

$$\begin{aligned}
& 16 A b^2 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 6 a^2 C \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 8 b^2 C \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + a(a+b)(8A-5C) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2(4a^2(A-C) - 2b^2(2A+C) + ab(8A+C)) \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \Big/ \\
& \left(4d \sqrt{\frac{1}{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}\right)
\end{aligned}$$

■ **Problem 1424: Result more than twice size of optimal antiderivative.**

$$\int (a+b \cos[c+dx])^{3/2} (A+C \cos[c+dx]^2) \sqrt{\sec[c+dx]} dx$$

Optimal (type 4, 613 leaves, 9 steps):

$$\begin{aligned}
& - \frac{1}{24 a b d \sqrt{\operatorname{Sec}[c+d x]}} (a-b) \sqrt{a+b} \left(3 a^2 C+8 b^2(3 A+2 C)\right) \sqrt{\operatorname{Cos}[c+d x]} \\
& \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}}+ \\
& \frac{1}{24 b d \sqrt{\operatorname{Sec}[c+d x]}} \sqrt{a+b} \left(48 a A b+24 A b^2+3 a^2 C+14 a b C+16 b^2 C\right) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}}-\frac{1}{8 b^2 d \sqrt{\operatorname{Sec}[c+d x]}} \\
& a \sqrt{a+b} \left(24 A b^2-a^2 C+12 b^2 C\right) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}}+\frac{a C \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{4 d \sqrt{\operatorname{Sec}[c+d x]}}+ \\
& \frac{C(a+b \operatorname{Cos}[c+d x])^{3 / 2} \operatorname{Sin}[c+d x]}{3 d \sqrt{\operatorname{Sec}[c+d x]}}+\frac{\left(3 a^2 C+8 b^2(3 A+2 C)\right) \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{24 b d}
\end{aligned}$$

Result (type 4, 1285 leaves):

$$\frac{\sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \left(\frac{1}{12} b C \operatorname{Sin}[c+d x]+\frac{7}{24} a C \operatorname{Sin}[2(c+d x)]+\frac{1}{12} b C \operatorname{Sin}[3(c+d x)]\right)}{d}+$$

$$\frac{1}{24 b d \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}$$

$$\begin{aligned}
& \left(24 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+24 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+3 a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+3 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+16 a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+ \right. \\
& 16 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]-48 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3-6 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3-32 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3-24 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+ \\
& \left. 24 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5-3 a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+3 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5-16 a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+16 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5\right)-
\end{aligned}$$

$$\begin{aligned}
& 144 a A b^2 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 6 a^3 C \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 72 a b^2 C \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 144 a A b^2 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 6 a^3 C \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 72 a b^2 C \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (24 A b^2 + 3 a^2 C + 16 b^2 C) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2 a b (24 a A - 48 A b + 7 a C - 26 b C) \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}
\end{aligned}$$

■ **Problem 1425: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \cos[c+dx])^{3/2} (A+C \cos[c+dx]^2)}{\sqrt{\sec[c+dx]}} dx$$

Optimal (type 4, 698 leaves, 10 steps):

$$\begin{aligned}
& - \frac{1}{64 b^2 d \sqrt{\text{Sec}[c + d x]}} (a - b) \sqrt{a + b} (80 A b^2 - 3 a^2 C + 52 b^2 C) \sqrt{\text{Cos}[c + d x]} \\
& \quad \text{Csc}[c + d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Cos}[c + d x]}}{\sqrt{a + b} \sqrt{\text{Cos}[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \text{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \text{Sec}[c + d x])}{a - b}} - \\
& \frac{1}{64 b^2 d \sqrt{\text{Sec}[c + d x]}} \sqrt{a + b} (3 a^3 C - 2 a^2 b C - 8 b^3 (4 A + 3 C) - 4 a b^2 (20 A + 13 C)) \sqrt{\text{Cos}[c + d x]} \text{Csc}[c + d x] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Cos}[c + d x]}}{\sqrt{a + b} \sqrt{\text{Cos}[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \text{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \text{Sec}[c + d x])}{a - b}} - \\
& \frac{1}{64 b^3 d \sqrt{\text{Sec}[c + d x]}} \sqrt{a + b} (3 a^4 C + 24 a^2 b^2 (2 A + C) + 16 b^4 (4 A + 3 C)) \sqrt{\text{Cos}[c + d x]} \text{Csc}[c + d x] \\
& \quad \text{EllipticPi}\left[\frac{a + b}{b}, \text{ArcSin}\left[\frac{\sqrt{a + b \text{Cos}[c + d x]}}{\sqrt{a + b} \sqrt{\text{Cos}[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \text{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \text{Sec}[c + d x])}{a - b}} - \\
& \frac{(3 a^2 C - 4 b^2 (4 A + 3 C)) \sqrt{a + b \text{Cos}[c + d x]} \text{Sin}[c + d x]}{32 b d \sqrt{\text{Sec}[c + d x]}} - \frac{a C (a + b \text{Cos}[c + d x])^{3/2} \text{Sin}[c + d x]}{8 b d \sqrt{\text{Sec}[c + d x]}} + \\
& \frac{C (a + b \text{Cos}[c + d x])^{5/2} \text{Sin}[c + d x]}{4 b d \sqrt{\text{Sec}[c + d x]}} + \frac{a (80 A b^2 - 3 a^2 C + 52 b^2 C) \sqrt{a + b \text{Cos}[c + d x]} \sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{64 b^2 d}
\end{aligned}$$

Result (type 4, 4000 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{a + b \text{Cos}[c + d x]} \sqrt{\text{Sec}[c + d x]} \\
& \left( \frac{3}{32} a C \text{Sin}[c + d x] + \frac{(16 A b^2 + a^2 C + 16 b^2 C) \text{Sin}[2(c + d x)]}{64 b} + \frac{3}{32} a C \text{Sin}[3(c + d x)] + \frac{1}{32} b C \text{Sin}[4(c + d x)] \right) + \\
& \left( \frac{a^2 A}{\sqrt{a + b \text{Cos}[c + d x]} \sqrt{\text{Sec}[c + d x]}} + \frac{A b^2}{2 \sqrt{a + b \text{Cos}[c + d x]} \sqrt{\text{Sec}[c + d x]}} + \frac{19 a^2 C}{32 \sqrt{a + b \text{Cos}[c + d x]} \sqrt{\text{Sec}[c + d x]}} + \right. \\
& \left. \frac{3 b^2 C}{8 \sqrt{a + b \text{Cos}[c + d x]} \sqrt{\text{Sec}[c + d x]}} + \frac{7 a A b \sqrt{\text{Sec}[c + d x]}}{8 \sqrt{a + b \text{Cos}[c + d x]}} - \frac{a^3 C \sqrt{\text{Sec}[c + d x]}}{128 b \sqrt{a + b \text{Cos}[c + d x]}} + \frac{19 a b C \sqrt{\text{Sec}[c + d x]}}{32 \sqrt{a + b \text{Cos}[c + d x]}} + \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{5 a A b \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{8 \sqrt{a+b \operatorname{Cos}[c+d x]}} - \frac{3 a^3 C \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{128 b \sqrt{a+b \operatorname{Cos}[c+d x]}} + \frac{13 a b C \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{32 \sqrt{a+b \operatorname{Cos}[c+d x]}} \right) \\
& \left( \frac{a(-80 A b^2 + 3 a^2 C - 52 b^2 C) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}}{64 b^2 \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}} \right) + \\
& \left( a(a+b)(-80 A b^2 + 3 a^2 C - 52 b^2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] + 2 b(-a^3 C - 4 a b^2(4 A + 3 C) + 8 b^3(4 A + 3 C) + \right. \\
& \quad \left. a^2(64 A b + 38 b C)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] + 2(3 a^4 C + 24 a^2 b^2(2 A + C) + 16 b^4(4 A + 3 C)) \\
& \quad \left. \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^4} \right) / \\
& \left( \left( 64 b^2(a+b) \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)^2 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) \right) / \\
& \left( \left( \frac{a(-80 A b^2 + 3 a^2 C - 52 b^2 C) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}}{128 b^2 \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}} \right) \right) - \\
& \left( a(a+b)(-80 A b^2 + 3 a^2 C - 52 b^2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& \quad \left. 2 b(-a^3 C - 4 a b^2(4 A + 3 C) + 8 b^3(4 A + 3 C) + a^2(64 A b + 38 b C)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left( 3 a^4 C + 24 a^2 b^2 (2 A + C) + 16 b^4 (4 A + 3 C) \right) \text{EllipticPi} \left[ -1, -\text{ArcSin} \left[ \text{Tan} \left[ \frac{1}{2} (c + d x) \right] \right], \frac{-a + b}{a + b} \right] \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \\
& \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^3 \sqrt{\frac{a + b + a \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 - b \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2}{1 + \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2}} \Big/ \left( 64 b^2 (a + b) \sqrt{\frac{1}{1 - \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2}} \right. \\
& \left. \left( -1 + \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right) \sqrt{\frac{a + b + a \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 - b \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2}{a + b}} \sqrt{1 - \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^4} \right) - \\
& \left( a (a + b) (-80 A b^2 + 3 a^2 C - 52 b^2 C) \text{EllipticE} \left[ \text{ArcSin} \left[ \text{Tan} \left[ \frac{1}{2} (c + d x) \right] \right], \frac{-a + b}{a + b} \right] + \right. \\
& 2 b (-a^3 C - 4 a b^2 (4 A + 3 C) + 8 b^3 (4 A + 3 C) + a^2 (64 A b + 38 b C)) \text{EllipticF} \left[ \text{ArcSin} \left[ \text{Tan} \left[ \frac{1}{2} (c + d x) \right] \right], \frac{-a + b}{a + b} \right] + \\
& \left. 2 \left( 3 a^4 C + 24 a^2 b^2 (2 A + C) + 16 b^4 (4 A + 3 C) \right) \text{EllipticPi} \left[ -1, -\text{ArcSin} \left[ \text{Tan} \left[ \frac{1}{2} (c + d x) \right] \right], \frac{-a + b}{a + b} \right] \right) \\
& \left( a \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \text{Tan} \left[ \frac{1}{2} (c + d x) \right] - b \text{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \text{Tan} \left[ \frac{1}{2} (c + d x) \right] \right) \\
& \sqrt{\frac{a + b + a \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 - b \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2}{1 + \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2}} \sqrt{1 - \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^4} \Big/ \\
& \left( 128 b^2 (a + b)^2 \sqrt{\frac{1}{1 - \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2}} \left( -1 + \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 \right) \left( \frac{a + b + a \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2 - b \text{Tan} \left[ \frac{1}{2} (c + d x) \right]^2}{a + b} \right)^{3/2} \right) - \\
& \left( a (a + b) (-80 A b^2 + 3 a^2 C - 52 b^2 C) \text{EllipticE} \left[ \text{ArcSin} \left[ \text{Tan} \left[ \frac{1}{2} (c + d x) \right] \right], \frac{-a + b}{a + b} \right] + \right. \\
& 2 b (-a^3 C - 4 a b^2 (4 A + 3 C) + 8 b^3 (4 A + 3 C) + a^2 (64 A b + 38 b C)) \text{EllipticF} \left[ \text{ArcSin} \left[ \text{Tan} \left[ \frac{1}{2} (c + d x) \right] \right], \frac{-a + b}{a + b} \right] + \\
& \left. 2 \left( 3 a^4 C + 24 a^2 b^2 (2 A + C) + 16 b^4 (4 A + 3 C) \right) \text{EllipticPi} \left[ -1, -\text{ArcSin} \left[ \text{Tan} \left[ \frac{1}{2} (c + d x) \right] \right], \frac{-a + b}{a + b} \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2-b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^4}\right] / \\
& \left( 64 b^2 (a+b) \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2-b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) - \\
& \left( a (a+b) (-80 A b^2+3 a^2 C-52 b^2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& 2 b\left(-a^3 C-4 a b^2(4 A+3 C)+8 b^3(4 A+3 C)+a^2(64 A b+38 b C)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \\
& \left. 2\left(3 a^4 C+24 a^2 b^2(2 A+C)+16 b^4(4 A+3 C)\right) \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \\
& \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2-b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^4}\right] / \\
& \left( 128 b^2 (a+b) \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2-b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) + \\
& \left( a (-80 A b^2+3 a^2 C-52 b^2 C) \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2-b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \left( \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} + \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1-\tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \right) / \left( 128 b^2 \left( \frac{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3/2} \right) - \\
& \left( a (-80 A b^2+3 a^2 C-52 b^2 C) \tan\left[\frac{1}{2}(c+dx)\right] \left( \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]-b \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2} - \right. \right. \\
& \left. \left. \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left(a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2-b \tan\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \right) /
\end{aligned}$$



$$\begin{aligned}
& \left( 128 b^2 \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \\
& \left( a(a+b)(-80Ab^2 + 3a^2C - 52b^2C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& \quad 2b(-a^3C - 4ab^2(4A+3C) + 8b^3(4A+3C) + a^2(64Ab+38bC)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \\
& \quad \left. 2(3a^4C + 24a^2b^2(2A+C) + 16b^4(4A+3C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^4} \left( \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} - \right. \\
& \quad \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] (a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2)}{(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2)^2} \right) \Bigg) / \\
& \left( 128 b^2 (a+b) \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right. \\
& \quad \left. \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \left( \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \quad \left. \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^4} \left( \frac{b(-a^3C - 4ab^2(4A+3C) + 8b^3(4A+3C) + a^2(64Ab+38bC)) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{1 - \frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} - \right. \right. \\
& \quad \left. \left. \frac{(3a^4C + 24a^2b^2(2A+C) + 16b^4(4A+3C)) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} (1 + \tan\left[\frac{1}{2}(c+dx)\right]^2)} \sqrt{1 - \frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) +
\end{aligned}$$

$$\left. \left. \frac{a(a+b) \left( -80Ab^2 + 3a^2C - 52b^2C \right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \frac{(-a+b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}{2\sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) \right) \left/ \right.$$

$$\left( 64b^2(a+b) \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left( -1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right)^2 \sqrt{\frac{a+b + a\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right)$$

■ **Problem 1426: Attempted integration timed out after 120 seconds.**

$$\int (a + b \cos[c + dx])^{5/2} (A + C \cos[c + dx]^2) \operatorname{Sec}[c + dx]^{13/2} dx$$

Optimal (type 4, 627 leaves, 9 steps):

$$\frac{1}{693 a^4 d \sqrt{\operatorname{Sec}[c + dx]}} 2 (a - b) b \sqrt{a + b} (8 A b^4 + 3 a^2 b^2 (17 A + 33 C) + a^4 (741 A + 957 C)) \sqrt{\operatorname{Cos}[c + dx]}$$

$$\operatorname{Csc}[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Cos}[c + dx]}}{\sqrt{a + b} \sqrt{\operatorname{Cos}[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \operatorname{Sec}[c + dx])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + dx])}{a - b}} +$$

$$\frac{1}{693 a^3 d \sqrt{\operatorname{Sec}[c + dx]}} 2 (a - b) \sqrt{a + b} (6 a A b^3 + 8 A b^4 + 15 a^4 (9 A + 11 C) + 3 a^2 b^2 (19 A + 33 C) - 6 a^3 b (101 A + 132 C))$$

$$\sqrt{\operatorname{Cos}[c + dx]} \operatorname{Csc}[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Cos}[c + dx]}}{\sqrt{a + b} \sqrt{\operatorname{Cos}[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \operatorname{Sec}[c + dx])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + dx])}{a - b}} -$$

$$\frac{2 (4 A b^4 - 15 a^4 (9 A + 11 C) - a^2 b^2 (205 A + 297 C)) \sqrt{a + b \operatorname{Cos}[c + dx]} \operatorname{Sec}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{693 a^2 d} +$$

$$\frac{2 b (3 A b^2 + a^2 (229 A + 297 C)) \sqrt{a + b \operatorname{Cos}[c + dx]} \operatorname{Sec}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{693 a d} +$$

$$\frac{2 (5 A b^2 + 3 a^2 (9 A + 11 C)) \sqrt{a + b \operatorname{Cos}[c + dx]} \operatorname{Sec}[c + dx]^{7/2} \operatorname{Sin}[c + dx]}{231 d} +$$

$$\frac{10 A b (a + b \operatorname{Cos}[c + dx])^{3/2} \operatorname{Sec}[c + dx]^{9/2} \operatorname{Sin}[c + dx]}{99 d} + \frac{2 A (a + b \operatorname{Cos}[c + dx])^{5/2} \operatorname{Sec}[c + dx]^{11/2} \operatorname{Sin}[c + dx]}{11 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1427: Attempted integration timed out after 120 seconds.**

$$\int (a + b \cos [c + d x])^{5/2} (A + C \cos [c + d x]^2) \sec [c + d x]^{11/2} dx$$

Optimal (type 4, 544 leaves, 8 steps):

$$\begin{aligned} & - \frac{1}{315 a^3 d \sqrt{\sec [c + d x]}} 2 (a - b) \sqrt{a + b} (10 A b^4 - 21 a^4 (7 A + 9 C) - 3 a^2 b^2 (93 A + 161 C)) \sqrt{\cos [c + d x]} \\ & \quad \text{Csc}[c + d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \sec [c + d x])}{a + b}} \sqrt{\frac{a (1 + \sec [c + d x])}{a - b}} - \\ & \frac{1}{315 a^2 d \sqrt{\sec [c + d x]}} 2 (a - b) \sqrt{a + b} (10 A b^3 + 21 a^3 (7 A + 9 C) + 15 a b^2 (11 A + 21 C) - 6 a^2 b (19 A + 28 C)) \sqrt{\cos [c + d x]} \\ & \quad \text{Csc}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \sec [c + d x])}{a + b}} \sqrt{\frac{a (1 + \sec [c + d x])}{a - b}} + \\ & \frac{2 b (5 A b^2 + a^2 (163 A + 231 C)) \sqrt{a + b \cos [c + d x]} \sec [c + d x]^{3/2} \sin [c + d x]}{315 a d} + \\ & \frac{2 (15 A b^2 + 7 a^2 (7 A + 9 C)) \sqrt{a + b \cos [c + d x]} \sec [c + d x]^{5/2} \sin [c + d x]}{315 d} + \\ & \frac{10 A b (a + b \cos [c + d x])^{3/2} \sec [c + d x]^{7/2} \sin [c + d x]}{63 d} + \frac{2 A (a + b \cos [c + d x])^{5/2} \sec [c + d x]^{9/2} \sin [c + d x]}{9 d} \end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1431: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos [c + d x])^{5/2} (A + C \cos [c + d x]^2) \sec [c + d x]^{3/2} dx$$

Optimal (type 4, 669 leaves, 10 steps):

$$\begin{aligned}
& \frac{1}{24 a d \sqrt{\operatorname{Sec}[c+d x]}} (a-b) \sqrt{a+b} \left( a^2 (48 A-33 C)-8 b^2 (3 A+2 C) \right) \sqrt{\operatorname{Cos}[c+d x]} \\
& \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \\
& \frac{1}{24 d \sqrt{\operatorname{Sec}[c+d x]}} \sqrt{a+b} \left( a^2 (48 A-33 C)-8 b^2 (3 A+2 C)-2 a b (72 A+13 C) \right) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{8 b d \sqrt{\operatorname{Sec}[c+d x]}} \\
& 5 a \sqrt{a+b} (8 A b^2+(a^2+4 b^2) C) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{a b (8 A-3 C) \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{4 d \sqrt{\operatorname{Sec}[c+d x]}} - \frac{b (6 A-C) (a+b \operatorname{Cos}[c+d x])^{3 / 2} \operatorname{Sin}[c+d x]}{3 d \sqrt{\operatorname{Sec}[c+d x]}} - \\
& \frac{\left( a^2 (48 A-33 C)-8 b^2 (3 A+2 C) \right) \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{24 d} + \frac{2 A (a+b \operatorname{Cos}[c+d x])^{5 / 2} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{d}
\end{aligned}$$

Result (type 4, 1405 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \left( \frac{1}{12} (24 a^2 A+b^2 C) \operatorname{Sin}[c+d x] + \frac{13}{24} a b C \operatorname{Sin}[2(c+d x)] + \frac{1}{12} b^2 C \operatorname{Sin}[3(c+d x)] \right) + \\
& \frac{1}{24 d \left( 1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right)^{3 / 2}} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \left( -48 a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] -48 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] +24 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] +24 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] +33 a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + \right. \\
& 33 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] +16 a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] +16 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] +96 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 -48 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - \\
& 66 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 -32 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 +48 a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 -48 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 -24 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + \\
& \left. 24 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 -33 a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 +33 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 -16 a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 +16 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \right)
\end{aligned}$$

$$\begin{aligned}
& 240 a A b^2 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 30 a^3 C \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 120 a b^2 C \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 240 a A b^2 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 30 a^3 C \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 120 a b^2 C \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - (a+b) (a^2 (48 A - 33 C) - 8 b^2 (3 A + 2 C)) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 2 a (24 a^2 (A - C) + a b (72 A + 13 C) - 2 b^2 (36 A + 19 C)) \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}
\end{aligned}$$

■ **Problem 1433:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \cos[c + dx])^{5/2} (A + C \cos[c + dx]^2)}{\sqrt{\sec[c + dx]}} dx$$

Optimal (type 4, 806 leaves, 11 steps):

$$\frac{1}{1920 a b^2 d \sqrt{\sec[c + dx]}} (a - b) \sqrt{a + b} (45 a^4 C - 256 b^4 (5 A + 4 C) - 12 a^2 b^2 (220 A + 141 C)) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \frac{1}{1920 b^2 d \sqrt{\sec[c + dx]}}$$

$$\sqrt{a + b} (45 a^4 C - 30 a^3 b C - 256 b^4 (5 A + 4 C) - 12 a^2 b^2 (220 A + 141 C) - 8 a b^3 (260 A + 193 C)) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \frac{1}{128 b^3 d \sqrt{\sec[c + dx]}}$$

$$a \sqrt{a + b} (3 a^4 C + 40 a^2 b^2 (2 A + C) + 80 b^4 (4 A + 3 C)) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \operatorname{EllipticPi}\left[\frac{a + b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right]$$

$$\sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \frac{a(240 A b^2 - 15 a^2 C + 172 b^2 C) \sqrt{a + b \cos[c + dx]} \sin[c + dx]}{320 b d \sqrt{\sec[c + dx]}}$$

$$\frac{(15 a^2 C - 16 b^2 (5 A + 4 C)) (a + b \cos[c + dx])^{3/2} \sin[c + dx]}{240 b d \sqrt{\sec[c + dx]}} - \frac{3 a C (a + b \cos[c + dx])^{5/2} \sin[c + dx]}{40 b d \sqrt{\sec[c + dx]}} +$$

$$\frac{C (a + b \cos[c + dx])^{7/2} \sin[c + dx]}{5 b d \sqrt{\sec[c + dx]}} - \frac{(45 a^4 C - 256 b^4 (5 A + 4 C) - 12 a^2 b^2 (220 A + 141 C)) \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} \sin[c + dx]}{1920 b^2 d}$$

Result (type 4, 2064 leaves):

$$\frac{1}{d} \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} \left( \frac{1}{960} (80 A b^2 + 93 a^2 C + 88 b^2 C) \sin[c + dx] + \frac{a(1040 A b^2 + 15 a^2 C + 1024 b^2 C) \sin[2(c + dx)]}{1920 b} + \right.$$

$$\left. \frac{1}{960} (80 A b^2 + 93 a^2 C + 100 b^2 C) \sin[3(c + dx)] + \frac{21}{320} a b C \sin[4(c + dx)] + \frac{1}{80} b^2 C \sin[5(c + dx)] \right) +$$

$$\left( \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}} \right.$$

$$\left. \left( -2640 a^3 A b^2 \tan\left[\frac{1}{2}(c + dx)\right] - 2640 a^2 A b^3 \tan\left[\frac{1}{2}(c + dx)\right] - 1280 a A b^4 \tan\left[\frac{1}{2}(c + dx)\right] - 1280 A b^5 \tan\left[\frac{1}{2}(c + dx)\right] + \right.$$

$$\begin{aligned}
& 45 a^5 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 45 a^4 b C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 1692 a^3 b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 1692 a^2 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \\
& 1024 a b^4 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 1024 b^5 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 5280 a^2 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 2560 A b^5 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 - \\
& 90 a^4 b C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 3384 a^2 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 2048 b^5 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 2640 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - \\
& 2640 a^2 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 1280 a A b^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 1280 A b^5 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 45 a^5 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + \\
& 45 a^4 b C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 1692 a^3 b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 1692 a^2 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 1024 a b^4 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - \\
& 1024 b^5 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 2400 a^3 A b^2 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 9600 a A b^4 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 90 a^5 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 1200 a^3 b^2 C \\
& \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 7200 \\
& a b^4 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 2400 a^3 A b^2 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 9600 a A b^4 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2
\end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 90a^5 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 1200a^3 b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 7200a b^4 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (45a^4 C - 256b^4 (5A+4C) - 12a^2 b^2 (220A+141C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2ab (15a^3 C - 6a^2 b (320A+191C) + 4ab^2 (260A+193C) - 8b^3 (380A+289C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \left. \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) / \\
& \left( 1920b^2 d \sqrt{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \left( b \left( -1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) - a \left( 1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right)
\end{aligned}$$

■ **Problem 1434: Unable to integrate problem.**

$$\int \frac{(A + C \cos[c+dx])^2 \operatorname{Sec}[c+dx]^{9/2}}{\sqrt{a+b \cos[c+dx]}} dx$$

Optimal (type 4, 469 leaves, 7 steps):



$$\begin{aligned}
& - \frac{1}{105 a^5 d \sqrt{\sec[c+dx]}} 4 (a-b) b \sqrt{a+b} (24 A b^2 + a^2 (22 A + 35 C)) \sqrt{\cos[c+dx]} \\
& \quad \text{Csc}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\
& \frac{1}{105 a^4 d \sqrt{\sec[c+dx]}} 2 \sqrt{a+b} (12 A A b^2 - 48 A b^3 - 5 a^3 (5 A + 7 C) - a^2 (44 A b + 70 b C)) \sqrt{\cos[c+dx]} \text{Csc}[c+dx] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{2 (24 A b^2 + 5 a^2 (5 A + 7 C)) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{105 a^3 d} - \\
& \frac{12 A b \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{5/2} \sin[c+dx]}{35 a^2 d} + \frac{2 A \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{7/2} \sin[c+dx]}{7 a d}
\end{aligned}$$

Result (type 8, 39 leaves):

$$\int \frac{(A + C \cos[c+dx])^2 \sec[c+dx]^{9/2}}{\sqrt{a+b \cos[c+dx]}} dx$$

■ **Problem 1435: Unable to integrate problem.**

$$\int \frac{(A + C \cos[c+dx])^2 \sec[c+dx]^{7/2}}{\sqrt{a+b \cos[c+dx]}} dx$$

Optimal (type 4, 394 leaves, 6 steps):

$$\begin{aligned}
& \frac{1}{15 a^4 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (8 A b^2 + 3 a^2 (3 A + 5 C)) \sqrt{\cos[c+dx]} \text{Csc}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \\
& \quad \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{15 a^3 d \sqrt{\sec[c+dx]}} 2 \sqrt{a+b} (2 a A b - 8 A b^2 - 3 a^2 (3 A + 5 C)) \sqrt{\cos[c+dx]} \\
& \quad \text{Csc}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\
& \frac{8 A b \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{15 a^2 d} + \frac{2 A \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{5/2} \sin[c+dx]}{5 a d}
\end{aligned}$$

Result (type 8, 39 leaves):

$$\int \frac{(A + C \cos[c + dx])^2 \sec[c + dx]^{7/2}}{\sqrt{a + b \cos[c + dx]}} dx$$

■ **Problem 1436: Unable to integrate problem.**

$$\int \frac{(A + C \cos[c + dx])^2 \sec[c + dx]^{5/2}}{\sqrt{a + b \cos[c + dx]}} dx$$

Optimal (type 4, 323 leaves, 5 steps):

$$-\frac{1}{3 a^3 d \sqrt{\sec[c + dx]}} 4 A (a - b) b \sqrt{a + b} \sqrt{\cos[c + dx]} \csc[c + dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \frac{1}{3 a^2 d \sqrt{\sec[c + dx]}}$$

$$2 \sqrt{a + b} (2 A b + a (A + 3 C)) \sqrt{\cos[c + dx]} \csc[c + dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right]$$

$$\sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \frac{2 A \sqrt{a + b \cos[c + dx]} \sec[c + dx]^{3/2} \sin[c + dx]}{3 a d}$$

Result (type 8, 39 leaves):

$$\int \frac{(A + C \cos[c + dx])^2 \sec[c + dx]^{5/2}}{\sqrt{a + b \cos[c + dx]}} dx$$

■ **Problem 1439: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c + dx]^2}{\sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]}} dx$$

Optimal (type 4, 515 leaves, 8 steps):

$$\frac{1}{4 b^2 d \sqrt{\operatorname{Sec}[c+d x]}}$$

$$3(a-b) \sqrt{a+b} C \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} -$$

$$\frac{1}{4 b^2 d \sqrt{\operatorname{Sec}[c+d x]}} (3 a-2 b) \sqrt{a+b} C \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{4 b^3 d \sqrt{\operatorname{Sec}[c+d x]}} \sqrt{a+b} (3 a^2 C+4 b^2(2 A+C)) \sqrt{\operatorname{Cos}[c+d x]}$$

$$\operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{C \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{2 b d \sqrt{\operatorname{Sec}[c+d x]}} - \frac{3 a C \sqrt{a+b} \operatorname{Cos}[c+d x] \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 b^2 d}$$

Result (type 4, 1399 leaves):

$$\frac{C \sqrt{a+b} \operatorname{Cos}[c+d x] \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[2(c+d x)]}{4 b d} +$$

$$\left(3 a^2 \sqrt{\frac{a-b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 3 a b \sqrt{\frac{a-b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 6 a b \sqrt{\frac{a-b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - 3 a^2 \sqrt{\frac{a-b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 +\right.$$

$$3 a b \sqrt{\frac{a-b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 16 i A b^2 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}$$

$$\sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 6 i a^2 C \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} +$$

$$8 i b^2 C \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}$$

$$\begin{aligned}
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+16 i A b^2 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+ \\
& 6 i a^2 C \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+8 i b^2 C \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+ \\
& 3 i a(a-b) C \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}- \\
& 2 i\left(4 A b^2+3 a^2 C-a b C+2 b^2 C\right) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a-b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}\right) / \\
& \left(4 b^2 \sqrt{\frac{a-b}{a+b}} d \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^4\right)\right)
\end{aligned}$$

■ **Problem 1440: Attempted integration timed out after 120 seconds.**

$$\int \frac{(A+C \operatorname{Cos}[c+d x])^2 \operatorname{Sec}[c+d x]^{7/2}}{(a+b \operatorname{Cos}[c+d x])^{3/2}} dx$$

Optimal (type 4, 534 leaves, 7 steps) :

$$\begin{aligned}
 & - \left( 2 (16 A b^4 - 2 a^2 b^2 (4 A - 5 C) - a^4 (3 A + 5 C)) \sqrt{\cos [c + d x]} \operatorname{Csc}[c + d x] \right. \\
 & \quad \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 5 a^5 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]} \right) - \\
 & \left( 2 (12 a A b^2 + 16 A b^3 + 2 a^2 b (2 A + 5 C) + a^3 (3 A + 5 C)) \sqrt{\cos [c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
 & \quad \left. \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 5 a^4 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]} \right) + \\
 & \frac{2 b (8 A b^2 - a^2 (3 A - 5 C)) \sqrt{a + b \cos [c + d x]} \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{5 a^3 (a^2 - b^2) d} + \frac{2 (A b^2 + a^2 C) \operatorname{Sec}[c + d x]^{5/2} \operatorname{Sin}[c + d x]}{a (a^2 - b^2) d \sqrt{a + b \cos [c + d x]}} - \\
 & \frac{2 (6 A b^2 - a^2 (A - 5 C)) \sqrt{a + b \cos [c + d x]} \operatorname{Sec}[c + d x]^{5/2} \operatorname{Sin}[c + d x]}{5 a^2 (a^2 - b^2) d}
 \end{aligned}$$

Result (type 1, 1 leaves) :

???

■ **Problem 1441: Unable to integrate problem.**

$$\int \frac{(A + C \cos [c + d x]^2) \operatorname{Sec}[c + d x]^{5/2}}{(a + b \cos [c + d x])^{3/2}} dx$$

Optimal (type 4, 432 leaves, 6 steps) :

$$\begin{aligned}
& \left( 2 b (8 A b^2 - a^2 (5 A - 3 C)) \sqrt{\cos [c + d x]} \operatorname{Csc}[c + d x] \right. \\
& \quad \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a^4 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]} \right) + \\
& \left( 2 (6 a A b + 8 A b^2 + a^2 (A + 3 C)) \sqrt{\cos [c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a^3 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]} \right) + \\
& \frac{2 (A b^2 + a^2 C) \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{a (a^2 - b^2) d \sqrt{a + b \cos [c + d x]}} - \frac{2 (4 A b^2 - a^2 (A - 3 C)) \sqrt{a + b \cos [c + d x]} \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{3 a^2 (a^2 - b^2) d}
\end{aligned}$$

Result (type 8, 39 leaves):

$$\int \frac{(A + C \cos [c + d x])^2 \operatorname{Sec}[c + d x]^{5/2}}{(a + b \cos [c + d x])^{3/2}} dx$$

■ **Problem 1442: Unable to integrate problem.**

$$\int \frac{(A + C \cos [c + d x])^2 \operatorname{Sec}[c + d x]^{3/2}}{(a + b \cos [c + d x])^{3/2}} dx$$

Optimal (type 4, 348 leaves, 5 steps):

$$\begin{aligned}
& - \left( 2 (2 A b^2 - a^2 (A - C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \right. \\
& \quad \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( a^3 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]} \right) - \\
& \left( 2 (2 A b + a (A - C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \right. \\
& \quad \left. \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( a^2 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]} \right) + \frac{2 (A b^2 + a^2 C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{a (a^2 - b^2) d \sqrt{a + b \cos[c + d x]}}
\end{aligned}$$

Result (type 8, 39 leaves) :

$$\int \frac{(A + C \cos[c + d x])^2 \operatorname{Sec}[c + d x]^{3/2}}{(a + b \cos[c + d x])^{3/2}} dx$$

■ **Problem 1443: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + C \cos[c + d x])^2 \sqrt{\operatorname{Sec}[c + d x]}}{(a + b \cos[c + d x])^{3/2}} dx$$

Optimal (type 4, 481 leaves, 7 steps) :

$$\begin{aligned}
& \left( 2 (A b^2 + a^2 C) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} \right) / \\
& \left( a^2 b \sqrt{a+b} d \sqrt{\operatorname{Sec}[c+d x]} \right) + \\
& \left( 2 (A b - a C) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} \right) / \\
& \left( a b \sqrt{a+b} d \sqrt{\operatorname{Sec}[c+d x]} \right) - \frac{1}{b^2 d \sqrt{\operatorname{Sec}[c+d x]}} \\
& 2 \sqrt{a+b} C \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{2(A b^2 + a^2 C) \sqrt{\operatorname{Sec}[c+d x]} \sin [c+d x]}{b(a^2 - b^2) d \sqrt{a+b \cos [c+d x]}}
\end{aligned}$$

Result (type 4, 1027 leaves):

$$\begin{aligned}
& \frac{\sqrt{a+b \cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \left( \frac{2(A b^2 + a^2 C) \sin [c+d x]}{a b (a^2 - b^2)} + \frac{2(A b^2 \sin [c+d x] + a^2 C \sin [c+d x])}{b(-a^2 + b^2)(a+b \cos [c+d x])} \right)}{d} \\
& \left( 2 \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \left( a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \right. \right. \\
& 2 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - 2 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + \\
& a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 2 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 2 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \left. \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 2 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right)
\end{aligned}$$



$$\begin{aligned}
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2ab^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b)(Ab^2+a^2C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - ab(a+b)(A+C) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \left. \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) / \\
& \left( b(a^3 - ab^2) d \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right)
\end{aligned}$$

■ **Problem 1444: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c+dx]^2}{(a+b \cos[c+dx])^{3/2} \sqrt{\sec[c+dx]}} dx$$

Optimal (type 4, 563 leaves, 8 steps):

$$\begin{aligned}
& - \left( (2 A b^2 + 3 a^2 C - b^2 C) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / (a b^2 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]}) + \\
& \left( (2 A b^2 + a(3 a + b) C) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / (a b^2 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]}) + \frac{1}{b^3 d \sqrt{\operatorname{Sec}[c + d x]}} \\
& 3 a \sqrt{a + b} C \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticPi}\left[\frac{a + b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \\
& \quad \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} - \frac{2(A b^2 + a^2 C) \sin[c + d x]}{b(a^2 - b^2) d \sqrt{a + b \cos[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}} + \\
& \quad \frac{(2 A b^2 + 3 a^2 C - b^2 C) \sqrt{a + b \cos[c + d x]} \sqrt{\operatorname{Sec}[c + d x]} \sin[c + d x]}{b^2(a^2 - b^2) d}
\end{aligned}$$

Result (type 4, 1163 leaves):

$$\begin{aligned}
& \frac{\sqrt{a + b \cos[c + d x]} \sqrt{\operatorname{Sec}[c + d x]} \left( \frac{2(A b^2 + a^2 C) \sin[c + d x]}{b^2(a^2 - b^2)} - \frac{2(a A b^2 \sin[c + d x] + a^3 C \sin[c + d x])}{b^2(a^2 - b^2)(a + b \cos[c + d x])} \right)}{d} + \\
& \left( \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + d x)\right]^2}} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + d x)\right]^2 - b \tan\left[\frac{1}{2}(c + d x)\right]^2}{1 + \tan\left[\frac{1}{2}(c + d x)\right]^2}} \right. \\
& \quad \left( 2 a A b^2 \tan\left[\frac{1}{2}(c + d x)\right] + 2 A b^3 \tan\left[\frac{1}{2}(c + d x)\right] + 3 a^3 C \tan\left[\frac{1}{2}(c + d x)\right] + 3 a^2 b C \tan\left[\frac{1}{2}(c + d x)\right] - a b^2 C \tan\left[\frac{1}{2}(c + d x)\right] - \right. \\
& \quad b^3 C \tan\left[\frac{1}{2}(c + d x)\right] - 4 A b^3 \tan\left[\frac{1}{2}(c + d x)\right]^3 - 6 a^2 b C \tan\left[\frac{1}{2}(c + d x)\right]^3 + 2 b^3 C \tan\left[\frac{1}{2}(c + d x)\right]^3 - 2 a A b^2 \tan\left[\frac{1}{2}(c + d x)\right]^5 + \\
& \quad \left. 2 A b^3 \tan\left[\frac{1}{2}(c + d x)\right]^5 - 3 a^3 C \tan\left[\frac{1}{2}(c + d x)\right]^5 + 3 a^2 b C \tan\left[\frac{1}{2}(c + d x)\right]^5 + a b^2 C \tan\left[\frac{1}{2}(c + d x)\right]^5 - b^3 C \tan\left[\frac{1}{2}(c + d x)\right]^5 \right)
\end{aligned}$$

$$\begin{aligned}
& 6 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 6 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 6 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 6 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (2 A b^2+3 a^2 C-b^2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2 b(a+b) (A b+a C) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \left. \left. \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) / \\
& \left( b^2(-a^2+b^2) d \sqrt{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left( b \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) - a \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \right) \right)
\end{aligned}$$

■ **Problem 1445: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Cos}[c + dx]^2}{(a + b \operatorname{Cos}[c + dx])^{3/2} \operatorname{Sec}[c + dx]^{3/2}} dx$$

Optimal (type 4, 664 leaves, 9 steps):

$$\begin{aligned}
& \left( (8 A b^2 + 15 a^2 C - 7 b^2 C) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / (4 b^3 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]}) - \\
& \left( (8 A b^2 + (15 a^2 + 5 a b - 2 b^2) C) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / (4 b^3 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]}) - \frac{1}{4 b^4 d \sqrt{\operatorname{Sec}[c + d x]}} \\
& \sqrt{a + b} (8 A b^2 + 15 a^2 C + 4 b^2 C) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticPi}\left[\frac{a + b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \\
& \quad \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} - \frac{2(A b^2 + a^2 C) \sin[c + d x]}{b(a^2 - b^2) d \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x]^{3/2}} + \\
& \frac{(4 A b^2 + 5 a^2 C - b^2 C) \sqrt{a + b \cos[c + d x]} \sin[c + d x]}{2 b^2 (a^2 - b^2) d \sqrt{\operatorname{Sec}[c + d x]}} - \frac{a(8 A b^2 + 15 a^2 C - 7 b^2 C) \sqrt{a + b \cos[c + d x]} \sqrt{\operatorname{Sec}[c + d x]} \sin[c + d x]}{4 b^3 (a^2 - b^2) d}
\end{aligned}$$

Result (type 4, 40202 leaves) : Display of huge result suppressed!

■ **Problem 1446: Attempted integration timed out after 120 seconds.**

$$\int \frac{(A + C \cos[c + d x])^2 \operatorname{Sec}[c + d x]^{5/2}}{(a + b \cos[c + d x])^{5/2}} dx$$

Optimal (type 4, 589 leaves, 7 steps) :

$$\begin{aligned}
& - \left( 4 b (8 A b^4 + a^4 (4 A - 3 C) - a^2 b^2 (14 A - C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a^5 \sqrt{a + b} (a^2 - b^2) d \sqrt{\operatorname{Sec}[c + d x]} \right) - \\
& \left( 2 (12 a A b^3 + 16 A b^4 - 2 a^2 b^2 (8 A - C) - a^4 (A + 3 C) - a^3 (9 A b - 3 b C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \right. \\
& \quad \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \\
& \quad \left( 3 a^4 \sqrt{a + b} (a^2 - b^2) d \sqrt{\operatorname{Sec}[c + d x]} \right) + \frac{2 (A b^2 + a^2 C) \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{3 a (a^2 - b^2) d (a + b \cos[c + d x])^{3/2}} + \\
& \quad \frac{4 (5 a^2 A b^2 - 3 A b^4 + 2 a^4 C) \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{3 a^2 (a^2 - b^2)^2 d \sqrt{a + b \cos[c + d x]}} + \\
& \quad \frac{2 (8 A b^4 + a^4 (A - 5 C) - a^2 b^2 (13 A - C)) \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{3 a^3 (a^2 - b^2)^2 d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1447: Attempted integration timed out after 120 seconds.**

$$\int \frac{(A + C \cos[c + d x])^2 \operatorname{Sec}[c + d x]^{3/2}}{(a + b \cos[c + d x])^{5/2}} dx$$

Optimal (type 4, 489 leaves, 6 steps):

$$\begin{aligned}
& \left( 2 \left( 8 A b^4 + 3 a^4 (A - C) - a^2 b^2 (15 A + C) \right) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a^4 \sqrt{a + b} (a^2 - b^2) d \sqrt{\operatorname{Sec}[c + d x]} \right) + \\
& \left( 2 \left( 6 a A b^2 + 8 A b^3 - 3 a^3 (A - C) - a^2 b (9 A + C) \right) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a^3 \sqrt{a + b} (a^2 - b^2) d \sqrt{\operatorname{Sec}[c + d x]} \right) + \\
& \frac{2 (A b^2 + a^2 C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{3 a (a^2 - b^2) d (a + b \cos[c + d x])^{3/2}} - \frac{4 (2 A b^4 - a^4 C - a^2 b^2 (4 A + C)) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{3 a^2 (a^2 - b^2)^2 d \sqrt{a + b \cos[c + d x]}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1448: Unable to integrate problem.**

$$\int \frac{(A + C \cos[c + d x])^2 \sqrt{\operatorname{Sec}[c + d x]}}{(a + b \cos[c + d x])^{5/2}} dx$$

Optimal (type 4, 456 leaves, 6 steps):

$$\begin{aligned}
& - \left( 4 b (A b^2 - a^2 (3 A + 2 C)) \sqrt{\cos [c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}} \right], -\frac{a + b}{a - b} \right] \right. \\
& \quad \left. \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a^3 (a - b) (a + b)^{3/2} d \sqrt{\operatorname{Sec}[c + d x]} \right) - \\
& \left( 2 (2 A b^2 + 3 a b (A + C) - a^2 (3 A + C)) \sqrt{\cos [c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a + b \cos [c + d x]}}{\sqrt{a + b} \sqrt{\cos [c + d x]}} \right], -\frac{a + b}{a - b} \right] \right. \\
& \quad \left. \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a^2 \sqrt{a + b} (a^2 - b^2) d \sqrt{\operatorname{Sec}[c + d x]} \right) + \\
& \frac{2 (A b^2 + a^2 C) \sin [c + d x]}{3 a (a^2 - b^2) d (a + b \cos [c + d x])^{3/2} \sqrt{\operatorname{Sec}[c + d x]}} + \frac{4 b (A b^2 - a^2 (3 A + 2 C)) \sqrt{\operatorname{Sec}[c + d x]} \sin [c + d x]}{3 a (a^2 - b^2)^2 d \sqrt{a + b \cos [c + d x]}}
\end{aligned}$$

Result (type 8, 39 leaves):

$$\int \frac{(A + C \cos [c + d x])^2 \sqrt{\operatorname{Sec}[c + d x]}}{(a + b \cos [c + d x])^{5/2}} dx$$

■ **Problem 1449: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos [c + d x]^2}{(a + b \cos [c + d x])^{5/2} \sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 618 leaves, 8 steps):

$$\begin{aligned}
& - \left( 2 (A b^4 - 3 a^4 C + a^2 b^2 (3 A + 7 C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a^2 b^2 \sqrt{a + b} (a^2 - b^2) d \sqrt{\operatorname{Sec}[c + d x]} \right) - \\
& \left( 2 (A b^3 + 3 a^3 C + a^2 b C - 3 a b^2 (A + 2 C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a (a - b) b^2 (a + b)^{3/2} d \sqrt{\operatorname{Sec}[c + d x]} \right) - \\
& \frac{1}{b^3 d \sqrt{\operatorname{Sec}[c + d x]}} 2 \sqrt{a + b} C \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticPi}\left[\frac{a + b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \\
& \quad \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} - \\
& \frac{2 (A b^2 + a^2 C) \sin[c + d x]}{3 b (a^2 - b^2) d (a + b \cos[c + d x])^{3/2} \sqrt{\operatorname{Sec}[c + d x]}} + \frac{2 (A b^4 - 3 a^4 C + a^2 b^2 (3 A + 7 C)) \sqrt{\operatorname{Sec}[c + d x]} \sin[c + d x]}{3 b^2 (a^2 - b^2)^2 d \sqrt{a + b \cos[c + d x]}}
\end{aligned}$$

Result (type 4, 1588 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{a + b \cos[c + d x]} \sqrt{\operatorname{Sec}[c + d x]} \left( \frac{2 (-3 a^2 A b^2 - A b^4 + 3 a^4 C - 7 a^2 b^2 C) \sin[c + d x]}{3 a b^2 (a^2 - b^2)^2} - \right. \\
& \quad \left. \frac{2 (a A b^2 \sin[c + d x] + a^3 C \sin[c + d x])}{3 b^2 (-a^2 + b^2) (a + b \cos[c + d x])^2} + \frac{4 (a^2 A b^2 \sin[c + d x] + A b^4 \sin[c + d x] - 2 a^4 C \sin[c + d x] + 4 a^2 b^2 C \sin[c + d x])}{3 b^2 (-a^2 + b^2)^2 (a + b \cos[c + d x])} \right) + \\
& \left( 2 \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + d x)\right]^2}} \left( 3 a^3 A b^2 \tan\left[\frac{1}{2}(c + d x)\right] + 3 a^2 A b^3 \tan\left[\frac{1}{2}(c + d x)\right] + a A b^4 \tan\left[\frac{1}{2}(c + d x)\right] + A b^5 \tan\left[\frac{1}{2}(c + d x)\right] - \right. \right. \\
& \quad \left. \left. 3 a^5 C \tan\left[\frac{1}{2}(c + d x)\right] - 3 a^4 b C \tan\left[\frac{1}{2}(c + d x)\right] + 7 a^3 b^2 C \tan\left[\frac{1}{2}(c + d x)\right] + 7 a^2 b^3 C \tan\left[\frac{1}{2}(c + d x)\right] - \right. \right. \\
& \quad \left. \left. 6 a^2 A b^3 \tan\left[\frac{1}{2}(c + d x)\right]^3 - 2 A b^5 \tan\left[\frac{1}{2}(c + d x)\right]^3 + 6 a^4 b C \tan\left[\frac{1}{2}(c + d x)\right]^3 - 14 a^2 b^3 C \tan\left[\frac{1}{2}(c + d x)\right]^3 - \right. \right.
\end{aligned}$$



$$\begin{aligned}
& 3 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 3 a^2 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - a A b^4 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + A b^5 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + \\
& 3 a^5 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 3 a^4 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 7 a^3 b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 7 a^2 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \\
& 6 a^5 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 12 a^3 b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 6 a b^4 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 6 a^5 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 12 a^3 b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 6 a b^4 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - (a+b) (-A b^4 + 3 a^4 C - a^2 b^2 (3 A + 7 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& a b (a+b) (2 a^2 C - 3 a b (A+C) - b^2 (A+3 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]
\end{aligned}$$

$$\left( \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) /$$

$$\left( 3 a b^2 (a^2 - b^2)^2 d \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right)$$

■ **Problem 1450: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \cos[c+dx]^2}{(a+b \cos[c+dx])^{5/2} \sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 710 leaves, 9 steps):

$$\left( (8 A b^4 - (15 a^4 - 26 a^2 b^2 + 3 b^4) C) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b}}\right], -\frac{a+b}{a-b}\right] \right.$$

$$\left. \sqrt{\frac{a(1 - \sec[c+dx])}{a+b}} \sqrt{\frac{a(1 + \sec[c+dx])}{a-b}} \right) / (3 a (a-b) b^3 (a+b)^{3/2} d \sqrt{\sec[c+dx]}) -$$

$$\left( (6 A b^4 - a b^3 (2 A - 3 C) - 15 a^4 C - 5 a^3 b C + 21 a^2 b^2 C) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b}}\right], -\frac{a+b}{a-b}\right] \right.$$

$$\left. \sqrt{\frac{a(1 - \sec[c+dx])}{a+b}} \sqrt{\frac{a(1 + \sec[c+dx])}{a-b}} \right) / (3 a (a-b) b^3 (a+b)^{3/2} d \sqrt{\sec[c+dx]}) +$$

$$\frac{1}{b^4 d \sqrt{\sec[c+dx]}} 5 a \sqrt{a+b} C \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1 - \sec[c+dx])}{a+b}} \sqrt{\frac{a(1 + \sec[c+dx])}{a-b}} - \frac{2 (A b^2 + a^2 C) \sin[c+dx]}{3 b (a^2 - b^2) d (a+b \cos[c+dx])^{3/2} \sec[c+dx]^{3/2}} +$$

$$\frac{2 (3 A b^4 - 5 a^4 C + a^2 b^2 (A + 9 C)) \sin[c+dx]}{3 b^2 (a^2 - b^2)^2 d \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]}} - \frac{(8 A b^4 - (15 a^4 - 26 a^2 b^2 + 3 b^4) C) \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]} \sin[c+dx]}{3 b^3 (a^2 - b^2)^2 d}$$

Result (type 4, 1609 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} \left( -\frac{4(-2Ab^4 + 3a^4c - 5a^2b^2c) \sin[c + dx]}{3b^3(a^2 - b^2)^2} + \right. \\
& \left. \frac{2(a^2Ab^2 \sin[c + dx] + a^4c \sin[c + dx])}{3b^3(-a^2 + b^2)(a + b \cos[c + dx])^2} + \frac{2(a^3Ab^2 \sin[c + dx] - 5aAb^4 \sin[c + dx] + 7a^5c \sin[c + dx] - 11a^3b^2c \sin[c + dx])}{3b^3(-a^2 + b^2)^2(a + b \cos[c + dx])} \right) + \\
& \left( \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}} \right. \\
& \left( 8aAb^4 \tan\left[\frac{1}{2}(c + dx)\right] + 8Ab^5 \tan\left[\frac{1}{2}(c + dx)\right] - 15a^5c \tan\left[\frac{1}{2}(c + dx)\right] - 15a^4bc \tan\left[\frac{1}{2}(c + dx)\right] + 26a^3b^2c \tan\left[\frac{1}{2}(c + dx)\right] + \right. \\
& 26a^2b^3c \tan\left[\frac{1}{2}(c + dx)\right] - 3ab^4c \tan\left[\frac{1}{2}(c + dx)\right] - 3b^5c \tan\left[\frac{1}{2}(c + dx)\right] - 16Ab^5 \tan\left[\frac{1}{2}(c + dx)\right]^3 + 30a^4bc \tan\left[\frac{1}{2}(c + dx)\right]^3 - \\
& 52a^2b^3c \tan\left[\frac{1}{2}(c + dx)\right]^3 + 6b^5c \tan\left[\frac{1}{2}(c + dx)\right]^3 - 8aAb^4 \tan\left[\frac{1}{2}(c + dx)\right]^5 + 8Ab^5 \tan\left[\frac{1}{2}(c + dx)\right]^5 + 15a^5c \tan\left[\frac{1}{2}(c + dx)\right]^5 - \\
& 15a^4bc \tan\left[\frac{1}{2}(c + dx)\right]^5 - 26a^3b^2c \tan\left[\frac{1}{2}(c + dx)\right]^5 + 26a^2b^3c \tan\left[\frac{1}{2}(c + dx)\right]^5 + 3ab^4c \tan\left[\frac{1}{2}(c + dx)\right]^5 - 3b^5c \tan\left[\frac{1}{2}(c + dx)\right]^5 - \\
& 30a^5c \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a+b}} + \\
& 60a^3b^2c \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a+b}} - \\
& 30ab^4c \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a+b}} - \\
& 30a^5c \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{-a+b}{a+b}\right] \tan\left[\frac{1}{2}(c + dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \\
& \left. \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a+b}} + 60a^3b^2c \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{-a+b}{a+b}\right] \tan\left[\frac{1}{2}(c + dx)\right]^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 30 a b^4 \operatorname{CEllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& (a+b) \left(-8 A b^4 + (15 a^4 - 26 a^2 b^2 + 3 b^4) c\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 b (a+b) \left(3 A b^3 - 5 a^3 c + 3 a^2 b c + a b^2 (A+6 c)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \left. \left. \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) \right) \\
& \left( 3 b^3 (a^2 - b^2)^2 d \sqrt{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left( b \left( -1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) - a \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right)
\end{aligned}$$

▪ **Problem 1482: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+B \cos[c+dx] + C \cos[c+dx]^2) \operatorname{Sec}[c+dx]^{7/2}}{a+b \cos[c+dx]} dx$$

Optimal (type 4, 294 leaves, 9 steps):

$$\begin{aligned}
& \frac{2(5Ab^2 - 5abB + a^2(3A+5C)) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{5a^3d} - \\
& \frac{2(Ab - aB) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{3a^2d} - \\
& \frac{2b(Ab^2 - a(bB - aC)) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{a^3(a+b)d} + \\
& \frac{2(5Ab^2 - 5abB + a^2(3A+5C)) \sqrt{\operatorname{Sec}[c+dx]} \sin[c+dx]}{5a^3d} - \frac{2(Ab - aB) \operatorname{Sec}[c+dx]^{3/2} \sin[c+dx]}{3a^2d} + \frac{2A \operatorname{Sec}[c+dx]^{5/2} \sin[c+dx]}{5ad}
\end{aligned}$$

Result (type 4, 698 leaves):

$$\begin{aligned}
& -\frac{1}{30 a^3 d} \left( -\left( 2 \left( 18 a^3 A + 40 a A b^2 - 40 a^2 b B + 30 a^3 C \right) \operatorname{Cos}[c + d x]^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c + d x]}\right], -1\right] \right. \right. \\
& \quad \left. \left. (b + a \operatorname{Sec}[c + d x]) \sqrt{1 - \operatorname{Sec}[c + d x]^2} \operatorname{Sin}[c + d x] \right) \right) / \left( (b + a \operatorname{Cos}[c + d x]) (1 - \operatorname{Cos}[c + d x]^2) \right) + \\
& \quad \left( 2 \left( 19 a^2 A b + 45 A b^3 - 10 a^3 B - 45 a b^2 B + 45 a^2 b C \right) \operatorname{Cos}[c + d x]^2 \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c + d x]}\right], -1\right] + \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c + d x]}\right], -1\right] \right) (b + a \operatorname{Sec}[c + d x]) \sqrt{1 - \operatorname{Sec}[c + d x]^2} \operatorname{Sin}[c + d x] \right) / \\
& \quad \left( a (a + b \operatorname{Cos}[c + d x]) (1 - \operatorname{Cos}[c + d x]^2) \right) + \left( (9 a^2 A b + 15 A b^3 - 15 a b^2 B + 15 a^2 b C) \operatorname{Cos}[2(c + d x)] (b + a \operatorname{Sec}[c + d x]) \right. \\
& \quad \left( -4 a b + 4 a b \operatorname{Sec}[c + d x]^2 - 4 a b \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c + d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c + d x]} \sqrt{1 - \operatorname{Sec}[c + d x]^2} + \right. \\
& \quad 2 (2 a - b) b \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c + d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c + d x]} \sqrt{1 - \operatorname{Sec}[c + d x]^2} + \\
& \quad 4 a^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c + d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c + d x]} \sqrt{1 - \operatorname{Sec}[c + d x]^2} - \\
& \quad \left. \left. 2 b^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c + d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c + d x]} \sqrt{1 - \operatorname{Sec}[c + d x]^2} \right) \operatorname{Sin}[c + d x] \right) / \\
& \quad \left( a b^2 (a + b \operatorname{Cos}[c + d x]) (1 - \operatorname{Cos}[c + d x]^2) \sqrt{\operatorname{Sec}[c + d x]} (2 - \operatorname{Sec}[c + d x]^2) \right) \Big) + \\
& \quad \frac{\sqrt{\operatorname{Sec}[c + d x]} \left( \frac{2(3 a^2 A + 5 A b^2 - 5 a b B + 5 a^2 C) \operatorname{Sin}[c + d x]}{5 a^3} + \frac{2 \operatorname{Sec}[c + d x] (-A b \operatorname{Sin}[c + d x] + a B \operatorname{Sin}[c + d x])}{3 a^2} + \frac{2 A \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{5 a} \right)}{d}
\end{aligned}$$

■ **Problem 1483: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \operatorname{Cos}[c + d x] + C \operatorname{Cos}[c + d x]^2) \operatorname{Sec}[c + d x]^{5/2}}{a + b \operatorname{Cos}[c + d x]} dx$$

Optimal (type 4, 218 leaves, 8 steps):

$$\begin{aligned}
& \frac{2 (A b - a B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{a^2 d} + \frac{2 A \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 a d} + \\
& \frac{2 (A b^2 - a (b B - a C)) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticPi}\left[\frac{2 b}{a + b}, \frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{a^2 (a + b) d} - \\
& \frac{2 (A b - a B) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{a^2 d} + \frac{2 A \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{3 a d}
\end{aligned}$$

Result (type 4, 600 leaves):

$$\frac{1}{6 a^2 d} \left( - \left( 2 (8 a A b - 6 a^2 B) \cos [c+d x]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] (b+a \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / \right. \\ \left. (b(a+b \cos [c+d x]) (1-\cos [c+d x]^2)) + \right. \\ \left. \left( 2 (2 a^2 A + 9 A b^2 - 9 a b B + 6 a^2 C) \cos [c+d x]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right) \right) / \right. \\ \left. (b+a \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / (a(a+b \cos [c+d x]) (1-\cos [c+d x]^2)) + \\ \left( (3 A b^2 - 3 a b B) \cos [2(c+d x)] (b+a \sec [c+d x]) \left( -4 a b + 4 a b \sec [c+d x]^2 - 4 a b \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \right. \right. \\ \left. \left. \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + 2 (2 a - b) b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + \right. \right. \\ \left. \left. 4 a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} - \right. \right. \\ \left. \left. 2 b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} \right) \sin [c+d x] \right) / \\ \left. \left( a b^2 (a+b \cos [c+d x]) (1-\cos [c+d x]^2) \sqrt{\sec [c+d x]} (2-\sec [c+d x]^2) \right) \right) + \frac{\sqrt{\sec [c+d x]} \left( \frac{2(-A b+a B) \sin [c+d x]}{a^2} + \frac{2 A \tan [c+d x]}{3 a} \right)}{d}$$

■ **Problem 1486: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos [c+d x] + C \cos [c+d x]^2}{(a+b \cos [c+d x]) \sqrt{\sec [c+d x]}} dx$$

Optimal (type 4, 207 leaves, 7 steps):

$$\frac{2 (b B - a C) \sqrt{\cos [c+d x]} \operatorname{EllipticE} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{b^2 d} + \\ \frac{2 (b^2 (3 A + C) - 3 a (b B - a C)) \sqrt{\cos [c+d x]} \operatorname{EllipticF} \left[ \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{3 b^3 d} - \\ \frac{2 a (A b^2 - a (b B - a C)) \sqrt{\cos [c+d x]} \operatorname{EllipticPi} \left[ \frac{2 b}{a+b}, \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{b^3 (a+b) d} + \frac{2 C \sin [c+d x]}{3 b d \sqrt{\sec [c+d x]}}$$

Result (type 4, 560 leaves):

$$\frac{1}{6bd} \left( - \left( 2(6Ab + 2bC) \cos[c+dx]^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] (b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / \right. \\ \left. (b(a+b \cos[c+dx]) (1-\cos[c+dx]^2)) + \right. \\ \left. \left( 2(3bB - aC) \cos[c+dx]^2 \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] + \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \right) \right. \right. \\ \left. \left. (b+a \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / (a(a+b \cos[c+dx]) (1-\cos[c+dx]^2)) + \right. \\ \left. \left( (3bB - 3aC) \cos[2(c+dx)] (b+a \sec[c+dx]) \left( -4ab + 4ab \sec[c+dx]^2 - 4ab \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \right. \right. \right. \\ \left. \left. \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + 2(2a-b)b \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + \right. \right. \\ \left. \left. 4a^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} - \right. \right. \\ \left. \left. 2b^2 \operatorname{EllipticPi} \left[ -\frac{a}{b}, -\operatorname{ArcSin} \left[ \sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \right) \sin[c+dx] \right) / \\ \left. \left( ab^2 (a+b \cos[c+dx]) (1-\cos[c+dx]^2) \sqrt{\sec[c+dx]} (2-\sec[c+dx]^2) \right) \right) + \frac{c \sqrt{\sec[c+dx]} \sin[2(c+dx)]}{3bd}$$

■ **Problem 1487: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c+dx] + C \cos[c+dx]^2}{(a+b \cos[c+dx]) \sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 270 leaves, 8 steps):

$$\frac{2(5Ab^2 - 5abB + 5a^2C + 3b^2C) \sqrt{\cos[c+dx]} \operatorname{EllipticE} \left[ \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{5b^3d} + \\ \frac{2(3a^2bB + b^3B - 3a^3C - ab^2(3A+C)) \sqrt{\cos[c+dx]} \operatorname{EllipticF} \left[ \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{3b^4d} + \\ \frac{2a^2(Ab^2 - a(bB - aC)) \sqrt{\cos[c+dx]} \operatorname{EllipticPi} \left[ \frac{2b}{a+b}, \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{b^4(a+b)d} + \frac{2C \sin[c+dx]}{5bd \sec[c+dx]^{3/2}} + \frac{2(bB - aC) \sin[c+dx]}{3b^2d \sqrt{\sec[c+dx]}}$$

Result (type 4, 632 leaves):

$$\begin{aligned}
& -\frac{1}{30 b^2 d} \\
& \left( -\left( 2 (-10 b^2 B - 8 a b C) \cos[c + d x]^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] (b + a \sec[c + d x]) \sqrt{1 - \sec[c + d x]^2} \sin[c + d x] \right) \right. \\
& \quad \left. (b (a + b \cos[c + d x]) (1 - \cos[c + d x]^2)) + \left( 2 (-15 A b^2 + 5 a b B - 5 a^2 C - 9 b^2 C) \cos[c + d x]^2 \right. \right. \\
& \quad \left. \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] \right) \right. \\
& \quad \left. (b + a \sec[c + d x]) \sqrt{1 - \sec[c + d x]^2} \sin[c + d x] \right) \left/ (a (a + b \cos[c + d x]) (1 - \cos[c + d x]^2)) + \right. \\
& \quad \left( (-15 A b^2 + 15 a b B - 15 a^2 C - 9 b^2 C) \cos[2(c + d x)] (b + a \sec[c + d x]) \left( -4 a b + 4 a b \sec[c + d x]^2 - \right. \right. \\
& \quad \left. \left. 4 a b \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] \sqrt{\sec[c + d x]} \sqrt{1 - \sec[c + d x]^2} + 2(2 a - b) b \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] \right. \right. \\
& \quad \left. \left. \sqrt{\sec[c + d x]} \sqrt{1 - \sec[c + d x]^2} + 4 a^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] \sqrt{\sec[c + d x]} \sqrt{1 - \sec[c + d x]^2} - \right. \right. \\
& \quad \left. \left. 2 b^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] \sqrt{\sec[c + d x]} \sqrt{1 - \sec[c + d x]^2} \right) \sin[c + d x] \right) \left/ \right. \\
& \quad \left. \left( a b^2 (a + b \cos[c + d x]) (1 - \cos[c + d x]^2) \sqrt{\sec[c + d x]} (2 - \sec[c + d x]^2) \right) \right) + \\
& \frac{\sqrt{\sec[c + d x]} \left( \frac{C \sin[c + d x]}{10 b} + \frac{(b B - a C) \sin[2(c + d x)]}{3 b^2} + \frac{C \sin[3(c + d x)]}{10 b} \right)}{d}
\end{aligned}$$

■ **Problem 1488: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + d x] + C \cos[c + d x]^2}{(a + b \cos[c + d x]) \sec[c + d x]^{5/2}} dx$$

Optimal (type 4, 345 leaves, 9 steps):

$$\begin{aligned}
& \frac{2 (5 a^2 b B + 3 b^3 B - 5 a^3 C - a b^2 (5 A + 3 C)) \sqrt{\cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{5 b^4 d} - \frac{1}{21 b^5 d} \\
& 2 (21 a^3 b B + 7 a b^3 B - 21 a^4 C - 7 a^2 b^2 (3 A + C) - b^4 (7 A + 5 C)) \sqrt{\cos[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]} - \\
& \frac{2 a^3 (A b^2 - a (b B - a C)) \sqrt{\cos[c + d x]} \operatorname{EllipticPi}\left[\frac{2 b}{a + b}, \frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{b^5 (a + b) d} + \\
& \frac{2 C \sin[c + d x]}{7 b d \sec[c + d x]^{5/2}} + \frac{2 (b B - a C) \sin[c + d x]}{5 b^2 d \sec[c + d x]^{3/2}} + \frac{2 (7 A b^2 - 7 a b B + 7 a^2 C + 5 b^2 C) \sin[c + d x]}{21 b^3 d \sqrt{\sec[c + d x]}}
\end{aligned}$$

Result (type 4, 713 leaves):



$$\begin{aligned}
& -\frac{1}{210 b^3 d} \left( -\left( 2 (-70 A b^3 - 56 a b^2 B + 56 a^2 b C - 50 b^3 C) \cos [c+d x]^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \right. \right. \\
& \quad \left. \left. (b+a \operatorname{Sec}[c+d x]) \sqrt{1-\operatorname{Sec}[c+d x]^2} \sin [c+d x]\right) / (b(a+b \cos [c+d x]) (1-\cos [c+d x]^2)) + \right. \\
& \quad \left( 2 (35 a A b^2 - 35 a^2 b B - 63 b^3 B + 35 a^3 C + 13 a b^2 C) \cos [c+d x]^2 \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{a}{b}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \right) (b+a \operatorname{Sec}[c+d x]) \sqrt{1-\operatorname{Sec}[c+d x]^2} \sin [c+d x]\right) / (a(a+b \cos [c+d x]) (1-\cos [c+d x]^2)) + \right. \\
& \quad \left( (105 a A b^2 - 105 a^2 b B - 63 b^3 B + 105 a^3 C + 63 a b^2 C) \cos [2(c+d x)] (b+a \operatorname{Sec}[c+d x]) \left( -4 a b + 4 a b \operatorname{Sec}[c+d x]^2 - \right. \right. \\
& \quad \left. \left. 4 a b \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} + 2(2 a-b) b \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \right. \right. \\
& \quad \left. \left. \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} + 4 a^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} - \right. \right. \\
& \quad \left. \left. 2 b^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2}\right) \sin [c+d x]\right) / \\
& \quad \left. \left( a b^2 (a+b \cos [c+d x]) (1-\cos [c+d x]^2) \sqrt{\operatorname{Sec}[c+d x]} (2-\operatorname{Sec}[c+d x]^2) \right) \right) + \frac{1}{d} \\
& \quad \sqrt{\operatorname{Sec}[c+d x]} \left( \frac{(b B - a C) \sin [c+d x]}{10 b^2} + \frac{(14 A b^2 - 14 a b B + 14 a^2 C + 13 b^2 C) \sin [2(c+d x)]}{42 b^3} + \right. \\
& \quad \frac{(b B - a C) \sin [3(c+d x)]}{10 b^2} + \\
& \quad \left. \frac{C \sin [4(c+d x)]}{28 b} \right)
\end{aligned}$$

■ **Problem 1491: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+B \cos [c+d x]+C \cos [c+d x]^2) \sqrt{\operatorname{Sec}[c+d x]}}{(a+b \cos [c+d x])^2} dx$$

Optimal (type 4, 303 leaves, 7 steps):

$$\begin{aligned}
& \frac{(A b^2 - a (b B - a C)) \sqrt{\cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{a b (a^2 - b^2) d} - \\
& \frac{(A b^2 - a b B - a^2 C + 2 b^2 C) \sqrt{\cos[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{b^2 (a^2 - b^2) d} - \frac{1}{a (a - b) b^2 (a + b)^2 d} \\
& \frac{(A b^4 + a^3 b B + a b^3 B + a^4 C - 3 a^2 b^2 (A + C)) \sqrt{\cos[c + d x]} \operatorname{EllipticPi}\left[\frac{2 b}{a + b}, \frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]} +}{a (a^2 - b^2) d (a + b \cos[c + d x]) \sqrt{\sec[c + d x]}} \\
& \frac{(A b^2 - a (b B - a C)) \sin[c + d x]}{a (a^2 - b^2) d (a + b \cos[c + d x]) \sqrt{\sec[c + d x]}}
\end{aligned}$$

Result (type 4, 688 leaves):

$$\begin{aligned}
& \frac{1}{4 a (-a + b) (a + b) d} \\
& \left( - \left( 2 (4 a A b - 4 a^2 B + 4 a b C) \cos[c + d x]^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] (b + a \sec[c + d x]) \sqrt{1 - \sec[c + d x]^2} \right. \right. \\
& \quad \left. \left. \sin[c + d x] \right) / (b (a + b \cos[c + d x]) (1 - \cos[c + d x]^2)) + \right. \\
& \left( 2 (-4 a^2 A + 3 A b^2 + a b B - a^2 C) \cos[c + d x]^2 \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] \right) \right. \\
& \quad \left. (b + a \sec[c + d x]) \sqrt{1 - \sec[c + d x]^2} \sin[c + d x] \right) / (a (a + b \cos[c + d x]) (1 - \cos[c + d x]^2)) + \\
& \left( (A b^2 - a b B + a^2 C) \cos[2(c + d x)] (b + a \sec[c + d x]) \left( -4 a b + 4 a b \sec[c + d x]^2 - 4 a b \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] \right. \right. \\
& \quad \left. \left. \sqrt{\sec[c + d x]} \sqrt{1 - \sec[c + d x]^2} + 2 (2 a - b) b \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] \sqrt{\sec[c + d x]} \sqrt{1 - \sec[c + d x]^2} + \right. \right. \\
& \quad \left. \left. 4 a^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] \sqrt{\sec[c + d x]} \sqrt{1 - \sec[c + d x]^2} - \right. \right. \\
& \quad \left. \left. 2 b^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] \sqrt{\sec[c + d x]} \sqrt{1 - \sec[c + d x]^2} \right) \sin[c + d x] \right) / \\
& \quad \left. \left( a b^2 (a + b \cos[c + d x]) (1 - \cos[c + d x]^2) \sqrt{\sec[c + d x]} (2 - \sec[c + d x]^2) \right) \right) + \\
& \frac{\sqrt{\sec[c + d x]} \left( \frac{(A b^2 - a b B + a^2 C) \sin[c + d x]}{a b (a^2 - b^2)} + \frac{A b^2 \sin[c + d x] - a b B \sin[c + d x] + a^2 C \sin[c + d x]}{b (-a^2 + b^2) (a + b \cos[c + d x])} \right)}{d}
\end{aligned}$$

■ **Problem 1492: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + d x] + C \cos[c + d x]^2}{(a + b \cos[c + d x])^2 \sqrt{\sec[c + d x]}} dx$$

Optimal (type 4, 311 leaves, 7 steps):

$$\frac{(Ab^2 - abB + 3a^2C - 2b^2C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{b^2(a^2 - b^2)d} +$$

$$\frac{(a^2bB - 2b^3B - 3a^3C + ab^2(A+4C)) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{b^3(a^2 - b^2)d} - \frac{1}{(a-b)b^3(a+b)^2d}$$

$$\frac{(Ab^4 + a^3bB - 3ab^3B - 3a^4C + a^2b^2(A+5C)) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]} -}{b(a^2 - b^2)d(a+b\cos[c+dx])\sqrt{\sec[c+dx]}}$$

$$\frac{(Ab^2 - a(bB - aC)) \sin[c+dx]}{b(a^2 - b^2)d(a+b\cos[c+dx])\sqrt{\sec[c+dx]}}$$

Result (type 4, 695 leaves):

$$\frac{1}{4(a-b)b(a+b)d}$$

$$\left( - \left( 2(4aAb - 4b^2B + 4abC) \cos[c+dx]^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] (b+a\sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \right. \right.$$

$$\left. \left. \sin[c+dx] \right) / (b(a+b\cos[c+dx])(1-\cos[c+dx]^2)) + \right.$$

$$\left( 2(-Ab^2 + abB + a^2C - 2b^2C) \cos[c+dx]^2 \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right) \right.$$

$$\left. (b+a\sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / (a(a+b\cos[c+dx])(1-\cos[c+dx]^2)) +$$

$$\left( (Ab^2 - abB + 3a^2C - 2b^2C) \cos[2(c+dx)] (b+a\sec[c+dx]) \left( -4ab + 4ab\sec[c+dx]^2 - 4ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right. \right.$$

$$\left. \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + 2(2a-b)b \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + \right.$$

$$4a^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} -$$

$$\left. 2b^2 \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \right) \sin[c+dx] \Big/$$

$$\left( ab^2(a+b\cos[c+dx])(1-\cos[c+dx]^2) \sqrt{\sec[c+dx]} (2-\sec[c+dx]^2) \right) +$$

$$\frac{\sqrt{\sec[c+dx]} \left( \frac{(Ab^2 - abB + a^2C) \sin[c+dx]}{b^2(-a^2+b^2)} + \frac{-aAb^2 \sin[c+dx] + a^2bB \sin[c+dx] - a^3C \sin[c+dx]}{b^2(-a^2+b^2)(a+b\cos[c+dx])} \right)}{d}$$

■ **Problem 1502: Attempted integration timed out after 120 seconds.**

$$\int \sqrt{a+b\cos[c+dx]} (A+B\cos[c+dx] + C\cos[c+dx]^2) \sec[c+dx]^{11/2} dx$$

Optimal (type 4, 592 leaves, 8 steps):

$$\begin{aligned}
& - \frac{1}{315 a^5 d \sqrt{\text{Sec}[c + d x]}} 2 (a - b) \sqrt{a + b} \left( 16 A b^4 - 57 a^3 b B - 24 a b^3 B + 6 a^2 b^2 (4 A + 7 C) - 21 a^4 (7 A + 9 C) \right) \sqrt{\text{Cos}[c + d x]} \\
& \quad \text{Csc}[c + d x] \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{a + b \text{Cos}[c + d x]}}{\sqrt{a + b} \sqrt{\text{Cos}[c + d x]}} \right], -\frac{a + b}{a - b} \right] \sqrt{\frac{a (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \text{Sec}[c + d x])}{a - b}} - \\
& \frac{1}{315 a^4 d \sqrt{\text{Sec}[c + d x]}} 2 (a - b) \sqrt{a + b} \left( 16 A b^3 + 12 a b^2 (A - 2 B) + 6 a^2 b (6 A - 3 B + 7 C) + 3 a^3 (49 A - 25 B + 63 C) \right) \sqrt{\text{Cos}[c + d x]} \\
& \quad \text{Csc}[c + d x] \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{a + b \text{Cos}[c + d x]}}{\sqrt{a + b} \sqrt{\text{Cos}[c + d x]}} \right], -\frac{a + b}{a - b} \right] \sqrt{\frac{a (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \text{Sec}[c + d x])}{a - b}} + \\
& \frac{2 (8 A b^3 + 75 a^3 B - 12 a b^2 B + a^2 b (13 A + 21 C)) \sqrt{a + b \text{Cos}[c + d x]} \text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{315 a^3 d} - \\
& \frac{2 (6 A b^2 - 9 a b B - 7 a^2 (7 A + 9 C)) \sqrt{a + b \text{Cos}[c + d x]} \text{Sec}[c + d x]^{5/2} \text{Sin}[c + d x]}{315 a^2 d} + \\
& \frac{2 (A b + 9 a B) \sqrt{a + b \text{Cos}[c + d x]} \text{Sec}[c + d x]^{7/2} \text{Sin}[c + d x]}{63 a d} + \frac{2 A \sqrt{a + b \text{Cos}[c + d x]} \text{Sec}[c + d x]^{9/2} \text{Sin}[c + d x]}{9 d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1503: Attempted integration timed out after 120 seconds.**

$$\int \sqrt{a + b \text{Cos}[c + d x]} (A + B \text{Cos}[c + d x] + C \text{Cos}[c + d x]^2) \text{Sec}[c + d x]^{9/2} dx$$

Optimal (type 4, 487 leaves, 7 steps):

$$\frac{1}{105 a^4 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (8 A b^3 + 63 a^3 B - 14 a b^2 B + a^2 b (19 A + 35 C)) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{1}{105 a^3 d \sqrt{\sec[c+dx]}}$$

$$2 (a-b) \sqrt{a+b} (8 A b^2 + 2 a b (3 A - 7 B) + a^2 (25 A - 63 B + 35 C)) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{2(4 A b^2 - 7 a b B - 5 a^2 (5 A + 7 C)) \sqrt{a+b} \cos[c+dx] \sec[c+dx]^{3/2} \sin[c+dx]}{105 a^2 d} +$$

$$\frac{2(A b + 7 a B) \sqrt{a+b} \cos[c+dx] \sec[c+dx]^{5/2} \sin[c+dx]}{35 a d} + \frac{2 A \sqrt{a+b} \cos[c+dx] \sec[c+dx]^{7/2} \sin[c+dx]}{7 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1504: Attempted integration timed out after 120 seconds.**

$$\int \sqrt{a+b \cos[c+dx]} (A+B \cos[c+dx]+C \cos[c+dx]^2) \sec[c+dx]^{7/2} dx$$

Optimal (type 4, 400 leaves, 6 steps):

$$-\frac{1}{15 a^3 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (2 A b^2 - 5 a b B - 3 a^2 (3 A + 5 C)) \sqrt{\cos[c+dx]}$$

$$\operatorname{Csc}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{15 a^2 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (2 A b + a (9 A - 5 B + 15 C)) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2(A b + 5 a B) \sqrt{a+b} \cos[c+dx] \sec[c+dx]^{3/2} \sin[c+dx]}{15 a d} + \frac{2 A \sqrt{a+b} \cos[c+dx] \sec[c+dx]^{5/2} \sin[c+dx]}{5 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1505: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \cos[c + dx]} (A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^{5/2} dx$$

Optimal (type 4, 467 leaves, 7 steps):

$$\frac{1}{3 a^2 d \sqrt{\sec[c + dx]}} 2 (a - b) \sqrt{a + b} (A b + 3 a B) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right]$$

$$\sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \frac{1}{3 a d \sqrt{\sec[c + dx]}} 2 \sqrt{a + b} (b(A - 3 B) - a(A - 3 B + 3 C)) \sqrt{\cos[c + dx]}$$

$$\operatorname{Csc}[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} -$$

$$\frac{1}{d \sqrt{\sec[c + dx]}} 2 \sqrt{a + b} C \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \operatorname{EllipticPi}\left[\frac{a + b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right]$$

$$\sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \frac{2 A \sqrt{a + b \cos[c + dx]} \sec[c + dx]^{3/2} \sin[c + dx]}{3 d}$$

Result (type 4, 5188 leaves):

$$\frac{\sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} \left( \frac{2(A b + 3 a B) \sin[c + dx]}{3 a} + \frac{2}{3} A \tan[c + dx] \right)}{d}$$

$$\left( 2 \left( -\frac{A b}{3 \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]}} - \frac{a B}{\sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]}} + \right. \right.$$

$$\frac{b C}{\sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]}} + \frac{a A \sqrt{\sec[c + dx]}}{3 \sqrt{a + b \cos[c + dx]}} - \frac{A b^2 \sqrt{\sec[c + dx]}}{3 a \sqrt{a + b \cos[c + dx]}} + \frac{a C \sqrt{\sec[c + dx]}}{\sqrt{a + b \cos[c + dx]}} -$$

$$\left. \frac{A b^2 \cos[2(c + dx)] \sqrt{\sec[c + dx]}}{3 a \sqrt{a + b \cos[c + dx]}} - \frac{b B \cos[2(c + dx)] \sqrt{\sec[c + dx]}}{\sqrt{a + b \cos[c + dx]}} \right) \sqrt{\cos\left[\frac{1}{2}(c + dx)\right]^2 \sec[c + dx]}$$

$$\left( 12 a b C \sqrt{\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \sqrt{\frac{a + b \cos[c + dx]}{(a + b)(1 + \cos[c + dx])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{-a + b}{a + b}\right] + \right.$$

$$\left. 2(a + b)(A b + 3 a B) \sqrt{\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \sqrt{\frac{a + b \cos[c + dx]}{(a + b)(1 + \cos[c + dx])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{-a + b}{a + b}\right] \sec\left[\frac{1}{2}(c + dx)\right]^2 - \right.$$

$$\begin{aligned}
& 2 a (b (A + 3 B - 3 C) + a (A + 3 (B + C))) \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] \\
& \sec \left[\frac{1}{2} (c + d x)\right]^2 + a A b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 3 a^2 B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 3 a b B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + \\
& 12 a b C \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 - \\
& 2 A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 - 6 a b B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 - a A b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 + \\
& A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 - 3 a^2 B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 + 3 a b B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 \Big) \Big) / \\
& \left( 3 a d \sqrt{a + b \cos [c + d x]} \left(\sec \left[\frac{1}{2} (c + d x)\right]^2\right)^{3/2} \left( -\frac{1}{3 a (a + b \cos [c + d x])^{3/2} \left(\sec \left[\frac{1}{2} (c + d x)\right]^2\right)^{3/2}} b \sqrt{\cos \left[\frac{1}{2} (c + d x)\right]^2 \sec [c + d x]} \right. \right. \\
& \left. \left. \sin [c + d x] \left( 12 a b C \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] + 2 (a + b) \right. \right. \right. \\
& \left. \left. (A b + 3 a B) \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] \sec \left[\frac{1}{2} (c + d x)\right]^2 - \right. \right. \\
& \left. \left. 2 a (b (A + 3 B - 3 C) + a (A + 3 (B + C))) \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] \right. \right. \\
& \left. \left. \sec \left[\frac{1}{2} (c + d x)\right]^2 + a A b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 3 a^2 B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 3 a b B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + \right. \right. \\
& \left. \left. 12 a b C \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{a + b \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{-a + b}{a + b}\right] \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 - \right. \right. \\
& \left. \left. 2 A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 - 6 a b B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 - a A b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 + A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 - 3 a^2 B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 3 a b B \tan\left[\frac{1}{2}(c+d x)\right]^5 + \frac{1}{a \sqrt{a+b \cos [c+d x]} \left(\sec\left[\frac{1}{2}(c+d x)\right]\right)^{3/2}} \sqrt{\cos\left[\frac{1}{2}(c+d x)\right]^2 \sec [c+d x] \tan\left[\frac{1}{2}(c+d x)\right]} \\
& \left(12 a b C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& 2(a+b)(A b+3 a B) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sec\left[\frac{1}{2}(c+d x)\right]^2 - \\
& 2 a(b(A+3 B-3 C)+a(A+3(B+C))) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sec\left[\frac{1}{2}(c+d x)\right]^2 + a A b \tan\left[\frac{1}{2}(c+d x)\right] + A b^2 \tan\left[\frac{1}{2}(c+d x)\right] + 3 a^2 B \tan\left[\frac{1}{2}(c+d x)\right] + 3 a b B \tan\left[\frac{1}{2}(c+d x)\right] + \\
& 12 a b C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \tan\left[\frac{1}{2}(c+d x)\right]^2 - \\
& 2 A b^2 \tan\left[\frac{1}{2}(c+d x)\right]^3 - 6 a b B \tan\left[\frac{1}{2}(c+d x)\right]^3 - a A b \tan\left[\frac{1}{2}(c+d x)\right]^5 + A b^2 \tan\left[\frac{1}{2}(c+d x)\right]^5 - \\
& \left. 3 a^2 B \tan\left[\frac{1}{2}(c+d x)\right]^5 + 3 a b B \tan\left[\frac{1}{2}(c+d x)\right]^5\right) - \frac{1}{3 a \sqrt{a+b \cos [c+d x]} \left(\sec\left[\frac{1}{2}(c+d x)\right]\right)^{3/2}} \\
& 2 \sqrt{\cos\left[\frac{1}{2}(c+d x)\right]^2 \sec [c+d x]} \left(\frac{1}{2} a A b \sec\left[\frac{1}{2}(c+d x)\right]^2 + \frac{1}{2} A b^2 \sec\left[\frac{1}{2}(c+d x)\right]^2 + \frac{3}{2} a^2 B \sec\left[\frac{1}{2}(c+d x)\right]^2 + \right. \\
& \left. \frac{3}{2} a b B \sec\left[\frac{1}{2}(c+d x)\right]^2 + \frac{1}{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}} 6 a b C \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \right) \\
& \left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2} - \frac{\sin [c+d x]}{1+\cos [c+d x]}\right) + \frac{1}{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}} (a+b)(A b+3 a B) \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}
\end{aligned}$$



$$\text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\frac{\text{Cos}[c+dx] \text{Sin}[c+dx]}{(1+\text{Cos}[c+dx])^2} - \frac{\text{Sin}[c+dx]}{1+\text{Cos}[c+dx]}\right) - \frac{1}{\sqrt{\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}}}$$

$$a(b(A+3B-3C) + a(A+3(B+C))) \sqrt{\frac{a+b \text{Cos}[c+dx]}{(a+b)(1+\text{Cos}[c+dx])}} \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right]$$

$$\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\frac{\text{Cos}[c+dx] \text{Sin}[c+dx]}{(1+\text{Cos}[c+dx])^2} - \frac{\text{Sin}[c+dx]}{1+\text{Cos}[c+dx]}\right) + \frac{1}{\sqrt{\frac{a+b \text{Cos}[c+dx]}{(a+b)(1+\text{Cos}[c+dx])}}} 6abc \sqrt{\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}}$$

$$\text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \left(-\frac{b \text{Sin}[c+dx]}{(a+b)(1+\text{Cos}[c+dx])} + \frac{(a+b \text{Cos}[c+dx]) \text{Sin}[c+dx]}{(a+b)(1+\text{Cos}[c+dx])^2}\right) +$$

$$\frac{1}{\sqrt{\frac{a+b \text{Cos}[c+dx]}{(a+b)(1+\text{Cos}[c+dx])}}} (a+b)(Ab+3aB) \sqrt{\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right]$$

$$\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(-\frac{b \text{Sin}[c+dx]}{(a+b)(1+\text{Cos}[c+dx])} + \frac{(a+b \text{Cos}[c+dx]) \text{Sin}[c+dx]}{(a+b)(1+\text{Cos}[c+dx])^2}\right) - \frac{1}{\sqrt{\frac{a+b \text{Cos}[c+dx]}{(a+b)(1+\text{Cos}[c+dx])}}}$$

$$a(b(A+3B-3C) + a(A+3(B+C))) \sqrt{\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2$$

$$\left(-\frac{b \text{Sin}[c+dx]}{(a+b)(1+\text{Cos}[c+dx])} + \frac{(a+b \text{Cos}[c+dx]) \text{Sin}[c+dx]}{(a+b)(1+\text{Cos}[c+dx])^2}\right) + 2(a+b)(Ab+3aB) \sqrt{\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} \sqrt{\frac{a+b \text{Cos}[c+dx]}{(a+b)(1+\text{Cos}[c+dx])}}$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \text{Tan}\left[\frac{1}{2}(c+dx)\right] - 2a(b(A+3B-3C) + a(A+3(B+C)))$$

$$\sqrt{\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} \sqrt{\frac{a+b \text{Cos}[c+dx]}{(a+b)(1+\text{Cos}[c+dx])}} \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \text{Tan}\left[\frac{1}{2}(c+dx)\right] +$$

$$12abc \sqrt{\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} \sqrt{\frac{a+b \text{Cos}[c+dx]}{(a+b)(1+\text{Cos}[c+dx])}} \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2$$

$$\begin{aligned}
& \tan\left[\frac{1}{2}(c+dx)\right] - 3Ab^2 \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^2 - 9abB \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^2 + \frac{1}{\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}}} \\
& 6abC \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \left(\frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} - \frac{\sin[c+dx]}{1+\cos[c+dx]}\right) \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 + \frac{1}{\sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} 6abC \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \left(-\frac{b\sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(a+b\cos[c+dx])\sin[c+dx]}{(a+b)(1+\cos[c+dx])^2}\right) \tan\left[\frac{1}{2}(c+dx)\right]^2 - \frac{5}{2}aAb \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^4 + \\
& \frac{5}{2}Ab^2 \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^4 - \frac{15}{2}a^2B \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^4 + \frac{15}{2}abB \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^4 - \\
& \frac{a(b(A+3B-3C) + a(A+3(B+C))) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sec\left[\frac{1}{2}(c+dx)\right]^4}{\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{1-\frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} - \\
& \frac{6abC \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sec\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{1-\frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} - \\
& \frac{6abC \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{1-\frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} + \left((a+b)(Ab+3aB) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}}\right. \\
& \left. \sqrt{\frac{a+b\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sec\left[\frac{1}{2}(c+dx)\right]^4 \sqrt{1-\frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}\right) / \left(\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}\right) -
\end{aligned}$$

$$\left( \left( 12 a b C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]+2(a+b)\right. \right.$$

$$(A b+3 a B) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 -$$

$$2 a(b(A+3 B-3 C)+a(A+3(B+C))) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2+a A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+3 a^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+3 a b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+$$

$$12 a b C \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{a+b \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 -$$

$$2 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3-6 a b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3-a A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5-3 a^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+$$

$$3 a b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5\left(-\cos \left[\frac{1}{2}(c+d x)\right] \operatorname{Sec}[c+d x] \sin \left[\frac{1}{2}(c+d x)\right]+\cos \left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]\right) \Bigg) /$$

$$\left( 3 a \sqrt{a+b \cos [c+d x]} \left( \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right)^{3 / 2} \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]} \right) \Bigg)$$

- **Problem 1507: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+b \cos [c+d x]}(A+B \cos [c+d x]+C \cos [c+d x]^2) \sqrt{\operatorname{Sec}[c+d x]} d x$$

Optimal (type 4, 543 leaves, 8 steps):

$$\begin{aligned}
& - \frac{1}{4 a b d \sqrt{\operatorname{Sec}[c+d x]}} (a-b) \sqrt{a+b} (4 b B+a C) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{4 b d \sqrt{\operatorname{Sec}[c+d x]}} \\
& \sqrt{a+b} (8 A b+a C+2 b(2 B+C)) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{4 b^2 d \sqrt{\operatorname{Sec}[c+d x]}} \sqrt{a+b} (8 A b^2+4 a b B-a^2 C+4 b^2 C) \sqrt{\operatorname{Cos}[c+d x]} \\
& \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} + \\
& \frac{C \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{2 d \sqrt{\operatorname{Sec}[c+d x]}} + \frac{(4 b B+a C) \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 b d}
\end{aligned}$$

Result (type 4, 1816 leaves):

$$\begin{aligned}
& \frac{C \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[2(c+d x)]}{4 d} + \\
& \left( -4 a b \sqrt{\frac{a-b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 4 b^2 \sqrt{\frac{a-b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a^2 \sqrt{\frac{a-b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \right. \\
& a b \sqrt{\frac{a-b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 8 b^2 \sqrt{\frac{a-b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 2 a b \sqrt{\frac{a-b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + \\
& 4 a b \sqrt{\frac{a-b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 4 b^2 \sqrt{\frac{a-b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + a^2 \sqrt{\frac{a-b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \\
& a b \sqrt{\frac{a-b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 16 i A b^2 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \left. \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 8 i a b B \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], -\frac{a+b}{a-b}\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 i a^2 \operatorname{CEllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 8 i b^2 \operatorname{CEllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 16 i a b^2 \operatorname{EllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 8 i a b \operatorname{BEllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 i a^2 \operatorname{CEllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 8 i b^2 \operatorname{CEllipticPi}\left[\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& i (a-b) (4 b B + a C) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], -\frac{a+b}{a-b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)
\end{aligned}$$

$$\sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+2 i(a-b)(4 A b+(a+2 b) C) \operatorname{EllipticF}\left[\operatorname{ArcSinh}\left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right],-\frac{a+b}{a-b}\right]$$

$$\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \Bigg/$$

$$\left(4 b \sqrt{\frac{a-b}{a+b}} d \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3 / 2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}\right)$$

■ **Problem 1508: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \operatorname{Cos}[c+dx]}(A+B \operatorname{Cos}[c+dx]+C \operatorname{Cos}[c+dx]^2)}{\sqrt{\operatorname{Sec}[c+dx]}} dx$$

Optimal (type 4, 646 leaves, 9 steps):

$$-\frac{1}{24 a b^2 d \sqrt{\operatorname{Sec}[c+dx]}}(a-b) \sqrt{a+b}\left(8 b^2(3 A+2 C)+3 a(2 b B-a C)\right) \sqrt{\operatorname{Cos}[c+dx]}$$

$$\operatorname{Csc}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}}+$$

$$\frac{1}{24 b^2 d \sqrt{\operatorname{Sec}[c+dx]}} \sqrt{a+b}\left(24 A b^2+(a+2 b)(6 b B-3 a C+8 b C)\right) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Csc}[c+dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}}+\frac{1}{8 b^3 d \sqrt{\operatorname{Sec}[c+dx]}}$$

$$\sqrt{a+b}\left(2 a^2 b B-8 b^3 B-a^3 C-4 a b^2(2 A+C)\right) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+dx]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+dx]}}\right],-\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+dx])}{a-b}}+\frac{(2 b B-a C) \sqrt{a+b \operatorname{Cos}[c+dx]} \operatorname{Sin}[c+dx]}{4 b d \sqrt{\operatorname{Sec}[c+dx]}}+$$

$$\frac{C(a+b \operatorname{Cos}[c+dx])^{3 / 2} \operatorname{Sin}[c+dx]}{3 b d \sqrt{\operatorname{Sec}[c+dx]}}+\frac{\left(8 b^2(3 A+2 C)+3 a(2 b B-a C)\right) \sqrt{a+b \operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{24 b^2 d}$$

Result (type 4, 3904 leaves):

$$\frac{\sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]} \left( \frac{1}{12} C \sin [c+d x] + \frac{(6 b B+a C) \sin [2(c+d x)]}{24 b} + \frac{1}{12} C \sin [3(c+d x)] \right)}{d} +$$

$$\left( \frac{a A}{\sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]}} + \frac{b B}{2 \sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]}} + \frac{7 a C}{12 \sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]}} + \frac{A b \sqrt{\sec [c+d x]}}{2 \sqrt{a+b \cos [c+d x]}} +$$

$$\frac{3 a B \sqrt{\sec [c+d x]}}{8 \sqrt{a+b \cos [c+d x]}} - \frac{a^2 C \sqrt{\sec [c+d x]}}{48 b \sqrt{a+b \cos [c+d x]}} + \frac{b C \sqrt{\sec [c+d x]}}{3 \sqrt{a+b \cos [c+d x]}} + \frac{A b \cos [2(c+d x)] \sqrt{\sec [c+d x]}}{2 \sqrt{a+b \cos [c+d x]}} +$$

$$\left. \frac{a B \cos [2(c+d x)] \sqrt{\sec [c+d x]}}{8 \sqrt{a+b \cos [c+d x]}} - \frac{a^2 C \cos [2(c+d x)] \sqrt{\sec [c+d x]}}{16 b \sqrt{a+b \cos [c+d x]}} + \frac{b C \cos [2(c+d x)] \sqrt{\sec [c+d x]}}{3 \sqrt{a+b \cos [c+d x]}} \right)$$

$$\left( \frac{(24 A b^2 + 6 a b B - 3 a^2 C + 16 b^2 C) \tan \left[ \frac{1}{2} (c+d x) \right] \sqrt{\frac{a+b+a \tan \left[ \frac{1}{2} (c+d x) \right]^2 - b \tan \left[ \frac{1}{2} (c+d x) \right]^2}{1+\tan \left[ \frac{1}{2} (c+d x) \right]^2}}}{24 b^2 \sqrt{\frac{1+\tan \left[ \frac{1}{2} (c+d x) \right]^2}{1-\tan \left[ \frac{1}{2} (c+d x) \right]^2}}} +$$

$$\left( (a+b) (-24 A b^2 - 6 a b B + 3 a^2 C - 16 b^2 C) \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \tan \left[ \frac{1}{2} (c+d x) \right] \right] \right], \frac{-a+b}{a+b} \right) +$$

$$2 b (12 b^2 B - a^2 C + 2 a b (12 A - 3 B + 7 C)) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \tan \left[ \frac{1}{2} (c+d x) \right] \right] \right], \frac{-a+b}{a+b} \right] + 6 (-2 a^2 b B + 8 b^3 B + a^3 C + 4 a b^2 (2 A + C))$$

$$\operatorname{EllipticPi} \left[ -1, -\operatorname{ArcSin} \left[ \tan \left[ \frac{1}{2} (c+d x) \right] \right] \right], \frac{-a+b}{a+b} \right) \sqrt{\frac{a+b+a \tan \left[ \frac{1}{2} (c+d x) \right]^2 - b \tan \left[ \frac{1}{2} (c+d x) \right]^2}{1+\tan \left[ \frac{1}{2} (c+d x) \right]^2}} \sqrt{1-\tan \left[ \frac{1}{2} (c+d x) \right]^4} \right) /$$

$$\left( 24 b^2 (a+b) \sqrt{\frac{1}{1-\tan \left[ \frac{1}{2} (c+d x) \right]^2}} \left( -1+\tan \left[ \frac{1}{2} (c+d x) \right] \right)^2 \sqrt{\frac{a+b+a \tan \left[ \frac{1}{2} (c+d x) \right]^2 - b \tan \left[ \frac{1}{2} (c+d x) \right]^2}{a+b}} \right) \right) /$$

$$\begin{aligned}
& \left( d \left( \frac{(24 A b^2 + 6 a b B - 3 a^2 C + 16 b^2 C) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}}{48 b^2 \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}} \right) \right. \\
& \left( (a+b) (-24 A b^2 - 6 a b B + 3 a^2 C - 16 b^2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& 2 b (12 b^2 B - a^2 C + 2 a b (12 A - 3 B + 7 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \\
& \left. 6 (-2 a^2 b B + 8 b^3 B + a^3 C + 4 a b^2 (2 A + C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \left. \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) / \left( 24 b^2 (a+b) \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \left. (-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right])^2 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \right) \right) - \\
& \left( (a+b) (-24 A b^2 - 6 a b B + 3 a^2 C - 16 b^2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& 2 b (12 b^2 B - a^2 C + 2 a b (12 A - 3 B + 7 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + 6 (-2 a^2 b B + 8 b^3 B + a^3 C + 4 a b^2 (2 A + C)) \\
& \left. \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \left( a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \\
& \left. \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \right) / \\
& \left( 48 b^2 (a+b)^2 \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left( -1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right)^2 \left( \frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right)^{3/2} \right) -
\end{aligned}$$



$$\begin{aligned}
& \left( (a+b) (-24 A b^2 - 6 a b B + 3 a^2 C - 16 b^2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& 2 b (12 b^2 B - a^2 C + 2 a b (12 A - 3 B + 7 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \\
& \left. 6 (-2 a^2 b B + 8 b^3 B + a^3 C + 4 a b^2 (2 A + C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \Big/ \\
& \left( 24 b^2 (a+b) \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) - \\
& \left( (a+b) (-24 A b^2 - 6 a b B + 3 a^2 C - 16 b^2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& 2 b (12 b^2 B - a^2 C + 2 a b (12 A - 3 B + 7 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \\
& \left. 6 (-2 a^2 b B + 8 b^3 B + a^3 C + 4 a b^2 (2 A + C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \Big/ \\
& \left( 48 b^2 (a+b) \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) - \\
& \left( (24 A b^2 + 6 a b B - 3 a^2 C + 16 b^2 C) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \Bigg/ \left( 48 b^2 \left( \frac{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3/2} \right) + \\
& \left( (24 A b^2 + 6 a b B - 3 a^2 C + 16 b^2 C) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left( \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} - \right. \right. \\
& \left. \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left( a + b + a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)}{\left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \right) \Bigg/ \\
& \left( 48 b^2 \sqrt{\frac{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a + b + a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \\
& \left( (a+b) (-24 A b^2 - 6 a b B + 3 a^2 C - 16 b^2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& 2 b (12 b^2 B - a^2 C + 2 a b (12 A - 3 B + 7 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \\
& \left. 6 (-2 a^2 b B + 8 b^3 B + a^3 C + 4 a b^2 (2 A + C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}^4 \left( \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} - \right. \\
& \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left( a + b + a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)}{\left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \Bigg/ \\
& \left( 48 b^2 (a+b) \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left( -1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \sqrt{\frac{a + b + a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \left( \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \right. \\
& \left. \left( \frac{b(12b^2B - a^2C + 2ab(12A - 3B + 7C)) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{1-\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} - \frac{3(-2a^2bB + 8b^3B + a^3C + 4ab^2(2A+C)) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} (1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2) \sqrt{1-\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} \right) + \right. \\
& \left. \left. \frac{(a+b)(-24Ab^2 - 6abB + 3a^2C - 16b^2C) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}{2\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) \right) \Bigg/ \\
& \left. \left( 24b^2(a+b) \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) \Bigg)
\end{aligned}$$

■ **Problem 1509: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \operatorname{Cos}[c+dx]} (A+B \operatorname{Cos}[c+dx] + C \operatorname{Cos}[c+dx]^2)}{\operatorname{Sec}[c+dx]^{3/2}} dx$$

Optimal (type 4, 766 leaves, 10 steps):

$$\frac{1}{192 a b^3 d \sqrt{\sec[c+dx]}} (a-b) \sqrt{a+b} (24 a^2 b B - 128 b^3 B - 15 a^3 C - 4 a b^2 (12 A + 7 C)) \sqrt{\cos[c+dx]}$$

$$\operatorname{Csc}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{192 b^3 d \sqrt{\sec[c+dx]}} \sqrt{a+b} (15 a^3 C - 2 a^2 b (12 B + 5 C) + 4 a b^2 (12 A + 4 B + 7 C) + 8 b^3 (12 A + 16 B + 9 C)) \sqrt{\cos[c+dx]}$$

$$\operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{64 b^4 d \sqrt{\sec[c+dx]}} \sqrt{a+b} (8 a^3 b B + 32 a b^3 B - 5 a^4 C - 8 a^2 b^2 (2 A + C) + 16 b^4 (4 A + 3 C)) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx]$$

$$\operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{C(a+b \cos[c+dx])^{3/2} \sin[c+dx]}{4 b d \sec[c+dx]^{3/2}} + \frac{(16 A b^2 - 8 a b B + 5 a^2 C + 12 b^2 C) \sqrt{a+b \cos[c+dx]} \sin[c+dx]}{32 b^2 d \sqrt{\sec[c+dx]}} +$$

$$\frac{(8 b B - 5 a C) (a+b \cos[c+dx])^{3/2} \sin[c+dx]}{24 b^2 d \sqrt{\sec[c+dx]}} - \frac{(24 a^2 b B - 128 b^3 B - 15 a^3 C - 4 a b^2 (12 A + 7 C)) \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]} \sin[c+dx]}{192 b^3 d}$$

Result (type 4, 4399 leaves):

$$\frac{1}{d} \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]}$$

$$\left( \frac{(8 b B + a C) \sin[c+dx]}{96 b} + \frac{(48 A b^2 + 8 a b B - 5 a^2 C + 48 b^2 C) \sin[2(c+dx)]}{192 b^2} + \frac{(8 b B + a C) \sin[3(c+dx)]}{96 b} + \frac{1}{32} C \sin[4(c+dx)] \right) +$$

$$\left( \frac{A b}{2 \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]}} + \frac{7 a B}{12 \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]}} + \frac{a^2 C}{96 b \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]}} + \right.$$

$$\left. \frac{3 b C}{8 \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]}} + \frac{3 a A \sqrt{\sec[c+dx]}}{8 \sqrt{a+b \cos[c+dx]}} - \frac{a^2 B \sqrt{\sec[c+dx]}}{48 b \sqrt{a+b \cos[c+dx]}} + \frac{b B \sqrt{\sec[c+dx]}}{3 \sqrt{a+b \cos[c+dx]}} + \right.$$

$$\begin{aligned}
& \frac{25 a C \sqrt{\operatorname{Sec}[c+d x]}}{96 \sqrt{a+b \operatorname{Cos}[c+d x]}} + \frac{5 a^3 C \sqrt{\operatorname{Sec}[c+d x]}}{384 b^2 \sqrt{a+b \operatorname{Cos}[c+d x]}} + \frac{a A \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{8 \sqrt{a+b \operatorname{Cos}[c+d x]}} - \frac{a^2 B \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{16 b \sqrt{a+b \operatorname{Cos}[c+d x]}} + \\
& \left. \frac{b B \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{3 \sqrt{a+b \operatorname{Cos}[c+d x]}} + \frac{7 a C \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{96 \sqrt{a+b \operatorname{Cos}[c+d x]}} + \frac{5 a^3 C \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{128 b^2 \sqrt{a+b \operatorname{Cos}[c+d x]}} \right) \\
& \left( \left( (a+b) \left( -24 a^2 b B + 128 b^3 B + 15 a^3 C + 4 a b^2 (12 A + 7 C) \right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] - \right. \right. \\
& \left. \left. 2 b \left( 5 a^3 C + 2 a^2 b (-4 B + C) + 24 b^3 (4 A + 3 C) - 4 a b^2 (12 A - 28 B + 9 C) \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] + \right. \right. \\
& \left. \left. 6 \left( -8 a^3 b B - 32 a b^3 B + 5 a^4 C + 8 a^2 b^2 (2 A + C) - 16 b^4 (4 A + 3 C) \right) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \right) \right. \\
& \left. \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^4} \right) / \\
& \left( 192 b^3 (a+b) \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \left( -1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) + \\
& \left. \frac{(48 a A b^2 - 24 a^2 b B + 128 b^3 B + 15 a^3 C + 28 a b^2 C) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \sqrt{a + \frac{b-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}}{192 b^3 \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right) / \\
& \left( \left( (a+b) \left( -24 a^2 b B + 128 b^3 B + 15 a^3 C + 4 a b^2 (12 A + 7 C) \right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2b \left( 5a^3C + 2a^2b(-4B+C) + 24b^3(4A+3C) - 4ab^2(12A-28B+9C) \right) \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \\
& 6 \left( -8a^3bB - 32ab^3B + 5a^4C + 8a^2b^2(2A+C) - 16b^4(4A+3C) \right) \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \text{Tan}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \Big/ \left( 192b^3(a+b) \right. \\
& \left. \sqrt{\frac{1}{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left( -1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \right) + \\
& \left( (a+b) \left( -24a^2bB + 128b^3B + 15a^3C + 4ab^2(12A+7C) \right) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - \right. \\
& 2b \left( 5a^3C + 2a^2b(-4B+C) + 24b^3(4A+3C) - 4ab^2(12A-28B+9C) \right) \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \\
& \left. 6 \left( -8a^3bB - 32ab^3B + 5a^4C + 8a^2b^2(2A+C) - 16b^4(4A+3C) \right) \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \\
& \left( a \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \text{Tan}\left[\frac{1}{2}(c+dx)\right] - b \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \text{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \\
& \sqrt{\frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \Big/ \\
& \left( 384b^3(a+b)^2 \sqrt{\frac{1}{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left( -1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \left( \frac{a+b+a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right)^{3/2} \right) + \\
& \left( (a+b) \left( -24a^2bB + 128b^3B + 15a^3C + 4ab^2(12A+7C) \right) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - \right. \\
& 2b \left( 5a^3C + 2a^2b(-4B+C) + 24b^3(4A+3C) - 4ab^2(12A-28B+9C) \right) \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \\
& \left. 6 \left( -8a^3bB - 32ab^3B + 5a^4C + 8a^2b^2(2A+C) - 16b^4(4A+3C) \right) \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2-b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^4}\right) / \\
& \left( 192 b^3 (a+b) \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2-b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) + \\
& \left( (a+b) (-24 a^2 b B+128 b^3 B+15 a^3 C+4 a b^2 (12 A+7 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - \right. \\
& \quad 2 b (5 a^3 C+2 a^2 b (-4 B+C)+24 b^3 (4 A+3 C)-4 a b^2 (12 A-28 B+9 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \\
& \quad \left. 6 (-8 a^3 b B-32 a b^3 B+5 a^4 C+8 a^2 b^2 (2 A+C)-16 b^4 (4 A+3 C)) \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \\
& \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2-b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^4}\right) / \\
& \left( 384 b^3 (a+b) \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2-b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) + \\
& \left( (48 a A b^2-24 a^2 b B+128 b^3 B+15 a^3 C+28 a b^2 C) \tan\left[\frac{1}{2}(c+dx)\right] \right. \\
& \quad \left. \left( -\frac{b \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2} - \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] (b-b \tan\left[\frac{1}{2}(c+dx)\right]^2)}{(1+\tan\left[\frac{1}{2}(c+dx)\right]^2)^2} \right) \right) / \\
& \left( 384 b^3 \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1+\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{a+\frac{b-b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) - \\
& \left( (48 a A b^2-24 a^2 b B+128 b^3 B+15 a^3 C+28 a b^2 C) \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{a+\frac{b-b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( 384 b^3 \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \right) + \\
& \frac{(48 a A b^2 - 24 a^2 b B + 128 b^3 B + 15 a^3 C + 28 a b^2 C) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{a + \frac{b - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}}}{384 b^3 \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} - \\
& \left( (48 a A b^2 - 24 a^2 b B + 128 b^3 B + 15 a^3 C + 28 a b^2 C) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \left. \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{a + \frac{b - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) / \left( 384 b^3 \sqrt{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right) - \\
& \left( (a+b) (-24 a^2 b B + 128 b^3 B + 15 a^3 C + 4 a b^2 (12 A + 7 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] - \right. \\
& \left. 2 b (5 a^3 C + 2 a^2 b (-4 B + C) + 24 b^3 (4 A + 3 C) - 4 a b^2 (12 A - 28 B + 9 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& \left. 6 (-8 a^3 b B - 32 a b^3 B + 5 a^4 C + 8 a^2 b^2 (2 A + C) - 16 b^4 (4 A + 3 C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}^4 \left( \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} - \right. \\
& \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] (a+b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2)}{(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2)^2} \right) / \\
& \left( 384 b^3 (a+b) \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right)
\end{aligned}$$



$$\begin{aligned}
& \left( \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) - \left( \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}^4 \left( -\frac{b\left(5a^3C+2a^2b(-4B+C)+24b^3(4A+3C)-4ab^2(12A-28B+9C)\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{1-\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} \right) - \\
& \frac{3\left(-8a^3bB-32ab^3B+5a^4C+8a^2b^2(2A+C)-16b^4(4A+3C)\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{1-\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} + \left( (a+b)\left(-24a^2bB+128b^3B+ \right. \right. \\
& \left. \left. 15a^3C+4ab^2(12A+7C)\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) / \left( 2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) \left. \right) / \\
& \left( 192b^3(a+b) \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \left. \right) \left. \right)
\end{aligned}$$

■ **Problem 1510: Attempted integration timed out after 120 seconds.**

$$\int (a+b \operatorname{Cos}[c+dx])^{3/2} (A+B \operatorname{Cos}[c+dx]+C \operatorname{Cos}[c+dx]^2) \operatorname{Sec}[c+dx]^{11/2} dx$$

Optimal (type 4, 590 leaves, 8 steps):

$$\frac{1}{315 a^4 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} \left( 8 A b^4 + 246 a^3 b B - 18 a b^3 B + 21 a^4 (7 A + 9 C) + 3 a^2 b^2 (11 A + 21 C) \right) \sqrt{\cos[c+dx]}$$

$$\operatorname{Csc}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{315 a^3 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} \left( 8 A b^3 + 6 a b^2 (A-3 B) + 3 a^2 b (13 A - 57 B + 21 C) - 3 a^3 (49 A - 25 B + 63 C) \right) \sqrt{\cos[c+dx]}$$

$$\operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} -$$

$$\frac{2 \left( 4 A b^3 - 75 a^3 B - 9 a b^2 B - 2 a^2 b (44 A + 63 C) \right) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{315 a^2 d} +$$

$$\frac{2 \left( 3 A b^2 + 72 a b B + 7 a^2 (7 A + 9 C) \right) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{5/2} \sin[c+dx]}{315 a d} +$$

$$\frac{2 (A b + 3 a B) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{7/2} \sin[c+dx]}{21 d} + \frac{2 A (a+b \cos[c+dx])^{3/2} \sec[c+dx]^{9/2} \sin[c+dx]}{9 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1511: Attempted integration timed out after 120 seconds.**

$$\int (a+b \cos[c+dx])^{3/2} (A+B \cos[c+dx]+C \cos[c+dx]^2) \sec[c+dx]^{9/2} dx$$

Optimal (type 4, 490 leaves, 7 steps):

$$\begin{aligned}
& - \frac{1}{105 a^3 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (6 A b^3 - 63 a^3 B - 21 a b^2 B - 2 a^2 b (41 A + 70 C)) \sqrt{\cos[c+dx]} \\
& \quad \text{Csc}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\
& \frac{1}{105 a^2 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} (6 A b^2 - a^2 (25 A - 63 B + 35 C) + 3 a b (19 A - 7 B + 35 C)) \sqrt{\cos[c+dx]} \text{Csc}[c+dx] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b} \cos[c+dx]}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{2 (3 A b^2 + 42 a b B + 5 a^2 (5 A + 7 C)) \sqrt{a+b} \cos[c+dx] \sec[c+dx]^{3/2} \sin[c+dx]}{105 a d} + \\
& \frac{2 (3 A b + 7 a B) \sqrt{a+b} \cos[c+dx] \sec[c+dx]^{5/2} \sin[c+dx]}{35 d} + \frac{2 A (a+b \cos[c+dx])^{3/2} \sec[c+dx]^{7/2} \sin[c+dx]}{7 d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1514: Result more than twice size of optimal antiderivative.**

$$\int (a+b \cos[c+dx])^{3/2} (A+B \cos[c+dx] + C \cos[c+dx]^2) \sec[c+dx]^{3/2} dx$$

Optimal (type 4, 595 leaves, 9 steps):

$$\begin{aligned}
& \frac{1}{4 a d \sqrt{\operatorname{Sec}[c+d x]}} (a-b) \sqrt{a+b} (8 a A-4 b B-5 a C) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{4 d \sqrt{\operatorname{Sec}[c+d x]}} \sqrt{a+b} (a(8 A-8 B-5 C)-2 b(8 A+2 B+C)) \sqrt{\operatorname{Cos}[c+d x]} \\
& \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{4 b d \sqrt{\operatorname{Sec}[c+d x]}} \\
& \sqrt{a+b} (8 A b^2+12 a b B+3 a^2 C+4 b^2 C) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Cos}[c+d x]}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right], -\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{b(4 A-C) \sqrt{a+b} \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{2 d \sqrt{\operatorname{Sec}[c+d x]}} - \\
& \frac{(8 a A-4 b B-5 a C) \sqrt{a+b} \operatorname{Cos}[c+d x] \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d} + \frac{2 A(a+b \operatorname{Cos}[c+d x])^{3/2} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{d}
\end{aligned}$$

Result (type 4, 1469 leaves):

$$\begin{aligned}
& \frac{\sqrt{a+b} \operatorname{Cos}[c+d x] \sqrt{\operatorname{Sec}[c+d x]} (2 a A \operatorname{Sin}[c+d x] + \frac{1}{4} b C \operatorname{Sin}[2(c+d x)])}{d} + \\
& \left( 8 a^2 A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 8 a A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 4 a b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 4 b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 5 a^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \right. \\
& 5 a b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 16 a A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 8 b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 10 a b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - 8 a^2 A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + \\
& 8 a A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 4 a b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 4 b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 5 a^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 5 a b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + \\
& 16 A b^2 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 24 a b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 6 a^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} +
\end{aligned}$$

$$\begin{aligned}
& 8 b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 16 A b^2 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 24 a b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 6 a^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 8 b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + (a+b) (8 a A - 4 b B - 5 a C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 \left(4 a^2 (A+B-C) - 2 b^2 (2 A+C) + a b (8 A-8 B+C)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \Big/ \\
& \left(4 d \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}\right)
\end{aligned}$$

■ **Problem 1516:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \cos[c + dx])^{3/2} (A + B \cos[c + dx] + C \cos[c + dx]^2)}{\sqrt{\sec[c + dx]}} dx$$

Optimal (type 4, 764 leaves, 10 steps):

$$\begin{aligned} & - \frac{1}{192 a b^2 d \sqrt{\sec[c + dx]}} (a - b) \sqrt{a + b} (24 a^2 b B + 128 b^3 B - 9 a^3 C + 12 a b^2 (20 A + 13 C)) \sqrt{\cos[c + dx]} \\ & \quad \text{Csc}[c + dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \\ & \frac{1}{192 b^2 d \sqrt{\sec[c + dx]}} \sqrt{a + b} (9 a^3 C - 6 a^2 b (4 B + C) - 8 b^3 (12 A + 16 B + 9 C) - 4 a b^2 (60 A + 28 B + 39 C)) \sqrt{\cos[c + dx]} \\ & \quad \text{Csc}[c + dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \\ & \frac{1}{64 b^3 d \sqrt{\sec[c + dx]}} \sqrt{a + b} (8 a^3 b B - 96 a b^3 B - 3 a^4 C - 24 a^2 b^2 (2 A + C) - 16 b^4 (4 A + 3 C)) \sqrt{\cos[c + dx]} \text{Csc}[c + dx] \\ & \quad \text{EllipticPi}\left[\frac{a + b}{b}, \text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \\ & \frac{(4 b^2 (4 A + 3 C) + a (8 b B - 3 a C)) \sqrt{a + b \cos[c + dx]} \sin[c + dx]}{32 b d \sqrt{\sec[c + dx]}} + \frac{(8 b B - 3 a C) (a + b \cos[c + dx])^{3/2} \sin[c + dx]}{24 b d \sqrt{\sec[c + dx]}} + \\ & \frac{C (a + b \cos[c + dx])^{5/2} \sin[c + dx]}{4 b d \sqrt{\sec[c + dx]}} + \frac{(24 a^2 b B + 128 b^3 B - 9 a^3 C + 12 a b^2 (20 A + 13 C)) \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} \sin[c + dx]}{192 b^2 d} \end{aligned}$$

Result (type 4, 4478 leaves):

$$\begin{aligned} & \frac{1}{d} \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} \left( \frac{1}{96} (8 b B + 9 a C) \sin[c + dx] + \right. \\ & \quad \left. \frac{(48 A b^2 + 56 a b B + 3 a^2 C + 48 b^2 C) \sin[2(c + dx)]}{192 b} + \frac{1}{96} (8 b B + 9 a C) \sin[3(c + dx)] + \frac{1}{32} b C \sin[4(c + dx)] \right) + \\ & \left( \frac{a^2 A}{\sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]}} + \frac{A b^2}{2 \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]}} + \frac{13 a b B}{12 \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]}} + \right. \end{aligned}$$

$$\begin{aligned}
& \frac{19 a^2 C}{32 \sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]}} + \frac{3 b^2 C}{8 \sqrt{a+b \cos [c+d x]} \sqrt{\sec [c+d x]}} + \frac{7 a A b \sqrt{\sec [c+d x]}}{8 \sqrt{a+b \cos [c+d x]}} + \\
& \frac{17 a^2 B \sqrt{\sec [c+d x]}}{48 \sqrt{a+b \cos [c+d x]}} + \frac{b^2 B \sqrt{\sec [c+d x]}}{3 \sqrt{a+b \cos [c+d x]}} - \frac{a^3 C \sqrt{\sec [c+d x]}}{128 b \sqrt{a+b \cos [c+d x]}} + \frac{19 a b C \sqrt{\sec [c+d x]}}{32 \sqrt{a+b \cos [c+d x]}} + \\
& \frac{5 a A b \cos [2(c+d x)] \sqrt{\sec [c+d x]}}{8 \sqrt{a+b \cos [c+d x]}} + \frac{a^2 B \cos [2(c+d x)] \sqrt{\sec [c+d x]}}{16 \sqrt{a+b \cos [c+d x]}} + \frac{b^2 B \cos [2(c+d x)] \sqrt{\sec [c+d x]}}{3 \sqrt{a+b \cos [c+d x]}} - \\
& \left( \frac{3 a^3 C \cos [2(c+d x)] \sqrt{\sec [c+d x]}}{128 b \sqrt{a+b \cos [c+d x]}} + \frac{13 a b C \cos [2(c+d x)] \sqrt{\sec [c+d x]}}{32 \sqrt{a+b \cos [c+d x]}} \right) \left( \frac{1}{192 b^2 \sqrt{\frac{1+\tan \left[\frac{1}{2}(c+d x)\right]^2}{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}}} \right) \\
& (24 a^2 b B + 128 b^3 B - 9 a^3 C + 12 a b^2 (20 A + 13 C)) \tan \left[ \frac{1}{2}(c+d x) \right] \sqrt{\frac{a+b+a \tan \left[\frac{1}{2}(c+d x)\right]^2 - b \tan \left[\frac{1}{2}(c+d x)\right]^2}{1+\tan \left[\frac{1}{2}(c+d x)\right]^2}} + \\
& \left( (a+b) (-24 a^2 b B - 128 b^3 B + 9 a^3 C - 12 a b^2 (20 A + 13 C)) \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \tan \left[ \frac{1}{2}(c+d x) \right] \right] \right], \frac{-a+b}{a+b} \right) + \\
& 2 b (-3 a^3 C + 24 b^3 (4 A + 3 C) - 4 a b^2 (12 A - 52 B + 9 C) + 2 a^2 b (96 A - 28 B + 57 C)) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \tan \left[ \frac{1}{2}(c+d x) \right] \right] \right], \frac{-a+b}{a+b} \right) + \\
& 6 (-8 a^3 b B + 96 a b^3 B + 3 a^4 C + 24 a^2 b^2 (2 A + C) + 16 b^4 (4 A + 3 C)) \operatorname{EllipticPi} \left[ -1, -\operatorname{ArcSin} \left[ \tan \left[ \frac{1}{2}(c+d x) \right] \right] \right], \frac{-a+b}{a+b} \right) \\
& \left. \sqrt{\frac{a+b+a \tan \left[\frac{1}{2}(c+d x)\right]^2 - b \tan \left[\frac{1}{2}(c+d x)\right]^2}{1+\tan \left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^4} \right) / \\
& \left( 192 b^2 (a+b) \sqrt{\frac{1}{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}} \left( -1+\tan \left[\frac{1}{2}(c+d x)\right]^2 \right) \sqrt{\frac{a+b+a \tan \left[\frac{1}{2}(c+d x)\right]^2 - b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) \right) /
\end{aligned}$$

$$\left( d \left( \frac{1}{384 b^2 \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}}} (24 a^2 b B + 128 b^3 B - 9 a^3 C + 12 a b^2 (20 A + 13 C)) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right. \right.$$

$$\left. \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} - \right.$$

$$\left( (a+b) (-24 a^2 b B - 128 b^3 B + 9 a^3 C - 12 a b^2 (20 A + 13 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right.$$

$$2 b (-3 a^3 C + 24 b^3 (4 A + 3 C) - 4 a b^2 (12 A - 52 B + 9 C) + 2 a^2 b (96 A - 28 B + 57 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] +$$

$$6 (-8 a^3 b B + 96 a b^3 B + 3 a^4 C + 24 a^2 b^2 (2 A + C) + 16 b^4 (4 A + 3 C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right]$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) / \left( 192 b^2 (a+b) \right.$$

$$\left. \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^4} \right) -$$

$$\left( (a+b) (-24 a^2 b B - 128 b^3 B + 9 a^3 C - 12 a b^2 (20 A + 13 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right.$$

$$2 b (-3 a^3 C + 24 b^3 (4 A + 3 C) - 4 a b^2 (12 A - 52 B + 9 C) + 2 a^2 b (96 A - 28 B + 57 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] +$$

$$6 (-8 a^3 b B + 96 a b^3 B + 3 a^4 C + 24 a^2 b^2 (2 A + C) + 16 b^4 (4 A + 3 C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right]$$

$$\left( a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right)$$



$$\begin{aligned}
& \left( \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \right) / \\
& \left( 384 b^2 (a+b)^2 \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2 \left(\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right)^{3/2} \right) - \\
& \left( (a+b) \left(-24 a^2 b B-128 b^3 B+9 a^3 C-12 a b^2(20 A+13 C)\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& \quad 2 b\left(-3 a^3 C+24 b^3(4 A+3 C)-4 a b^2(12 A-52 B+9 C)+2 a^2 b(96 A-28 B+57 C)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \\
& \quad \left. 6\left(-8 a^3 b B+96 a b^3 B+3 a^4 C+24 a^2 b^2(2 A+C)+16 b^4(4 A+3 C)\right) \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \right) / \\
& \left( 192 b^2 (a+b) \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) - \\
& \left( (a+b) \left(-24 a^2 b B-128 b^3 B+9 a^3 C-12 a b^2(20 A+13 C)\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \right. \\
& \quad 2 b\left(-3 a^3 C+24 b^3(4 A+3 C)-4 a b^2(12 A-52 B+9 C)+2 a^2 b(96 A-28 B+57 C)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] + \\
& \quad \left. 6\left(-8 a^3 b B+96 a b^3 B+3 a^4 C+24 a^2 b^2(2 A+C)+16 b^4(4 A+3 C)\right) \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \right) \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( 384 b^2 (a+b) \left( -1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2 \right) \sqrt{\frac{a+b+a \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2 - b \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}{a+b}} \right) - \\
& \left( (24 a^2 b B + 128 b^3 B - 9 a^3 C + 12 a b^2 (20 A + 13 C)) \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] \sqrt{\frac{a+b+a \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2 - b \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}{1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}} \right. \\
& \left. \left( \frac{\operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]}{1 - \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2} + \frac{\operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2 \right)}{\left( 1 - \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2 \right)^2} \right) \right) / \\
& \left( 384 b^2 \left( \frac{1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}{1 - \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2} \right)^{3/2} \right) + \left( (24 a^2 b B + 128 b^3 B - 9 a^3 C + 12 a b^2 (20 A + 13 C)) \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] \right. \\
& \left. \left( \frac{a \operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] - b \operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2} - \right. \right. \\
& \left. \left. \frac{\operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] \left( a+b+a \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2 - b \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2 \right)}{\left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2 \right)^2} \right) \right) / \\
& \left( 384 b^2 \sqrt{\frac{1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}{1 - \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}} \sqrt{\frac{a+b+a \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2 - b \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}{1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}} \right) + \\
& \left( (a+b) (-24 a^2 b B - 128 b^3 B + 9 a^3 C - 12 a b^2 (20 A + 13 C)) \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] \right], \frac{-a+b}{a+b} \right] + \right. \\
& 2 b (-3 a^3 C + 24 b^3 (4 A + 3 C) - 4 a b^2 (12 A - 52 B + 9 C) + 2 a^2 b (96 A - 28 B + 57 C)) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] \right], \frac{-a+b}{a+b} \right] + \\
& \left. 6 (-8 a^3 b B + 96 a b^3 B + 3 a^4 C + 24 a^2 b^2 (2 A + C) + 16 b^4 (4 A + 3 C)) \operatorname{EllipticPi} \left[ -1, -\operatorname{ArcSin} \left[ \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] \right], \frac{-a+b}{a+b} \right] \right) \\
& \sqrt{1 - \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2}^4 \left( \frac{a \operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right] - b \operatorname{Sec} \left[ \frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[ \frac{1}{2} (c+dx) \right]^2} - \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left(a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \right) \right) / \left( 384 b^2 (a+b) \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \left. \left. \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) \right) \right) + \\
& \left( \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left( b(-3a^3C+24b^3(4A+3C) - 4ab^2(12A-52B+9C) + \right. \right. \right. \\
& \left. \left. \left. 2a^2b(96A-28B+57C) \sec\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right) / \left( \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{1-\frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) - \\
& \frac{3(-8a^3bB+96ab^3B+3a^4C+24a^2b^2(2A+C)+16b^4(4A+3C)) \sec\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{1-\frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} + \left( (a+b)(-24a^2bB-128b^3B+ \right. \right. \\
& \left. \left. \left. 9a^3C-12ab^2(20A+13C) \sec\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) / \left( 2\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \right) \right) \right) / \\
& \left. \left. \left. \left( 192b^2(a+b) \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 1517: Attempted integration timed out after 120 seconds.**

$$\int (a+b \cos[c+dx])^{5/2} (A+B \cos[c+dx] + C \cos[c+dx]^2) \sec[c+dx]^{13/2} dx$$

Optimal (type 4, 705 leaves, 9 steps) :

$$\frac{1}{3465 a^4 d \sqrt{\sec [c+d x]}} 2 (a-b) \sqrt{a+b} \left(40 A b^5+1617 a^5 B+3069 a^3 b^2 B-110 a b^4 B+15 a^2 b^3 (17 A+33 C)+15 a^4 b (247 A+319 C)\right) \sqrt{\cos [c+d x]}$$

$$\operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}+\frac{1}{3465 a^3 d \sqrt{\sec [c+d x]}}$$

$$2(a-b) \sqrt{a+b} \left(40 A b^4+10 a b^3(3 A-11 B)+15 a^2 b^2(19 A-121 B+33 C)+3 a^4(225 A-539 B+275 C)-6 a^3 b(505 A-209 B+660 C)\right)$$

$$\sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}}-$$

$$\frac{1}{3465 a^2 d} 2\left(20 A b^4-1793 a^3 b B-55 a b^3 B-75 a^4(9 A+11 C)-5 a^2 b^2(205 A+297 C)\right) \sqrt{a+b \cos [c+d x]} \sec [c+d x]^{3 / 2} \sin [c+d x]+$$

$$\frac{2\left(15 A b^3+539 a^3 B+825 a b^2 B+5 a^2 b(229 A+297 C)\right) \sqrt{a+b \cos [c+d x]} \sec [c+d x]^{5 / 2} \sin [c+d x]}{3465 a d}+$$

$$\frac{2\left(5 A b^2+44 a b B+3 a^2(9 A+11 C)\right) \sqrt{a+b \cos [c+d x]} \sec [c+d x]^{7 / 2} \sin [c+d x]}{231 d}+$$

$$\frac{2(5 A b+11 a B)(a+b \cos [c+d x])^{3 / 2} \sec [c+d x]^{9 / 2} \sin [c+d x]}{99 d}+\frac{2 A(a+b \cos [c+d x])^{5 / 2} \sec [c+d x]^{11 / 2} \sin [c+d x]}{11 d}$$

Result (type 1, 1 leaves) :

???

■ **Problem 1518: Attempted integration timed out after 120 seconds.**

$$\int (a+b \cos [c+d x])^{5 / 2}(A+B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]^{11 / 2} d x$$

Optimal (type 4, 592 leaves, 8 steps) :

$$\begin{aligned}
& - \frac{1}{315 a^3 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} \left( 10 A b^4 - 435 a^3 b B - 45 a b^3 B - 21 a^4 (7 A + 9 C) - 3 a^2 b^2 (93 A + 161 C) \right) \sqrt{\cos[c+dx]} \\
& \quad \operatorname{Csc}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \\
& \frac{1}{315 a^2 d \sqrt{\sec[c+dx]}} 2 (a-b) \sqrt{a+b} \left( 10 A b^3 + 15 a b^2 (11 A - 3 B + 21 C) - 6 a^2 b (19 A - 60 B + 28 C) + 3 a^3 (49 A - 25 B + 63 C) \right) \\
& \quad \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{2 \left( 5 A b^3 + 75 a^3 B + 135 a b^2 B + a^2 b (163 A + 231 C) \right) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{3/2} \sin[c+dx]}{315 a d} + \\
& \frac{2 \left( 15 A b^2 + 90 a b B + 7 a^2 (7 A + 9 C) \right) \sqrt{a+b \cos[c+dx]} \sec[c+dx]^{5/2} \sin[c+dx]}{315 d} + \\
& \frac{2 (5 A b + 9 a B) (a+b \cos[c+dx])^{3/2} \sec[c+dx]^{7/2} \sin[c+dx]}{63 d} + \frac{2 A (a+b \cos[c+dx])^{5/2} \sec[c+dx]^{9/2} \sin[c+dx]}{9 d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1521: Result more than twice size of optimal antiderivative.**

$$\int (a+b \cos[c+dx])^{5/2} (A+B \cos[c+dx]+C \cos[c+dx]^2) \sec[c+dx]^{5/2} dx$$

Optimal (type 4, 682 leaves, 10 steps):

$$\begin{aligned}
& \frac{1}{12 a d \sqrt{\operatorname{Sec}[c+d x]}} (a-b) \sqrt{a+b} \left(24 a^2 B-12 b^2 B+a b(56 A-27 C)\right) \sqrt{\operatorname{Cos}[c+d x]} \\
& \operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \\
& \frac{1}{12 d \sqrt{\operatorname{Sec}[c+d x]}} \sqrt{a+b} \left(a b(56 A-72 B-27 C)-6 b^2(12 A+2 B+C)-8 a^2(A-3 B+3 C)\right) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{4 d \sqrt{\operatorname{Sec}[c+d x]}} \\
& \sqrt{a+b} \left(8 A b^2+20 a b B+15 a^2 C+4 b^2 C\right) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Cos}[c+d x]}}{\sqrt{a+b} \sqrt{\operatorname{Cos}[c+d x]}}\right],-\frac{a+b}{a-b}\right] \\
& \sqrt{\frac{a(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{b(8 A b+4 a B-b C) \sqrt{a+b \operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{2 d \sqrt{\operatorname{Sec}[c+d x]}} - \\
& \frac{\left(24 a^2 B-12 b^2 B+a b(56 A-27 C)\right) \sqrt{a+b \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{12 d} + \\
& \frac{2(5 A b+3 a B)(a+b \operatorname{Cos}[c+d x])^{3 / 2} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{3 d} + \frac{2 A(a+b \operatorname{Cos}[c+d x])^{5 / 2} \operatorname{Sec}[c+d x]^{3 / 2} \operatorname{Sin}[c+d x]}{3 d}
\end{aligned}$$

Result (type 4, 1640 leaves):

$$\begin{aligned}
& \left(56 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+56 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+24 a^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]+24 a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]-12 a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]-\right. \\
& 12 b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]-27 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]-27 a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]-112 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3-48 a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3+ \\
& 24 b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3+54 a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3-56 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+56 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5-24 a^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+ \\
& 24 a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+12 a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5-12 b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+27 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5-27 a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5+ \\
& \left.48 A b^3 \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right],-\frac{a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+\right.
\end{aligned}$$

$$\begin{aligned}
& 120 a b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 90 a^2 b C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 24 b^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 48 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 120 a b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 90 a^2 b C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 24 b^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + (a+b) (24 a^2 B - 12 b^2 B + a b (56 A - 27 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 (4 a^2 b (7 A + 9 B - 9 C) - 6 b^3 (2 A + C) + 3 a b^2 (12 A - 12 B + C) + 4 a^3 (A + 3 (B + C))) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \Big/
\end{aligned}$$

$$\left( 12 d \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \left(-1 + \tan\left[\frac{1}{2}(c + dx)\right]\right)^2 \left(1 + \tan\left[\frac{1}{2}(c + dx)\right]\right)^2 \right)^{3/2} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}} + \frac{1}{d} \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} \left( \frac{2}{3} a (7Ab + 3aB) \sin[c + dx] + \frac{1}{4} b^2 C \sin[2(c + dx)] + \frac{2}{3} a^2 A \tan[c + dx] \right)$$

■ **Problem 1522: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cos[c + dx])^{5/2} (A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^{3/2} dx$$

Optimal (type 4, 707 leaves, 10 steps):

$$\begin{aligned} & - \frac{1}{24 a d \sqrt{\sec[c + dx]}} (a - b) \sqrt{a + b} (54 a b B - a^2 (48 A - 33 C) + 8 b^2 (3 A + 2 C)) \sqrt{\cos[c + dx]} \\ & \quad \text{Csc}[c + dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \\ & \frac{1}{24 d \sqrt{\sec[c + dx]}} \sqrt{a + b} (a^2 (48 A - 48 B - 33 C) - 4 b^2 (6 A + 3 B + 4 C) - 2 a b (72 A + 27 B + 13 C)) \sqrt{\cos[c + dx]} \\ & \quad \text{Csc}[c + dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \\ & \frac{1}{8 b d \sqrt{\sec[c + dx]}} \sqrt{a + b} (30 a^2 b B + 8 b^3 B + 5 a^3 C + 20 a b^2 (2 A + C)) \sqrt{\cos[c + dx]} \text{Csc}[c + dx] \\ & \quad \text{EllipticPi}\left[\frac{a + b}{b}, \text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \\ & \frac{b(8 a A - 2 b B - 3 a C) \sqrt{a + b \cos[c + dx]} \sin[c + dx]}{4 d \sqrt{\sec[c + dx]}} - \frac{b(6 A - C) (a + b \cos[c + dx])^{3/2} \sin[c + dx]}{3 d \sqrt{\sec[c + dx]}} + \\ & \frac{(54 a b B - a^2 (48 A - 33 C) + 8 b^2 (3 A + 2 C)) \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} \sin[c + dx]}{24 d} + \frac{2 A (a + b \cos[c + dx])^{5/2} \sqrt{\sec[c + dx]} \sin[c + dx]}{d} \end{aligned}$$

Result (type 4, 1940 leaves):

$$\frac{1}{d} \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} \left( \frac{1}{12} (24 a^2 A + b^2 C) \sin[c + dx] + \frac{1}{24} b (6 b B + 13 a C) \sin[2(c + dx)] + \frac{1}{12} b^2 C \sin[3(c + dx)] \right) +$$



$$\begin{aligned}
& \left( 48 a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 48 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 24 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 24 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 54 a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \right. \\
& 54 a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 33 a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 33 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 16 a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 16 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \\
& 96 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 48 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 108 a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 66 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 32 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 - \\
& 48 a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 48 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 24 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 24 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 54 a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - \\
& \left. 54 a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 33 a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 33 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 16 a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 16 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + \right. \\
& 240 a A b^2 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 180 a^2 b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 48 b^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 30 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 120 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 240 a A b^2 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 180 a^2 b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 48 b^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right]
\end{aligned}$$

$$\begin{aligned}
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 30 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 120 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (-54 a b B + a^2 (48 A - 33 C) - 8 b^2 (3 A + 2 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 (-12 b^3 B + 24 a^3 (A+B-C) + a^2 b (72 A - 72 B + 13 C) - 2 a b^2 (36 A - 3 B + 19 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \Big/ \\
& \left(24 d \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b+a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}}\right)
\end{aligned}$$

■ **Problem 1524: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a+b \cos[c+dx])^{5/2} (A+B \cos[c+dx] + C \cos[c+dx]^2)}{\sqrt{\sec[c+dx]}} dx$$

Optimal (type 4, 894 leaves, 11 steps):

$$\begin{aligned}
& - \frac{1}{1920 a b^2 d \sqrt{\sec[c+dx]}} (a-b) \sqrt{a+b} \left( 150 a^3 b B + 2840 a b^3 B - 45 a^4 C + 256 b^4 (5 A + 4 C) + 12 a^2 b^2 (220 A + 141 C) \right) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{1920 b^2 d \sqrt{\sec[c+dx]}} \\
& \sqrt{a+b} \left( 45 a^4 C - 30 a^3 b (5 B + C) - 16 b^4 (80 A + 45 B + 64 C) - 8 a b^3 (260 A + 355 B + 193 C) - 4 a^2 b^2 (660 A + 295 B + 423 C) \right) \\
& \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{1}{128 b^3 d \sqrt{\sec[c+dx]}} \sqrt{a+b} \left( 10 a^4 b B - 240 a^2 b^3 B - 96 b^5 B - 3 a^5 C - 40 a^3 b^2 (2 A + C) - 80 a b^4 (4 A + 3 C) \right) \sqrt{\cos[c+dx]} \\
& \operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \\
& \frac{\left( 50 a^2 b B + 120 b^3 B - 15 a^3 C + 4 a b^2 (60 A + 43 C) \right) \sqrt{a+b \cos[c+dx]} \operatorname{Sin}[c+dx]}{320 b d \sqrt{\sec[c+dx]}} + \\
& \frac{\left( 80 A b^2 + 50 a b B - 15 a^2 C + 64 b^2 C \right) (a+b \cos[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{240 b d \sqrt{\sec[c+dx]}} + \\
& \frac{\left( 10 b B - 3 a C \right) (a+b \cos[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{40 b d \sqrt{\sec[c+dx]}} + \frac{C (a+b \cos[c+dx])^{7/2} \operatorname{Sin}[c+dx]}{5 b d \sqrt{\sec[c+dx]}} + \frac{1}{1920 b^2 d} \\
& \left( 150 a^3 b B + 2840 a b^3 B - 45 a^4 C + 256 b^4 (5 A + 4 C) + 12 a^2 b^2 (220 A + 141 C) \right) \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]} \operatorname{Sin}[c+dx]
\end{aligned}$$

Result (type 4, 803 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} \\
& \left( \frac{1}{960} (80 A b^2 + 170 a b B + 93 a^2 C + 88 b^2 C) \sin[c + dx] + \frac{(1040 a A b^2 + 590 a^2 b B + 480 b^3 B + 15 a^3 C + 1024 a b^2 C) \sin[2(c + dx)]}{1920 b} + \right. \\
& \left. \frac{1}{960} (80 A b^2 + 170 a b B + 93 a^2 C + 100 b^2 C) \sin[3(c + dx)] + \frac{1}{320} b (10 b B + 21 a C) \sin[4(c + dx)] + \frac{1}{80} b^2 C \sin[5(c + dx)] \right) + \\
& \frac{1}{1920 b^2 d} \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}} \\
& \left( (150 a^3 b B + 2840 a b^3 B - 45 a^4 C + 256 b^4 (5 A + 4 C) + 12 a^2 b^2 (220 A + 141 C)) \tan\left[\frac{1}{2}(c + dx)\right] + \right. \\
& \left. \frac{1}{\sqrt{\frac{a-b}{a+b}}} \left( a - a \tan\left[\frac{1}{2}(c + dx)\right]^4 + b \left(-1 + \tan\left[\frac{1}{2}(c + dx)\right]^2\right)^2 \right) \right. \\
& \left. i \left( (a - b) (-150 a^3 b B - 2840 a b^3 B + 45 a^4 C - 256 b^4 (5 A + 4 C) - 12 a^2 b^2 (220 A + 141 C)) \operatorname{EllipticE}\left[ i \operatorname{ArcSinh}\left[ \sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c + dx)\right] \right] \right], \right. \right. \\
& \left. \left. - \frac{a+b}{a-b} \right) - 2(a-b) (-720 b^4 B - 30 a^3 b (5 B - C) + 45 a^4 C - 4 a^2 b^2 (180 A + 185 B + 129 C) - 8 a b^3 (220 A + 45 B + 161 C)) \right. \\
& \left. \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[ \sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c + dx)\right] \right] \right], -\frac{a+b}{a-b} \right) + 30 (-10 a^4 b B + 240 a^2 b^3 B + 96 b^5 B + 3 a^5 C + \\
& \left. 40 a^3 b^2 (2 A + C) + 80 a b^4 (4 A + 3 C)) \operatorname{EllipticPi}\left[ \frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[ \sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(c + dx)\right] \right] \right], -\frac{a+b}{a-b} \right) \\
& \left. \left( -1 - \tan\left[\frac{1}{2}(c + dx)\right]^2 \right) \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} \right)
\end{aligned}$$

■ **Problem 1525: Attempted integration timed out after 120 seconds.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^{9/2}}{\sqrt{a + b \cos[c + dx]}} dx$$

Optimal (type 4, 506 leaves, 7 steps):

$$\begin{aligned} & - \frac{1}{105 a^5 d \sqrt{\sec[c + dx]}} 2 (a - b) \sqrt{a + b} (48 A b^3 - 63 a^3 B - 56 a b^2 B + a^2 (44 A b + 70 b C)) \sqrt{\cos[c + dx]} \\ & \quad \text{Csc}[c + dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \\ & \frac{1}{105 a^4 d \sqrt{\sec[c + dx]}} 2 \sqrt{a + b} (48 A b^3 - 4 a b^2 (3 A + 14 B) + a^3 (25 A - 63 B + 35 C) + 2 a^2 b (22 A + 7 (B + 5 C))) \sqrt{\cos[c + dx]} \\ & \quad \text{Csc}[c + dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \\ & \frac{2 (24 A b^2 - 28 a b B + 5 a^2 (5 A + 7 C)) \sqrt{a + b \cos[c + dx]} \sec[c + dx]^{3/2} \sin[c + dx]}{105 a^3 d} - \\ & \frac{2 (6 A b - 7 a B) \sqrt{a + b \cos[c + dx]} \sec[c + dx]^{5/2} \sin[c + dx]}{35 a^2 d} + \frac{2 A \sqrt{a + b \cos[c + dx]} \sec[c + dx]^{7/2} \sin[c + dx]}{7 a d} \end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1526: Attempted integration timed out after 120 seconds.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^{7/2}}{\sqrt{a + b \cos[c + dx]}} dx$$

Optimal (type 4, 412 leaves, 6 steps):

$$\frac{1}{15 a^4 d \sqrt{\sec [c+d x]}} 2(a-b) \sqrt{a+b} \left(8 A b^2-10 a b B+3 a^2(3 A+5 C)\right) \sqrt{\cos [c+d x]}$$

$$\operatorname{Csc}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} -$$

$$\frac{1}{15 a^3 d \sqrt{\sec [c+d x]}} 2 \sqrt{a+b} \left(8 A b^2-2 a b(A+5 B)+a^2(9 A-5 B+15 C)\right) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} -$$

$$\frac{2(4 A b-5 a B) \sqrt{a+b \cos [c+d x]} \sec [c+d x]^{3 / 2} \sin [c+d x]}{15 a^2 d} + \frac{2 A \sqrt{a+b \cos [c+d x]} \sec [c+d x]^{5 / 2} \sin [c+d x]}{5 a d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1527: Unable to integrate problem.**

$$\int \frac{(A+B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]^{5 / 2}}{\sqrt{a+b \cos [c+d x]}} d x$$

Optimal (type 4, 333 leaves, 5 steps):

$$-\frac{1}{3 a^3 d \sqrt{\sec [c+d x]}} 2(a-b) \sqrt{a+b} (2 A b-3 a B) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \frac{1}{3 a^2 d \sqrt{\sec [c+d x]}}$$

$$2 \sqrt{a+b} (2 A b+a(A-3 B+3 C)) \sqrt{\cos [c+d x]} \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos [c+d x]}}{\sqrt{a+b} \sqrt{\cos [c+d x]}}\right],-\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec [c+d x])}{a+b}} \sqrt{\frac{a(1+\sec [c+d x])}{a-b}} + \frac{2 A \sqrt{a+b \cos [c+d x]} \sec [c+d x]^{3 / 2} \sin [c+d x]}{3 a d}$$

Result (type 8, 47 leaves):

$$\int \frac{(A+B \cos [c+d x]+C \cos [c+d x]^2) \sec [c+d x]^{5 / 2}}{\sqrt{a+b \cos [c+d x]}} d x$$

■ **Problem 1530: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c + dx] + C \cos[c + dx]^2}{\sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]}} dx$$

Optimal (type 4, 545 leaves, 8 steps):

$$\begin{aligned} & -\frac{1}{4 a b^2 d \sqrt{\sec[c + dx]}} (a - b) \sqrt{a + b} (4 b B - 3 a C) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \\ & \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \frac{1}{4 b^2 d \sqrt{\sec[c + dx]}} \\ & \sqrt{a + b} (3 a C - 2 b (2 B + C)) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \\ & \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \frac{1}{4 b^3 d \sqrt{\sec[c + dx]}} \sqrt{a + b} (8 A b^2 - 4 a b B + 3 a^2 C + 4 b^2 C) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \\ & \operatorname{EllipticPi}\left[\frac{a + b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b} \sqrt{\cos[c + dx]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} + \\ & \frac{C \sqrt{a + b \cos[c + dx]} \operatorname{Sin}[c + dx]}{2 b d \sqrt{\sec[c + dx]}} + \frac{(4 b B - 3 a C) \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} \operatorname{Sin}[c + dx]}{4 b^2 d} \end{aligned}$$

Result (type 4, 1376 leaves):

$$\begin{aligned} & \frac{C \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} \operatorname{Sin}[2(c + dx)]}{4 b d} + \left( \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}} \right. \\ & \left( -4 a b B \tan\left[\frac{1}{2}(c + dx)\right] - 4 b^2 B \tan\left[\frac{1}{2}(c + dx)\right] + 3 a^2 C \tan\left[\frac{1}{2}(c + dx)\right] + 3 a b C \tan\left[\frac{1}{2}(c + dx)\right] + 8 b^2 B \tan\left[\frac{1}{2}(c + dx)\right]^3 - \right. \\ & \left. 6 a b C \tan\left[\frac{1}{2}(c + dx)\right]^3 + 4 a b B \tan\left[\frac{1}{2}(c + dx)\right]^5 - 4 b^2 B \tan\left[\frac{1}{2}(c + dx)\right]^5 - 3 a^2 C \tan\left[\frac{1}{2}(c + dx)\right]^5 + 3 a b C \tan\left[\frac{1}{2}(c + dx)\right]^5 + \right. \\ & \left. 16 A b^2 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{-a + b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} \right) \end{aligned}$$

$$\begin{aligned}
& 8 a b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 6 a^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 8 b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 16 A b^2 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 8 a b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 6 a^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 8 b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& (a+b)(-4 b B+3 a C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)^2 \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 2 b(4 A b-a C+2 b C) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right]
\end{aligned}$$



$$\left( \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) /$$

$$\left( 4b^2 d \sqrt{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \left( b \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - a \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \right) \right)$$

■ **Problem 1531: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \cos[c+dx] + C \cos^2[c+dx]}{\sqrt{a+b \cos[c+dx]} \sec^3[c+dx]} dx$$

Optimal (type 4, 653 leaves, 9 steps):

$$-\frac{1}{24ab^3d\sqrt{\sec[c+dx]}}(a-b)\sqrt{a+b}(24Ab^2-18abB+15a^2C+16b^2C)\sqrt{\cos[c+dx]}$$

$$\operatorname{Csc}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{24b^3d\sqrt{\sec[c+dx]}}\sqrt{a+b}(24Ab^2-18abB+12b^2B+15a^2C-10abC+16b^2C)\sqrt{\cos[c+dx]}\operatorname{Csc}[c+dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b}}\right], -\frac{a+b}{a-b}\right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{1}{8b^4d\sqrt{\sec[c+dx]}}$$

$$\sqrt{a+b}(6a^2bB+8b^3B-5a^3C-4ab^2(2A+C))\sqrt{\cos[c+dx]}\operatorname{Csc}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b}}\right], -\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} + \frac{C\sqrt{a+b \cos[c+dx]}\sin[c+dx]}{3bd\sec^3[c+dx]} +$$

$$\frac{(6bB-5aC)\sqrt{a+b \cos[c+dx]}\sin[c+dx]}{12b^2d\sqrt{\sec[c+dx]}} + \frac{(24Ab^2-18abB+15a^2C+16b^2C)\sqrt{a+b \cos[c+dx]}\sqrt{\sec[c+dx]}\sin[c+dx]}{24b^3d}$$

Result (type 4, 1837 leaves):

$$\frac{\sqrt{a+b \cos[c+dx]}\sqrt{\sec[c+dx]}\left(\frac{C \sin[c+dx]}{12b} + \frac{(6bB-5aC)\sin[2(c+dx)]}{24b^2} + \frac{C \sin[3(c+dx)]}{12b}\right)}{d}$$

$$\begin{aligned}
& \left( \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left( 24 a A b^2 \tan\left[\frac{1}{2}(c+dx)\right] + 24 A b^3 \tan\left[\frac{1}{2}(c+dx)\right] - 18 a^2 b B \tan\left[\frac{1}{2}(c+dx)\right] - 18 a b^2 B \tan\left[\frac{1}{2}(c+dx)\right] + 15 a^3 C \tan\left[\frac{1}{2}(c+dx)\right] + \right. \\
& 15 a^2 b C \tan\left[\frac{1}{2}(c+dx)\right] + 16 a b^2 C \tan\left[\frac{1}{2}(c+dx)\right] + 16 b^3 C \tan\left[\frac{1}{2}(c+dx)\right] - 48 A b^3 \tan\left[\frac{1}{2}(c+dx)\right]^3 + 36 a b^2 B \tan\left[\frac{1}{2}(c+dx)\right]^3 - \\
& 30 a^2 b C \tan\left[\frac{1}{2}(c+dx)\right]^3 - 32 b^3 C \tan\left[\frac{1}{2}(c+dx)\right]^3 - 24 a A b^2 \tan\left[\frac{1}{2}(c+dx)\right]^5 + 24 A b^3 \tan\left[\frac{1}{2}(c+dx)\right]^5 + 18 a^2 b B \tan\left[\frac{1}{2}(c+dx)\right]^5 - \\
& 18 a b^2 B \tan\left[\frac{1}{2}(c+dx)\right]^5 - 15 a^3 C \tan\left[\frac{1}{2}(c+dx)\right]^5 + 15 a^2 b C \tan\left[\frac{1}{2}(c+dx)\right]^5 - 16 a b^2 C \tan\left[\frac{1}{2}(c+dx)\right]^5 + 16 b^3 C \tan\left[\frac{1}{2}(c+dx)\right]^5 + \\
& 48 a A b^2 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 36 a^2 b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 48 b^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 30 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 24 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 48 a A b^2 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left. \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 36 a^2 b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 48b^3 \text{B EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 30a^3 \text{C EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 24ab^2 \text{C EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (24Ab^2 - 18abB + 15a^2C + 16b^2C) \text{EllipticE}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2b (12b^2B + 5a^2C + 2ab(-3B+C)) \text{EllipticF}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b+a\tan\left[\frac{1}{2}(c+dx)\right]^2 - b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(24b^3d \sqrt{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(b \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - a \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right)
\end{aligned}$$

■ **Problem 1533: Attempted integration timed out after 120 seconds.**

$$\int \frac{(A + B \cos[c+dx] + C \cos[c+dx]^2) \sec[c+dx]^{7/2}}{(a + b \cos[c+dx])^{3/2}} dx$$

Optimal (type 4, 585 leaves, 7 steps):

$$\begin{aligned}
& - \left( 2 (48 A b^4 + 25 a^3 b B - 40 a b^3 B - 6 a^2 b^2 (4 A - 5 C) - 3 a^4 (3 A + 5 C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \right. \\
& \quad \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / (15 a^5 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]}) - \\
& \left( 2 (48 A b^3 + 4 a b^2 (9 A - 10 B) + 6 a^2 b (2 A - 5 B + 5 C) + a^3 (9 A - 5 B + 15 C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \right. \\
& \quad \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / (15 a^4 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]}) + \\
& \frac{2 (24 A b^3 + 5 a^3 B - 20 a b^2 B - a^2 (9 A b - 15 b C)) \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{15 a^3 (a^2 - b^2) d} + \\
& \frac{2 (A b^2 - a (b B - a C)) \operatorname{Sec}[c + d x]^{5/2} \operatorname{Sin}[c + d x]}{a (a^2 - b^2) d \sqrt{a + b \cos[c + d x]}} - \\
& \frac{2 (6 A b^2 - 5 a b B - a^2 (A - 5 C)) \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x]^{5/2} \operatorname{Sin}[c + d x]}{5 a^2 (a^2 - b^2) d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1534: Attempted integration timed out after 120 seconds.**

$$\int \frac{(A + B \cos[c + d x] + C \cos[c + d x]^2) \operatorname{Sec}[c + d x]^{5/2}}{(a + b \cos[c + d x])^{3/2}} dx$$

Optimal (type 4, 464 leaves, 6 steps):

$$\begin{aligned}
& \left( 2 (8 A b^3 + 3 a^3 B - 6 a b^2 B - a^2 (5 A b - 3 b C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \right. \\
& \quad \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a^4 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]} \right) + \\
& \left( 2 (8 A b^2 + 6 a b (A - B) + a^2 (A - 3 B + 3 C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a^3 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]} \right) + \\
& \frac{2 (A b^2 - a (b B - a C)) \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{a (a^2 - b^2) d \sqrt{a + b \cos[c + d x]}} - \frac{2 (4 A b^2 - 3 a b B - a^2 (A - 3 C)) \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{3 a^2 (a^2 - b^2) d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1535: Attempted integration timed out after 120 seconds.**

$$\int \frac{(A + B \cos[c + d x] + C \cos[c + d x]^2) \operatorname{Sec}[c + d x]^{3/2}}{(a + b \cos[c + d x])^{3/2}} dx$$

Optimal (type 4, 362 leaves, 5 steps):

$$\begin{aligned}
& - \left( 2 (2 A b^2 - a b B - a^2 (A - C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \right. \\
& \quad \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( a^3 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]} \right) - \\
& \left( 2 (2 A b + a (A - B - C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \right. \\
& \quad \left. \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( a^2 \sqrt{a + b} d \sqrt{\operatorname{Sec}[c + d x]} \right) + \frac{2 (A b^2 - a (b B - a C)) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{a (a^2 - b^2) d \sqrt{a + b \cos[c + d x]}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1536: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sqrt{\sec[c + dx]}}{(a + b \cos[c + dx])^{3/2}} dx$$

Optimal (type 4, 496 leaves, 7 steps):

$$\left( 2 (Ab^2 - a(bB - aC)) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b}}\right], -\frac{a + b}{a - b}\right] \right.$$

$$\left. \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} \right) / (a^2 b \sqrt{a + b} d \sqrt{\sec[c + dx]}) +$$

$$\left( 2 (Ab + bB - aC) \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b}}\right], -\frac{a + b}{a - b}\right] \right.$$

$$\left. \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}} \sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} \right) / (ab \sqrt{a + b} d \sqrt{\sec[c + dx]}) - \frac{1}{b^2 d \sqrt{\sec[c + dx]}}$$

$$2 \sqrt{a + b} C \sqrt{\cos[c + dx]} \operatorname{Csc}[c + dx] \operatorname{EllipticPi}\left[\frac{a + b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + dx]}}{\sqrt{a + b}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + dx])}{a + b}}$$

$$\sqrt{\frac{a(1 + \sec[c + dx])}{a - b}} - \frac{2 (Ab^2 - a(bB - aC)) \sqrt{\sec[c + dx]} \operatorname{Sin}[c + dx]}{b (a^2 - b^2) d \sqrt{a + b \cos[c + dx]}}$$

Result (type 4, 1141 leaves):

$$\frac{\sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]} \left( \frac{2 (Ab^2 - a(bB + a^2C)) \operatorname{Sin}[c + dx]}{ab (a^2 - b^2)} + \frac{2 (Ab^2 \operatorname{Sin}[c + dx] - abB \operatorname{Sin}[c + dx] + a^2C \operatorname{Sin}[c + dx])}{b (-a^2 + b^2) (a + b \cos[c + dx])} \right)}{d}$$

$$\left( 2 \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \left( aAb^2 \tan\left[\frac{1}{2}(c + dx)\right] + Ab^3 \tan\left[\frac{1}{2}(c + dx)\right] - a^2bB \tan\left[\frac{1}{2}(c + dx)\right] - ab^2B \tan\left[\frac{1}{2}(c + dx)\right] + a^3C \tan\left[\frac{1}{2}(c + dx)\right] \right) + \right.$$

$$\left. a^2bC \tan\left[\frac{1}{2}(c + dx)\right] - 2Ab^3 \tan\left[\frac{1}{2}(c + dx)\right]^3 + 2ab^2B \tan\left[\frac{1}{2}(c + dx)\right]^3 - 2a^2bC \tan\left[\frac{1}{2}(c + dx)\right]^3 - aAb^2 \tan\left[\frac{1}{2}(c + dx)\right]^5 + \right.$$

$$\begin{aligned}
& A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + \\
& 2 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 2 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 2 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 2 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& (a+b)\left(A b^2+a(-b B+a C)\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - a b(a+b)(A-B+C) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \left.\left.\left.\left.\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right)\sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}\right)\right]\right)\right) / \\
& \left(b\left(a^3-a b^2\right) d\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right)\right)^{3 / 2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}
\end{aligned}$$

■ **Problem 1537: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+B \operatorname{Cos}[c+d x]+C \operatorname{Cos}[c+d x]^2}{(a+b \operatorname{Cos}[c+d x])^{3 / 2} \sqrt{\operatorname{Sec}[c+d x]}} d x$$

Optimal (type 4, 595 leaves, 8 steps):

$$\begin{aligned}
& - \left( (2Ab^2 - 2abB + 3a^2C - b^2C) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \right. \\
& \quad \left. \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}} \right], -\frac{a+b}{a-b} \right] \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} \right) / (ab^2 \sqrt{a+b} d \sqrt{\sec[c+dx]}) + \\
& \left( (2Ab^2 - a(b(2B-C) - 3aC)) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}} \right], -\frac{a+b}{a-b} \right] \right. \\
& \quad \left. \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} \right) / (ab^2 \sqrt{a+b} d \sqrt{\sec[c+dx]}) - \frac{1}{b^3 d \sqrt{\sec[c+dx]}} \\
& \sqrt{a+b} (2bB - 3aC) \sqrt{\cos[c+dx]} \operatorname{Csc}[c+dx] \operatorname{EllipticPi} \left[ \frac{a+b}{b}, \operatorname{ArcSin} \left[ \frac{\sqrt{a+b \cos[c+dx]}}{\sqrt{a+b} \sqrt{\cos[c+dx]}} \right], -\frac{a+b}{a-b} \right] \\
& \quad \sqrt{\frac{a(1-\sec[c+dx])}{a+b}} \sqrt{\frac{a(1+\sec[c+dx])}{a-b}} - \frac{2(Ab^2 - a(bB - aC)) \sin[c+dx]}{b(a^2 - b^2) d \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]}} + \\
& \quad \frac{(2Ab^2 - 2abB + 3a^2C - b^2C) \sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]} \sin[c+dx]}{b^2(a^2 - b^2) d}
\end{aligned}$$

Result (type 4, 1683 leaves):

$$\frac{\sqrt{a+b \cos[c+dx]} \sqrt{\sec[c+dx]} \left( \frac{2(Ab^2 - abB + a^2C) \sin[c+dx]}{b^2(-a^2+b^2)} - \frac{2(aAb^2 \sin[c+dx] - a^2bB \sin[c+dx] + a^3C \sin[c+dx])}{b^2(-a^2+b^2)(a+b \cos[c+dx])} \right)}{d}$$

$$\left( \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b + a \tan\left[\frac{1}{2}(c+dx)\right]^2 - b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right)$$

$$\begin{aligned}
& \left( -2aAb^2 \tan\left[\frac{1}{2}(c+dx)\right] - 2Ab^3 \tan\left[\frac{1}{2}(c+dx)\right] + 2a^2bB \tan\left[\frac{1}{2}(c+dx)\right] + 2ab^2B \tan\left[\frac{1}{2}(c+dx)\right] - 3a^3C \tan\left[\frac{1}{2}(c+dx)\right] - \right. \\
& \quad \left. 3a^2bC \tan\left[\frac{1}{2}(c+dx)\right] + ab^2C \tan\left[\frac{1}{2}(c+dx)\right] + b^3C \tan\left[\frac{1}{2}(c+dx)\right] + 4Ab^3 \tan\left[\frac{1}{2}(c+dx)\right]^3 - 4ab^2B \tan\left[\frac{1}{2}(c+dx)\right]^3 + \right. \\
& \quad \left. 6a^2bC \tan\left[\frac{1}{2}(c+dx)\right]^3 - 2b^3C \tan\left[\frac{1}{2}(c+dx)\right]^3 + 2aAb^2 \tan\left[\frac{1}{2}(c+dx)\right]^5 - 2Ab^3 \tan\left[\frac{1}{2}(c+dx)\right]^5 - 2a^2bB \tan\left[\frac{1}{2}(c+dx)\right]^5 + \right.
\end{aligned}$$



$$\begin{aligned}
& 2 a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 3 a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 3 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + \\
& 4 a^2 b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 4 b^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 6 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 6 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 4 a^2 b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 4 b^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 6 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 6 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& (a+b)\left(2 A b^2-2 a b B+3 a^2 C-b^2 C\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{-a+b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right)
\end{aligned}$$

$$\sqrt{\frac{a + b + a \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} + 2b(a + b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right], \frac{-a + b}{a + b}\right]$$

$$\sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right) \sqrt{\frac{a + b + a \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 - b \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} \right) /$$

$$\left(b^2(-a^2 + b^2) d \sqrt{1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2} \left(b \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right) - a \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right)\right)\right)$$

■ **Problem 1538: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \operatorname{Cos}[c + dx] + C \operatorname{Cos}[c + dx]^2}{(a + b \operatorname{Cos}[c + dx])^{3/2} \operatorname{Sec}[c + dx]^{3/2}} dx$$

Optimal (type 4, 720 leaves, 9 steps):

$$\begin{aligned}
& - \left( (12 a^2 b B - 4 b^3 B - a b^2 (8 A - 7 C) - 15 a^3 C) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \right. \\
& \quad \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + d x])}{a + b}} \sqrt{\frac{a(1 + \sec[c + d x])}{a - b}} \right) / (4 a b^3 \sqrt{a + b} d \sqrt{\sec[c + d x]}) - \\
& \left( (8 A b^2 - a b (12 B - 5 C) + 15 a^2 C - 2 b^2 (2 B + C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a(1 - \sec[c + d x])}{a + b}} \sqrt{\frac{a(1 + \sec[c + d x])}{a - b}} \right) / (4 b^3 \sqrt{a + b} d \sqrt{\sec[c + d x]}) - \\
& \frac{1}{4 b^4 d \sqrt{\sec[c + d x]}} \sqrt{a + b} (8 A b^2 - 12 a b B + 15 a^2 C + 4 b^2 C) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \\
& \operatorname{EllipticPi}\left[\frac{a + b}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \sqrt{\frac{a(1 - \sec[c + d x])}{a + b}} \sqrt{\frac{a(1 + \sec[c + d x])}{a - b}} - \\
& \frac{2 (A b^2 - a (b B - a C)) \sin[c + d x]}{b (a^2 - b^2) d \sqrt{a + b \cos[c + d x]} \sec[c + d x]^{3/2}} + \frac{(4 A b^2 - 4 a b B + 5 a^2 C - b^2 C) \sqrt{a + b \cos[c + d x]} \sin[c + d x]}{2 b^2 (a^2 - b^2) d \sqrt{\sec[c + d x]}} + \\
& \frac{(12 a^2 b B - 4 b^3 B - a b^2 (8 A - 7 C) - 15 a^3 C) \sqrt{a + b \cos[c + d x]} \sqrt{\sec[c + d x]} \sin[c + d x]}{4 b^3 (a^2 - b^2) d}
\end{aligned}$$

Result (type 4, 656 leaves):

$$\frac{1}{d} \sqrt{a + b \cos[c + dx]} \sqrt{\sec[c + dx]}$$

$$\left( \frac{2a(Ab^2 - abB + a^2C) \sin[c + dx]}{b^3(a^2 - b^2)} + \frac{2(a^2Ab^2 \sin[c + dx] - a^3bB \sin[c + dx] + a^4C \sin[c + dx])}{b^3(-a^2 + b^2)(a + b \cos[c + dx])} + \frac{C \sin[2(c + dx)]}{4b^2} \right) +$$

$$\frac{1}{4b^3(a^2 - b^2)d} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}} \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}}$$

$$\left( -(-12a^2bB + 4b^3B + ab^2(8A - 7C) + 15a^3C) \tan\left[\frac{1}{2}(c + dx)\right] + \frac{1}{\sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\frac{a + b + a \tan\left[\frac{1}{2}(c + dx)\right]^2 - b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}}}} \right.$$

$$i \sqrt{\frac{a - b}{a + b}} \left( (-12a^2bB + 4b^3B + ab^2(8A - 7C) + 15a^3C) \operatorname{EllipticE}\left[ i \operatorname{ArcSinh}\left[ \sqrt{\frac{a - b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right] \right], -\frac{a + b}{a - b} \right] - \right.$$

$$2(-2a^2b(6B - 5C) + 15a^3C + 2b^3(2A + C) + ab^2(8A - 8B + C)) \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[ \sqrt{\frac{a - b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right] \right], -\frac{a + b}{a - b} \right] +$$

$$\left. 2(a + b)(8Ab^2 - 12abB + 15a^2C + 4b^2C) \operatorname{EllipticPi}\left[ \frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[ \sqrt{\frac{a - b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right] \right], -\frac{a + b}{a - b} \right] \left( -1 - \tan\left[\frac{1}{2}(c + dx)\right] \right)^2 \right)$$

■ **Problem 1539: Attempted integration timed out after 120 seconds.**

$$\int \frac{(A + B \cos[c + dx] + C \cos[c + dx]^2) \sec[c + dx]^{5/2}}{(a + b \cos[c + dx])^{5/2}} dx$$

Optimal (type 4, 660 leaves, 7 steps):

$$\begin{aligned}
& - \left( 2 (16 A b^5 - 3 a^5 B + 15 a^3 b^2 B - 8 a b^4 B - 2 a^2 b^3 (14 A - C) + a^4 (8 A b - 6 b C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticE} \left[ \right. \right. \\
& \quad \left. \left. \operatorname{ArcSin} \left[ \frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b}} \right], -\frac{a + b}{a - b} \right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a^5 \sqrt{a + b} (a^2 - b^2) d \sqrt{\operatorname{Sec}[c + d x]} \right) - \\
& \left( 2 (16 A b^4 + 4 a b^3 (3 A - 2 B) - 3 a^3 b (3 A - 3 B - C) - 2 a^2 b^2 (8 A + 3 B - C) - a^4 (A - 3 B + 3 C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \right. \\
& \quad \left. \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b}} \right], -\frac{a + b}{a - b} \right] \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \\
& \quad \left( 3 a^4 \sqrt{a + b} (a^2 - b^2) d \sqrt{\operatorname{Sec}[c + d x]} \right) + \frac{2 (A b^2 - a (b B - a C)) \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{3 a (a^2 - b^2) d (a + b \cos[c + d x])^{3/2}} + \\
& \quad \frac{2 (10 a^2 A b^2 - 6 A b^4 - 7 a^3 b B + 3 a b^3 B + 4 a^4 C) \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{3 a^2 (a^2 - b^2)^2 d \sqrt{a + b \cos[c + d x]}} + \frac{1}{3 a^3 (a^2 - b^2)^2 d} \\
& \quad 2 (8 A b^4 + 8 a^3 b B - 4 a b^3 B + a^4 (A - 5 C) - a^2 b^2 (13 A - C)) \sqrt{a + b \cos[c + d x]} \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1540: Attempted integration timed out after 120 seconds.**

$$\int \frac{(A + B \cos[c + d x] + C \cos[c + d x]^2) \operatorname{Sec}[c + d x]^{3/2}}{(a + b \cos[c + d x])^{5/2}} dx$$

Optimal (type 4, 535 leaves, 6 steps):

$$\begin{aligned}
& \left( 2 \left( 8 A b^4 + 6 a^3 b B - 2 a b^3 B + 3 a^4 (A - C) - a^2 b^2 (15 A + C) \right) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b}} \right], -\frac{a + b}{a - b} \right] \right. \\
& \quad \left. \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a^4 \sqrt{a + b} (a^2 - b^2) d \sqrt{\operatorname{Sec}[c + d x]} \right) + \\
& \left( 2 \left( 8 A b^3 + 2 a b^2 (3 A - B) - 3 a^3 (A - B - C) - a^2 b (9 A + 3 B + C) \right) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b}} \right], -\frac{a + b}{a - b} \right] \right. \\
& \quad \left. \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a^3 \sqrt{a + b} (a^2 - b^2) d \sqrt{\operatorname{Sec}[c + d x]} \right) + \\
& \frac{2 (A b^2 - a (b B - a C)) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{3 a (a^2 - b^2) d (a + b \cos[c + d x])^{3/2}} - \frac{2 (4 A b^4 + 5 a^3 b B - a b^3 B - 2 a^4 C - 2 a^2 b^2 (4 A + C)) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{3 a^2 (a^2 - b^2)^2 d \sqrt{a + b \cos[c + d x]}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1541: Unable to integrate problem.**

$$\int \frac{(A + B \cos[c + d x] + C \cos[c + d x]^2) \sqrt{\operatorname{Sec}[c + d x]}}{(a + b \cos[c + d x])^{5/2}} dx$$

Optimal (type 4, 495 leaves, 6 steps):

$$\begin{aligned}
& - \left( 2 (2 A b^3 + 3 a^3 B + a b^2 B - 2 a^2 b (3 A + 2 C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a^3 (a - b) (a + b)^{3/2} d \sqrt{\operatorname{Sec}[c + d x]} \right) - \\
& \left( 2 (2 A b^2 - a^2 (3 A + 3 B + C) + a b (3 A + B + 3 C)) \sqrt{\cos[c + d x]} \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \cos[c + d x]}}{\sqrt{a + b} \sqrt{\cos[c + d x]}}\right], -\frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{\frac{a (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Sec}[c + d x])}{a - b}} \right) / \left( 3 a^2 \sqrt{a + b} (a^2 - b^2) d \sqrt{\operatorname{Sec}[c + d x]} \right) + \\
& \frac{2 (A b^2 - a (b B - a C)) \sin[c + d x]}{3 a (a^2 - b^2) d (a + b \cos[c + d x])^{3/2} \sqrt{\operatorname{Sec}[c + d x]}} + \frac{2 (2 A b^3 + 3 a^3 B + a b^2 B - 2 a^2 b (3 A + 2 C)) \sqrt{\operatorname{Sec}[c + d x]} \sin[c + d x]}{3 a (a^2 - b^2)^2 d \sqrt{a + b \cos[c + d x]}}
\end{aligned}$$

Result (type 8, 47 leaves):

$$\int \frac{(A + B \cos[c + d x] + C \cos[c + d x]^2) \sqrt{\operatorname{Sec}[c + d x]}}{(a + b \cos[c + d x])^{5/2}} dx$$

## Test results for the 98 problems in "4.2.7 (d trig)^m (a+b (c cos)^n)^p.m"

- Problem 6: Result more than twice size of optimal antiderivative.

$$\int \frac{\sin[x]}{a - a \cos[x]^2} dx$$

Optimal (type 3, 8 leaves, 2 steps):

$$-\frac{\operatorname{ArcTanh}[\cos[x]]}{a}$$

Result (type 3, 21 leaves):

$$-\frac{\operatorname{Log}\left[\cos\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\sin\left[\frac{x}{2}\right]\right]}{a}$$

- Problem 7: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc}[x]}{a - a \cos[x]^2} dx$$

Optimal (type 3, 22 leaves, 3 steps) :

$$-\frac{\text{ArcTanh}[\text{Cos}[x]]}{2a} - \frac{\text{Cot}[x] \text{Csc}[x]}{2a}$$

Result (type 3, 51 leaves) :

$$\frac{-\frac{1}{8} \text{Csc}\left[\frac{x}{2}\right]^2 - \frac{1}{2} \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \frac{1}{2} \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] + \frac{1}{8} \text{Sec}\left[\frac{x}{2}\right]^2}{a}$$

■ **Problem 9: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[x]^3}{a - a \text{Cos}[x]^2} dx$$

Optimal (type 3, 35 leaves, 4 steps) :

$$-\frac{3 \text{ArcTanh}[\text{Cos}[x]]}{8a} - \frac{3 \text{Cot}[x] \text{Csc}[x]}{8a} - \frac{\text{Cot}[x] \text{Csc}[x]^3}{4a}$$

Result (type 3, 75 leaves) :

$$\frac{-\frac{3}{32} \text{Csc}\left[\frac{x}{2}\right]^2 - \frac{1}{64} \text{Csc}\left[\frac{x}{2}\right]^4 - \frac{3}{8} \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \frac{3}{8} \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] + \frac{3}{32} \text{Sec}\left[\frac{x}{2}\right]^2 + \frac{1}{64} \text{Sec}\left[\frac{x}{2}\right]^4}{a}$$

■ **Problem 11: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[x]^5}{a + b \text{Cos}[x]^2} dx$$

Optimal (type 3, 54 leaves, 4 steps) :

$$-\frac{(a+b)^2 \text{ArcTan}\left[\frac{\sqrt{b} \text{Cos}[x]}{\sqrt{a}}\right]}{\sqrt{a} b^{5/2}} + \frac{(a+2b) \text{Cos}[x]}{b^2} - \frac{\text{Cos}[x]^3}{3b}$$

Result (type 3, 116 leaves) :

$$\frac{-\frac{12(a+b)^2 \text{ArcTan}\left[\frac{\sqrt{b} - \sqrt{a+b} \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]}{\sqrt{a}} - \frac{12(a+b)^2 \text{ArcTan}\left[\frac{\sqrt{b} + \sqrt{a+b} \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]}{\sqrt{a}} + 3\sqrt{b} (4a+7b) \text{Cos}[x] - b^{3/2} \text{Cos}[3x]}{12b^{5/2}}$$

■ **Problem 12: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[x]^3}{a + b \text{Cos}[x]^2} dx$$

Optimal (type 3, 36 leaves, 3 steps) :



$$-\frac{(a+b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \cos[x]}{\sqrt{a}}\right]}{\sqrt{a} b^{3/2}} + \frac{\cos[x]}{b}$$

Result (type 3, 90 leaves):

$$\frac{-(a+b) \operatorname{ArcTan}\left[\frac{\sqrt{b}-\sqrt{a+b} \tan\left[\frac{x}{2}\right]}{\sqrt{a}}\right] - (a+b) \operatorname{ArcTan}\left[\frac{\sqrt{b}+\sqrt{a+b} \tan\left[\frac{x}{2}\right]}{\sqrt{a}}\right] + \sqrt{a} \sqrt{b} \cos[x]}{\sqrt{a} b^{3/2}}$$

■ **Problem 15: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc[x]^3}{a+b \cos[x]^2} dx$$

Optimal (type 3, 62 leaves, 5 steps):

$$-\frac{b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \cos[x]}{\sqrt{a}}\right]}{\sqrt{a} (a+b)^2} - \frac{(a+3b) \operatorname{ArcTanh}[\cos[x]]}{2(a+b)^2} - \frac{\cot[x] \csc[x]}{2(a+b)}$$

Result (type 3, 140 leaves):

$$\frac{1}{8 \sqrt{a} (a+b)^2} \left( -8 b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b}-\sqrt{a+b} \tan\left[\frac{x}{2}\right]}{\sqrt{a}}\right] - 8 b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b}+\sqrt{a+b} \tan\left[\frac{x}{2}\right]}{\sqrt{a}}\right] + \sqrt{a} \left( -(a+b) \csc\left[\frac{x}{2}\right]^2 - 4(a+3b) \left( \log\left[\cos\left[\frac{x}{2}\right]\right] - \log\left[\sin\left[\frac{x}{2}\right]\right] \right) + (a+b) \sec\left[\frac{x}{2}\right]^2 \right) \right)$$

■ **Problem 16: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc[x]^5}{a+b \cos[x]^2} dx$$

Optimal (type 3, 94 leaves, 6 steps):

$$-\frac{b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \cos[x]}{\sqrt{a}}\right]}{\sqrt{a} (a+b)^3} - \frac{(3a^2+10ab+15b^2) \operatorname{ArcTanh}[\cos[x]]}{8(a+b)^3} - \frac{(3a+7b) \cot[x] \csc[x]}{8(a+b)^2} - \frac{\cot[x] \csc[x]^3}{4(a+b)}$$

Result (type 3, 204 leaves):

$$\frac{1}{64 \sqrt{a} (a+b)^3} \left( -64 b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} - \sqrt{a+b} \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a}}\right] - 64 b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} + \sqrt{a+b} \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a}}\right] + \sqrt{a} \left( -2 (3 a^2 + 10 a b + 7 b^2) \operatorname{Csc}\left[\frac{x}{2}\right]^2 - (a+b)^2 \operatorname{Csc}\left[\frac{x}{2}\right]^4 - 8 (3 a^2 + 10 a b + 15 b^2) \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]\right] - \operatorname{Log}\left[\operatorname{Sin}\left[\frac{x}{2}\right]\right] \right) + 2 (3 a^2 + 10 a b + 7 b^2) \operatorname{Sec}\left[\frac{x}{2}\right]^2 + (a+b)^2 \operatorname{Sec}\left[\frac{x}{2}\right]^4 \right) \right)$$

■ **Problem 32: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[x]}{a+b \operatorname{Cos}[x]^2} dx$$

Optimal (type 3, 41 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}[\operatorname{Sin}[x]]}{a} - \frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sin}[x]}{\sqrt{a+b}}\right]}{a \sqrt{a+b}}$$

Result (type 3, 93 leaves):

$$\frac{-2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right] + 2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right] + \frac{\sqrt{b} \left( \operatorname{Log}\left[\sqrt{a+b} - \sqrt{b} \operatorname{Sin}[x]\right] - \operatorname{Log}\left[\sqrt{a+b} + \sqrt{b} \operatorname{Sin}[x]\right] \right)}{\sqrt{a+b}}}{2 a}$$

■ **Problem 33: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[x]^3}{a+b \operatorname{Cos}[x]^2} dx$$

Optimal (type 3, 59 leaves, 5 steps):

$$\frac{(a-2b) \operatorname{ArcTanh}[\operatorname{Sin}[x]]}{2 a^2} + \frac{b^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sin}[x]}{\sqrt{a+b}}\right]}{a^2 \sqrt{a+b}} + \frac{\operatorname{Sec}[x] \operatorname{Tan}[x]}{2 a}$$

Result (type 3, 152 leaves):

$$\frac{1}{4 a^2} \left( -2 (a-2b) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right] + 2 (a-2b) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right] - \frac{2 b^{3/2} \operatorname{Log}\left[\sqrt{a+b} - \sqrt{b} \operatorname{Sin}[x]\right]}{\sqrt{a+b}} + \frac{2 b^{3/2} \operatorname{Log}\left[\sqrt{a+b} + \sqrt{b} \operatorname{Sin}[x]\right]}{\sqrt{a+b}} + \frac{a}{\left(\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right)^2} - \frac{a}{\left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^2} \right)$$

■ **Problem 34: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[x]^5}{a + b \text{Cos}[x]^2} dx$$

Optimal (type 3, 90 leaves, 6 steps) :

$$\frac{(3 a^2 - 4 a b + 8 b^2) \text{ArcTanh}[\text{Sin}[x]]}{8 a^3} - \frac{b^{5/2} \text{ArcTanh}\left[\frac{\sqrt{b} \text{Sin}[x]}{\sqrt{a+b}}\right]}{a^3 \sqrt{a+b}} + \frac{(3 a - 4 b) \text{Sec}[x] \text{Tan}[x]}{8 a^2} + \frac{\text{Sec}[x]^3 \text{Tan}[x]}{4 a}$$

Result (type 3, 215 leaves) :

$$\frac{1}{16 a^3} \left( -2 (3 a^2 - 4 a b + 8 b^2) \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + 2 (3 a^2 - 4 a b + 8 b^2) \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \frac{8 b^{5/2} \text{Log}\left[\sqrt{a+b} - \sqrt{b} \text{Sin}[x]\right]}{\sqrt{a+b}} - \frac{8 b^{5/2} \text{Log}\left[\sqrt{a+b} + \sqrt{b} \text{Sin}[x]\right]}{\sqrt{a+b}} + \frac{a^2}{\left(\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right)^4} - \frac{a^2}{\left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^4} + \frac{a (-3 a + 4 b)}{\left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^2} + \frac{a (-3 a + 4 b)}{-1 + \text{Sin}[x]} \right)$$

■ **Problem 67: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cos}[x]}{\sqrt{1 + \text{Cos}[x]^2}} dx$$

Optimal (type 3, 9 leaves, 2 steps) :

$$\text{ArcSin}\left[\frac{\text{Sin}[x]}{\sqrt{2}}\right]$$

Result (type 3, 19 leaves) :

$$\text{ArcTan}\left[\frac{\sqrt{2} \text{Sin}[x]}{\sqrt{3 + \text{Cos}[2 x]}}\right]$$

■ **Problem 68: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cos}[5 + 3 x]}{\sqrt{3 + \text{Cos}[5 + 3 x]^2}} dx$$

Optimal (type 3, 15 leaves, 2 steps) :

$$\frac{1}{3} \text{ArcSin}\left[\frac{1}{2} \text{Sin}[5 + 3 x]\right]$$

Result (type 3, 31 leaves) :

$$\frac{1}{3} \text{ArcTan} \left[ \frac{\sqrt{2} \sin[5 + 3x]}{\sqrt{7 + \cos[2(5 + 3x)]}} \right]$$

- **Problem 69: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[x]}{\sqrt{4 - \cos[x]^2}} dx$$

Optimal (type 3, 9 leaves, 2 steps):

$$\text{ArcSinh} \left[ \frac{\sin[x]}{\sqrt{3}} \right]$$

Result (type 3, 21 leaves):

$$\text{ArcTanh} \left[ \frac{\sqrt{2} \sin[x]}{\sqrt{7 - \cos[2x]}} \right]$$

- **Problem 70: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{a + b \cos[x]^4} dx$$

Optimal (type 3, 487 leaves, 10 steps):

$$\frac{(\sqrt{a} + \sqrt{a+b}) \text{ArcTan} \left[ \frac{a^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b} - \sqrt{2} (a+b)^{3/4} \cot[x]}{a^{1/4} \sqrt{a+b+\sqrt{a}} \sqrt{a+b}} \right]}{2 \sqrt{2} a^{3/4} (a+b)^{1/4} \sqrt{a+b+\sqrt{a}} \sqrt{a+b}} - \frac{(\sqrt{a} + \sqrt{a+b}) \text{ArcTan} \left[ \frac{a^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b} + \sqrt{2} (a+b)^{3/4} \cot[x]}{a^{1/4} \sqrt{a+b+\sqrt{a}} \sqrt{a+b}} \right]}{2 \sqrt{2} a^{3/4} (a+b)^{1/4} \sqrt{a+b+\sqrt{a}} \sqrt{a+b}} -$$

$$\frac{(\sqrt{a} - \sqrt{a+b}) \text{Log} \left[ \sqrt{a} (a+b)^{1/4} - \sqrt{2} a^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b} \cot[x] + (a+b)^{3/4} \cot[x]^2 \right]}{4 \sqrt{2} a^{3/4} (a+b)^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b}} +$$

$$\frac{(\sqrt{a} - \sqrt{a+b}) \text{Log} \left[ \sqrt{a} (a+b)^{1/4} + \sqrt{2} a^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b} \cot[x] + (a+b)^{3/4} \cot[x]^2 \right]}{4 \sqrt{2} a^{3/4} (a+b)^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b}}$$

Result (type 3, 121 leaves):

$$\frac{\text{ArcTan} \left[ \frac{\sqrt{a} \tan[x]}{\sqrt{a+i} \sqrt{a} \sqrt{b}} \right]}{2 \sqrt{a} \sqrt{a+i} \sqrt{a} \sqrt{b}} - \frac{\text{ArcTanh} \left[ \frac{\sqrt{a} \tan[x]}{\sqrt{-a+i} \sqrt{a} \sqrt{b}} \right]}{2 \sqrt{a} \sqrt{-a+i} \sqrt{a} \sqrt{b}}$$

- **Problem 72: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{1 + \cos[x]^4} dx$$

Optimal (type 3, 292 leaves, 10 steps):

$$\frac{x}{2\sqrt{-1+\sqrt{2}}} + \frac{\text{ArcTan}\left[\frac{(-2+\sqrt{2})\cos[x]\sin[x]+\sqrt{-1+\sqrt{2}}(1-2\sin[x]^2)}{2+\sqrt{1+\sqrt{2}}+2\sqrt{-1+\sqrt{2}}\cos[x]\sin[x]+(-2+\sqrt{2})\sin[x]^2}\right]}{4\sqrt{-1+\sqrt{2}}} + \frac{\text{ArcTan}\left[\frac{(-2+\sqrt{2})\cos[x]\sin[x]+\sqrt{-1+\sqrt{2}}(-1+2\sin[x]^2)}{2+\sqrt{1+\sqrt{2}}-2\sqrt{-1+\sqrt{2}}\cos[x]\sin[x]+(-2+\sqrt{2})\sin[x]^2}\right]}{4\sqrt{-1+\sqrt{2}}} +$$

$$\frac{1}{8}\sqrt{-1+\sqrt{2}}\log\left[\sqrt{2}-2\sqrt{-1+\sqrt{2}}\cot[x]+2\cot[x]^2\right] - \frac{1}{8}\sqrt{-1+\sqrt{2}}\log\left[1+\sqrt{2(-1+\sqrt{2})}\cot[x]+\sqrt{2}\cot[x]^2\right]$$

Result (type 3, 45 leaves):

$$\frac{\text{ArcTan}\left[\frac{\tan[x]}{\sqrt{1-i}}\right]}{2\sqrt{1-i}} + \frac{\text{ArcTan}\left[\frac{\tan[x]}{\sqrt{1+i}}\right]}{2\sqrt{1+i}}$$

- **Problem 74: Result is not expressed in closed-form.**

$$\int \frac{1}{a + b \cos[x]^5} dx$$

Optimal (type 3, 494 leaves, 12 steps):

$$\frac{2 \text{ArcTan}\left[\frac{\sqrt{a^{1/5}-b^{1/5}} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+b^{1/5}}}\right]}{5 a^{4/5} \sqrt{a^{1/5}-b^{1/5}} \sqrt{a^{1/5}+b^{1/5}}} + \frac{2 \text{ArcTan}\left[\frac{\sqrt{a^{1/5}+(-1)^{1/5}b^{1/5}} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-(-1)^{1/5}b^{1/5}}}\right]}{5 a^{4/5} \sqrt{a^{1/5}-(-1)^{1/5}b^{1/5}} \sqrt{a^{1/5}+(-1)^{1/5}b^{1/5}}} + \frac{2 \text{ArcTan}\left[\frac{\sqrt{a^{1/5}-(-1)^{2/5}b^{1/5}} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+(-1)^{2/5}b^{1/5}}}\right]}{5 a^{4/5} \sqrt{a^{1/5}-(-1)^{2/5}b^{1/5}} \sqrt{a^{1/5}+(-1)^{2/5}b^{1/5}}} +$$

$$\frac{2 \text{ArcTan}\left[\frac{\sqrt{a^{1/5}+(-1)^{3/5}b^{1/5}} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-(-1)^{3/5}b^{1/5}}}\right]}{5 a^{4/5} \sqrt{a^{1/5}-(-1)^{3/5}b^{1/5}} \sqrt{a^{1/5}+(-1)^{3/5}b^{1/5}}} + \frac{2 \text{ArcTan}\left[\frac{\sqrt{a^{1/5}-(-1)^{4/5}b^{1/5}} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+(-1)^{4/5}b^{1/5}}}\right]}{5 a^{4/5} \sqrt{a^{1/5}-(-1)^{4/5}b^{1/5}} \sqrt{a^{1/5}+(-1)^{4/5}b^{1/5}}}$$

Result (type 7, 130 leaves):

$$\frac{8}{5} \text{RootSum}\left[b + 5 b \#1^2 + 10 b \#1^4 + 32 a \#1^5 + 10 b \#1^6 + 5 b \#1^8 + b \#1^{10} \&, \frac{2 \text{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right] \#1^3 - i \log\left[1 - 2 \cos[x] \#1 + \#1^2\right] \#1^3}{b + 4 b \#1^2 + 16 a \#1^3 + 6 b \#1^4 + 4 b \#1^6 + b \#1^8} \&\right]$$

- **Problem 75: Result is not expressed in closed-form.**

$$\int \frac{1}{a + b \cos[x]^6} dx$$

Optimal (type 3, 171 leaves, 7 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}+b^{1/3}} \cot[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}+b^{1/3}}} - \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}-(-1)^{1/3} b^{1/3}} \cot[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}-(-1)^{1/3} b^{1/3}}} - \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}+(-1)^{2/3} b^{1/3}} \cot[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}+(-1)^{2/3} b^{1/3}}}$$

Result (type 7, 146 leaves):

$$\frac{8}{3} \text{RootSum}\left[b + 6 b \#1 + 15 b \#1^2 + 64 a \#1^3 + 20 b \#1^3 + 15 b \#1^4 + 6 b \#1^5 + b \#1^6 \&, \frac{2 \text{ArcTan}\left[\frac{\sin[2x]}{\cos[2x]-\#1}\right] \#1^2 - i \text{Log}\left[1 - 2 \cos[2x] \#1 + \#1^2\right] \#1^2}{b + 5 b \#1 + 32 a \#1^2 + 10 b \#1^2 + 10 b \#1^3 + 5 b \#1^4 + b \#1^5} \&\right]$$

■ **Problem 76: Result is not expressed in closed-form.**

$$\int \frac{1}{a + b \cos[x]^8} dx$$

Optimal (type 3, 245 leaves, 9 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{(-a)^{1/4}-b^{1/4}} \cot[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4}-b^{1/4}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{(-a)^{1/4}-i b^{1/4}} \cot[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4}-i b^{1/4}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{(-a)^{1/4}+i b^{1/4}} \cot[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4}+i b^{1/4}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{(-a)^{1/4}+b^{1/4}} \cot[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4}+b^{1/4}}}$$

Result (type 7, 172 leaves):

$$8 \text{RootSum}\left[b + 8 b \#1 + 28 b \#1^2 + 56 b \#1^3 + 256 a \#1^4 + 70 b \#1^4 + 56 b \#1^5 + 28 b \#1^6 + 8 b \#1^7 + b \#1^8 \&, \frac{2 \text{ArcTan}\left[\frac{\sin[2x]}{\cos[2x]-\#1}\right] \#1^3 - i \text{Log}\left[1 - 2 \cos[2x] \#1 + \#1^2\right] \#1^3}{b + 7 b \#1 + 21 b \#1^2 + 128 a \#1^3 + 35 b \#1^3 + 35 b \#1^4 + 21 b \#1^5 + 7 b \#1^6 + b \#1^7} \&\right]$$

■ **Problem 77: Result is not expressed in closed-form.**

$$\int \frac{1}{a - b \cos[x]^5} dx$$

Optimal (type 3, 494 leaves, 12 steps):

$$\frac{2 \text{ArcTan}\left[\frac{\sqrt{a^{1/5}+b^{1/5}} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-b^{1/5}}}\right]}{5 a^{4/5} \sqrt{a^{1/5}-b^{1/5}} \sqrt{a^{1/5}+b^{1/5}}} + \frac{2 \text{ArcTan}\left[\frac{\sqrt{a^{1/5}-(-1)^{1/5} b^{1/5}} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+(-1)^{1/5} b^{1/5}}}\right]}{5 a^{4/5} \sqrt{a^{1/5}-(-1)^{1/5} b^{1/5}} \sqrt{a^{1/5}+(-1)^{1/5} b^{1/5}}} + \frac{2 \text{ArcTan}\left[\frac{\sqrt{a^{1/5}+(-1)^{2/5} b^{1/5}} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-(-1)^{2/5} b^{1/5}}}\right]}{5 a^{4/5} \sqrt{a^{1/5}-(-1)^{2/5} b^{1/5}} \sqrt{a^{1/5}+(-1)^{2/5} b^{1/5}}} +$$

$$\frac{2 \text{ArcTan}\left[\frac{\sqrt{a^{1/5}-(-1)^{3/5} b^{1/5}} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+(-1)^{3/5} b^{1/5}}}\right]}{5 a^{4/5} \sqrt{a^{1/5}-(-1)^{3/5} b^{1/5}} \sqrt{a^{1/5}+(-1)^{3/5} b^{1/5}}} + \frac{2 \text{ArcTan}\left[\frac{\sqrt{a^{1/5}+(-1)^{4/5} b^{1/5}} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-(-1)^{4/5} b^{1/5}}}\right]}{5 a^{4/5} \sqrt{a^{1/5}-(-1)^{4/5} b^{1/5}} \sqrt{a^{1/5}+(-1)^{4/5} b^{1/5}}}$$

Result (type 7, 130 leaves):

$$-\frac{8}{5} \text{RootSum}\left[b + 5 b \#1^2 + 10 b \#1^4 - 32 a \#1^5 + 10 b \#1^6 + 5 b \#1^8 + b \#1^{10} \&, \frac{2 \text{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right] \#1^3 - i \text{Log}\left[1 - 2 \cos[x] \#1 + \#1^2\right] \#1^3}{b + 4 b \#1^2 - 16 a \#1^3 + 6 b \#1^4 + 4 b \#1^6 + b \#1^8} \&\right]$$

■ **Problem 78: Result is not expressed in closed-form.**

$$\int \frac{1}{a - b \cos[x]^6} dx$$

Optimal (type 3, 175 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}-b^{1/3}} \cot[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}-b^{1/3}}} - \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}+(-1)^{1/3} b^{1/3}} \cot[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}+(-1)^{1/3} b^{1/3}}} - \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}-(-1)^{2/3} b^{1/3}} \cot[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}-(-1)^{2/3} b^{1/3}}}$$

Result (type 7, 146 leaves):

$$-\frac{8}{3} \text{RootSum}\left[b + 6 b \#1 + 15 b \#1^2 - 64 a \#1^3 + 20 b \#1^3 + 15 b \#1^4 + 6 b \#1^5 + b \#1^6 \&, \frac{2 \text{ArcTan}\left[\frac{\sin[2x]}{\cos[2x]-\#1}\right] \#1^2 - i \text{Log}\left[1 - 2 \cos[2x] \#1 + \#1^2\right] \#1^2}{b + 5 b \#1 - 32 a \#1^2 + 10 b \#1^2 + 10 b \#1^3 + 5 b \#1^4 + b \#1^5} \& \right]$$

■ **Problem 79: Result is not expressed in closed-form.**

$$\int \frac{1}{a - b \cos[x]^8} dx$$

Optimal (type 3, 213 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/4}-b^{1/4}} \cot[x]}{a^{1/8}}\right]}{4 a^{7/8} \sqrt{a^{1/4}-b^{1/4}}} - \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/4}-i b^{1/4}} \cot[x]}{a^{1/8}}\right]}{4 a^{7/8} \sqrt{a^{1/4}-i b^{1/4}}} - \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/4}+i b^{1/4}} \cot[x]}{a^{1/8}}\right]}{4 a^{7/8} \sqrt{a^{1/4}+i b^{1/4}}} - \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/4}+b^{1/4}} \cot[x]}{a^{1/8}}\right]}{4 a^{7/8} \sqrt{a^{1/4}+b^{1/4}}}$$

Result (type 7, 172 leaves):

$$-8 \text{RootSum}\left[b + 8 b \#1 + 28 b \#1^2 + 56 b \#1^3 - 256 a \#1^4 + 70 b \#1^4 + 56 b \#1^5 + 28 b \#1^6 + 8 b \#1^7 + b \#1^8 \&, \frac{2 \text{ArcTan}\left[\frac{\sin[2x]}{\cos[2x]-\#1}\right] \#1^3 - i \text{Log}\left[1 - 2 \cos[2x] \#1 + \#1^2\right] \#1^3}{b + 7 b \#1 + 21 b \#1^2 - 128 a \#1^3 + 35 b \#1^3 + 35 b \#1^4 + 21 b \#1^5 + 7 b \#1^6 + b \#1^7} \& \right]$$

■ **Problem 80: Result is not expressed in closed-form.**

$$\int \frac{1}{1 + \cos[x]^5} dx$$

Optimal (type 3, 223 leaves, 11 steps):

$$\frac{2 \operatorname{ArcTan}\left[\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\operatorname{Tan}\left[\frac{x}{2}\right]\right]}{5\sqrt{1-(-1)^{4/5}}} + \frac{2 \operatorname{ArcTan}\left[\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\operatorname{Tan}\left[\frac{x}{2}\right]\right]}{5\sqrt{1+(-1)^{3/5}}} -$$

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{\frac{-1-(-1)^{3/5}}{1+(-1)^{3/5}}}}\right]}{5\sqrt{-1+(-1)^{2/5}}} - \frac{2\sqrt{\frac{-1-(-1)^{3/5}}{1+(-1)^{3/5}}}\operatorname{ArcTanh}\left[\sqrt{\frac{-1-(-1)^{3/5}}{1+(-1)^{3/5}}}\operatorname{Tan}\left[\frac{x}{2}\right]\right]}{5(1+(-1)^{3/5})} + \frac{\operatorname{Sin}[x]}{5(1+\operatorname{Cos}[x])}$$

Result (type 7, 378 leaves):

$$-\frac{1}{10}\operatorname{RootSum}\left[1-2\#1+8\#1^2-14\#1^3+30\#1^4-14\#1^5+8\#1^6-2\#1^7+\#1^8\ \&, \right.$$

$$\frac{1}{-1+8\#1-21\#1^2+60\#1^3-35\#1^4+24\#1^5-7\#1^6+4\#1^7}\left(2\operatorname{ArcTan}\left[\frac{\operatorname{Sin}[x]}{\operatorname{Cos}[x]-\#1}\right]-i\operatorname{Log}\left[1-2\operatorname{Cos}[x]\#1+\#1^2\right]-8\operatorname{ArcTan}\left[\frac{\operatorname{Sin}[x]}{\operatorname{Cos}[x]-\#1}\right]\#1+\right.$$

$$4i\operatorname{Log}\left[1-2\operatorname{Cos}[x]\#1+\#1^2\right]\#1+30\operatorname{ArcTan}\left[\frac{\operatorname{Sin}[x]}{\operatorname{Cos}[x]-\#1}\right]\#1^2-15i\operatorname{Log}\left[1-2\operatorname{Cos}[x]\#1+\#1^2\right]\#1^2-80\operatorname{ArcTan}\left[\frac{\operatorname{Sin}[x]}{\operatorname{Cos}[x]-\#1}\right]\#1^3+$$

$$40i\operatorname{Log}\left[1-2\operatorname{Cos}[x]\#1+\#1^2\right]\#1^3+30\operatorname{ArcTan}\left[\frac{\operatorname{Sin}[x]}{\operatorname{Cos}[x]-\#1}\right]\#1^4-15i\operatorname{Log}\left[1-2\operatorname{Cos}[x]\#1+\#1^2\right]\#1^4-8\operatorname{ArcTan}\left[\frac{\operatorname{Sin}[x]}{\operatorname{Cos}[x]-\#1}\right]\#1^5+$$

$$\left.4i\operatorname{Log}\left[1-2\operatorname{Cos}[x]\#1+\#1^2\right]\#1^5+2\operatorname{ArcTan}\left[\frac{\operatorname{Sin}[x]}{\operatorname{Cos}[x]-\#1}\right]\#1^6-i\operatorname{Log}\left[1-2\operatorname{Cos}[x]\#1+\#1^2\right]\#1^6\right)\ \& \left. + \frac{1}{5}\operatorname{Tan}\left[\frac{x}{2}\right]\right]$$

■ **Problem 82: Result is not expressed in closed-form.**

$$\int \frac{1}{1+\operatorname{Cos}[x]^8} dx$$

Optimal (type 3, 129 leaves, 9 steps):

$$-\frac{\operatorname{ArcTan}\left[\sqrt{1-(-1)^{1/4}}\operatorname{Cot}[x]\right]}{4\sqrt{1-(-1)^{1/4}}}-\frac{\operatorname{ArcTan}\left[\sqrt{1+(-1)^{1/4}}\operatorname{Cot}[x]\right]}{4\sqrt{1+(-1)^{1/4}}}-\frac{\operatorname{ArcTan}\left[\sqrt{1-(-1)^{3/4}}\operatorname{Cot}[x]\right]}{4\sqrt{1-(-1)^{3/4}}}-\frac{\operatorname{ArcTan}\left[\sqrt{1+(-1)^{3/4}}\operatorname{Cot}[x]\right]}{4\sqrt{1+(-1)^{3/4}}}$$

Result (type 7, 141 leaves):

$$8\operatorname{RootSum}\left[1+8\#1+28\#1^2+56\#1^3+326\#1^4+56\#1^5+28\#1^6+8\#1^7+\#1^8\ \&, \right.$$

$$\frac{2\operatorname{ArcTan}\left[\frac{\operatorname{Sin}[2x]}{\operatorname{Cos}[2x]-\#1}\right]\#1^3-i\operatorname{Log}\left[1-2\operatorname{Cos}[2x]\#1+\#1^2\right]\#1^3}{1+7\#1+21\#1^2+163\#1^3+35\#1^4+21\#1^5+7\#1^6+\#1^7}\ \& \left. \right]$$

■ **Problem 83: Result is not expressed in closed-form.**

$$\int \frac{1}{1-\operatorname{Cos}[x]^5} dx$$

Optimal (type 3, 205 leaves, 11 steps):



$$\frac{2 \operatorname{ArcTan}\left[\sqrt{\frac{1-(-1)^{1/5}}{1+(-1)^{1/5}}}\tan\left[\frac{x}{2}\right]\right]}{5\sqrt{1-(-1)^{2/5}}} + \frac{2 \operatorname{ArcTan}\left[\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\tan\left[\frac{x}{2}\right]\right]}{5\sqrt{1+(-1)^{1/5}}} - \frac{2 \operatorname{ArcTanh}\left[\frac{\tan\left[\frac{x}{2}\right]}{\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right]}{5\sqrt{-1+(-1)^{4/5}}} + \frac{2 \operatorname{ArcTanh}\left[\sqrt{\frac{-1+(-1)^{4/5}}{1-(-1)^{4/5}}}\tan\left[\frac{x}{2}\right]\right]}{5\sqrt{-1-(-1)^{3/5}}} - \frac{\sin[x]}{5(1-\cos[x])}$$

Result (type 7, 378 leaves):

$$-\frac{1}{5}\operatorname{Cot}\left[\frac{x}{2}\right] + \frac{1}{10}\operatorname{RootSum}\left[1+2\#1+8\#1^2+14\#1^3+30\#1^4+14\#1^5+8\#1^6+2\#1^7+\#1^8\ \&, \right. \\ \left. \frac{1}{1+8\#1+21\#1^2+60\#1^3+35\#1^4+24\#1^5+7\#1^6+4\#1^7}\left(2\operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right]-i\operatorname{Log}\left[1-2\cos[x]\#1+\#1^2\right]+ \right. \\ \left. 8\operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right]\#1-4i\operatorname{Log}\left[1-2\cos[x]\#1+\#1^2\right]\#1+30\operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right]\#1^2-15i\operatorname{Log}\left[1-2\cos[x]\#1+\#1^2\right]\#1^2+ \right. \\ \left. 80\operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right]\#1^3-40i\operatorname{Log}\left[1-2\cos[x]\#1+\#1^2\right]\#1^3+30\operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right]\#1^4-15i\operatorname{Log}\left[1-2\cos[x]\#1+\#1^2\right]\#1^4+ \right. \\ \left. 8\operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right]\#1^5-4i\operatorname{Log}\left[1-2\cos[x]\#1+\#1^2\right]\#1^5+2\operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right]\#1^6-i\operatorname{Log}\left[1-2\cos[x]\#1+\#1^2\right]\#1^6\right)\ \&]$$

■ **Problem 88: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{1-\cos[x]^2}\tan[x]\,dx$$

Optimal (type 3, 20 leaves, 5 steps):

$$\operatorname{ArcTanh}\left[\sqrt{\sin[x]^2}\right]-\sqrt{\sin[x]^2}$$

Result (type 3, 47 leaves):

$$-\operatorname{Csc}[x]\sqrt{\sin[x]^2}\left(\operatorname{Log}\left[\cos\left[\frac{x}{2}\right]-\sin\left[\frac{x}{2}\right]\right]-\operatorname{Log}\left[\cos\left[\frac{x}{2}\right]+\sin\left[\frac{x}{2}\right]\right]+\sin[x]\right)$$

■ **Problem 91: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[x]}{\sqrt{1-\cos[x]^2}}\,dx$$

Optimal (type 3, 9 leaves, 4 steps):

$$\operatorname{ArcTanh}\left[\sqrt{\sin[x]^2}\right]$$

Result (type 3, 44 leaves):

$$\frac{\left(-\operatorname{Log}\left[\cos\left[\frac{x}{2}\right]-\sin\left[\frac{x}{2}\right]\right]+\operatorname{Log}\left[\cos\left[\frac{x}{2}\right]+\sin\left[\frac{x}{2}\right]\right]\right)\sin[x]}{\sqrt{\sin[x]^2}}$$

■ **Problem 92: Result is not expressed in closed-form.**

$$\int \frac{\tan[x]^3}{a + b \cos[x]^3} dx$$

Optimal (type 3, 153 leaves, 11 steps):

$$-\frac{b^{2/3} \operatorname{ArcTan}\left[\frac{a^{1/3} - 2b^{1/3} \cos[x]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{5/3}} + \frac{\operatorname{Log}[\cos[x]]}{a} + \frac{b^{2/3} \operatorname{Log}[a^{1/3} + b^{1/3} \cos[x]]}{3 a^{5/3}} -$$

$$\frac{b^{2/3} \operatorname{Log}[a^{2/3} - a^{1/3} b^{1/3} \cos[x] + b^{2/3} \cos[x]^2]}{6 a^{5/3}} - \frac{\operatorname{Log}[a + b \cos[x]^3]}{3 a} + \frac{\operatorname{Sec}[x]^2}{2 a}$$

Result (type 7, 217 leaves):

$$\frac{1}{6 a} \left( 6 \left( \operatorname{Log}[\cos[x]] + \operatorname{Log}\left[\operatorname{Sec}\left[\frac{x}{2}\right]^2\right] \right) - \right.$$

$$2 \operatorname{RootSum}\left[ a + b + 3 a \#1 - 3 b \#1 + 3 a \#1^2 + 3 b \#1^2 + a \#1^3 - b \#1^3 \&, \left( a \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{x}{2}\right]^2\right] + b \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{x}{2}\right]^2\right] + 2 a \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{x}{2}\right]^2\right] \#1 + \right.$$

$$\left. 4 b \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{x}{2}\right]^2\right] \#1 + a \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{x}{2}\right]^2\right] \#1^2 - b \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{x}{2}\right]^2\right] \#1^2 \right) / \left( a - b + 2 a \#1 + 2 b \#1 + a \#1^2 - b \#1^2 \right) \& \left. \right] + 3 \operatorname{Sec}[x]^2$$

■ **Problem 93: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \cos[x]^3} \tan[x] dx$$

Optimal (type 3, 45 leaves, 5 steps):

$$\frac{2}{3} \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \cos[x]^3}}{\sqrt{a}}\right] - \frac{2}{3} \sqrt{a + b \cos[x]^3}$$

Result (type 3, 668 leaves):

$$-\left( \left( \sqrt{4 a + 3 b \cos[x] + b \cos[3 x]} \left( b + a (\operatorname{Sec}[x]^2)^{3/2} - \sqrt{a} \sqrt{b} \operatorname{ArcSinh}\left[\frac{\sqrt{a} (\operatorname{Sec}[x]^2)^{3/4}}{\sqrt{b}}\right] (\operatorname{Sec}[x]^2)^{3/4} \sqrt{1 + \frac{a (\operatorname{Sec}[x]^2)^{3/2}}{b}} \right) \right. \right.$$

$$\left. \left. \operatorname{Tan}[x] \sqrt{\cos[x]^4 \left( a + b \sqrt{\operatorname{Sec}[x]^2} + 2 a \operatorname{Tan}[x]^2 + a \operatorname{Tan}[x]^4 \right)} \right) / \left( 3 \left( b + a (\operatorname{Sec}[x]^2)^{3/2} \right) \right)$$

$$\left( \frac{1}{(b + a (\sec[x]^2)^{3/2})^2} 2 a (\sec[x]^2)^{3/2} \left( b + a (\sec[x]^2)^{3/2} - \sqrt{a} \sqrt{b} \operatorname{ArcSinh} \left[ \frac{\sqrt{a} (\sec[x]^2)^{3/4}}{\sqrt{b}} \right] \right) (\sec[x]^2)^{3/4} \sqrt{1 + \frac{a (\sec[x]^2)^{3/2}}{b}} \right)$$

$$\tan[x] \sqrt{\cos[x]^4 \left( a + b \sqrt{\sec[x]^2} + 2 a \tan[x]^2 + a \tan[x]^4 \right)} - \frac{1}{3 (b + a (\sec[x]^2)^{3/2})}$$

$$2 \left( \frac{3}{2} a (\sec[x]^2)^{3/2} \tan[x] - \frac{3 a^{3/2} \operatorname{ArcSinh} \left[ \frac{\sqrt{a} (\sec[x]^2)^{3/4}}{\sqrt{b}} \right] (\sec[x]^2)^{9/4} \tan[x]}{2 \sqrt{b} \sqrt{1 + \frac{a (\sec[x]^2)^{3/2}}{b}}} - \frac{3}{2} \sqrt{a} \sqrt{b} \operatorname{ArcSinh} \left[ \frac{\sqrt{a} (\sec[x]^2)^{3/4}}{\sqrt{b}} \right] \right)$$

$$(\sec[x]^2)^{3/4} \sqrt{1 + \frac{a (\sec[x]^2)^{3/2}}{b}} \tan[x] \left( \sqrt{\cos[x]^4 \left( a + b \sqrt{\sec[x]^2} + 2 a \tan[x]^2 + a \tan[x]^4 \right)} - \right.$$

$$\left. \left( \left( b + a (\sec[x]^2)^{3/2} - \sqrt{a} \sqrt{b} \operatorname{ArcSinh} \left[ \frac{\sqrt{a} (\sec[x]^2)^{3/4}}{\sqrt{b}} \right] \right) (\sec[x]^2)^{3/4} \sqrt{1 + \frac{a (\sec[x]^2)^{3/2}}{b}} \right) (\cos[x]^4 \right.$$

$$\left. \left. \left( 4 a \sec[x]^2 \tan[x] + b \sqrt{\sec[x]^2} \tan[x] + 4 a \sec[x]^2 \tan[x]^3 \right) - 4 \cos[x]^3 \sin[x] \left( a + b \sqrt{\sec[x]^2} + 2 a \tan[x]^2 + a \tan[x]^4 \right) \right) \right) /$$

$$\left( 3 (b + a (\sec[x]^2)^{3/2}) \sqrt{\cos[x]^4 \left( a + b \sqrt{\sec[x]^2} + 2 a \tan[x]^2 + a \tan[x]^4 \right)} \right)$$

■ **Problem 94: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[x]}{\sqrt{a + b \cos[x]^3}} dx$$

Optimal (type 3, 28 leaves, 4 steps) :

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cos [x]^3}}{\sqrt{a}}\right]}{3 \sqrt{a}}$$

Result (type 3, 207 leaves) :

$$\left( 128 \sqrt{b} \operatorname{ArcSinh}\left[\frac{\sqrt{a} (\operatorname{Sec}[x]^2)^{3/4}}{\sqrt{b}}\right] \cos [x]^4 (\operatorname{Sec}[x]^2)^{3/4} \left( b \cos [x]^2 + a \sqrt{\operatorname{Sec}[x]^2} \right) \right. \\ \left. \sqrt{1 + \frac{a (\operatorname{Sec}[x]^2)^{3/2}}{b}} \left( a + 2 a \cot [x]^2 + \cot [x]^4 \left( a + b \sqrt{\operatorname{Sec}[x]^2} \right) \right) \sin [x]^4 \right) / \left( 3 \sqrt{a} \sqrt{4 a + 3 b \cos [x] + b \cos [3 x]} \right. \\ \left. \left( 32 a^2 + 10 b^2 + b^2 \cos [6 x] + 24 a b \sqrt{\operatorname{Sec}[x]^2} + 2 b \cos [4 x] \left( 3 b + 4 a \sqrt{\operatorname{Sec}[x]^2} \right) + b \cos [2 x] \left( 15 b + 32 a \sqrt{\operatorname{Sec}[x]^2} \right) \right) \right) \right)$$

- **Problem 95: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+b \cos [x]^4} \tan [x] dx$$

Optimal (type 3, 45 leaves, 5 steps) :

$$\frac{1}{2} \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cos [x]^4}}{\sqrt{a}}\right] - \frac{1}{2} \sqrt{a+b \cos [x]^4}$$

Result (type 4, 47997 leaves) : Display of huge result suppressed!

- **Problem 96: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\tan [x]}{\sqrt{a+b \cos [x]^4}} dx$$

Optimal (type 3, 28 leaves, 4 steps) :

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cos [x]^4}}{\sqrt{a}}\right]}{2 \sqrt{a}}$$

Result (type 4, 48584 leaves) : Display of huge result suppressed!

## Test results for the 21 problems in "4.2.8 (a+b cos)^m (c+d trig)^n.m"

- **Problem 9: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d \operatorname{Sec}[e + f x])^4}{a + b \operatorname{Cos}[e + f x]} dx$$

Optimal (type 3, 247 leaves, 12 steps):

$$\frac{2 (a c - b d)^4 \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{a+b}}\right]}{a^4 \sqrt{a-b} \sqrt{a+b} f} + \frac{d^3 (4 a c - b d) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{2 a^2 f} + \frac{d (2 a c - b d) (2 a^2 c^2 - 2 a b c d + b^2 d^2) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{a^4 f} + \frac{d^4 \operatorname{Tan}[e + f x]}{a f} + \frac{d^2 (6 a^2 c^2 - 4 a b c d + b^2 d^2) \operatorname{Tan}[e + f x]}{a^3 f} + \frac{d^3 (4 a c - b d) \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{2 a^2 f} + \frac{d^4 \operatorname{Tan}[e + f x]^3}{3 a f}$$

Result (type 3, 952 leaves):

$$\begin{aligned}
& - \frac{2(a c - b d)^4 \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{-a^2+b^2}}\right] \operatorname{Cos}[e+f x]^4 (c+d \operatorname{Sec}[e+f x])^4}{a^4 \sqrt{-a^2+b^2} f (d+c \operatorname{Cos}[e+f x])^4} + \frac{1}{2 a^4 f (d+c \operatorname{Cos}[e+f x])^4} \\
& (-8 a^3 c^3 d + 12 a^2 b c^2 d^2 - 4 a^3 c d^3 - 8 a b^2 c d^3 + a^2 b d^4 + 2 b^3 d^4) \operatorname{Cos}[e+f x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right] (c+d \operatorname{Sec}[e+f x])^4 + \\
& \frac{1}{2 a^4 f (d+c \operatorname{Cos}[e+f x])^4} (8 a^3 c^3 d - 12 a^2 b c^2 d^2 + 4 a^3 c d^3 + 8 a b^2 c d^3 - a^2 b d^4 - 2 b^3 d^4) \operatorname{Cos}[e+f x]^4 \\
& \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right] (c+d \operatorname{Sec}[e+f x])^4 + \frac{(12 a c d^3 + a d^4 - 3 b d^4) \operatorname{Cos}[e+f x]^4 (c+d \operatorname{Sec}[e+f x])^4}{12 a^2 f (d+c \operatorname{Cos}[e+f x])^4 (\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right])^2} + \\
& \frac{d^4 \operatorname{Cos}[e+f x]^4 (c+d \operatorname{Sec}[e+f x])^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]}{6 a f (d+c \operatorname{Cos}[e+f x])^4 (\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right])^3} + \frac{d^4 \operatorname{Cos}[e+f x]^4 (c+d \operatorname{Sec}[e+f x])^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]}{6 a f (d+c \operatorname{Cos}[e+f x])^4 (\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right])^3} + \\
& \frac{(-12 a c d^3 - a d^4 + 3 b d^4) \operatorname{Cos}[e+f x]^4 (c+d \operatorname{Sec}[e+f x])^4}{12 a^2 f (d+c \operatorname{Cos}[e+f x])^4 (\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right])^2} + \left(\operatorname{Cos}[e+f x]^4 (c+d \operatorname{Sec}[e+f x])^4 \right. \\
& \left. \left(18 a^2 c^2 d^2 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] - 12 a b c d^3 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 2 a^2 d^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 3 b^2 d^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\right) / \\
& \left(3 a^3 f (d+c \operatorname{Cos}[e+f x])^4 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\right) + \left(\operatorname{Cos}[e+f x]^4 (c+d \operatorname{Sec}[e+f x])^4 \right. \\
& \left. \left(18 a^2 c^2 d^2 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] - 12 a b c d^3 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 2 a^2 d^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 3 b^2 d^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\right) / \\
& \left(3 a^3 f (d+c \operatorname{Cos}[e+f x])^4 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\right)
\end{aligned}$$

■ **Problem 16: Unable to integrate problem.**

$$\int \frac{\sqrt{c+d \operatorname{Sec}[e+f x]}}{a+b \operatorname{Cos}[e+f x]} dx$$

Optimal (type 4, 213 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 \sqrt{c+d} \operatorname{Cot}[e+f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d \operatorname{Sec}[e+f x]}}{\sqrt{c+d}}\right], \frac{c+d}{c-d}\right] \sqrt{\frac{d(1-\operatorname{Sec}[e+f x])}{c+d}} \sqrt{-\frac{d(1+\operatorname{Sec}[e+f x])}{c-d}}}{a f} + \\
& \frac{2(a c - b d) \operatorname{EllipticPi}\left[\frac{2 a}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{1-\operatorname{Sec}[e+f x]}}{\sqrt{2}}\right], \frac{2 d}{c+d}\right] \sqrt{\frac{c+d \operatorname{Sec}[e+f x]}{c+d}} \operatorname{Tan}[e+f x]}{a(a+b) f \sqrt{c+d \operatorname{Sec}[e+f x]} \sqrt{-\operatorname{Tan}[e+f x]^2}}
\end{aligned}$$

Result (type 8, 29 leaves) :

$$\int \frac{\sqrt{c + d \operatorname{Sec}[e + f x]}}{a + b \operatorname{Cos}[e + f x]} dx$$

■ **Problem 17: Unable to integrate problem.**

$$\int \frac{1}{(a + b \operatorname{Cos}[e + f x]) \sqrt{c + d \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 4, 102 leaves, 2 steps) :

$$\frac{2 \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{1-\operatorname{Sec}[e+fx]}}{\sqrt{2}}\right], \frac{2d}{c+d}\right] \sqrt{\frac{c+d \operatorname{Sec}[e+fx]}{c+d}} \operatorname{Tan}[e+fx]}{(a+b) f \sqrt{c+d \operatorname{Sec}[e+fx]} \sqrt{-\operatorname{Tan}[e+fx]^2}}$$

Result (type 8, 29 leaves) :

$$\int \frac{1}{(a + b \operatorname{Cos}[e + f x]) \sqrt{c + d \operatorname{Sec}[e + f x]}} dx$$

## Test results for the 20 problems in "4.2.9 trig^m (a+b cos^n+c cos^(2 n))^p.m"

■ **Problem 5: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Csc}[x]^3}{a + b \operatorname{Cos}[x] + c \operatorname{Cos}[x]^2} dx$$

Optimal (type 3, 205 leaves, 10 steps) :

$$\frac{(b^4 + 2 c^2 (a + c)^2 - 2 b^2 c (2 a + c)) \operatorname{ArcTanh}\left[\frac{b+2 c \operatorname{Cos}[x]}{\sqrt{b^2-4 a c}}\right]}{\sqrt{b^2-4 a c} (a^2 - b^2 + 2 a c + c^2)^2} + \frac{(b - (a + c) \operatorname{Cos}[x]) \operatorname{Csc}[x]^2}{2 (a - b + c) (a + b + c)} +$$

$$\frac{(a + 2 b + 3 c) \operatorname{Log}[1 - \operatorname{Cos}[x]]}{4 (a + b + c)^2} - \frac{(a - 2 b + 3 c) \operatorname{Log}[1 + \operatorname{Cos}[x]]}{4 (a - b + c)^2} - \frac{b (b^2 - 2 c (a + c)) \operatorname{Log}[a + b \operatorname{Cos}[x] + c \operatorname{Cos}[x]^2]}{2 (a^2 - b^2 + 2 a c + c^2)^2}$$

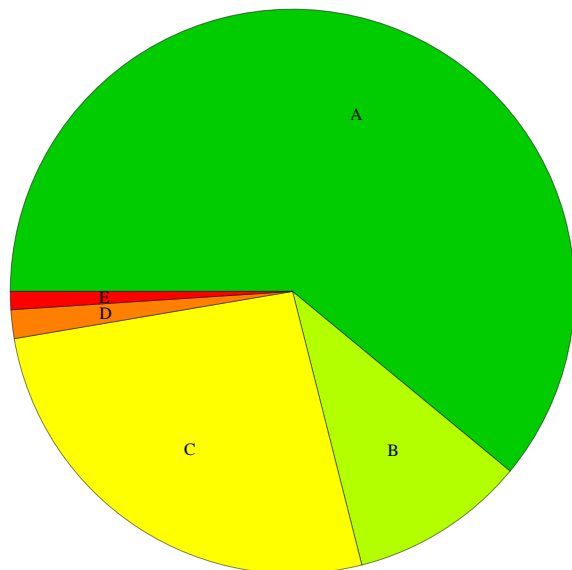
Result (type 3, 392 leaves) :

$$\begin{aligned}
& \frac{1}{8} \left( \frac{16 i (b^3 - 2 b c (a + c)) x}{(a - b + c)^2 (a + b + c)^2} + \frac{4 i (a - 2 b + 3 c) \text{ArcTan}[\text{Tan}[x]]}{(a - b + c)^2} - \frac{4 i (a + 2 b + 3 c) \text{ArcTan}[\text{Tan}[x]]}{(a + b + c)^2} - \frac{\text{Csc}\left[\frac{x}{2}\right]^2}{a + b + c} - \frac{2 (a - 2 b + 3 c) \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]^2\right]}{(a - b + c)^2} \right. \\
& \frac{4 \left( b^4 + 2 c^2 (a + c)^2 - 2 b^2 c (2 a + c) + b^3 \sqrt{b^2 - 4 a c} - 2 b c (a + c) \sqrt{b^2 - 4 a c} \right) \text{Log}\left[-b + \sqrt{b^2 - 4 a c} - 2 c \text{Cos}[x]\right]}{\sqrt{b^2 - 4 a c} (a^2 - b^2 + 2 a c + c^2)^2} - \\
& \frac{4 \left( -b^4 - 2 c^2 (a + c)^2 + 2 b^2 c (2 a + c) + b^3 \sqrt{b^2 - 4 a c} - 2 b c (a + c) \sqrt{b^2 - 4 a c} \right) \text{Log}\left[b + \sqrt{b^2 - 4 a c} + 2 c \text{Cos}[x]\right]}{\sqrt{b^2 - 4 a c} (a^2 - b^2 + 2 a c + c^2)^2} + \\
& \left. \frac{2 (a + 2 b + 3 c) \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]^2\right]}{(a + b + c)^2} + \frac{\text{Sec}\left[\frac{x}{2}\right]^2}{a - b + c} \right)
\end{aligned}$$



# Summary of Integration Test Results

4442 integration problems



A - 2709 optimal antiderivatives

B - 447 more than twice size of optimal antiderivatives

C - 1167 unnecessarily complex antiderivatives

D - 73 unable to integrate problems

E - 46 integration timeouts